Lattice QCD Meets Continuum: QCD Calculations in Flavour Physics
Siegen, 29.09-02.10.2014

Charm Physics on the Lattice

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Charm and lattice QCD

Historically:

- **early days of dynamical simulations** - charm observables considered testing ground: CLEO-c's mission statement "… our measurements of D and D_s meson decays to leptonic and semileptonic final states are crucial tests of the Lattice QCD techniques used to compute important heavy quark processes"

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- there has been an incredible amount of progress since:
  - field theory
  - algorithm
  - machine
- → **dynamical simulations got mature** (… and everybody started believing in lattice QCD …)
- level of precision reached now is making impact on SM phenomenology (CKM, quark masses, spectroscopy)

Lattice QCD

Formulate QCD on Euclidean discretised space-time

- provides gauge-invariant regularisation wt. cut-off $\propto a^{-1}$
- observables in terms of expectation value of discretised path integral

$$\langle 0|O|0\rangle \quad = \quad \frac{1}{\mathcal{Z}} \int \mathcal{D}[U,\psi,\bar{\psi}] O e^{-S_{\mathsf{lat}}[U,\psi,\bar{\psi}]}$$

- Evaluate discretised path integral in finite volume by means of Monte Carlo simulation

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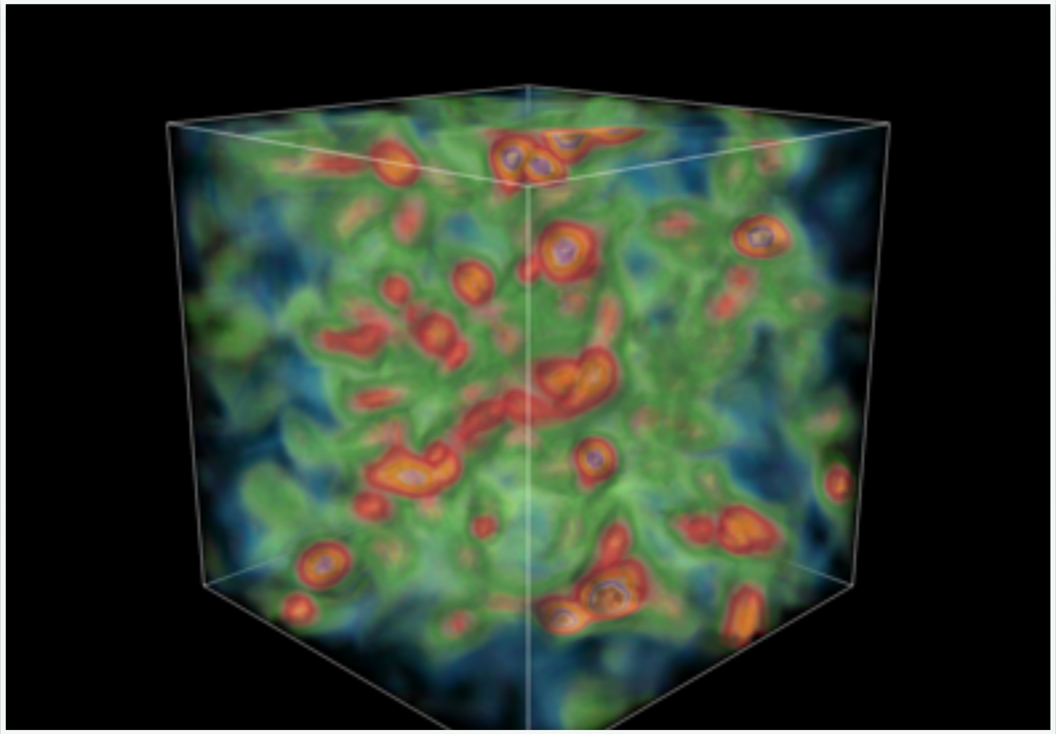
State of the art simulations

What we can do

- mass degenerate up and down quarks at their physical point
- physical strange and charm quarks ($\rightarrow N_f = 2,\ 2+1,\ 2+1+1$ QCD)
- bottom needs special treatment (talks by Kronfeld, Lepage, Shigemitsu, Sommer and Wingate)
- cut-off $a^{-1} \leq 4\mathrm{GeV}$
- volume $L \leq 6 fm$



action density of RBC/UKQCD physical point DWF ensemble

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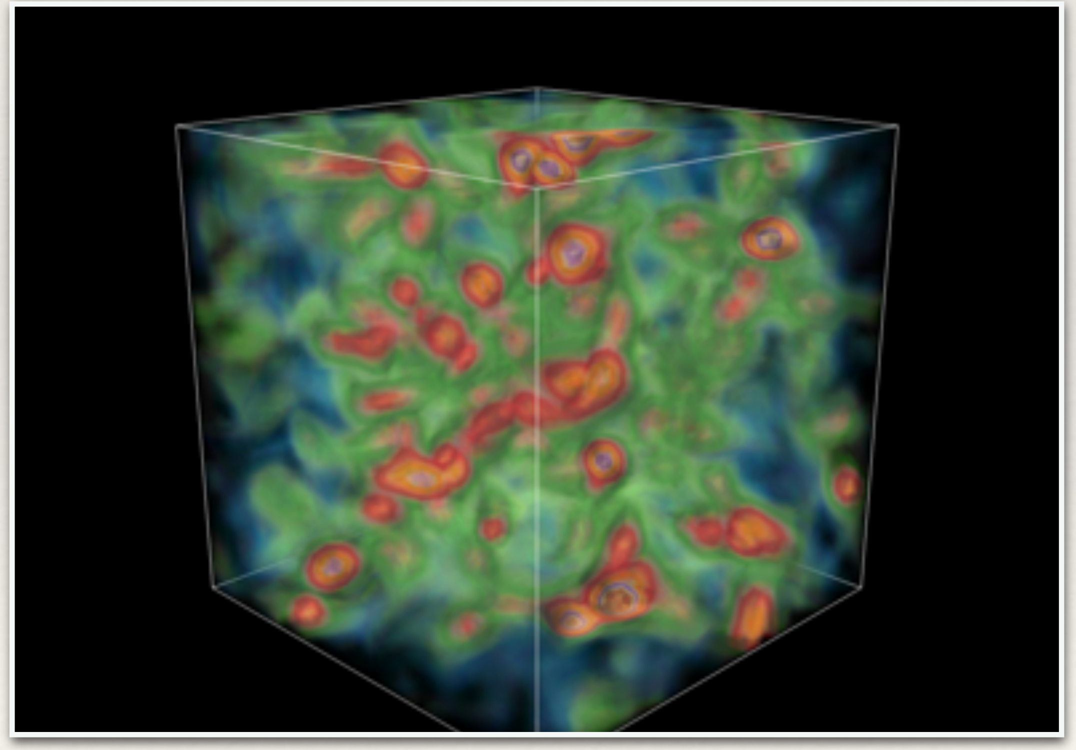
What comes next

- add isospin breaking
- add electromagnetism
  see e.g. Antonin Portelli's Lattice 2014 plenary



action density of RBC/UKQCD
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Standard, challenging, very challenging processes (with relevance for meson flavour physics)

- **Standard:** single incoming and/or outgoing pseudo-scalar states
  - $\pi, K, D_{(s)}, B_{(s)} \to \mathrm{QCD-vacuum}$
  - $\pi \to \pi, K \to \pi, D \to K, B \to \pi, ...$
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- **Challenging:** two initial/final hadronic states, one channel
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- **Very challenging - new ideas needed/no clue:**
  - multi-channel final states (hadronic D, B) (e.g. Hansen, Sharpe PRD86, 016007 (2012))
  - long-distance contributions in e.g. K, D-mixing (Bai et al. PRD113 2014)
  - transition MEs with vector final states (e.g. $B \to K^* ll$) (Briceño et al. arXiv:1406.5965)

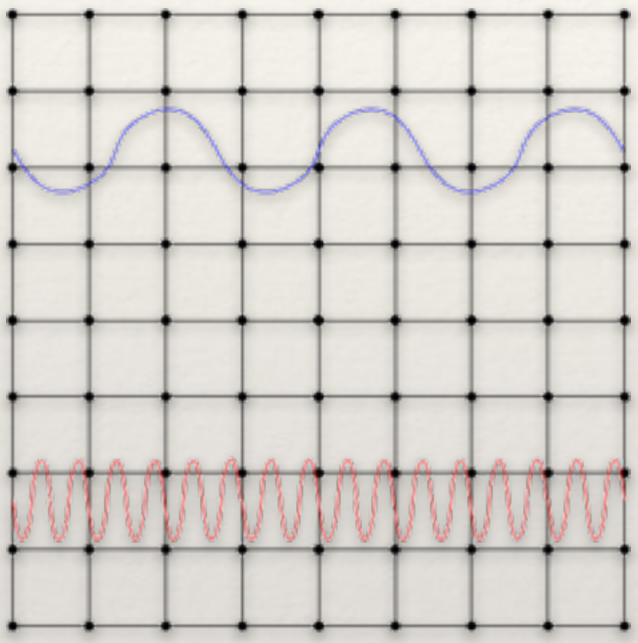
Lattice - systematic uncertainties

In practice need to control a number of sources of systematic uncertainties:

- **discretisation errors** (lattice spacing a)
  effects differ between heavy and light quarks
- **finite volume errors** (box size L)
- **quark mass extrapolation**
  until very recently mostly unphysical heavy light-quark masses
- **renormalisation, running**
- **heavy quark treatment**

charm - dominant systematics - cutoff effects

cutoff effects - suitable simulation parameters



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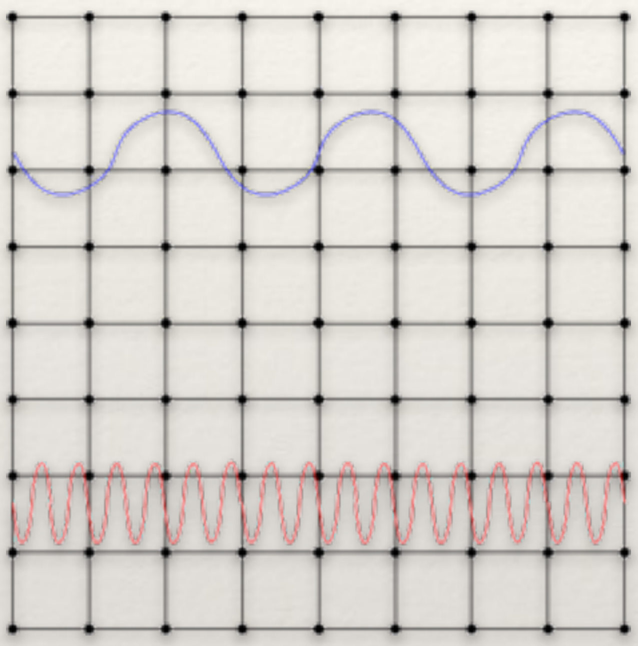

need to keep

$$a^{-1} \ll \text{ relevant scales } \ll L^{-1}$$

- for m_π=140MeV the constraint for controlled finite volume effects of $m_\pi L \gtrsim 4$ suggests L≈6fm
- for charm quarks to be well resolved $am_c < 1$ *e.g.* a^{-1} larger than ≈2.5GeV needed
- lattices with $L/a \gtrsim 80$ needed

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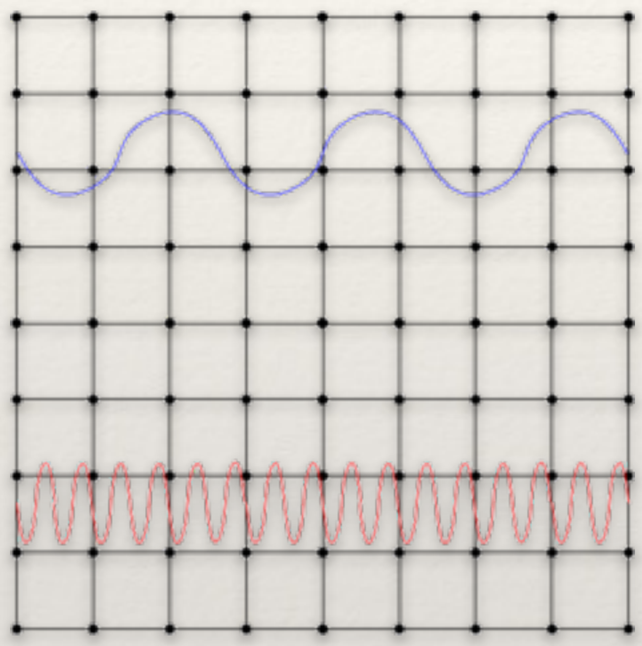

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Fulfilling all the constraints is just starting to happen
(e.g. first $96^3 \times 192$ have been generated (MILC)) in the meantime most collaborations
- weaken the finite volume effects by simulating unphysical heavy pions
- extrapolate from coarser lattices relying on assumptions for functional form of cutoff effects

charm - dominant systematics - cutoff effects

cutoff effects - improved actions

$$S_{\text{eff}} = \int d^4x \left\{ \mathcal{L}_0(x) + a\mathcal{L}_1(x) + a^2\mathcal{L}_2(x) + \ldots \right\}$$

Symanzik 1982,1983

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Symanzik 1982,1983

Devise *improved* lattice discretisations which include irrelevant higher-dimensional terms which reduce/remove cutoff effects:

- **improved Wilson/twisted mass:** non-perturbatively *O(a)*-improved
- **domain wall/overlap:** automatically *O(a)*-improved (chiral symmetry)
- **staggered Asqtad/HISQ:** smearing, improvement terms to remove a^2-errors leading taste violations $\alpha_s^2 a^2$, other lattice artefacts $\alpha_s a^2$ (with HISQ smaller coeffs as Asqtad); also leading $(am_c)^2$ and $\alpha_s(am_c)^2$ effects removed (leading in v quark velocity)

charm - dominant systematics - cutoff effects

in practice:

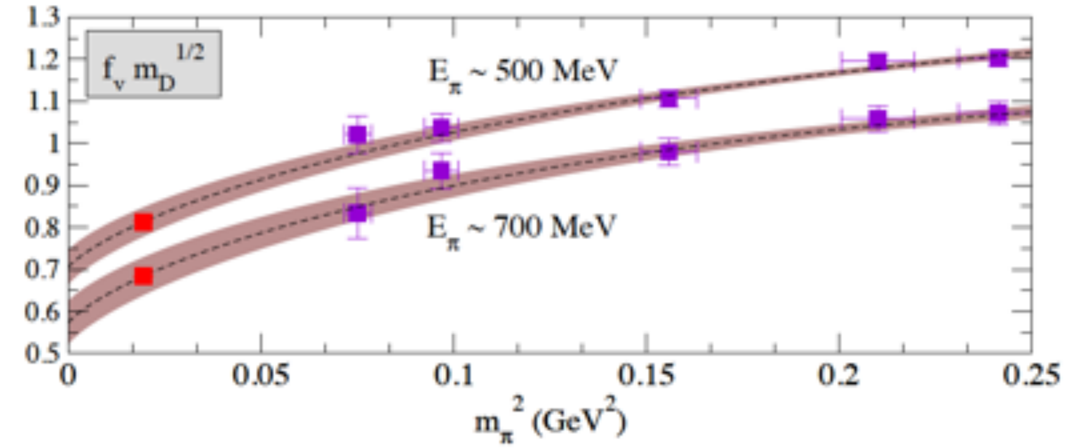
- ideally continuum limit with 3-4 lattice spacings down to ≈0.5fm
- some collaborations rely on scaling assumptions and extrapolate charm observables from rather coarse lattices, e.g. 2.2GeV≈1/0.09fm
- sometimes no continuum limit is taken at all

I consider it important to test scaling assumptions in detail;
this can be done cheaply and reliably in a quenched frame work (see later)

charm - dominant systematics - m_l-dependence

light quark mass dependence (in the case of simulations away from the physical point)

- sizeable m_l dependence possible in heavy-light quantities - most results in lattice charm physics are still from simulations with unphysical heavy light quark masses



ETM $D\to\pi$ vector form factor @ fixed pion energy

- extrapolation using Heavy meson chiral perturbation theory

static Wise PRD 45 1992,
Burdman, Donoghue PLB 280 1992,
Yan et al PRD 46 1992
1/M Boyd, Grinstein NPB 442 1992

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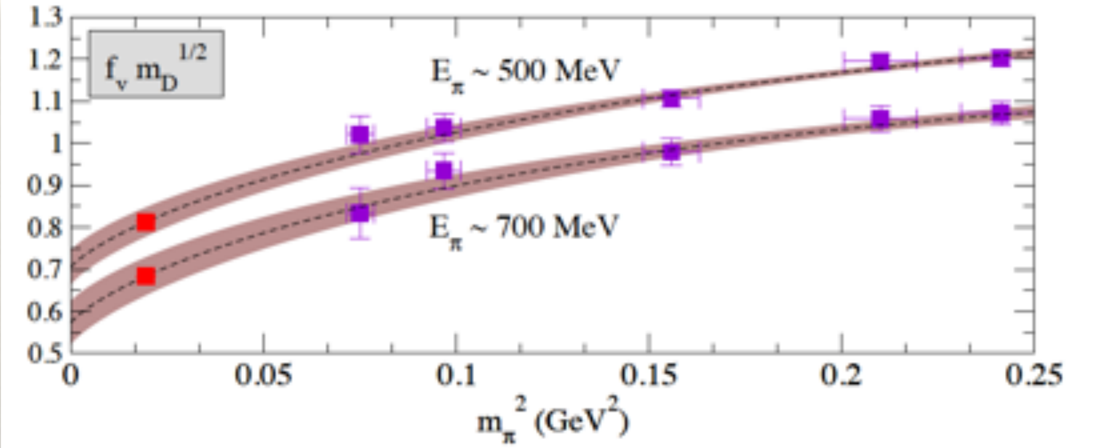

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in practice:

- **light quark mass:**
  ideally physical light quarks if not, HMChPT
- **combine light quark and a^2-extrapolation**
  augment HMChPT formulae by cut-off terms like a^2 or $(am_c)^2$ etc

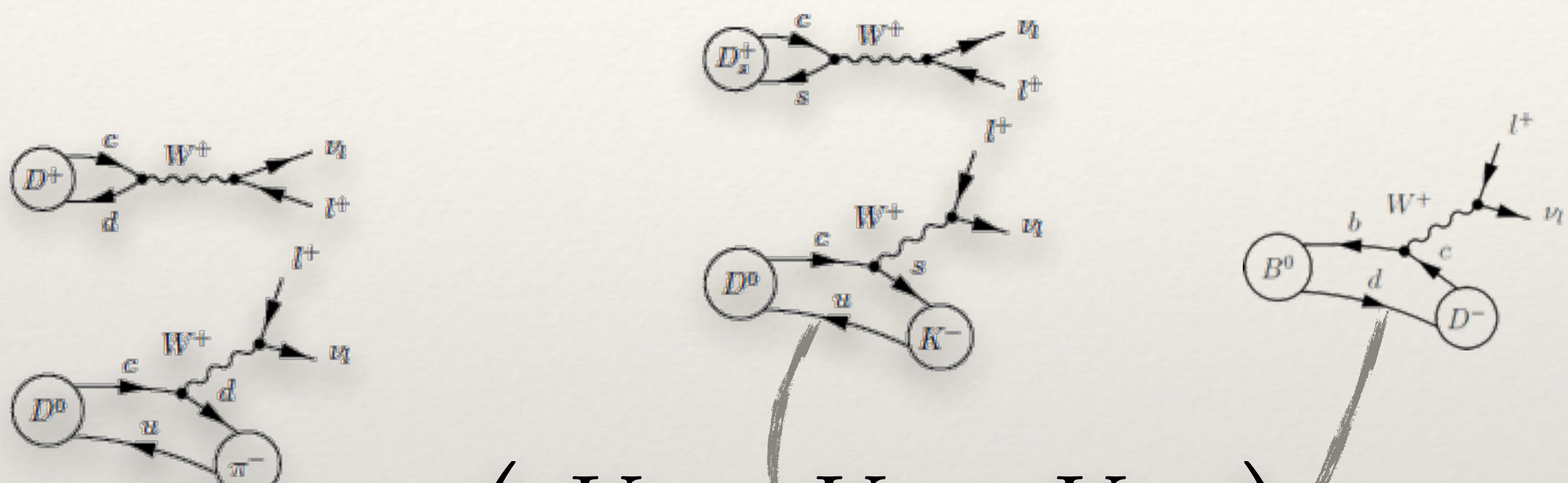
we know of ChPT's limitations, do we understand HMChPTs limitations?
is charm too light?

properties of typical simulations

Collaboration	N	action	a/fm	min m for stagg.)
ETM	2	tm	0.05,0.07,0.09,0.1	270
ETM	2+1+1	tm	0.06,0.08,0.09	211
HPQCD	2+1	Asqtad/HISQ	0.09,0.12	330
FNAL/MILC	2+1	Asqtad/HISQ	0.09,0.12,0.15	320
FNAL/MILC	2+1+1	HISQ	0.06,0.09,0.12,0.15	145
RBC/UKQCD	2+1	domain wall	0.07,0.09,0.11	physical

tree decays with charm

Determination of CKM elements



$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

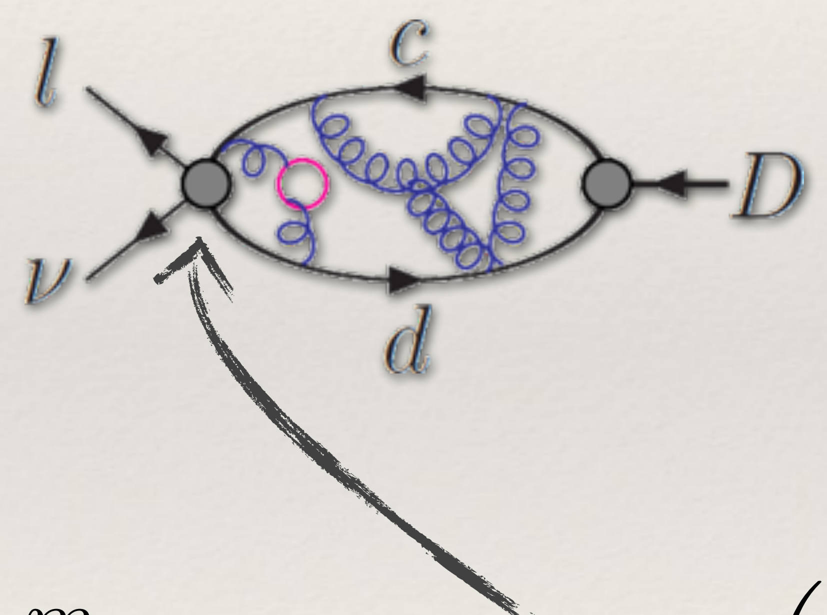
illustrations from
L. Lellouch's Les Houches
Lecture arXiv:1104.5484

CKM ME from tree-level decays

An example - leptonic D-decay

$$\Gamma_{\text{exp.}} = V_{\text{CKM}}(\text{WEAK})(\text{EM})(\text{STRONG})$$

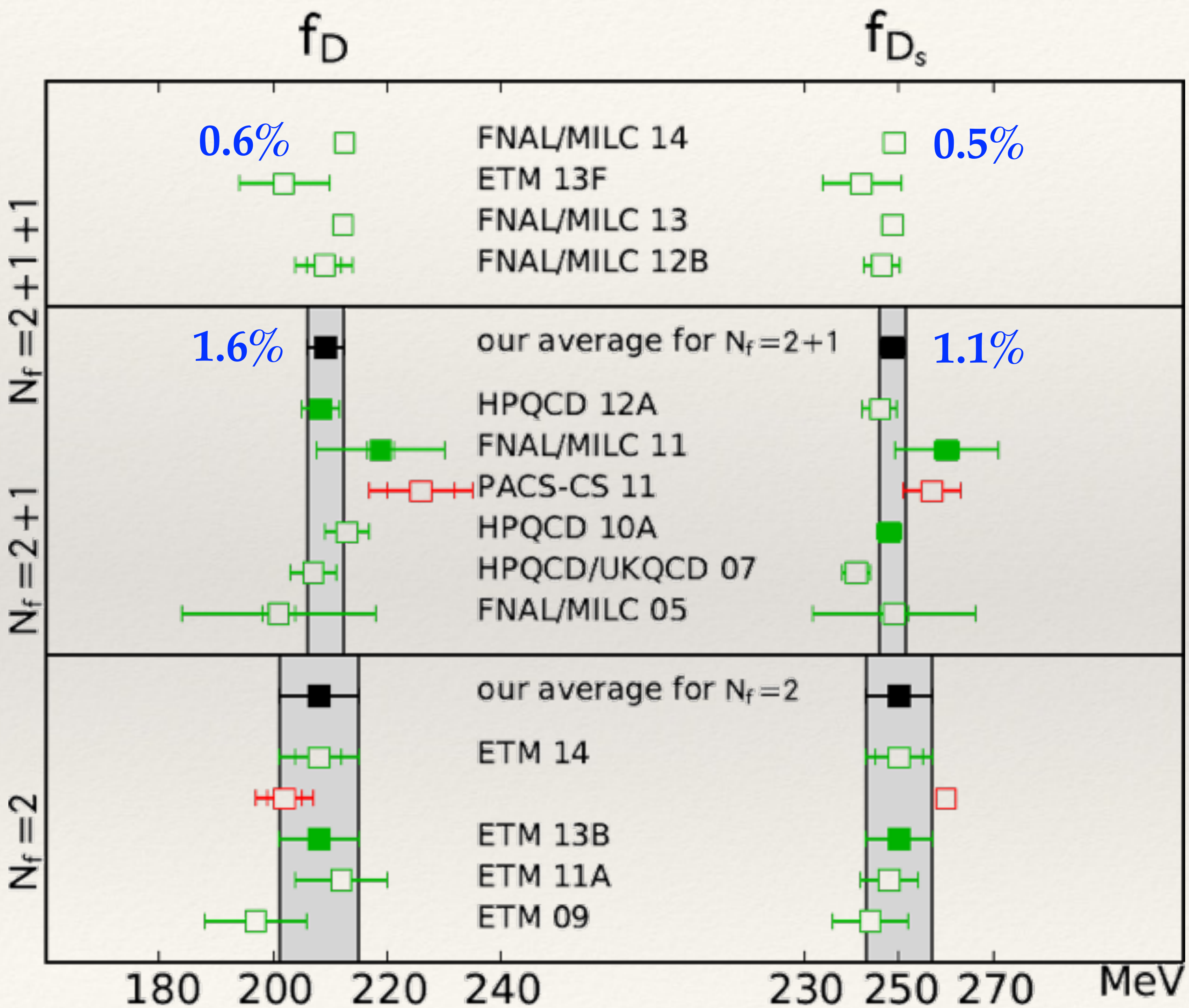
more specifically e.g tree level leptonic B decay



$$\Gamma(D \to l\nu_l) = \frac{m_D}{8\pi} G_F^2 f_D{}^2 |V_{cd}|^2 m_l^2 \left(1 - \frac{m_l^2}{m_D^2}\right)^2$$

Experimental measurement + theory prediction allows for extraction of CKM MEs

Leptonic $D_{(s)}$ meson decays



This plot: FLAG-2 plus some new results

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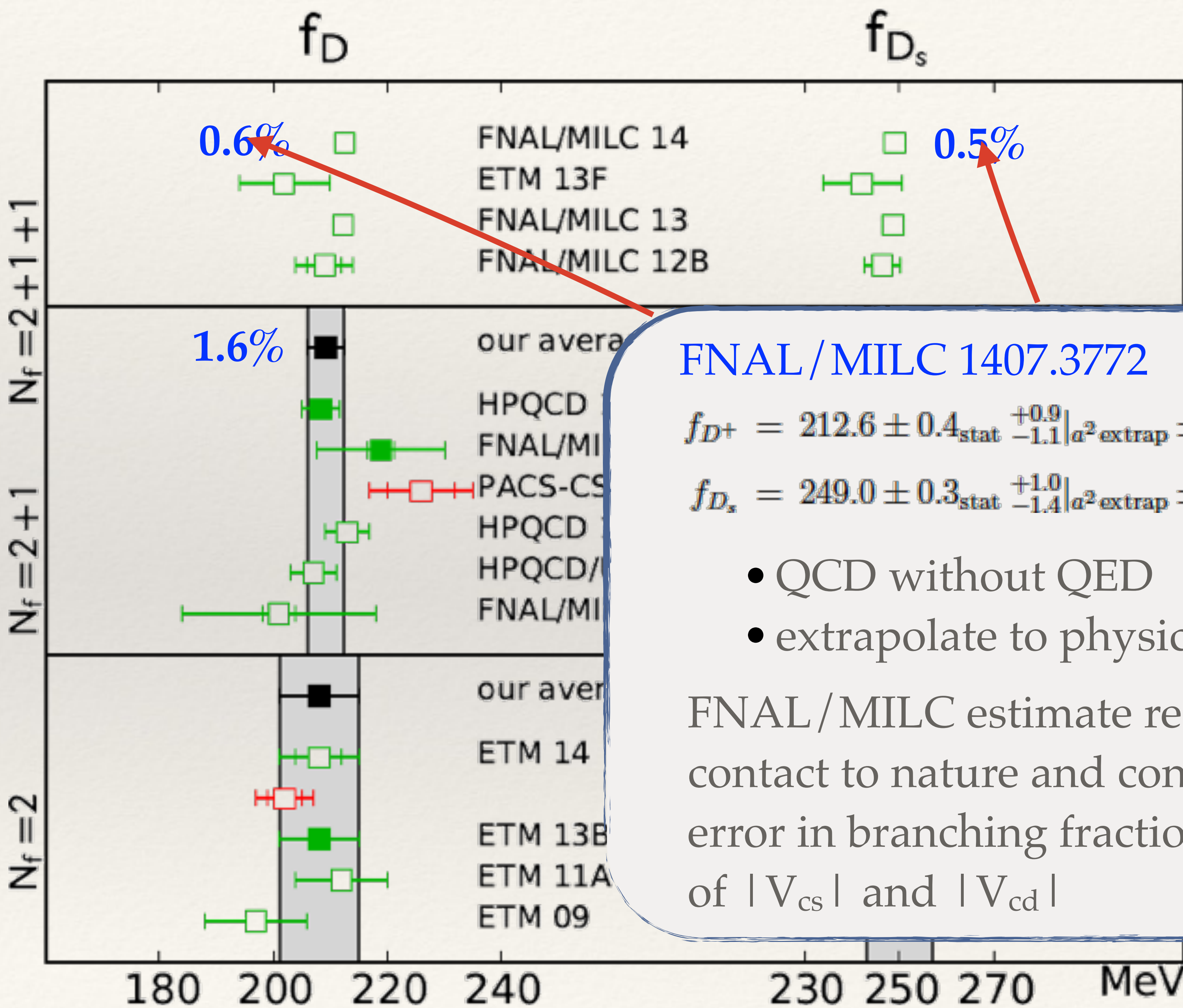


This plot: FLAG-2 plus some new results

FNAL / MILC 1407.3772

$$f_{D^+} = 212.6 \pm 0.4_{\text{stat}}\ {}^{+0.9}_{-1.1}|_{a^2\,\text{extrap}} \pm 0.3_{\text{FV}} \pm 0.1_{\text{EM}} \pm 0.3_{f_\pi\,\text{PDG}}\ \text{MeV}$$

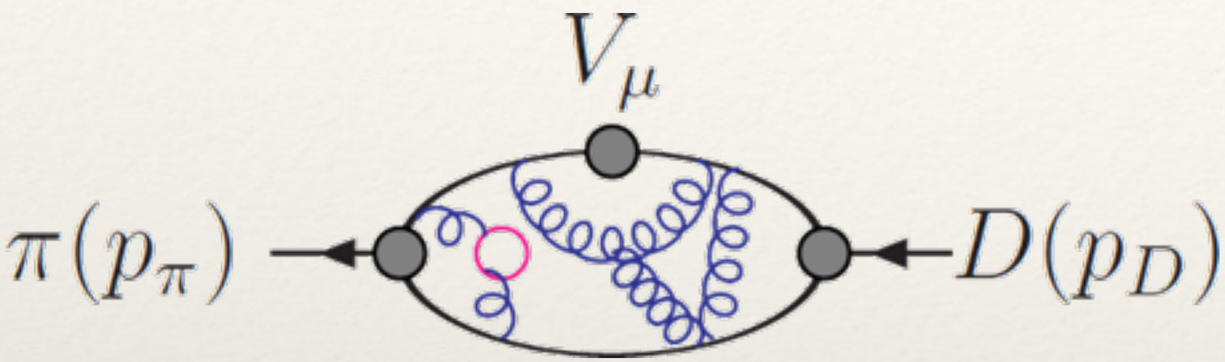
$$f_{D_s} = 249.0 \pm 0.3_{\text{stat}}\ {}^{+1.0}_{-1.4}|_{a^2\,\text{extrap}} \pm 0.2_{\text{FV}} \pm 0.1_{\text{EM}} \pm 0.4_{f_\pi\,\text{PDG}}\ \text{MeV}$$

- QCD without QED
- extrapolate to physical valence quark mass

FNAL / MILC estimate residual systematic in making contact to nature and conclude that experimental error in branching fraction dominant in determination of $|V_{cs}|$ and $|V_{cd}|$

CKM ME from tree-level decays

Semileptonic decay



$$\frac{d\Gamma(D \to P\ell\nu)}{dq^2} = \frac{G_F^2 |V_{cx}|^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 \sqrt{E_P^2 - m_P^2}}{q^4 m_D^2} \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) m_D^2 (E_P^2 - m_P^2) |f_+(q^2)|^2 \right.$$
$$\left. + \frac{3m_\ell^2}{8q^2} (m_D^2 - m_P^2)^2 |f_0(q^2)|^2 \right],$$

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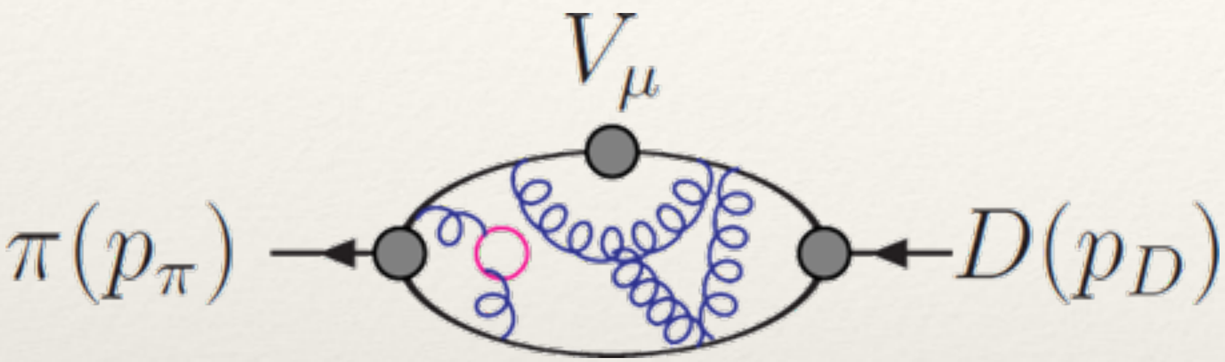

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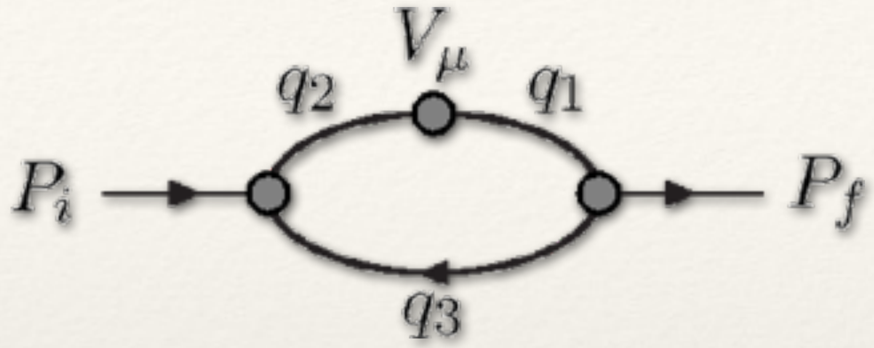
leptonic

- $D \to l\nu$, $D_s \to l\nu$

semi-leptonic

- $D \to \pi l\nu$, $D \to Kl\nu$
- $D_s \to \phi l\nu$, $D_s \to \eta l\nu$

transition MEs from Euclidean 3pt functions



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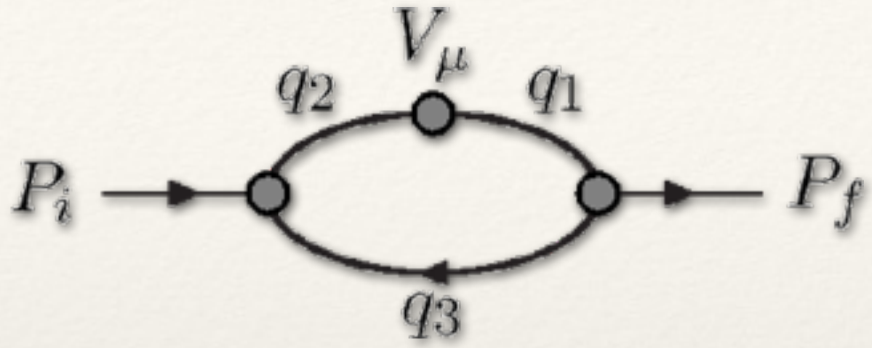

$$\langle P_f(\vec{p}_f)|V_\mu|P_i(\vec{p}_i)\rangle = f_+(q^2)\left(p_\mu^{P_i} - p_\mu^{P_f} - \frac{M_{P_f}^2 - M_{P_i}^2}{q^2}q_\mu\right) + f_0(q^2)\frac{M_{P_f}^2 - M_{P_i}^2}{q^2}q^\mu$$

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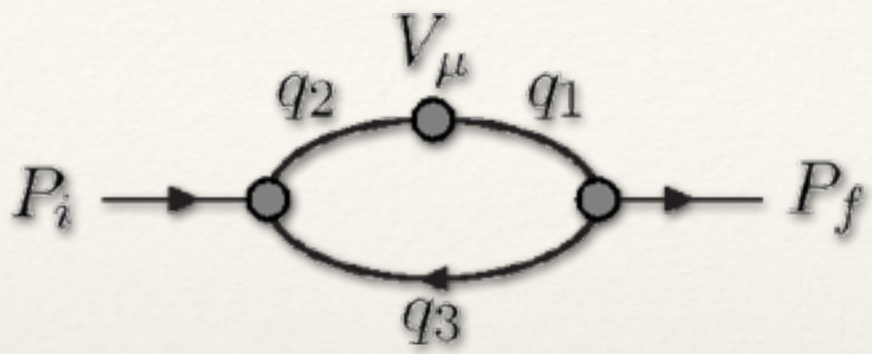

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$$\begin{aligned} C_{P_iP_f}(t_i,t,t_f,\vec{p}_i,\vec{p}_f) &= \sum_{\vec{x}_f,\vec{x}} e^{i\vec{p}_f(\vec{x}_f-\vec{x})}\langle O_f(t_f,\vec{x}_f)V_4(t,\vec{x})O_i^\dagger(t_i,\vec{0})\rangle \\ &= \frac{Z_iZ_f}{4E_iE_f}\langle P_f(\vec{p}_f)|V_4(0)|P_i(\vec{p}_i)\rangle e^{-E_i(t-t_i)-E_f(t_f-t)} \end{aligned}$$

semileptonic charm decay

Two phenomenology strategies:

- compute $f(q^2=0)$: use as absolute normalisation for parameterisation of exp. form factor
- compute $f(q^2)$: predict q^2-dependence - all experimental bins can be used in the analysis (e.g. $D\to K$: BaBar PRD 76 2007 10 bins, CLEO PRD 80 2009 9 bins)

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Extrapolations:

1) HMChPT (chiral & continuum extrapolation)
2) parameterisation of q^2-dependence (z-expansion)

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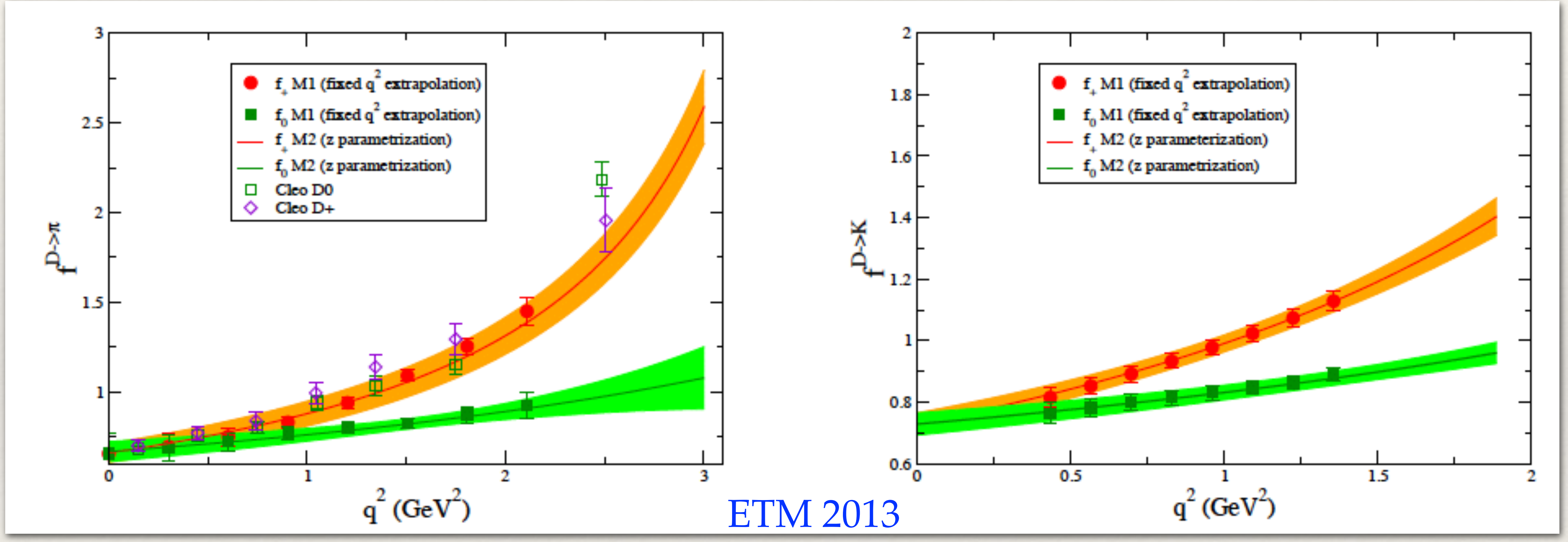
2) parameterisation of q^2-dependence (z-expansion)

HPQCD: combine 1) and 2) into modified z-expansion (where coefficients are a- and m_l-dependent

ETM

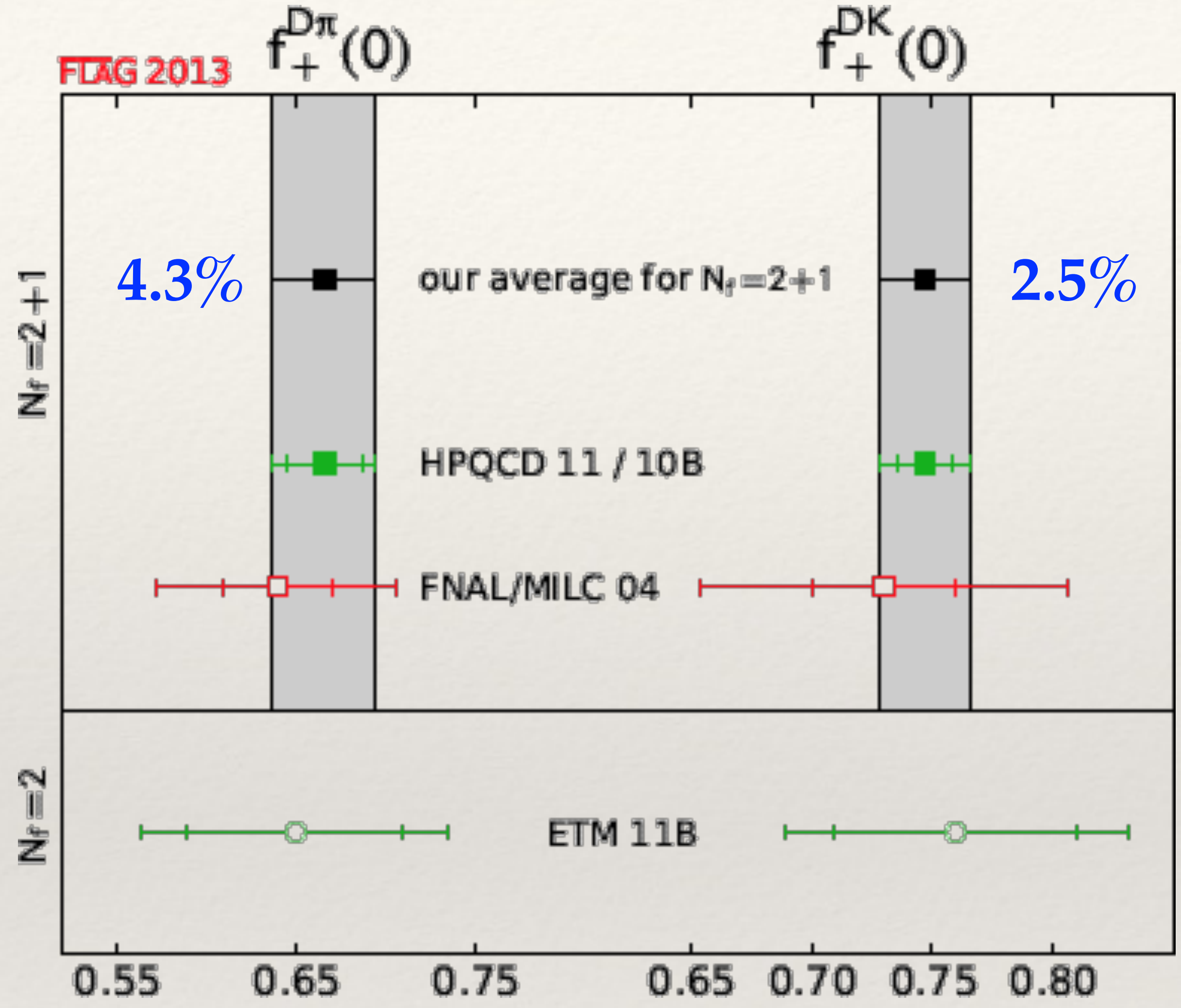
a) a- and m_l extrapolation at fixed, interpolated q^2 reference points
b) combine 1) and 2)

semileptonic charm decay



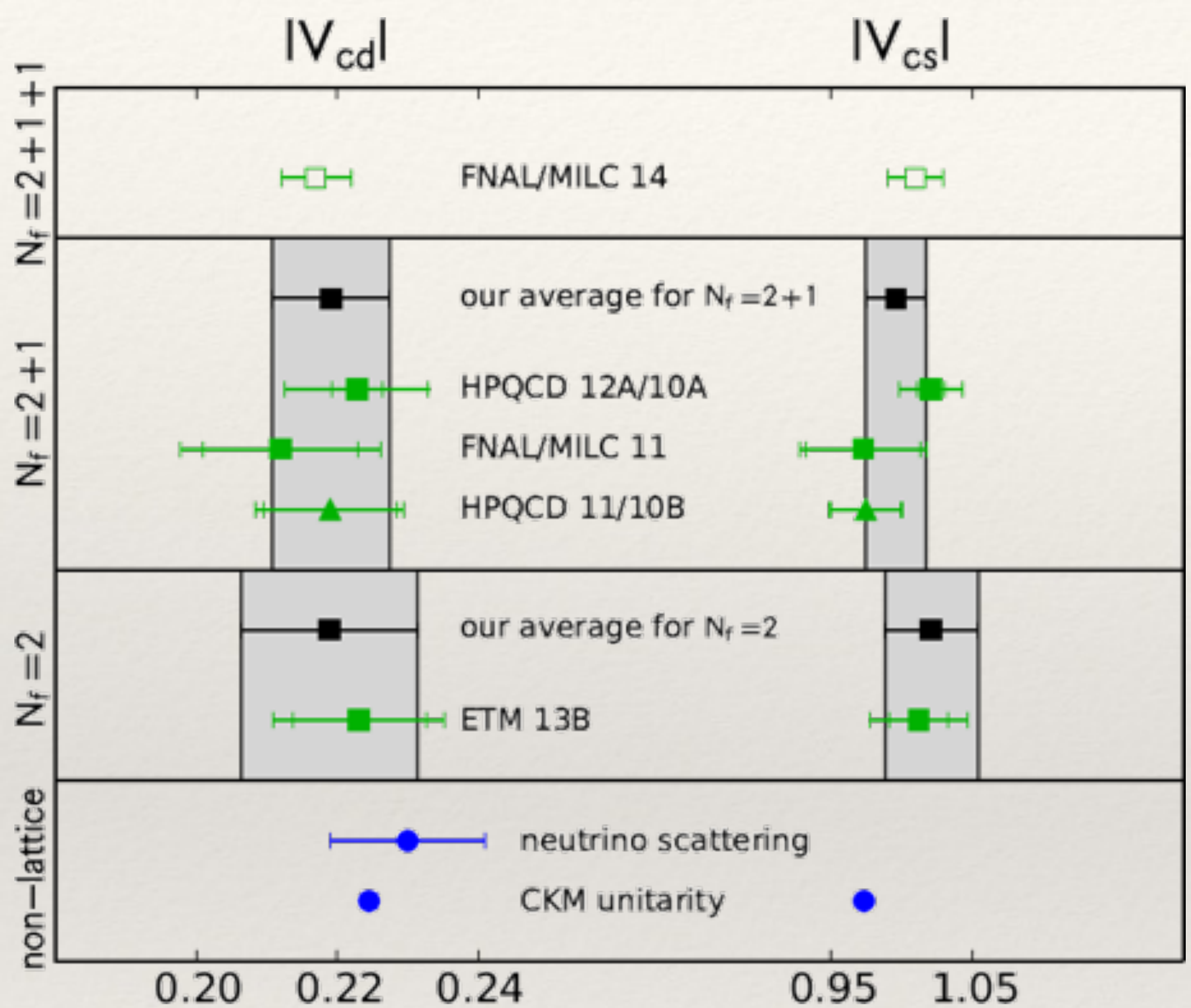
ETM 2013

Leptonic $D_{(s)}$ meson decays

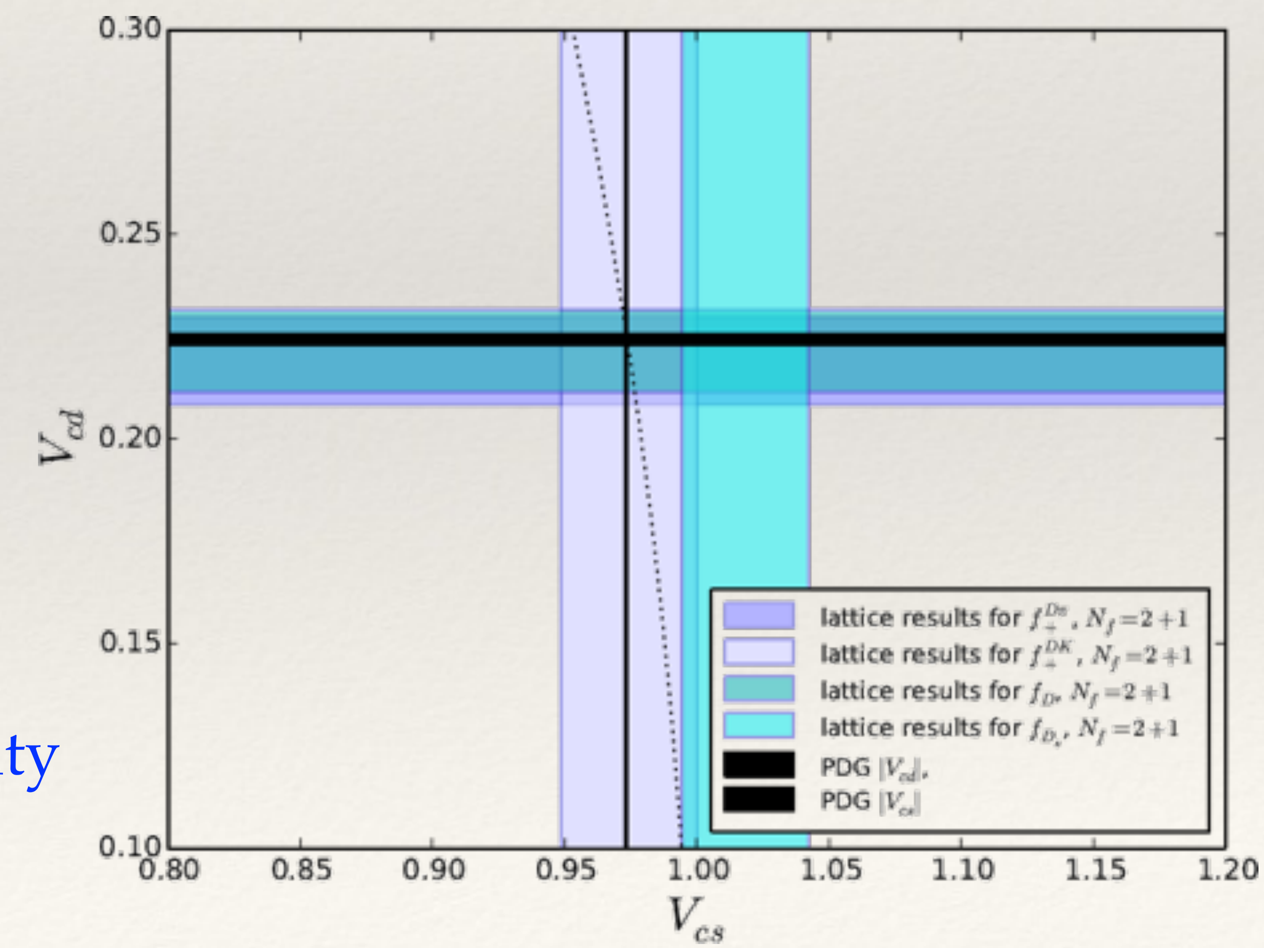


- continuum and chiral extrapolation dominant syst. uncertainties
- more activity needed in particular for semi-leptonics (see Lattice 2014)

Results for $|V_{cd}|$ and $|V_{cs}|$



- $|V_{cs}|$ from leptonic decays is slightly larger than from semileptonic decays
- $|V_{cs}|$ from leptonic decays is at tension with CKM-unitarity by 1.9σ (→HPQCD)



semi-leptonic channel consistent with unitarity
leptonic channel at tension

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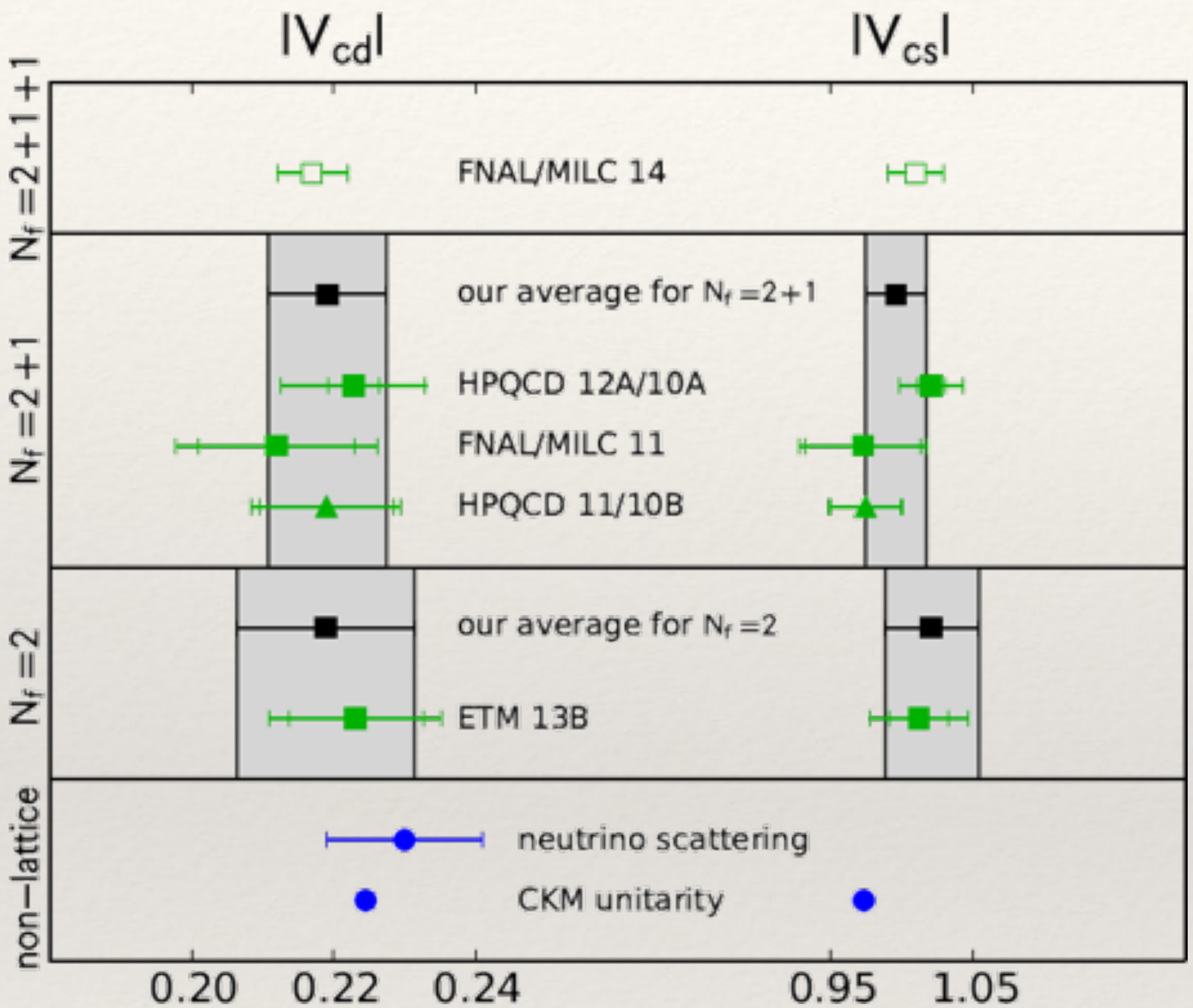
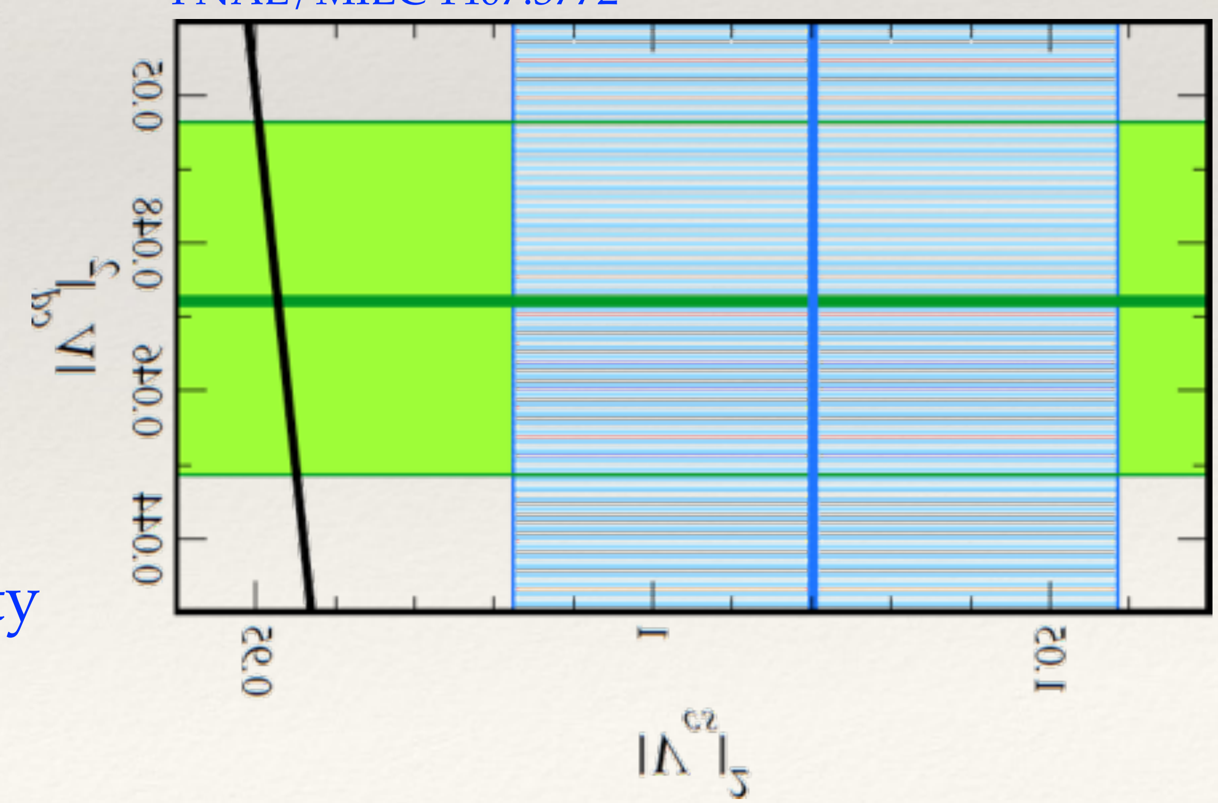


Tension persists with new FNAL/MILC data:

FNAL/MILC 1407.3772



semi-leptonic channel consistent with unitarity
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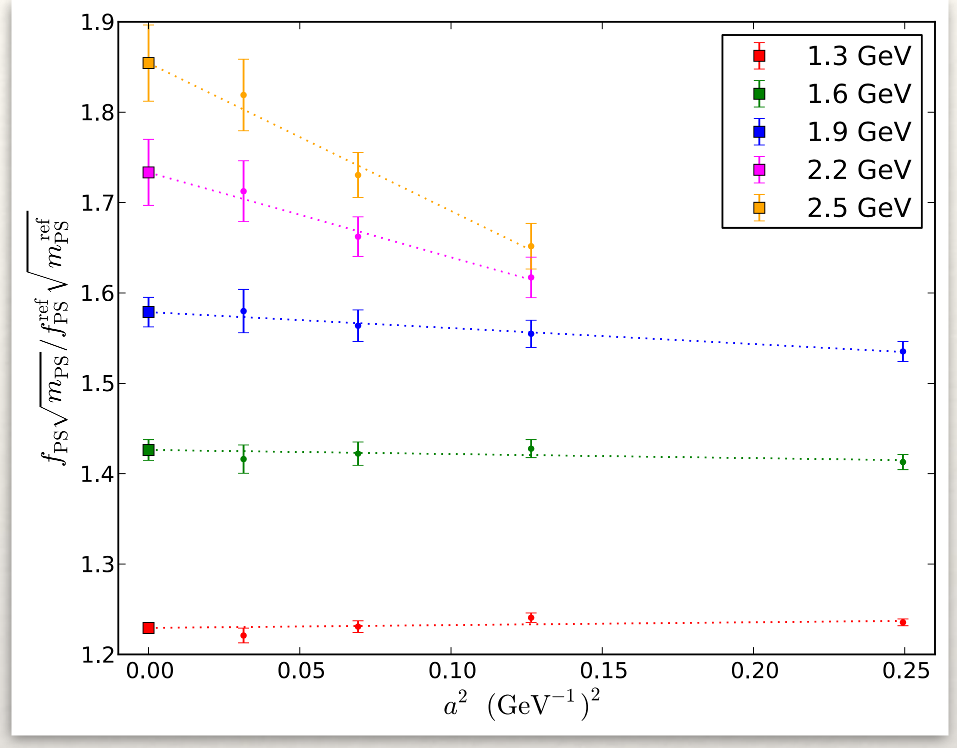
RBC/UKQCD N_f=2+1 charm programme

- Domain wall fermions as light **and** charm quark discretisation
  - automatically $O(a)$-improved
  - good chiral properties
- use RBC/UKQCD $N_f = 2{+}1$ DWF ensembles
  - a^{-1}=1.7, 2.3 GeV readily available, 3ish GeV ensemble under way
  - m_π physical ensembles in large volumes
- complementary to ongoing RBC/UKQCD B-physics program

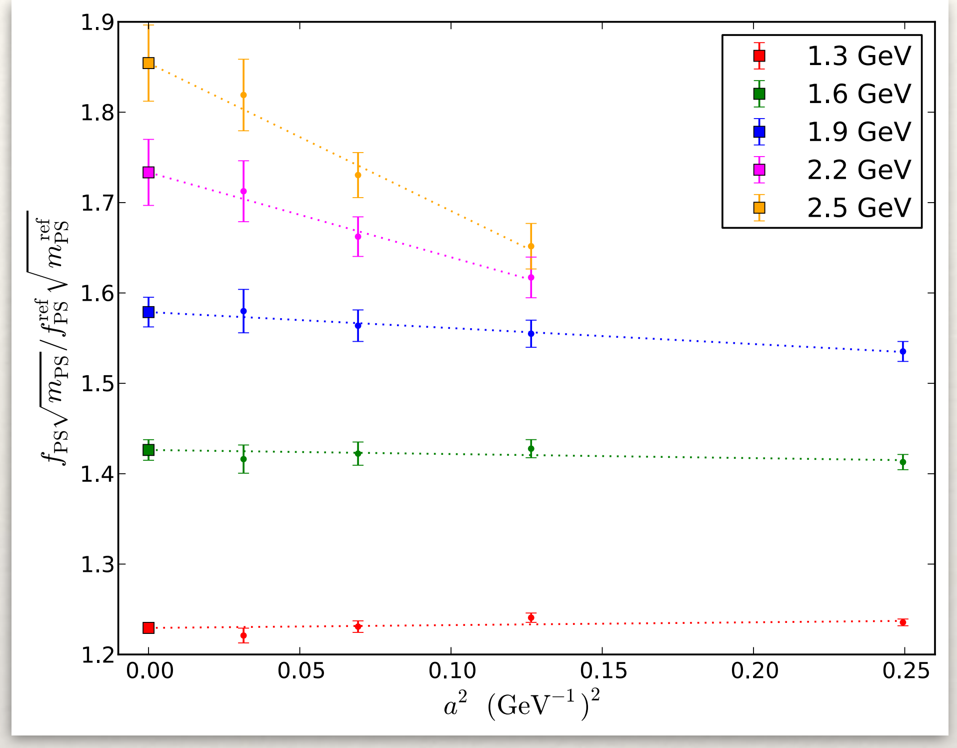
Our strategy

- **Quenched pilot study**
  - $a^{-1} \sim 2, 3, 4, 6$ GeV → scaling study
  - small volume L=1.6fm
  - parameter tuning
  - map out range of applicability
  - qualitative picture should apply also to dynamical case



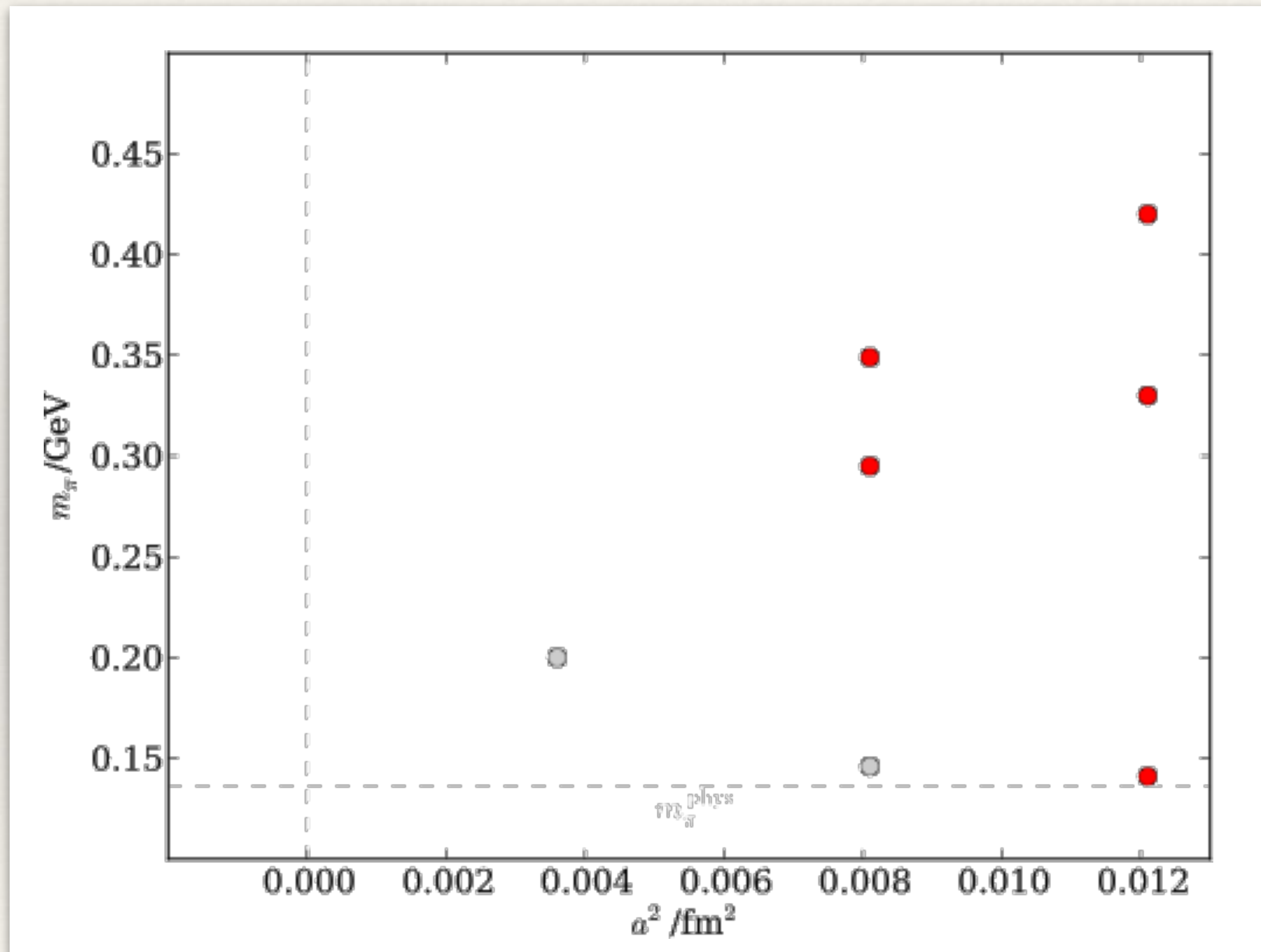
Our strategy

- **Quenched pilot study**
  - $a^{-1} \sim 2, 3, 4, 6$ GeV → scaling study
  - small volume L=1.6fm
  - parameter tuning
  - map out range of applicability
  - qualitative picture should apply also to dynamical case



- ***N_f= 2+1* DWF charm project (RBC/UKQCD)**
  - strategy: simulate several '*charm*' masses
  - inter/extrapolate to charm (and beyond?)
  - heavy-light, heavy-strange, heavy-heavy
  - leptonic/semileptonic decays, mixing (BSM), *g*-2, …

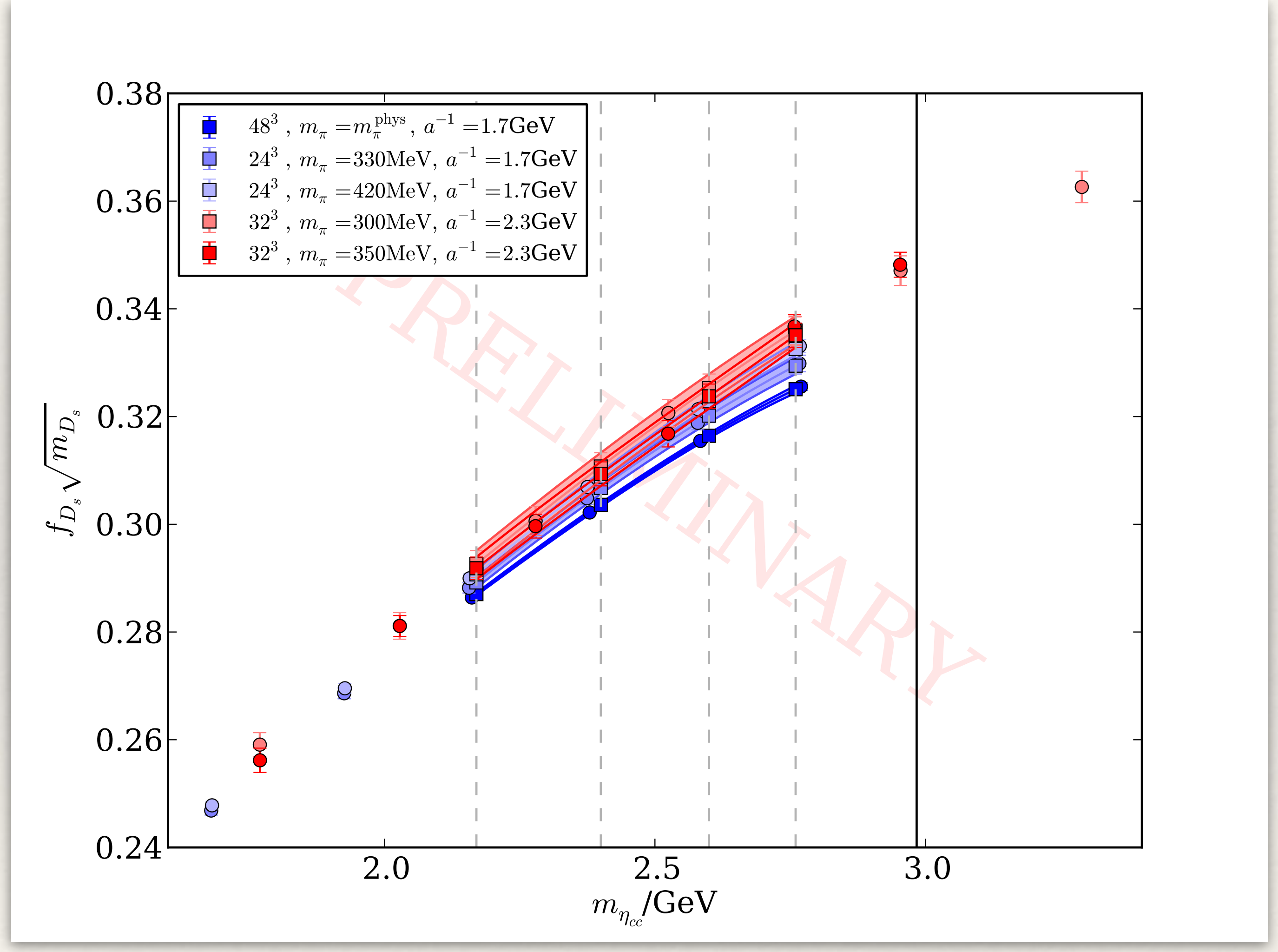
Dynamical case: simulation parameters



RBC/UKQCD DWF $N_{f=2+1}$ ensembles:
(Shamir/Moebius + Iwasaki)

L	a	m
48	1.7	physical
24	1.7	330
24	1.7	420
32	2.3	295
32	2.3	350
64	2.3	physical
	3.0	

Results interpolated to common η_{cc}-masses



1st STEP:
interpolation to common η_{cc}-masses

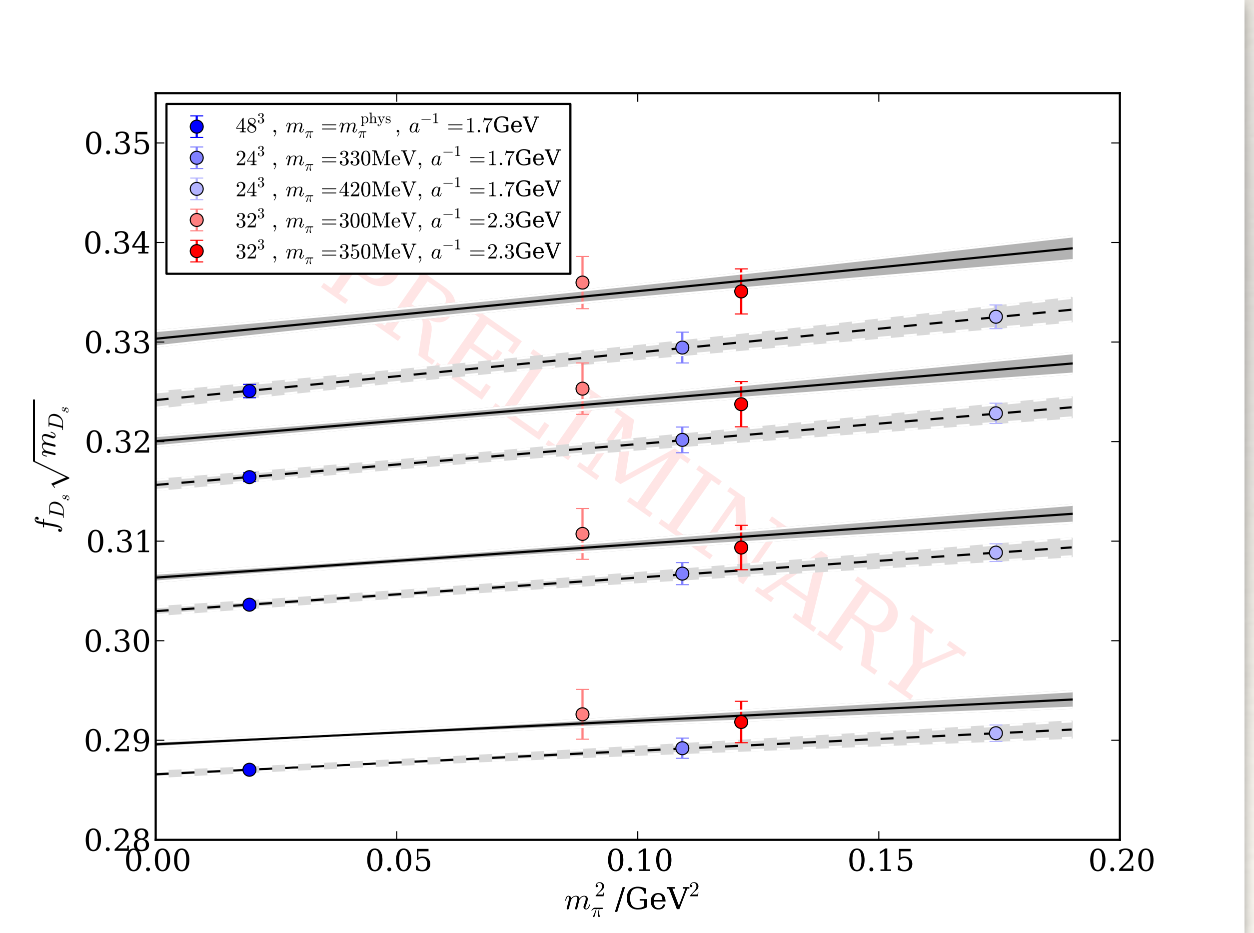
- $m_{\eta cc} \approx 2.2, 2.4, 2.6, 2.8$GeV
- polynomial ansatz, data benign
- interpolation points mostly close to data points
- solid vertical line: charm

Light quark mass dependence

2nd STEP: light quark mass *interpolation*

- Data on 1.7GeV covers m_π down to physical point - **NO curvature seen!!**
  → linear interpolation to physical point
- outlook: physical point on 2.3GeV lattice coming soon

D_s



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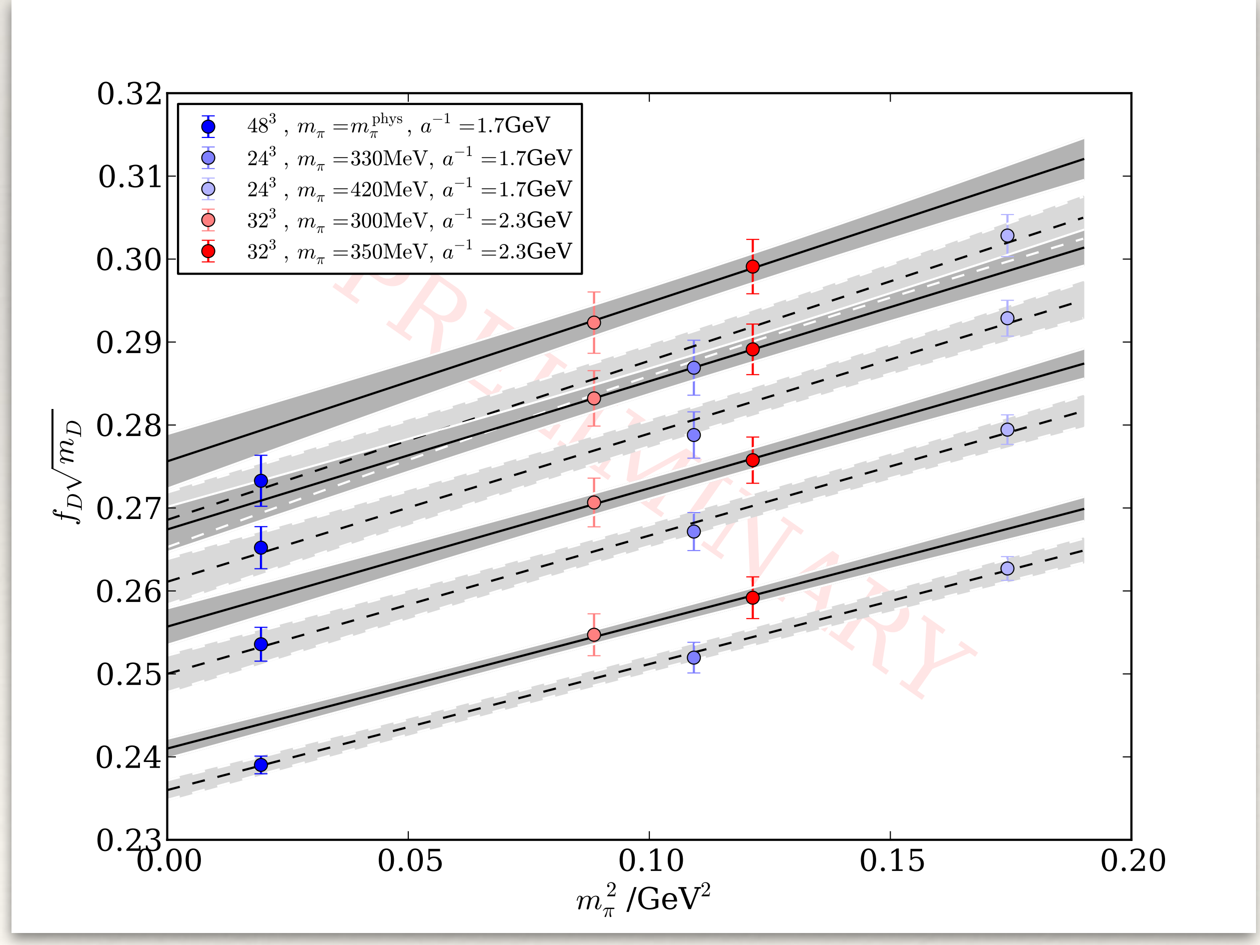
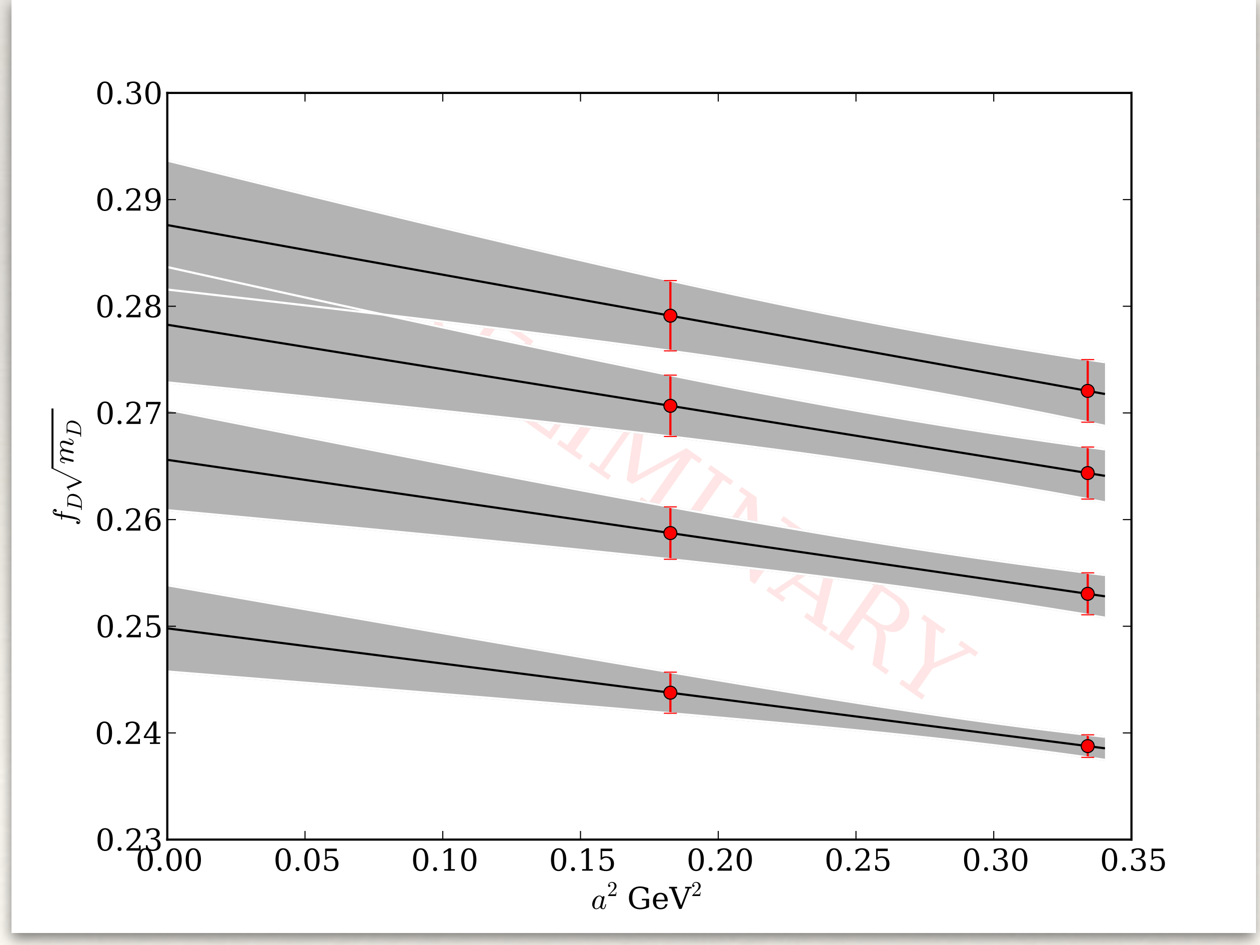


Continuum limit

3rd STEP: continuum limit:

- so far only 1.7 and 2.3GeV and therefore very preliminary
- quenched study suggests linear dependence on a^2
- but clearly **need 3rd lattice spacing for reliable predictions**
  → we are working on it

D_s



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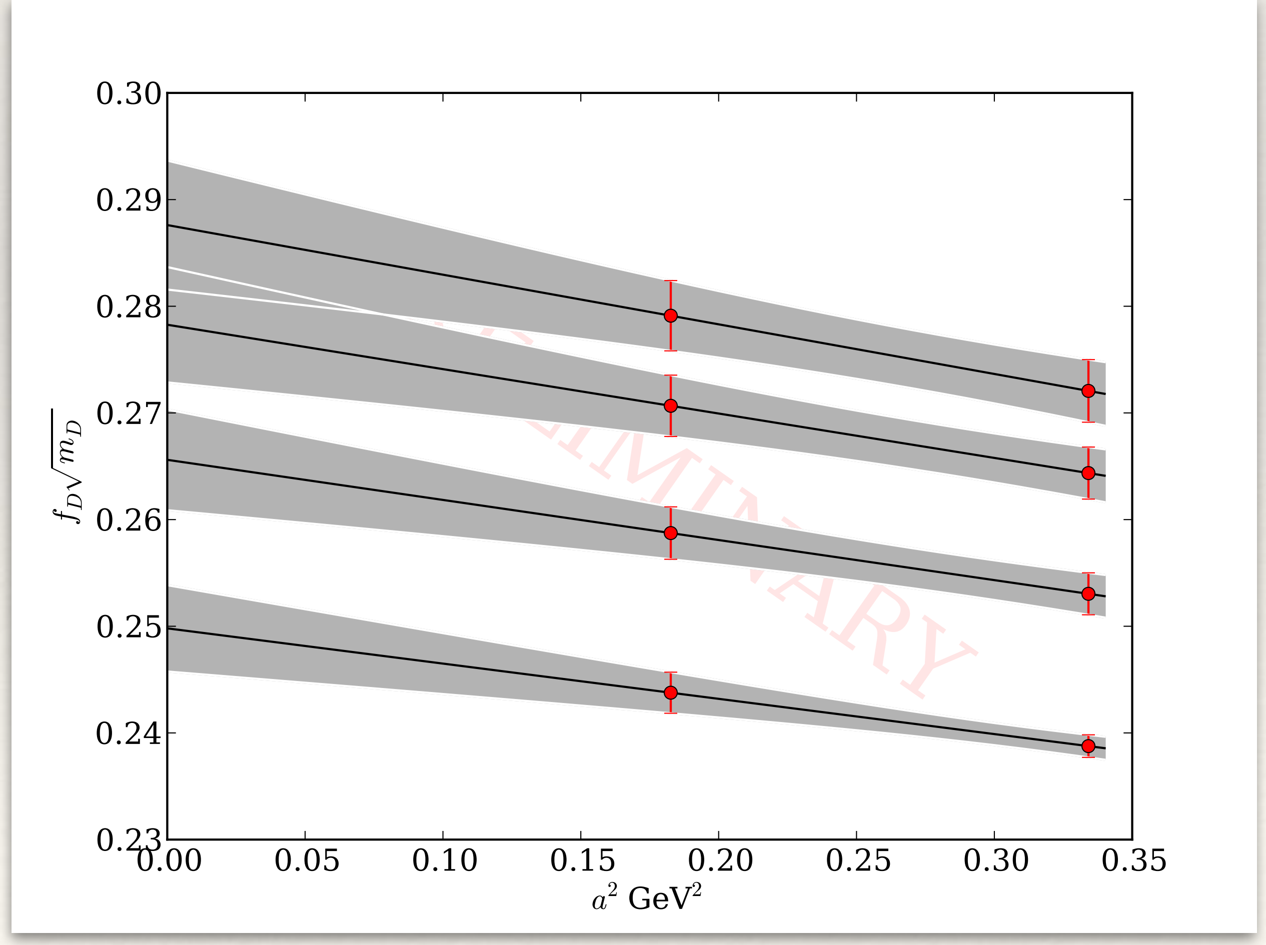
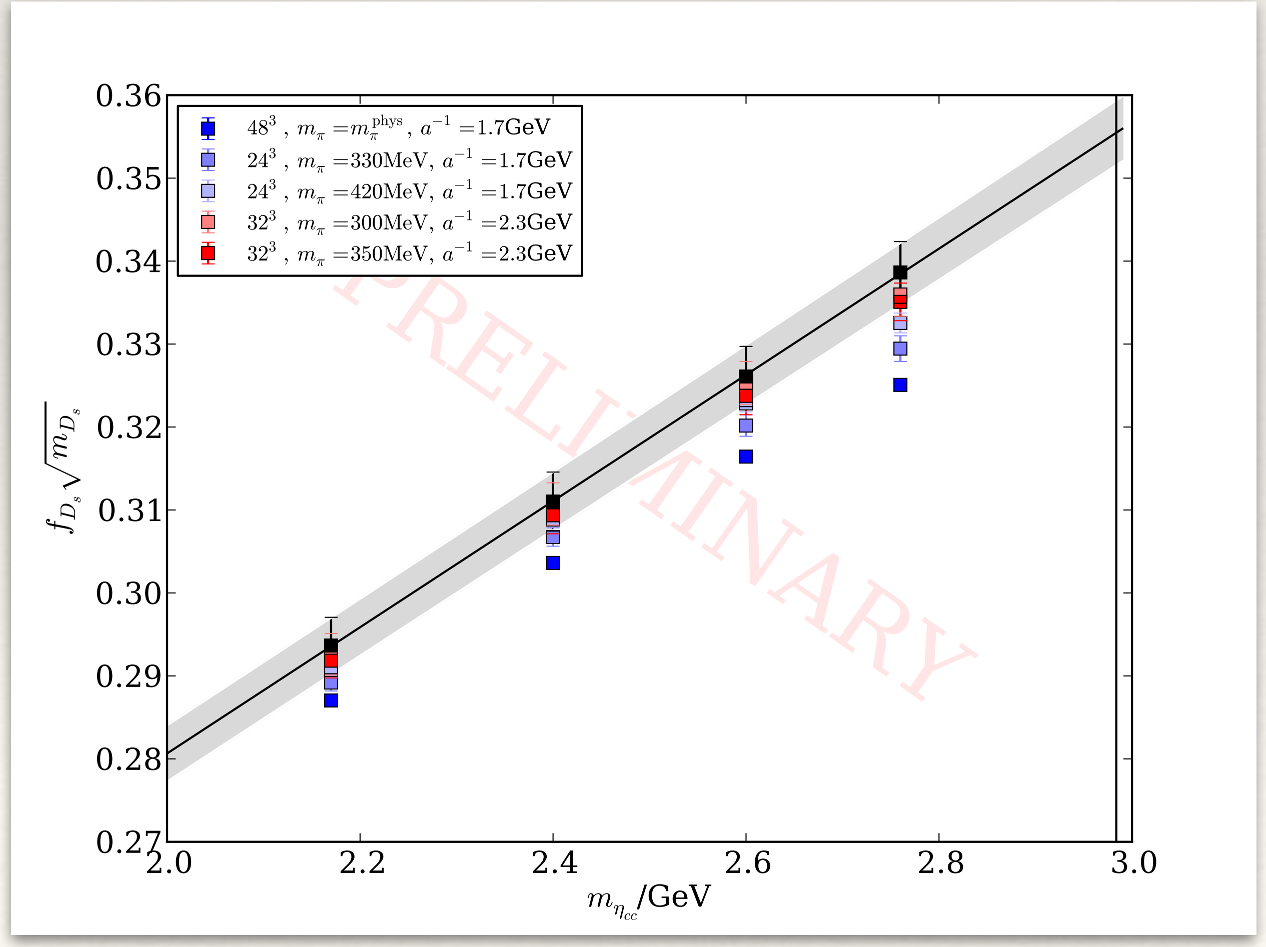


Extrapolation to charm

3rd STEP: extrapolation to charm

- looks quite linear but will study systematics
- already at this early stage astonishingly good statistical precision

D_s



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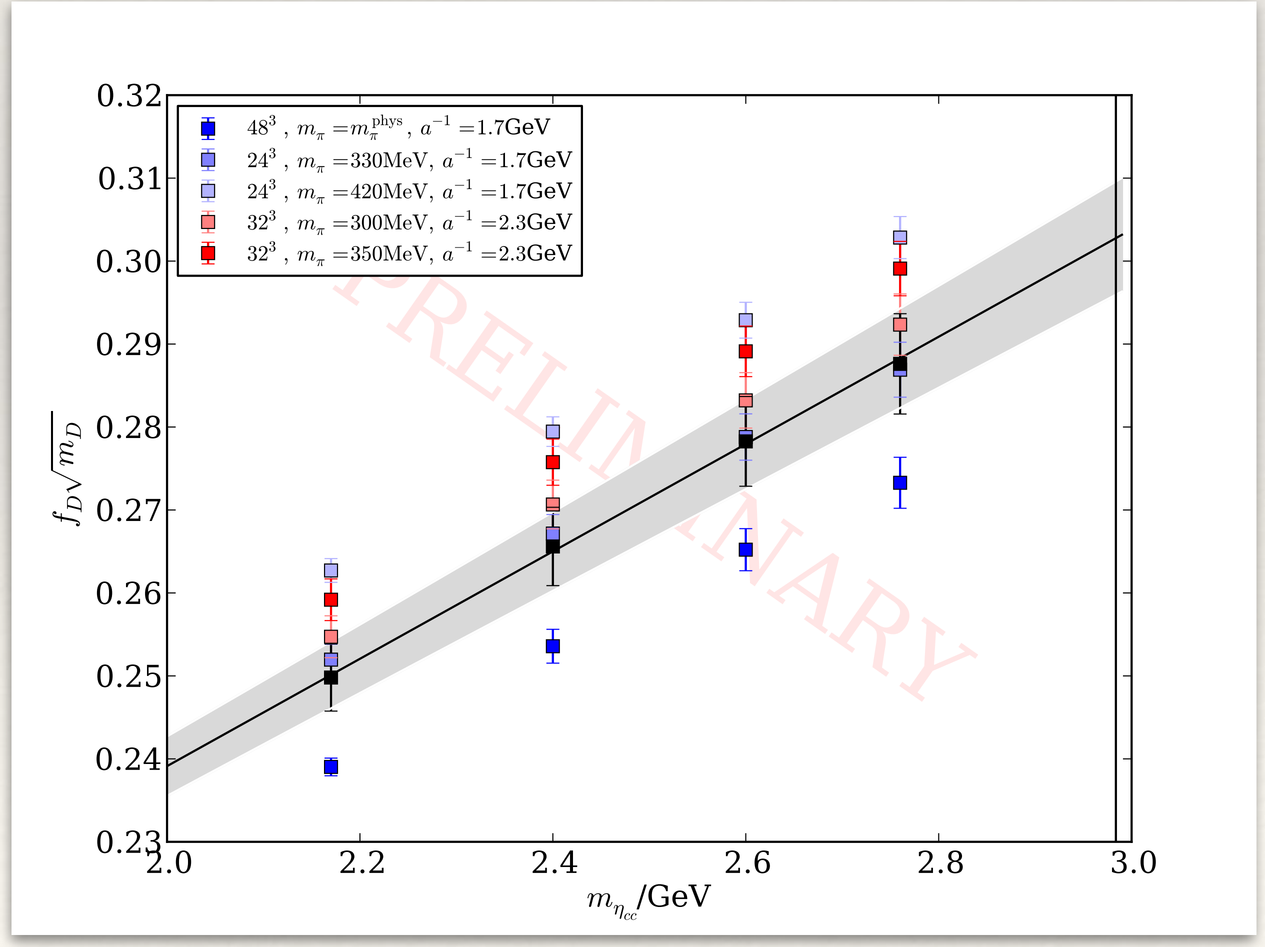


Comments

Results so far:

- benign mass-dependence, polynomial parameterisations and interpolation for light quark mass should work well
- charm not far away → short extrapolation to charm, good statistical properties
- all results preliminary, too early to provide numbers
- DWF excellent for charm!

Outlook:

- look into ratio method, interpolate to static, …
- other observables: quark masses, HVP, semi-leptonics, $D^0 - \bar{D}^0$ mixing

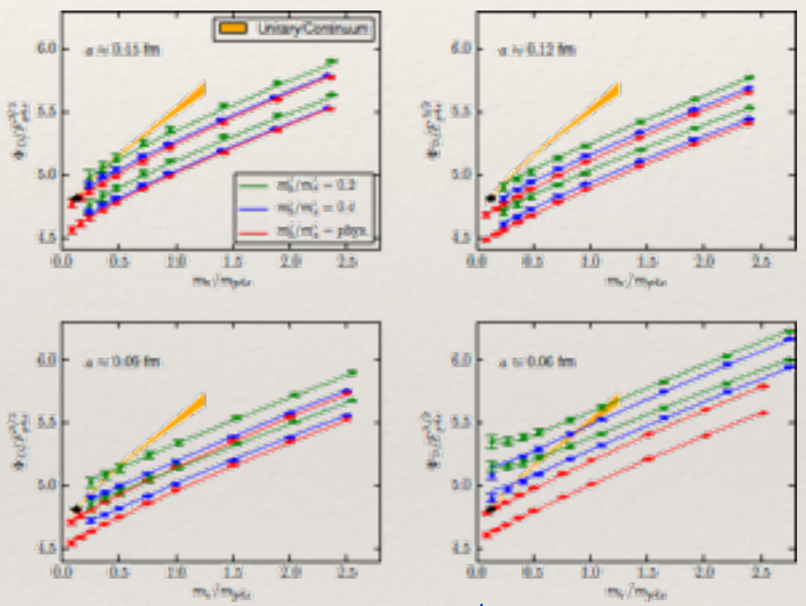
Paradigm change?

Results with physical point pions are quickly becoming standard

This could have significant consequences for the analysis strategies:

A. do a global analysis over physical AND unphysical point QCD simulations and make predictions in terms of global fit

C. take physical point result as what it is and use unphysical point results merely to correct (i.e. interpolate) for mistuning in quark masses



FNAL/MILC 1407.3772

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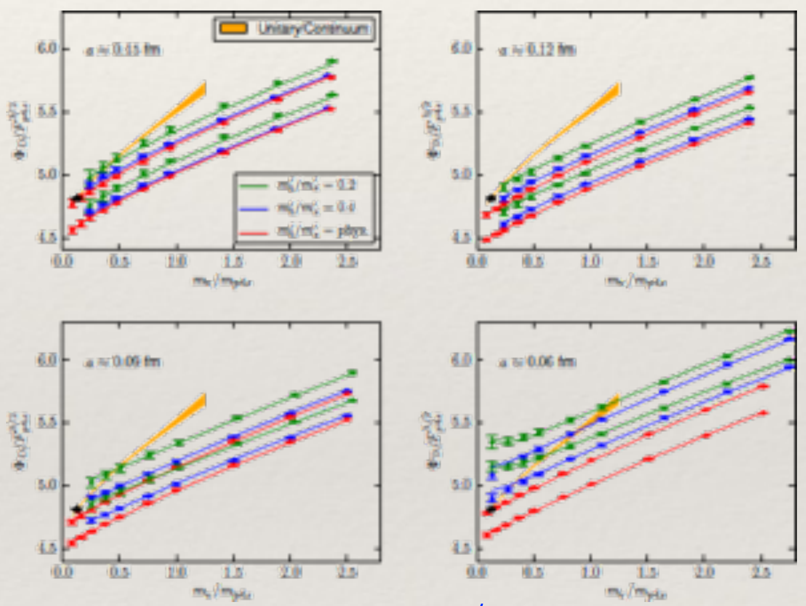

FNAL/MILC 1407.3772

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→ reduce stat. error

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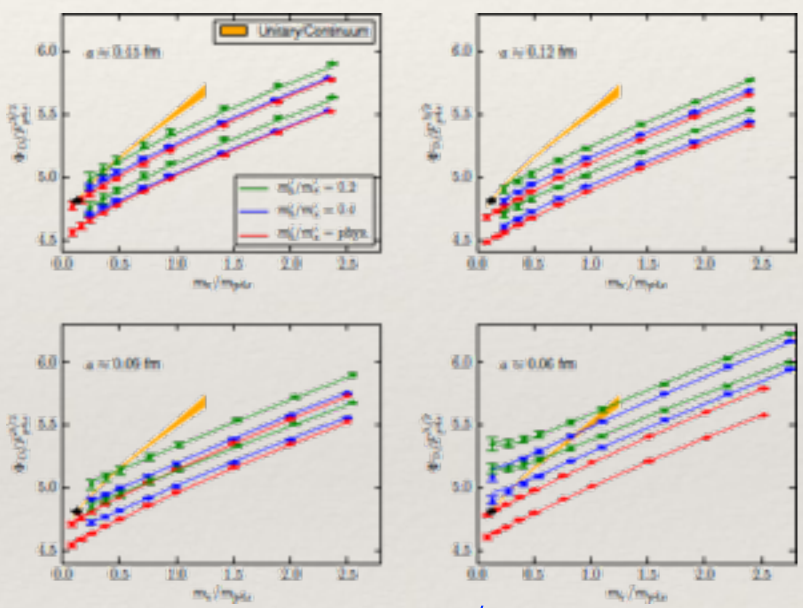

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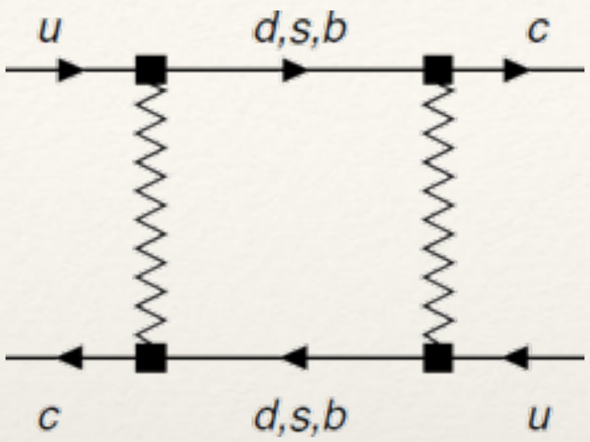
we have a choice…

charm mixing

SM and BSM mixing



- $D^0 - \overline{D}^0$-mixing (BaBar, Belle 2007) in SM model governed by V-A structure but there are 5 possible 4-quark operators:

$$Q_1 = [\bar{c}^a \gamma_\mu (1-\gamma_5) l^a][\bar{c}^b \gamma_\mu (1-\gamma_5) l^b],$$

$$\begin{aligned} Q_2 &= [\bar{c}^a (1-\gamma_5) l^a][\bar{c}^b (1-\gamma_5) l^b], & Q_4 &= [\bar{c}^a (1-\gamma_5) l^a][\bar{c}^b (1+\gamma_5) l^b], \\ Q_3 &= [\bar{c}^a (1-\gamma_5) l^b][\bar{c}^b (1-\gamma_5) l^a], & Q_5 &= [\bar{c}^a (1-\gamma_5) l^b][\bar{c}^b (1+\gamma_5) l^a] \end{aligned}$$

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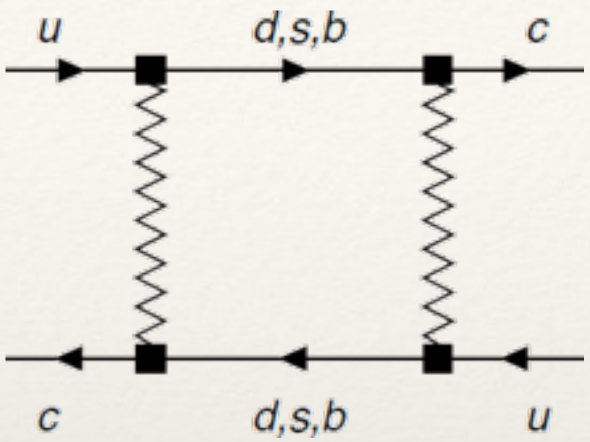

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$$Q_3 = [\bar{c}^a (1-\gamma_5) l^b][\bar{c}^b (1-\gamma_5) l^a], \quad Q_5 = [\bar{c}^a (1-\gamma_5) l^b][\bar{c}^b (1+\gamma_5) l^a]$$

- contrary to *K*- and *B*-mixing, SM-contribution long-distance dominated (*d*,*s*-loops)
- SM contribution *very real* → very strong constraints on *CP*-violation in NP

mixing (short distance)

- $D^0 - \bar{D}^0$

ETM Phys.Rev. D90 (2014) 014502

	B_1	B_2	B_3	B_4	B_5
$\overline{\mathrm{MS}}$ (3GeV)	0.75(02)	0.66(02)	0.96(05)	0.91(04)	1.10(05)
RI-MOM (3GeV)	0.74(02)	0.82(03)	1.21(06)	1.09(05)	1.35(06)

see also FNAL/MILC, Soni's talk

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Phenomenology:

$$\mathcal{H}_{\text{eff}}^{\Delta C=2} = \frac{1}{4}\sum_{i=1}^{5} C_i(\mu) Q_i(\mu)$$

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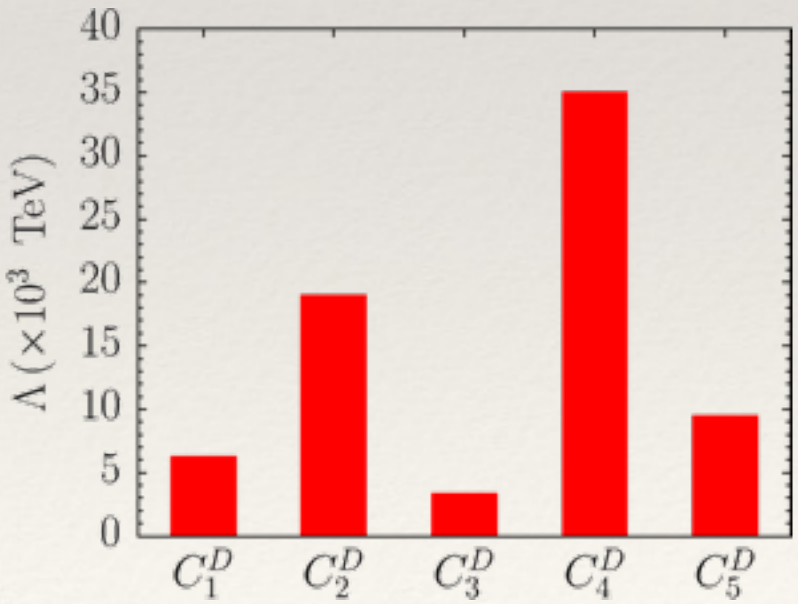
Phenomenology:

$$\mathcal{H}_{\text{eff}}^{\Delta C=2} = \frac{1}{4}\sum_{i=1}^{5} C_i(\mu)Q_i(\mu)$$

- SM analysis (e.g. UTfit 2008) provides constraints on C_i and hence the NP scale but relies on very old lattice input
- ETM updated constraints - e.g. generic strongly interacting or tree-level coupled NP

$$C_i(\Lambda) = \frac{F_i L_i}{\Lambda^2}$$



hadronic decays

two hadrons in a finite box

LHCb PRL108 (2012): CP-violation in $D^0 \to \pi^+\pi^-$, $D^0 \to K^+K^-$

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Hard problem for us:

- in a finite volume no asymptotic states can be defined
- a pair of pions is continuously rescattering and one expects discrete scattering energies
- Lüscher computed the finite volume pion scattering energies - the quantisation condition is

Lüscher, Comm. Math. Phys. 150, 1986; NPB 154, 1991

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$$n\pi - \delta_I(k_n) = \phi(q_n)$$

$$W_n = 2\sqrt{M_\pi^2 + k_n^2}$$

- $L=\infty$ scattering phase shift can be constructed from finite-L two particle energies (below 4-particle threshold)

the "simplest" case: $K \to \pi\pi$

- formalism extended to $K \to \pi\pi$, treat kaon as infinitely narrow resonance and use its effect on finite box scattering energies to predict $L=\infty$ scattering amplitudes

Lellouch and Lüscher, Comm. Math. Phys. 219, 2001

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- interesting phenomenology:

$$\begin{aligned} \langle \pi\pi(I=2)|H_W|K^0\rangle &= A_2 e^{i\delta_2} & \Delta = \tfrac{3}{2} \\ \langle \pi\pi(I=0)|H_W|K^0\rangle &= A_0 e^{i\delta_0} & \Delta = \tfrac{1}{2} \end{aligned}$$

$$\frac{|A_0|}{|A_2|} = 22.4$$ "$\Delta I=1/2$-rule"

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$$\frac{|A_0|}{|A_2|} = 22.4$$ “$\Delta I = 1/2$-rule”

- RBC/UKQCD championed calculation (RBC/UKQCD PRD84, 2011)
  - exploratory study: $m_K = 877\text{MeV} \quad m_\pi = 422\text{MeV} \quad \frac{\text{Re}A_0}{\text{Re}A_2} = 9.1(2.1)$
    $m_K = 622\text{MeV} \quad m_\pi = 329\text{MeV} \quad \frac{\text{Re}A_0}{\text{Re}A_2} = 12.0(1.7)$

  - study with physical parameters is under way

what about other channels?

Decay channels:

K

▾ **Hadronic modes**

Γ_1	$K_S^0 \to \pi^0\pi^0$	$(3.069 \pm .005) \times 10^{-1}$	209
Γ_2	$K_S^0 \to \pi^+\pi^-$	$(6.920 \pm .005) \times 10^{-1}$	206
Γ_3	$K_S^0 \to \pi^+\pi^-\pi^0$	$(3.5\ ^{+1.1}_{-0.9}) \times 10^{-7}$	133

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D

- ▸ Hadronic modes with one K
  - ▸ Fractions of many of the following modes with resonances have already appeared above as submodes of particular charged-particle modes.(Modes for which there are only upper limits and $\overline{K}^*(892)\rho$ submodes only appear below.)
- ▸ Hadronic modes with three K's
- ▸ Pionic modes
- ▸ Hadronic modes with a $K\overline{K}$ pair
  - ▸ Other $K\overline{K}X$ modes. They include all decay modes of the ϕ, η, and ω.

what about other channels?

Lüscher- and Lellouch-Lüscher-technique needs to be extended in a quantum field theoretical frame-work!

- multiple final 2→2-channels (e.g. $D \to \pi\pi$, KK, $\eta\eta$)
  Hansen, Sharpe PRD86, 2012
- 2→2 and 3→3 (finite volume scattering understood, relation to infinite volume amplitudes still unclear)
  Hansen, Sharpe 1408.5933
- 1→2 (e.g. $\gamma\pi \to \pi\pi$, $B \to K^{*}ll \to K\pi ll$)
  Briceño, Hansen, Walker-Loud arXiv:1406.5965

All this is brand new and needs to be digested and extended but at least there is cautious hope…

Summary

- … all that covers only a small fraction of the activity lattice charm physics (see links to Lattice 2014 plenary talks on next slide)

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- … all that covers only a small fraction of the activity lattice charm physics (see links to Lattice 2014 plenary talks on next slide)

- Some take away messages:
  - we are now simulating physical QCD parameters
  - light, strange and charm can be simulated with the same *light quark* discretisation
  - there is a large group of quantities which we can pre-/post-dict with an excellent control over systematic effects ($\rightarrow$ FLAG)
  - the *bread and butter* quantities involving charm are now being computed to a level of precision where
    - elm. and isospin effects need to be addressed
    - theory is not always limiting precision
  - there is a lot of interesting conceptual progress which raises our hopes that hadronic decays beyond K might at some point become feasible…

and 

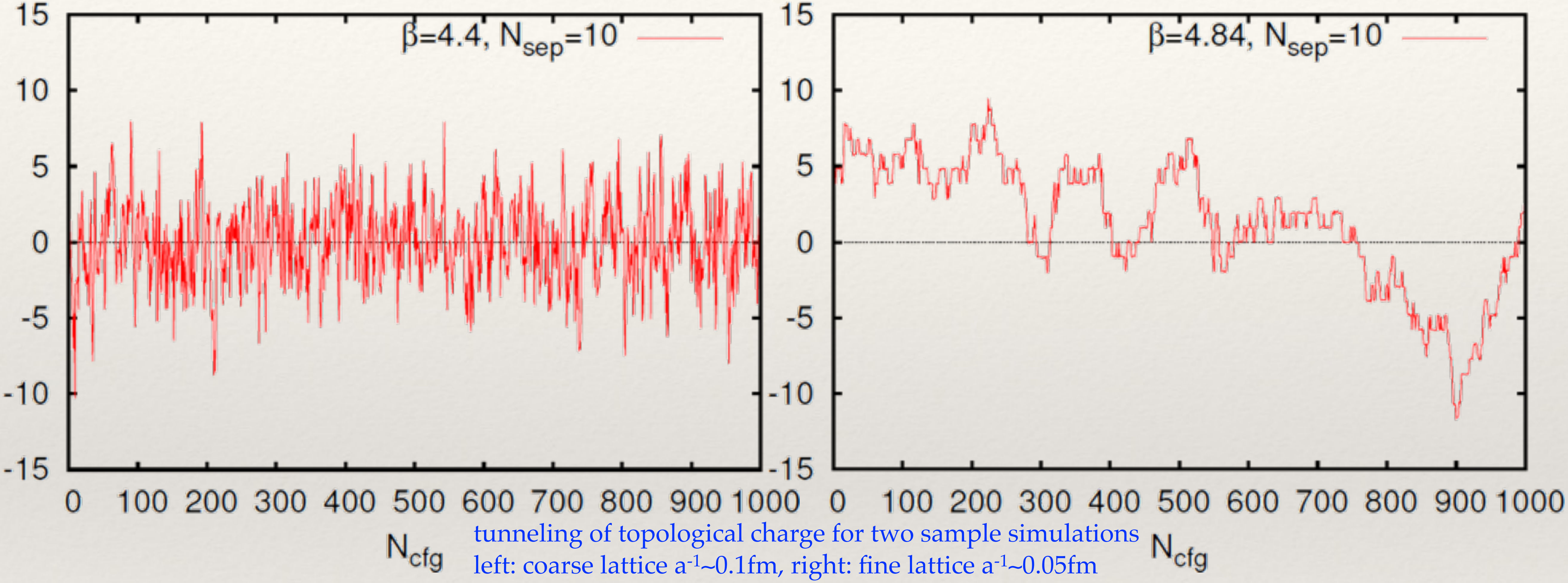
at Lattice 2014

	plenary speaker
c- and b-quark masses	Francesco Sanfilippo
hadron spectroscopy	Sasa Prelovsek
electro-weak matrix elements • leptonic • semi-leptonic (tree, rare) • mixing	Chris Bouchard
hadron structure	Martha Constantinou
isospin breaking	Antonin Portelli

The research leading to these results has received funding from the European Research Council under the European Community's Seventh Framework Programme (FP7/2007-2013) ERC grant agreement No 279757

Supplementary material

Critical slowing down



tunneling of topological charge for two sample simulations
left: coarse lattice a^{-1}~0.1fm, right: fine lattice a^{-1}~0.05fm

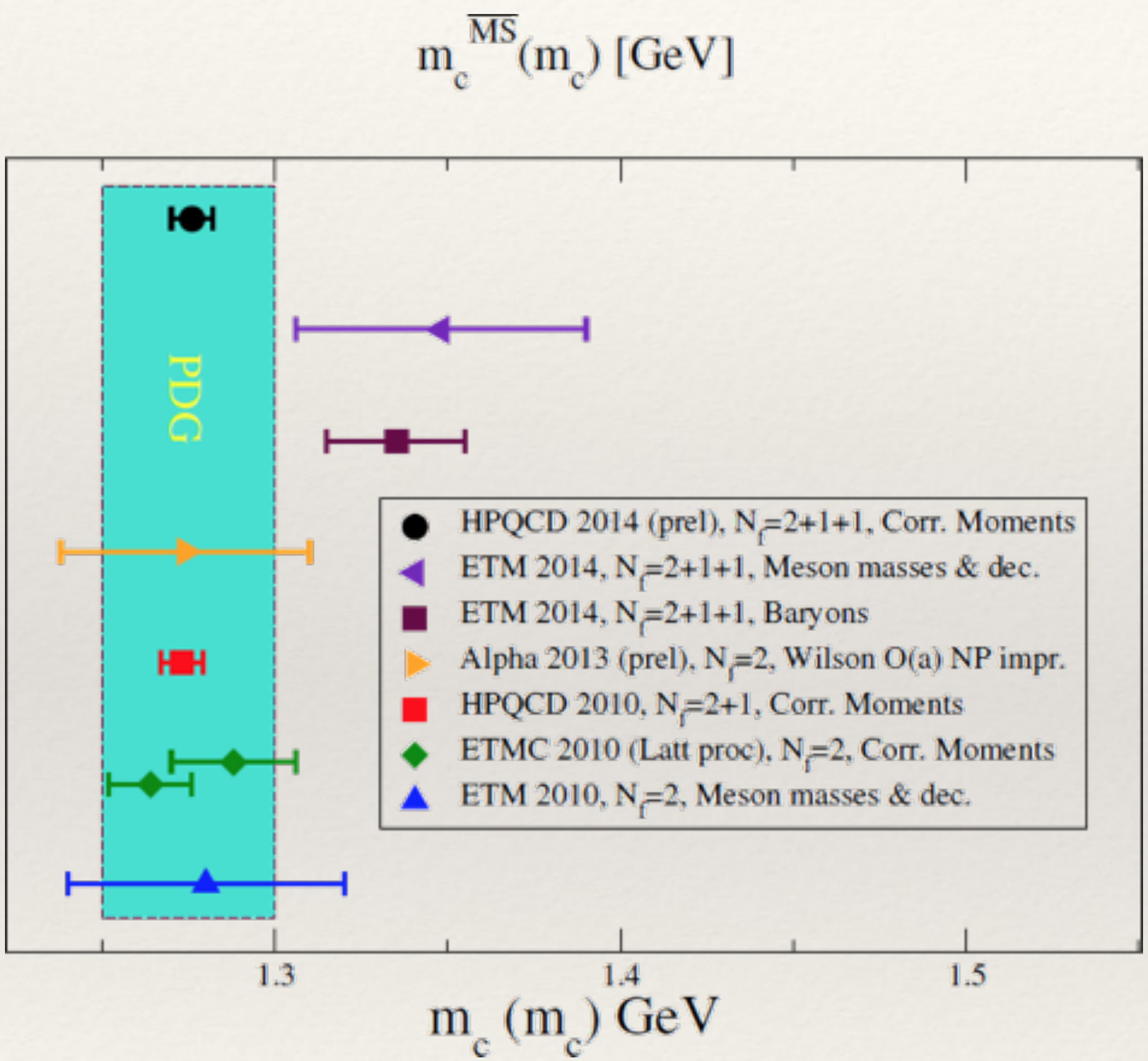
We have seen critical slowing down of algorithms beyond a^{-1}~4GeV

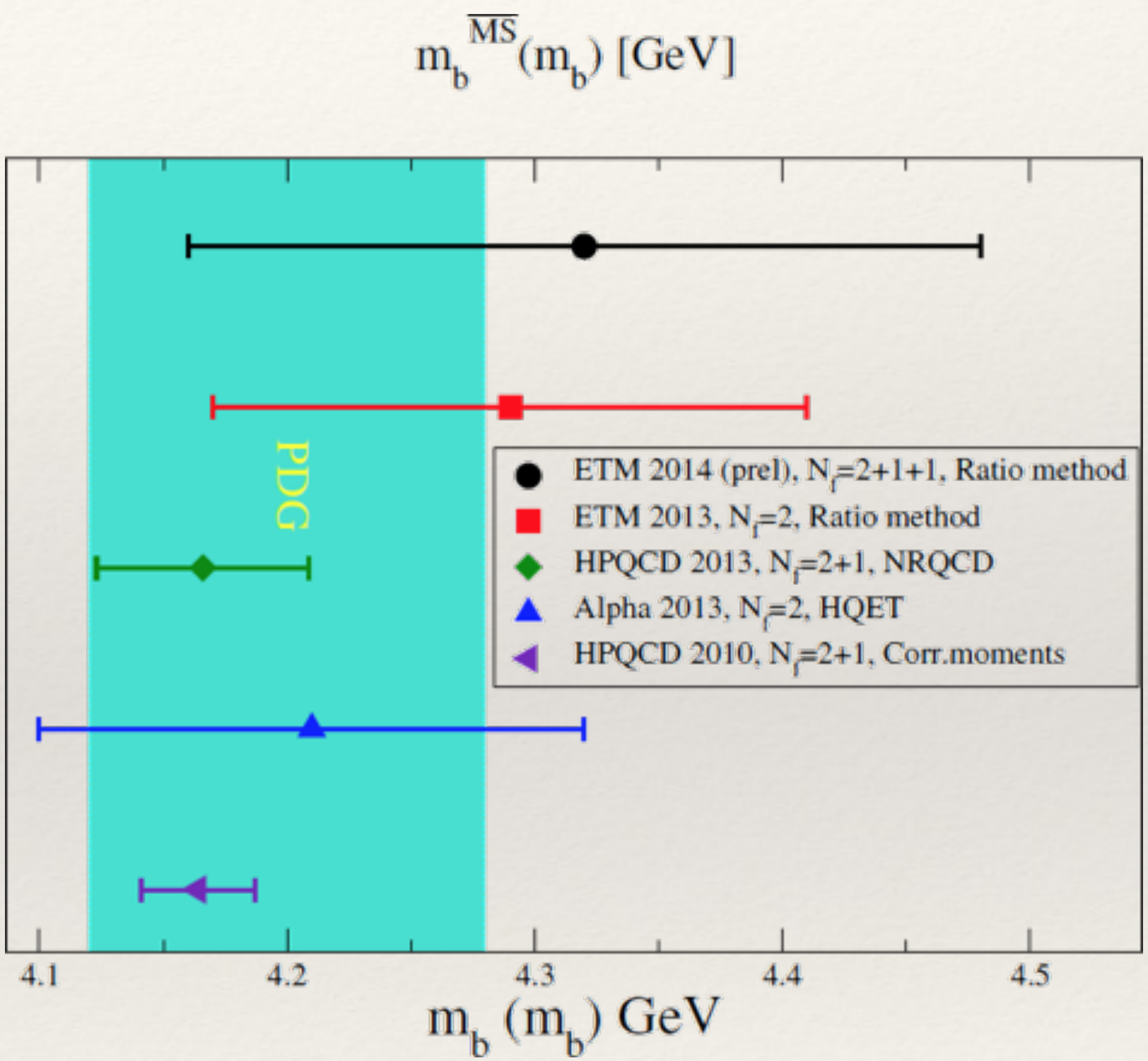
→ needs to be be considered for reliable estimation of stat. errors (ALPHA NP B845 (2011) 93-119)

→ needs to be assessed in detail for every individual ensemble / action

→ open boundary conditions (Lüscher, Schaefer, JHEP 1107 (2011) 36, McGlynn, Mawhinney arXiv:1406.4551)

Charm and bottom masses





Plots taken from F. Sanfilippo's Lattice 2014 plenary

Quarkonia

Main problems:

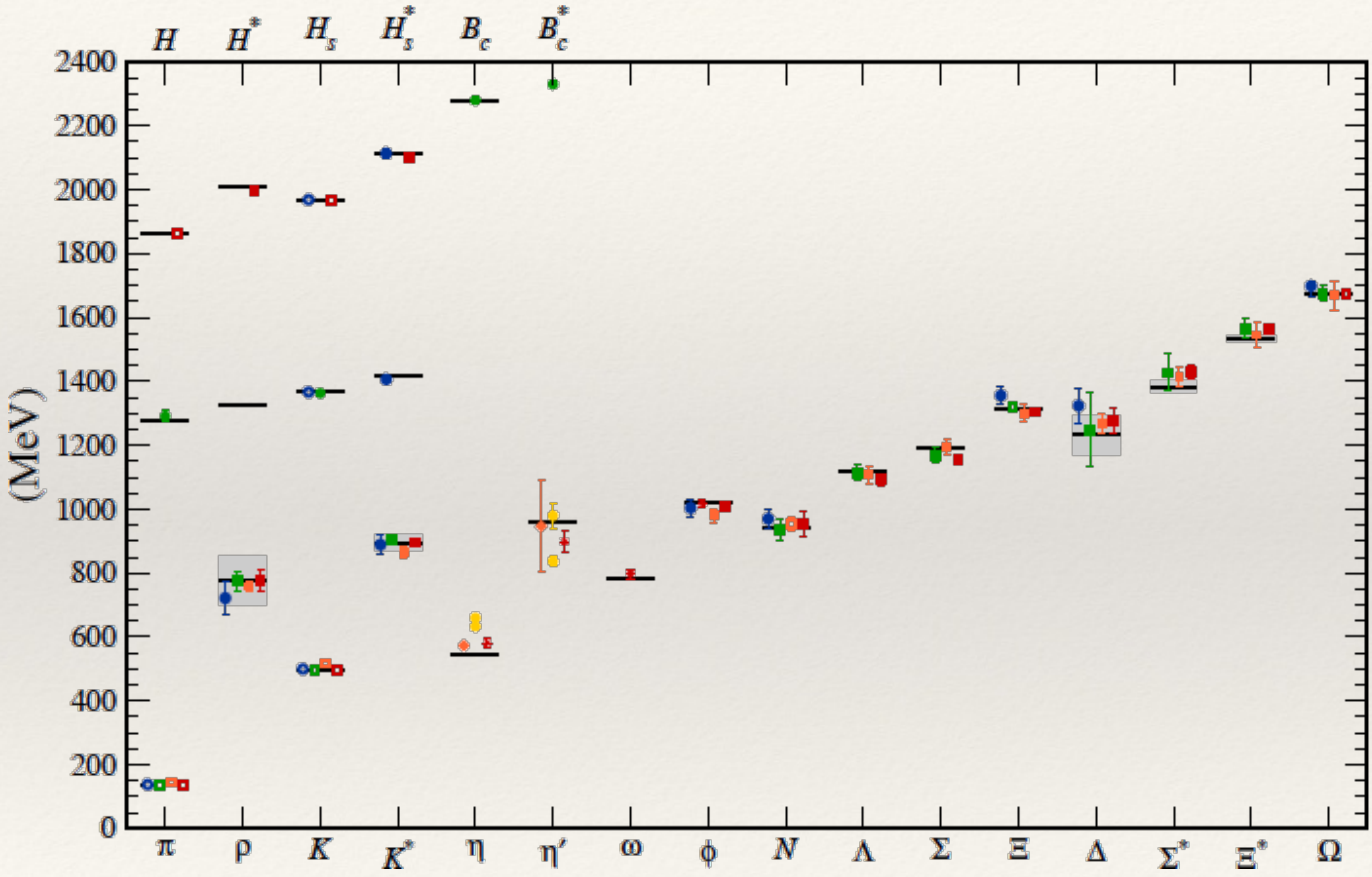
- project on the correct state (large set of bilinear operators)
- get a signal (→GEVP, need large statistics,
  most existing data for very *heavy pions*)
- deal with plethora of Wick contractions
- scattering in finite volume is hard (Lüscher Nucl.Phys. B354 (1991) 531-578)

What can be done

- precision: below threshold (low-lying charmonium)
- near or above threshold: *single hadron approximation*
- beyond: hard but interesting

Lattice 2014: Hadron spectroscopy Sasa Prelovsek

Some spectra from LQCD vs. experiment



Kronfeld, Ann. Rev. of Nucl. Part. Sci 2012 62

Results for $|V_{ub}|$

- leptonic decays: experimental input for $B \to \tau\nu_\tau$ from Belle and Babar → there is a tension:

	BaBar	Belle
BR(	*1.79(48)*	*0.96(26)*
|	*3.87(52)(9)*	*5.28(71)(12)*

FLAG combines this to $|V_{ub}|=4.18(52)(9)\times 10^{-3}$

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- semileptonic decays:
simultaneous analysis of lattice, Belle and BaBar results (here N_f=2+1 lattice input)

	BaBar	Belle
|	3.37(21)	3.47(22)

no FLAG average due to unknown exp. correlations

Heavy quark treatment:

✓ RHQ (tl $O(a)$ improved)

NRQCD (tl matched $O(1/m)$
improved through $O(a^2)$)

HQET (including 1/m and leading
cutoff effects at $O(a^2)$)

standard lattice actions
($O(a)$-improved)

★ ○ ■

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Continuum extrapolation:

★ ≥3 lattice spacings

& $a^2_{\mathrm{max}}/a^2_{\mathrm{min}} \geq 2$

& $D(a_{\mathrm{min}}) \leq 2\%$

& $\delta(a_{\mathrm{min}}) \leq 1$

○ two or more lattice spacings

& $a^2_{\mathrm{max}}/a^2_{\mathrm{min}} \geq 1.4$

& $D(a_{\mathrm{min}}) \leq 10\%$

& $\delta(a_{\mathrm{min}}) \leq 2$

■ otherwise

★ ○ ■

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○ two or more lattice spacings $\& \; a^2_{\rm max}/a^2_{\rm min} \geq 1.4$ $\& \; D(a_{\rm min}) \leq 10\%$ $\& \; \delta(a_{\rm min}) \leq 2$

■ otherwise

$D(a)$ relative difference between fintest lattice data and continuum limit

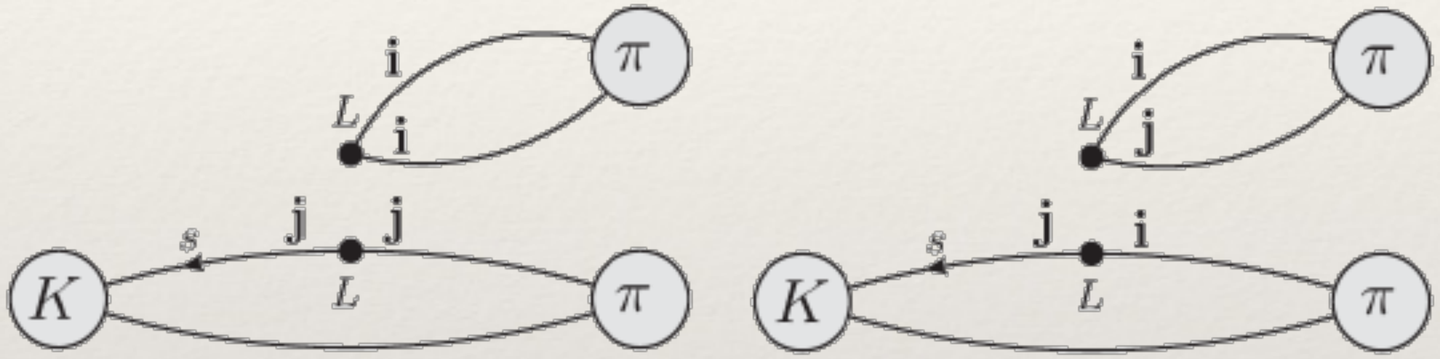
$\delta(a)$ deviation of fintest lattice data relative to the statistical and sysetmatic uncertainty of the calculation

Emerging understanding of the $\Delta I = \frac{1}{2}$ rule

- results of exploratory studies *RBC/UKQCD PRL 110 (2013)*:

  $m_K = 877\text{MeV} \quad m_\pi = 422\text{MeV}: \quad \frac{\text{Re}A_0}{\text{Re}A_2} = 9.1(2.1)$

  $m_K = 622\text{MeV} \quad m_\pi = 329\text{MeV}: \quad \frac{\text{Re}A_0}{\text{Re}A_2} = 12.0(1.7)$
- still some way to go but optimistic
- qualitative observations for $\text{Re}A_0/\text{Re}A_2$:
  - dominant contribution from contractions



Contraction ①. Contraction ②.

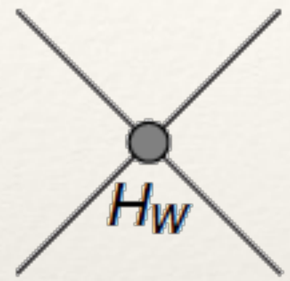
  - observation: (1)≈-(2)
  - $\text{Re}A_2 \propto$ (1)+(2) $\rightarrow$ suppression
  - $\text{Re}A_0 \propto$ 2(1)-(2) $\rightarrow$ enhancement
  - naively from color counting: (2)=$\frac{1}{3}$ (1)

Authors believe that this is a major factor for explaining the $\Delta I = 1/2$ rule from first principles

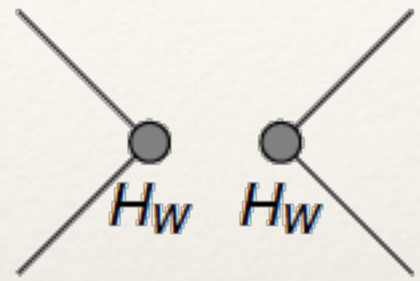
needs to be confirmed for physical kinematics and at the physical point

short vs. long distance

- mass difference ($\Delta S = 2$) receives short- and long-distance contributions
  Buras, Guadagonli, Isidori PLB 688 (2010)



single 4-quark OP
length scale $O(10^{-18}\text{m})$



two 4-quark OP
length scale $O(1/\Lambda_{\text{QCD}})$

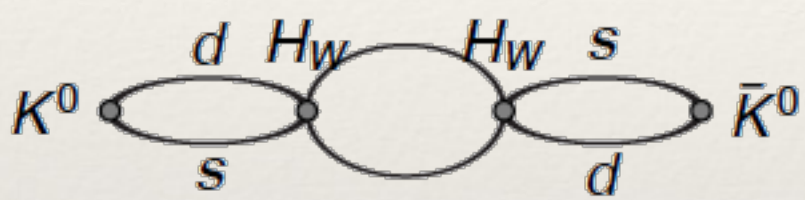
- $K_L - K_S$ mass difference $\Delta M_K = 3.486(6) \cdot 10^{-12}\text{MeV}$
- ΔM_K might be sensitive to new physics
- short-distance contributions $O(70\%)$ in NNLO ptQCD *Brod, Gorbahn, PRL 108 (2012)*
- rest (30%) SM long distance?
- convergence issues of PT? NNLO at m_c *Brod, Gorbahn, PRL 108 (2012)*
- lattice simulations with dynamical charm could shed ligth on this

long distance effects

efforts under way for long distance contribution

Christ, Izubuchi, Sachrajda, Soni, Yu arXiv:1212.5931

- ΔM_K long distance $\propto$ correlator involving two insertions of the weak Hamiltonian



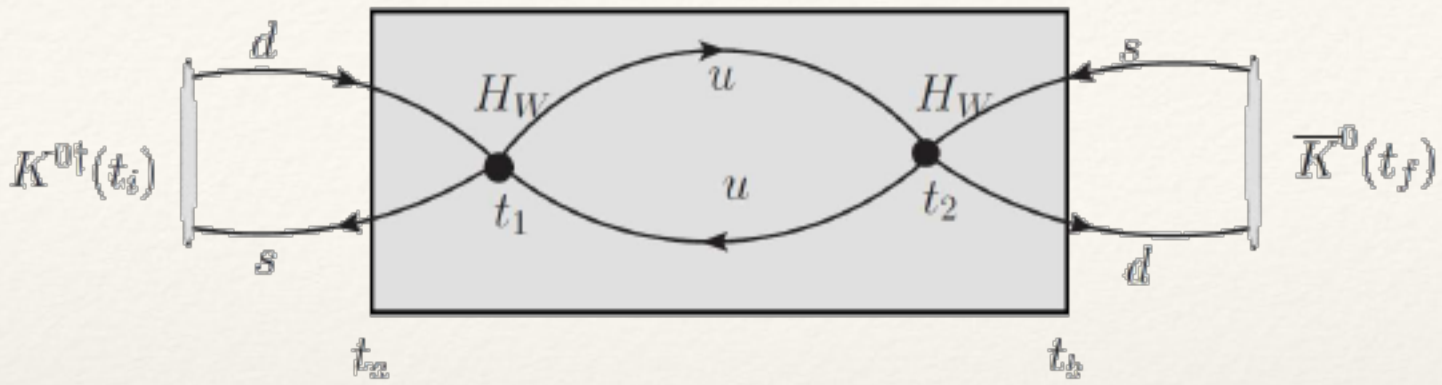
$$\int d^4x \int d^4y \langle h_2 | T\{H_W(x) H_W(y)| h_1\rangle$$

- lattice computation seems possible in principle
- no quadratic divergence in integration due to GIM in the presence of charm $N_f = 2 + 1 + 1$
- Lellouch-Lüscher type correction for finite volume effects in case where $\pi\pi$-state degenerate with K

applications: ϵ_K, rare decays

long distance effects

Christ, Izubuchi, Sachrajda, Soni, Yu arXiv:1212.5931



$$\mathcal{A} = N_K^2 e^{-M_K(t_f-t_i)} \left\{ \sum_{n\neq n_0} \frac{\langle \bar{K}^0|H_W|n\rangle\langle n|H_W|K^0\rangle}{M_K-E_n} \left(T + \frac{e^{(M_K-E_n)T}-1}{M_K-E_n} \right) + \frac{1}{2}\langle \bar{K}^0|H_W|n_0\rangle\langle n_0|H_W|K^0\rangle T^2 \right\} \quad (T = t_b - t_a + 1)$$

- term $\propto T \to$ signal
- constant term can be ignored
- exponentially decreasing ($E_n > M_K$) term negligible for large T
- exponentially increasing ($E_n < M_K$) term needs to be subtracted
- term $\propto T^2$ from $E_{n_0} = M_K$ needs to be removed in order to control finite volume effects Christ arXiv:1301.4239

Spectrum

- “precision spectrum” ($a \to 0$ and $L \to \infty$ limits taken) - far below strong threshold J/Ψ, $M_{\eta c}$
- “non-precision spectrum” ($a \to 0$ and $L \to \infty$ limits usually not taken) - near or above threshold, single hadron approximation (wacky)

beyond sinlge hadron approx: