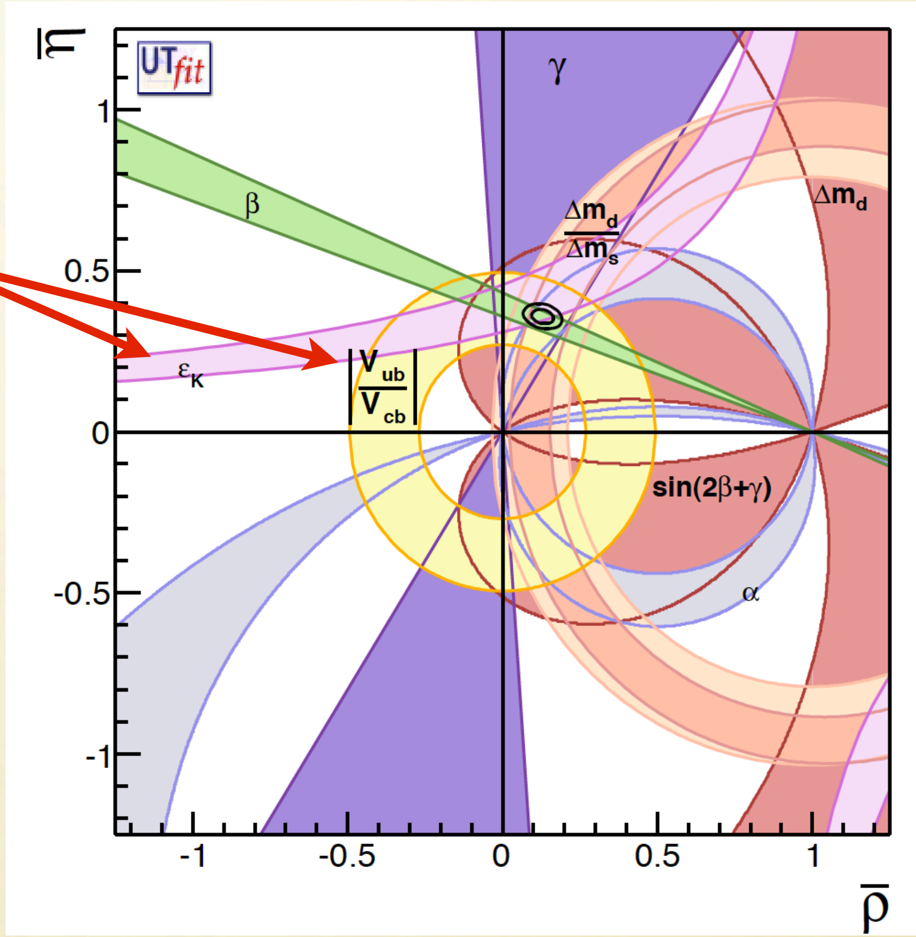
V_{cb} FROM INCLUSIVE SEMILEPTONIC B DECAYS

PAOLO GAMBINO
UNIVERSITÀ DI TORINO & INFN

CKM 2014, Vienna, 8/9/2014

Importance of $|V_{cb}|$

V_{cb} plays an important role in the determination of UT

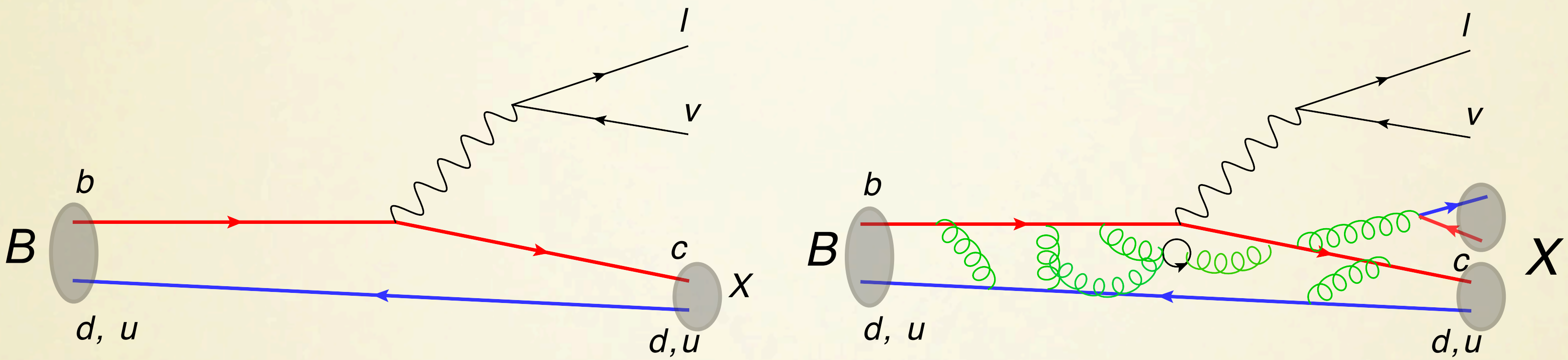


and in the prediction of FCNC:

$$\propto |V_{tb}V_{ts}|^2 \simeq |V_{cb}|^2\left[1+O(\lambda^2)\right]$$

Since several years, exclusive decays prefer smaller $|V_{ub}|$ and $|V_{cb}|$

Inclusive decays: basics



- ***Simple idea:*** inclusive decays do not depend on final state, long distance dynamics of the B meson factorizes. An OPE allows to express it in terms of B meson matrix elements of local operators

- The Wilson coefficients are perturbative, matrix elements of local ops parameterize non-pert physics: ***double series in α_s, Λ/m_b***

- Lowest order: decay of a free *b*, linear Λ/m_b absent. Depends on $m_{b,c}$, 2 parameters at $O(1/m_b^2)$, 2 more at $O(1/m_b^3)$...

$$\mu_\pi^2(\mu) = \frac{1}{2M_B}\left\langle B\left|\bar{b}\left(i\vec{D}\right)^2 b\right|B\right\rangle_\mu \qquad \mu_G^2(\mu) = \frac{1}{2M_B}\left\langle B\left|\bar{b}\frac{i}{2}\sigma_{\mu\nu}G^{\mu\nu}b\right|B\right\rangle_\mu$$

OBSERVABLES IN THE OPE

$$
\begin{aligned}
M = M_0 \Big[& 1 + c_1(r)\frac{\alpha_s}{\pi} + c_2(r)\frac{\alpha_s^2}{\pi^2} \\
& - \frac{\mu_\pi^2}{2m_b^2}\left(1 + c_\pi^{(1)}(r)\frac{\alpha_s}{\pi}\right) \\
& + \frac{\mu_G^2}{m_b^2}\left(c_G^{(0)}(r) + c_G^{(1)}(r)\frac{\alpha_s}{\pi}\right) \\
& + c_D(r)\frac{\rho_D^3}{m_b^3} + c_{LS}(r)\frac{\rho_{LS}^3}{m_b^3} \\
& + O\left(\alpha_s^3, \alpha_s^2\frac{\Lambda^2}{m_b^2}, \alpha_s\frac{\Lambda^3}{m_b^3}, \frac{\Lambda^4}{m_b^4}\right)\Big]
\end{aligned}
$$

NEW

$$r = \frac{m_c^2}{m_b^2}$$

OPE valid for inclusive enough measurements, away from perturbative singularities ➠ semileptonic width, moments

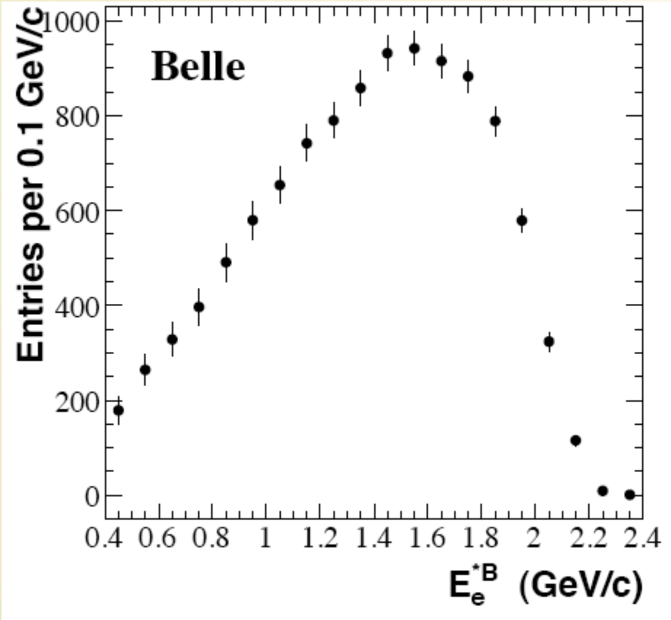
The fit presented here includes 6 non-pert parameters

$m_{b,c}, \quad \mu^2_{\pi,G}, \quad \rho^3_{D,LS}$

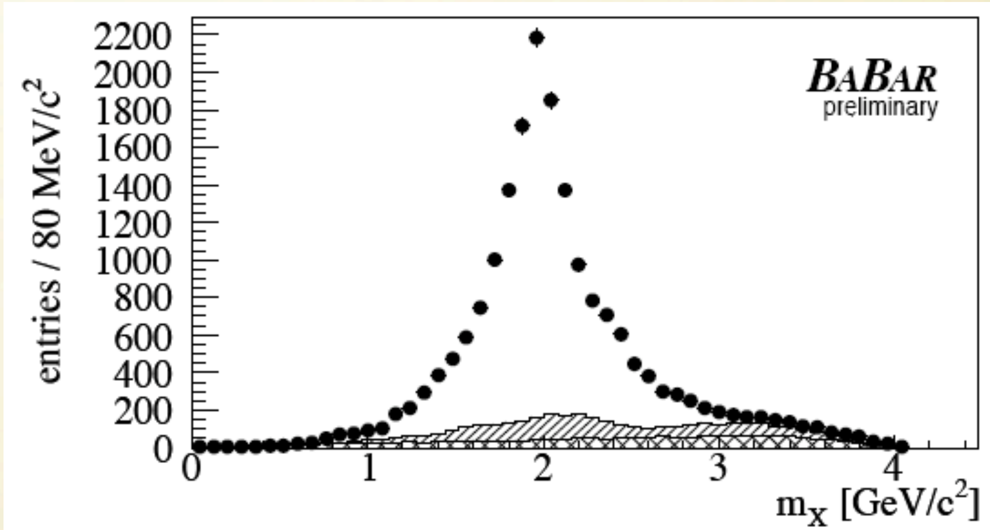
and all known corrections up to $O(\Lambda^3/m_b^3)$

Extraction of the OPE parameters

E_l spectrum



m_X spectrum



Global ***shape*** *parameters (first moments of the distributions) tell us about B structure, m_b and m_c, total* ***rate*** *about $|V_{cb}|$*

OPE parameters describe universal properties of the B meson and of the quarks → useful in many applications (rare decays, V_{ub},...)

Let's focus on:

1. Status of higher order corrections
2. Estimate of residual theoretical errors
3. Additional constraints in the fits

HIGHER ORDER EFFECTS

- Reliability of the method depends on our ability to control higher order effect and quark-hadron duality violations.

- **Purely perturbative corrections** complete at NNLO, small residual error Melnikov, Biswas, Czarnecki, Pak, PG

- **Higher power corrections** $O(1/m_Q^{4,5})$ known

  Mannel,Turczyk,Uraltsev 2010

- **Mixed corrections** perturbative corrections to power suppressed coefficients completed at $O(\alpha_s/m_b^2)$
  Becher, Boos, Lunghi, Alberti, Ewerth, Nandi, PG

Higher power corrections

Mannel,Turczyk,Uraltsev **1009.4622**

Proliferation of non-pert parameters and powers of $1/m_c$ starting $1/m^5$. At $1/m_b^4$

$$2M_B m_1 = \langle ((\vec{p})^2)^2 \rangle$$
$$2M_B m_2 = g^2 \langle \vec{E}^2 \rangle$$
$$2M_B m_3 = g^2 \langle \vec{B}^2 \rangle$$
$$2M_B m_4 = g \langle \vec{p} \cdot \mathrm{rot}\, \vec{B} \rangle$$

$$2M_B m_5 = g^2 \langle \vec{S} \cdot (\vec{E} \times \vec{E}) \rangle$$
$$2M_B m_6 = g^2 \langle \vec{S} \cdot (\vec{B} \times \vec{B}) \rangle$$
$$2M_B m_7 = g \langle (\vec{S} \cdot \vec{p})(\vec{p} \cdot \vec{B}) \rangle$$
$$2M_B m_8 = g \langle (\vec{S} \cdot \vec{B})(\vec{p})^2 \rangle$$
$$2M_B m_9 = g \langle \Delta(\vec{\sigma} \cdot \vec{B}) \rangle$$

can be estimated by **Lowest Lying State Saturation** approx by truncating

$$\langle B|O_1 O_2|B\rangle = \sum_n \langle B|O_1|n\rangle\langle n|O_2|B\rangle$$

In LLSA ***good convergence*** of the HQE. First fit with $1/m^{4,5}$:

$$\frac{\delta V_{cb}}{V_{cb}} \simeq -0.35\%$$

Turczyk,PG preliminary

NEW: Heinonen,Mannel 1407.4384 more systematic, discrepancies to be clarified

LLSA might set the scale of effect, not yet clear *how much it depends on assumptions on expectation values.* Large corrections to LLSA have been found.

Mannel, Uraltsev, PG, 2012

Allowing 80% gaussian deviations from LLSA seem to leave V_{cb} unaffected.

$O(\alpha_s/m_b^2)$ EFFECTS

Boos,Becher,Lunghi 2007
Ewerth,Nandi, PG 2009
Alberti,Ewerth,Nandi,PG 2012
Alberti,Nandi,PG 2013

Hadronic tensor
$$W^{\alpha\beta} = \frac{(2\pi)^3}{2m_B} \sum_{X_c} \delta^4(p_b - q - p_X) \langle \bar{B} | J_L^{\dagger\alpha} | X_c \rangle \langle X_c | J_L^\beta | \bar{B} \rangle$$

$$m_b W^{\alpha\beta} = -W_1 g^{\alpha\beta} + W_2 v^\alpha v^\beta + i W_3 \epsilon^{\alpha\beta\rho\sigma} v_\rho \hat{q}_\sigma + W_4 \hat{q}^\alpha \hat{q}^\beta + W_5 (v^\alpha \hat{q}^\beta + v^\beta \hat{q}^\beta)$$

$$W_i = W_i^{(0)} + \frac{\mu_\pi^2}{2m_b^2} W_i^{(\pi,0)} + \frac{\mu_G^2}{2m_b^2} W_i^{(G,0)} + \frac{C_F \alpha_s}{\pi} \left[W_i^{(1)} + \frac{\mu_\pi^2}{2m_b^2} W_i^{(\pi,1)} + \frac{\mu_G^2}{2m_b^2} W_i^{(G,1)} \right]$$

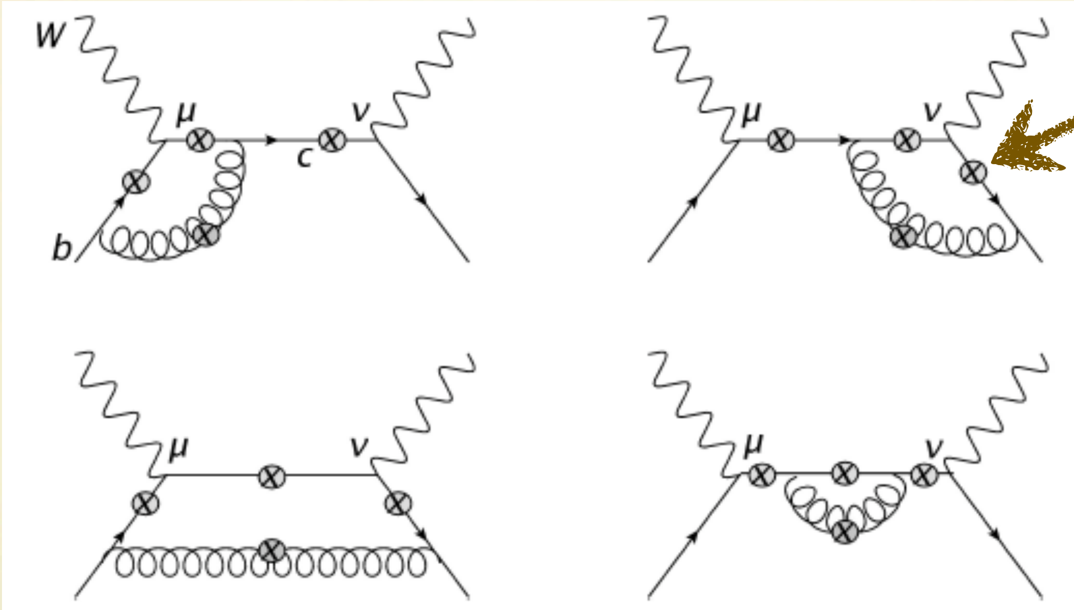
$W_i^{(\pi,n)}$ can be computed using **reparameterization invariance** which relates different orders in the HQET: e.g. for $i=3$ at all orders

$$W_3^{(\pi,n)} = \frac{5}{3} \hat{q}_0 \frac{dW_3^{(n)}}{d\hat{q}_0} - \frac{\hat{q}^2 - \hat{q}_0^2}{3} \frac{d^2 W_3^{(n)}}{d\hat{q}_0^2}$$

Manohar 2010

Proliferation of power divergences, up to $1/u^3$,
and complex kinematics (q^2, q_0, m_c, m_b) $W_i^{(G,1)}$ requires proper matching.

Matching at $O(\alpha_s)$



possible gluon insertions

QCD

HQET

$$\frac{2i}{\pi}\int d^4x\, e^{-iq\cdot x}\, T[J_L^{\dagger\mu}(x) J_L^{\nu}(0)] = \sum_i c^{(i)\mu\nu}_{\{\alpha\}}(v,q)\, O_i^{\{\alpha\}}(0)$$

Taylor expansion around on-shell b quark matched onto HQET local operators. Analytic formulae. RPI relations reproduced. Unlike μ_π, μ_G gets renormalized, therefore Wilson coefficients scale-dependent.

Numerical results

In on-shell scheme (m_b=*4.6*GeV, m_c=*1.15*GeV) without cuts

$$\Gamma_{B\to X_c\ell\nu} = \Gamma_0 \left[\left(1 - 1.78\,\frac{\alpha_s}{\pi}\right)\left(1 - \frac{\mu_\pi^2}{2m_b^2}\right) - \left(1.94 + 2.42\,\frac{\alpha_s}{\pi}\right)\frac{\mu_G^2(m_b)}{m_b^2}\right]$$

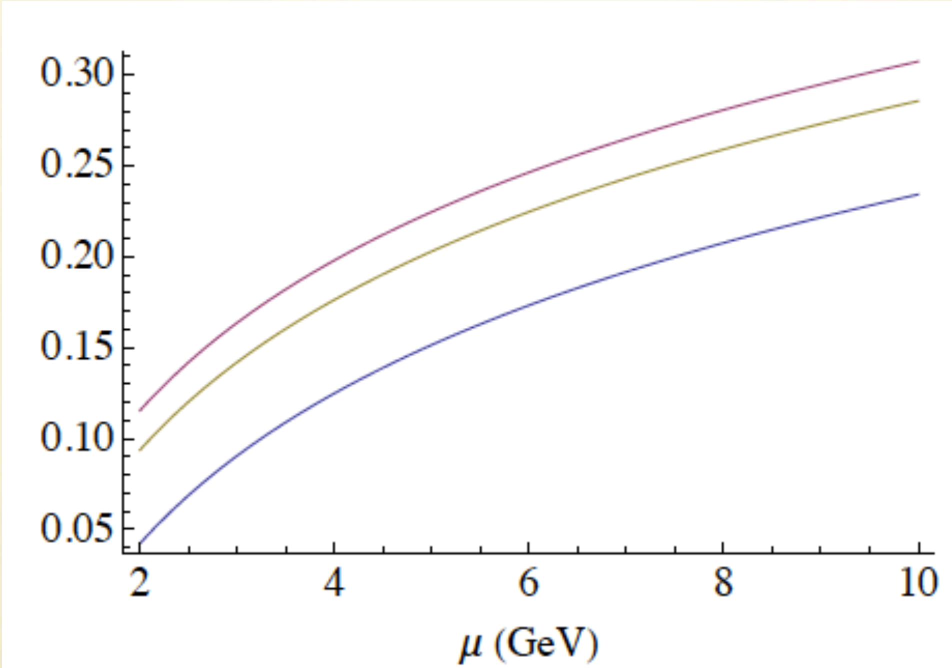
$$\langle E_\ell \rangle = 1.41\text{GeV}\left[\left(1 - 0.02\,\frac{\alpha_s}{\pi}\right)\left(1 + \frac{\mu_\pi^2}{2m_b^2}\right) - \left(1.19 + 4.20\,\frac{\alpha_s}{\pi}\right)\frac{\mu_G^2(m_b)}{m_b^2}\right]$$

$$\ell_2 = 0.183\,\text{GeV}^2\left[1 - 0.16\,\frac{\alpha_s}{\pi} + \left(4.98 - 0.37\,\frac{\alpha_s}{\pi}\right)\frac{\mu_\pi^2}{m_b^2} - \left(2.89 + 8.44\,\frac{\alpha_s}{\pi}\right)\frac{\mu_G^2(m_b)}{m_b^2}\right]$$

Similar results in the kinetic scheme. NLO corrections generally *O(15-20%)* of tree level coefficients, **shifts in some cases larger than experimental error**. Impact on V_{cb} requires new fit of semileptonic moments.

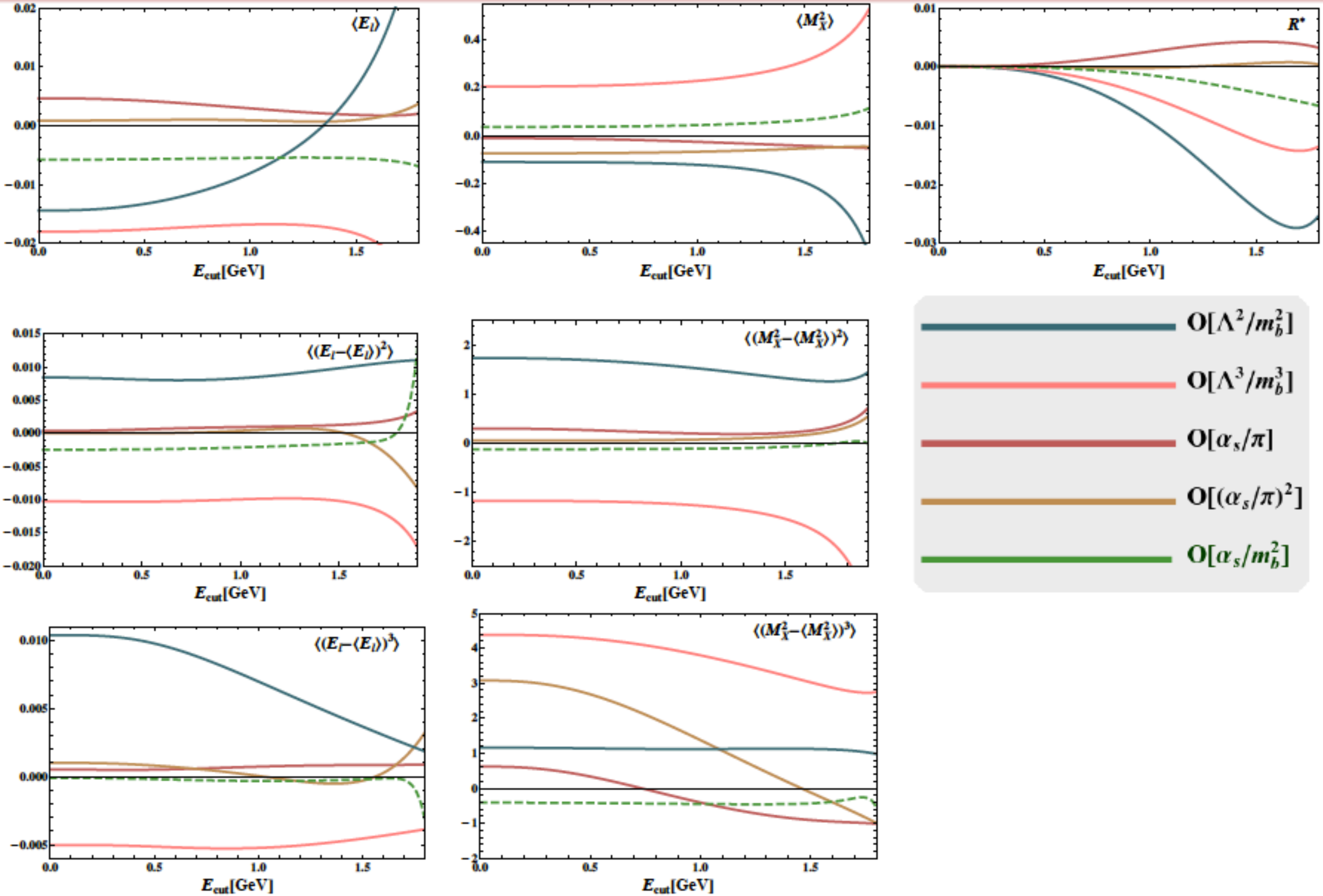
Mannel, Pivovarov, Rosenthal (1405.5072) have computed the μ_G correction to the width in the limit m_c=0 and find compatible result.

μ_G^2-SCALE DEPENDENCE

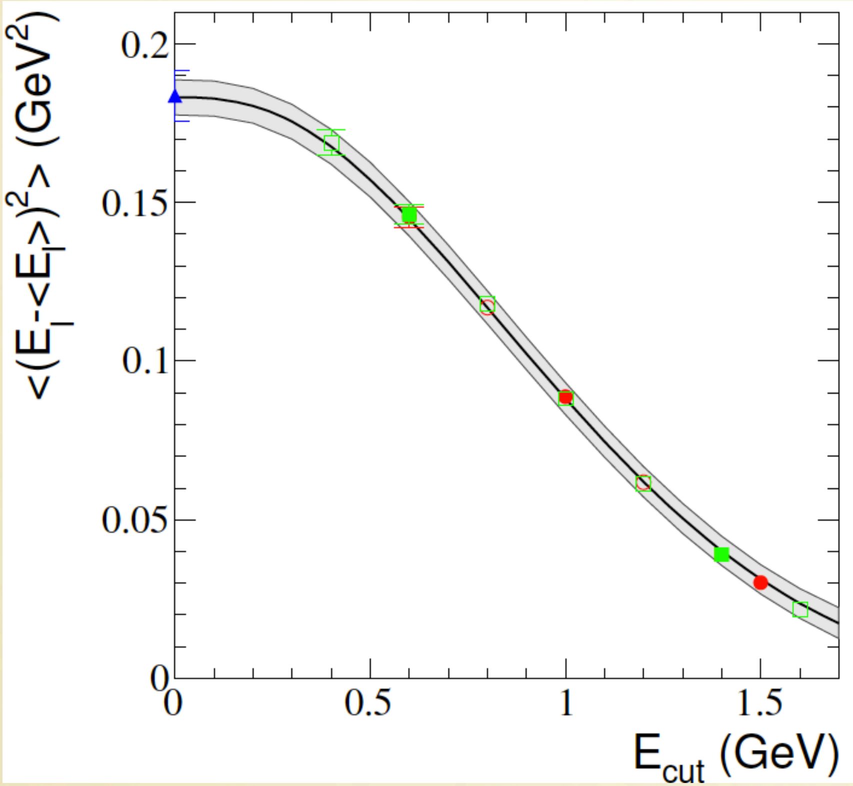


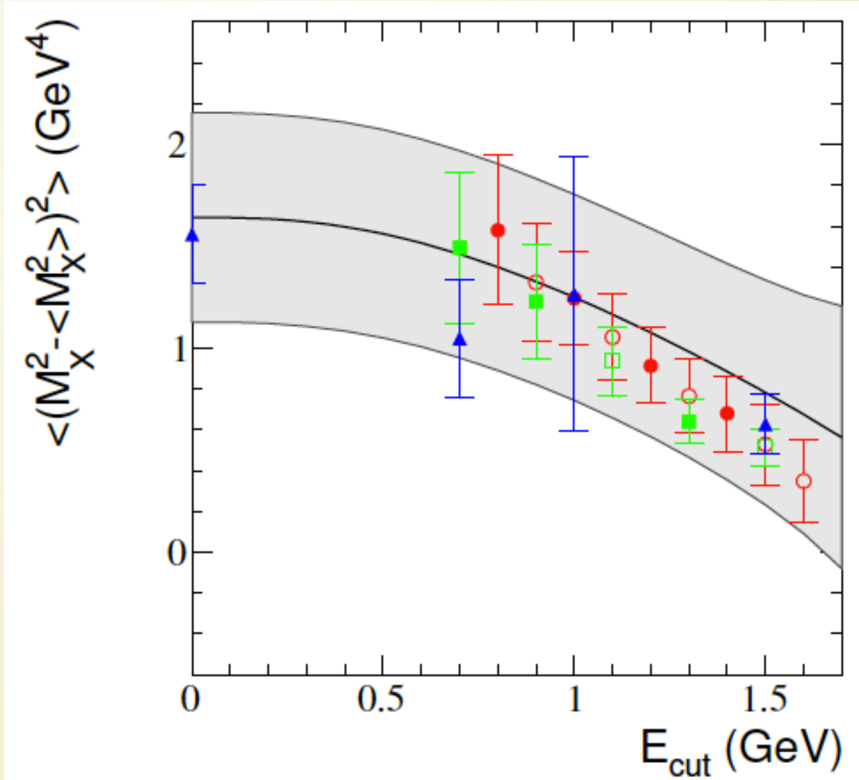
Relative NLO correction to the coefficients of μ_G in the width (blue), first (red) and second central (yellow) leptonic moments as a function of the renormalization scale. Smaller corrections for smaller scale.

New Contributions $\mathcal{O}(\alpha_s/m_b^2)$: R



Theoretical errors

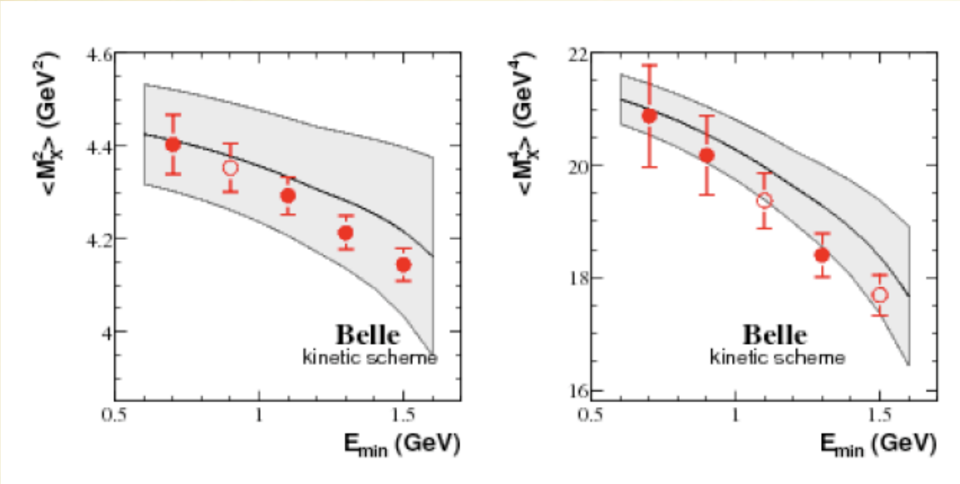


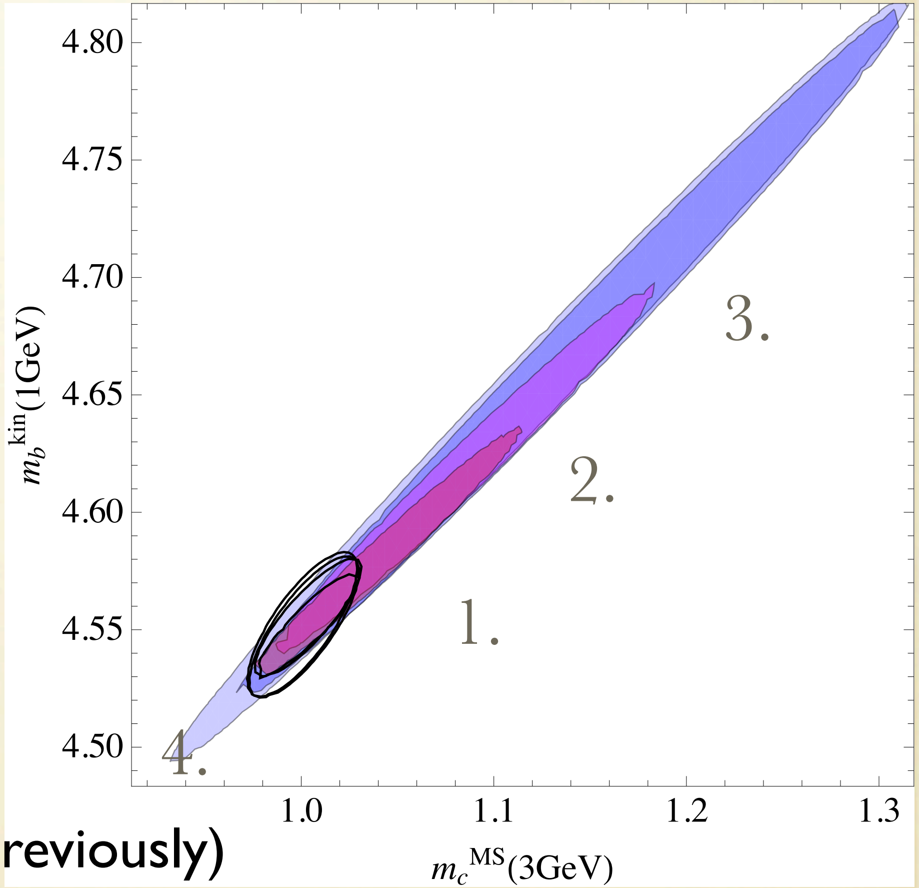


Theoretical errors are generally the **dominant** ones in the fits.
We estimate them in a **conservative** way by mimicking higher orders varying the parameters by fixed amounts.

Duality violation, expected to be suppressed, would manifest as inconsistency in the fit.

Theoretical correlations



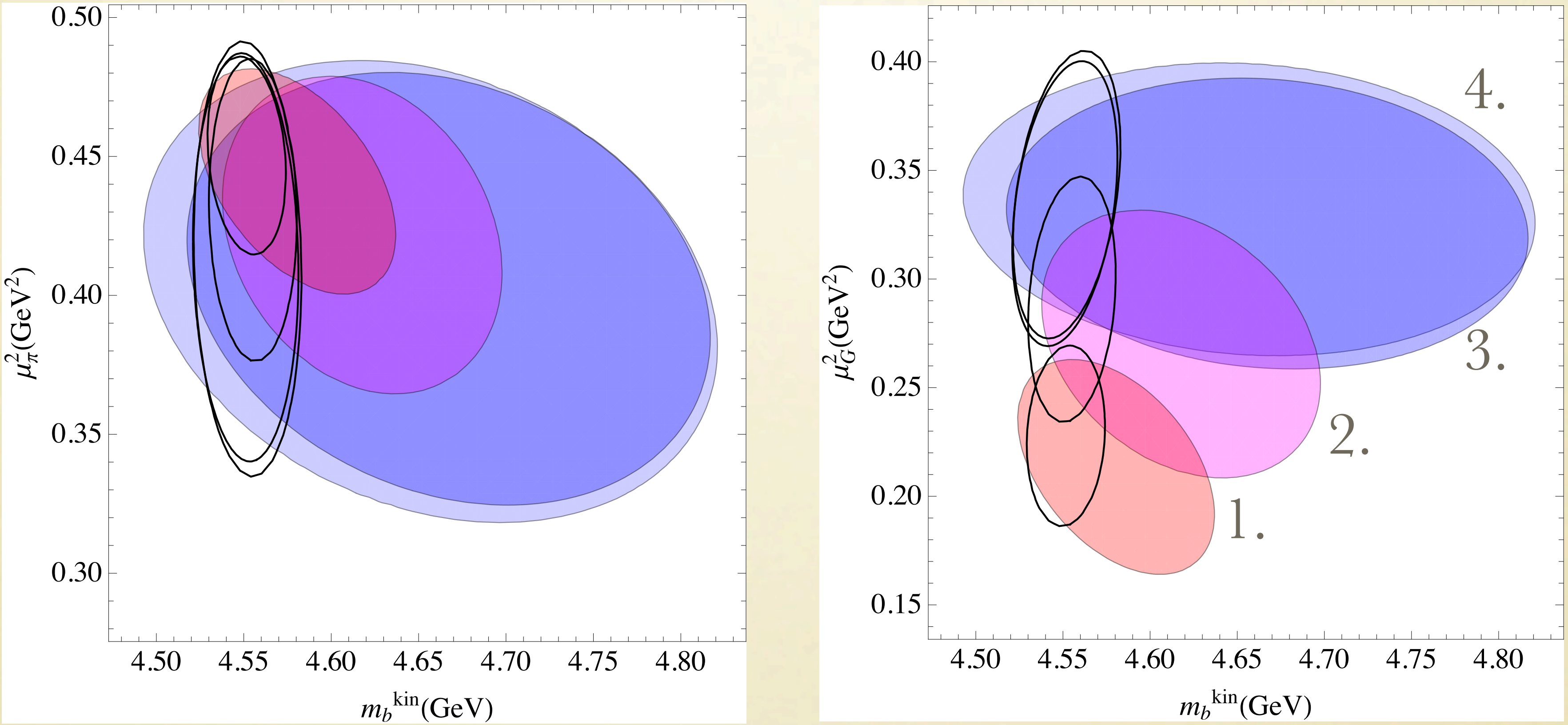


Correlations between theory errors of moments with different cuts difficult to estimate

1. 100% correlations (unrealistic but used previously)
2. corr. computed from low-order expressions
3. constant factor 0<ξ<1 for 100MeV step
4. same as 3. but larger for larger cuts

always assume different central moments uncorrelated
1. and 2. are strongly disfavored when new corrections are included

Theoretical correlations



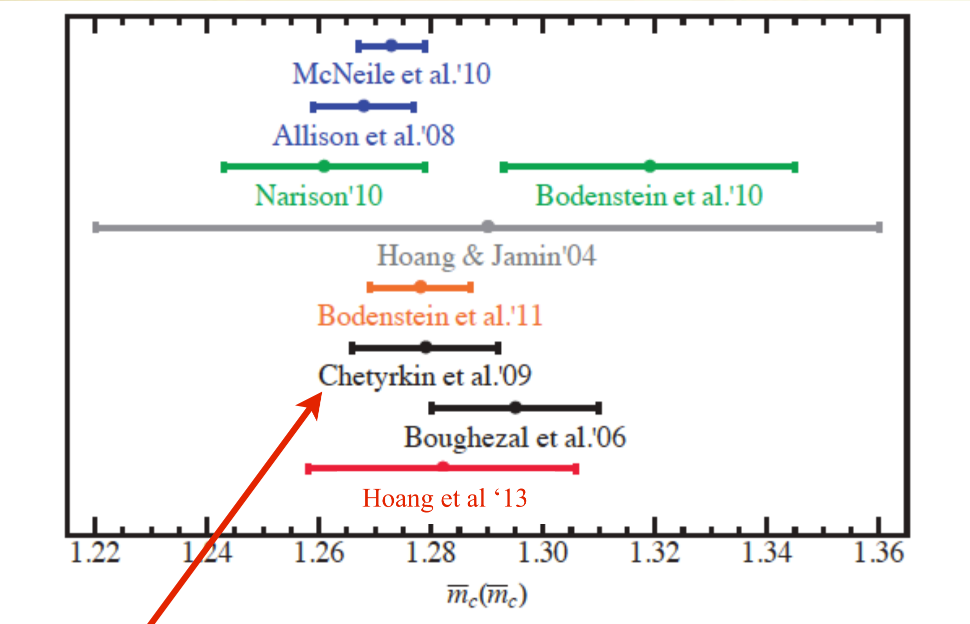
New semileptonic fit

Alberti, Healey, Nandi, PG

- updates the fit in Schwanda, PG, 1307.4551
- kinetic scheme calculation based on 1107.3100; hep-ph/0401063
- includes all $O(\alpha_s^2)$ corrections Czarnecki, Pak, Melnikov, Biswas, PG
- reassessment of theoretical errors, realistic correlations
- external constraints: precise heavy quark mass determinations, plus mild constraints on μ^2_G from hyperfine splitting and ρ^3_{LS} from sum rules

Previous fits: Buchmuller, Flaecher hep-ph/0507253,
Bauer et al, hep-ph/0408002 (1S scheme)

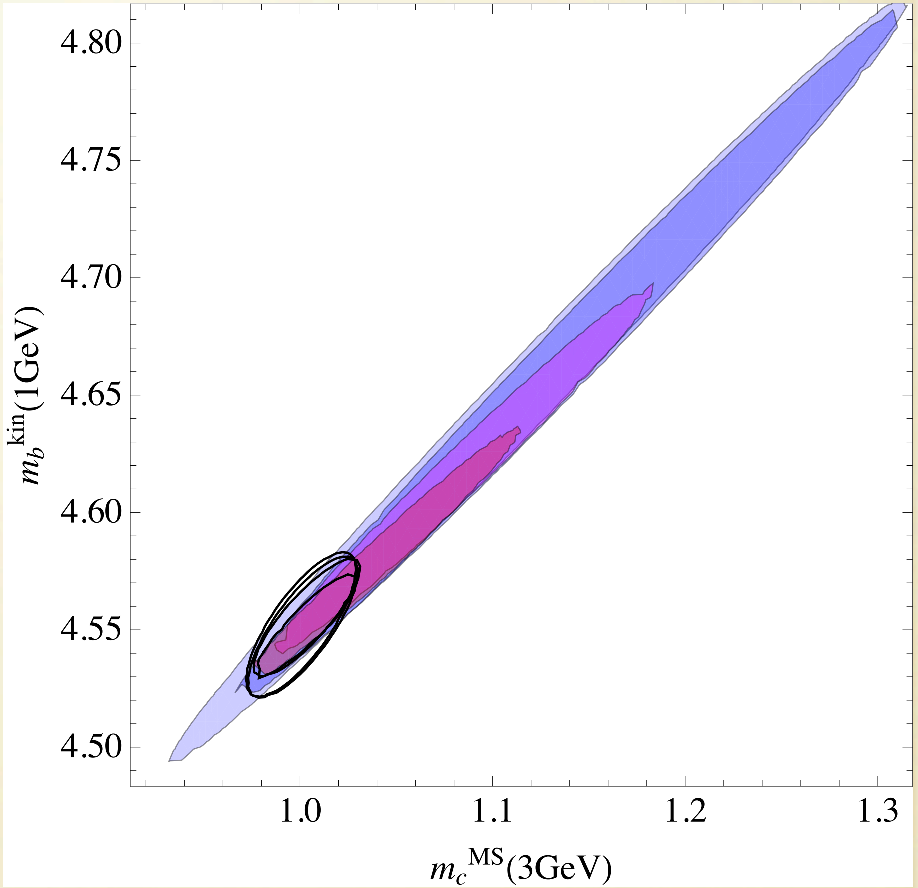
CHARM MASS DETERMINATIONS



our default choice

sum rules studies of $\sigma(e^+e^- \to \text{hadrons})$

almost all at NNNLO



Remarkable improvement in recent years.

m_c can be used as precise input to fix m_b instead of radiative moments

PRELIMINARY RESULTS

NEW

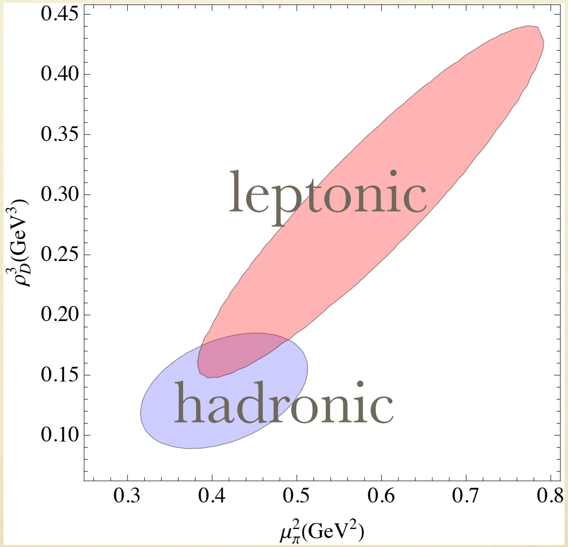
th corr scenario	m_b^{kin}	m_c (3GeV)	μ_π	ρ_D	μ_G	ρ_{LS}	BR(%)	$10^3 V_{cb}$
4.	4.539	0.988	0.454	0.149	0.296	-0.142	10.67	42.41
uncertainty	0.021	0.013	0.077	0.044	0.063	0.097	0.16	0.83

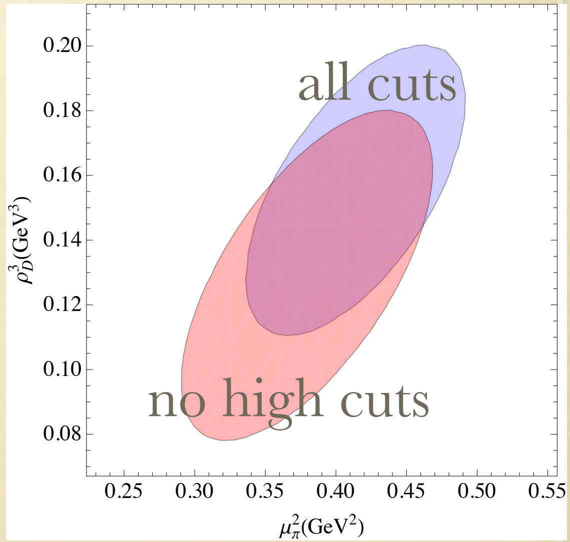
Schwanda PG 2013

th. corr. scenario	m_b^{kin}	$m_c^{(3GeV)}$	μ_π^2	ρ_D^3	μ_G^2	ρ_{LS}^3	$BR_{c\ell\nu}(\%)$	$10^3\,\lvert V_{cb}\rvert$
4.	4.541	0.987	0.414	0.154	0.340	-0.147	10.65	42.42
uncertainty	0.023	0.013	0.078	0.045	0.066	0.098	0.16	0.86

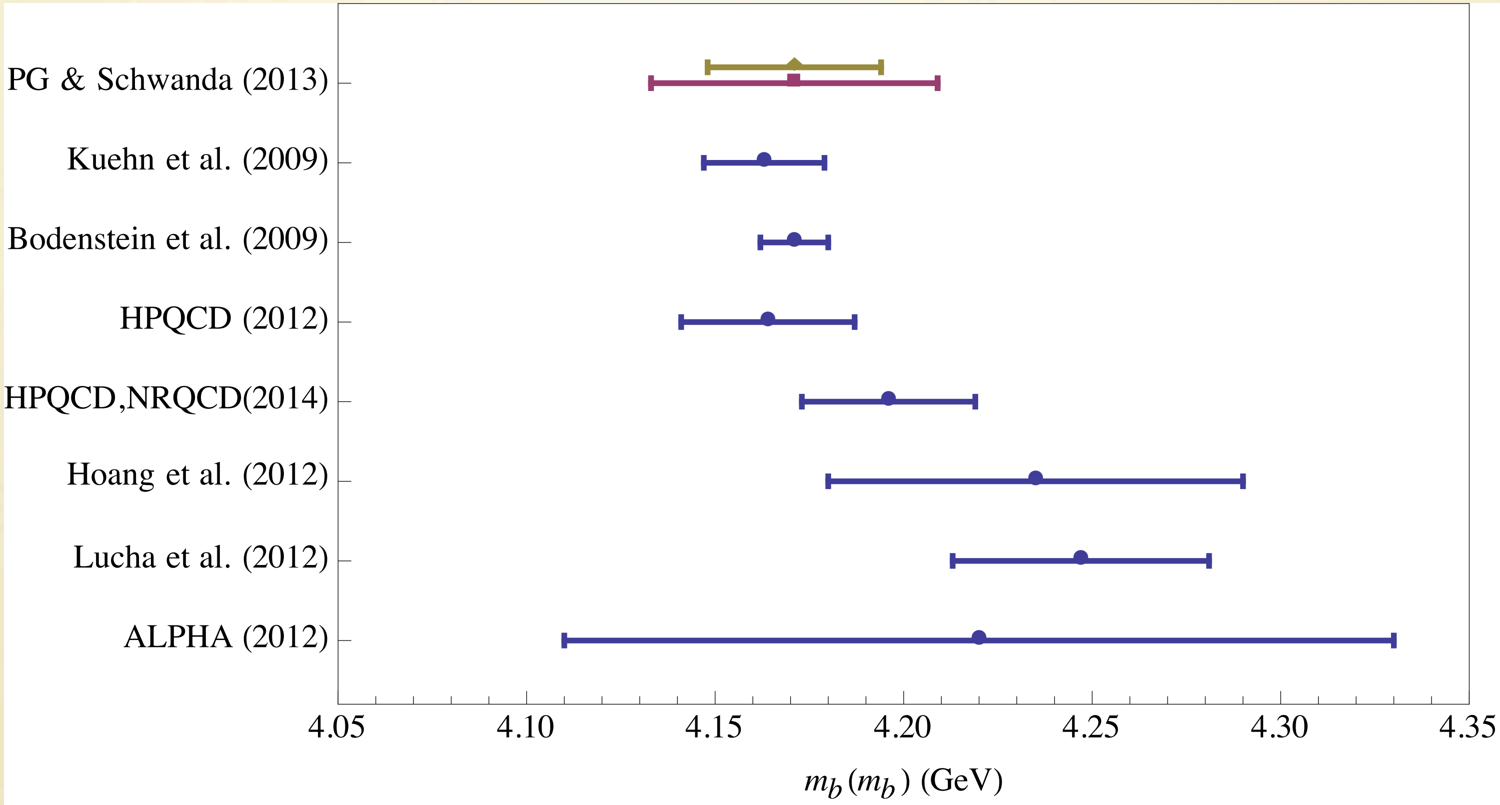
Without mass constraints $\quad m_b^{kin}(1\,\mathrm{GeV}) - 0.85\,\overline{m}_c(3\,\mathrm{GeV}) = 3.701 \pm 0.019\,\mathrm{GeV}$

- results depend little on assumption for correlations and choice of inputs, 2% determination of V_{cb}
- 20-30% determination of the OPE parameters





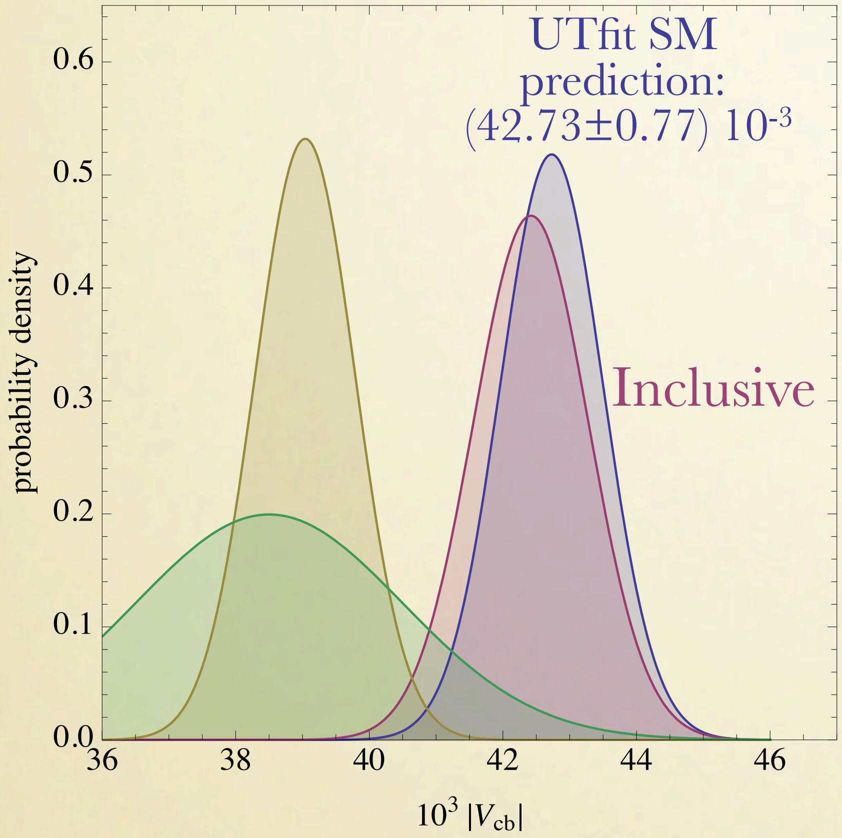
Results: bottom mass



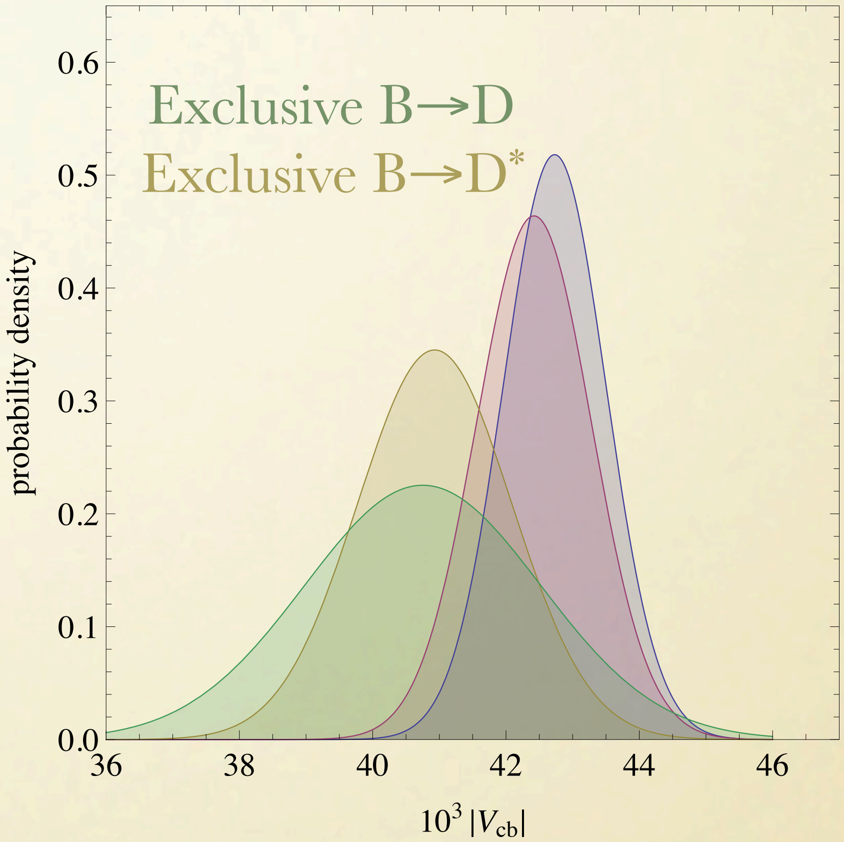
The fits give $\boldsymbol{m_b^{kin}}$**(1GeV)=4.539(21)GeV**, independent of th corr.

scheme translation error $m_b^{kin}(1\mathrm{GeV}) = m_b(m_b) + 0.37(3)\mathrm{GeV}$

V_{cb} VISUAL SUMMARY



Latest lattice results for exclusives (FNAL/MILC)



HQSR,HQE for exclusives Mannel, Uraltsev, PG

New physics?

The difference with FNAL/MILC is **quite large**: 3σ or about 8%. The perturbative corrections to inclusive total 5%, the power corrections about 4%.

Right Handed currents **disfavored** since

$$|V_{cb}|_{incl} \simeq |V_{cb}|\left(1 + \frac{1}{2}|\delta|^2\right)$$

$$|V_{cb}|_{B\to D^*} \simeq |V_{cb}|\left(1 - \delta\right)$$

$$|V_{cb}|_{B\to D} \simeq |V_{cb}|\left(1 + \delta\right)$$

$$\delta = \epsilon_R \frac{\tilde{V}_{cb}}{V_{cb}} \approx 0.08$$

Conclusions

- Theoretical efforts to improve the OPE approach to semileptonic decays continue. All effects $O(\alpha_s \Lambda^2/m_b^2)$ implemented. **No sign of inconsistency in this approach so far**. Calculation of $O(\alpha_s \Lambda^3/m_b^3)$ effects ongoing.
- Renewed activity on higher power corrections, unlikely to shift V_{cb} but need to be studied.
- **New fit** results: V_{cb} stable, competitive m_b determination based on precise m_c
- Exclusive/incl. tension in V_{cb} remains **large and mysterious** (3σ, 8%). It cannot be explained by right-handed current. Thorough investigations required at Belle-II.