FORM FACTORS FOR $B \to D, D^*, \ldots$ NON-LATTICE METHODS

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Lattice meets continuum, Siegen, 29/9/14

PLAN

- Main goals
- HQET relations
- Zero-recoil FF for $B \to D^*$ with HQSR
- Zero-recoil FF for $B \to D$ in HQE
- LCSR FFs, pQCD
- Interpolation formulas

Main goals

- V_{cb} determination possibly at a level competitive with lattice
- Tauonic decays: are $R(D)$, $R(D^*)$ 1-2-3?σ away from SM?
- Input for exclusive channel modelling in experiment, especially higher resonances, background estimates in precision V_{ub}, V_{cb} analyses

B→D and B→D* rates

$$\frac{d\Gamma^{B\to D\ell\bar{\nu}}}{dw} = \frac{G_F^2}{48\pi^3}|V_{cb}|^2(m_B+m_D)^2 m_D^3 (w^2-1)^{3/2}|\eta_{\rm ew}\mathcal{G}(w)|^2$$

w=v.v'

$$\frac{d\Gamma^{B\to D^*\ell\bar{\nu}}}{dw} = \frac{G_F^2 m_B^5}{48\pi^3}|V_{cb}|^2(w^2-1)^{1/2}(w+1)^2 r^3(1-r)^2 P(w)|\eta_{\rm ew}\mathcal{F}(w)|^2$$

$$P(w) = \left(1+\frac{4w}{w+1}\frac{1-2rw+r^2}{(1-r)^2}\right)$$

In <u>heavy quark limit</u> all FF related to a single Isgur-Wise function $\xi(w)$: $\mathcal{G}(w)=\mathcal{F}(w)$

$\xi(w)$ is then normalized at zero recoil, $\xi(1)=1$ *for all decays.*

Beyond the HM limit

- Beyond heavy mass limit: several subleading IW functions, corrections to normalization -- in some cases protected by Luke's theorem

  $\mathcal{F}(w)=1+O(1/m_Q^2)$, $\mathcal{G}(w)=1+O(1/m_Q)$ $m_Q=m_c,\ m_b$

- Since the '90s, a lot of work with QCD sum rules, quark models, mostly including $1/m_Q$ terms. Falk, Neubert, Ligeti, Nir...

- Unfortunately limited accuracy, unsuited to present & future situation

- While HQET provides essential guidance, higher **power corrections are not small** and must be accounted for reliably (combining with experimental/lattice data etc.)

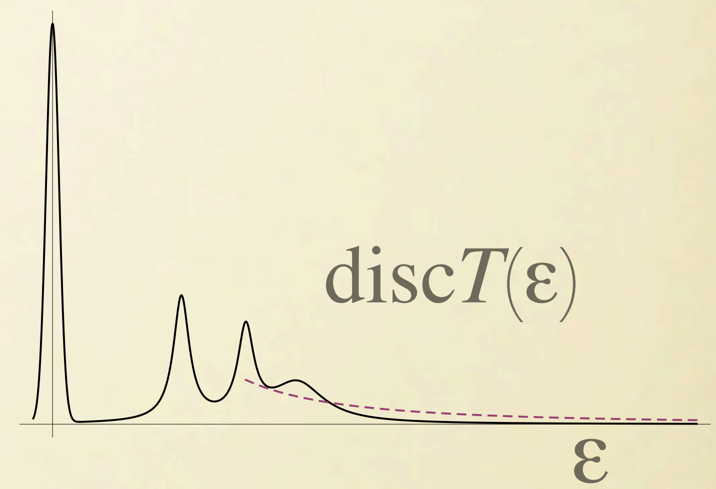
ZERO RECOIL SUM RULE

- Heavy quark sum rules put bounds on the zero recoil form factor F(1) for $B \to D^*$ Shifman,Vainshtein, Uraltsev 1996
- Recent calculation incorporates higher order effects and estimates inelastic contributions Mannel, Uraltsev, PG 2012
- Starting point OPE for axial vector current at zero recoil: expansion in $1/m_c$ and $1/m_b$
- Estimate of inelastic (non-resonant) contribution is the hard part

ZERO RECOIL SUM RULE

$$T(\varepsilon) = \frac{i}{6M_B} \int d^4x e^{-ix_0(M_B - M_{D^*} - \varepsilon)} \langle B | T J_A^k(x) J_{Ak}(0) | B \rangle$$

$$\varepsilon = M_X - M_{D^*}$$



$$I_0(\varepsilon_M) = -\frac{1}{2\pi i} \oint_{|\varepsilon| = \varepsilon_M} T(\varepsilon)\, d\varepsilon = \mathcal{F}^2(1) + I_{inel}(\varepsilon_M)$$

Inelastic non-resonant piece $I_{inel}(\varepsilon_M) = \frac{1}{2\pi i} \int_{0+}^{\varepsilon_M} \mathrm{disc}\, T(\varepsilon) d\varepsilon$

$$\mathcal{F}(1) = \sqrt{I_0(\varepsilon_M) - I_{inel}(\varepsilon_M)}$$

$$\mathcal{F}(1) \leq \sqrt{I_0(\varepsilon_M)}$$

Unitarity bound

OPE: Perturbative effects

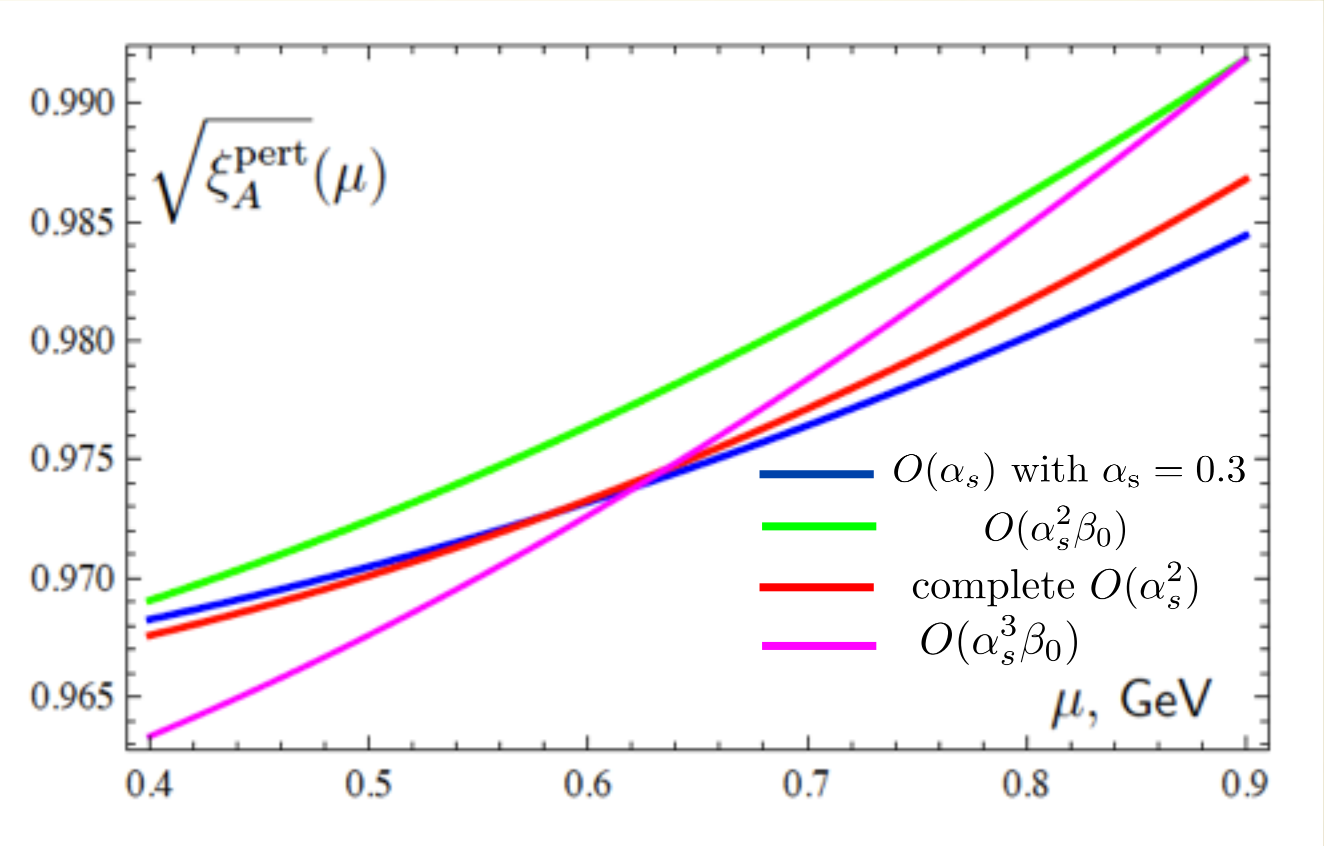
$$I_0(\varepsilon_M) \;=\; \xi_A^{\rm pert}(\varepsilon_M,\mu) + \sum_k C_k(\varepsilon_M,\mu)\,\frac{\frac{1}{2M_B}\langle B|O_k|B\rangle_\mu}{m_Q^{d_k-3}}$$

the cutoff μ separates pert and non-pert physics

Power corrections start with $1/m_c^2$ $\Lambda_{QCD} \ll \varepsilon_M,\, \mu \ll 2m_c$

We choose $\varepsilon_M = \mu = 0.75\text{GeV}$ and include 1, 2 loop and higher BLM corrections with no expansion in μ/m_c

$$\sqrt{\xi_A^{\rm pert}(0.75\text{GeV})} = 0.98 \pm 0.01$$



OPE: power corrections

$$\Delta_{1/m^2} = \frac{\mu_G^2}{3m_c^2} + \frac{\mu_\pi^2 - \mu_G^2}{4}\left(\frac{1}{m_c^2} + \frac{2}{3m_c m_b} + \frac{1}{m_b^2}\right),$$
$$\Delta_{1/m^3} = \frac{\rho_D^3 - \frac{1}{3}\rho_{LS}^3}{4m_c^3} + \frac{1}{12m_b}\left(\frac{1}{m_c^2} + \frac{1}{m_c m_b} + \frac{3}{m_b^2}\right)(\rho_D^3 + \rho_{LS}^3)$$

matrix elements from moments fits & sum rule constraints

$1/m^4$ and $1/m^5$ also known, matrix elements have been estimated by ground state saturation

Mannel, Turczyk, Uraltsev 2010
Heinonen, Mannel 2014
see Turczyk's talk

$$\Delta_A \simeq 0.090 + 0.029 - 0.023 - 0.013 + \ldots$$

heavy quark expansion converges reasonably well

Including all errors for ε_M=0.75GeV

$$\mathcal{F}(1) < 0.935$$

The inelastic contribution

$$I_1(\varepsilon_M) = -\frac{1}{2\pi i}\oint_{|\varepsilon|=\varepsilon_M} T(\varepsilon)\,\varepsilon\, d\varepsilon \qquad I_{inel}(\varepsilon_M) = \frac{I_1(\varepsilon_M)}{\bar{\varepsilon}}$$

$\bar{\varepsilon}$ represents the average excitation energy mainly controlled by the lowest radial $(1/2^+)$ and D-wave $(3/2^+)$ excitations, therefore about 700MeV

OPE:

$$I_1 = \frac{-(\rho^3_{\pi G}+\rho^3_A)}{3m_c^2} + \frac{-2\rho^3_{\pi\pi}-\rho^3_{\pi G}}{3m_c m_b} + \frac{\rho^3_{\pi\pi}+\rho^3_{\pi G}+\rho^3_S+\rho^3_A}{4}\left(\frac{1}{m_c^2}+\frac{2}{3m_c m_b}+\frac{1}{m_b^2}\right) + \mathcal{O}\left(\frac{1}{m_Q^3}\right)$$

in terms of little known non-local correlators of the form

$$\frac{i}{2M_B}\int d^4x \langle B|T\{O_i(x), O_j(x)\}|B\rangle, \qquad O \sim \bar{b}\,\pi_k\pi_l\, b$$

$$\rho^3_{\pi\pi} + \rho^3_{\pi G} + \rho^3_S + \rho^3_A \geq 0$$

each of them is integral of spectral function with specific spin structure e.g.

$$\rho^3_{\pi\pi} = \int_{\omega>0} d\omega\, \frac{\rho_p^{(\frac{1}{2}^+)}}{\omega}$$

ESTIMATING THE NON-LOCAL GUYS

Hyperfine splitting

$$\Delta M_Q^2 = M_{Q^*}^2 - M_Q^2 = \frac{4}{3} c_G(m_Q)\mu_G^2 + \frac{2}{3}\frac{\rho_{\pi G}^3 + \rho_A^3 - \rho_{LS}^3 + 2\bar{\Lambda}\mu_G^2}{m_Q} + O\left(\frac{1}{m_Q^2}\right)$$

Experimentally $\Delta M_B^2 \simeq \Delta M_D^2$ $\rho_{\pi G}^3 + \rho_A^3 \approx -0.45\text{GeV}^3$

within a ~25% uncertainty

From $\bar{M}_B$-$\bar{M}_D$ and moments fits $\rho_{\pi G}^3 + \rho_A^3 \lesssim (-0.33 \pm 0.17)\text{GeV}^3$

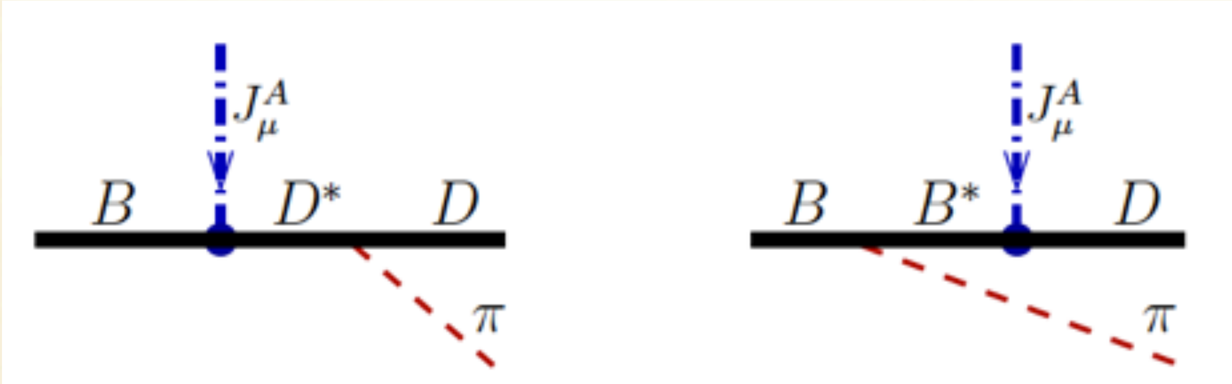
These are strong indications that non-local guys are larger than expected.
Based on a BPS expansion we get a minimum $I_{inel}(\varepsilon_M \sim 0.75\text{GeV}) \gtrsim 0.14 \pm 0.03$

using the lowest value of I_{inel} and interpreting the total uncertainty as gaussian

$$\mathcal{F}(1) = 0.86 \pm 0.02$$

in perfect agreement with inclusive V_{cb} which implies 0.85±0.03

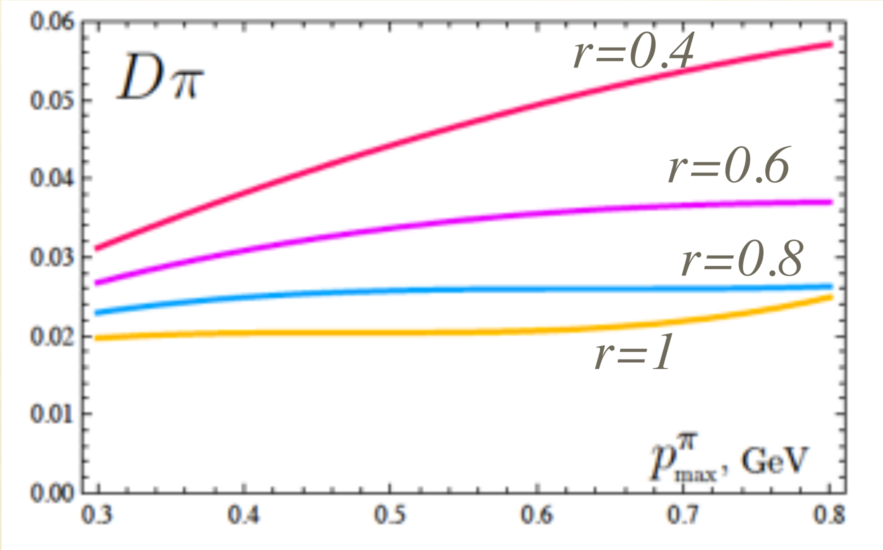
SUPPORTING EVIDENCE: CONTINUUM $D^{(*)}\pi$



- Continuum contributions are included in non-local correlators. They are $1/N_c$ suppressed, but enhanced by HQS breaking

- We computed them in the soft pion approximation, up to a cutoff on the pion momentum, including subleading effects

- $B \to D\pi$ dominates over $B \to D^*\pi$



$$r = \frac{g_{B^*B\pi}}{g_{D^*D\pi}}$$

$$I_{inel}^{D^{(*)}\pi} \approx 3 \div 5\%$$

"RADIAL" CONTRIBUTIONS TO TOTAL WIDTH

- Non-local guys determined by transitions to + parity light d.o.f.: $\frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+$
- Large I_{inel} implies strong transitions to "radial" excitations (radial & D-wave states)
- Assuming a single multiplet of "radials" for each j_q hyperfine splitting constrains the strength of B→"radials"
- At leading order in the heavy quark expansion and neglecting v dependence of the form factors one typically gets

$$\frac{\Gamma_{rad}}{\Gamma_{sl}} \approx 6 \div 7\%$$

suggesting "radials" contribute significantly to the broad resonances, a possible solution of 1/2 vs 3/2 puzzle see also Bernlochner, Ligeti, Turczyk

How to improve F(1) estimate

- Perturbative corrections to $1/m^2$ to I_0 available, unlikely to be relevant. $1/m^3$ will soon be available
- Better knowledge of local ME, up to dim 8. More recent fits lower slightly I_0
- Better knowledge of I_{inel} i.e. of local guys. Lattice could help: values of the hyperfine splitting for m_Q between charm and bottom could help fixing the slope in m_Q, and would have better converging HQE than M_D-M_{D*}.

$B \to D$ FORM FACTOR & BPS

Uraltsev, 2004

Proceeds via vector current, zero recoil ff receives $1/m$ corrections but all power correction vanish in the **BPS limit** where ground state satisfies

$$\vec{\sigma} \cdot \vec{\pi} |B\rangle = 0$$

In BPS limit heavy flavor symmetry holds at all orders in $1/m_Q$.

Hence G(1)=1 exactly and the IW function can be completely determined

Jugeau et al 2006 $\mu_\pi^2 = \mu_G^2 \quad \rho_D^3 = -\rho_{LS}^3 \quad \rho_A^3 + \rho_{\pi G}^3 + \rho_{\pi\pi}^3 + \rho_S^3 = 0$

One can expand around BPS in powers of $\beta = \frac{\mu_\pi^2 - \mu_G^2}{\mu_\pi^2}$

$G(1)=1+O(\beta^2)$ **at any order in $1/m_Q$**

including perturbative and $1/m_Q^2$ corrections for $\mu_\pi^2(1\text{GeV}) <\approx 0.45\text{GeV}^2$

$$G(1) = 1.04 \pm 0.01_{pert} \pm 0.01_{power}$$

Light-cone sum rules

- Apply only to $q^2 \approx 0$ *i.e. near* max recoil
- Faller et al. 0809.0222: finite m_c, expansion in $1/m_b$
- Limited accuracy (~25% on FFs)
- Reasonable agreement with experimental shapes
- Method applicable to higher resonances, see eg Bernlochner et al 1202.1834 (radials)
- Can be improved by computing radiative corrections and improving B meson DA

Faller et al. 0809.0222   consistent $G(w)$ in Fu et al, 1309.5723

Fig. 4.2 Comparison of the $B \to D^*$ form factor $h_{A_1}(w)$ calculated from LCSR at $w > 1.3$ (*solid*), with the fit of the BaBar data to the CLN parameterization (*long-dashed*). *Dotted* (*short-dashed*) lines indicate the estimated theoretical uncertainty (experimental fit error)

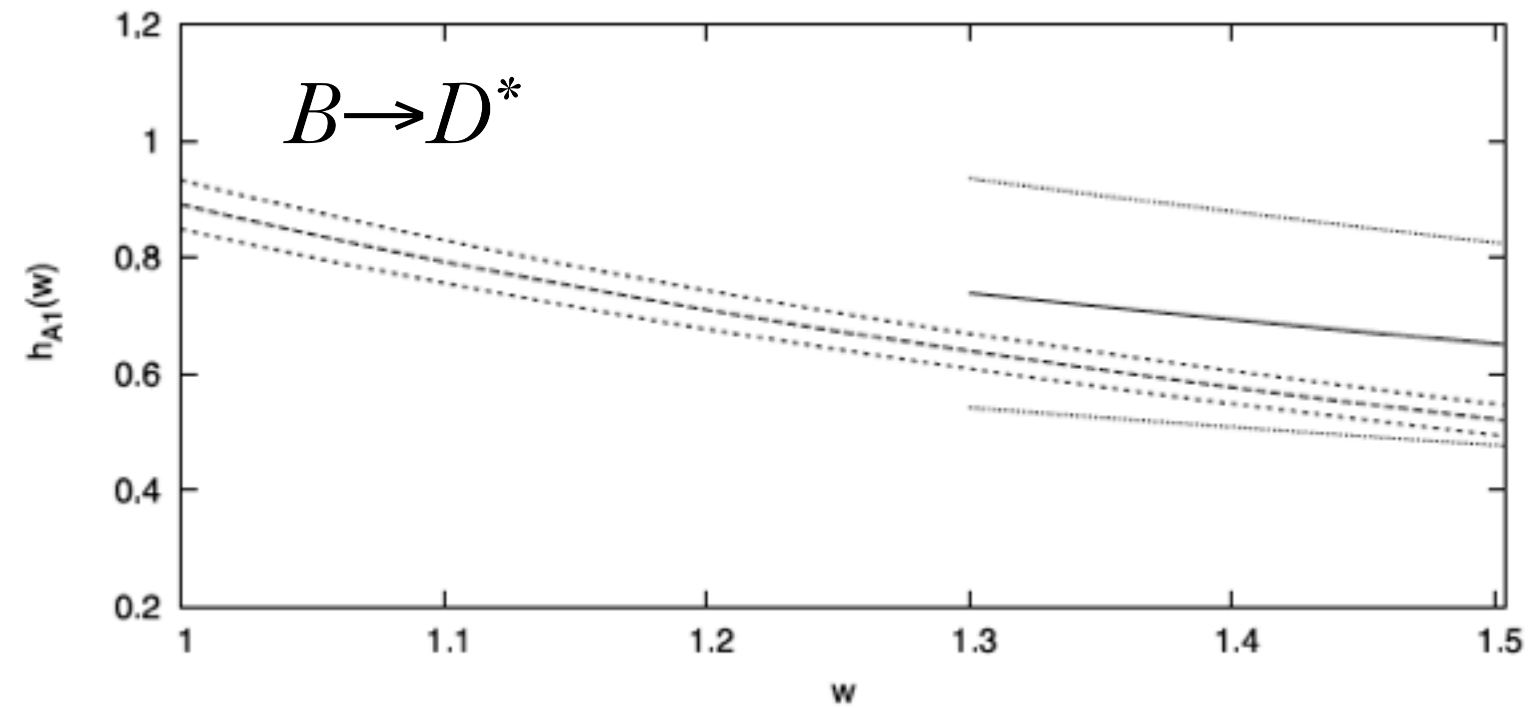
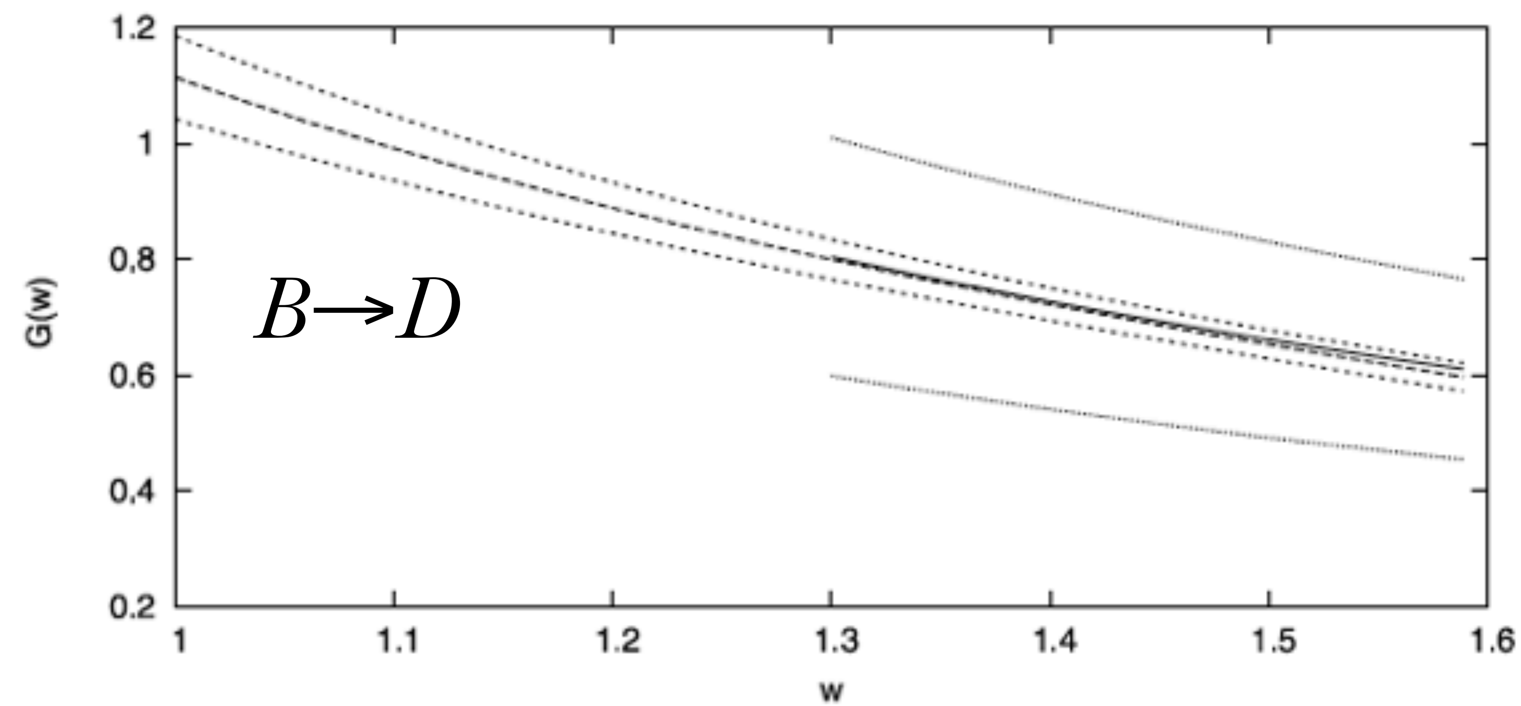


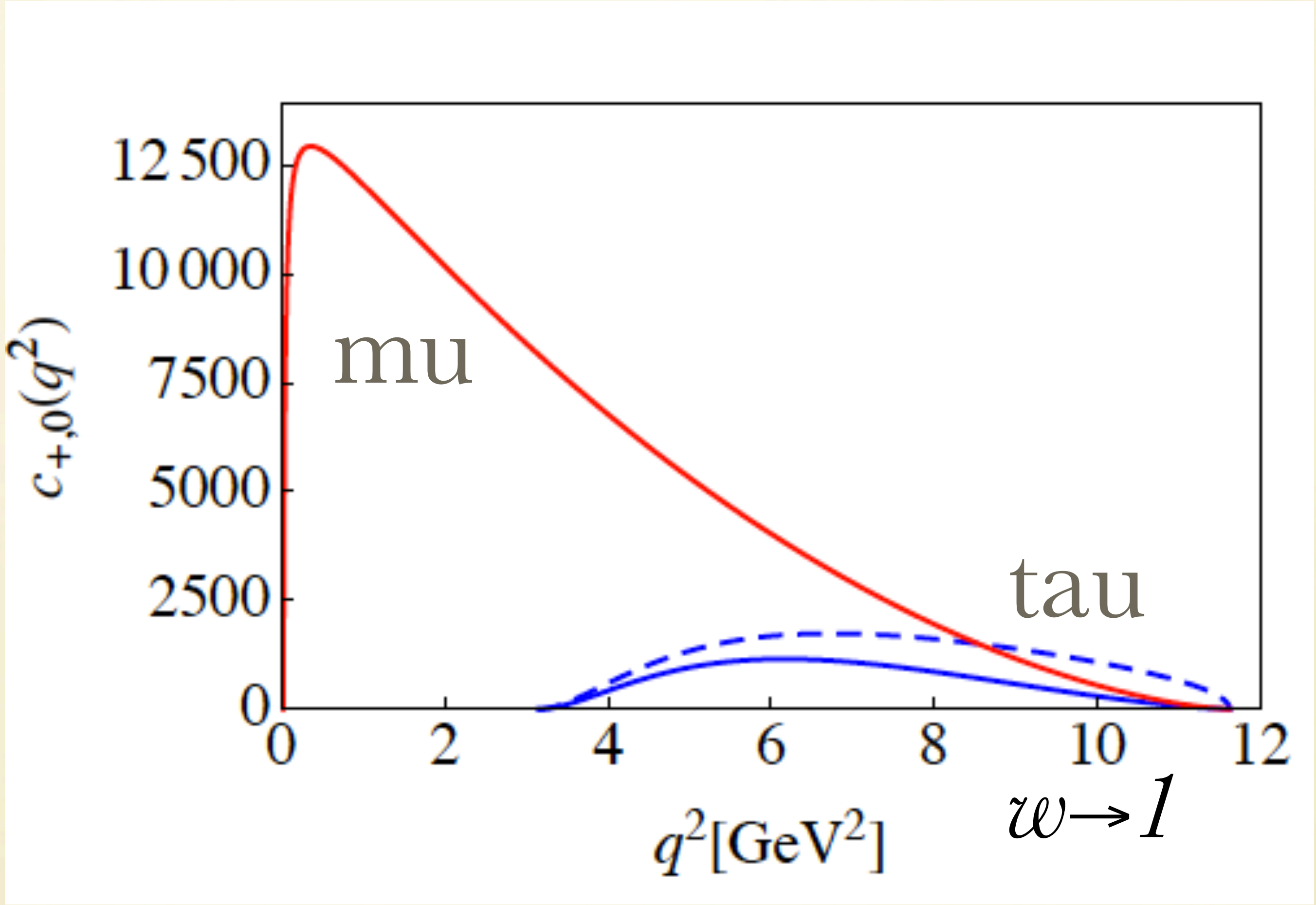
Fig. 4.4 The combination of $B \to D$ form factors $\mathcal{G}(w)$ calculated from LCSR at $w > 1.3$ (*solid*), compared with the fits of the BaBar data to the CLN parameterization (*long-dashed*). The *dotted* (*short-dashed*) lines indicate the theoretical uncertainty (experimental fit error)



pQCD factorization provides an estimate at max recoil including $1/m_Q$ corrections very similar in size and error   Kurimoto, Li, Sanda 2003

Phase space

phase space
in $B \to Dl\nu$



Most precise FF are at zero-recoil, where the rates vanish. Extrapolation is unavoidable now and will remain crucial for a long time.

CLN PARAMETERIZATION

- Caprini, Lellouch, Neubert proposed in 1998 a FF one-parameter form based on analiticity, unitarity and $1/m_Q$ HQET near zero-recoil. For $B \to D$

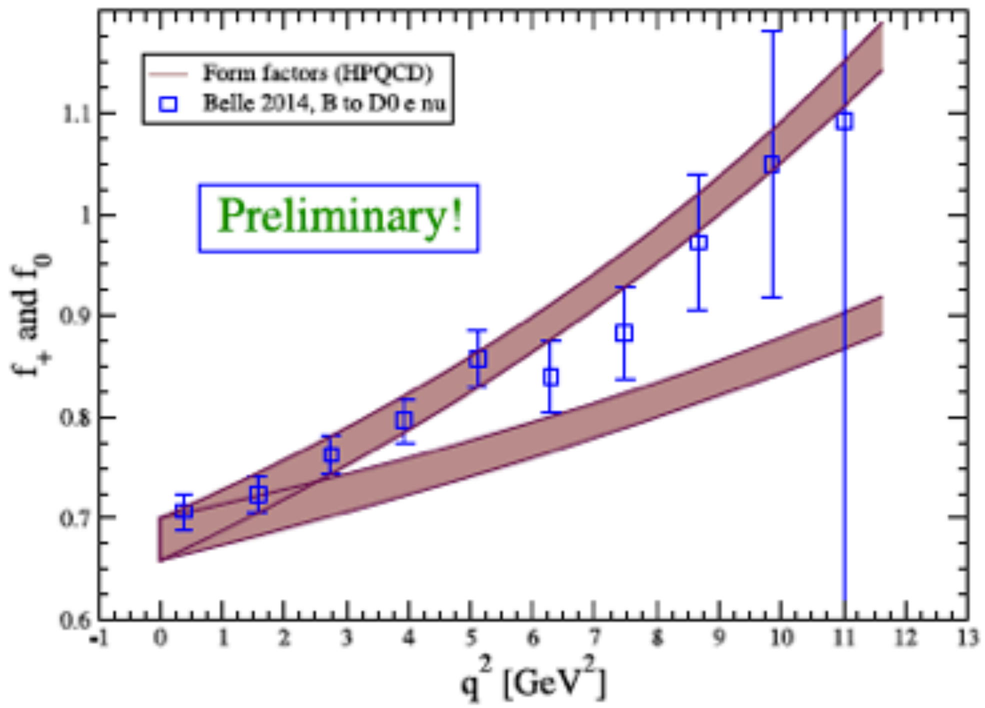
$$G(w) \quad = \quad G(1) \times [1 - 8\rho^2 z(w) + (51\rho^2 - 10) z(w)^2 - (252\rho^2 - 84) z(w)^3]$$

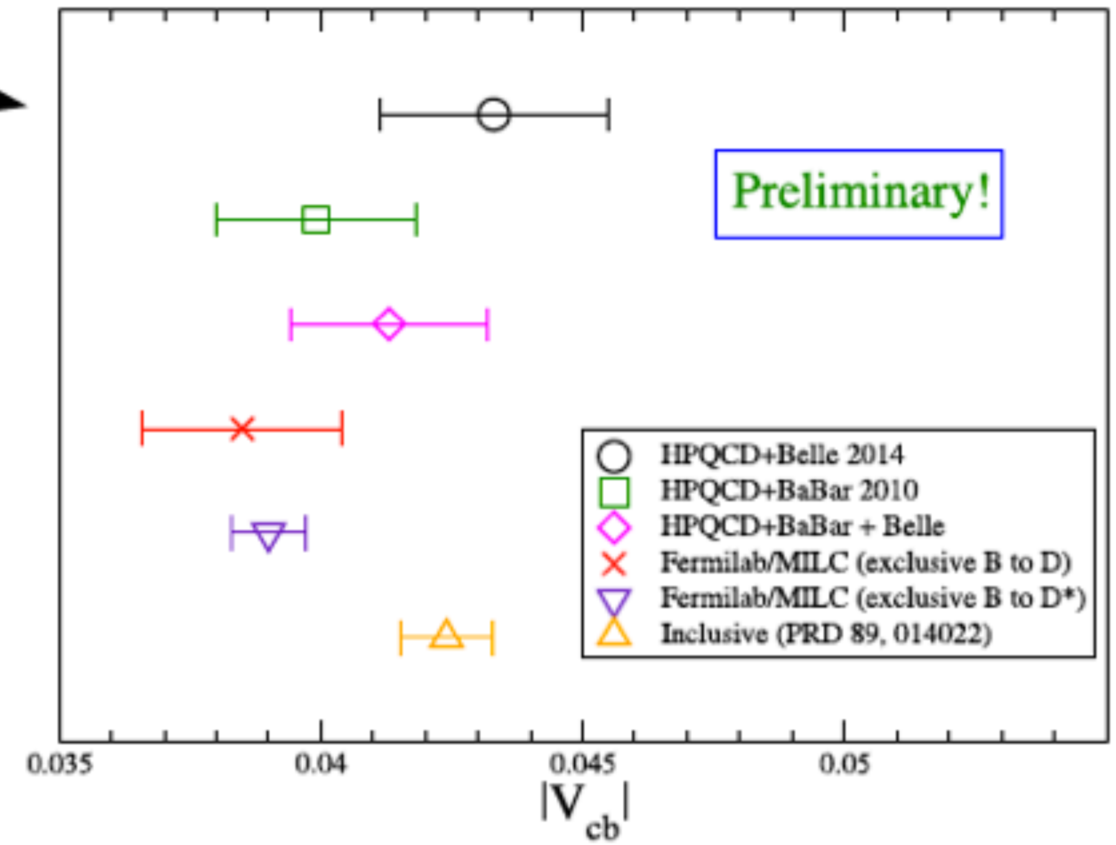
- A 2% error is quoted, but this does not include higher power corrections

- CLN is widely used by experiments and theorists

The future

- $B \rightarrow D^{(*)} \ell \nu$:
  - Error on form factor parametrization by Caprini et al not quantified (<2% is all we know)
  - Switch from using form factor parametrization and calculation of G(1) and F(1) to model-indepent fits fitting LQCD parameters and reconstructed data together
  - In contact with MILC and HPQCD





Plots by HPQCD, from private communication with Heechang Na. Preliminary fit using preliminary $B \rightarrow D \ell \nu$ data from Belle. MILC data points are from LATTICE 2013 proceedings [arXiv: 1312.0155]
Left plot shows example of fit for a subsample of preliminary $B \rightarrow D \ell \nu$.
Red bands are the form factors (f_+ and f_0) from a lattice simulation by HPQCD.

Summary

- Heavy quark sum rules: unitarity bound $F(1)<0.935$, including also $D^{(*)}\pi$ continuum $F(1)<0.91$.
- Hyperfine splitting in B and D implies strong transitions to radial/D wave states, with implications for higher D states. The resulting $F(1)=0.86(2)$ leads to $|V_{cb}|= 41.5(1.3)\ 10^{-3}$ in agreement with inclusive V_{cb}
- Uraltsev HQE estimate $G(1)=1.04(2)$ in HQE leads to $|V_{cb}|= 40.7(1.7)\ 10^{-3}$
- LCSR and pQCD constrain F,G at small q^2 or max recoil, but limited accuracy
- Parameterization of FF shape remains crucial. It should not rely on assumptions on power corrections.