$B \to D$, D^*, D^{**} Form Factors from Lattice QCD

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Lattice Meets Continuum: QCD Calculations in Flavor Physics

29 September 2014

Outline

- $|V_{cb}|$ from $B \to D^* l\nu$ at zero recoil—arXiv:1403.0635
- $|V_{cb}|$ from $B \to D l\nu$ at all recoil—arXiv:1312.0155 and in preparation
- Strange interlude
- Extra form factors for non-SM contributions to $B \to D^{(*)}\tau\nu$—arXiv:1206.4992
- Other applications—arXiv:1202.6346
- Radial and orbital excitations: $B \to D^{**} l\nu$ calculations
- Future Calculations

Numerical Lattice Gauge Theory

- The lattice provides a UV cutoff; a finite volume provides an IR cutoff; a finite Euclidean time leads to a nonzero temperature.
- Write a random number generator to create lattice gauge fields distributed with the weight of the functional integral.
- Fit correlations functions to get masses and matrix elements.
- Repeat several times while varying bare gauge coupling and bare masses.
- Find a trajectory with constant pion, kaon, D_s, B_s, masses (one for each quark) in dimensionless but physical units and obtain the continuum limit.
- Convert units to MeV.

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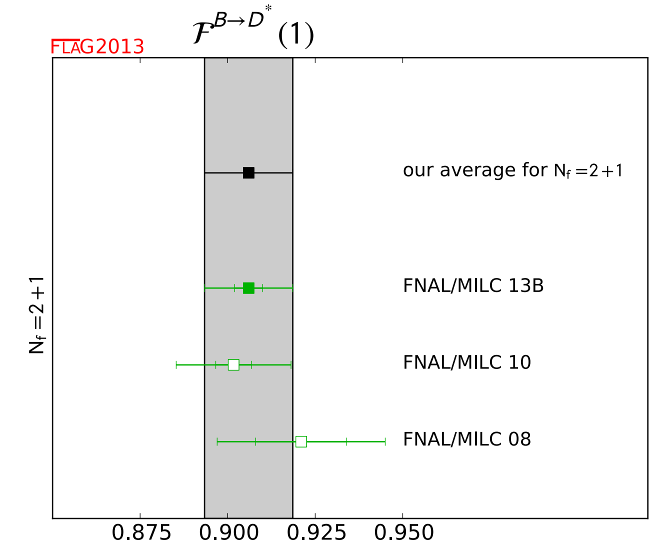
- Find … quark) in d…

requires wisdom and effective field theory

- Convert units to MeV.

$|V_{cb}|$ from $B \to D^* l\nu$ at zero recoil



Basic Formulas

- Matrix elements:

$$\frac{\langle D^*(p_{D^*},\varepsilon^{(\alpha)})|\mathscr{A}^\mu|B(p_B)\rangle}{\sqrt{2M_{D^*}}\qquad\qquad\sqrt{2M_B}} = \frac{i}{2}\varepsilon_\nu^{(\alpha)*}\left[g^{\mu\nu}(1+w)h_{A_1}(w) - v_B^\nu\left(v_B^\mu h_{A_2}(w) + v_{D^*}^\mu h_{A_3}(w)\right)\right],$$

$$\frac{\langle D^*(p_{D^*},\varepsilon^{(\alpha)})|\mathscr{V}^\mu|B(p_B)\rangle}{\sqrt{2M_{D^*}}\qquad\qquad\sqrt{2M_B}} = \frac{1}{2}\varepsilon^{\mu\nu}{}_{\rho\sigma}\varepsilon_\nu^{(\alpha)*}v_B^\rho v_{D^*}^\sigma h_V(w)$$

- Differential decay rate:

$$\frac{d\Gamma}{dw} = \frac{G_F^2 M_{D^*}^3}{4\pi^3}(M_B - M_{D^*})^2(w^2-1)^{1/2}|\eta_{\mathrm{EW}}|^2|V_{cb}|^2\chi(w)|\mathscr{F}(w)|^2,$$

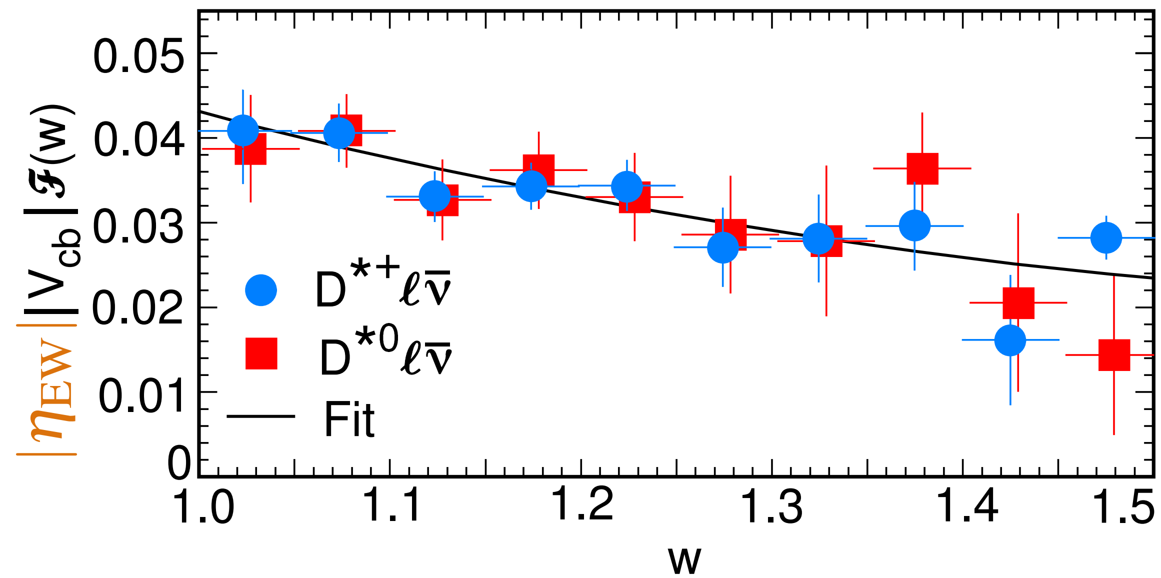
- Zero recoil when $w \equiv v_B \cdot v_{D^*} \to 1$. Then, $\chi(w{=}1) = 1$, and

$$\mathscr{F}(1) = h_{A_1}(1).$$

$|V_{cb}|$ Determination

- Divide $d\Gamma/dw$ by "known stuff" and phase space factor $(w^2-1)^{1/2}\,\chi(w)$.

- Extrapolate $w \to 1$ via a fit, guided by analyticity, unitarity, & HQET to obtain $|\eta_{\mathrm{EW}}||V_{cb}|\mathscr{F}(1)$.



CLEO, arXiv:hep-ex/0210040

- The factor $|\eta_{\mathrm{EW}}|$ includes a large log from $W/Z/\gamma$ boxes [Sirlin, 1982].

- It also accounts for long-distance soft photons, which are very small for charged B decays and "huge" (~2%) for neutral B decays.

- The form factor is calculated in QCD, e.g., with lattice gauge theory.

Error Budgets for $\mathscr{F}(1) = h_{A_1}(1)$, $|V_{cb}|$

Fermilab/MILC, arXiv:1403.0635

- Some entries are not very interesting: statistics, scale error, isospin.
- Discuss others in turn.
- Note in passing: r_1 is defined by

$$r_1^2 F(r_1) = 1$$

where F is the force between static sources.

  - Compute r_1/a on every lattice and form dimensionless quantities, $(r_1/a)(af_\pi)$.
  - Define $r_1 = (r_1 f_\pi)_{a\to 0}/f_\pi^{\text{PDG}}$.

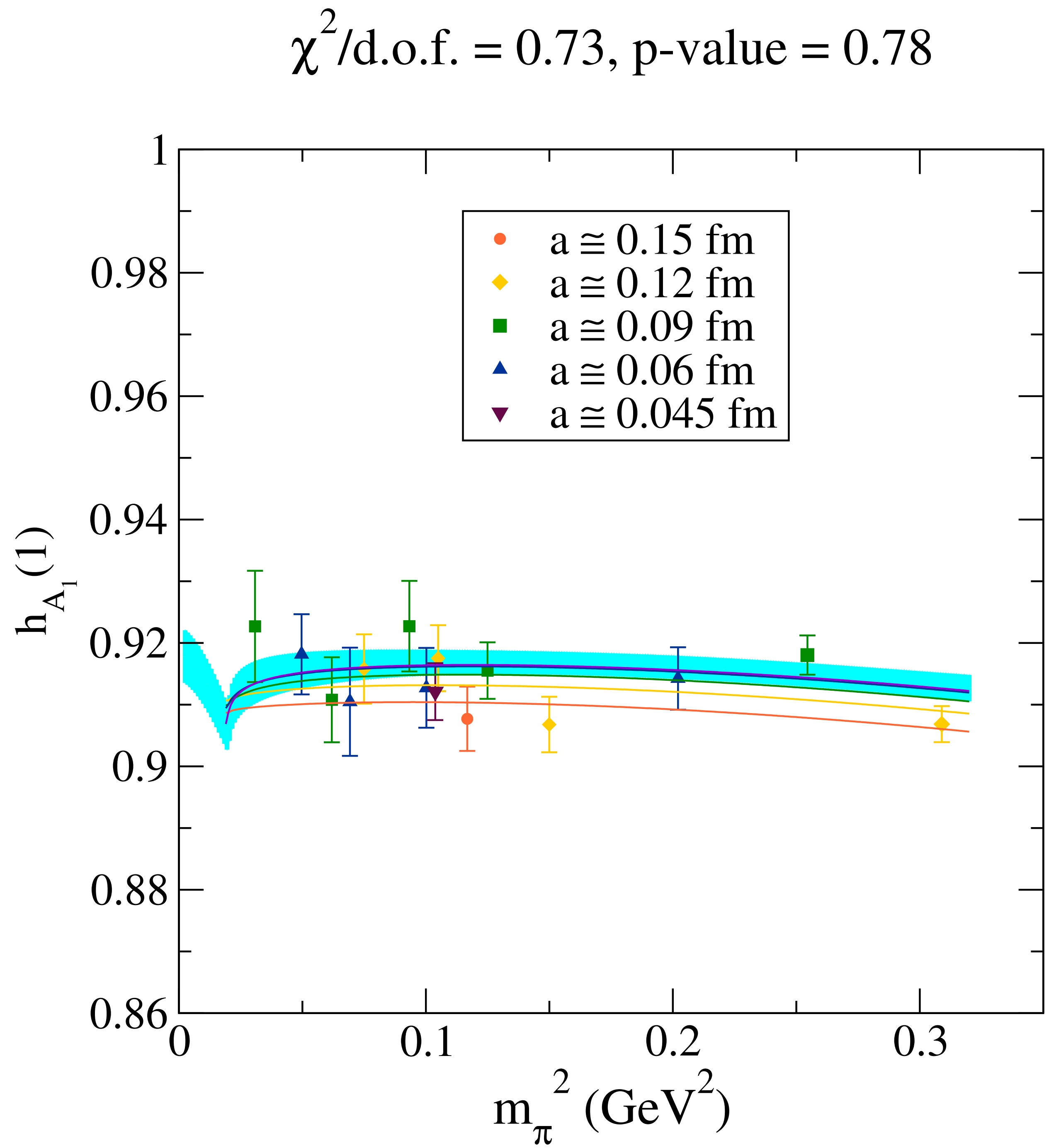
Uncertainty	$h_{A_1}(1)$
Statistics	0.4%
Scale (r_1) error	0.1%
χPT fits	0.5%
$g_{D^*D\pi}$	0.3%
Discretization errors	1.0%
Perturbation theory	0.4%
Isospin	0.1%
Total	1.4%

Uncertainty	$\lvert V_{cb}\rvert$
QCD	1.4%
QED	0.5%
Expt	1.3%

Chiral Perturbation Theory

Fermilab/MILC, arXiv:1403.0635

- The light quarks are too massive, so the pion cloud is too small.
- Fix with chiral perturbation theory:
  - NLO expression available;
  - includes mods for discretization;
  - model larger light masses with analytic terms of NNLO.
- Note the cusp from $D\pi$ threshold.



The $g_{D^*D\pi}$ Coupling

Fermilab/MILC, arXiv:1403.0635

- The chiral logs are proportional to

$$g^2_{D^*D\pi}/(4\pi f_\pi)^2$$

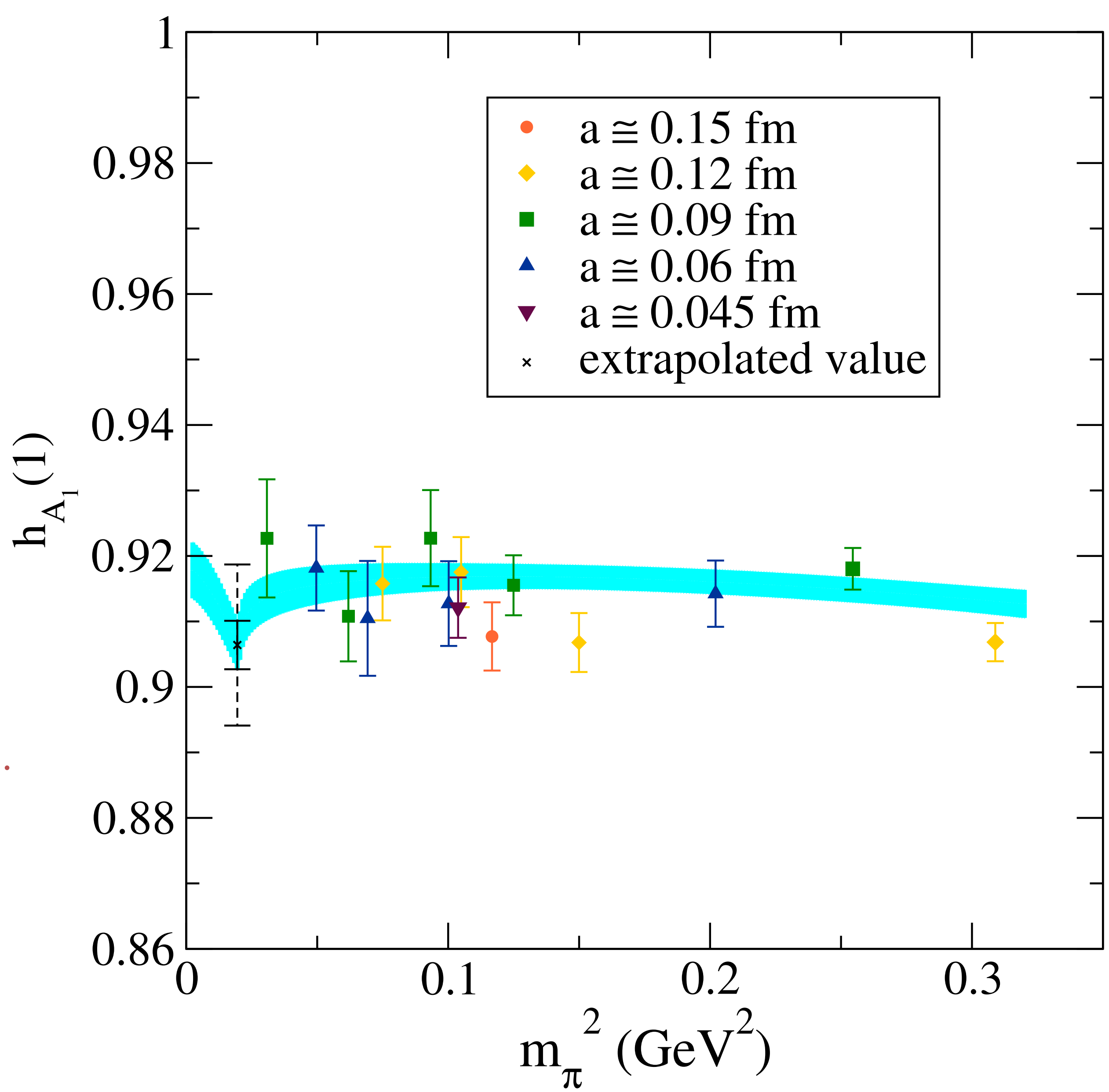
but ~10^{-3} where we have data.

- Take $g_{D^*D\pi} = 0.53 \pm 0.08$ based on

0.53(3)(3)	$n_f = 2$	arXiv:1210.5410
0.55(6)	$a \neq 0$	arXiv:1210.0869
0.57(5)(6)	$B^*B\pi$	arXiv:1311.2251
0.45(5)	static	arXiv:1203.3378
0.570(6)	BaBar	arXiv:1304.5657

- Need much larger $g_{D^*D\pi}$ or huge NNLO log to reach, e.g., 0.87.

$$(0.4_{\text{choices}} + 0.1_{\text{NNLO log}})_{\chi\text{PT}} \oplus 0.3_{g_{D^*D\pi}}$$



HQET Theory of Cutoff Effects

- LGT with Wilson fermions and continuum QCD enjoy heavy-quark symmetry:
  - lattice version is stronger: the radiative corrections are HQ symmetric.
- Therefore, both can be described by HQET:

$$\mathscr{L}_{\text{LGT}} \doteq \bar{h}(iv\cdot D - m_1)h + \frac{\bar{h}D_\perp^2 h}{2m_2} + \frac{\bar{h}s\cdot Bh}{2m_B} + \frac{\bar{h}[D_\perp^\alpha, iE_\alpha]h}{8m_D^2} + \frac{\bar{h}s_{\alpha\beta}\{D_\perp^\alpha, iE^\beta\}h}{4m_E^2} + \cdots,$$

$$\mathscr{L}_{\text{QCD}} \doteq \bar{h}(iv\cdot D - m)h + \frac{\bar{h}D_\perp^2 h}{2m} + \frac{z_B\bar{h}s\cdot Bh}{2m} + \frac{z_D\bar{h}[D_\perp^\alpha, iE_\alpha]h}{8m^2} + \frac{z_E\bar{h}s_{\alpha\beta}\{D_\perp^\alpha, iE^\beta\}h}{4m^2} + \cdots.$$

$$V^\mu \doteq \bar{C}_{V_\parallel}^{\text{LGT}} v^\mu \bar{c}_{v'} b_v + \bar{C}_{V_\perp}^{\text{LGT}} \bar{c}_{v'} i\gamma_\perp^\mu b_v + \bar{C}_{V_{v'}}^{\text{LGT}} v_\perp^{\prime\mu} \bar{c}_{v'} b_v - \sum_{a=1}^{14} \bar{B}_{Va}^{\text{LGT}} \bar{\mathscr{Q}}_{Va}^\mu + \cdots,$$

$$\mathscr{V}^\mu \doteq \bar{C}_{V_\parallel}^{\text{QCD}} v^\mu \bar{c}_{v'} b_v + \bar{C}_{V_\perp}^{\text{QCD}} \bar{c}_{v'} i\gamma_\perp^\mu b_v + \bar{C}_{V_{v'}}^{\text{QCD}} v_\perp^{\prime\mu} \bar{c}_{v'} b_v - \sum_{a=1}^{14} \bar{B}_{Va}^{\text{QCD}} \bar{\mathscr{Q}}_{Va}^\mu + \cdots$$

Matching

$$V^\mu \doteq \bar{C}^{\rm LGT}_{V_\parallel} v^\mu \bar{c}_{v'} b_v + \bar{C}^{\rm LGT}_{V_\perp} \bar{c}_{v'} i\gamma^\mu_\perp b_v + \bar{C}^{\rm LGT}_{V_{v'}} v'^\mu_\perp \bar{c}_{v'} b_v - \sum_{a=1}^{14} \bar{B}^{\rm LGT}_{Va} \bar{\mathscr{Q}}^\mu_{Va} + \cdots,$$

$$\mathscr{V}^\mu \doteq \bar{C}^{\rm QCD}_{V_\parallel} v^\mu \bar{c}_{v'} b_v + \bar{C}^{\rm QCD}_{V_\perp} \bar{c}_{v'} i\gamma^\mu_\perp b_v + \bar{C}^{\rm QCD}_{V_{v'}} v'^\mu_\perp \bar{c}_{v'} b_v - \sum_{a=1}^{14} \bar{B}^{\rm QCD}_{Va} \bar{\mathscr{Q}}^\mu_{Va} + \cdots$$

- Then the current $Z_V V$ is well matched if

$$Z_{V^\mu} = \frac{\bar{C}^{\rm QCD}_{V_\parallel}}{\bar{C}^{\rm LGT}_{V_\parallel}}$$

- Note that HQET here is a scaffolding: we compute the Z_V directly.

- In practice: compute degenerate-mass Z_V nonperturbatively and use one-loop PT for $\rho_{A^i_{cb}} = Z_{A^i_{cb}} Z^{-1/2}_{V^4_{bb}} Z^{-1/2}_{V^4_{cc}}$.

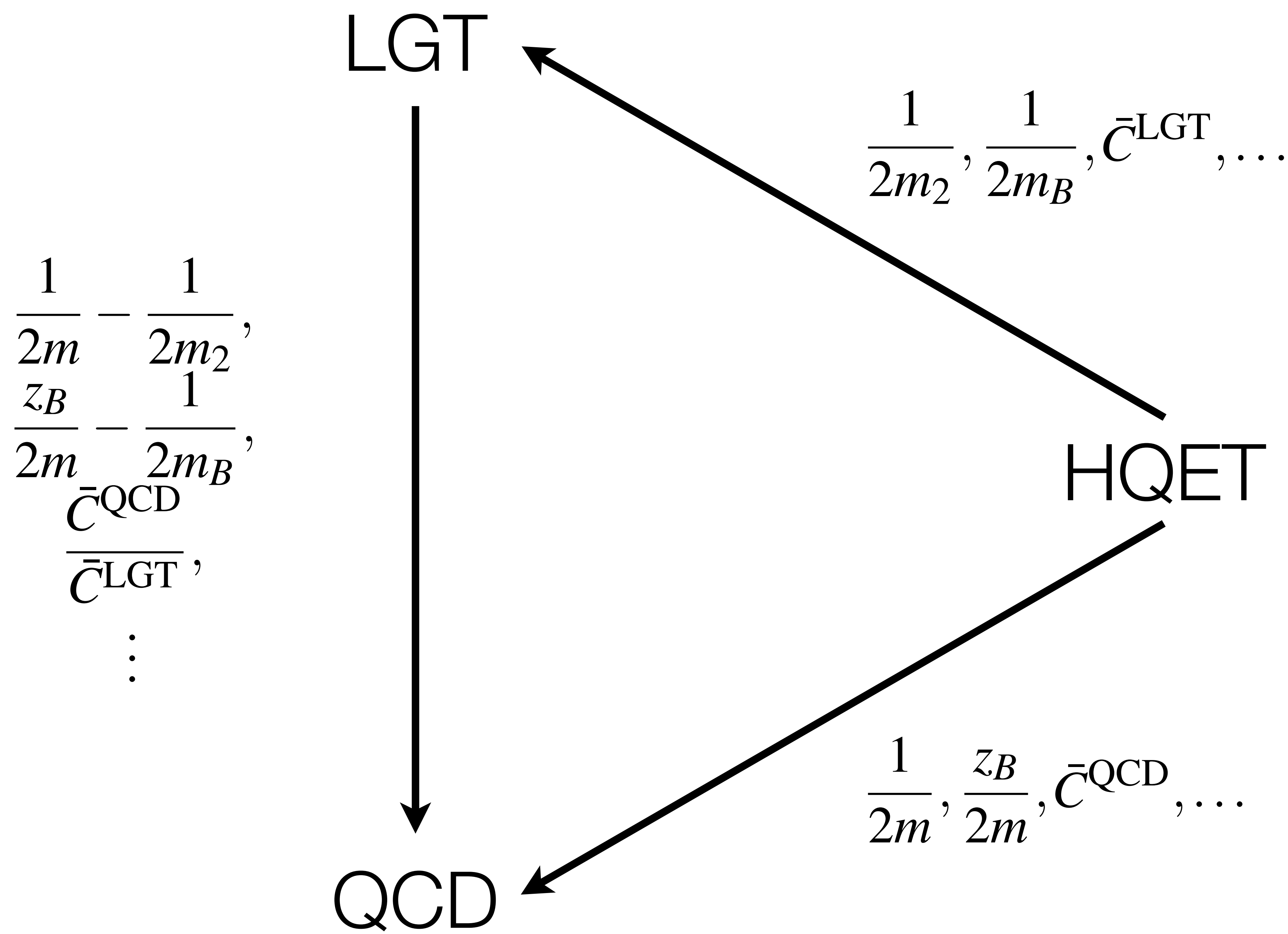
Discretization Effects

$$\mathscr{L}_{\mathrm{LGT}} \doteq \bar{h}(iv\cdot D - m_1)h + \frac{\bar{h}D_\perp^2 h}{2m_2} + \frac{\bar{h}s\cdot Bh}{2m_B} + \frac{\bar{h}[D_\perp^\alpha, iE_\alpha]h}{8m_D^2} + \frac{\bar{h}s_{\alpha\beta}\{D_\perp^\alpha, iE^\beta\}h}{4m_E^2} + \cdots,$$

$$\mathscr{L}_{\mathrm{QCD}} \doteq \bar{h}(iv\cdot D - m)h + \frac{\bar{h}D_\perp^2 h}{2m} + \frac{z_B\bar{h}s\cdot Bh}{2m} + \frac{z_D\bar{h}[D_\perp^\alpha, iE_\alpha]h}{8m^2} + \frac{z_E\bar{h}s_{\alpha\beta}\{D_\perp^\alpha, iE^\beta\}h}{4m^2} + \cdots.$$

- To estimate discretization effects, take the difference between the HQET descriptions of LGT and QCD.
- Again, HQET becomes a scaffolding:
  - coefficients depend on how HQET is UV regulated and renormalized;
  - dependence drops out in the difference.

HQET Matching



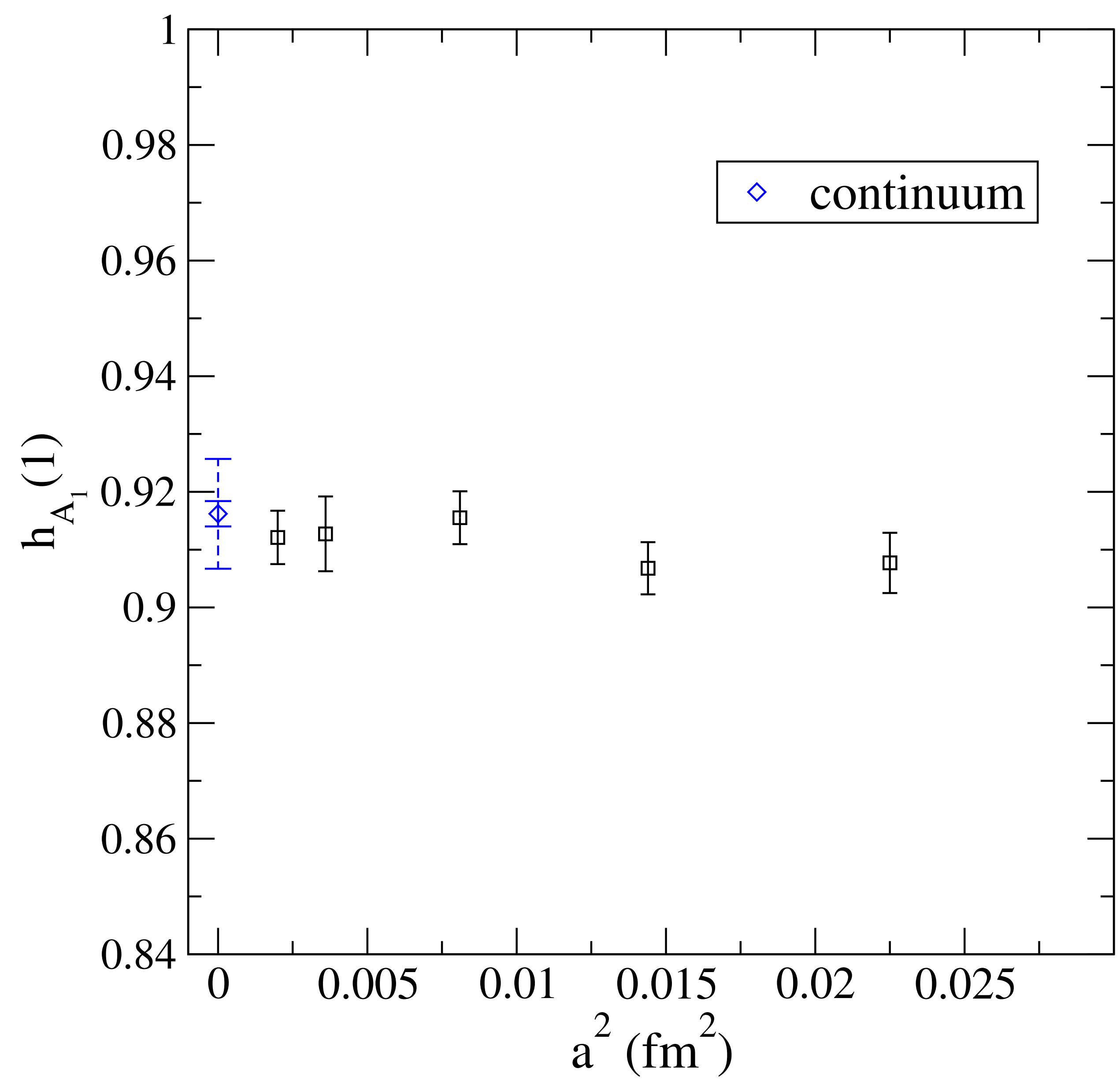
In practice

- Dim 5 (4) in Lagrangian (current), use Ansatz for unknown one-loop coefficient difference.
- Dim 6 (4), use known tree-level coefficient difference.
- Use "reasonable" values for HQET quantities: $\mu_G^2 = (603\ \mathrm{MeV})^2$

$$\mu_\pi^2 = (651 \pm 32\ \mathrm{MeV})^2$$

$$|R_1 + 3R_2 + 3E| \lesssim (450\ \mathrm{MeV})^2$$

- Compare to data.



EW and QED

- The "Sirlin factor" is well known throughout semileptonic decays:

$$\eta_{\rm EW} = 1 + \frac{\alpha}{\pi}\left[\ln\frac{m_W}{\mu} + \tan^2\theta_W \frac{m_W^2}{m_Z^2 - m_W^2}\ln\frac{m_Z}{m_W}\right]$$

$$= 1 + \frac{\alpha}{\pi}\ln\frac{m_Z}{\mu} \qquad \text{(SM)}$$

- At long distances, there are also universal radiative corrections, which are quite large when the final state has Coulomb attraction:

$$\frac{\alpha}{2\pi}(i\pi)(-i\pi) = \frac{1}{2}\pi\alpha$$

- Experiments must separate neutral and charged B decays.

- Finally, there are radiative corrections for photons of energy $\Lambda_{\rm QCD}$.

$|V_{cb}|$ from $B \to Dl\nu$ at all recoil

Basic Formulas

- Matrix element:

$$\frac{\langle D(p_D)|\mathscr{V}^\mu|B(p_B)\rangle}{\sqrt{M_B M_D}} = h_+(w)(v_B+v_D)^\mu + h_-(w)(v_B-v_D)^\mu$$

- Differential decay rate:

$$\frac{d\Gamma}{dw} = \frac{G_F^2 M_D^3}{48\pi^3}(M_B+M_D)^2(w^2-1)^{3/2}|\eta_{\mathrm{EW}}|^2|V_{cb}|^2\mathscr{G}(w)^2,$$

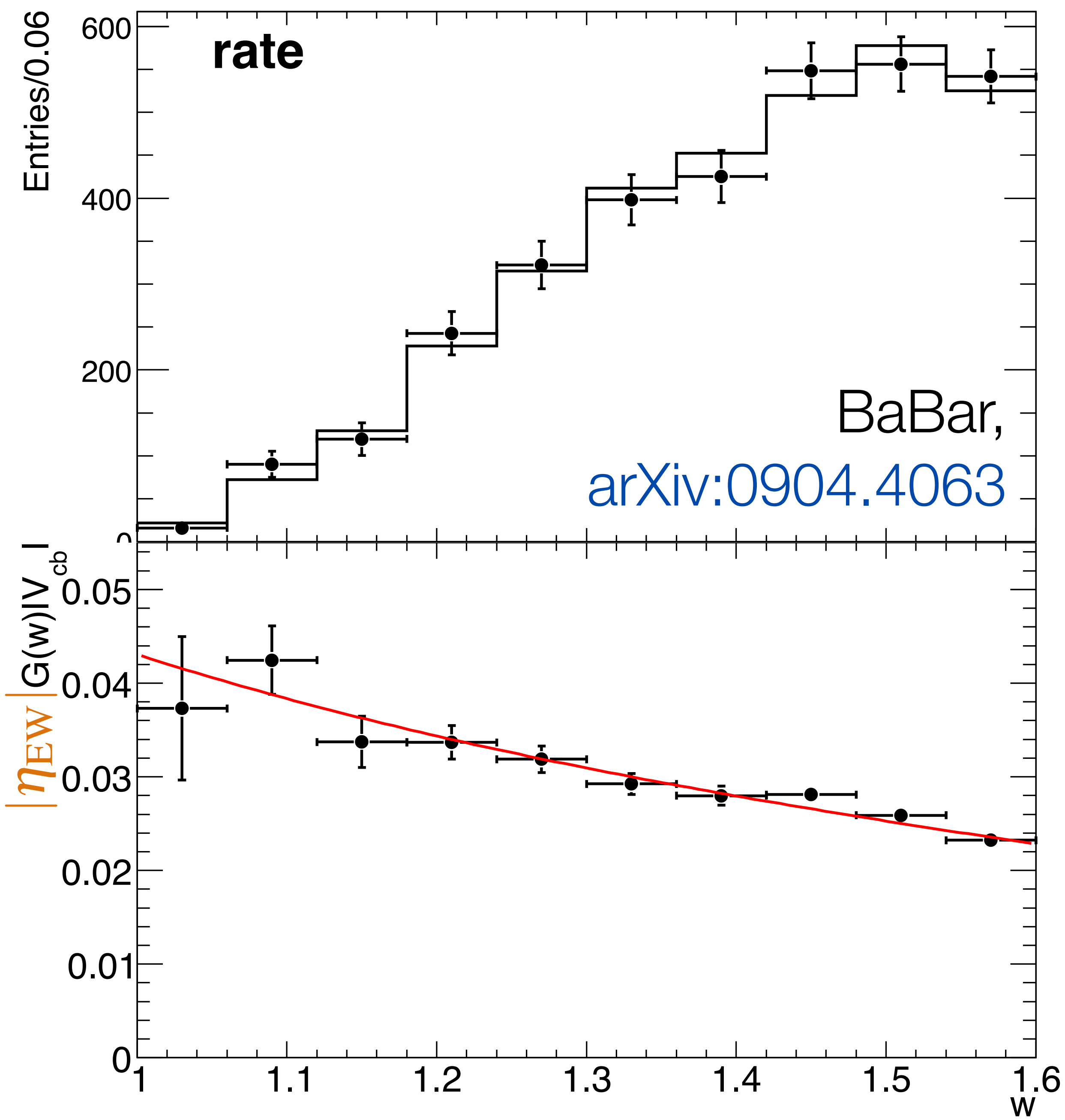
where

$$\begin{aligned}\mathscr{G}(w) &= h_+(w) + \frac{M_B-M_D}{M_B+M_D}h_-(w)\\ &= \frac{2\sqrt{M_B M_D}}{M_B+M_D}f_+(q^2)\end{aligned}$$

with $q^2 = M_B^2 + M_D^2 - 2wM_BM_D$.

$|V_{cb}|$ Determination

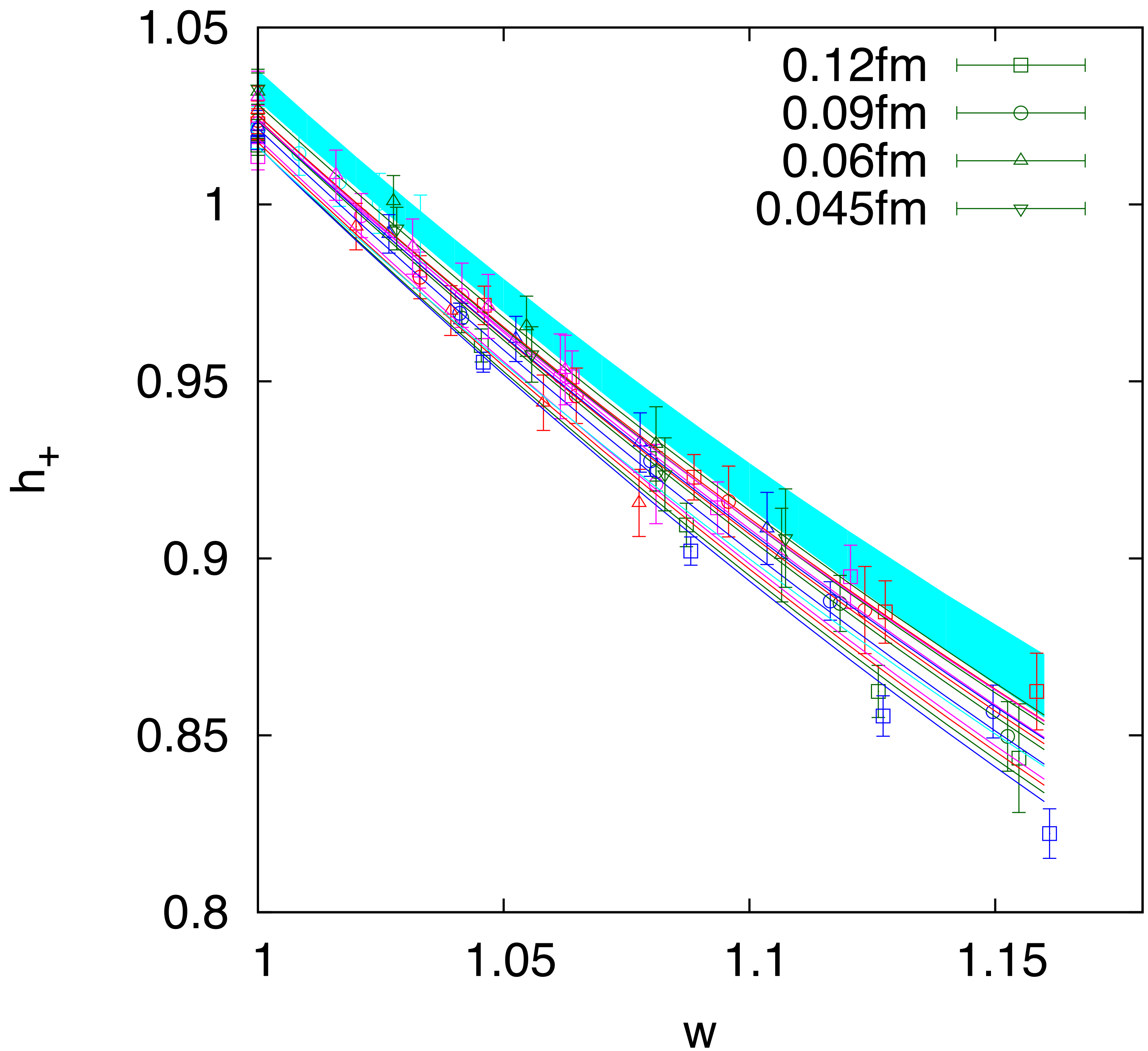
- Could follow same procedure, but now phase space factor is $(w^2-1)^{3/2}$.
- Also $h_-(1) \neq 0$: HQS constraint weak.
- As with $K \to \pi$, $D \to K$, $B \to \pi$, etc., obtain full q^2 aka w dependence.
- Simultaneous fit to experimental data & lattice QCD: obtain $|V_{cb}|$ from relative normalization.



Chiral-continuum extrapolation

Fermilab/MILC, arXiv:1312.0155

- As before, use NLO logs, NNLO analytic, and try various combinations.
- Cyan band shows result for $a \to 0$, $m^2_{\mathrm{PS}} \to m^2_\pi$.
- After 1312.0155, we found a better way to match the spatial component of the current: significant change in h_-, but small change in f_+ and f_0.



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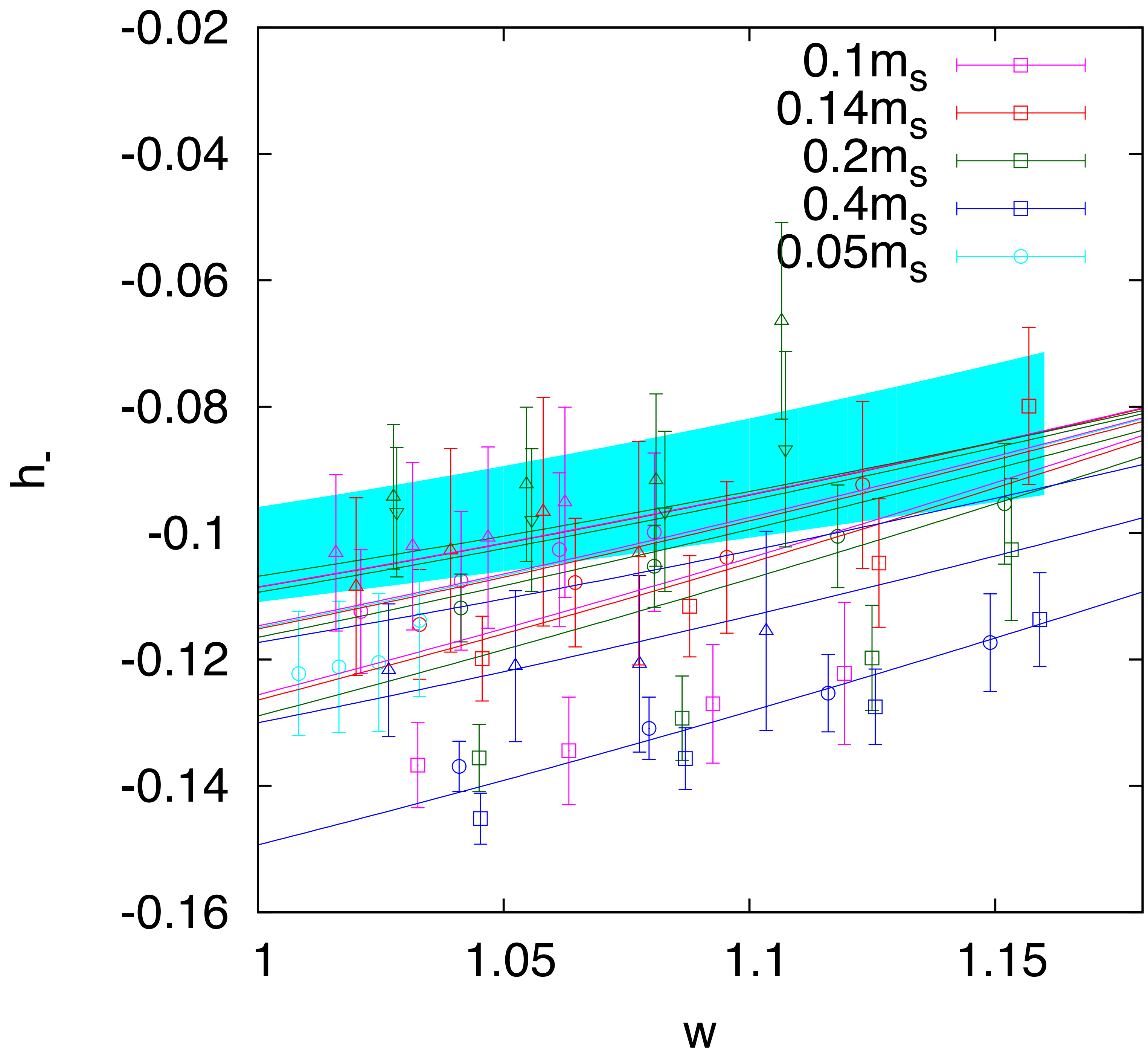
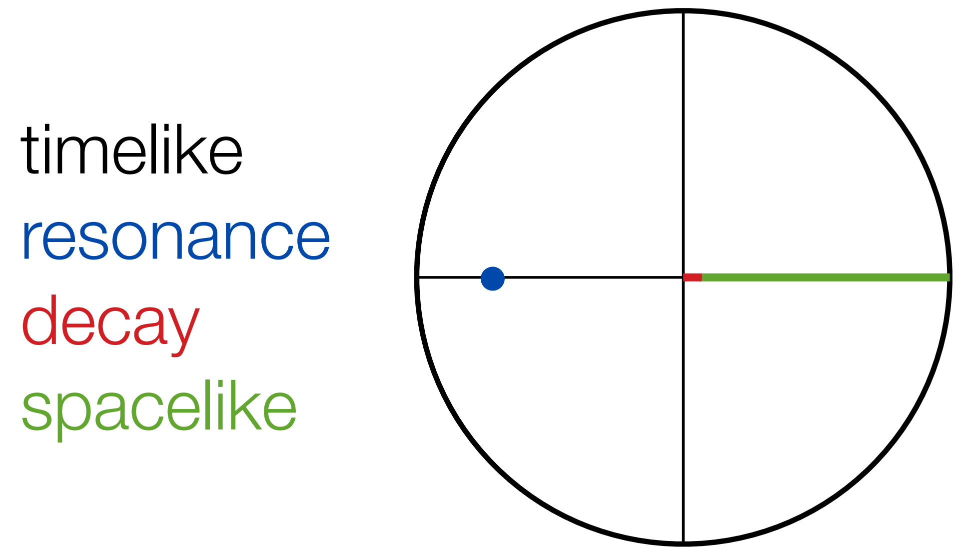


Extrapolate over all w

Fermilab/MILC, arXiv:1312.0155

- Map the whole q^2 axis onto the unit disk:

$$z(w) = \frac{\sqrt{1+w}-\sqrt{2}}{\sqrt{1+w}+\sqrt{2}} \in [0, 0.0645]$$



- Write

$$f_i(z) = \frac{1}{P_i(z)\phi_i(z)} \sum_{n=0}^{\infty} a_{i,n} z^n, \quad i = +, 0$$

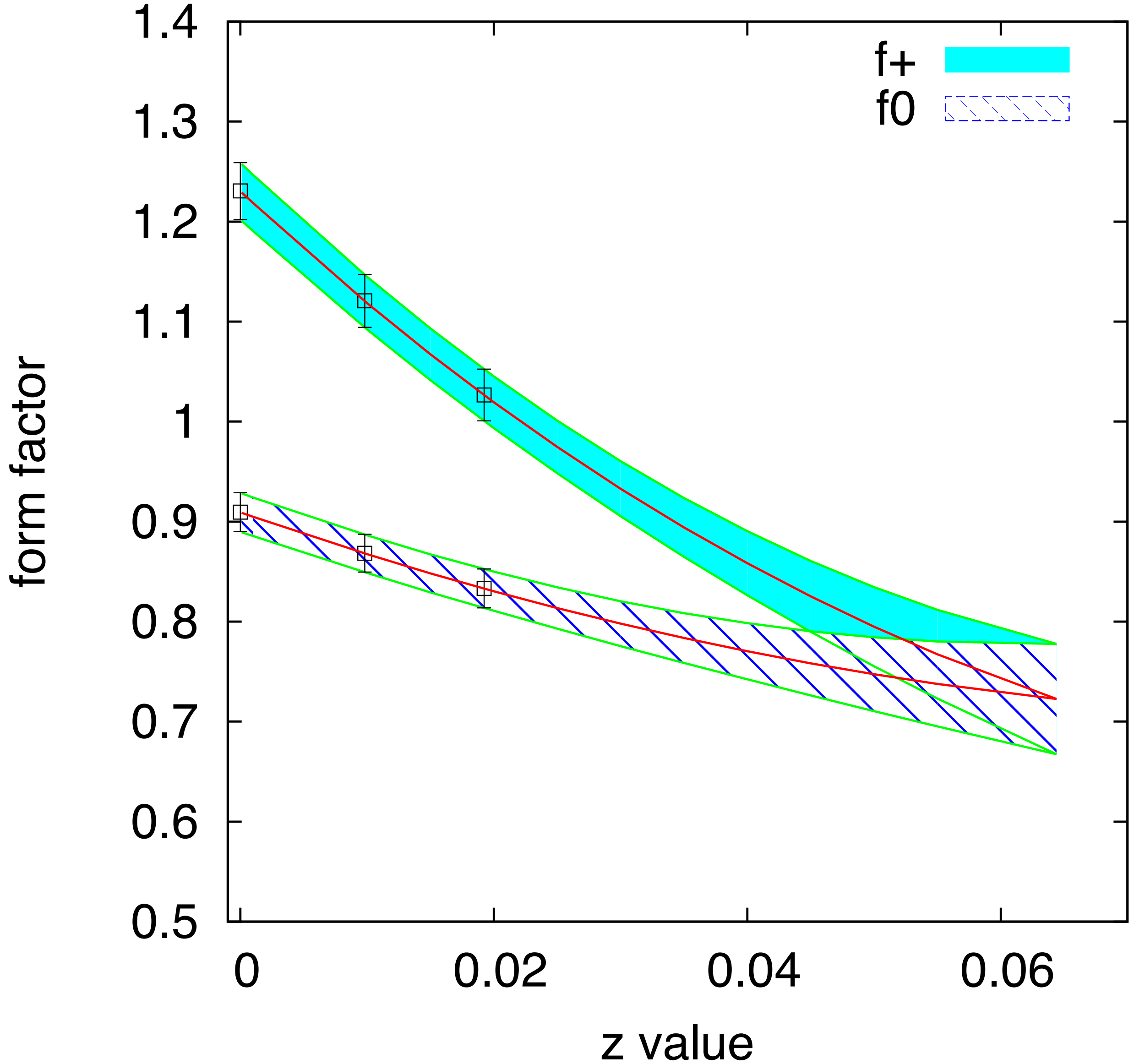
where the “Blaschke” factor $P_i(z)$ remove subthreshold poles, and $\phi_i(z)$ is a convenient “outer” function.

- Unitarity constrains the expansion coefficients. With our choice for $\phi_i(z)$,

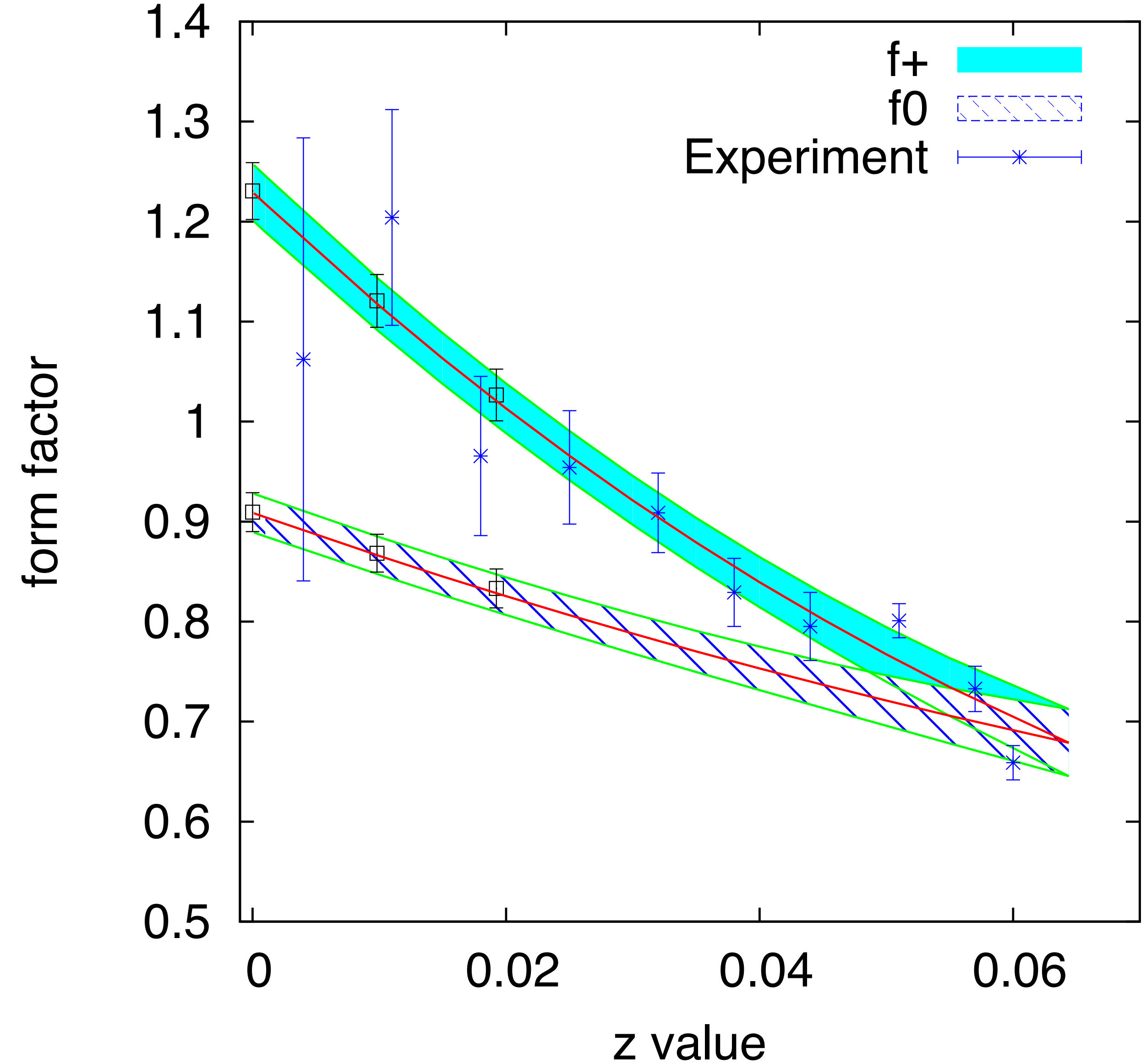
$$\sum_{n=0}^{N} |a_{i,n}|^2 \le 1$$

$|V_{cb}|$ Determination

Fermilab/MILC, arXiv:1312.0155



- Fit imposing kinematic constraint.

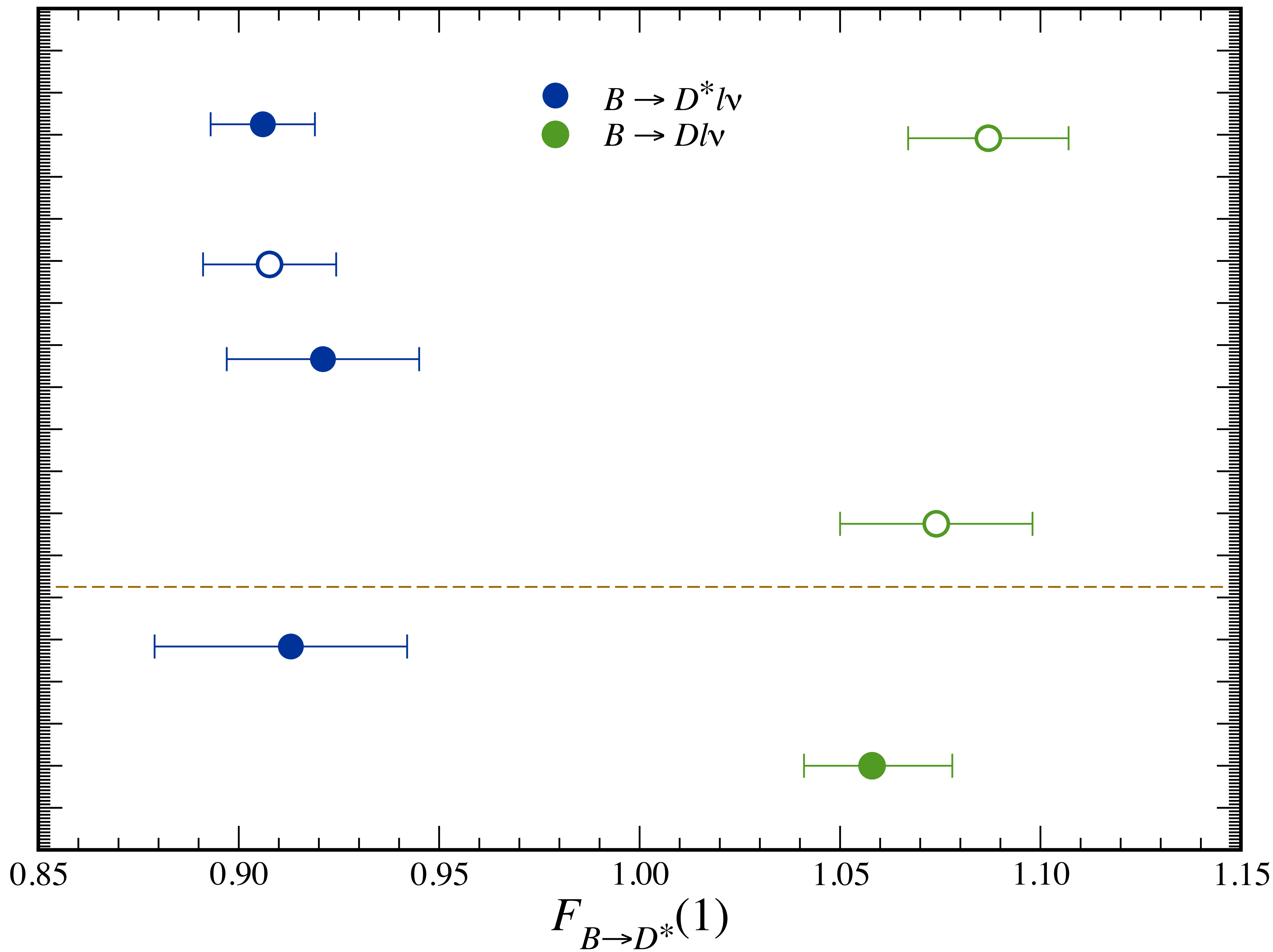


- Refit with floating norm for expt.

Consistency

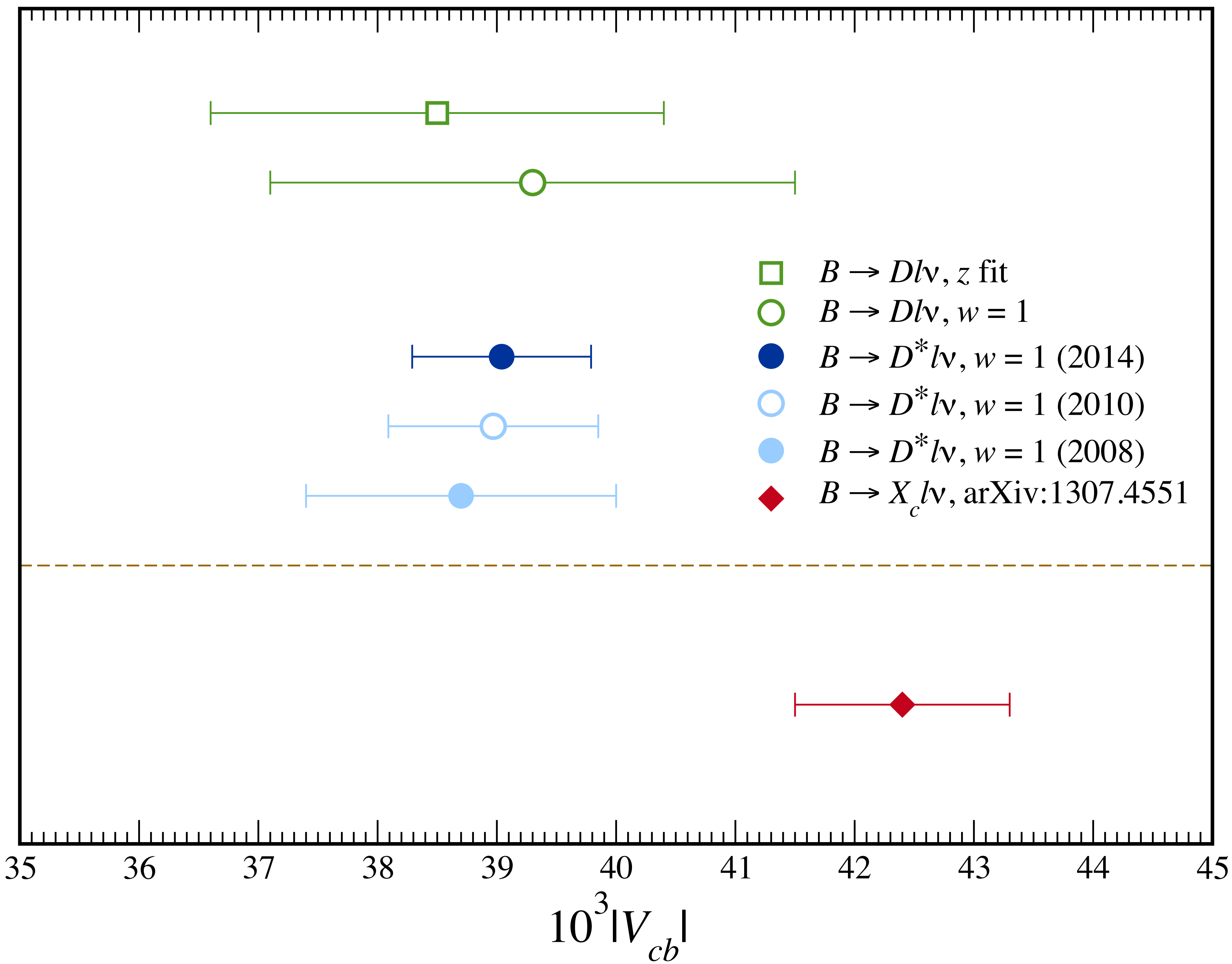
$n_f = 2 + 1$

quenched



Results for $|V_{cb}|$

$10^3|V_{cb}|$
$38.5 \pm 1.9_{\mathrm{exp+QCD}} \pm 0.2_{\mathrm{QED}}$
$39.3 \pm 2.2_{\mathrm{exp+QCD}} \pm 0.2_{\mathrm{QED}}$
$39.04 \pm 0.49 \pm 0.53 \pm 0.19$
$38.97 \pm 0.49 \pm 0.75 \pm 0.19$
$38.7 \pm 0.9 \pm 1.0 (\pm 0.19)$
42.42 ± 0.86
arXiv:1307.4551



$$B_s \to D_s l\nu$$

Strange Spectator

- Lattice QCD with strange quarks needs less of a chiral extrapolation.

- Atoui et alia [arXiv:1310.5238] compute zero-recoil form factors ($n_f = 2$):

$$\mathscr{G}(1) = 1.052(46), \quad \frac{f_0(q^2)}{f_+(q^2)} = 0.77(2), \quad \frac{f_T(q^2)}{f_+(q^2)} = 1.08(7)$$

at $q^2 = 11.6\ \mathrm{GeV}^2$.

- Employ a sequence of ratios designed to be close to 1, and guaranteed to be 1 when $m_h = m_c$ and when $m_h \to \infty$. Here $m_h = \lambda^h m_c$, such that $\lambda^9 m_c = m_b$.

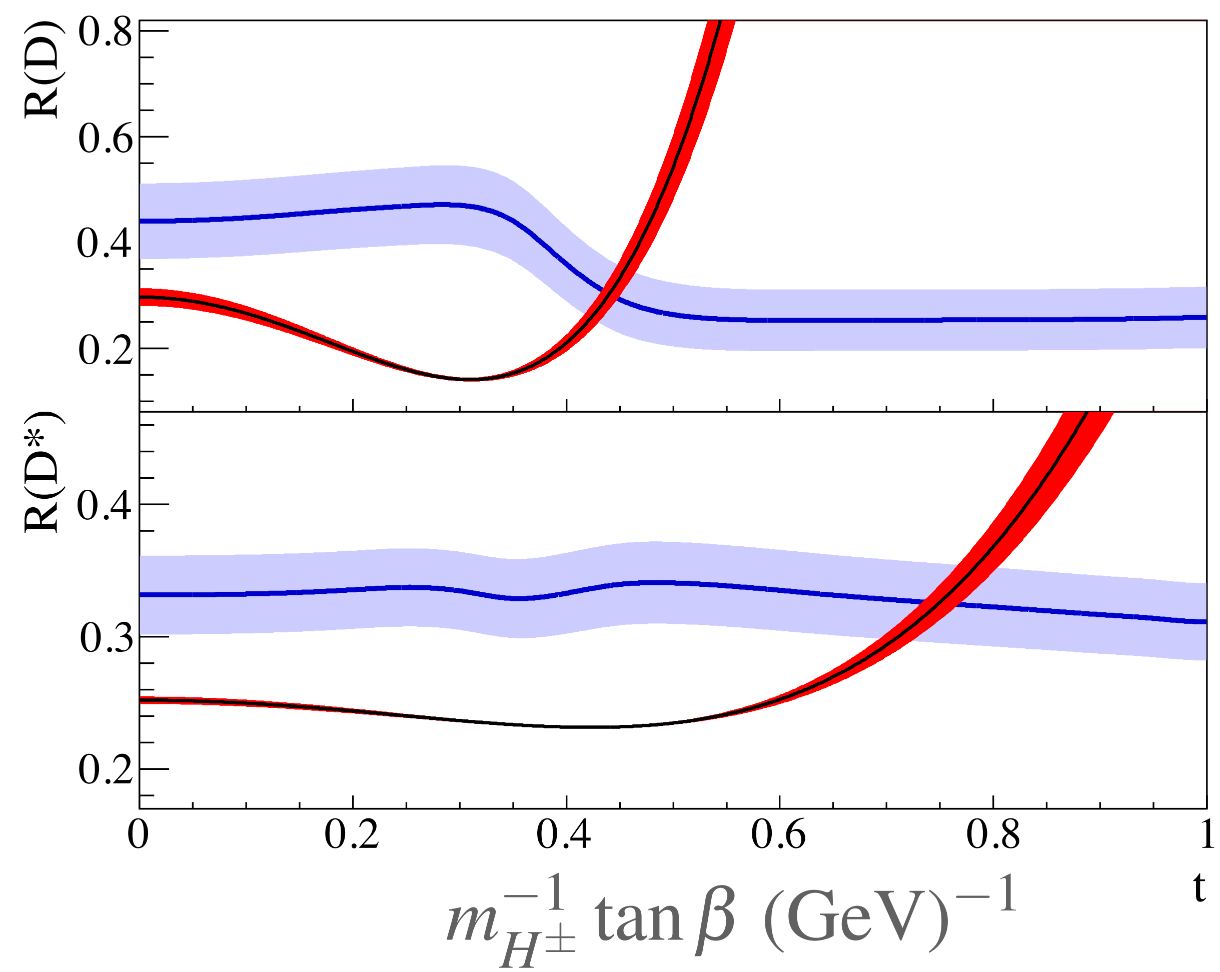
- Employed interpolations in $1/m_h$ omit logarithmic m_h dependence.

Extra form factors for non-SM $B \to D^{(*)}\tau\nu$

New Physics in $B \to D^{(*)}\tau\nu$

BaBar, arXiv:1205.5442

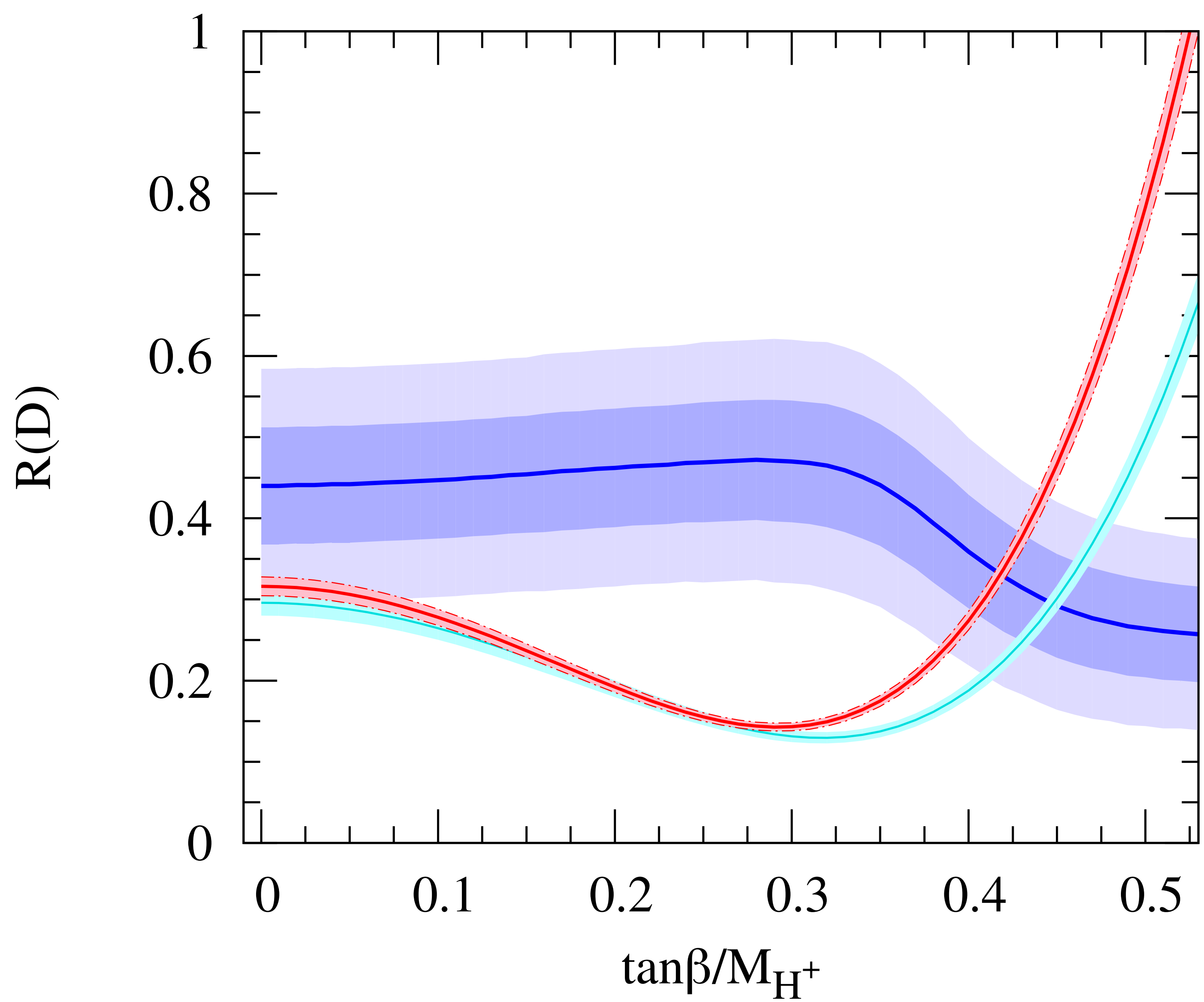
- BaBar has presented evidence for an excess in both channels:
    - 2.0σ for $R(D)$;
    - 2.7σ for $R(D^*)$;
    - 3.4σ combined.
- Using estimates of form factors from HQET and quenched QCD.



Form Factors for $B \to D^{(*)}\tau\nu$

Fermilab/MILC, arXiv:1206.4992

- With $n_f = 2 + 1$ lattice QCD, the tension lessens a bit:
  - 1.7σ for $R(D)$.
- Similar conclusions in arXiv:1206.4977.
- Analogous work for $R(D^*)$ yet to be carried out.



Other applications

Form Factor Ratios for $\mathrm{BR}(B_s \to \mu^+\mu^-)$

Fleischer, Serra, Tuning, arXiv:1004.3982, arXiv:1012.2784

- Use nonleptonic B and B_s decays to determine the fragmentation ratio f_s/f_d:
  - needed to measure $\mathrm{BR}(B_s \to \mu^+\mu^-)$ at hadron colliders.
- With a factorization assumption, need the following two ratios:

$$\frac{f_0^{(s)}(M_\pi^2)}{f_0^{(d)}(M_K^2)} \qquad \frac{f_0^{(s)}(M_\pi^2)}{f_0^{(d)}(M_\pi^2)}$$

- With a subset of the data used above for , Fermilab/MILC computed these ratios [arXiv:1202.6346], yielding

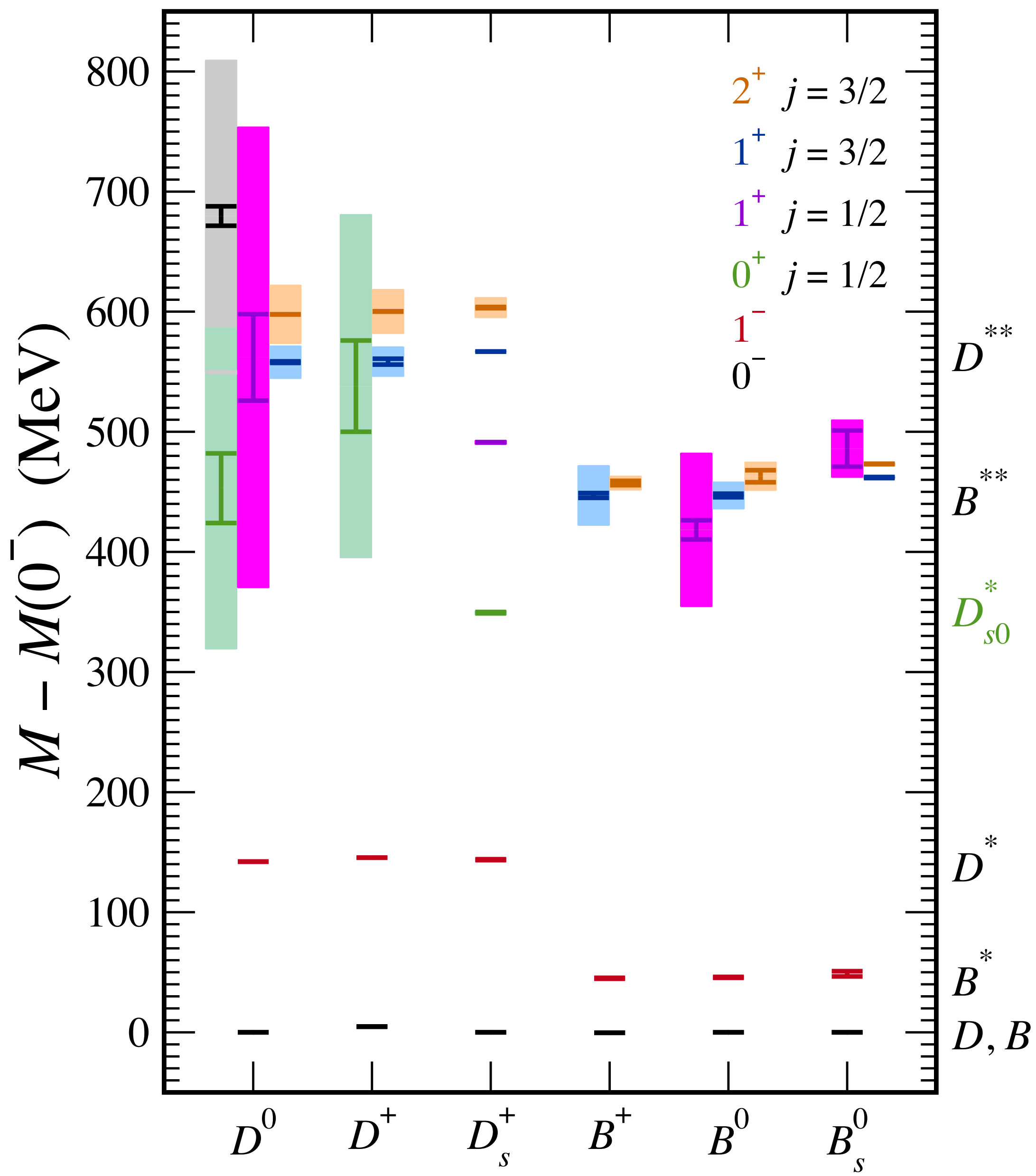
$$\frac{f_s}{f_d} = 0.283(27)_{\mathrm{stat}}(19)_{\mathrm{syst}}(24)_{\mathrm{theo}} \quad \text{or} \quad 0.286(16)_{\mathrm{stat}}(21)_{\mathrm{syst}}(26)_{\mathrm{lat}}(22)_{\mathrm{NE}}$$

which is competitive with other (experimental) methods.

$B \rightarrow D^{**} l\nu$ Calculations

Radial and Orbital Excitation Spectrum

- Decays to D^{**} seen in experiments.
- Rich excitation spectrum of D mesons:
  - $j = 3/2$ are wide (aka narrow);
  - $j = 1/2$ are very wide (aka wide).
- Poses re-scattering challenges in Euclidean finite volume:
  - spectroscopy still leading edge;
  - form factors in factorization, e.g., arXiv:1301.7336: $f_{D'}/f_D = 0.57(16)$.



Uraltsev Sum Rules

- Two form factors in $B \to D^{**} l\nu$: τ_j, $j = 3/2,\ 1/2$.
- Uraltsev sum rule: $U = \sum_n \left|\tau_{3/2}^{(n)}(1)\right|^2 - \left|\tau_{1/2}^{(n)}(1)\right|^2 = \frac{1}{4}$, sum over excitations.
- Pilot calculations of the form factors in static limit, with incomplete error budgets (as the authors state):
  - $U^{(0)} = 0.13(8)(?)$ [hep-lat/0406031];
  - $U^{(0)} = 0.17–0.21$ [arXiv:0903.2298].
- Suggests the lowest-lying contributions nearly saturate the sum rule.

Future calculations

b quarks as light quarks

- Until now, all lattice treatments of heavy quarks use HQET (even those who say they don't).

- But now, several lattice groups are planning (or have started) to generate a suite of ensembles with $m_b a < 1$, even $m_b a \ll 1$.

- This transition has already happened for lattice D physics, which now resembles K physics more so than B physics.

- Some of the uncertainties will simply disappear or simplify: matching driven by conserved densities; ensembles with physical light quarks; ....

- Remains to be seen how well the current round of calculations hold up.