NP-HQET

R. Sommer

based on work of the ALPHA Collaboration

F. Bahr, F. Bernardoni, B. Blossier, J. Bulava, M. Della Morte,
M. Donnellan, S. Dürr, P. Fritzsch, A. Gérardin, N. Garron, D. Hesse,
J. Heitger, G. von Hippel, A. Jüttner, A. Joseph, H. B. Meyer,
M. Papinutto, A. Ramos, J. Rolf, A. Shindler, H. Simma, T. Mendes

HELMHOLTZ | ASSOCIATION DESY SFB TR9 NIC ALPHA Collaboration

Why non-perturbative?

- EFTs such as HQET have power divergences

$$(aM)^{-n}$$

  which must be subtracted non-perturbatively in order to have a continuum limit

- Power (1/M) corrections are only defined when the leading term is computed non-perturbatively

- late asymptotics of QCD perturbation theory for heavy-light physics

All of this is taken care of by NP HQET:

NP matching of HQET and QCD
No predictions are lost

$\mathcal{O}_{\rm kin}(x) = \overline{\psi}_{\rm h}(x)\vec{D}^2\psi_{\rm h}(x)$

- power divergences
  $$\frac{g_0^{2L}}{a^n} \sim \frac{1}{\log(a\Lambda_{\rm QCD})^{L} a^n}$$
  need NP subtraction
  $n = 1, 2$

e.g.

$$(\mathcal{O}_{\rm kin})_{\rm R}(z) = Z_{\mathcal{O}_{\rm kin}}\left(\mathcal{O}_{\rm kin}(z) + \frac{c_1}{a}\,\overline{\psi}_{\rm h}(z)D_0\psi_{\rm h}(z) + \frac{c_2(g_0)}{a^2}\,\overline{\psi}_{\rm h}(z)\psi_{\rm h}(z)\right)$$

- power corrections
  $$(\alpha(m))^L \overset{m\to\infty}{\gg} \frac{\Lambda_{\rm QCD}}{m}$$
  need NP leading terms to define power corrections

It is in general not enough to compute Wilson coefficients in perturbation theory

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- late asymptotics of QCD perturbation theory for heavy-light physics ← discuss a bit more

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On QCD PT for heavy-light systems

On QCD PT for heavy-light systems

at the leading order in 1/M

mass dependence has a PT expansion

NP constant

$$\Phi_{\rm RGI} = \exp\left\{-\int^{\bar{g}(\mu)} dx\, \frac{\gamma(x)}{\beta(x)}\right\} \Phi(\mu) = \underbrace{Z_{\rm RGI}(g_0)}_{\text{known, ALPHA Collaboration}} \times \underbrace{\Phi(g_0)}_{\text{bare ME}}$$

$$\equiv \left[2b_0\bar{g}(\mu)^2\right]^{-\gamma_0/2b_0} \exp\left\{-\int_0^{\bar{g}(\mu)} dx \left[\frac{\gamma(x)}{\beta(x)} - \frac{\gamma_0}{b_0 x}\right]\right\} \Phi(\mu)$$

$$\Phi^{\rm QCD} = C_{\rm PS}(M/\Lambda) \times \Phi_{\rm RGI} \;\; + \mathrm{O}(1/M)$$

γ : AD in HQET

$$C_{\rm PS}(M/\Lambda) = \exp\left\{\int^{g_\star(M/\Lambda)} dx\, \frac{\gamma_{\rm match}(x)}{\beta(x)}\right\}$$

with

Λ : Lambda-para

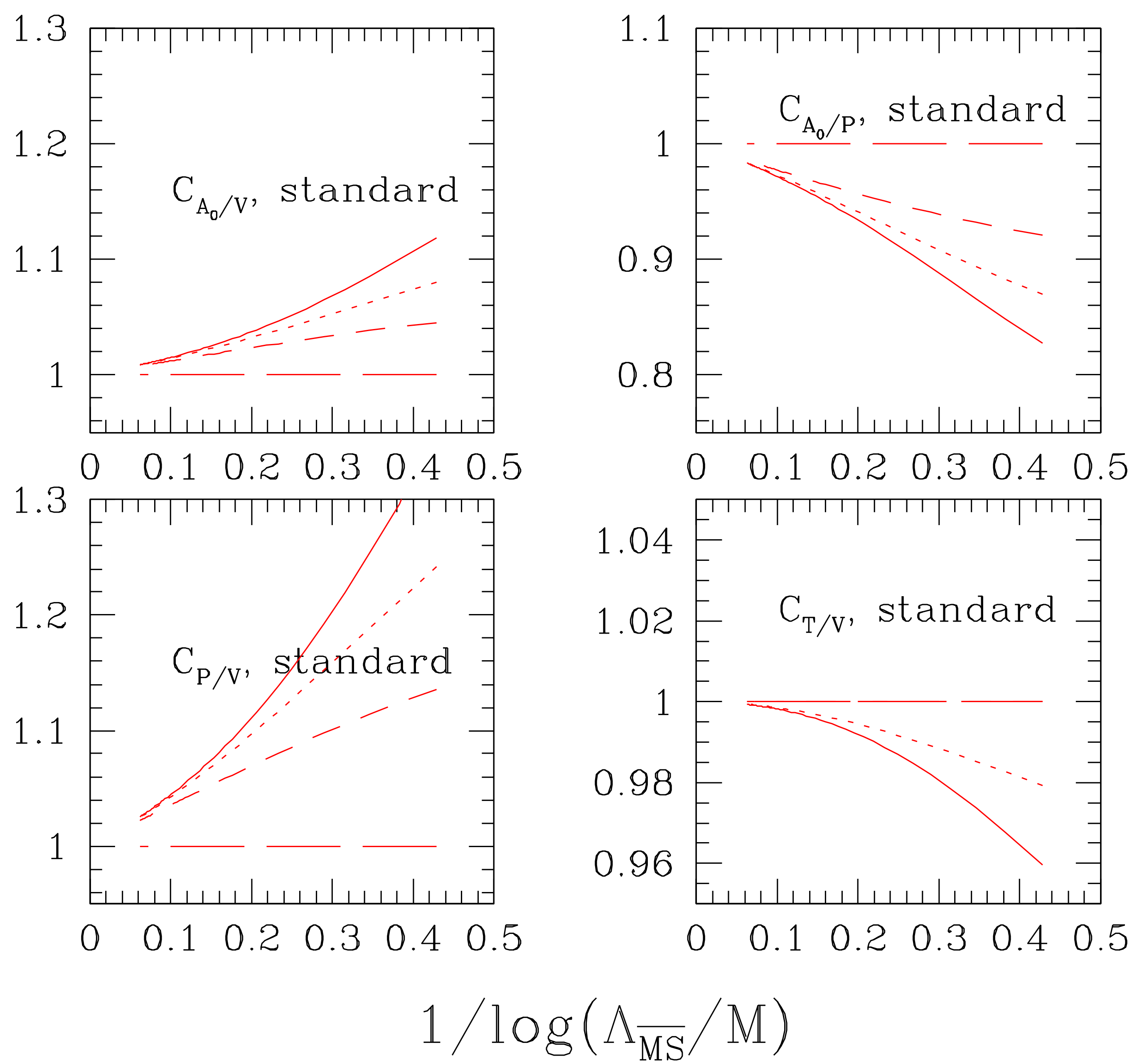
$$\frac{\Lambda}{M} = \exp\left\{-\int^{g_\star(M/\Lambda)} dx\, \frac{1-\tau(x)}{\beta(x)}\right\}, \quad \to \quad g_\star(M/\Lambda)$$

M : RGI quark mass

$\gamma_{\rm match}$: describes the mass dependence

$g_\star$: $\mu = m_\star = \overline{m}(m_\star)\,,\; g_\star = \bar{g}(m_\star)$

Compare different orders



We actually show $C_{\Gamma/\Gamma'} = C^{\Gamma}_{\rm match}(m,\mu)/C^{\Gamma'}_{\rm match}(m,\mu)$

B-physics: $\Lambda_{\overline{\rm MS}}/M_{\rm b} \approx 0.04$
$-1/\log(\Lambda_{\overline{\rm MS}}/M_{\rm b}) \approx 0.3$

Perturbation theory is badly behaved
for charm quarks very badly
$-1/\log(\Lambda_{\overline{\rm MS}}/M_{\rm c}) \approx 0.5$

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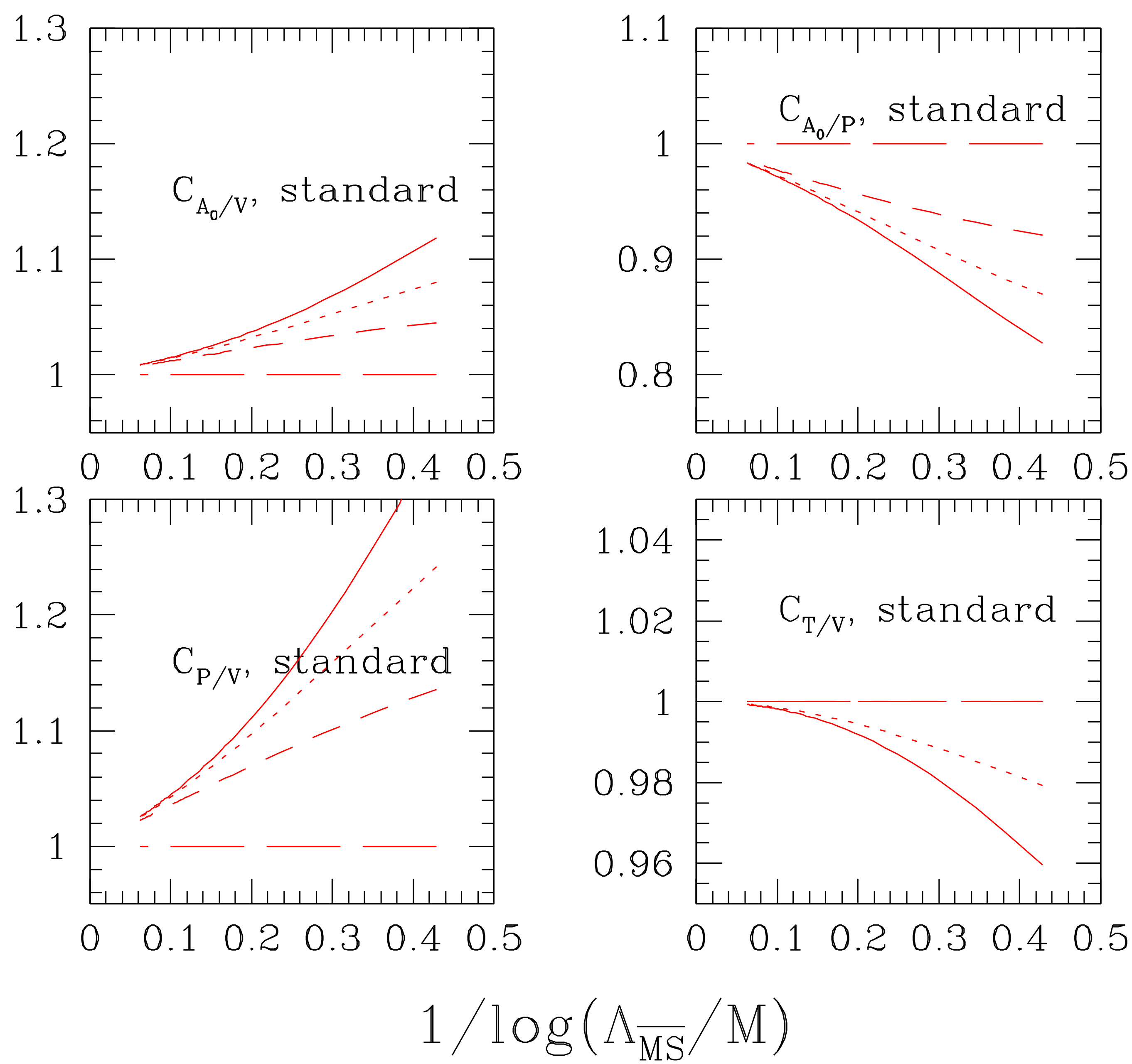

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- b-mass not in asymptotic convergence region

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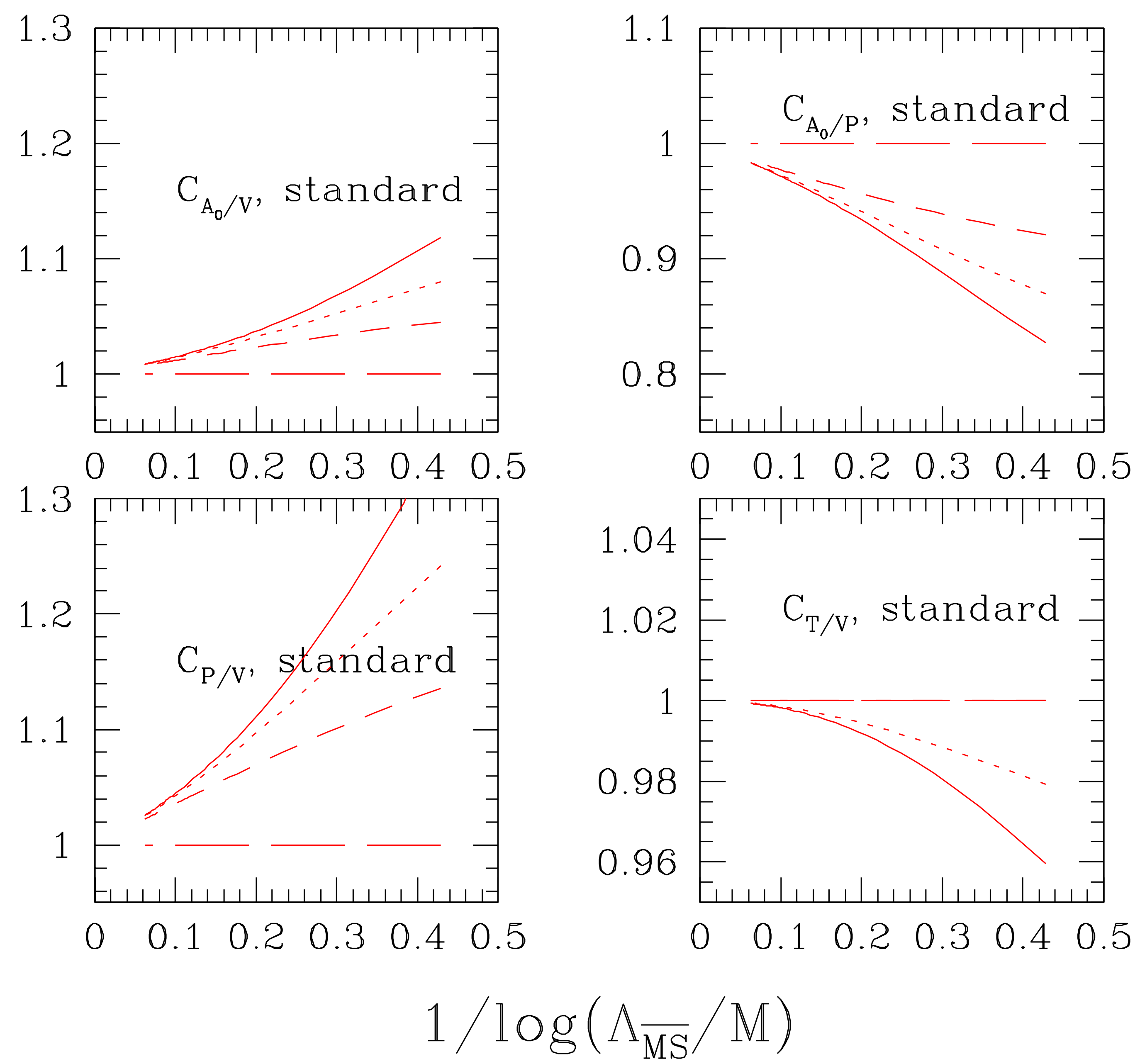

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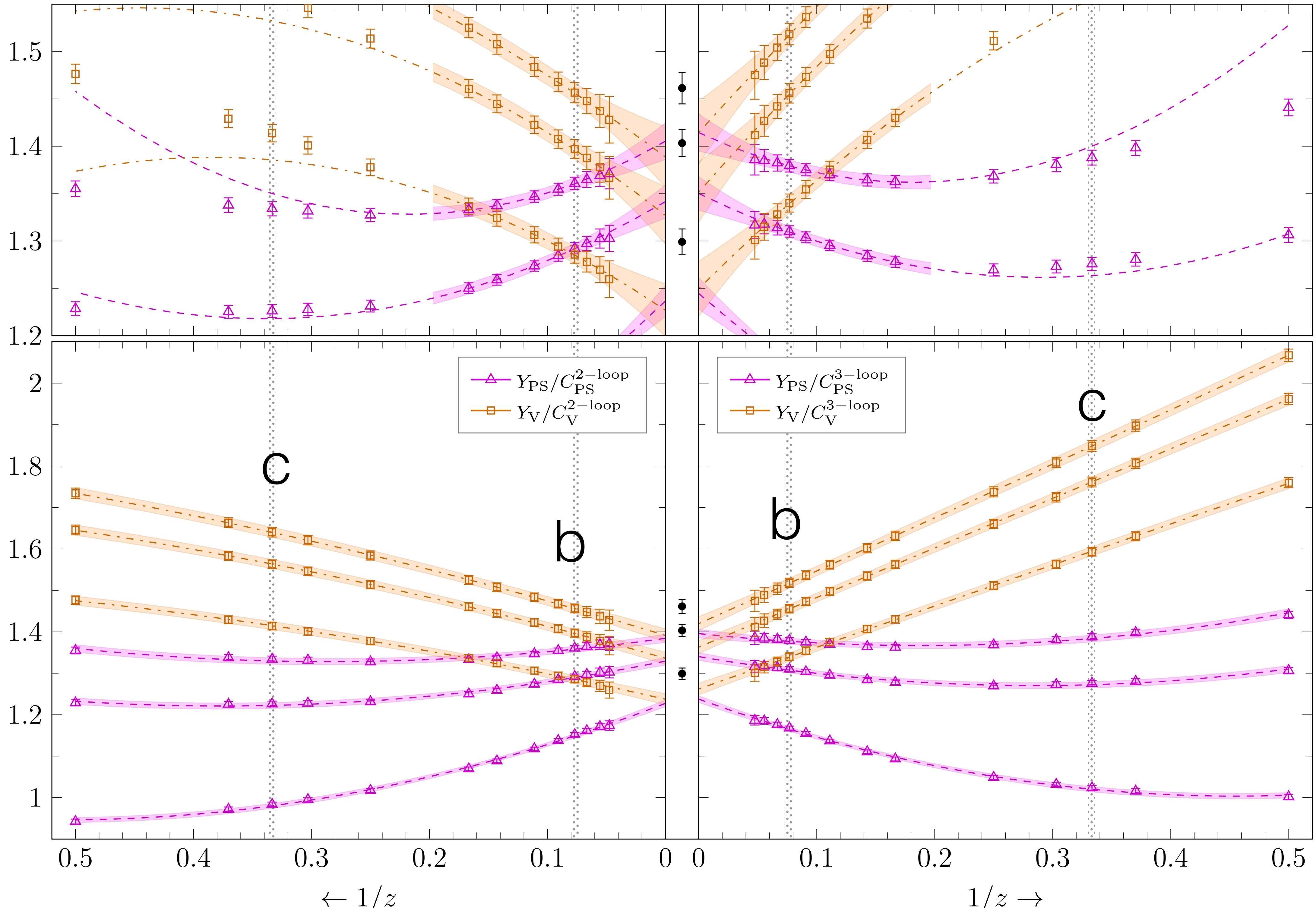
- b-mass not in asymptotic convergence region
- This is a worry for perturbative matching and renormalisation

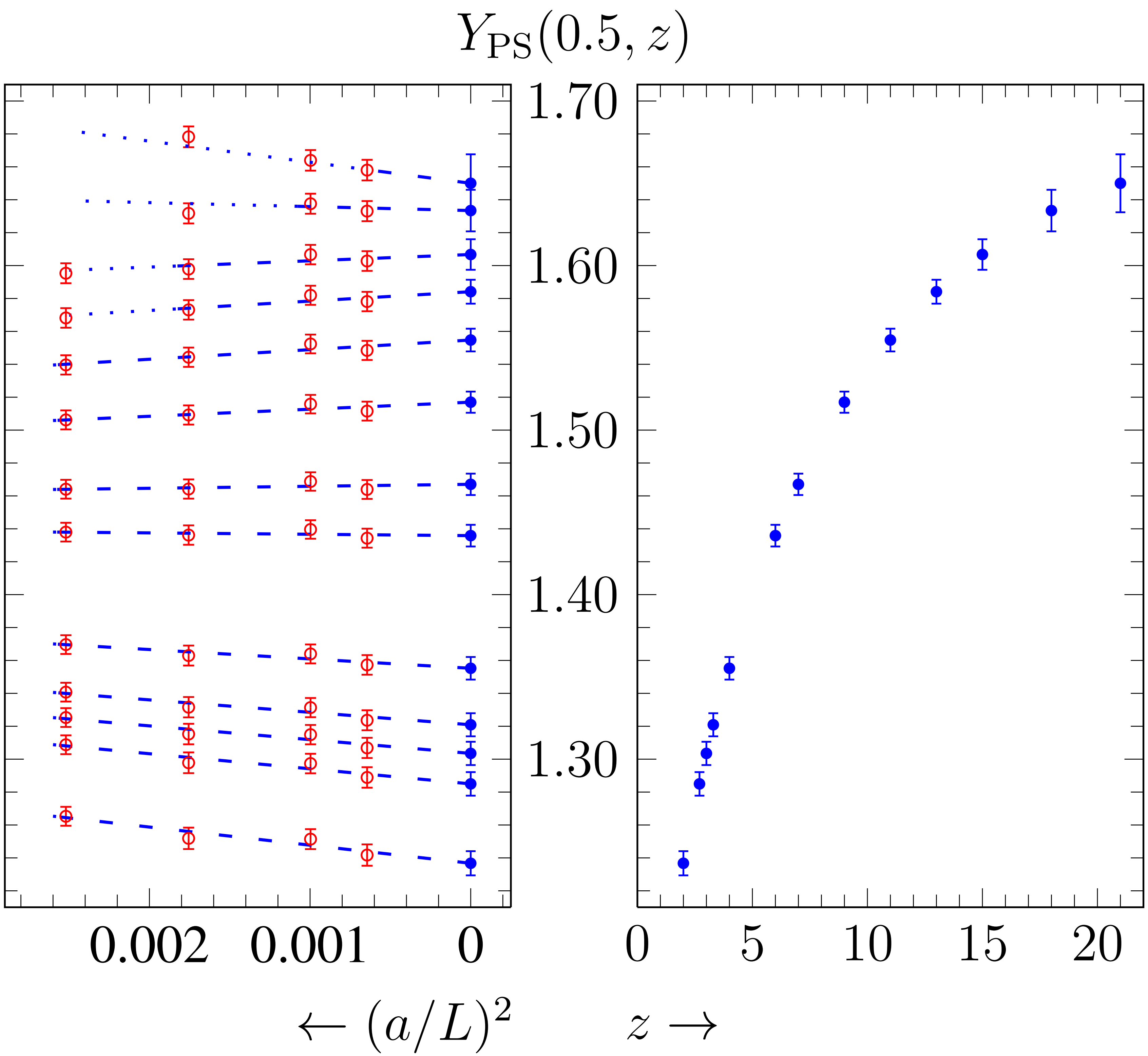
Compare extrapolation of QCD — direct static

P. Fritzsch, N. Garron and J. Heitger (not yet published): finite volume matrix elements of HL currents



z=M L_1, L_1=0.5fm

(Based on continuum extrapolations)



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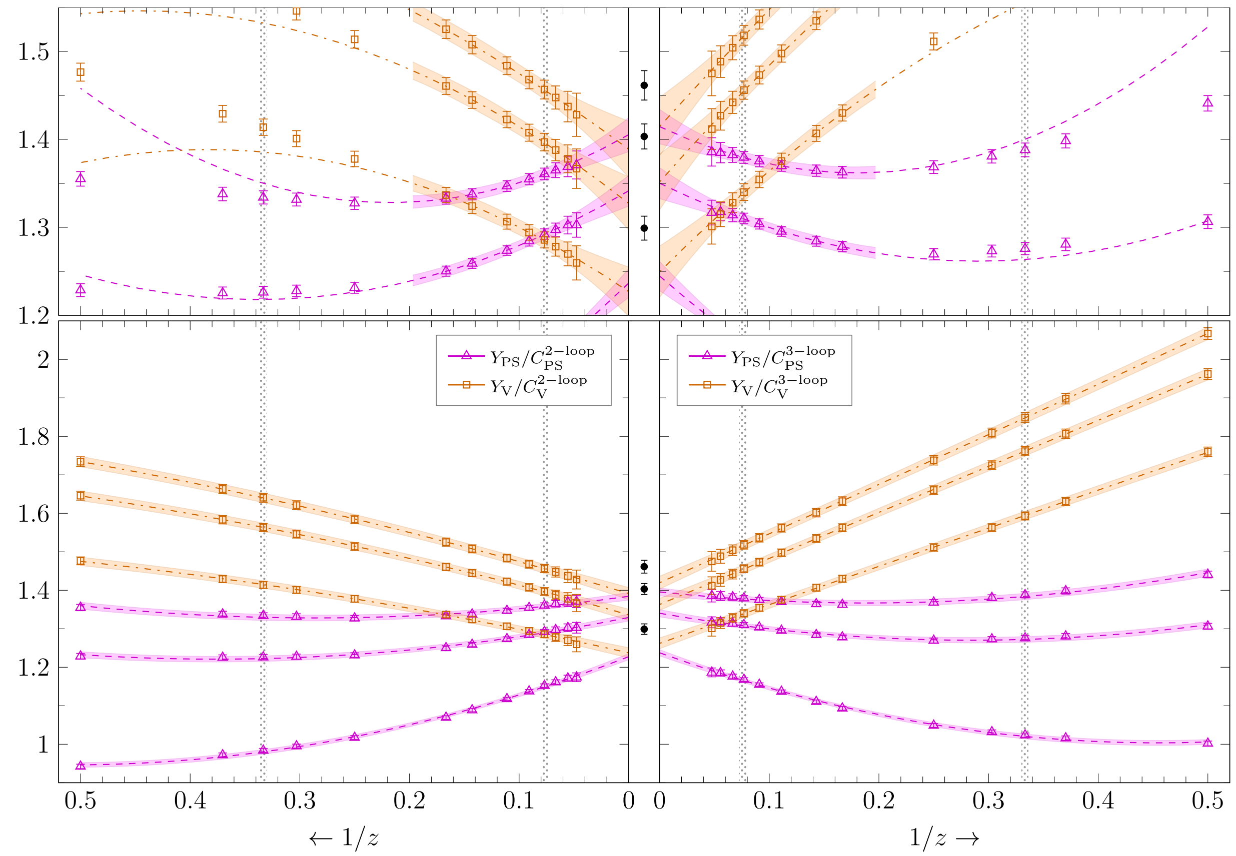

- picture looks different depending on the order of PT
- extrapolation to static limit not that convincing
- what error to associate to perturbative matching

NP HQET

Path integral with weight (directly on the lattice)

$$W_{\rm HQET} \equiv \exp(-a^4 \sum_x [\mathcal{L}_{\rm light}(x) + \mathcal{L}_{\rm h}^{\rm stat}(x)]) \times \left\{ 1 + a^4 \sum_x (\omega_{\rm kin}\, \mathcal{O}_{\rm kin}(x) + \omega_{\rm spin}\, \mathcal{O}_{\rm spin}(x)) \right\}$$

This yields

$$\langle \mathcal{O} \rangle = \langle \mathcal{O} \rangle_{\rm stat} + \omega_{\rm kin} a^4 \sum_x \langle \mathcal{O} \mathcal{O}_{\rm kin}(x) \rangle_{\rm stat} + \omega_{\rm spin} a^4 \sum_x \langle \mathcal{O} \mathcal{O}_{\rm spin}(x) \rangle_{\rm stat} \equiv \langle \mathcal{O} \rangle_{\rm stat} + \omega_{\rm kin} \langle \mathcal{O} \rangle_{\rm kin} + \omega_{\rm spin} \langle \mathcal{O} \rangle_{\rm spin}\,,$$

with

$$\langle \mathcal{O} \rangle_{\rm stat} = \frac{1}{\mathcal{Z}} \int_{\rm fields} \mathcal{O} \exp(-a^4 \sum_x [\mathcal{L}_{\rm light}(x) + \mathcal{L}_{\rm h}^{\rm stat}(x)]) \quad \Leftarrow$$

renormalizable

$$\mathcal{L}_{\rm h}^{\rm stat} = \overline{\psi}_{\rm h}\, [D_0 + \delta m]\, \psi_{\rm h}$$

The weight is expanded because then the theory is renormalizable

$$[\, \mathcal{O}_{\rm kin}(x) = \overline{\psi}_{\rm h}(x)\, \mathbf{D}^2\, \psi_{\rm h}(x)\,, \quad \mathcal{O}_{\rm spin}(x) = \overline{\psi}_{\rm h}(x)\, \boldsymbol{\sigma} \cdot \mathbf{B}(x)\, \psi_{\rm h}(x)\ ]$$

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with

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NP matching: QCD — HQET

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A finite volume, recursive strategy

NP matching: QCD — HQET

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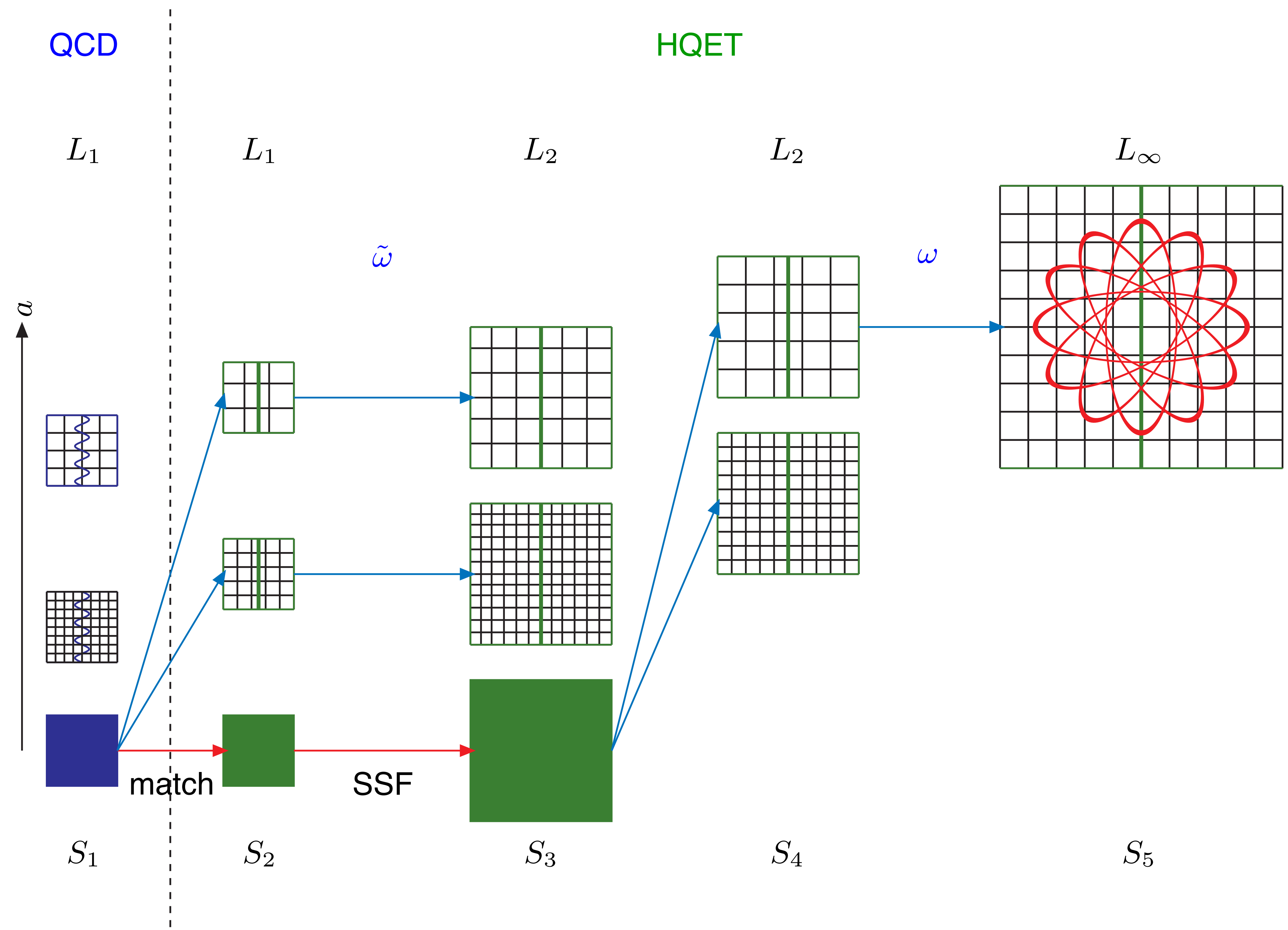


$L_1 \approx 0.5\,\mathrm{fm}$ (now)

NP matching: QCD = HQET

1. Lagrangian + currents

QCD — HQET parameter free — HQET bare “correlation functions” — HQET parameters

$$\Phi_i^{\rm QCD}(L, m_{\rm h}, 0) \;= \eta_i(L,a) + \varphi_i^j(L,a)\,\omega_j(M,a) + \mathcal{O}(1/m_{\rm h}^2)$$

(M=mh)

structure:

$$\varphi = \left(\begin{array}{c|c|c|c|c} \varphi_1^1 & * & * & 0 & 0 \\ \hline 0 & * & 0 & 0 & 0 \\ \hline 0 & * & * & 0 & 0 \\ \hline 0 & * & 0 & * & 0 \\ \hline 0 & * & 0 & 0 & * \end{array}\right)$$

mass | kin, spin | current 1 | current 2 | current 3

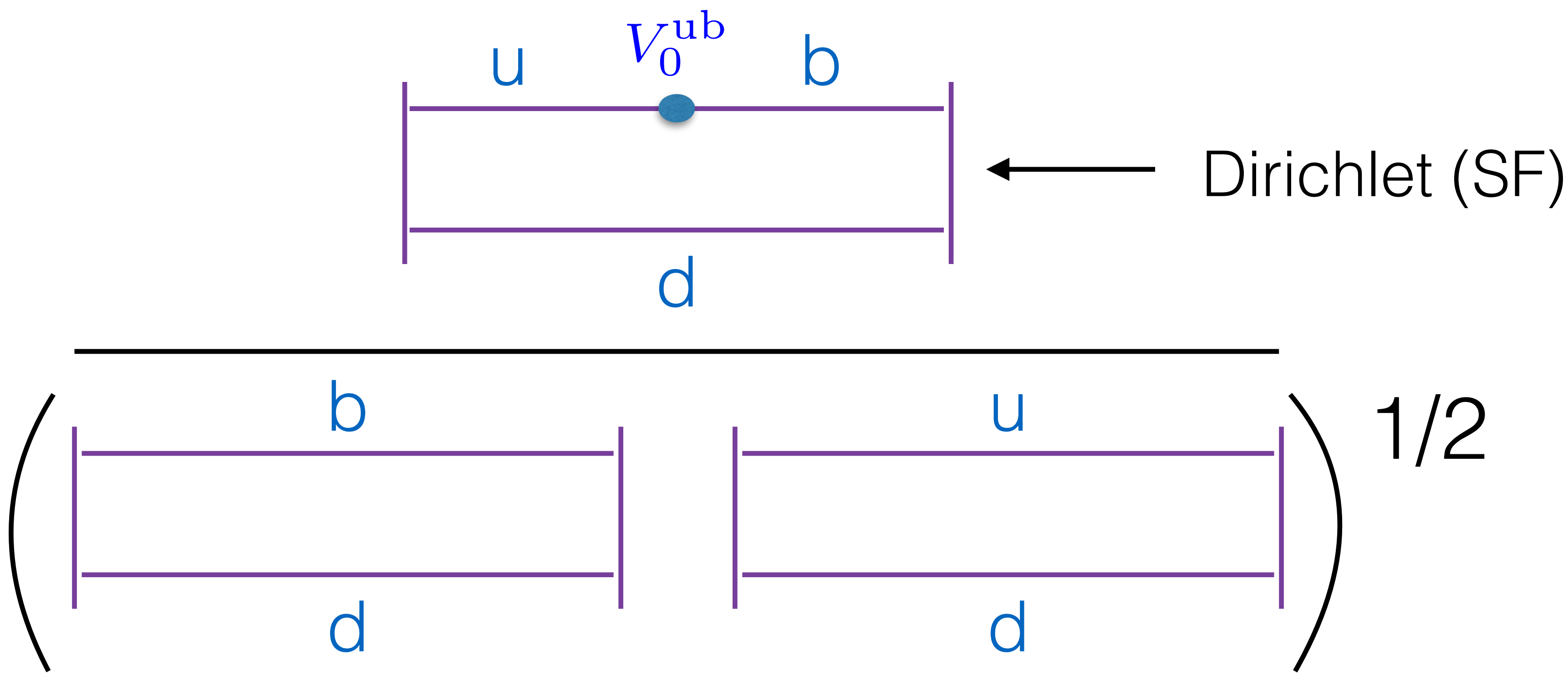
notation from

Michele Della Morte, Samantha Dooling, Jochen Heitger, Dirk Hesse, Hubert Simma JHEP 1405 (2014) 060

NP matching: QCD — HQET

2. Example of a Φ_i

$$\Phi^{\mathrm{QCD}} \sim -Z_{\mathrm{V}} \frac{F_{\mathrm{V}_0}(T/2;\theta,z)}{[F_1^{\mathrm{ud}}(\theta)F_1^{\mathrm{bd}}(\theta,z)]^{1/2}}$$



Dirk Hesse, Rainer Sommer JHEP 1302 (2013) 115

θ/L quark momentum kinematical parameter

NP matching: QCD — HQET

3. complete set of parameters with heavy-light flavour currents

i	ω_i	origin
1, 2, 3	$m_{\rm bare},\ \omega_{\rm kin},\ \omega_{\rm spin}$	$\mathscr{L}^{\rm HQET}$
4, …, 6	$c_{{\rm A}_{0,1}},\ c_{{\rm A}_{0,2}},\ \ln Z^{\rm HQET}_{A_0}$	$A_0^{\rm HQET}$
7, …, 11	$c_{{\rm A}_{k,1}},\ c_{{\rm A}_{k,2}},\ c_{{\rm A}_{k,3}},\ c_{{\rm A}_{k,4}},\ \ln Z^{\rm HQET}_{\vec{\rm A}}$	$A_k^{\rm HQET}$
12 …, 14	$c_{{\rm V}_{0,1}},\ c_{{\rm V}_{0,2}},\ \ln Z^{\rm HQET}_{V_0}$	$V_0^{\rm HQET}$
15, …, 19	$c_{{\rm V}_{k,1}},\ c_{{\rm V}_{k,2}},\ c_{{\rm V}_{k,3}},\ c_{{\rm V}_{k,4}},\ \ln Z^{\rm HQET}_{\vec{\rm V}}$	$V_k^{\rm HQET}$

Michele Della Morte, Samantha Dooling, Jochen Heitger, Dirk Hesse, Hubert Simma

JHEP 1405 (2014) 060

Status

- determination of action, time component of axial current N_f=2
- strategy for action+all currents
  - tree level investigation
  - one-loop investigation
  - decision on kinematical parameters
- results for
  - quark mass, decay constants
- preliminary static computation for B_s -> K

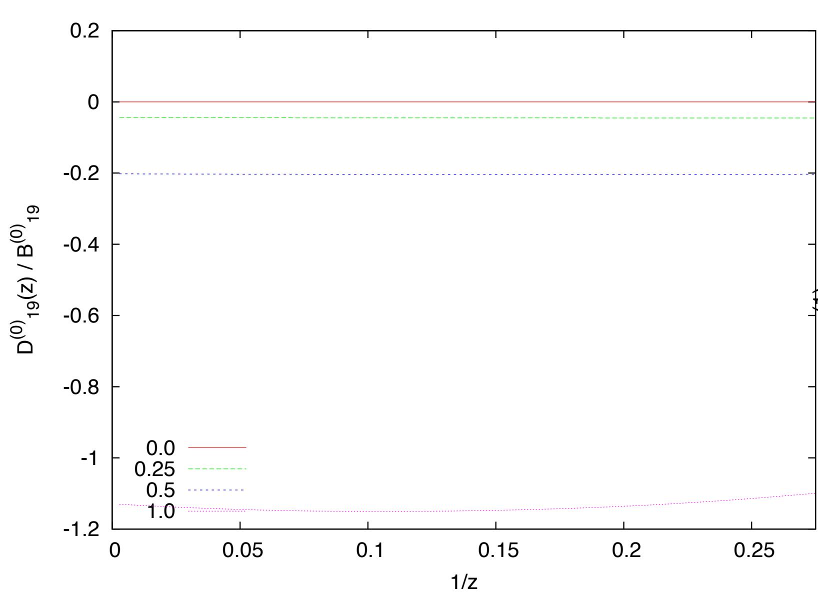
Investigation of matching conditions

$$\Phi^{\rm QCD}(z) = B + C\frac{1}{z} + O(1/z^2) \qquad X = X^{(0)} + X^{(1)}g^2 + \ldots$$

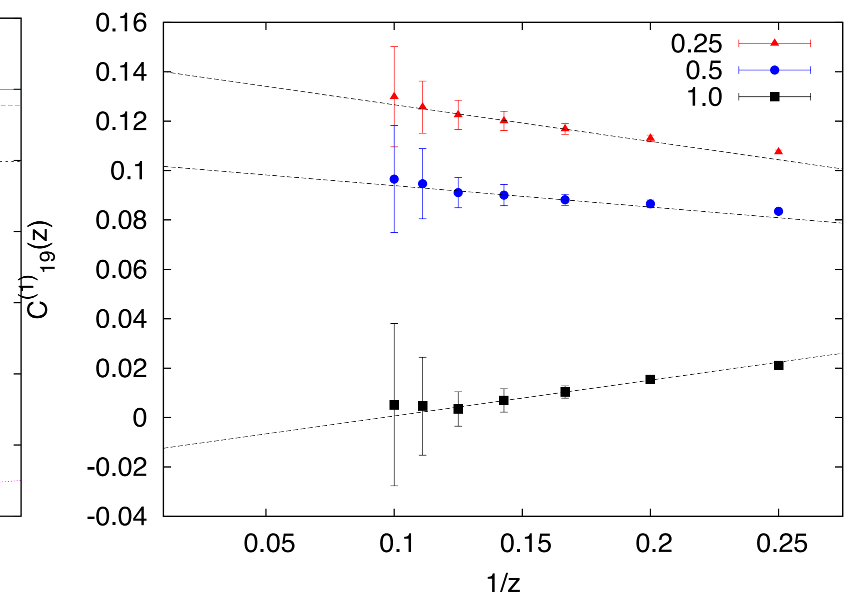
How large is $O(1/z^2)$?

$$B = \lim_{z\to\infty} \Phi^{\rm QCD}(z) \qquad C = \lim_{z\to\infty} z\,[\Phi^{\rm QCD}(z) - B] \qquad D = \lim_{z\to\infty} z^2\,[\Phi^{\rm QCD}(z) - B - C\frac{1}{z}]$$

$D^{(0)}/B^{(0)}$



$C^{(1)}$



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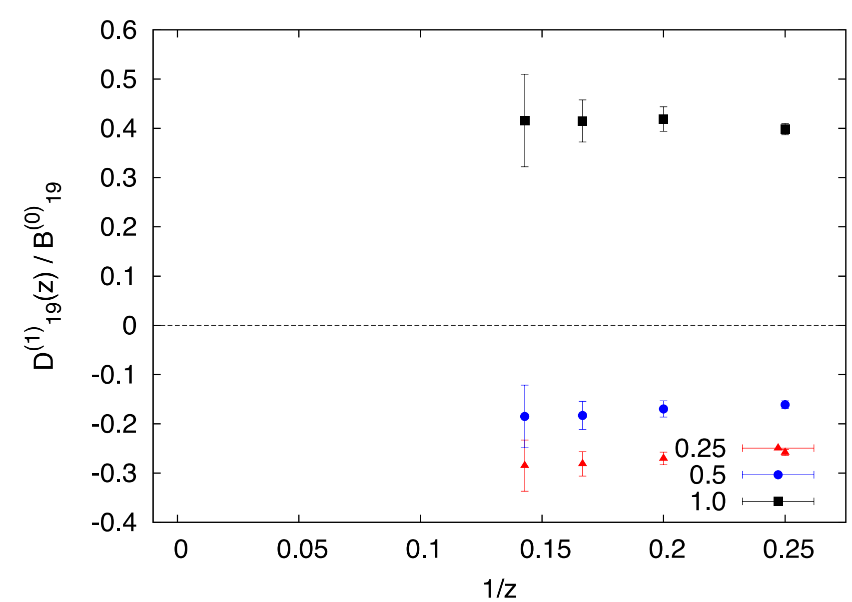

Michele Della Morte, Samantha Dooling, Jochen Heitger, Dirk Hesse, Hubert Simma **JHEP 1405 (2014) 060**

Hesse + S.; P. Korcyl (unpublished)

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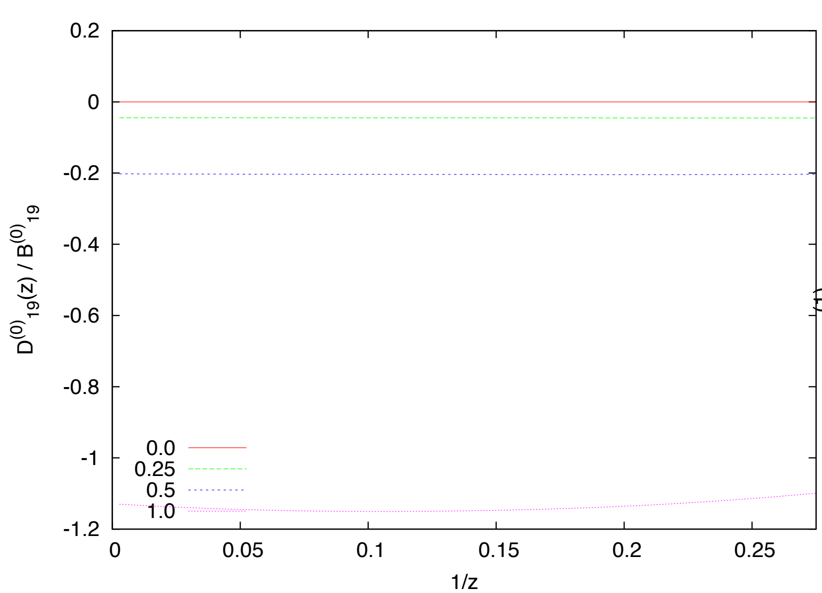
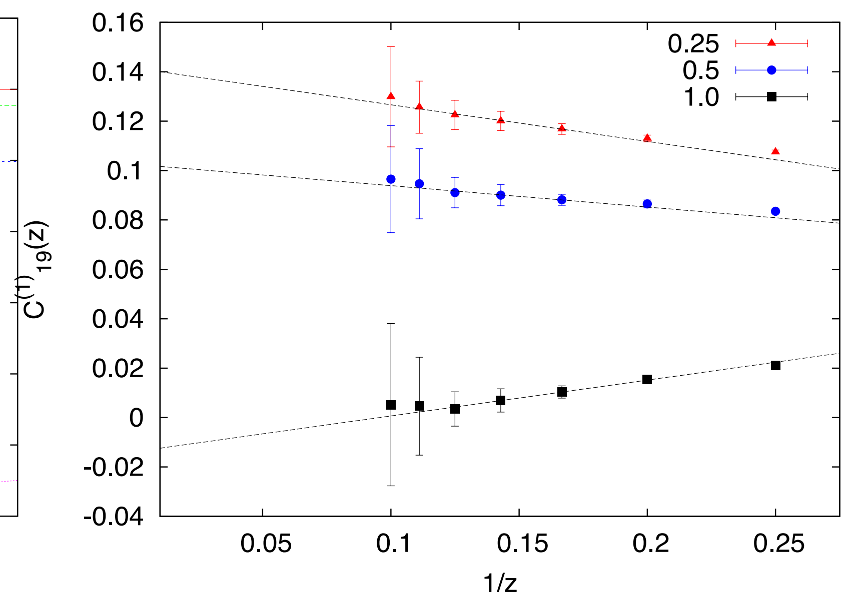
no log(z)/z

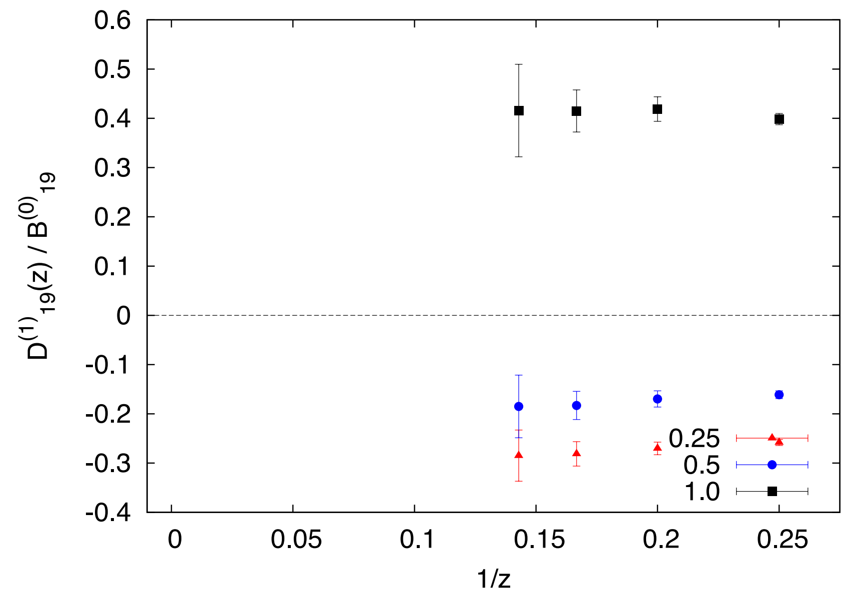
$D^{(0)}/B^{(0)}$



$C^{(1)}$



$D^{(1)}/B^{(0)}$



Michele Della Morte, Samantha Dooling, Jochen Heitger, Dirk Hesse, Hubert Simma — JHEP 1405 (2014) 060

Hesse + S.; P. Korcyl (unpublished)

Results: 1. A Summary of published results

Precision lattice QCD computation of the $B^*B\pi$ coupling

Fabio Bernardoni[a], John Bulava[b], Michael Donnellan[a], Rainer Sommer[a] arXiv:1404.6951

The b-quark mass from non-perturbative $N_\mathrm{f} = 2$ Heavy Quark Effective Theory at $\mathrm{O}(1/m_\mathrm{h})$

ALPHA Collaboration

Fabio Bernardoni[a], Benoît Blossier[b], John Bulava[c], Michele Della Morte[d], Patrick Fritzsch[e,*], Nicolas Garron[c], Antoine Gérardin[b], Jochen Heitger[f], Georg von Hippel[g], Hubert Simma[a], Rainer Sommer[a]

Physics Letters B 730 (2014) 171–177

Decay constants of B-mesons from non-perturbative HQET with two light dynamical quarks

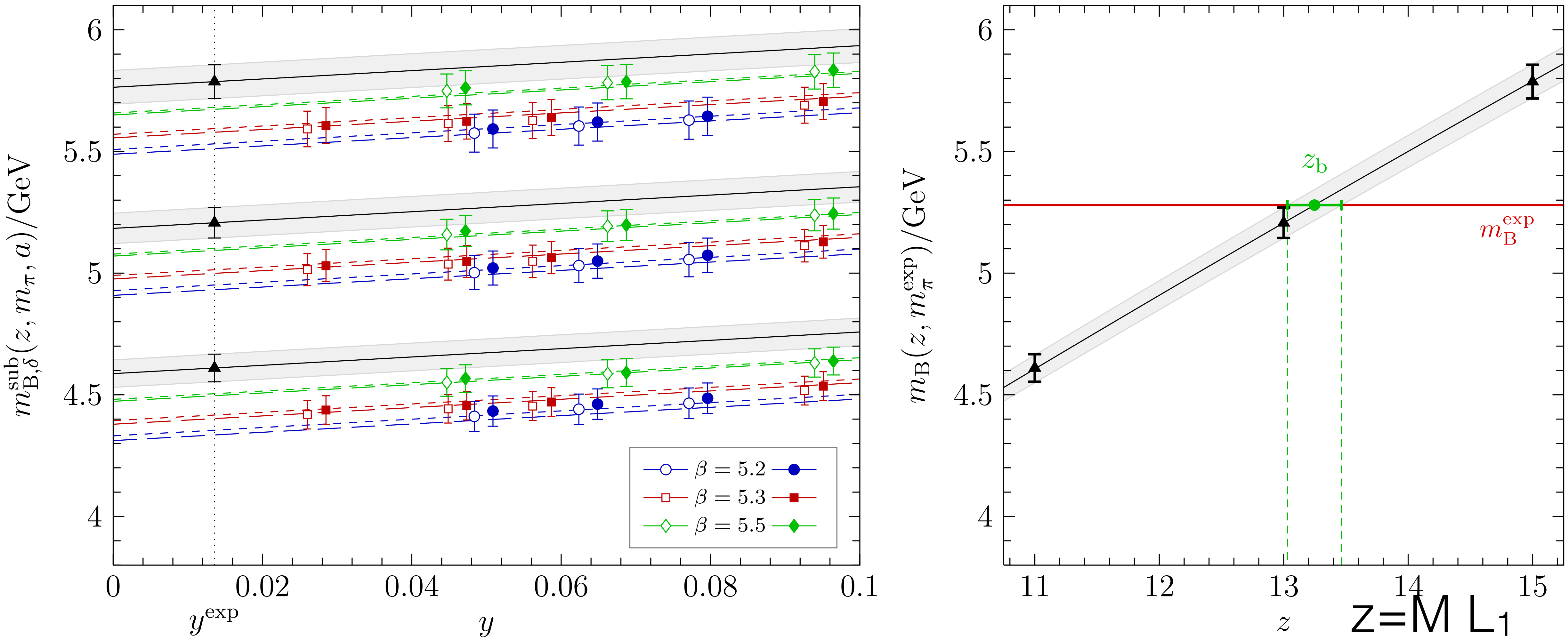
ALPHA Collaboration

Fabio Bernardoni[a], Benoît Blossier[b], John Bulava[c], Michele Della Morte[d,e], Patrick Fritzsch[f,*], Nicolas Garron[c], Antoine Gérardin[b], Jochen Heitger[g], Georg von Hippel[h], Hubert Simma[a], Rainer Sommer[a]

Physics Letters B 735 (2014) 349–356

Results: parameters, b-quark mass, N_f=2

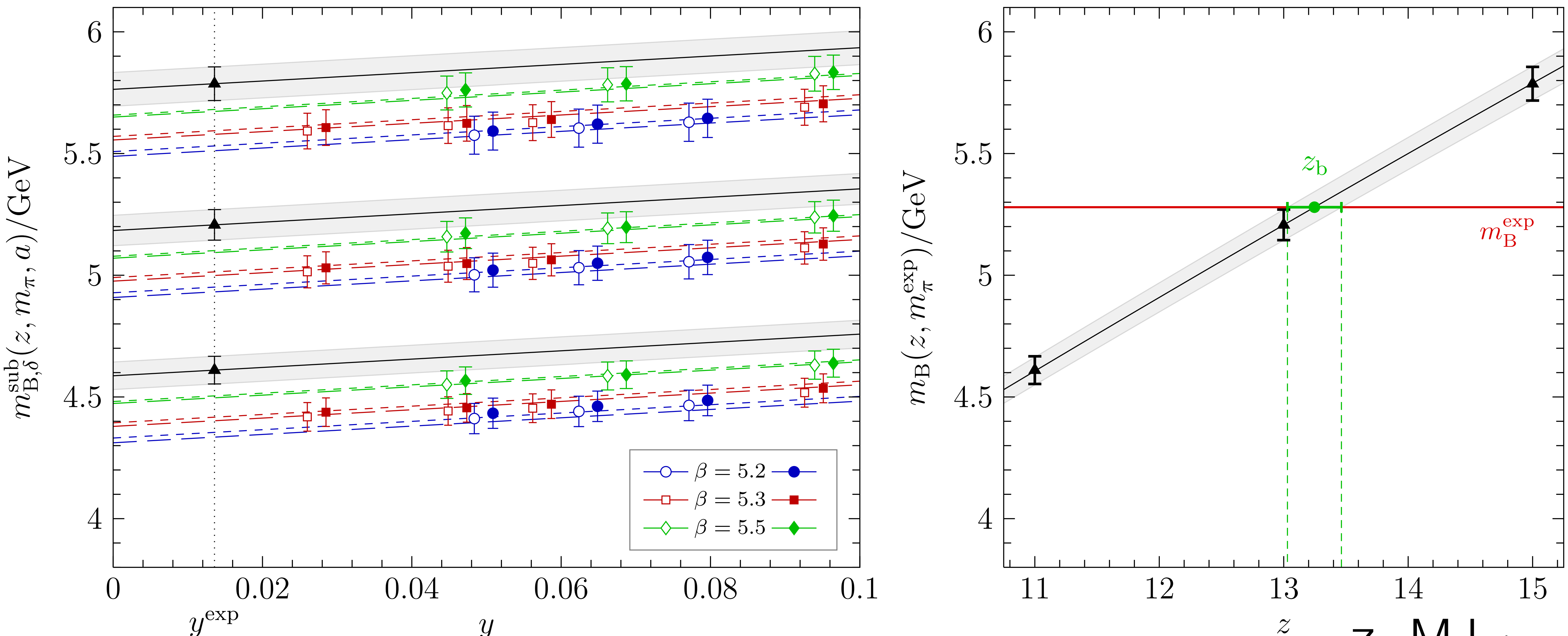
$$m^{\mathrm{sub}}_{\mathrm{B},\delta}(z,y,a) \equiv m_{\mathrm{B},\delta}(z,m_\pi,a) + \frac{3\hat{g}^2}{16\pi}\left(\frac{m_\pi^3}{f_\pi^2} - \frac{(m_\pi^{\mathrm{exp}})^3}{(f_\pi^{\mathrm{exp}})^2}\right)$$



N_f	Ref.	M	$\overline{m}_{\overline{\mathrm{MS}}}(\overline{m}_{\overline{\mathrm{MS}}})$	$\overline{m}_{\overline{\mathrm{MS}}}(4\,\mathrm{GeV})$	$\overline{m}_{\overline{\mathrm{MS}}}(2\,\mathrm{GeV})$	$\Lambda_{\overline{\mathrm{MS}}}$[MeV]
0	[36]	6.76(9)	4.35(5)	4.39(6)	4.87(8)	238(19) [69]
2	this work	6.58(17)	4.21(11)	4.25(12)	4.88(15)	310(20) [55]
5	PDG13 [1]	7.50(8)	4.18(3)	4.22(4)	4.91(5)	212(8) [1]

Results: 1. A Summary of published results

- determine parameters, determine the b-quark mass, N_f=2



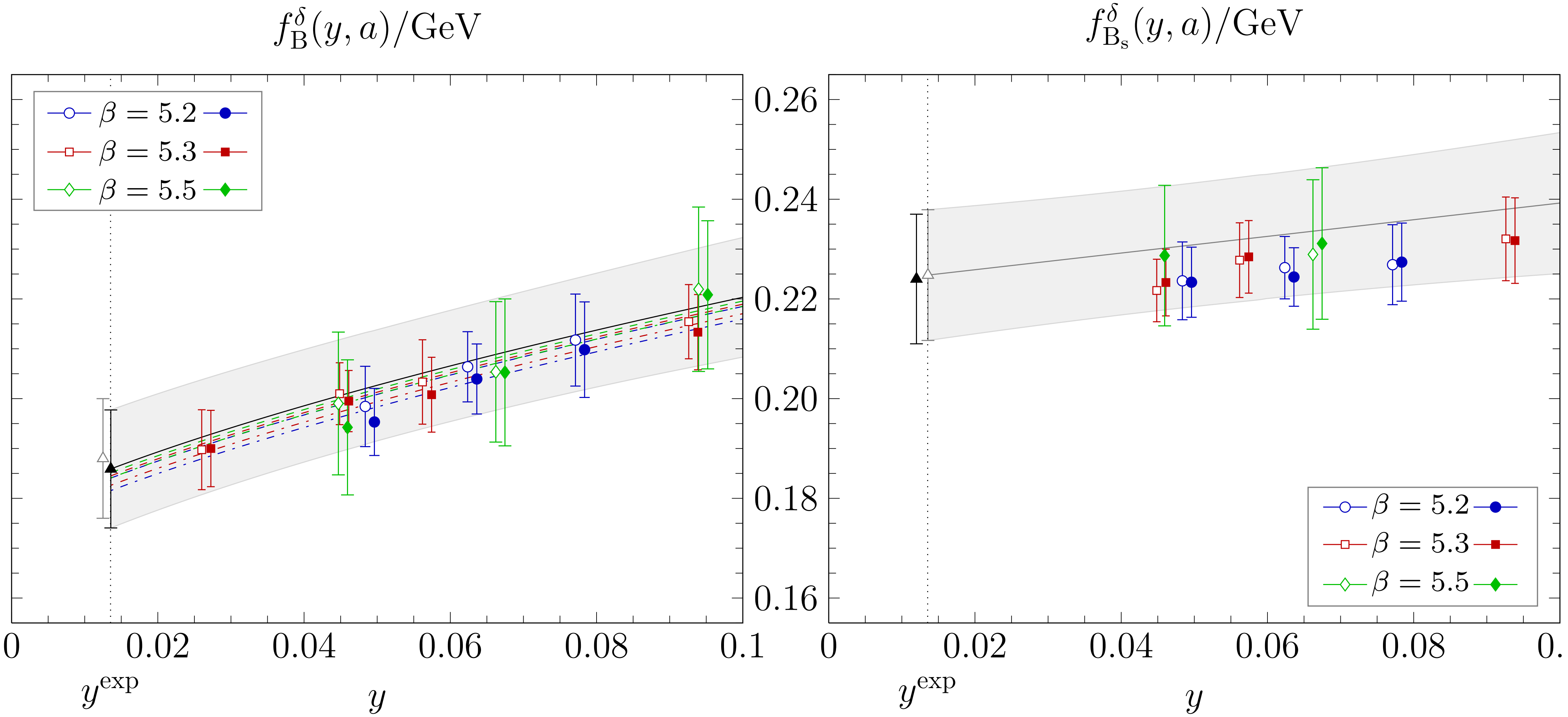
$z = M L_1$

Partial contributions $(\sigma_i/\sigma)^2$ to the accumulated error σ of z_b. Only error sources contributing with a relative squared uncertainty $(\sigma_i/\sigma)^2 > 0.5\%$ are listed. The ensemble A3 did not appear in Table 1 since it enters through the scale setting procedure [54,55] only.

Source i	A3	G8	N5	N6	O7	Z_A	ω^{HQET}
$(\sigma_i/\sigma)^2$ [%]	1.2	0.9	2.6	5.9	5.6	20.6	61.6

Results: 1. A Summary of published results

- determine decay constants, N_f=2



Results: 1. A Summary of published results

- determine decay constants, N_f=2

$$f_{\mathrm{B}}^{\mathrm{stat}} = 190(5)(2)_\chi \ \mathrm{MeV}, \qquad \frac{f_{\mathrm{B_s}}^{\mathrm{stat}}}{f_{\mathrm{B}}^{\mathrm{stat}}} = 1.189(24)(30)_\chi,$$

$$f_{\mathrm{B_s}}^{\mathrm{stat}} = 226(6)(9)_\chi \ \mathrm{MeV}.$$

$$f_{\mathrm{B}} = 186(13) \ \mathrm{MeV}, \qquad f_{\mathrm{B_s}}/f_{\mathrm{B}} = 1.203(65),$$

$$f_{\mathrm{B_s}} = 224(14) \ \mathrm{MeV}.$$

- tiny NLO (1/M) corrections

- the same for the quark mass:

$$\left[\overline{m}_{\mathrm{b}}^{\overline{\mathrm{MS}}}\left(\overline{m}_{\mathrm{b}}^{\overline{\mathrm{MS}}}\right)\right]^{\mathrm{stat}} = 4.21(11) \ \mathrm{GeV} \qquad \overline{m}_{\mathrm{b}}^{\overline{\mathrm{MS}}}\left(\overline{m}_{\mathrm{b}}^{\overline{\mathrm{MS}}}\right) = 4.21(11) \ \mathrm{GeV}$$

- there are other indications that HQET is an excellent (asymptotic) expansion for b-quarks at appropriate kinematics

There are more and interesting applications to come

From talk F. Bahr at CKM 2014 (last week):

Form factors for $B_s \to K\ell\nu$ decays in Lattice QCD

Felix Bahr

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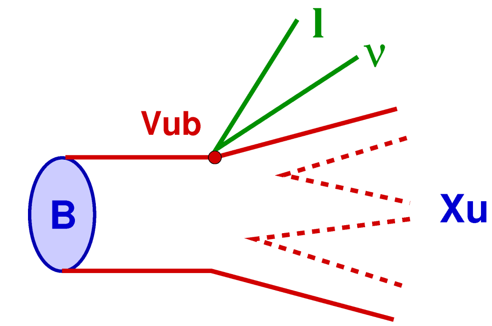
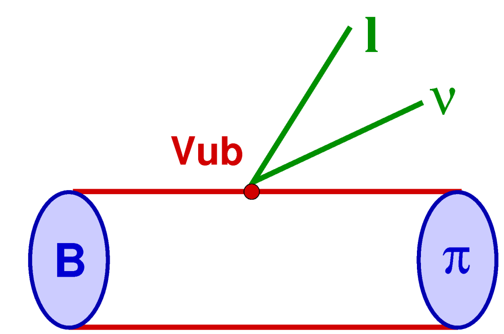
September 10, 2014

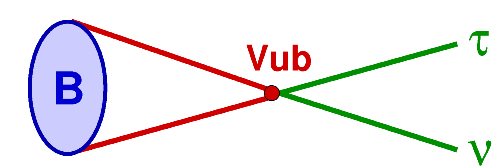
In collaboration with: F. Bernardoni, J. Bulava, A. Joseph, A. Ramos, H. Simma, R. Sommer

V_{ub} puzzle

- Determination of $|V_{ub}|$
- $\sim 3\sigma$ discrepancy [PDG] :
  - Inclusive $B \to X_u \ell \nu$:
    $V_{ub} = (4.41 \pm 0.15^{+0.15}_{-0.17}) \times 10^{-3}$
  - Exclusive $B \to \pi \ell \nu$: $V_{ub} = (3.28 \pm 0.29) \times 10^{-3}$
  - from $B \to \tau \nu$ via f_B: $V_{ub} = (4.22 \pm 0.42) \times 10^{-3}$
- theoretical and experimental input needed
- This talk: Non-perturbative determination of form factors for $B_s \to K \ell \nu$ decay







Based on a lot of complicated theory (assumptions)

e.g. HMrstCh PT

e.g. HPChPT inspired factorization of Eq. (19) allows a simultaneous chiral, continuum, and kinematic extrapolation of lattice data at arbitrary energies. Because the chi-

Our approach to semi-leptonic decays

Our approach to semi-leptonic decays

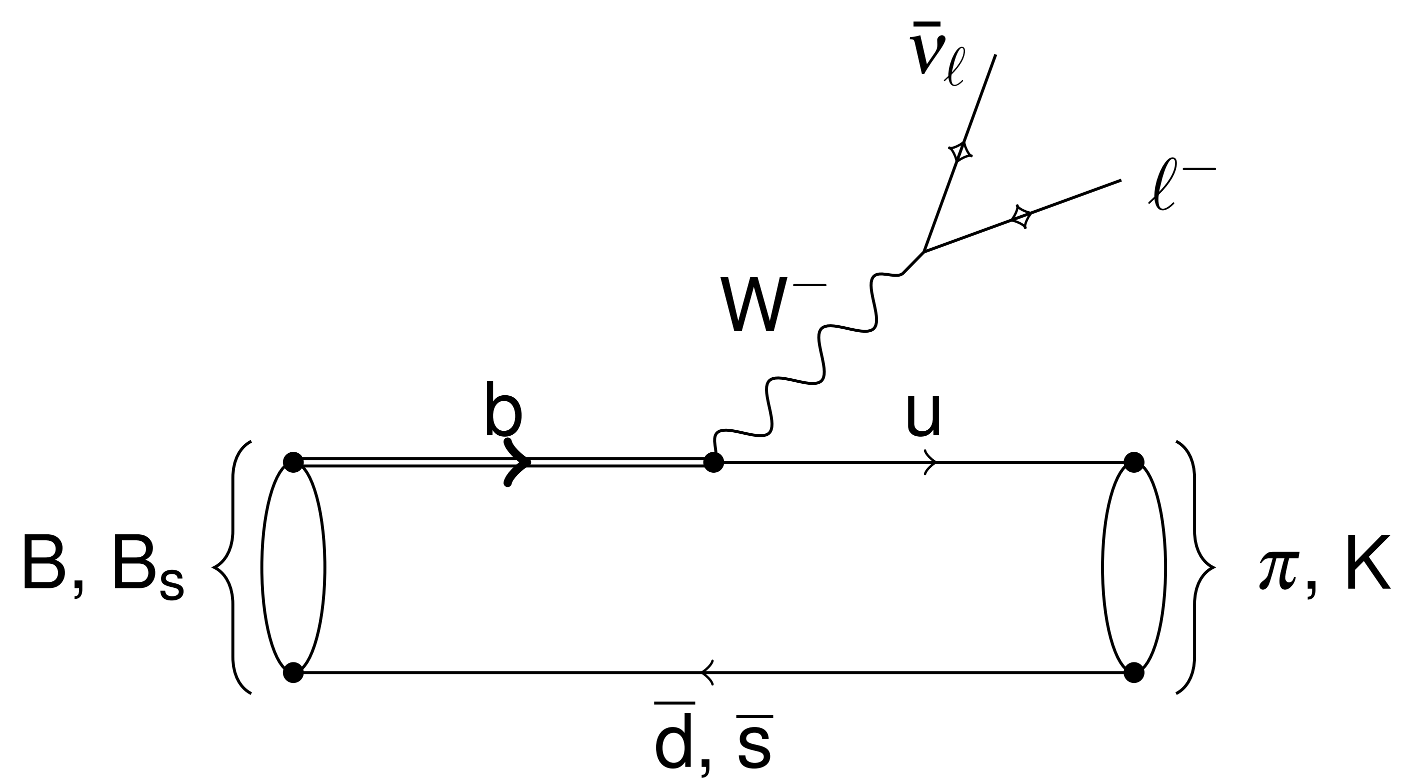
- fixed kinematics (q^2)
- improved Wilson fermions
- HQET at (N)LO for b-quark (NP matched)
- maybe separate chiral and continuum extrapolation

- At the moment we have just a check
  - and still 2 dynamical quarks
  - and only the leading order in 1/M
  - and renormalisation only as
    (worry about PT exists)

$$\Phi^{\mathrm{QCD}} = C_{\mathrm{V}}(M/\Lambda)\Phi^{\mathrm{RGI}}$$

3-loop PT

Semi-leptonic decays $\mathrm{B} \to \pi\ell\nu$, $\mathrm{B_s} \to \mathrm{K}\ell\nu$



$\mathrm{B_s} \to \mathrm{K}$:

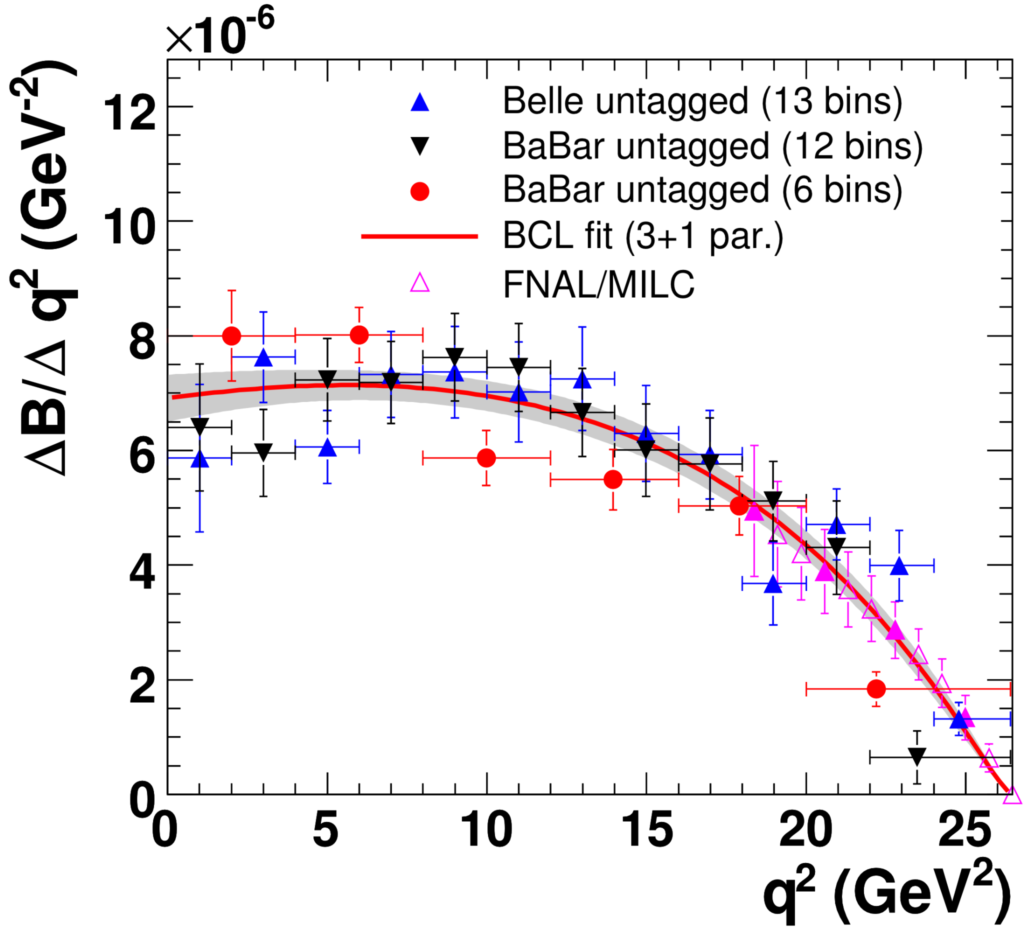
- no experimental data *yet* – predictions
- easier on the lattice (valence $m_K = m_K^{\mathrm{phys}}$ computationally less expensive than for the π)
- not far from $\mathrm{B} \to \pi$

$$\langle \mathrm{K}(p_\mathrm{K}^\mu) | V^\mu | \mathrm{B_s}(p_{\mathrm{B_s}}^\mu) \rangle = f_+(q^2) \left[p_{\mathrm{B_s}}^\mu + p_\mathrm{K}^\mu - \frac{m_{\mathrm{B_s}}^2 - m_\mathrm{K}^2}{q^2} q^\mu \right] + f_0(q^2) \frac{m_{\mathrm{B_s}}^2 - m_\mathrm{K}^2}{q^2} q^\mu$$

Experimental decay rates

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_{B_s}^3} \lambda^{3/2}(q^2) \left| f_+(q^2) \right|^2$$

$$\lambda(q^2) = (m_{B_s}^2 + m_K^2 - q^2)^2 - 4 m_{B_s}^2 m_K^2$$



- experimentally measured decay rate
- form factor $f_+(q^2)$ computed in LQCD
- $\Rightarrow$ determine V_{ub}

Parameterisation of $f(q^2) \times V_{\rm ub}$

Our ultimate plan:
BCL-Parameterisation [Bourrely, Caprini, Lellouch '09] :

$$f_+(q^2) = \frac{1}{1 - q^2/m^2_{B_s^*}} \sum_{k=0}^{K-1} b_k \left[z^k(q^2) - (-1)^{k-K} \frac{k}{K} z^K(q^2) \right]$$

- Correlated, combined fit of our data and experimental data
- Minimise $\chi^2 = \chi^2_{\rm th} + \chi^2_{\rm exp}$
- fit parameters b_k, $V_{\rm ub}$

Extrapolations

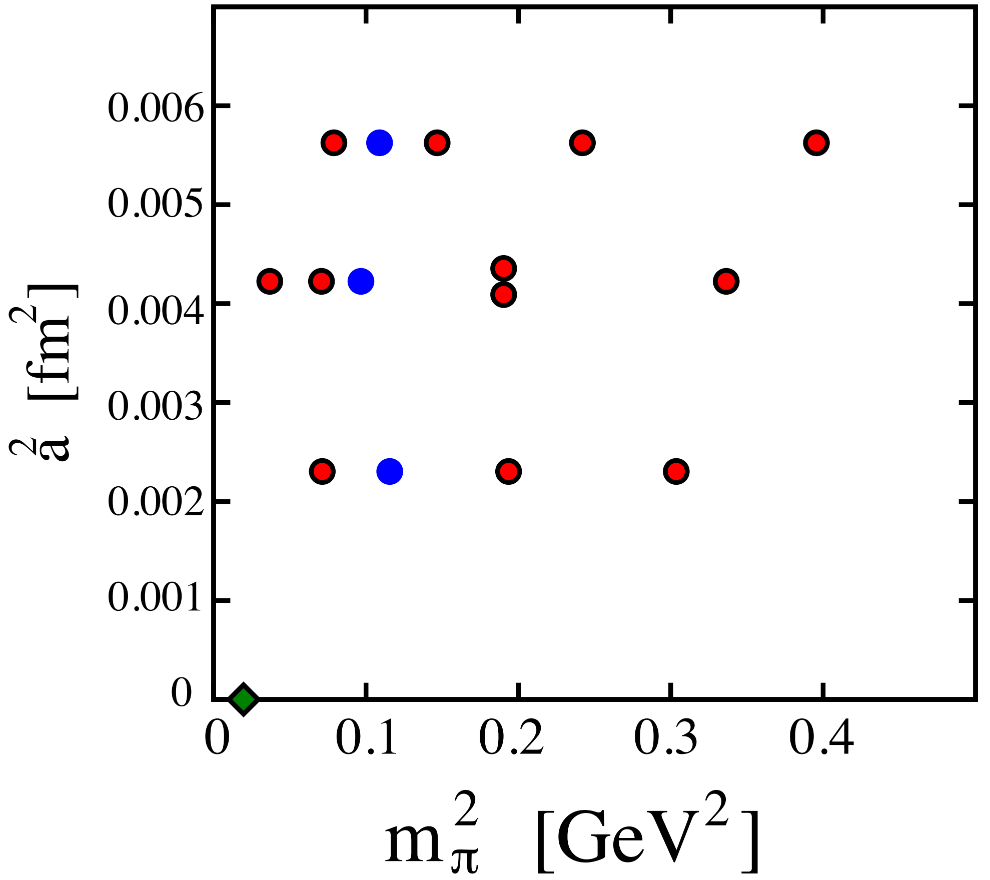
At fixed q^2, achieved by "twisting" [Bedaque '04] the s quark:
$\psi(x+L\hat{k}) = \mathrm{e}^{\mathrm{i}\,\theta_k}\,\psi(x)$

$\vec{p}^{\theta} = (2\pi\vec{n}+\vec{\theta})/L$ freely tuneable $\rightarrow$ heavy quark twisting (keep B_s in rest frame)

- continuum, $a \rightarrow 0$
- chiral, $m_\pi \rightarrow m_\pi^{\mathrm{phys}}$

Ensembles and simulation

- non-perturbatively $O(a)$ improved Wilson fermions
- $N_f = 2$ CLS ensembles
- scale setting via f_K [Fritzsch et al. '12]
- $m_\pi L \gtrsim 4$
- Error estimates taking into account autocorrelations [Schaefer et al. '12]



id	$T \times L^3$	a [fm]	m_π [MeV]	$m_\pi L$	# meas.	# target
A5	64×32^3	0.0749(8)	330	4.0	500	500
F6	96×48^3	0.0652(6)	310	5.0	254	500
N6	96×48^3	0.0483(4)	340	4.0	220	500

- keep $m_K / f_K = \text{phys.}$
- for now: one value of q^2 only, $q^2 = 21.23\,\text{GeV}^2$

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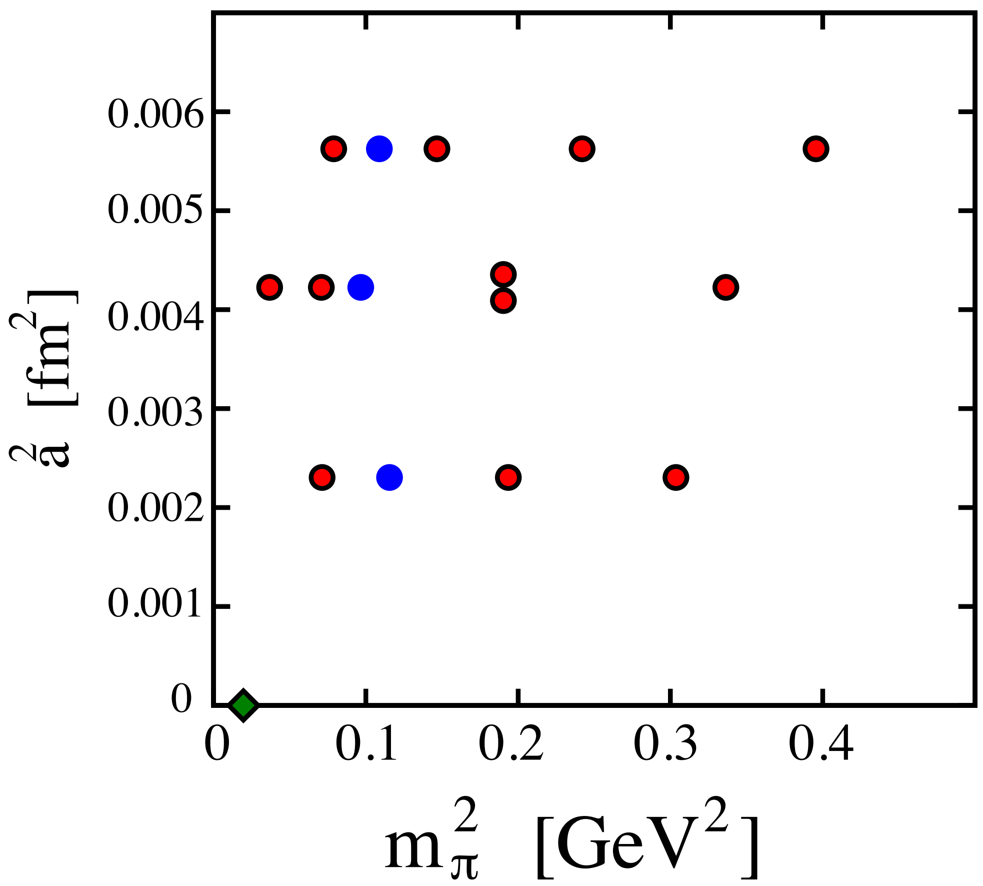

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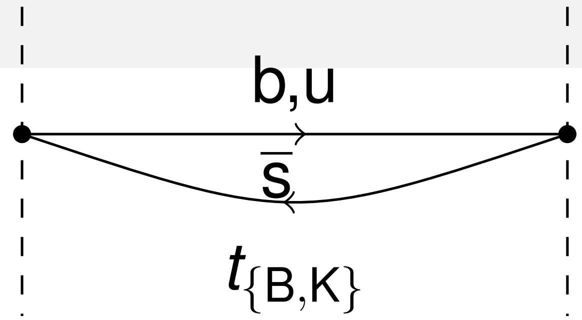
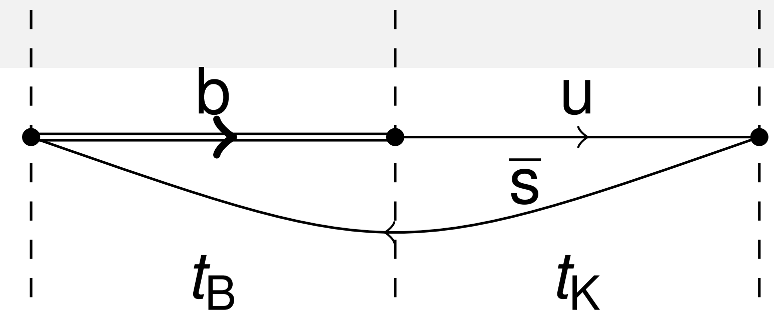
below a=0.045 fm: topological freezing with PBC

- keep $m_K / f_K = $ phys.
- for now: one value of q^2 only, $q^2 = 21.23\,\mathrm{GeV}^2$

Obtaining the form factor

$\mathcal{C}^{\{B,K\}} \sim$ 

$\mathcal{C}^{3} \sim$ 

Ratio – plateaux

$$\langle \mathrm{K}(p_\mathrm{K}^\theta)|V^\mu|\mathrm{B_s}(0)\rangle = \lim_{T,t_\mathrm{B},t_\mathrm{K}\to\infty} \frac{\mathcal{C}^3_\mu(t_\mathrm{K},t_\mathrm{B})}{\sqrt{\mathcal{C}^\mathrm{K}(t_\mathrm{K})\mathcal{C}^\mathrm{B}(t_\mathrm{B})}} \mathrm{e}^{E_\mathrm{K} t_\mathrm{K}/2}\,\mathrm{e}^{E_\mathrm{B} t_\mathrm{B}/2} \equiv \lim_{T,t_\mathrm{B},t_\mathrm{K}\to\infty} f_\mu^{\mathrm{ratio}}(q^2)$$

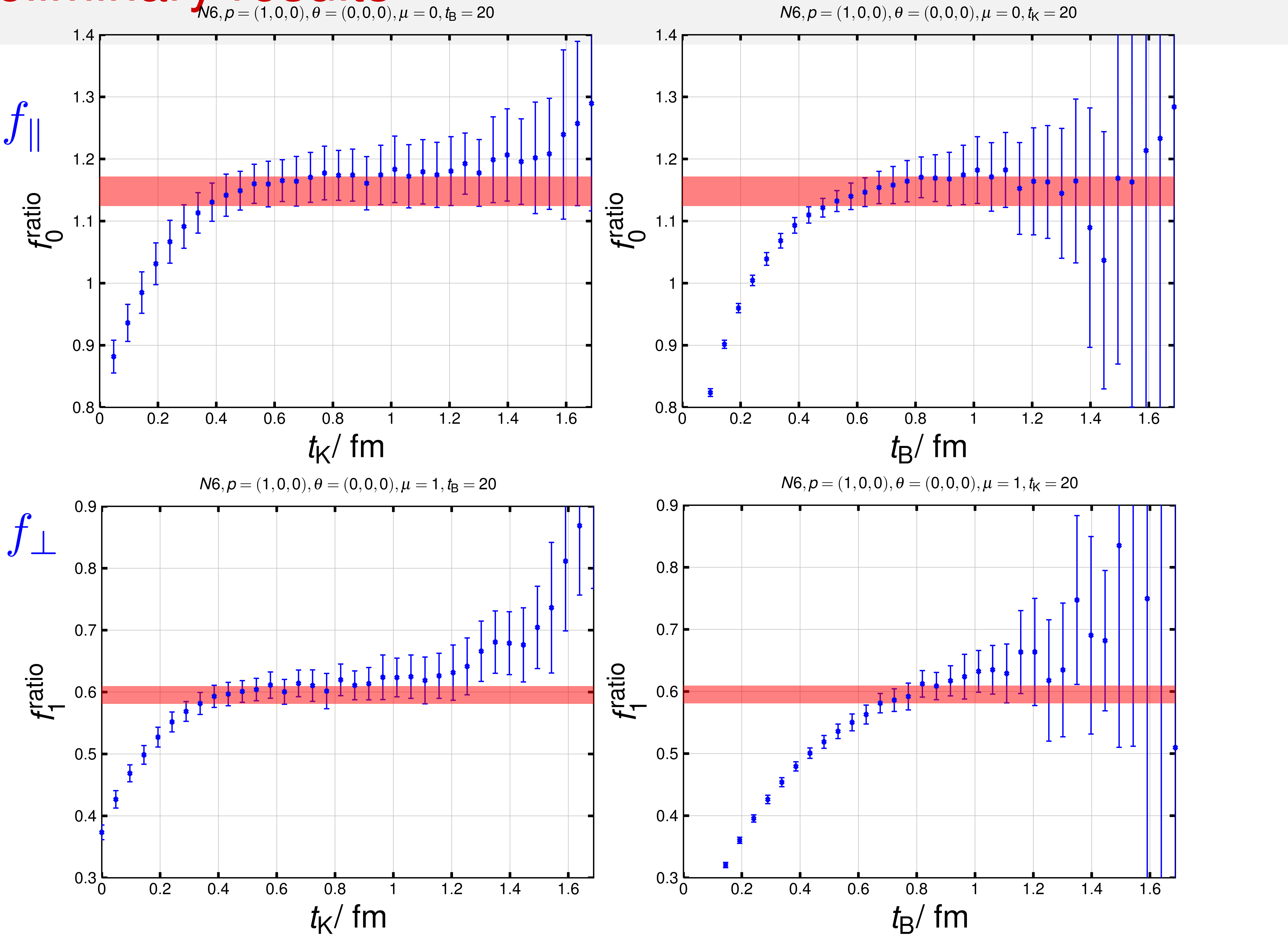
Factorising Fit

Combined fit to ground and first excited state of $\mathcal{C}^3, \mathcal{C}^\mathrm{B}$

$$\begin{cases} \mathcal{C}^3_{\mu i}(t_\mathrm{B},t_\mathrm{K}) & = \sum_{n,m} \beta_i^{(n)} \varphi_\mu^{(n,m)} \kappa^{(m)}\,\mathrm{e}^{-E_\mathrm{B}^{(n)} t_\mathrm{B}}\,\mathrm{e}^{-E_\mathrm{K}^{(m)} t_\mathrm{K}}, \\ \mathcal{C}^\mathrm{B}_{ij}(t_\mathrm{B}) & = \sum_n \beta_i^{(n)} \beta_j^{(n)}\,\mathrm{e}^{-E_\mathrm{B}^{(n)} t_\mathrm{B}} \\ \mathcal{C}^\mathrm{K}(t_\mathrm{K}) & = \sum_m (\kappa^{(m)})^2\,\mathrm{e}^{-E_\mathrm{K}^{(m)} t_\mathrm{K}} \end{cases} \qquad \varphi_\mu^{(1,1)} \sim f_+(q^2)$$

- Gaussian smearing, $\psi_\mathrm{l}^{\mathrm{sm}}(x) = (1+\kappa\Delta)^{N_\mathrm{it}}\psi_\mathrm{l}(x)$, $N_\mathrm{it} \leftrightarrow$ wavefunctions
- random noise sources, full time dilution

Preliminary results



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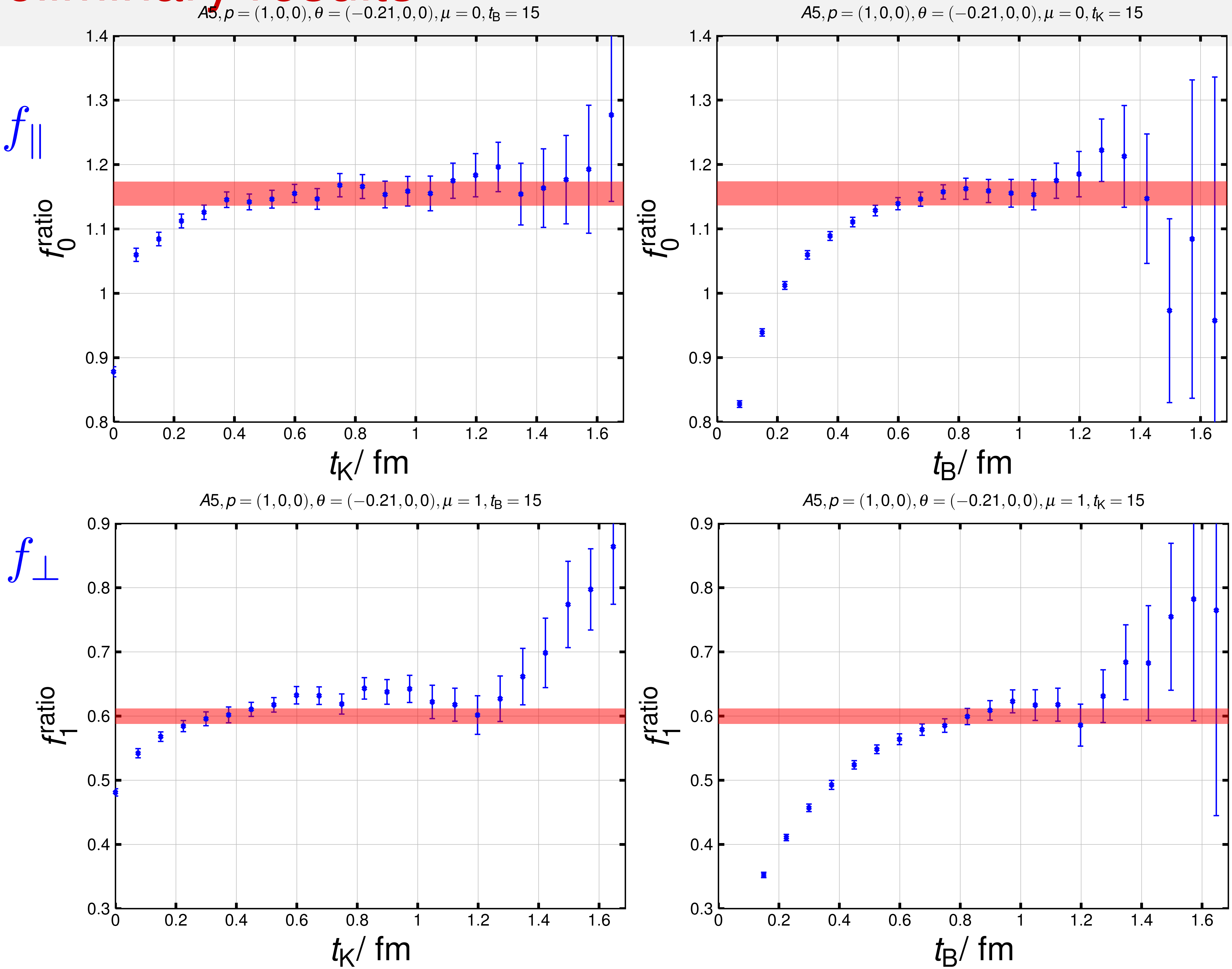
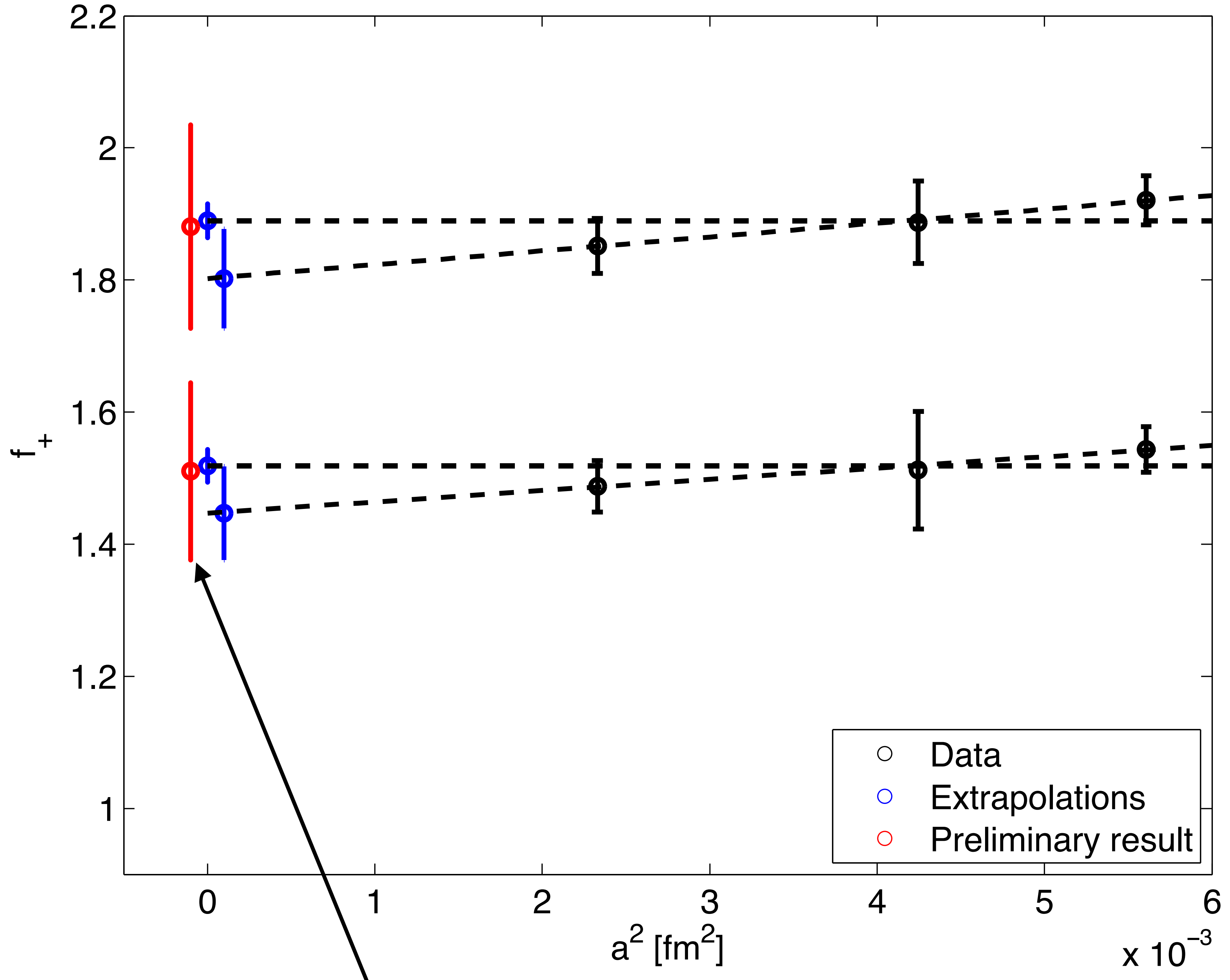


Preliminary continuum extrapolation



$$f_+ = \frac{1}{\sqrt{2M_{B_s}}}\left(f_\parallel + (M_{B_s} - E_K)f_\perp\right)$$

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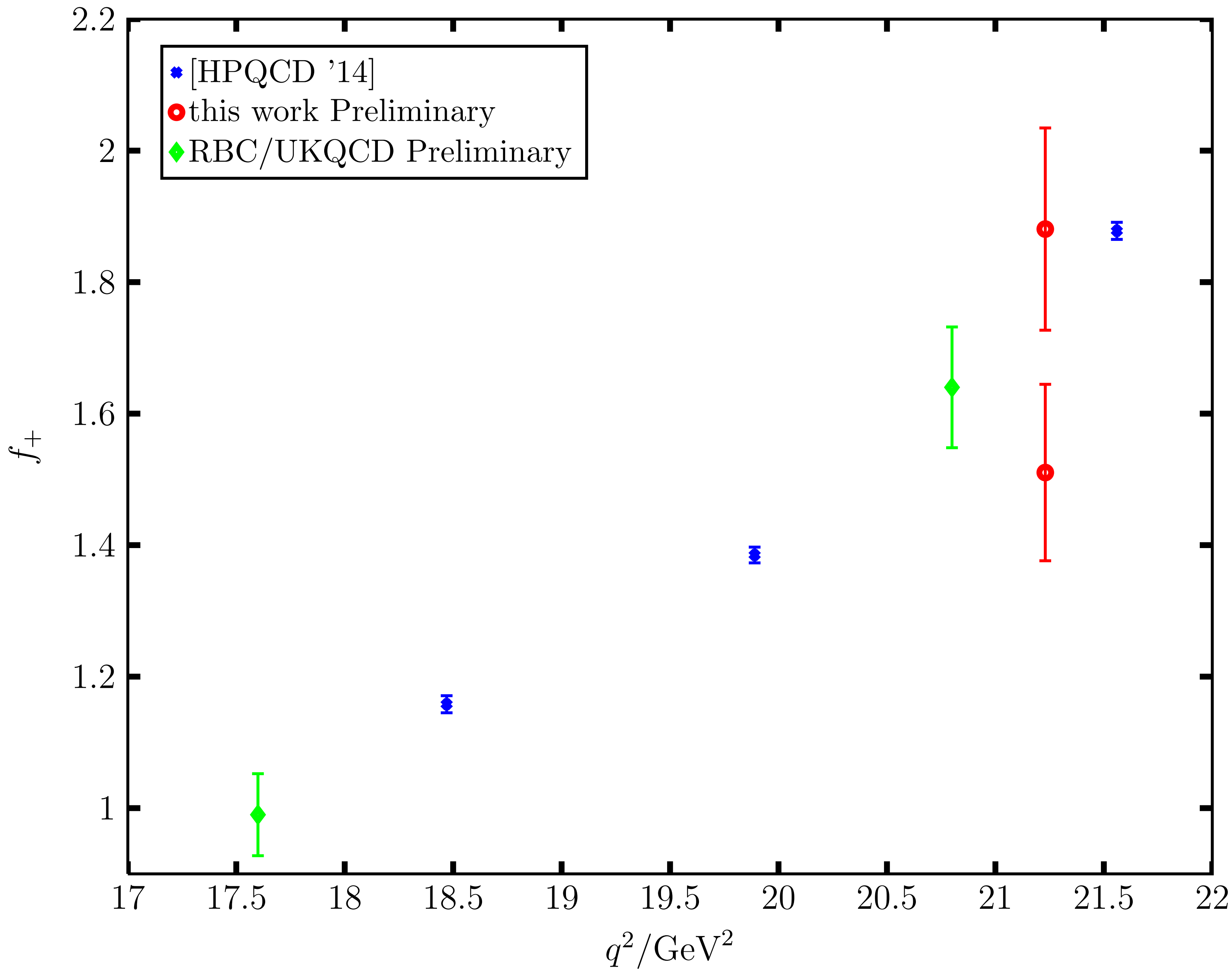
errors a bit large (sorry: preliminary)

A comparison



- HPQCD 2014
  a=0.09fm
  m_{pi}=320MeV
  perturbative renormalization
- ALPHA, preliminary
  continuum extrapolation
  m_{pi}=340MeV
  NP renormalization
- RBC/UKQCD, preliminary
  chiral + continuum extrapol.
  perturbative renormalization

Preliminary conclusion (V_{ub} puzzle)

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- ✓ form factors are rather stable (reliable)
- ✓ the puzzle remains
- ➡ theory for inclusive rate?
- ➡ or new physics?

General conclusion

General conclusion

- NP HQET works in practice
- the V_{ub} puzzle remains
- definitive results for phenomenology
  require more work (N_f=2+1, form factor B->pi)

Discussion

request by Giulia Ricciardi

I mean claryfing the NP
approach (e.g. matching, discretization effects, how m itself is defined)
and comparing with other methods. Expecially for the latter

QCD	HQET in static approx.
$Z_{\rm A}\,\langle f\vert A_0(x)\vert i\rangle_{\rm QCD}$	$Z_{\rm A}^{\rm stat}(\mu)\langle f\vert A_0^{\rm stat}(x)\vert i\rangle_{\rm stat}$
$\Phi^{\rm QCD}(m)$	$\Phi(\mu)$

$$\Phi_{\rm RGI} = \exp\left\{-\int^{\bar g(\mu)} {\rm d}x\,\frac{\gamma(x)}{\beta(x)}\right\}\Phi(\mu) = \underbrace{Z_{\rm RGI}(g_0)}_{\text{known, ALPHA Collaboration}} \times \underbrace{\Phi(g_0)}_{\text{bare ME}}$$

β : beta-fct

$$\equiv \left[\,2b_0\bar g(\mu)^2\,\right]^{-\gamma_0/2b_0} \exp\left\{-\int_0^{\bar g(\mu)} {\rm d}x\left[\frac{\gamma(x)}{\beta(x)} - \frac{\gamma_0}{b_0 x}\right]\right\}\Phi(\mu)$$

$$\Phi^{\rm QCD} = C_{\rm PS}(M/\Lambda)\times\Phi_{\rm RGI}$$

γ : AD in HQET

$$C_{\rm PS}(M/\Lambda) = \exp\left\{\int^{g_\star(M/\Lambda)} {\rm d}x\,\frac{\gamma_{\rm match}(x)}{\beta(x)}\right\}$$

with

Λ : Lambda-para

$$\frac{\Lambda}{M} = \exp\left\{-\int^{g_\star(M/\Lambda)} {\rm d}x\,\frac{1-\tau(x)}{\beta(x)}\right\}, \qquad \rightarrow \qquad g_\star(M/\Lambda)$$

M : RGI quark mass

$$\Phi^{\mathrm{QCD}} = C_{\mathrm{PS}}(M/\Lambda) \times \Phi_{\mathrm{RGI}}$$

in perturbation theory:

error = $(\alpha(m))^L$

NLO 1/M correction: $\frac{\Lambda_{\mathrm{QCD}}}{m}$

NLO correction undefined with perturbative C:

$$(\alpha(m))^L \overset{m\to\infty}{\gg} \frac{\Lambda_{\mathrm{QCD}}}{m}$$

In lattice regularisation:

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power divergences

$$\frac{g_0^{2L}}{a^n} \sim \frac{1}{\log(a\Lambda_{\rm QCD})^L a^n}$$

need NP subtraction

$n = 1, 2$

e.g.

$$(\mathcal{O}_{\rm kin})_{\rm R}(z) = Z_{\mathcal{O}_{\rm kin}}\left(\mathcal{O}_{\rm kin}(z) + \frac{c_1}{a}\,\overline{\psi}_{\rm h}(z) D_0 \psi_{\rm h}(z) + \frac{c_2(g_0)}{a^2}\,\overline{\psi}_{\rm h}(z)\psi_{\rm h}(z)\right)$$

remains divergent
at any order in PT

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Can't use PT for c_1 , c_2

NP HQET

Which mass do we expand in?

Mass is a parameter,
but only formally
an expansion parameter

Matching

$$\Phi_i^{\mathrm{HQET}}(\{\omega_i(g_0, aM_{\mathrm{b}})\}) = \Phi_i^{\mathrm{QCD}}(M_{\mathrm{b}})$$

bare parameters
of HQET Lagrangian

power divergent
no direct physical
relevance

QCD RGI mass
no ambiguity
no scheme
dependence

equivalent to MSbar
mass for $\mu \to \infty$

The confusion (maybe?)

- start with some "derivation" of HQET Lagrangian
  - integrating out
  - FTW trafo
  which is essentially classical, contains "the mass"

- but this just serves to find/motivate the form of the Lagrangian

- NP interpretation of the Lagrangian
  - operators of increasing dimension
  - with free coefficients $\omega_i(g_0, aM_\mathrm{b}) \quad \sim M_\mathrm{b}^{-(d_\mathcal{O}-4)}$
  - respecting the symmetries

↑ any mass