Heavy Quark Expansion Matrix Elements: Continuum Approaches

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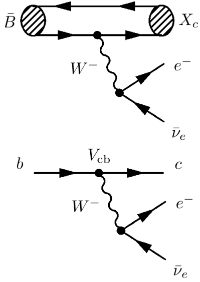
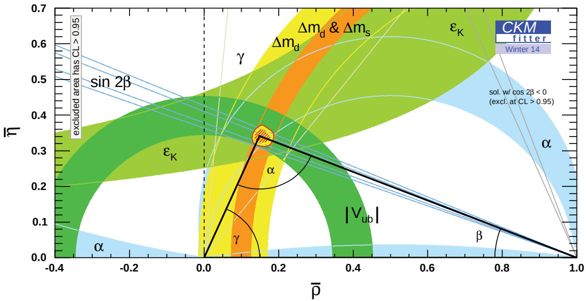
Lattice Meets Continuum: QCD Calculations in Flavour Physics
Monday, September 29th, 2014

Outline

Consider inclusive semi-leptonic decay $B \to X_c \ell \bar{\nu}_\ell$





Input of $|V_{cb}|$

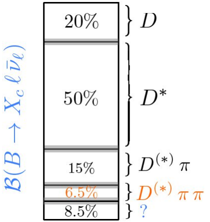
- Important ingredient for UT: $V_{\rm ub}^* V_{\rm ud} + V_{\rm tb}^* V_{\rm td} + V_{\rm cb}^* V_{\rm cd} = 0$
- Determination of ϵ_K depends on $|V_{\rm cb}|^4$: $\sim 35\%$ of error budget!
  [1009.0947 [hep-ph], 0805.3887 [hep-ph]]

Charm state X_c	$\mathcal{B}(B^+ \to X_c\,\ell^+\,\nu)$
D	$(2.31 \pm 0.09)\,\%$
D^*	$(5.63 \pm 0.18)\,\%$
$\sum D^{(*)}$	$\mathbf{(7.94 \pm 0.20)\,\%}$
$D_0^* \to D\,\pi$	$(0.41 \pm 0.08)\,\%$
$D_1^* \to D^*\,\pi$	$(0.45 \pm 0.09)\,\%$
$D_1 \to D^*\,\pi$	$(0.43 \pm 0.03)\,\%$
$D_2^* \to D^{(*)}\,\pi$	$(0.41 \pm 0.03)\,\%$
$\sum D^{**} \to D^*\pi$	$\mathbf{(1.70 \pm 0.12)\,\%}$
$D\,\pi$	$(0.66 \pm 0.08)\,\%$
$D^*\,\pi$	$(0.87 \pm 0.10)\,\%$
$\sum D^*\pi$	$\mathbf{(1.53 \pm 0.13)\,\%}$
$\sum D^{(*)} + \sum D^*\pi$	$\mathbf{(9.47 \pm 0.24)\,\%}$
$\sum D^{(*)} + \sum D^{**} \to D^{(*)}\pi$	$\mathbf{(9.64 \pm 0.23)\,\%}$
Inclusive X_c	$\mathbf{(10.92 \pm 0.16)\,\%}$

broad states $(0.86 \pm 0.12)\%$

narrow states $(0.84 \pm 0.04)\%$

Gap $\sim 5 - 7\sigma$ [Bernlochner,ST,CKM2012]



New Measurement of

$$B \to D^{(*)}\pi^+\pi^-\ell\bar{\nu}_\ell$$

Reduces Gap to $\sim 3\sigma$
[Bernlochner,CKM2014]

Results from [PDG 2014]

$$|V_{cb}|^{\text{excl.}} = (39.5 \pm 0.8) \cdot 10^{-3}$$
$$|V_{cb}|^{\text{incl.}} = (42.4 \pm 0.9) \cdot 10^{-3}$$

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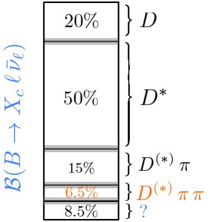

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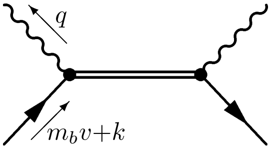
Decay Rate

$$d\Gamma = 16\pi\, G_F^2\, |V_{cb}|^2\, W_{\mu\nu}\, L^{\mu\nu}\, d\phi$$

- Starting point: $W_{\mu\nu} = -\frac{1}{\pi}\mathrm{Im}\, T_{\mu\nu}$

  Correlator of two hadronic currents

$$iT_{\mu\nu} = \frac{1}{2M_B}\int d^4x\, e^{-ix(m_b v - q)}$$
$$\times \langle B|\bar{b}_v(x)\Gamma_\nu^\dagger\, \mathrm{T}\left[c(x)\bar{c}(0)\right]\Gamma_\mu b_v(0)|B\rangle$$



Expansion of the Rate

- Rate can be written as

$$\Gamma = \Gamma_0 + \frac{1}{m_b^2}\sum_i C_2^i(\alpha_s)\,\mathcal{O}_5^i + \frac{1}{m_b^3}\sum_i C_3^i\,\mathcal{O}_6^i + \ldots$$

- Each Wilson Coefficient C_j^i has a power series in α_s

$\Rightarrow$ Combined expansion in α_s and $1/m_b$: Heavy Quark Expansion

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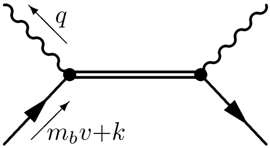

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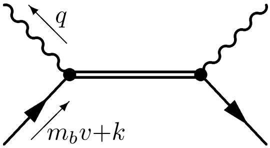

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Background Field Method

Heavy Quark Matrix Elements

$$2M_B\, T_{\mu\nu} = \langle B(p)|\bar{b}_v \gamma_\nu P_L S_{\mathrm{BGF}} \gamma_\mu P_L b_v |B(p)\rangle$$

Parametrize Background Field Propagator

- Remove only large momentum: $p_b = m_b v + k,\ b_v(x) = e^{im_b v\cdot x} b(x)$
- Background field propagator:

$$iS_{\mathrm{BGF}} = \frac{i}{m_b \not{v} - \not{q} + i\not{D} - m_c}$$

- HQE corresponds to expand S_{BGF} in small quantitiy $i\not{D}$

$$S_{\mathrm{BGF}} = \sum_{n=0} (-1)^n \frac{1}{\not{Q} - m_c}\left(i\not{D}\frac{1}{\not{Q} - m_c}\right)^n$$

$\Rightarrow$ Keeps track on the ordering of the covariant derivatives

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General Structure in each Order

- "Trace-formulae": Non-perturbative input in Dimension $n+3$

$$\langle B(p)|\bar{b}_{v,\alpha}(iD_{\mu_1})\ldots(iD_{\mu_n})b_{v,\beta}|B(p)\rangle = \sum_i \hat{\Gamma}^{(i)}_{\beta\alpha} A^{(i)}_{\mu_1\mu_2\cdots\mu_n}$$

- "Off-shellness" The imaginary part is given by

$$-\frac{1}{\pi}\text{Im}\frac{1}{(Q^2 - m_c^2 + i\epsilon)^{n+1}} = \frac{(-1)^n}{n!}\delta^{(n)}(Q^2 - m_c^2)$$

Comment on Uncertainties

- Truncation of Series and Parametric Uncertainties
- Duality Violation [Shifman, hep-ph/0009131]
  1. In Instanton model suppressed by $1/m_b^3$
  2. Supposedly will result in inconsitent fit ⇒ Currently not observed

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Current Status: Theory

α_s^n \ $1/m_b^n$	0	2	3	4	5
0	•	•	•	•[a]	•[b]
1	•	•[c]	–	–	–
2	•[d]	–	–	–	–
3	∘[e]	–	–	–	–

a [Dassinger,Mannel,ST] JHEP 0703 (2007) 087

b [Mannel,Uraltsev,ST]: JHEP 1011 (2010) 109

c [Becher, Boos,Lunghi] μ_π^2 JHEP 0712, 062 (2007)
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e Only BLM corrections / special kinematical point

Topics Addressed in this Talk: Heavy Quark Matrix Elements

- Explicit Corrections to order $1/m_b^4$ and $1/m_b^5$
- Subtleties concerning final state charm quark m_c

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Non-Perturbative Parameter

To Order $1/m_b^2$

$$2M_B\mu_\pi^2 = -\langle B(p)|\bar{b}_v(iD_\perp)^2 b_v|B(p)\rangle$$
$$\hat{=} \langle \mathbf{p}^2 \rangle$$
$$2M_B\mu_G^2 = 1/2\,\langle B(p)|\bar{b}_v\left[(iD_\perp^\mu),(iD_\perp^\nu)\right](-i\sigma_{\mu\nu})b_v|B(p)\rangle$$
$$\hat{=} \langle \mathbf{s}\cdot\mathbf{B} \rangle$$

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$$2M_B\rho_D^3 = 1/2\,\langle B(p)|\bar{b}_v\Big[(iD_{\perp,\mu}),\left[(iv\cdot D),(iD_\perp^\mu)\right]\Big]b_v|B(p)\rangle$$
$$\hat{=} \langle \nabla\cdot\mathbf{E} \rangle$$
$$2M_B\rho_{LS}^3 = 1/2\,\langle B(p)|\bar{b}_v\Big\{(iD_\perp^\mu),\left[(iv\cdot D),(iD_\perp^\nu)\right]\Big\}(-i\sigma_{\mu\nu})b_v|B(p)\rangle$$
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Higher Orders

Dimension - 7: $1/m_b^4$

- 4 Spin independent parameter
- 5 Spin dependent parameters

Dimension - 8: $1/m_b^5$

- Proliferation of parameters
- 8 Spin independent parameter
- 10 Spin dependent parameter

Problem in Experiment

- All parameters have to be extracted from correlated measurements

$\Rightarrow$ Not reliably possible

$\Rightarrow$ Estimate parameters and use this to estimate influence

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Lowest-Lying State Approximation (LLSA)

[Mannel,Uraltsev,ST,JHEP 1011 (2010) 109], improved [Heinonen,Mannel,1407.4384]

- Insert complete set to decompose matrix elements

$$\sum_n |n\rangle\langle n| = \sum_{\text{pol}} \int \tilde{\mathrm{d}p} \left[|1^+, \frac{1}{2}\rangle\langle 1^+, \frac{1}{2}| + |1^+, \frac{3}{2}\rangle\langle 1^+, \frac{3}{2}| \right] + \dots$$

$\Rightarrow$ Express higher order M.E. through product of lower orders

Master Equation

[Heinonen,Mannel,1407.4384]

$$\sum_{k=0}^{\infty} \sum_n (2\pi)^3 \delta^3(p_n^\perp) \left(\frac{-\epsilon_n}{\omega}\right)^k \langle B(p_B)|\bar{b}_v \mathcal{P}_1 Q_v|n\rangle \langle n|\bar{Q}_v \mathcal{P}_2 \Gamma b_v|B(p_B)\rangle$$

$$= \sum_{k=0}^{\infty} \langle B(p_B)|\bar{b}_v \mathcal{P}_1 \left(\frac{i\, v\cdot D}{\omega}\right)^k \left(\frac{1+\slashed{v}}{2}\right) \mathcal{P}_2 \Gamma b_v|B(p_B)\rangle$$

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Known Parameters [Heinonen,Mannel], [Fit] [Gambino,Schwanda, PRD89,014022]

$\mu_\pi^2 = 0.414\ \text{GeV}^2\ \ \mu_G^2 = 0.340\ \text{GeV}^2\ \ \epsilon_{1/2} = 0.390\ \text{GeV}\ \ \epsilon_{3/2} = 0.476\ \text{GeV}$

$$\rho_D^3 = \frac{1}{3}\epsilon_{1/2}(\mu_\pi^2 - \mu_G^2) + \frac{1}{3}\epsilon_{3/2}(2\mu_\pi^2 + \mu_G^2) = 0.21\ \text{GeV}^3 \quad [0.154 \pm 0.045]$$

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A Comment on Precision [Heinonen,Mannel]

- Some minor discrepancies to be clarified
- Estimate of Series Truncation $\sim$ Duality violation

$\Rightarrow$ Model with sum of infinitly narrow resonances [Shifman,hep-ph/0009131]

- Relative Uncertainty of truncating after first term and analytic sum

$$\left[\frac{\pi^2}{6}\right] - 1 \sim 64\%$$

- Large corrections to LLSA have previously been found
[Gaminbo,Mannel,Uraltsev,JHEP,1210,169 (2012)]

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Numerical Example Dim=7

[Mannel,Uraltsev,ST,JHEP 1011 (2010) 109]

$$2M_B\, m_1 = \langle \bar{B}|\bar{b}_v\, iD_\rho iD_\sigma iD_\lambda iD_\delta\, b_v|\bar{B}\rangle\, \frac{1}{3}\left(\Pi^{\rho\sigma}\Pi^{\lambda\delta} + \Pi^{\rho\lambda}\Pi^{\sigma\delta} + \Pi^{\rho\delta}\Pi^{\sigma\lambda}\right)$$

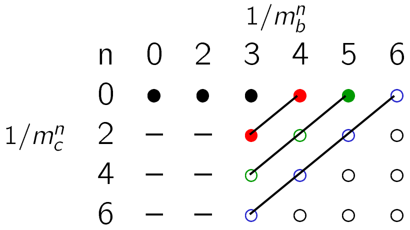
$$2M_B\, m_4 = \langle \bar{B}|\bar{b}_v \left\{ iD_\rho, \left[iD_\sigma, \left[iD_\lambda, iD_\delta \right]\right]\right\} b_v|\bar{B}\rangle \Pi^{\sigma\lambda}\Pi^{\rho\delta}$$

$$2M_B\, m_8 = \langle \bar{B}|\bar{b}_v \left\{ \left\{ iD_\rho, iD_\sigma \right\}, \left[iD_\lambda, iD_\delta \right]\right\} \left(- i\sigma_{\alpha\beta}\right) b_v|\bar{B}\rangle \Pi^{\rho\sigma}\Pi^{\alpha\lambda}\Pi^{\beta\delta}$$

$$2M_B\, m_9 = \langle \bar{B}|\bar{b}_v \left[iD_\rho, \left[iD_\sigma, \left[iD_\lambda, iD_\delta \right]\right]\right] \left(- i\sigma_{\alpha\beta}\right) b_v|\bar{B}\rangle\ \Pi^{\rho\beta}\Pi^{\lambda\alpha}\Pi^{\sigma\delta}\,,$$

Singlet param.	m_1	m_2	m_3		m_4
Fact. estimate	$\frac{5}{9}\left(\mu_\pi^2\right)^2$	$-\bar{\epsilon}\rho_D^3$	$-\frac{2}{3}\left(\mu_G^2\right)^2$		$\left(\mu_G^2\right)^2 + \frac{4}{3}\left(\mu_\pi^2\right)^2$
Value / GeV^4	0.113	-0.06	-0.82		0.393
Norm Factor	1	1	4		8
Triplet param.	m_5	m_6	m_7	m_8	m_9
Fact. estimate	$-\bar{\epsilon}\rho_{LS}^3$	$\frac{2}{3}\left(\mu_G^2\right)^2$	$-\frac{8}{3}\mu_G^2\mu_\pi^2$	$-8\mu_G^2\mu_\pi^2$	$\left(\mu_G^2\right)^2 - \frac{10}{3}\mu_G^2\mu_\pi^2$
Value / GeV^4	0.060	0.082	-0.420	-1.260	-0.403
Norm Factor	1	4	8	8	8

Subtlety in the $1/m_Q$ expansion



- Expansion in both heavy quark masses m_b and $m_c \approx \sqrt{m_b \Lambda}$

$\Rightarrow$ Some Higher Order terms formally belong formally to lower orders

- Starting at leading order $\frac{\Lambda^3}{m_b^3}\left(\log \frac{m_c^2}{m_b^2} + \frac{\Lambda^2}{m_c^2} + \ldots\right)$

$\Rightarrow$ Leading to systematical effects

$\Rightarrow$ Computation and estimation of higher orders and these effects

[Bigi,Uraltsev,Zwicky[hep-ph/0511158], Breidenbach,Feldmann,Mannel,ST[0805.0971], Bigi,Mannel,Uraltev,ST[0911.3322]]

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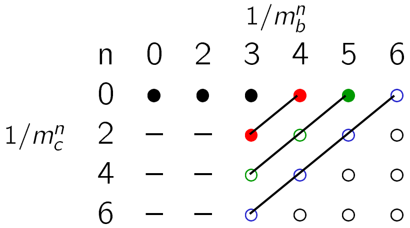

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Hadronic Tensor for "IC"

Starting Point

$$W_{\mu\nu} = \frac{1}{2M_B} \sum_{X_c} \langle \bar{B} | J^\dagger_{q,\nu}(x) | X_c \rangle \langle X_c | J_{q,\mu}(0) | \bar{B} \rangle (2\pi)^3 \delta^4 (p_B - q - p_{X_c})$$

Rewrite for "Intrinsic Charm" Contribution

- Use translational invariance and expand in local operators

$$2M_B\, W^{IC}_{\mu\nu} = (2\pi)^3\, \delta^4(q - m_b v)\, \langle \bar{B}(p) | (\bar{b}_v\, \gamma_\nu P_L\, c)\, (\bar{c}\, \gamma_\mu P_L\, b_v) | \bar{B}(p) \rangle$$
$$+ (2\pi)^3 \left(\frac{\partial}{\partial q_\alpha} \delta^4(q - m_b v) \right) \langle \bar{B}(p) | (i\partial_\alpha\, \bar{b}_v \gamma_\nu P_L c)(\bar{c} \gamma_\mu P_L b_v) | \bar{B}(p) \rangle$$
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Scenarios

Scenario I

- Consider charm-quark as heavy

$\Rightarrow$ $m_b \sim m_c \gg \Lambda_{\text{QCD}}$

Scenario II

- Consider charm-quark as semi-heavy

$\Rightarrow$ $m_b \gg m_c \gg \Lambda_{\text{QCD}}$

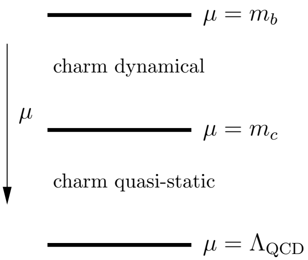
Scenario III

- Consider charm-quark as light

$\Rightarrow$ $m_b \gg m_c \gtrsim \Lambda_{\text{QCD}}$

Scenario II: $m_b \gg m_c \gg \Lambda_{QCD}$

- 2 matching steps



Difference to Scenario I

- Resum logarithmic terms $\ln m_c/m_b$ into short-distance coefficient functions
- Expand analytic terms in powers of $m_c/m_b \sim \sqrt{\Lambda_{QCD}/m_b} \sim 0.3$

$\Rightarrow$ Reproduces Scenario I

Scenario II: $m_b \gg m_c \gg \Lambda_{QCD}$

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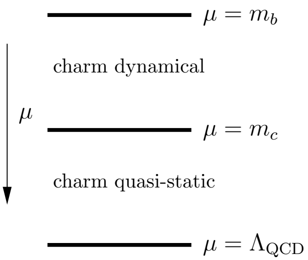

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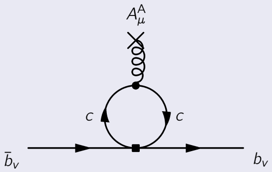
Mixing of Operators [Scenario 2/3]

Dimension 6 Intrinsic Charm



- Generates mixing into ρ_D^3

$\Rightarrow$ Renormalization group flow

$$\frac{d}{d\ln\mu}\begin{pmatrix}\rho_D(\mu)\\ T_1(\mu)\\ T_2(\mu)\end{pmatrix} = -\begin{pmatrix}0 & 0 & 0\\ -2/3 & 0 & 0\\ 4/3 & 0 & 0\end{pmatrix}\begin{pmatrix}\rho_D(\mu)\\ T_1(\mu)\\ T_2(\mu)\end{pmatrix}$$

Dimension 7 Intrinsic Charm

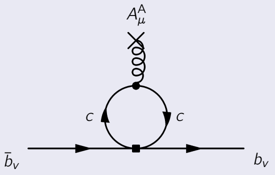
- Generates mixing into $m_c^4 \bar{b}_v b_v$

$\Rightarrow$ RGE flow

- $1/m_c^{2n}$ terms also reproduced [hep-ph/0511158]

Mixing of Operators [Scenario 2/3]

Dimension 6 Intrinsic Charm



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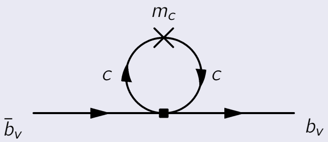

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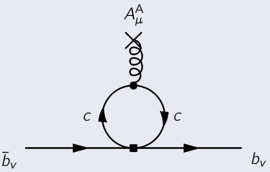

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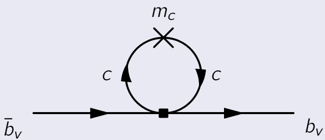

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Measurement Procedure I

Extraction of Heavy Quark Parameters

- Use normalization to cancel out prefactors
- Need completely integrated hadronic phase-space
- Sufficient number of observables for all different parameters

Definition of Observables

1. Electron energy spectrum

$$\mathrm{BR}(E_e) = \frac{1}{\int \frac{\mathrm{d}\Gamma}{\mathrm{d}E_e}\mathrm{d}E_E}\frac{\mathrm{d}\Gamma}{\mathrm{d}E_e}$$

2. Moments of electron energy and hadronic invariant mass

$$\langle E_e^n M_X^m\rangle(E_{\mathrm{cut}}) = \frac{1}{\int_{E_e>E_{\mathrm{cut}}}\frac{\mathrm{d}^2\Gamma}{\mathrm{d}E_e\mathrm{d}M_X}\mathrm{d}E_e\mathrm{d}M_X}\int_{E_e>E_{\mathrm{cut}}} E_e^n M_X^m\frac{\mathrm{d}^2\Gamma}{\mathrm{d}E_e\mathrm{d}M_X}\mathrm{d}E_e\mathrm{d}M_X$$

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Measurement Procedure II

Extraction of V_{cb}

- Heavy Quark parameters known from fit to moments and spectra
- Normalisation to partial branching fraction determines $|V_{cb}|$

$$\Gamma = \frac{G_F^2 |V_{cb}|^2 m_b^5}{192\pi^3} f(m_c, m_b, \mu_\pi^2, \ldots)$$

Remarks

- E_{cut} restricts phase-space
- ⇒ Reduces validity of HQE
- Highly correlated measurement
- ⇒ Limits reasonable order of non-perturbative expansion

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Most Recent Data

[BABAR, Phys.Rev.D81:032003,2010]

- Experimental errors are competative with theoretical errors
- General uncertainty due to operators with charm content [hep-ph/0511158]

Fit	$\lvert V_{cb}\rvert$	m_b/GeV	m_c/GeV
RESULT	41.91	4.566	1.101
Δ_{exp}	0.48	0.034	0.045
Δ_{theo}	0.38	0.041	0.064
$\Delta\Gamma_{sl}$	0.59		
Δ_{tot}	0.85	0.055	0.078

Used in Analysis

- Non-perturbative corrections up to $1/m_b^3$
- Electroweak corrections: Estimated $1 + A_{\text{EW}} \approx 1.014$
- Perturbative contributions: Using α_s, $\alpha_s^2\beta_0$ and $\alpha_s^3\beta_0^2$ to leading order in $1/m_b$: $A_{\text{pert}} \approx 0.908$

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[Gambino, Schwanda,Phys.Rev. D89,014022]

$$\lvert V_{cb}\rvert = (42.42 \pm 0.86) \cdot 10^{-3} \qquad m_b = (4.541 \pm 0.023)\ \text{GeV}$$

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Generic Effects on $|V_{cb}|$

Direct effect

- Additional terms in branching ratio

$\Rightarrow$ Change value of $|V_{cb}|$ directly

Indirect Effect

- Use estimate of higher-order parameters
- Value fixed by moment $\mathcal{M}^{(6)}$ up to dimension six
- Compensate effect by change of heavy quark parameter in $\mathcal{M}^{(6)}$

$$\delta m_b = -\frac{\delta \mathcal{M}^{(8)}}{\frac{\partial \mathcal{M}^{(6)}}{\partial m_b}}, \qquad \delta \mu_\pi^2 = -\frac{\delta \mathcal{M}^{(8)}}{\frac{\partial \mathcal{M}^{(6)}}{\partial \mu_\pi^2}}, \qquad \delta \rho_D^3 = -\frac{\delta \mathcal{M}^{(8)}}{\frac{\partial \mathcal{M}^{(6)}}{\partial \rho_D^3}}$$

$\Rightarrow$ Results in indirect change of $|V_{cb}|$

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Direct Effect on Branching Fraction

[Mannel,Uraltsev,ST,JHEP 1011 (2010) 109]

Naive Assumption

- Definition: $\delta\Gamma_{1/m^k} = \Gamma_{1/m^k} - \Gamma_{1/m^{k-1}}$ and Γ_{parton} leading order

$$\frac{\delta\Gamma_{1/m^2}}{\Gamma_{\text{parton}}} = -4.3\% \qquad \frac{\delta\Gamma_{1/m^3}}{\Gamma_{\text{parton}}} = -3.0\%$$

$$\frac{\delta\Gamma_{1/m^4}}{\Gamma_{\text{parton}}} = 0.75\% \qquad \frac{\delta\Gamma_{1/m^5}}{\Gamma_{\text{parton}}} = 0.6\% \qquad \frac{\delta\Gamma^{\text{IC}}}{\Gamma_{\text{parton}}} = 0.7\%$$

Implication for $|V_{cb}|$

$$\frac{\delta\Gamma_{1/m^4} + \delta\Gamma_{1/m^5}}{\Gamma_{\text{parton}}} \simeq 1.3\%$$

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Indirect Effect on V_{cb} from Selected Moments

Results for $\langle E_e \rangle$

$$\delta m_b = -33 \text{ MeV}\,, \qquad \delta\mu_\pi^2 = -0.39 \text{ GeV}^2\,, \qquad \delta\rho_D^3 = 0.15 \text{ GeV}^3$$

$$\Rightarrow \frac{\delta|V_{cb}|}{|V_{cb}|} = 0.022 \qquad \Rightarrow \frac{\delta|V_{cb}|}{|V_{cb}|} = -0.005 \qquad \Rightarrow \frac{\delta|V_{cb}|}{|V_{cb}|} = 0.014$$

Results for $\langle M_X^2 \rangle$

$$\delta m_b = -17 \text{ MeV}\,, \qquad \delta\mu_\pi^2 = -0.12 \text{ GeV}^2\,, \qquad \delta\rho_D^3 = 0.086 \text{ GeV}^3$$

$$\Rightarrow \frac{\delta|V_{cb}|}{|V_{cb}|} = 0.011 \qquad \Rightarrow \frac{\delta|V_{cb}|}{|V_{cb}|} = -0.0015 \qquad \Rightarrow \frac{\delta|V_{cb}|}{|V_{cb}|} = 0.008$$

- Combining everything we expact a net increase of $|V_{cb}|$

$$\frac{\delta|V_{cb}|}{|V_{cb}|} \approx +(0.3 \div 0.5)\%$$

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$$\Rightarrow \frac{\delta|V_{cb}|}{|V_{cb}|} = 0.022 \qquad \Rightarrow \frac{\delta|V_{cb}|}{|V_{cb}|} = -0.005 \qquad \Rightarrow \frac{\delta|V_{cb}|}{|V_{cb}|} = 0.014$$

Results for $\langle M_X^2 \rangle$

$$\delta m_b = -17 \text{ MeV}\,, \qquad \delta\mu_\pi^2 = -0.12 \text{ GeV}^2\,, \qquad \delta\rho_D^3 = 0.086 \text{ GeV}^3$$

$$\Rightarrow \frac{\delta|V_{cb}|}{|V_{cb}|} = 0.011 \qquad \Rightarrow \frac{\delta|V_{cb}|}{|V_{cb}|} = -0.0015 \qquad \Rightarrow \frac{\delta|V_{cb}|}{|V_{cb}|} = 0.008$$

- Combining everything we expact a net increase of $|V_{cb}|$

$$\frac{\delta|V_{cb}|}{|V_{cb}|} \approx +(0.3 \div 0.5)\%$$

Indirect Effect on V_{cb} from Selected Moments

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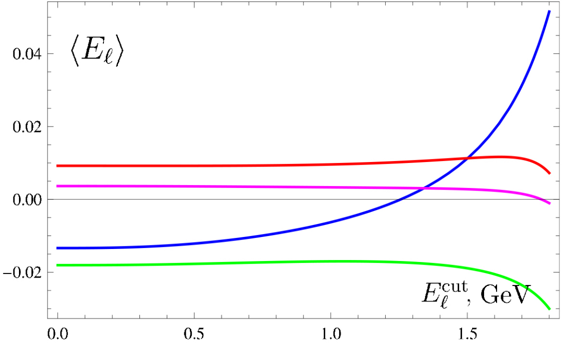
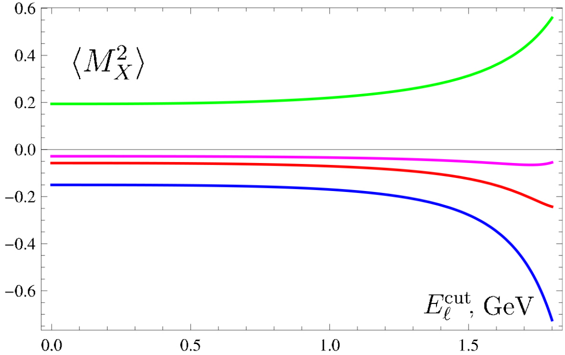
Effect of Electron Energy Cut



Legend of Different Order Contributions

- Blue: $1/m_b^2$
- Red: $1/m_b^4$
- Green: $1/m_b^3$
- Magenta: $1/m_b^5$

Effect of Electron Energy Cut



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Recent Fit Results [Gambino,CKM2014], [Alberti,Gambino,Healy,Nandi]

PRELIMINARY RESULTS

NEW

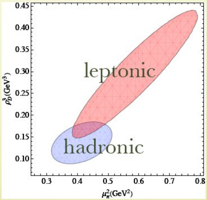
th corr scenario	m_b^{kin}	m_c (3GeV)	μ^2_π	ρ^3_D	μ^2_G	ρ^3_{LS}	BR(%)	$10^3 \lvert V_{cb} \rvert$
4.	4.539	0.988	0.454	0.149	0.296	-0.142	10.67	42.41
uncertainty	0.021	0.013	0.077	0.044	0.063	0.097	0.16	0.83

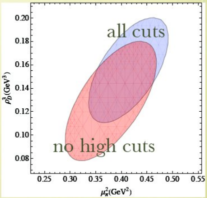
Schwanda PG 2013

th. corr. scenario	m_b^{kin}	$m_c^{(3GeV)}$	μ_π^2	ρ_D^3	μ_G^2	ρ_{LS}^3	$BR_{c\ell\nu}(\%)$	$10^3 \lvert V_{cb} \rvert$
4.	4.541	0.987	0.414	0.154	0.340	-0.147	10.65	42.42
uncertainty	0.023	0.013	0.078	0.045	0.066	0.098	0.16	0.86

Without mass constraints $m_b^{kin}(1\,\text{GeV}) - 0.85\,\overline{m}_c(3\,\text{GeV}) = 3.701 \pm 0.019\,\text{GeV}$

- results depend little on assumption for correlations and choice of inputs, 2% determination of V_{cb}
- 20-30% determination of the OPE parameters





Summary

- Heavy Quark Expansion of inclusive decays

$\Rightarrow$ HQE matrix element of form $\langle B|\bar{b}_v iD \ldots iDb_v|B\rangle$

- Estimated size of unknown parameters (LLSA)
- Estimated impact on $|V_{cb}|$ extraction

$\Rightarrow$ Improving knowledge on ME crucial for $< 0.5{-}1\%$ theo. uncertainty

Future Plans

- Fit including Dim=7,8 parameters [Gambino, ST]

$\Rightarrow$ First hint $\frac{\delta V_{cb}}{V_{cb}} \simeq -0.35\%$

$\Rightarrow$ Allowing 80% gaussian deviations seem to leave V_{cb} unaffected

- More elaborate estimate of higher order parameters

$\Rightarrow$ Including radiative corrections, higher terms? [Heinonen, Mannel]

- Combined α_s/m_b^3 correction to Darwin term

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Backup Slides

$$2M_B\, m_1 = \langle \bar{B}|\bar{b}_v\, iD_\rho iD_\sigma iD_\lambda iD_\delta\, b_v|\bar{B}\rangle\ \frac{1}{3}\left(\Pi^{\rho\sigma}\Pi^{\lambda\delta} + \Pi^{\rho\lambda}\Pi^{\sigma\delta} + \Pi^{\rho\delta}\Pi^{\sigma\lambda}\right)$$

$$2M_B\, m_2 = \langle \bar{B}|\bar{b}_v\, \left[iD_\rho, iD_\sigma\right]\left[iD_\lambda, iD_\delta\right]\, b_v|\bar{B}\rangle\ \Pi^{\rho\delta} v^\sigma v^\lambda$$

$$2M_B\, m_3 = \langle \bar{B}|\bar{b}_v\, \left[iD_\rho, iD_\sigma\right]\left[iD_\lambda, iD_\delta\right]\, b_v|B\rangle\ \Pi^{\rho\lambda}\Pi^{\sigma\delta}$$

$$2M_B\, m_4 = \langle \bar{B}|\bar{b}_v\, \left\{iD_\rho, \left[iD_\sigma, \left[iD_\lambda, iD_\delta\right]\right]\right\}\, b_v|\bar{B}\rangle \Pi^{\sigma\lambda}\Pi^{\rho\delta}$$

$$2M_B\, m_5 = \langle \bar{B}|\bar{b}_v\, \left[iD_\rho, iD_\sigma\right]\left[iD_\lambda, iD_\delta\right]\left(-i\sigma_{\alpha\beta}\right)\, b_v|\bar{B}\rangle\ \Pi^{\alpha\rho}\Pi^{\beta\delta} v^\sigma v^\lambda$$

$$2M_B\, m_6 = \langle \bar{B}|\bar{b}_v\, \left[iD_\rho, iD_\sigma\right]\left[iD_\lambda, iD_\delta\right]\left(-i\sigma_{\alpha\beta}\right)\, b_v|\bar{B}\rangle\ \Pi^{\alpha\sigma}\Pi^{\beta\lambda}\Pi^{\rho\delta}$$

$$2M_B\, m_7 = \langle \bar{B}|\bar{b}_v\, \left\{\left\{iD_\rho, iD_\sigma\right\}, \left[iD_\lambda, iD_\delta\right]\right\}\left(-i\sigma_{\alpha\beta}\right)\, b_v|\bar{B}\rangle\ \Pi^{\sigma\lambda}\Pi^{\alpha\rho}\Pi^{\beta\delta}$$

$$2M_B\, m_8 = \langle \bar{B}|\bar{b}_v\, \left\{\left\{iD_\rho, iD_\sigma\right\}, \left[iD_\lambda, iD_\delta\right]\right\}\left(-i\sigma_{\alpha\beta}\right)\, b_v|\bar{B}\rangle \Pi^{\rho\sigma}\Pi^{\alpha\lambda}\Pi^{\beta\delta}$$

$$2M_B\, m_9 = \langle \bar{B}|\bar{b}_v\, \left[iD_\rho, \left[iD_\sigma, \left[iD_\lambda, iD_\delta\right]\right]\right]\left(-i\sigma_{\alpha\beta}\right)\, b_v|\bar{B}\rangle\ \Pi^{\rho\beta}\Pi^{\lambda\alpha}\Pi^{\sigma\delta}\,,$$

$$
\begin{aligned}
2M_B r_1 &= \langle \bar{B}|\bar{b}_v \, iD_\rho \, (iv \cdot D)^3 \, iD^\rho \, b_v|\bar{B}\rangle \\
2M_B r_2 &= \langle \bar{B}|\bar{b}_v \, iD_\rho \, (iv \cdot D) \, iD^\rho \, iD_\sigma \, iD^\sigma \, b_v|\bar{B}\rangle \\
2M_B r_3 &= \langle \bar{B}|\bar{b}_v \, iD_\rho \, (iv \cdot D) \, iD_\sigma \, iD^\rho \, iD^\sigma \, b_v|\bar{B}\rangle \\
2M_B r_4 &= \langle \bar{B}|\bar{b}_v \, iD_\rho \, (iv \cdot D) \, iD_\sigma \, iD^\sigma \, iD^\rho \, b_v|\bar{B}\rangle \\
2M_B r_5 &= \langle \bar{B}|\bar{b}_v \, iD_\rho \, iD^\rho \, (iv \cdot D) \, iD_\sigma \, iD^\sigma \, b_v|\bar{B}\rangle \\
2M_B r_6 &= \langle \bar{B}|\bar{b}_v \, iD_\rho \, iD_\sigma \, (iv \cdot D) \, iD^\sigma \, iD^\rho \, b_v|\bar{B}\rangle \\
2M_B r_7 &= \langle \bar{B}|\bar{b}_v \, iD_\rho \, iD_\sigma \, (iv \cdot D) \, iD^\rho \, iD^\sigma \, b_v|\bar{B}\rangle
\end{aligned}
$$

$$2M_B r_8 = \langle \bar{B} | \bar{b}_v \, iD_\mu \, (iv \cdot D)^3 \, iD_\nu \, (-i\sigma^{\mu\nu}) \, b_v | \bar{B} \rangle$$
$$2M_B r_9 = \langle \bar{B} | \bar{b}_v \, iD_\mu \, (iv \cdot D) \, iD_\nu \, iD_\rho \, iD^\rho \, (-i\sigma^{\mu\nu}) \, b_v | \bar{B} \rangle$$
$$2M_B r_{10} = \langle \bar{B} | \bar{b}_v \, iD_\rho \, (iv \cdot D) \, iD^\rho \, iD_\mu \, iD_\nu \, (-i\sigma^{\mu\nu}) \, b_v | \bar{B} \rangle$$
$$2M_B r_{11} = \langle \bar{B} | \bar{b}_v \, iD_\rho \, (iv \cdot D) \, iD_\mu \, iD^\rho \, iD_\nu \, (-i\sigma^{\mu\nu}) \, b_v | \bar{B} \rangle$$
$$2M_B r_{12} = \langle \bar{B} | \bar{b}_v \, iD_\mu \, (iv \cdot D) \, iD_\rho \, iD_\nu \, iD^\rho \, (-i\sigma^{\mu\nu}) \, b_v | \bar{B} \rangle$$
$$2M_B r_{13} = \langle \bar{B} | \bar{b}_v \, iD_\rho \, (iv \cdot D) \, iD_\mu \, iD_\nu \, iD^\rho \, (-i\sigma^{\mu\nu}) \, b_v | \bar{B} \rangle$$
$$2M_B r_{14} = \langle \bar{B} | \bar{b}_v \, iD_\mu \, (iv \cdot D) \, iD_\rho \, iD^\rho \, iD_\nu \, (-i\sigma^{\mu\nu}) \, b_v | \bar{B} \rangle$$
$$2M_B r_{15} = \langle \bar{B} | \bar{b}_v \, iD_\mu \, iD_\nu \, (iv \cdot D) \, iD_\rho \, iD^\rho \, (-i\sigma^{\mu\nu}) \, b_v | \bar{B} \rangle$$
$$2M_B r_{16} = \langle \bar{B} | \bar{b}_v \, iD_\rho \, iD_\mu \, (iv \cdot D) \, iD_\nu \, iD^\rho \, (-i\sigma^{\mu\nu}) \, b_v | \bar{B} \rangle$$
$$2M_B r_{17} = \langle \bar{B} | \bar{b}_v \, iD_\mu \, iD_\rho \, (iv \cdot D) \, iD^\rho \, iD_\nu \, (-i\sigma^{\mu\nu}) \, b_v | \bar{B} \rangle$$
$$2M_B r_{18} = \langle \bar{B} | \bar{b}_v \, iD_\rho \, iD_\mu \, (iv \cdot D) \, iD^\rho \, iD_\nu \, (-i\sigma^{\mu\nu}) \, b_v | \bar{B} \rangle$$

Truncation Uncertainty Estimate

- Rewrite Master Formulae as spectral density integral

$$\sum_{k=0}^{\infty}\sum_{n}(2\pi)^3\delta^3(p_n^\perp)\left(\frac{-\epsilon_n}{\omega}\right)^k \langle B(p_B)|\bar{b}_v\mathcal{P}_1 Q_v|n\rangle\,\langle n|\bar{Q}_v\mathcal{P}_2\Gamma b_v|B(p_B)\rangle$$

$$= \int\frac{\mathrm{d}\omega'}{2\pi}\frac{\rho(\omega')}{\omega-\omega'} := \Delta(\omega)$$

- Represent spectral density as sum infintely many narrow resonances

$$\rho(\omega) = \sum_n g(n)\delta(\omega - n\Lambda)$$

- Then the master equation is given by

$$\Delta(\omega) = \frac{1}{2\pi}\sum_n g(n)\frac{1}{\omega - n\Lambda}$$

- Impose radial wave function behaviour for resonances $g(n) = g_0 1/n^2$

$$\Delta(\omega) = \frac{g_0}{2\pi\Lambda}\frac{1}{(\omega/\Lambda)^2}\left[\gamma + \psi(1-\omega/\Lambda) + \frac{\pi^2}{6}\omega/\Lambda\right]$$

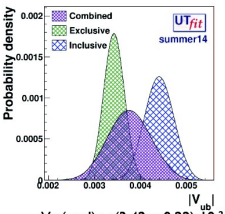
UT Fit V_{ub} and V_{cb} [Derkach ICHEP 2014]

The relative ratio of CKM elements is easily calculable:

$$\left|\frac{V_{ub}}{V_{cb}}\right| = \frac{\lambda}{1 - \frac{\lambda^2}{2}}\sqrt{\bar{\rho}^2 + \bar{\eta}^2}$$

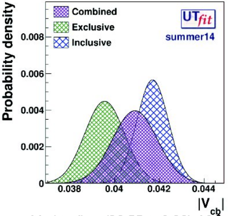
QCD corrections to be considered

- inclusive measurements: OPE
- exclusive measurements: form-factors from lattice QCD



V_{ub}(excl) = (3.42 ± 0.22) 10^{-3}
V_{ub}(incl) = (4.40 ± 0.31) 10^{-3}
V_{ub} = (3.75 ± 0.46) 10^{-3}

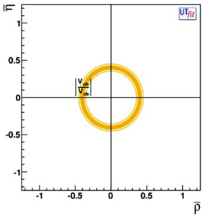
~1.9 σ discrepancy



V_{cb}(excl) = (39.55 ± 0.88) 10^{-3}
V_{cb}(incl) = (41.7 ± 0.7) 10^{-3}
V_{cb} = (40.9 ± 1.0) 10^{-3}

~2.5 σ discrepancy

D. Derkach UTFIT@ICHEP2014

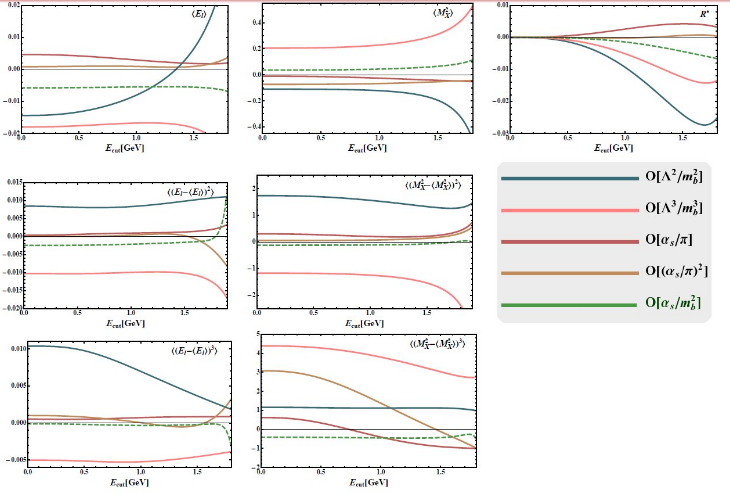


There is still an inconsistency between inclusive and exclusive measurements. We take this into account inflating the combined uncertainty (a-la PDG).

11

α_s/m_b Corrections and Fit [Healy ICHEP 2014]

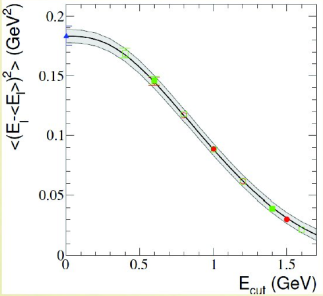
New Contributions $\mathcal{O}(\alpha_s/m_b^2)$: R^*

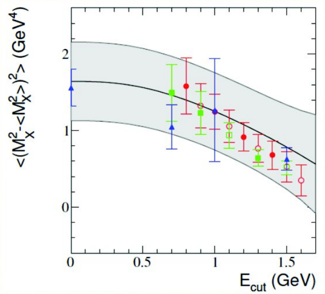


Kristopher J. Healey | ICHEP2014 | 8/ 11

α_s/m_b Corrections [Gambino CKM 2014]

THEORETICAL ERRORS





Theoretical errors are generally the **dominant** ones in the fits.
We estimate them in a **conservative** way by mimicking higher orders varying the parameters by fixed amounts.

Duality violation, expected to be suppressed, would manifest as inconsistency in the fit.

α_s/m_b Corrections [Gambino CKM 2014]

$O(\alpha_s/m_b^2)$ EFFECTS

Boos,Becher,Lunghi 2007
Ewerth,Nandi, PG 2009
Alberti,Ewerth,Nandi,PG 2012
Alberti,Nandi,PG 2013

Hadronic tensor $$W^{\alpha\beta} = \frac{(2\pi)^3}{2m_B} \sum_{X_c} \delta^4(p_b - q - p_X) \langle \bar{B} | J_L^{\dagger\alpha} | X_c \rangle \langle X_c | J_L^\beta | \bar{B} \rangle$$

$$m_b W^{\alpha\beta} = -W_1 g^{\alpha\beta} + W_2 v^\alpha v^\beta + i W_3 \epsilon^{\alpha\beta\rho\sigma} v_\rho \hat{q}_\sigma + W_4 \hat{q}^\alpha \hat{q}^\beta + W_5 (v^\alpha \hat{q}^\beta + v^\beta \hat{q}^\beta)$$

$$W_i = W_i^{(0)} + \frac{\mu_\pi^2}{2m_b^2} W_i^{(\pi,0)} + \frac{\mu_G^2}{2m_b^2} W_i^{(G,0)} + \frac{C_F \alpha_s}{\pi} \left[W_i^{(1)} + \frac{\mu_\pi^2}{2m_b^2} W_i^{(\pi,1)} + \frac{\mu_G^2}{2m_b^2} W_i^{(G,1)} \right]$$

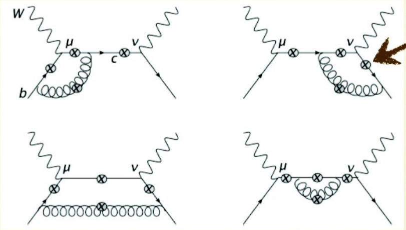
$W_i^{(\pi,n)}$ can be computed using **reparameterization invariance** which relates different orders in the HQET: e.g. for $i=3$ at all orders

$$W_3^{(\pi,n)} = \frac{5}{3} \hat{q}_0 \frac{dW_3^{(n)}}{d\hat{q}_0} - \frac{\hat{q}^2 - \hat{q}_0^2}{3} \frac{d^2 W_3^{(n)}}{d\hat{q}_0^2}$$ Manohar 2010

Proliferation of power divergences, up to $1/u^3$,
and complex kinematics (q^2, q_0, m_c, m_b) $W_i^{(G,1)}$ requires proper matching.

α_s/m_b Corrections [Gambino CKM 2014]

MATCHING AT O(α_s)



possible gluon insertions

QCD

HQET

$$\frac{2i}{\pi}\int d^4x\, e^{-iq\cdot x}\, T[J_L^{\dagger\mu}(x)J_L^{\nu}(0)] = \sum_i c^{(i)\mu\nu}_{\{\alpha\}}(v,q)\, O_i^{\{\alpha\}}(0)$$

Taylor expansion around on-shell b quark matched onto HQET local operators. Analytic formulae. RPI relations reproduced. Unlike μ_π, μ_G gets renormalized, therefore Wilson coefficients scale-dependent.

α_s/m_b Corrections [Gambino CKM 2014]

Numerical results

In on-shell scheme (m_b=4.6GeV, m_c=1.15GeV) without cuts

$$\Gamma_{B\to X_c\ell\nu} = \Gamma_0 \left[\left(1 - 1.78\,\frac{\alpha_s}{\pi}\right)\left(1 - \frac{\mu_\pi^2}{2m_b^2}\right) - \left(1.94 + 2.42\,\frac{\alpha_s}{\pi}\right)\frac{\mu_G^2(m_b)}{m_b^2}\right]$$

$$\langle E_\ell \rangle = 1.41\mathrm{GeV} \left[\left(1 - 0.02\,\frac{\alpha_s}{\pi}\right)\left(1 + \frac{\mu_\pi^2}{2m_b^2}\right) - \left(1.19 + 4.20\,\frac{\alpha_s}{\pi}\right)\frac{\mu_G^2(m_b)}{m_b^2}\right]$$

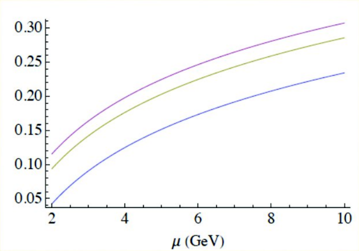
$$\ell_2 = 0.183\,\mathrm{GeV}^2 \left[1 - 0.16\,\frac{\alpha_s}{\pi} + \left(4.98 - 0.37\,\frac{\alpha_s}{\pi}\right)\frac{\mu_\pi^2}{m_b^2} - \left(2.89 + 8.44\,\frac{\alpha_s}{\pi}\right)\frac{\mu_G^2(m_b)}{m_b^2}\right]$$

Similar results in the kinetic scheme. NLO corrections generally *O(15-20%)* of tree level coefficients, **shifts in some cases larger than experimental error**. Impact on V_{cb} requires new fit of semileptonic moments.

Mannel, Pivovarov, Rosenthal (1405.5072) have computed the μ_G correction to the width in the limit m_c=0 and find compatible result.

α_s/m_b Corrections [Gambino CKM 2014]

μ_G^2-SCALE DEPENDENCE



Relative NLO correction to the coefficients of μ_G in the width (blue), first (red) and second central (yellow) leptonic moments as a function of the renormalization scale. Smaller corrections for smaller scale.

Setup of the Differential Rate

Double Differential Rate

- Consider differential rate in $v{\cdot}p$ and p^2, where $p = m_b v - q$

$$\frac{d^2\Gamma}{dv{\cdot}p\,dp^2} = \frac{G_F^2|V_{cb}|^2}{6\pi^3}\sqrt{(v{\cdot}p)^2 - p^2}\; W^{\mu\nu}$$
$$\times \left[m_b^2\left(v_\mu v_\nu - g_{\mu\nu}\right) - 2m_b\left(\frac{v_\mu p_\nu + v_\nu p_\mu}{2} - g_{\mu\nu} v{\cdot}p\right) + p_\mu p_\nu - g_{\mu\nu}p^2 \right]$$

- Hadronic tensor
- From leptonic tensor

General Structure

$$\frac{d^2\Gamma}{dv{\cdot}p\,dp^2} = \frac{G_F^2|V_{cb}|^2}{6\pi^3}\sqrt{(v{\cdot}p)^2 - p^2}\sum_{n=0}^{\infty} P_n(v{\cdot}p, p^2, m_c)\,\delta^{(n)}(p^2 - m_c^2)$$

- P_n is a polynomial containing $\langle B(p)|\bar{b}_{v,\alpha}(iD_{\mu_1})\dots(iD_{\mu_n})b_{v,\beta}|B(p)\rangle$

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Expansion in $1/m_c$

Origin

$$\int \mathrm{d}v\!\cdot\! p \sqrt{(v\!\cdot\! p)^2 - p^2}\; v\!\cdot\! p^{2n} (p^2)^k = C_n\, (p^2)^{n+k+1} \log p^2 + \text{regular}$$

- Project out most singular contribution

Determine Leading Order

- We have in the order $1/m_b^i$ (for simplicity $n = k = 0$)

$$\Gamma \sim \int \mathrm{d}p^2\, m_c^{i+1} p^2 \log p^2\, \delta^{(i)}(p^2 - m_c^2)$$
$$\sim (m_c^2)^{\frac{3-i}{2}}$$

$\Rightarrow$ For $i = 5$ the first $1/(m_b^3 m_c^2)$ terms appear

Expansion in $1/m_c$

Origin

$$\int \mathrm{d}v{\cdot}p\sqrt{(v{\cdot}p)^2 - p^2}\; v{\cdot}p^{2n}(p^2)^k = C_n\;(p^2)^{n+k+1}\log p^2 + \text{regular}$$

- Project out most singular contribution

Determine Leading Order

- We have in the order $1/m_b^i$ (for simplicity $n = k = 0$)

$$\Gamma \sim \int \mathrm{d}p^2\, m_c^{i+1} p^2 \log p^2\, \delta^{(i)}(p^2 - m_c^2)$$
$$\sim (m_c^2)^{\frac{3-i}{2}}$$

⇒ For $i = 5$ the first $1/(m_b^3 m_c^2)$ terms appear

Scenario I: $m_b \sim m_c \gg \Lambda_{\mathrm{QCD}}$

Calculable Part

- Integrate out (hard) quantum fluctuations with virtuality of $\mathcal{O}(m_{b,c})$

⇒ Only light-degrees of freedom remain:
  - light quarks
  - gluons
  - quasi-static b-quark field in HQET
- Short-distance matching coefficients and phase space integrals are functions of fixed ratio $\rho = m_c^2/m_b^2$

Non-Perturbative Part

- At $\mu < m_c$: Operators with charm-quark do not appear in a standard renormalization scheme like e.g. $\overline{\mathrm{MS}}$
- They correspond to $\langle \bar{B}|\bar{b}_v \ldots c_{\mathrm{static}}\, \bar{c}_{\mathrm{static}} \ldots b_v|\bar{B}\rangle \equiv 0$
- Matches to zero because of $m_b + 2m_c + \Delta E_{\mathrm{soft}} > m_B$

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Scenario III: $m_b \gg m_c \gtrsim \Lambda_{QCD}$

- Charm-quark effects cannot be integrated out perturbatively

$\Rightarrow$ Define proper power counting

Consequences

- Genuine intrinsic-charm operators exist
  - $\Rightarrow$ Hadronic matrix elements of this operators have to be defined at μ_0 with $m_b \geq \mu_0 \gg m_c$
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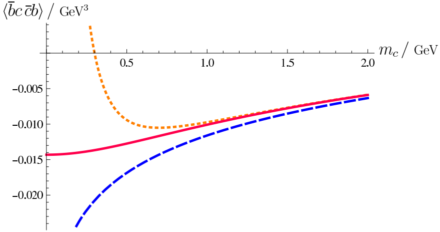
Model: Weak Annihilation in $b \to u$ Transitions



- Blue: Leading Log from order $1/m_b^3$
- Yellow: Including $1/(m_b^3 m_c^2)$ Corrections
- Red: Model (s.b.)

Model for "Weak-Annihilation" Operator

- Szenario 3: Four quark operator appears: "Weak-Annihilation"
- Renormalization group inspired model

$$\frac{1}{2M_B}\langle B|\bar{b}\gamma^k(1-\gamma_5)c\,\bar{c}\gamma^k(1-\gamma_5)b|B\rangle = \frac{\rho_D^3}{m_b^3}\ln\frac{m_b^2}{m_c^2+\Lambda^2}$$

- $\Lambda \approx 0.7$ GeV from comparisson with expansion up to $1/m_b^5$

$\Rightarrow$ Estimate of $\mathcal{O}(-3\%)$ contribution in $B \to X_u \ell \bar{\nu}_\ell$ decays

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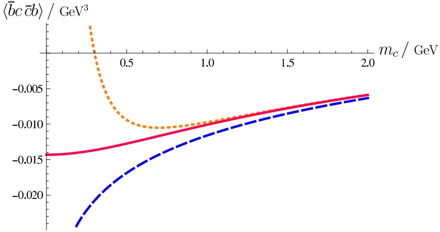

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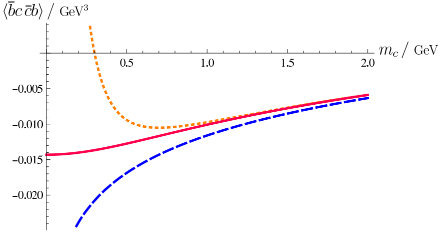

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