Heavy Quark mass determinations: non-lattice methods

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In collaboration with A. Hoang and B. Dehnadi (U. Vienna),

JHEP09 (2013) 103 + work in progress

Lattice meets continuum. Siegen.

30-09-2014

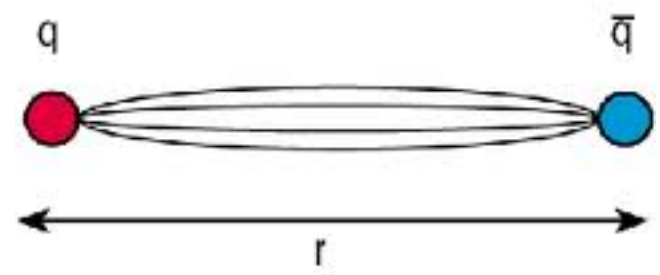
Outline

- Motivation & Introduction
- Charm mass from relativistic sum rules
  - Experimental data
  - Perturbative uncertainties
- Charm mass from pseudo-scalar correlator
- Bottom mass determinations
  - Experimental data
  - Perturbative uncertainties
- Conclusions and Outlook

Introduction

Theoretical remarks

Confinement: m_q not a physical observable



Parameter in QCD Lagrangian ⟶ formal definition (as for α_s)

$$\mathcal{L}_{\rm QCD} = \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} + \sum_f \bar{q}_f (\not{D} - m_f) q_f$$

Theoretical remarks

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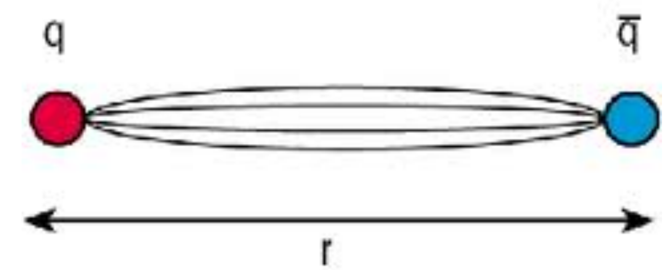


Parameter in QCD Lagrangian ⟶ formal definition (as for α_s)

$$\mathcal{L}_{\mathrm{QCD}} = \frac{1}{4} G^a_{\mu\nu} G_a^{\mu\nu} + \sum_f \bar{q}_f (\not{D} - m_f) q_f$$

Renormalization and scheme dependent object ⟶ $\delta m_q < \Lambda_{\mathrm{QCD}}$ possible

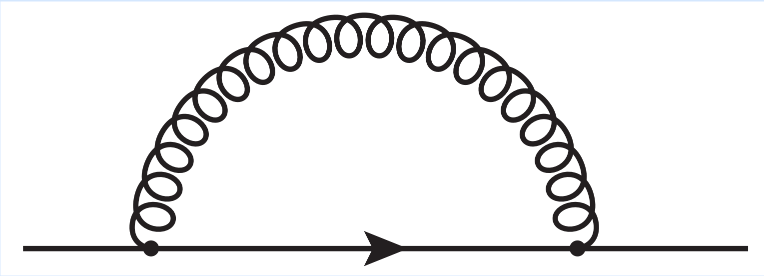
In general running mass (RG evolution)

Theoretical remarks

position of pole of propagator

$$m_{\text{pole}} = m_{\text{short-distance}} + \delta m$$

mass in
short distance scheme



δm defines the scheme and running

Theoretical remarks

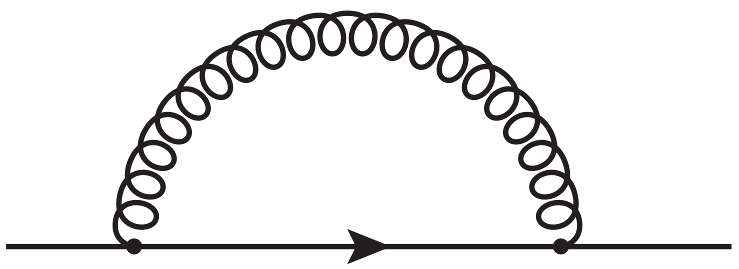
position of pole of propagator

$$m_{\text{pole}} = m_{\text{short-distance}} + \delta m$$

does not suffer from $\mathcal{O}(\Lambda_{\text{QCD}})$ ambiguity

$$\delta m = \mu \sum_{n=1} \alpha_s^{n+1} 2^n \beta_0^n \, n!$$

Contains renormalon



δm defines the scheme and running

infinitely many schemes !!!

Some schemes better than others…

best choice: process dependent

Theoretical remarks

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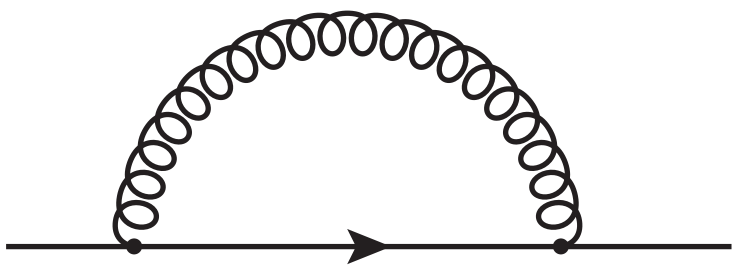


δm defines the scheme and running

infinitely many schemes !!!

Some schemes better than others…

best choice: process dependent

$\overline{\rm MS}$ scheme

- Short-distance scheme
- Standard mass for comparison $\overline{m}_q(\overline{m}_q)$
- And free of renormalon ambiguities

Short-distance masses in general have an ambiguity ~ $\mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{m_q}\right)$

top	0.5 - 1	MeV
bottom	20 - 50	MeV
charm	60 - 150	MeV

provably better in $\overline{\rm MS}$ scheme

Motivation

Why high precision?

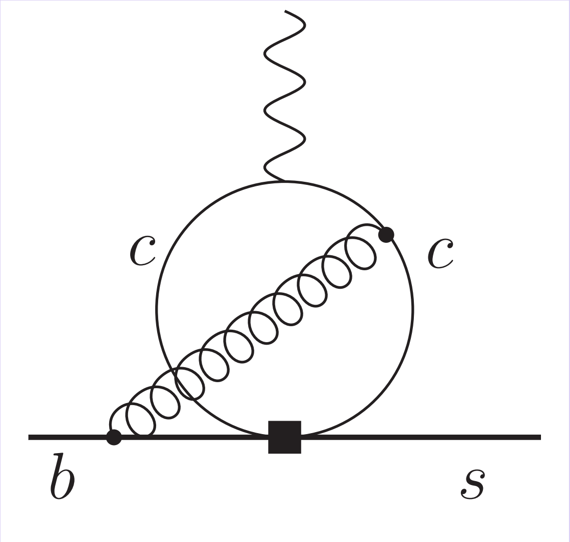
Strong dependence in flavor processes

constrains new physics

$B \to X_s\,\gamma$

strong charm mass dependence
in NLO matrix elements

[Missiak & Gambino]



$K^+ \to \pi^+\,\nu\,\bar{\nu}$

NNLO QCD computations for charm distributions

$B \to X_u\,\ell\,\nu_\ell$

$$\Gamma = G_F^2 |V_{ub}|^2 m_b^5 \left(1 + \alpha_s + \alpha_s^2 + \ldots\right)$$

$$\frac{\delta V_{us}}{V_{us}} \sim 2.5\,\frac{\delta m_b}{m_b}$$

Why high precision?

$K^+ \to \pi^+ \nu \bar{\nu}$ NNLO QCD computations for charm distributions

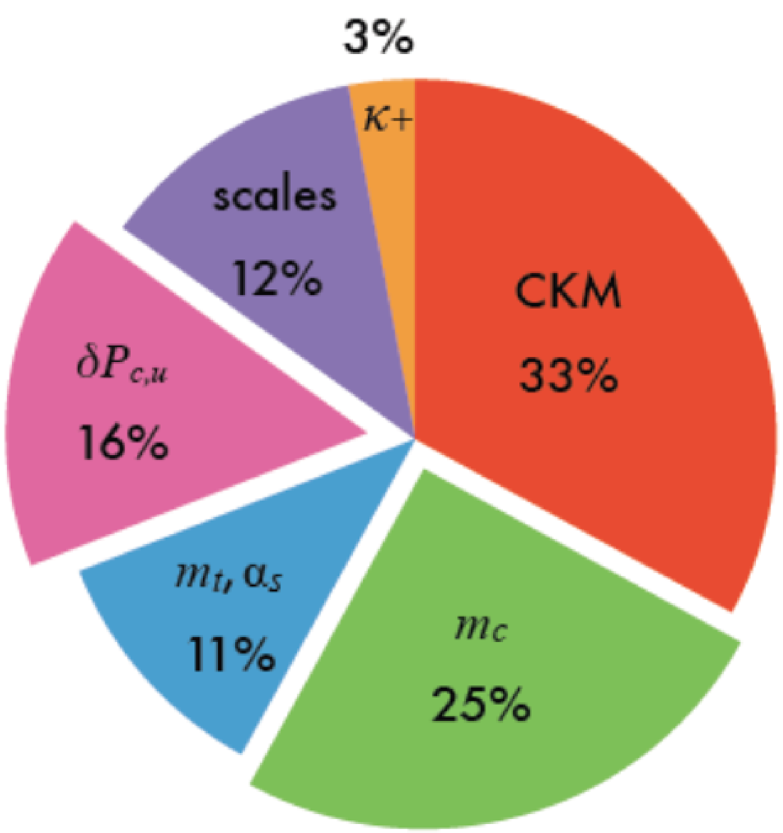
SM prediction(s) of $K^+ \to \pi^+ \nu\bar{\nu}$: error budget

$$\mathcal{B}(K^+ \to \pi^+ \nu\bar{\nu}(\gamma)) = \{8.57 \pm 0.93, 8.51 \pm 0.73, 8.04 \pm 0.98\} \times 10^{-11}$$

$m_c(m_c) = (1.30 \pm 0.05)$ GeV

$m_c(m_c) = (1.286 \pm 0.013)$ GeV *

$m_c(m_c) = (1.224 \pm 0.017 \pm 0.054)$ GeV †



*Kühn et al. '07 †Hoang & Manohar '05

from U. Haisch (2008)

Why high precision?

$K^+ \to \pi^+ \nu \bar{\nu}$ NNLO QCD computations for charm distributions

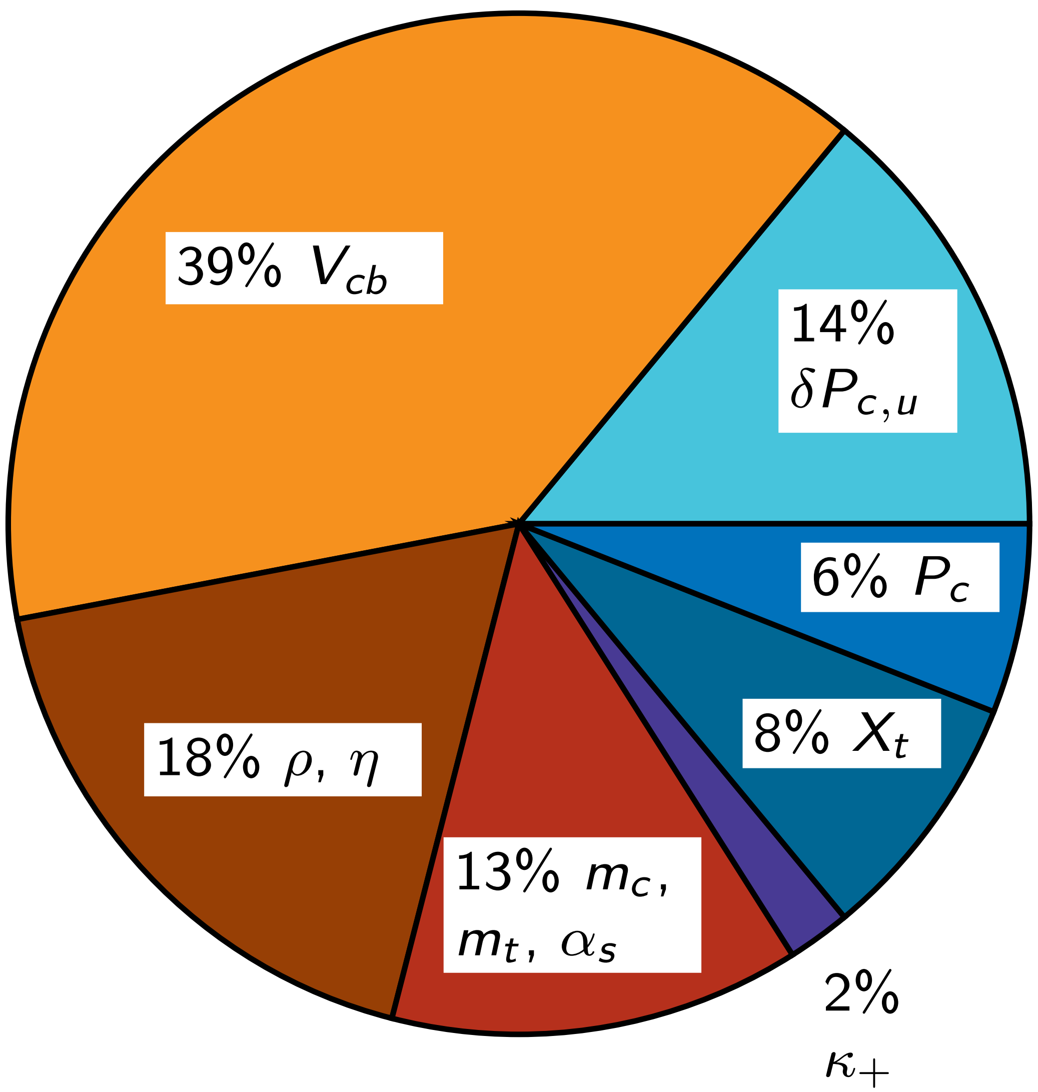
Error from m_c has gone down by a factor of two in few years.

Also central value has decreased

$\mathrm{Br}^{\mathrm{th}}(K^+) = 7.81(75)(29) \times 10^{-11}$
$\mathrm{Br}^{\mathrm{exp}}(K^+) = (17.3^{+11.5}_{-10.5}) \times 10^{-11}$

[E787, E949 '08]

$K^+ \to \pi^+ \nu \bar{\nu}$: Error Budget



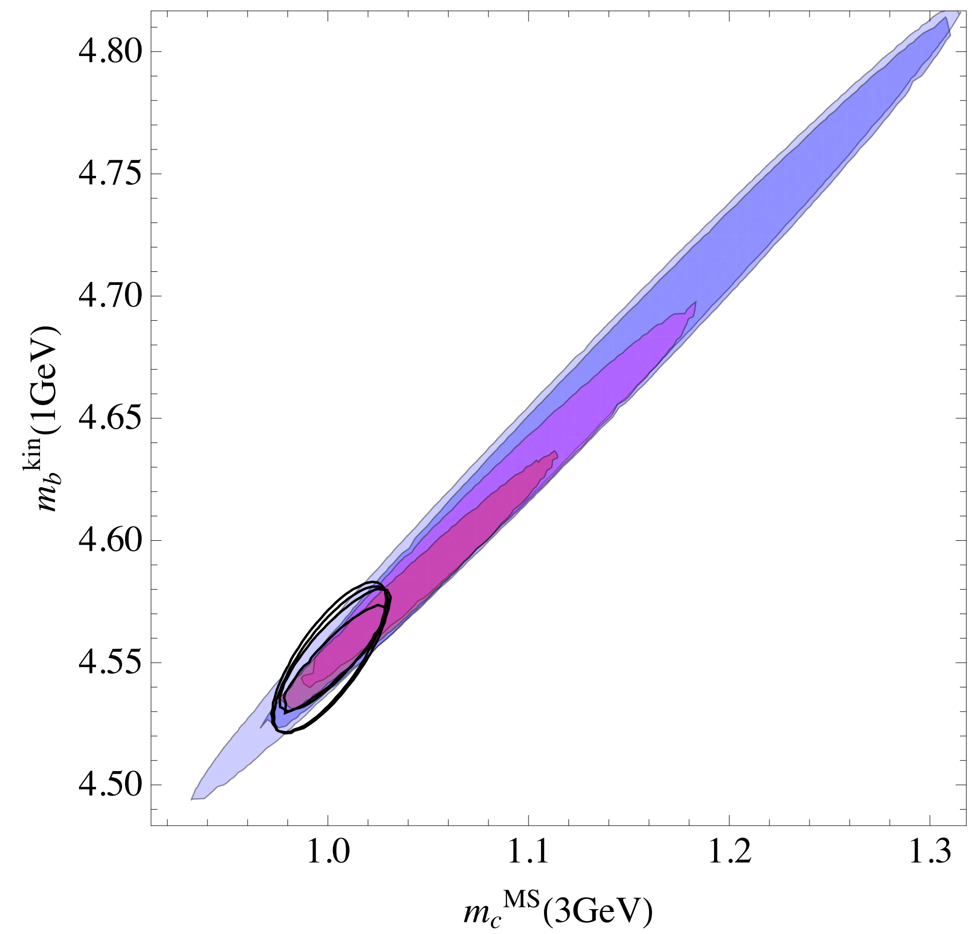
from Joachim Brod (CKM 2014)

Why high precision?

Inclusive B decays [Gambino Schwanda (2013)]

Analysis at $\mathcal{O}(\alpha_s^2)$

- very strong degeneracy m_c vs m_b
- Simple fit for both masses is hard
- Take m_c as input and determine m_b



	$\overline{m}_c(3\,\mathrm{GeV})$	$m_b^{kin}(1\,\mathrm{GeV})$	$\overline{m}_b(\overline{m}_b)$
Chetyrkin et al	0.986(13) [11]	4.541(23)	4.171(38)
Allison et al	0.986(6) [12]	4.540(20)	4.170(36)
Our analysis	0.994(26) [13]	4.549(29)	4.179(42)

Charm mass determination

Charm mass determinations

[K. A. Olive et al., PDG (2014)]

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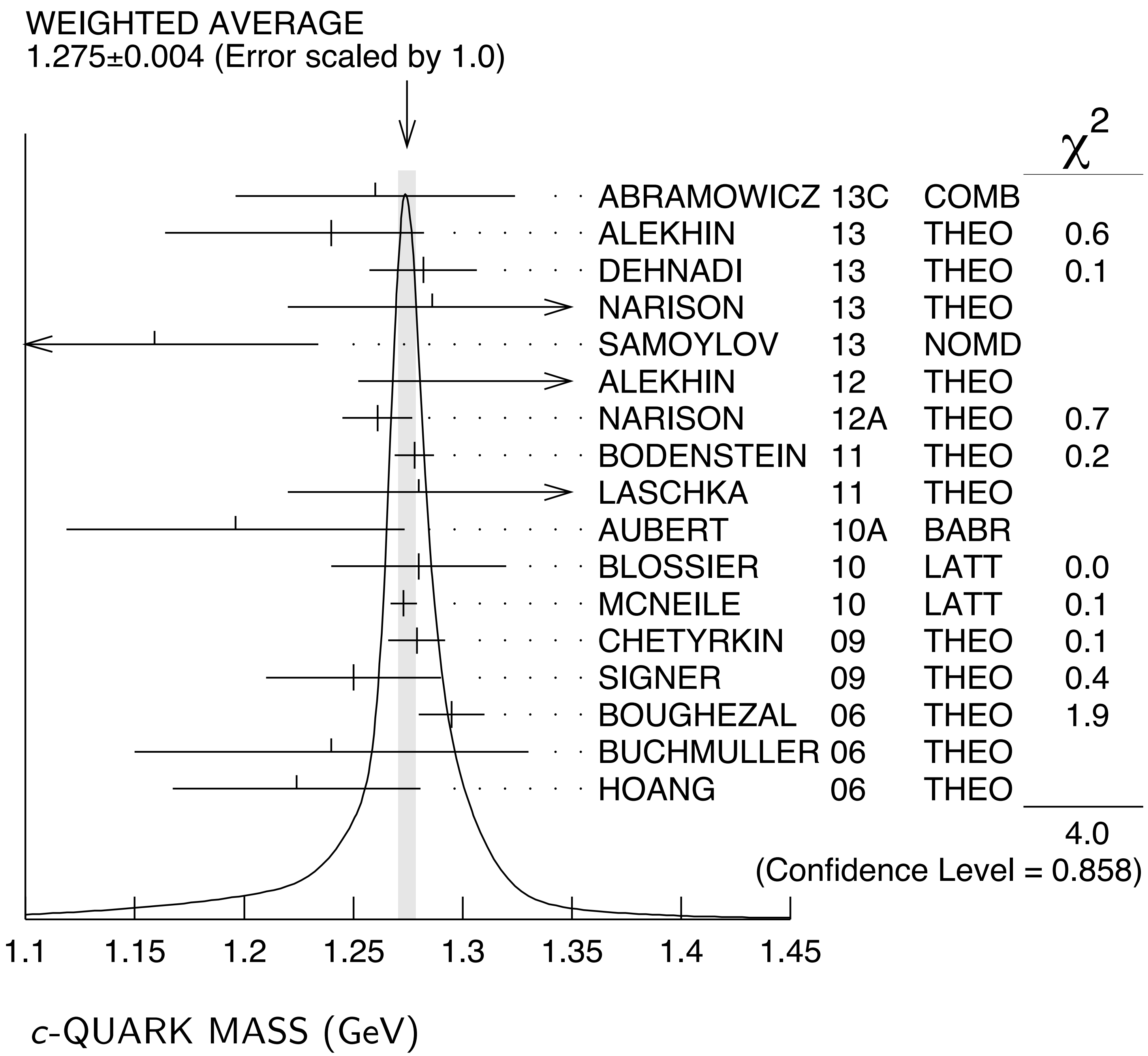

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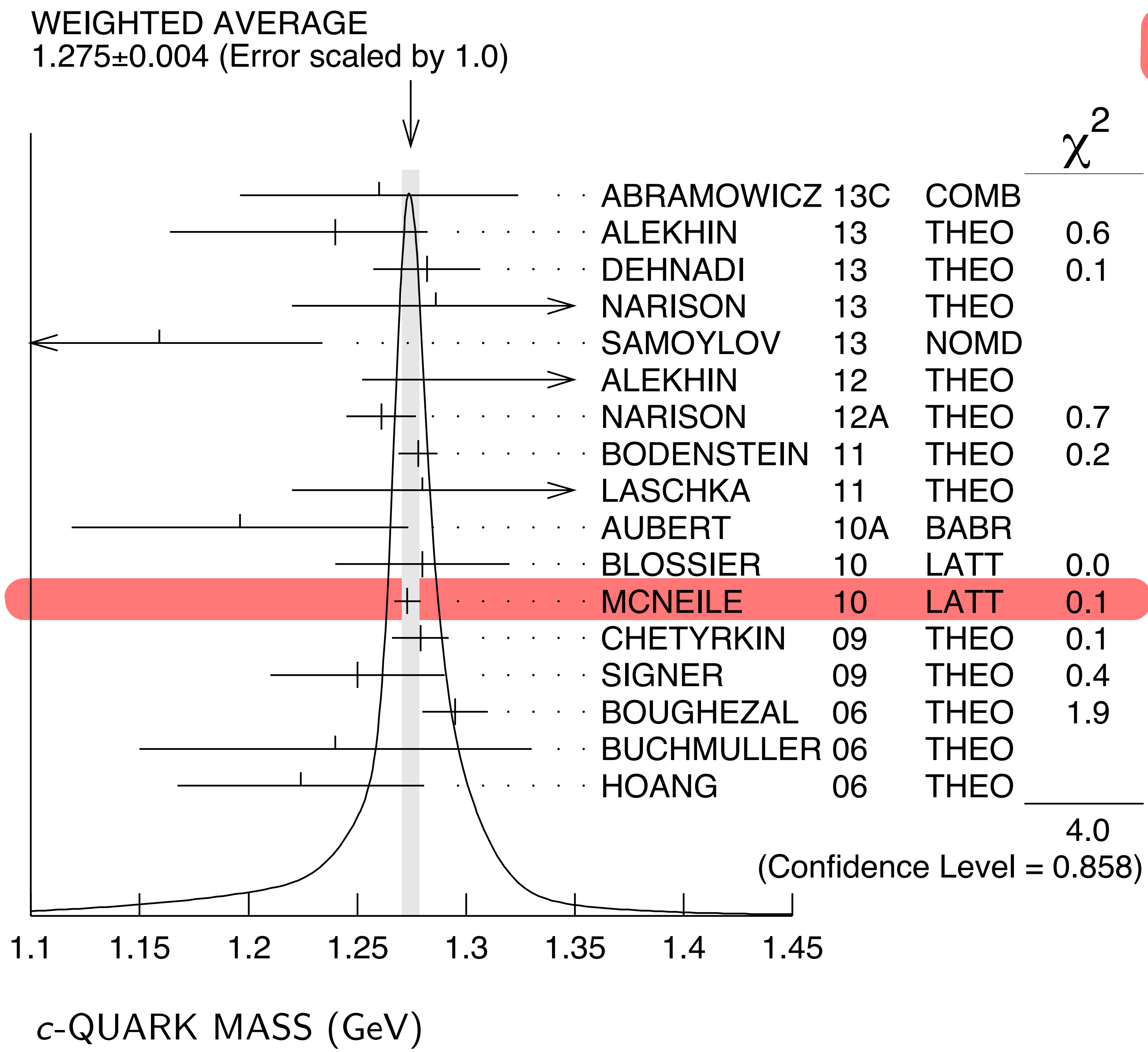

Current analyses with smallest error

1. *[HPQCD, McNeile et al (2010)]*

$$\overline{m}_c(\overline{m}_c) = 1.2758 \pm 0.0058\,\mathrm{GeV}$$

2. *[HPQCD, Chakraborty et al (2014)]*

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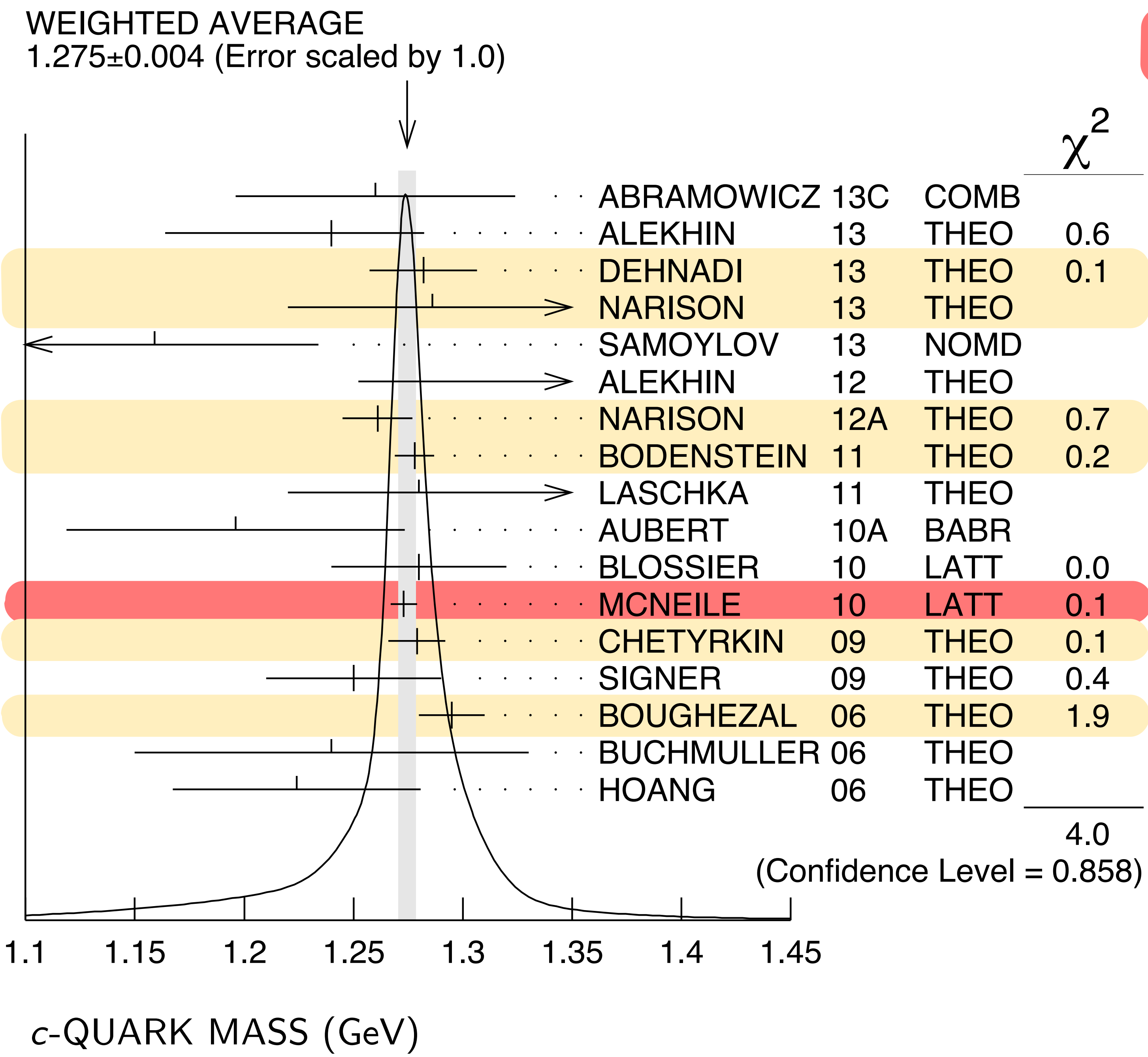

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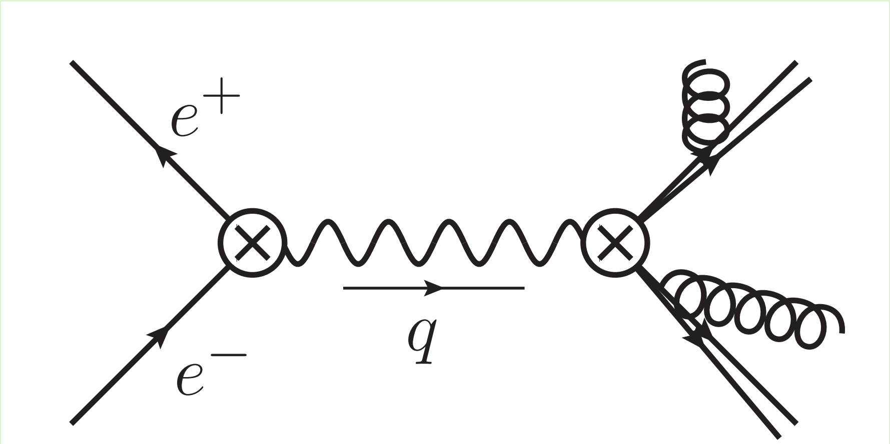
QCD sum rules (this talk)

Charm mass from Vector Correlator

QCD sum rules

Total hadronic cross section

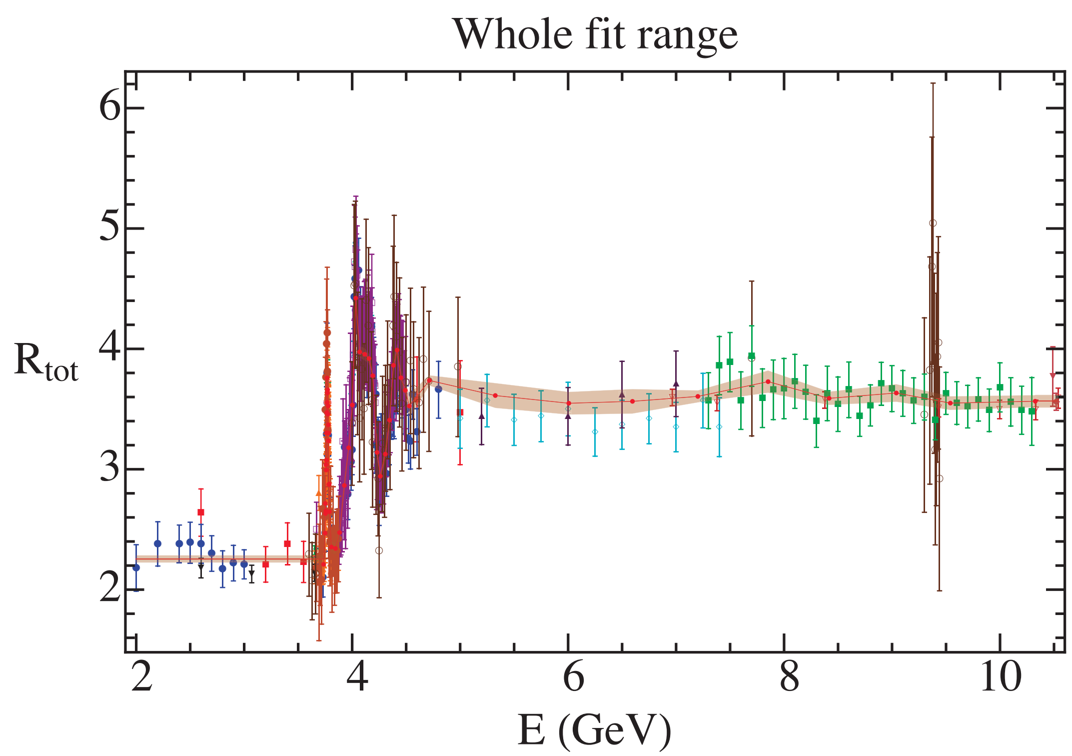
$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



QCD sum rules

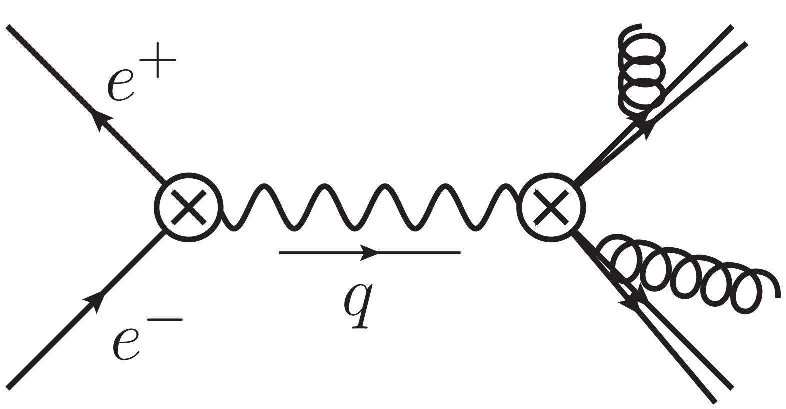
Total hadronic cross section

$$R(s) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$



- Some smearing is necessary for perturbation theory to have any chance to describe data

- We also need to design the observable to be maximally sensitive to the heavy quark mass



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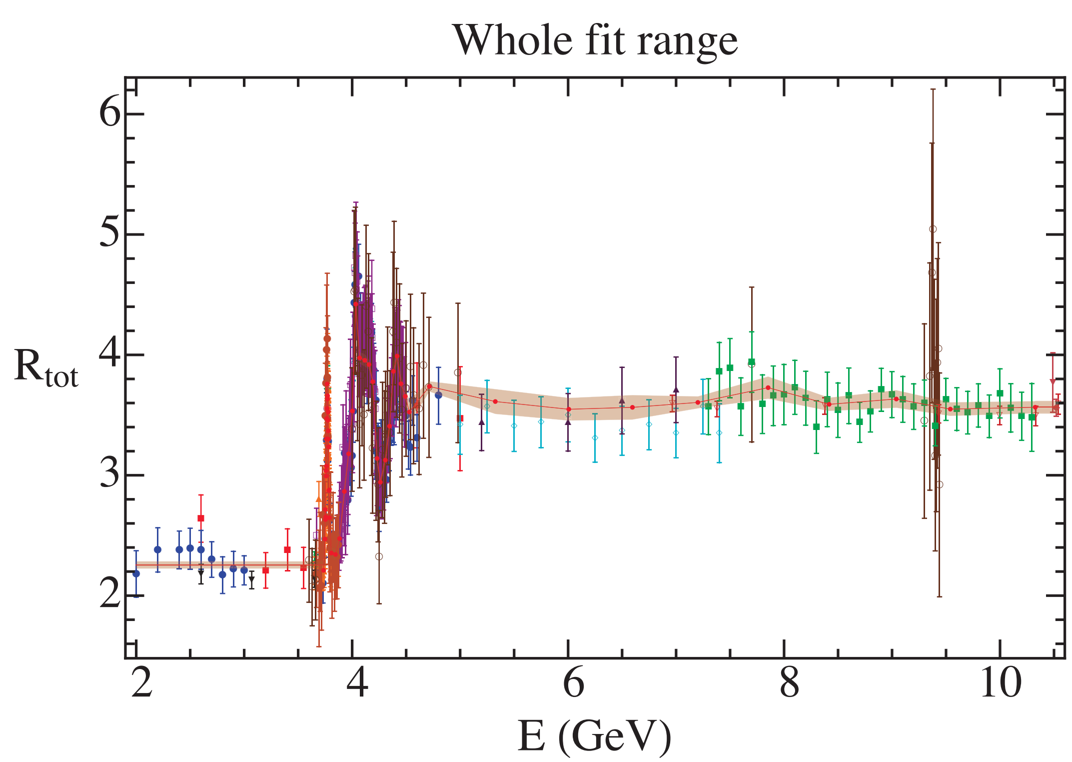

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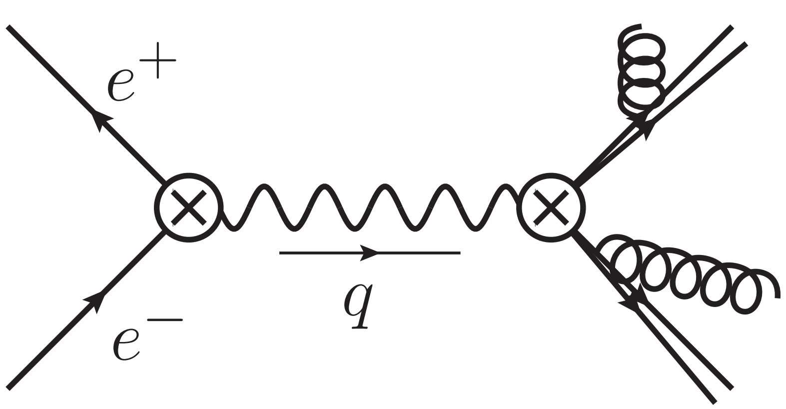

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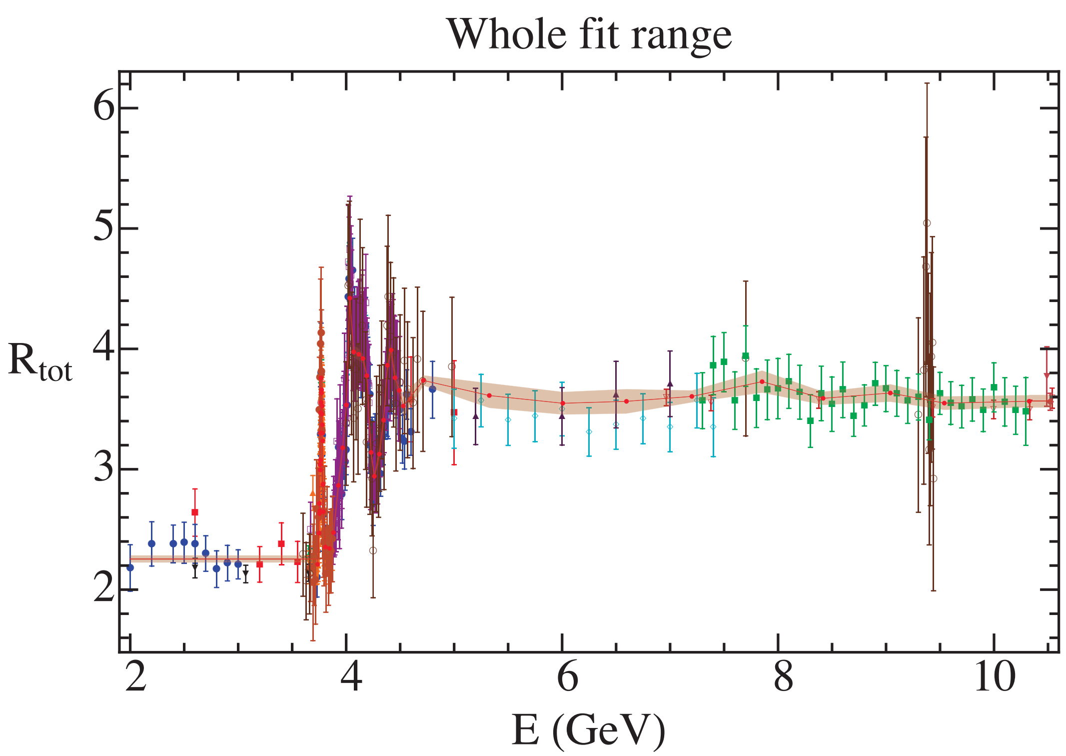
change of variables



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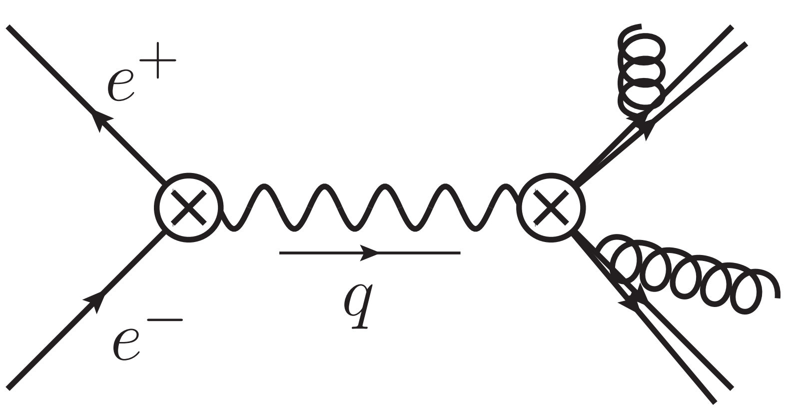

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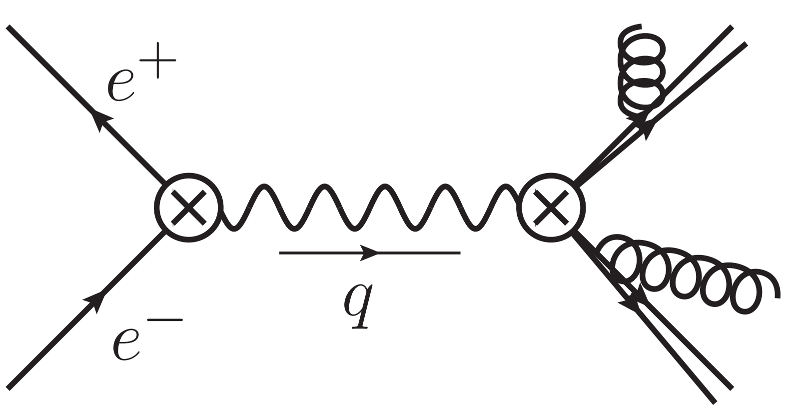

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Vector current (electromagnetic)

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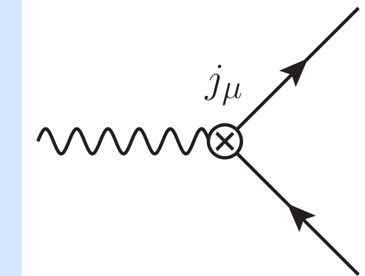

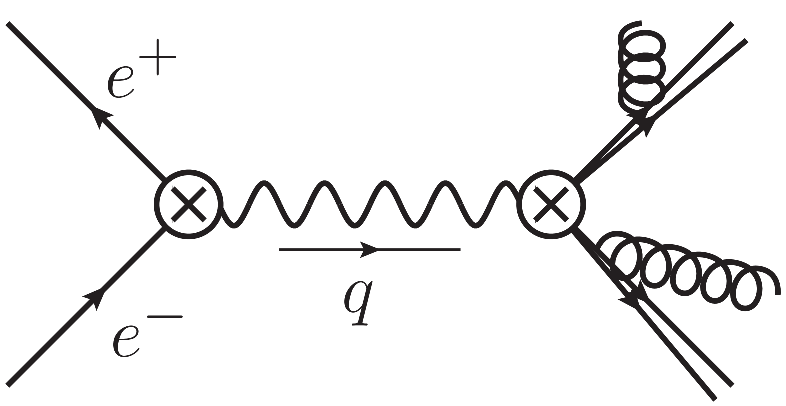

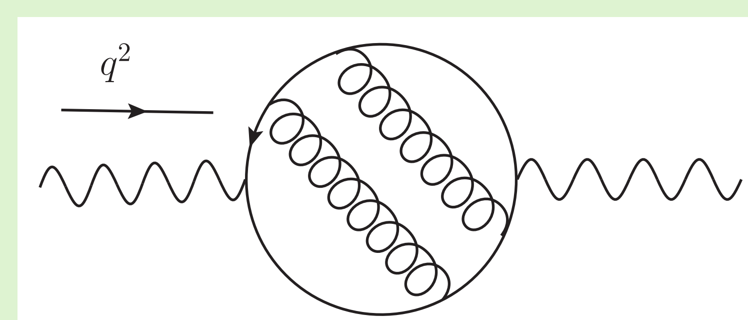

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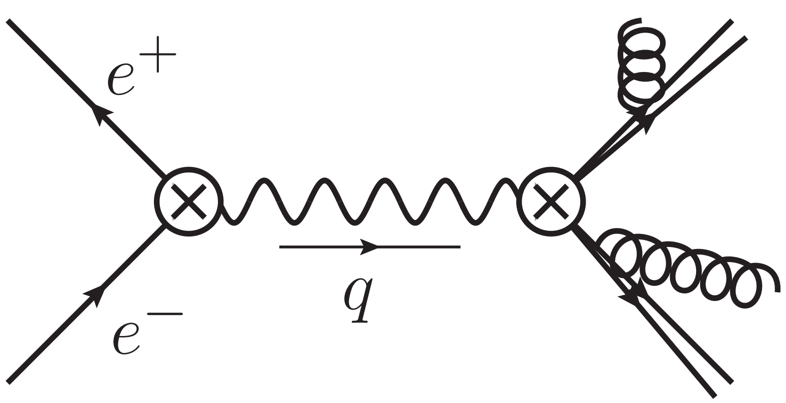
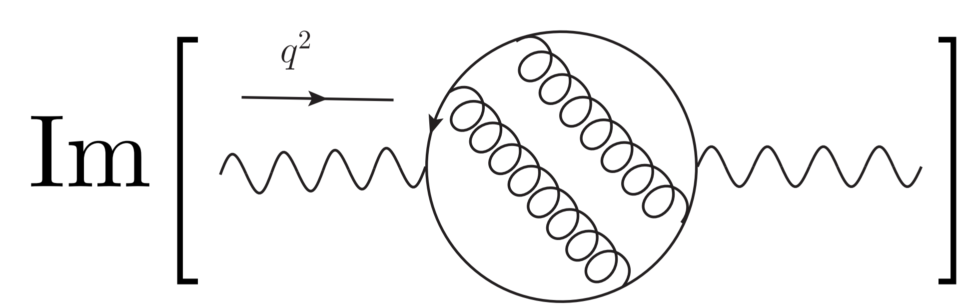
Optical theorem

electric charge

$$R(s) = 12\,\pi\, Q^2\, \mathrm{Im}\,\Pi(s + i\,0^+)$$



$\propto$



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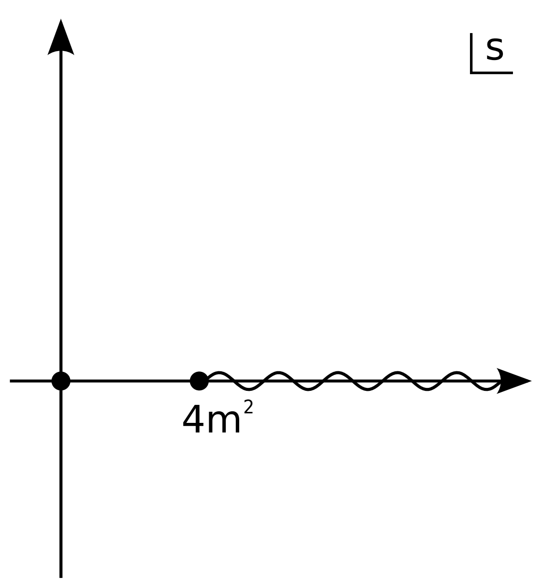

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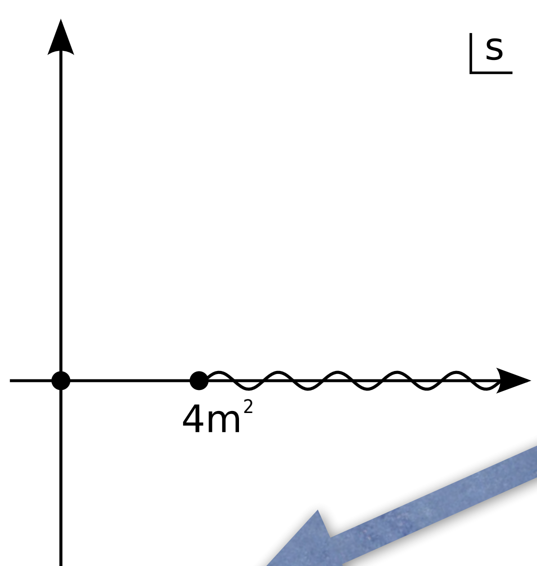

$$\Pi(q^2 \sim 0) = \frac{1}{12\,\pi^2\, Q^2} \sum_{n=0}^{\infty} M_n q^{2n} \quad \Longleftrightarrow \quad M_n^{\mathrm{th}} = \frac{12\pi^2 Q^2}{n!} \frac{\mathrm{d}^n}{\mathrm{d}q^{2n}} \Pi(q^2)\Big|_{q^2=0}$$

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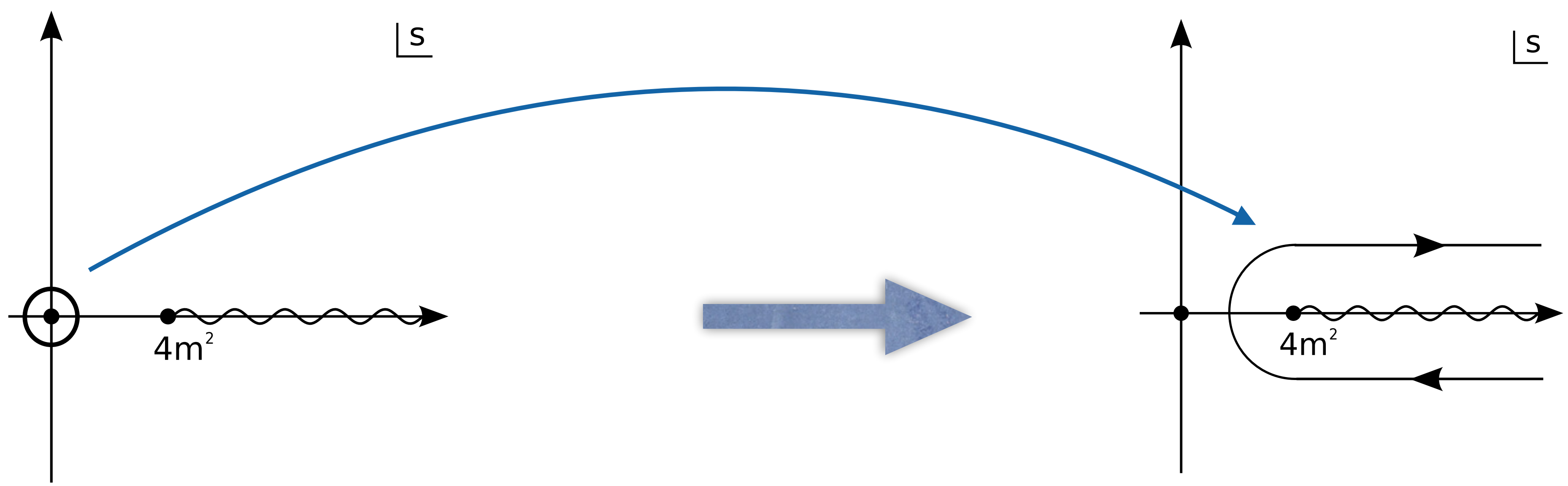

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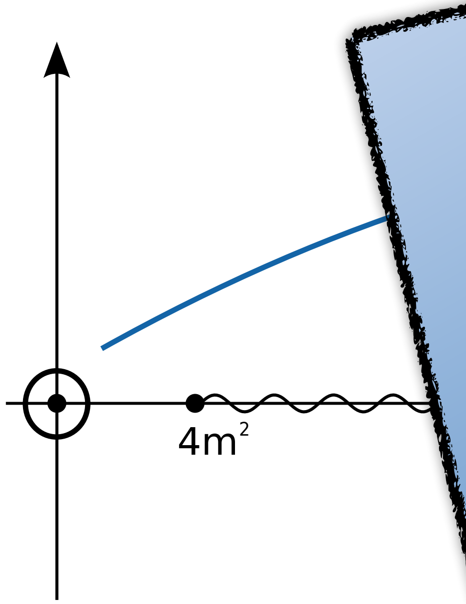
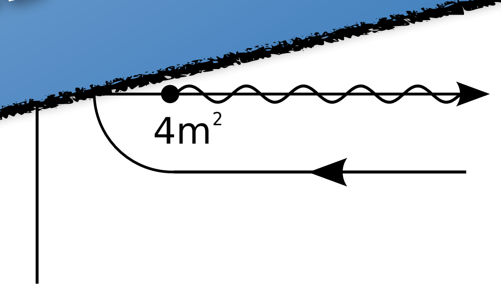


Di… …on

$$\ldots \int_{4m^2}^{\infty} \mathrm{d}s\, \frac{R(s)}{s(s-q^2)}$$



Duality bound:

$n < m_c/\Lambda_{QCD} \sim 2$

$n < m_b/\Lambda_{QCD} \sim 4$

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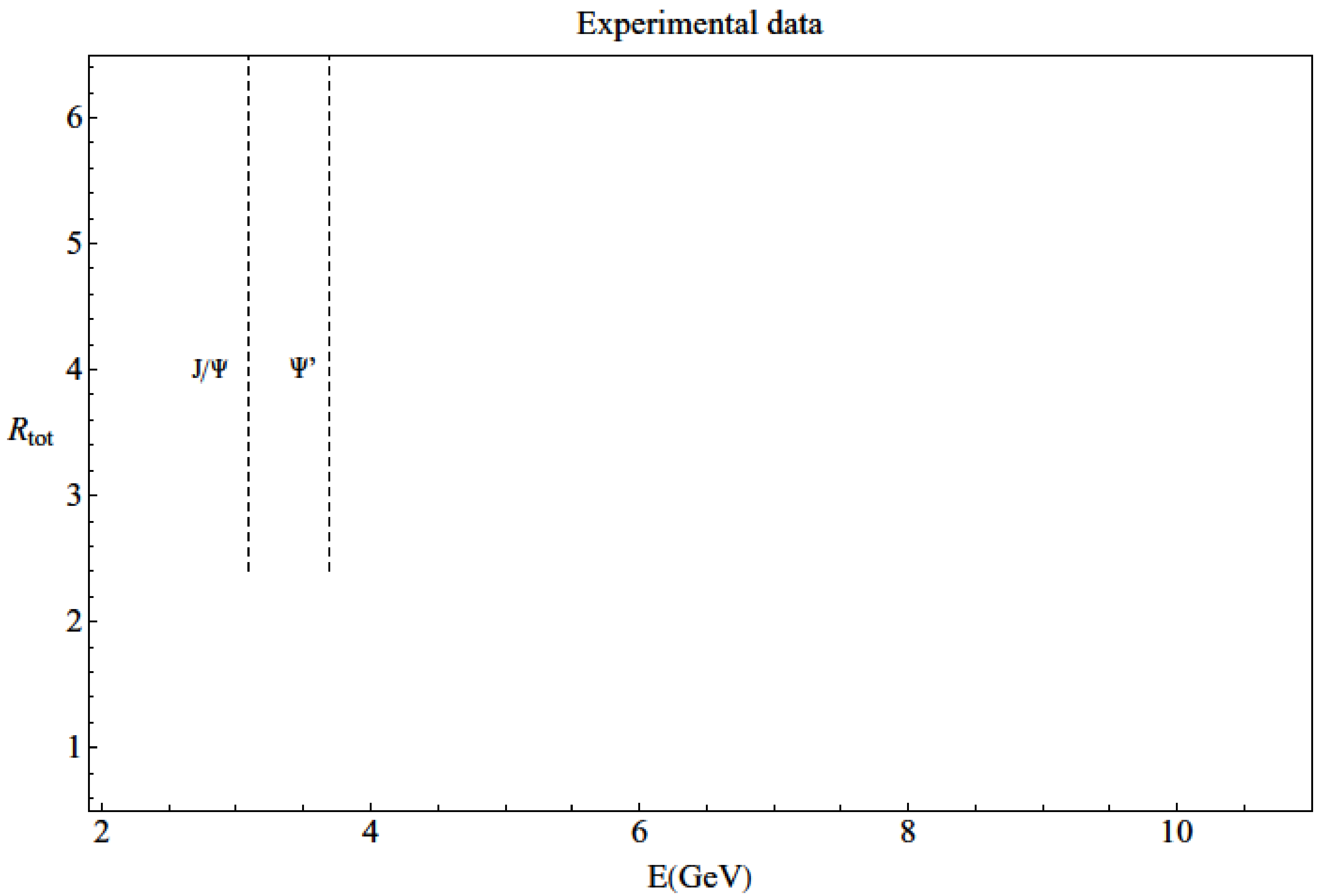
Experimental data for charm

Experimental data: charm

Narrow resonances

	J/Ψ	$\psi(2S)$
M (GeV)	3.096916(11)	3.686093(34)
Γ_{ee} (keV)	5.55(14)	2.48(6)
$(\alpha/\alpha(M))^2$	0.957785	0.95554

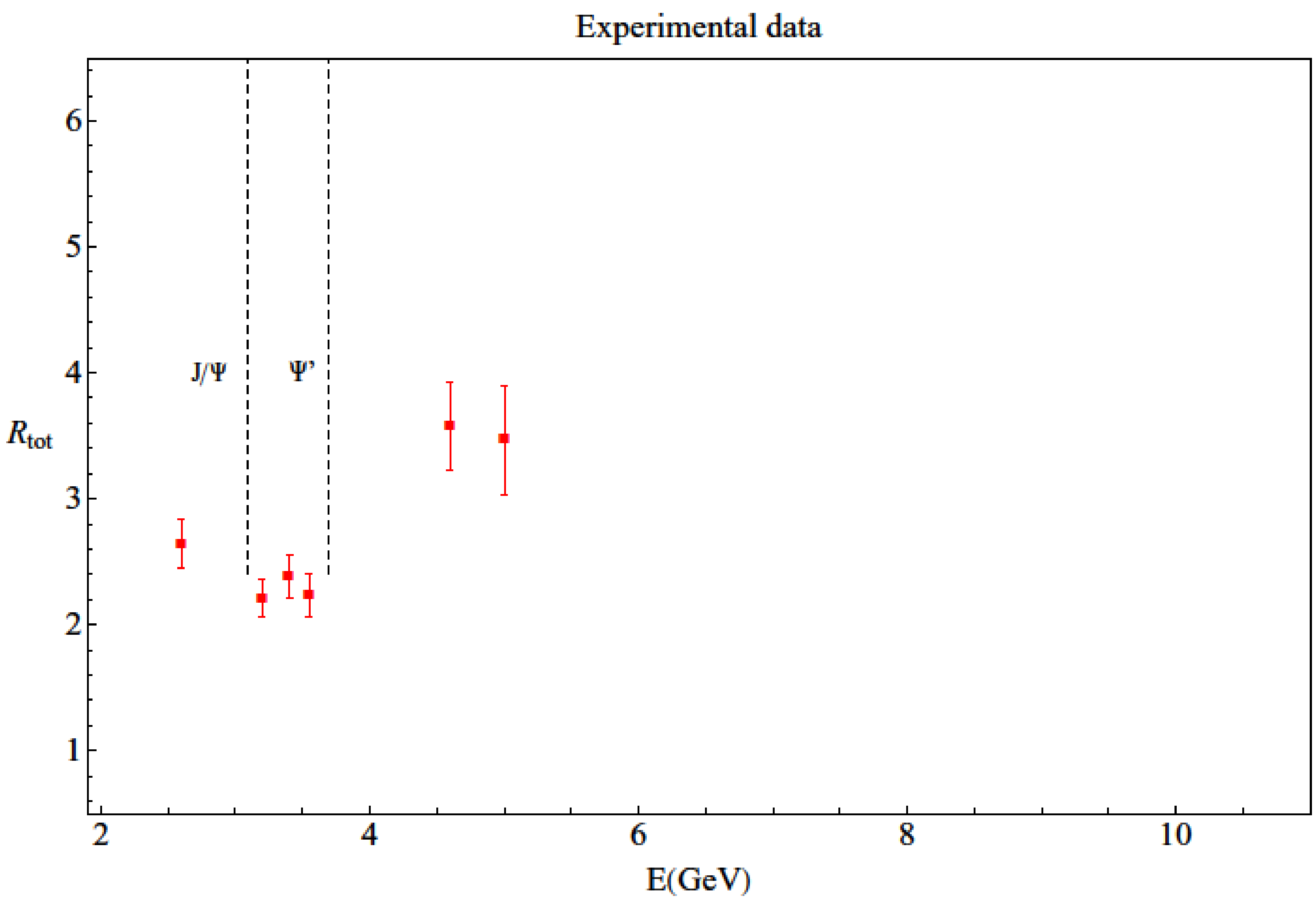


$$M_n^{\rm res} = \frac{9\,\pi\,\Gamma_{ee}}{\alpha(M)^2 M^{2n+1}}$$

Narrow-width approximation

Experimental data: charm

Sub-threshold and threshold   BES 1999



Experimental data: charm

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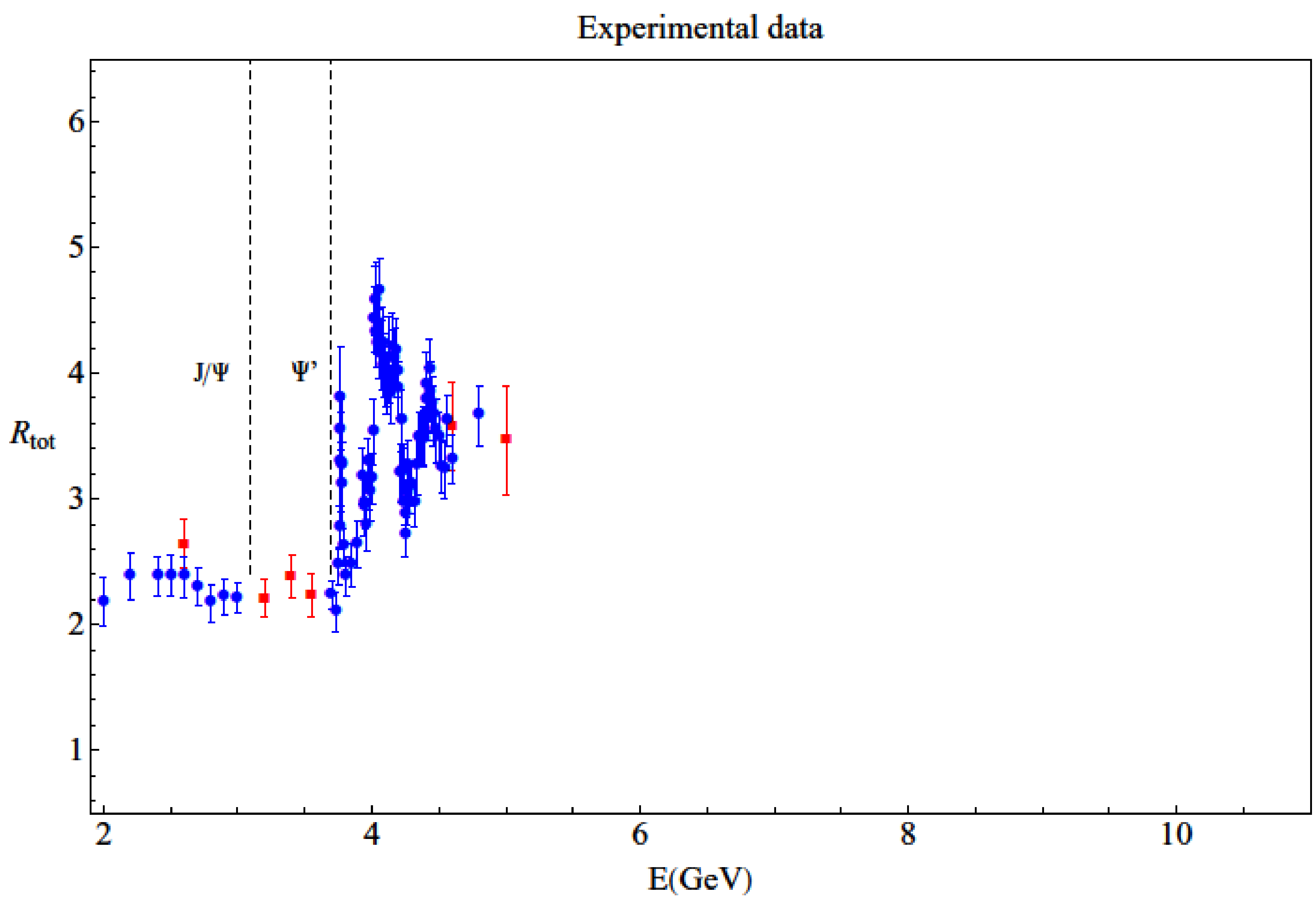

Experimental data: charm

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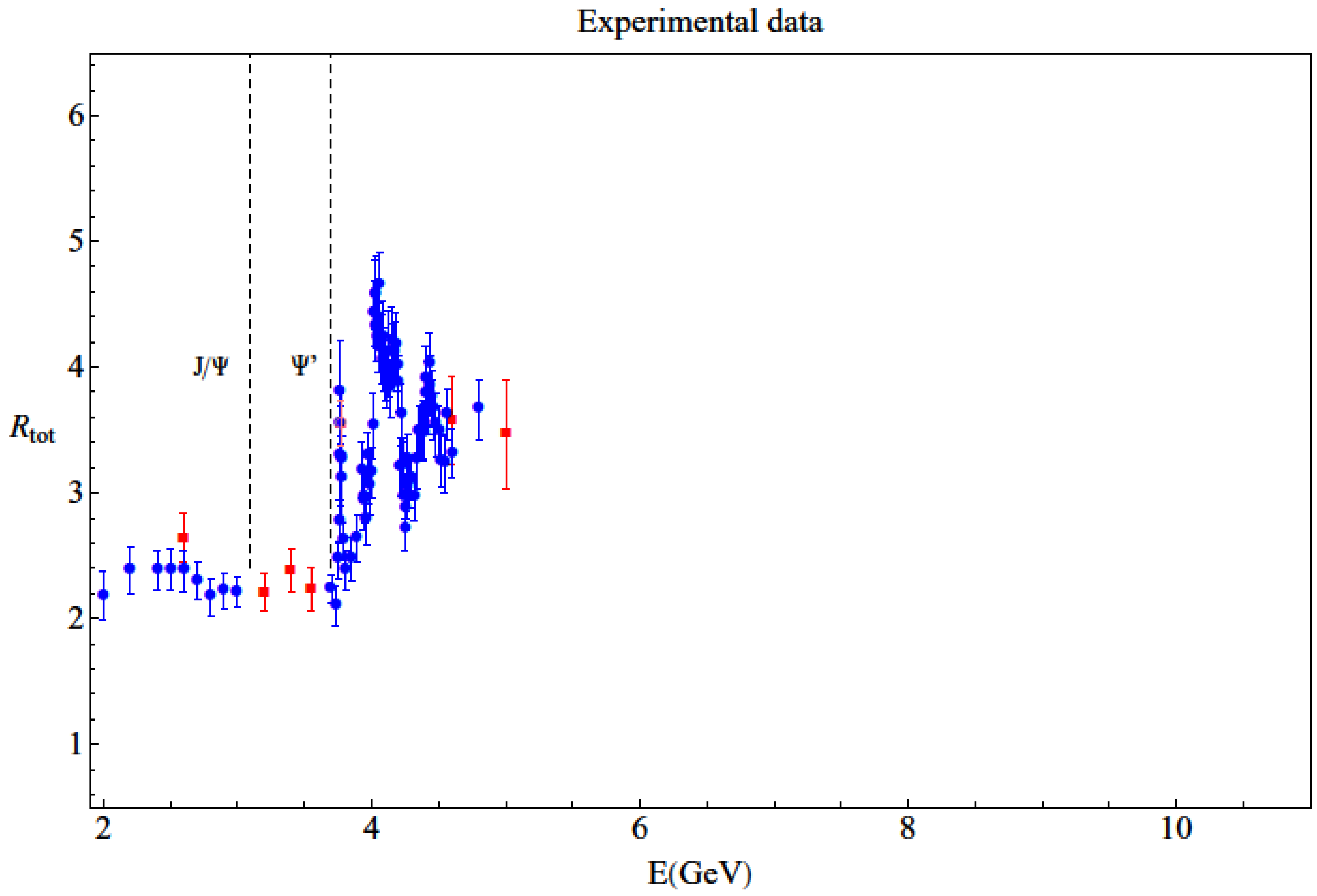
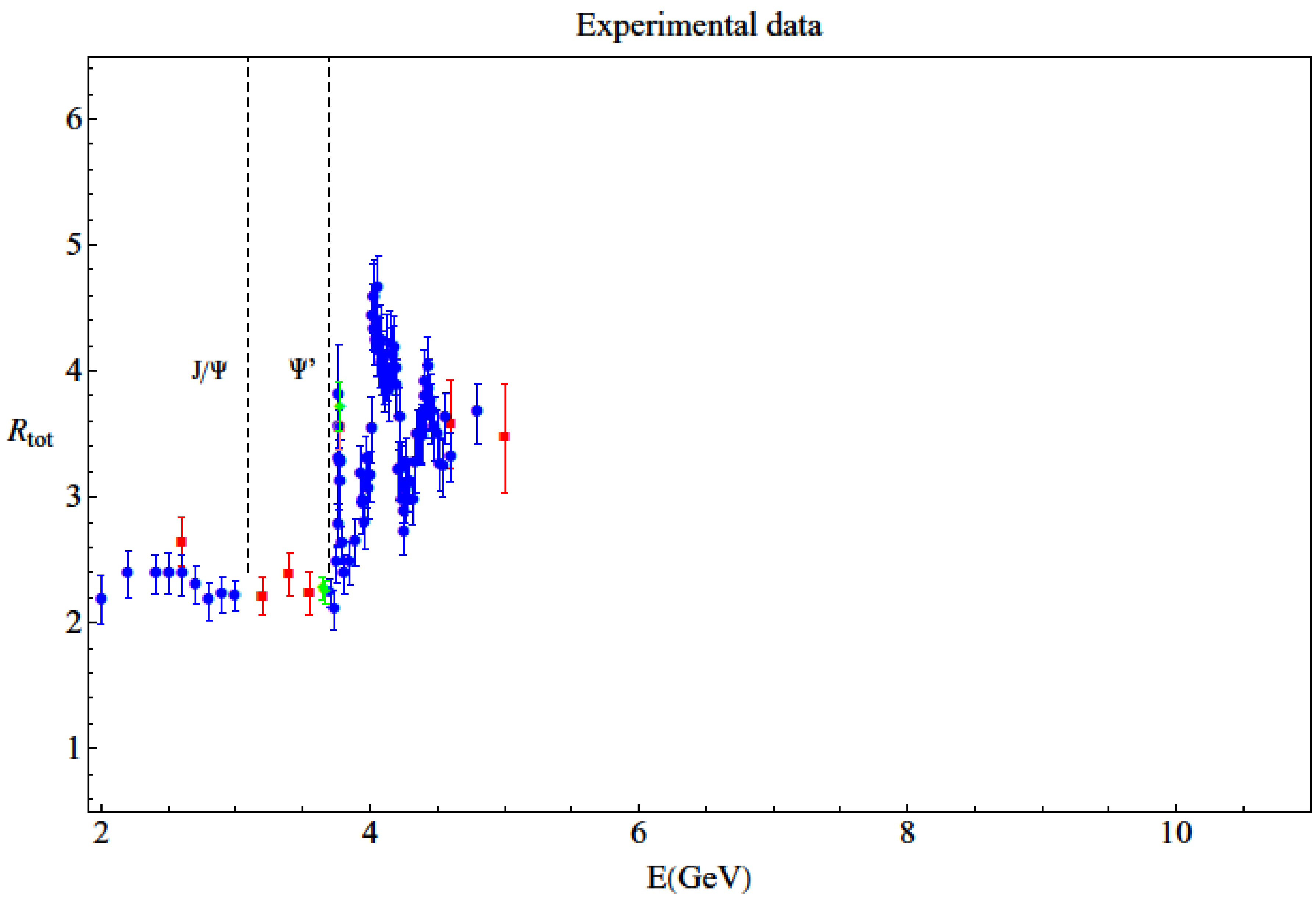


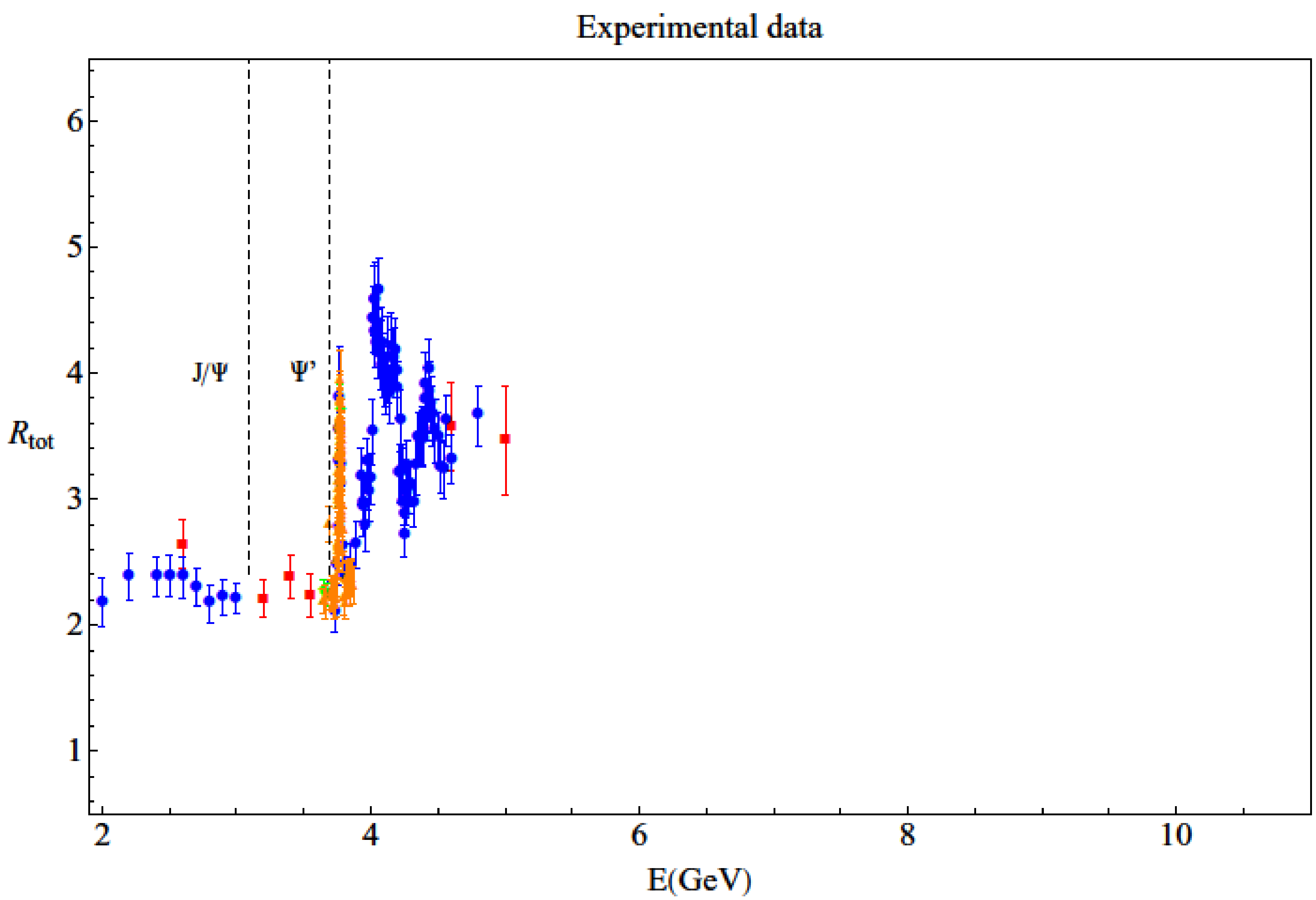
Experimental data: charm

Sub-threshold and threshold BES 2006 (I)



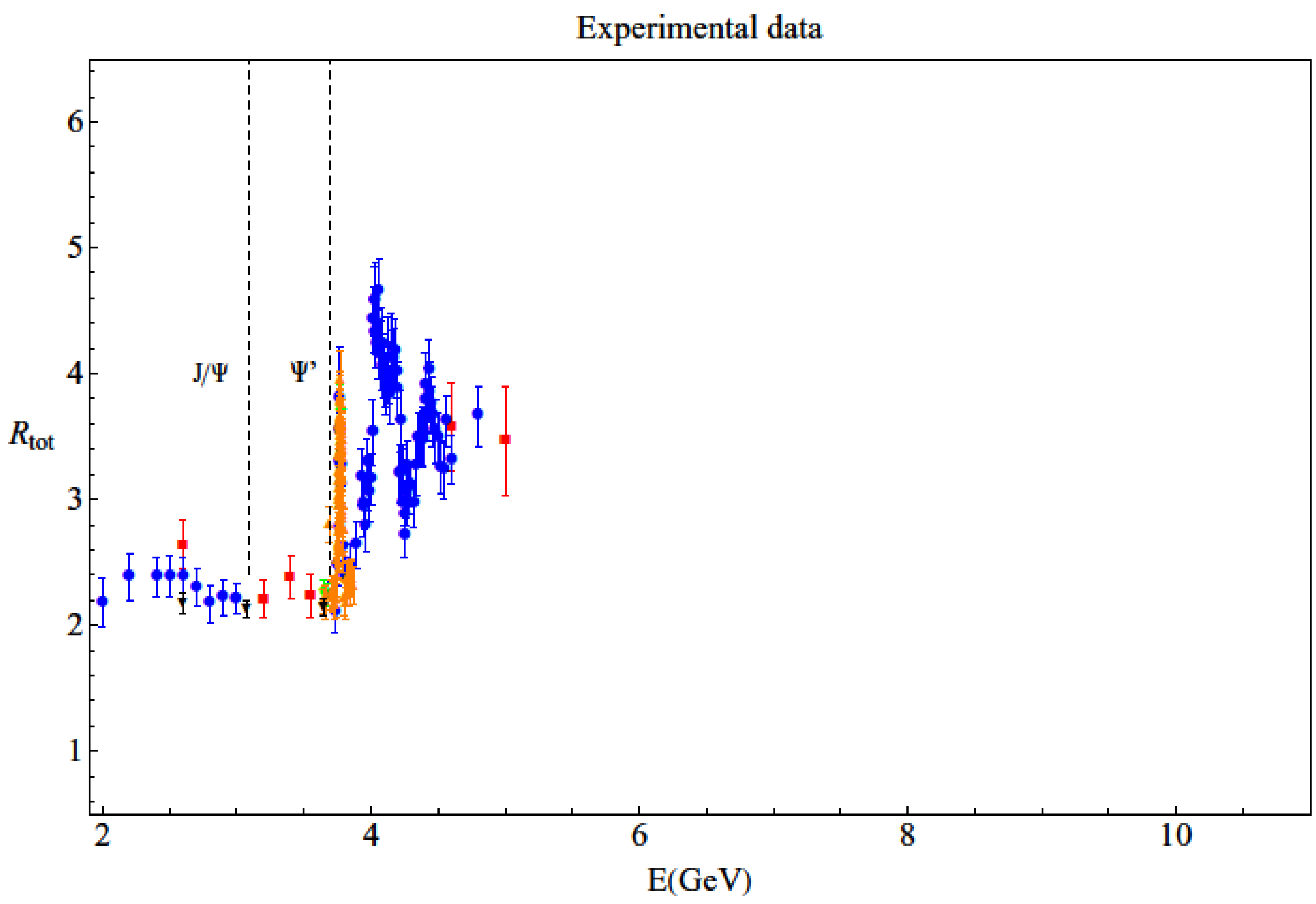
Experimental data: charm

Sub-threshold and threshold   BES 2006 (II)



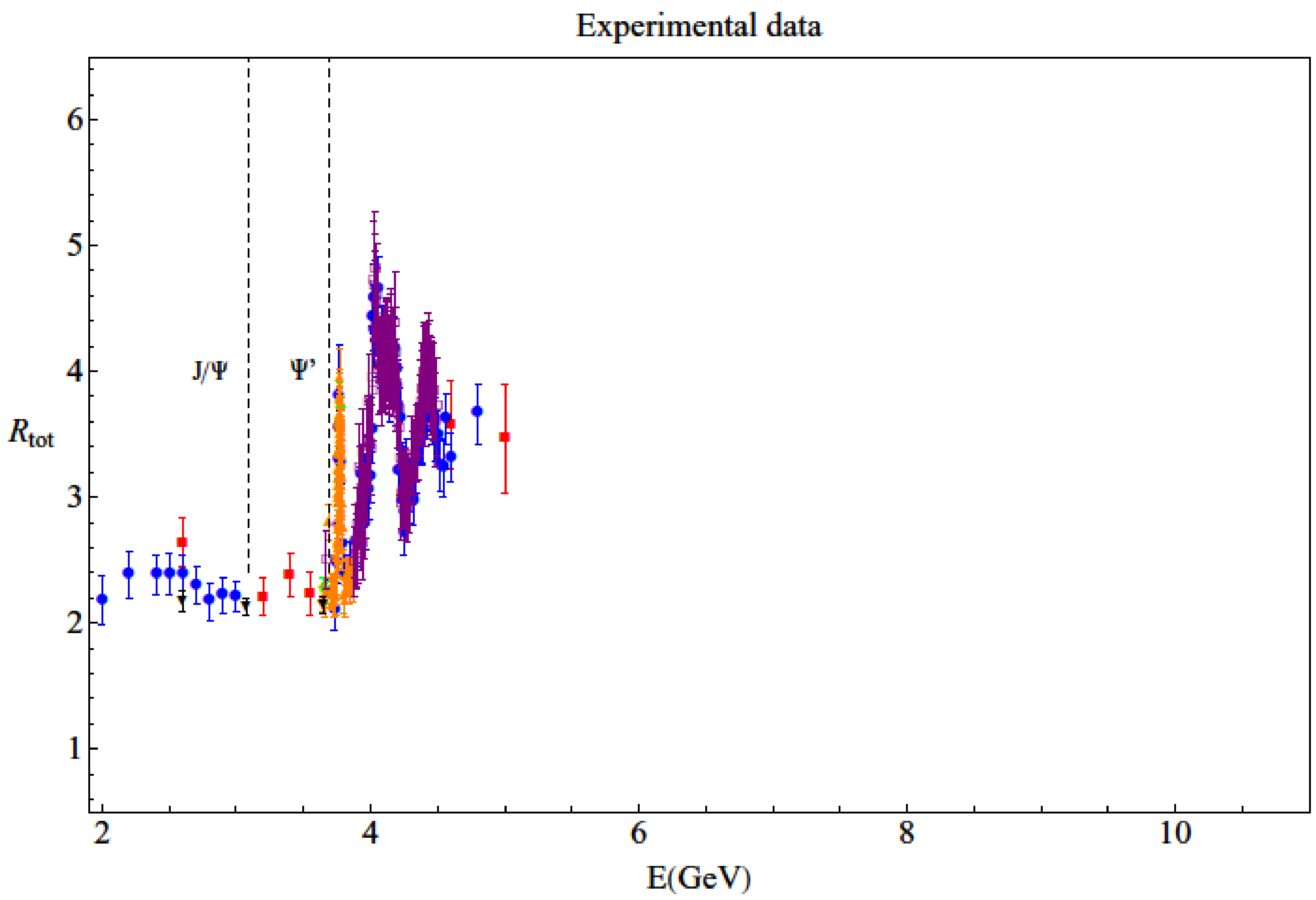
Experimental data: charm

Sub-threshold and threshold BES 2009



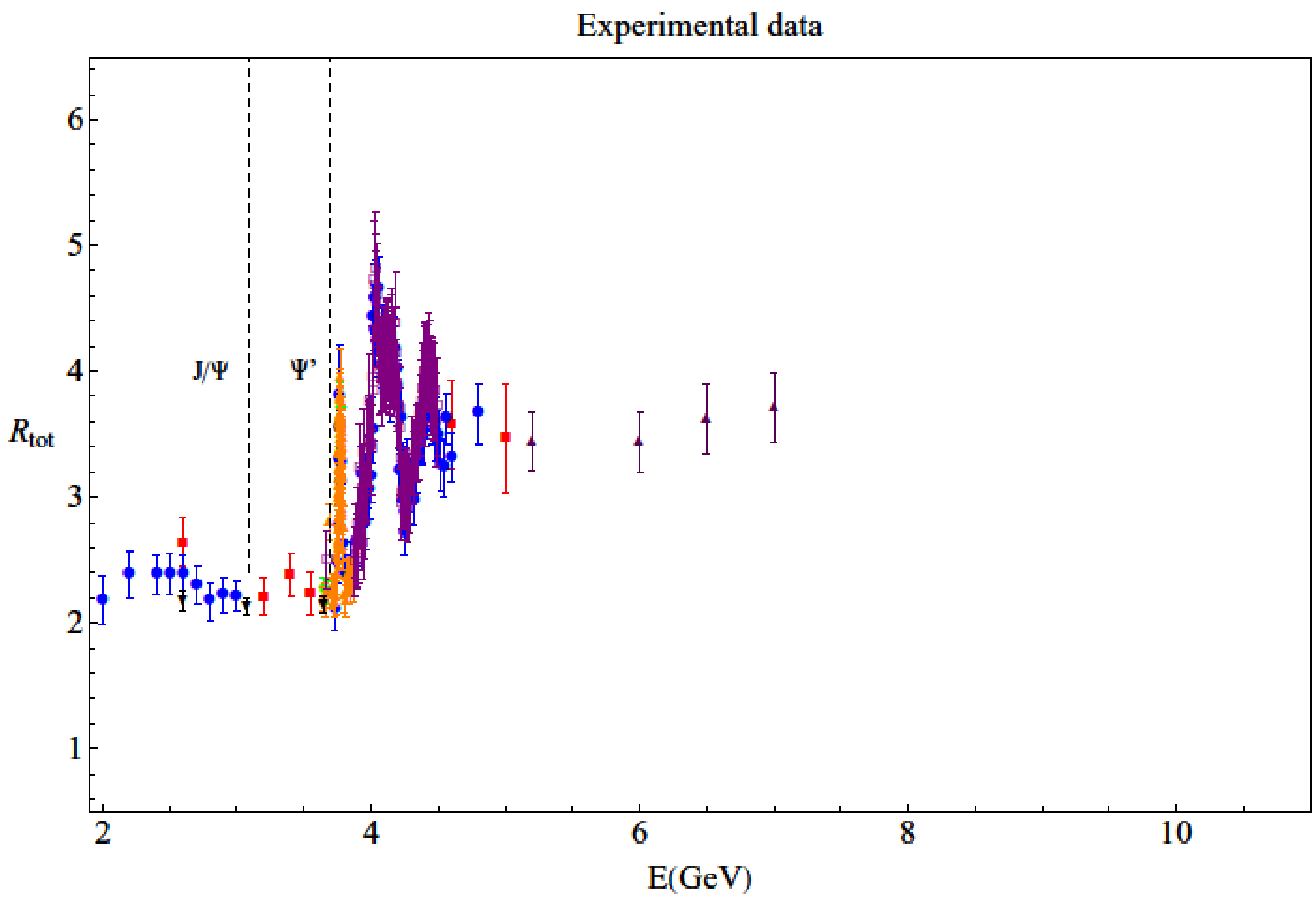
Experimental data: charm

Sub-threshold and threshold **Crystal Ball 1986**



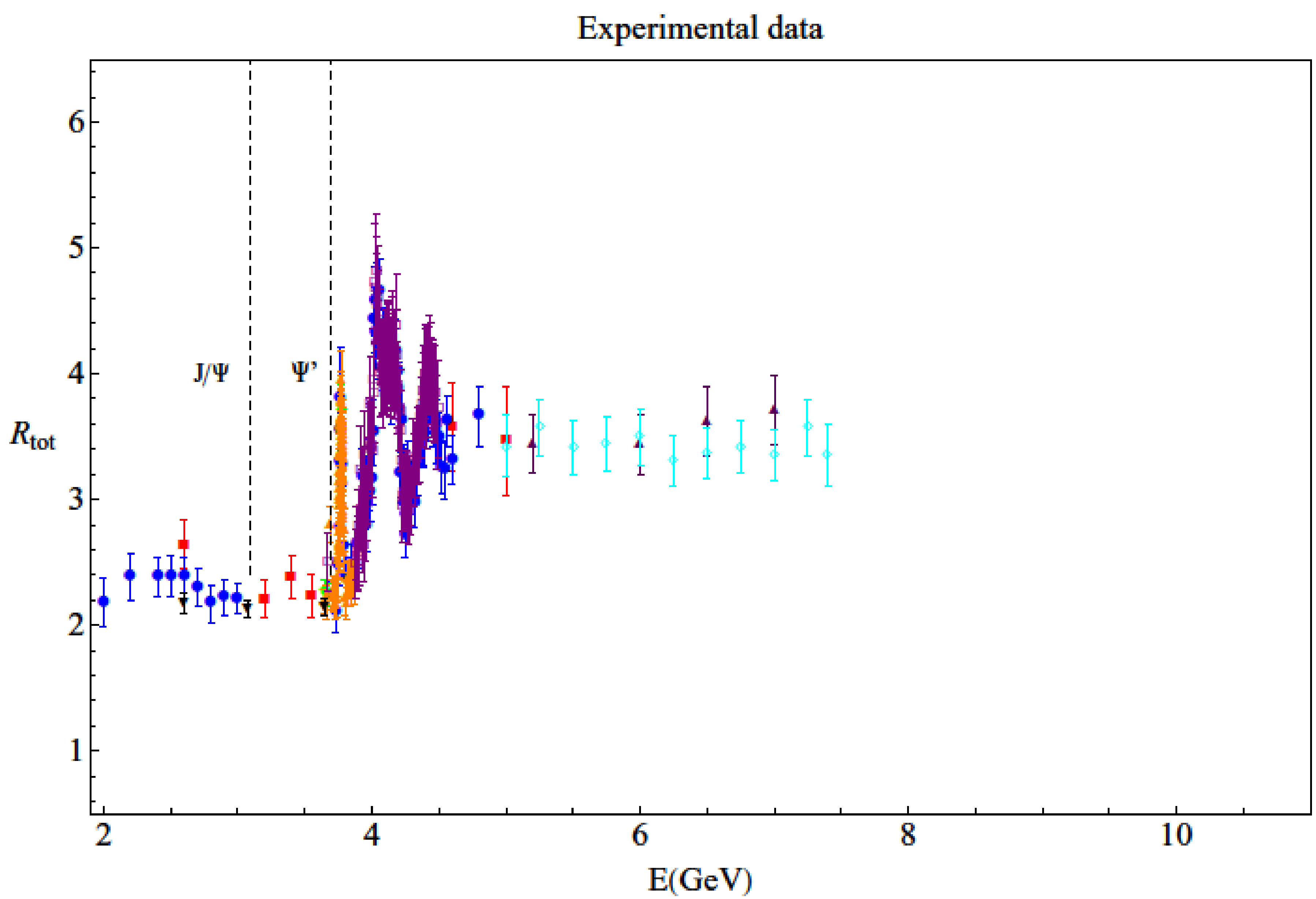
Experimental data: charm

Gap region **Crystal Ball 1990 (I)**



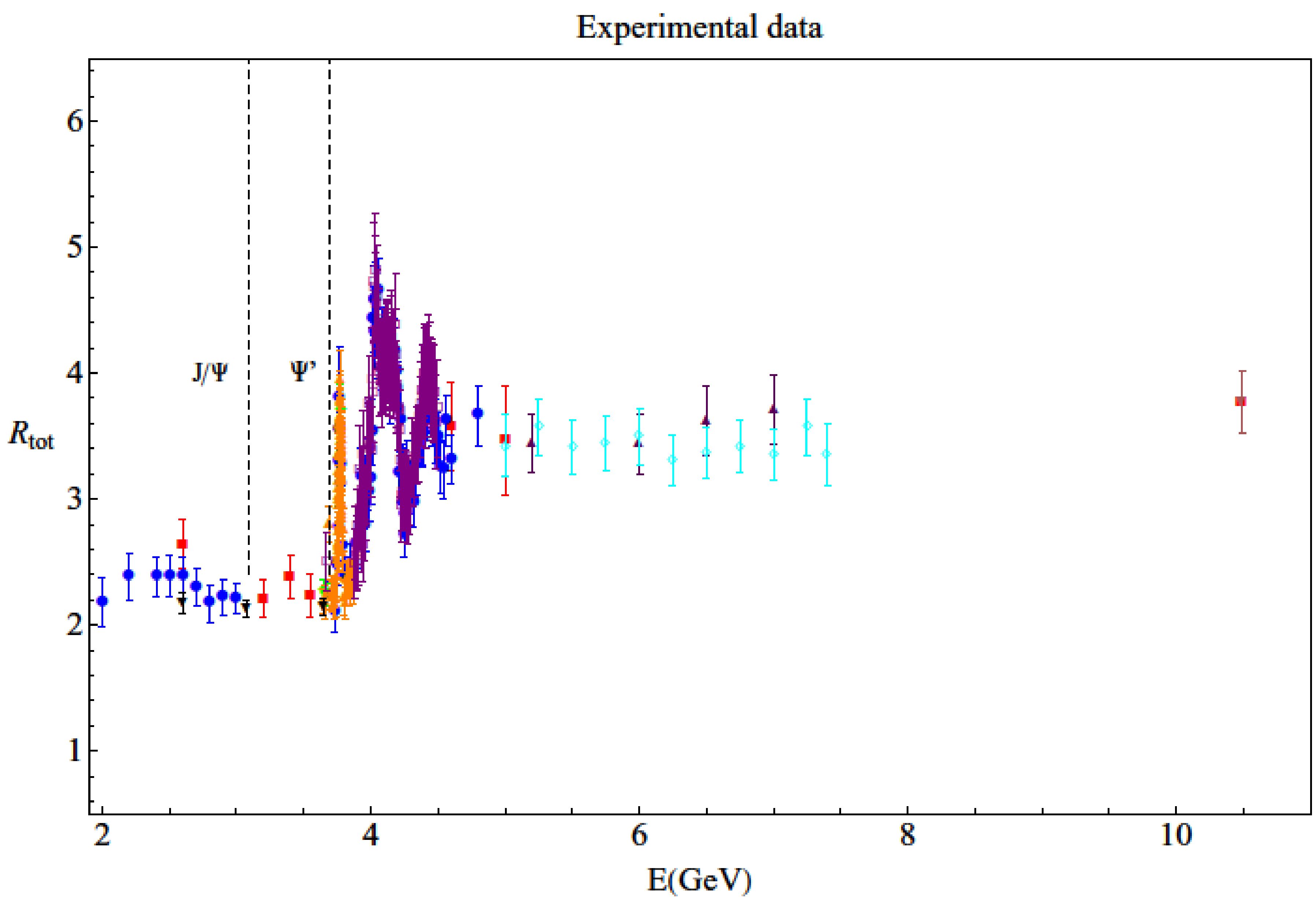
Experimental data: charm

Gap region **Crystal Ball 1990 (II)**



Experimental data: charm

High energy region **CLEO 1979**



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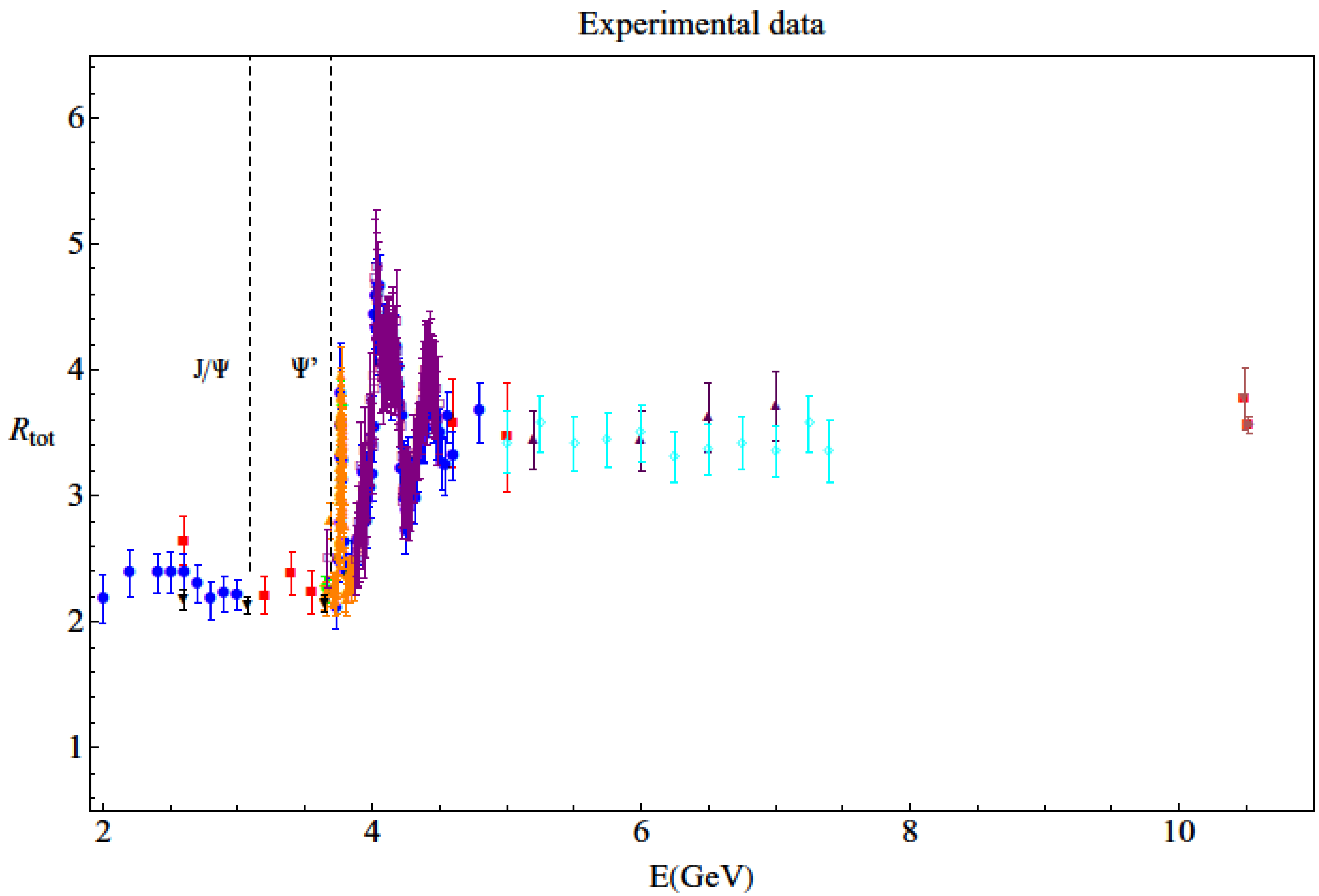
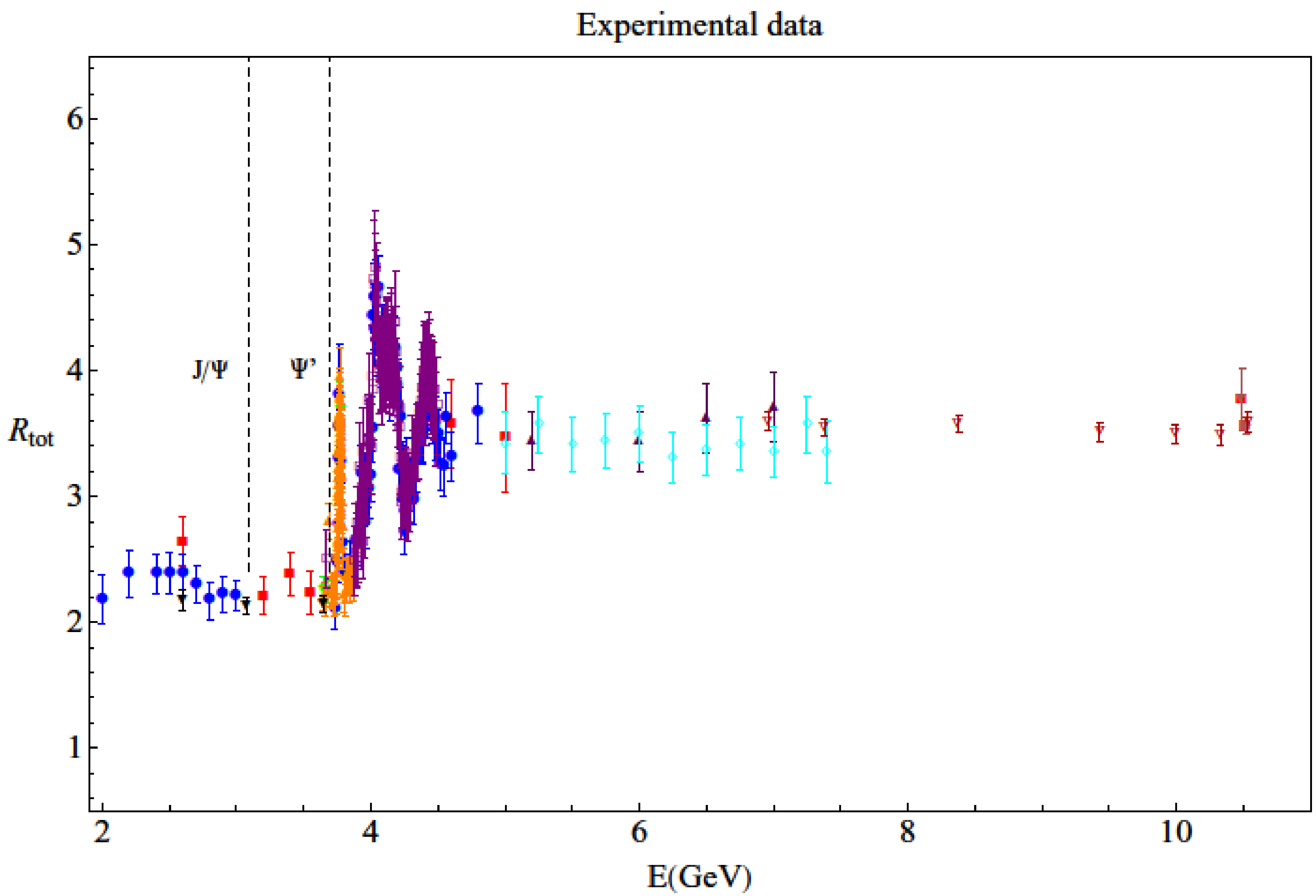


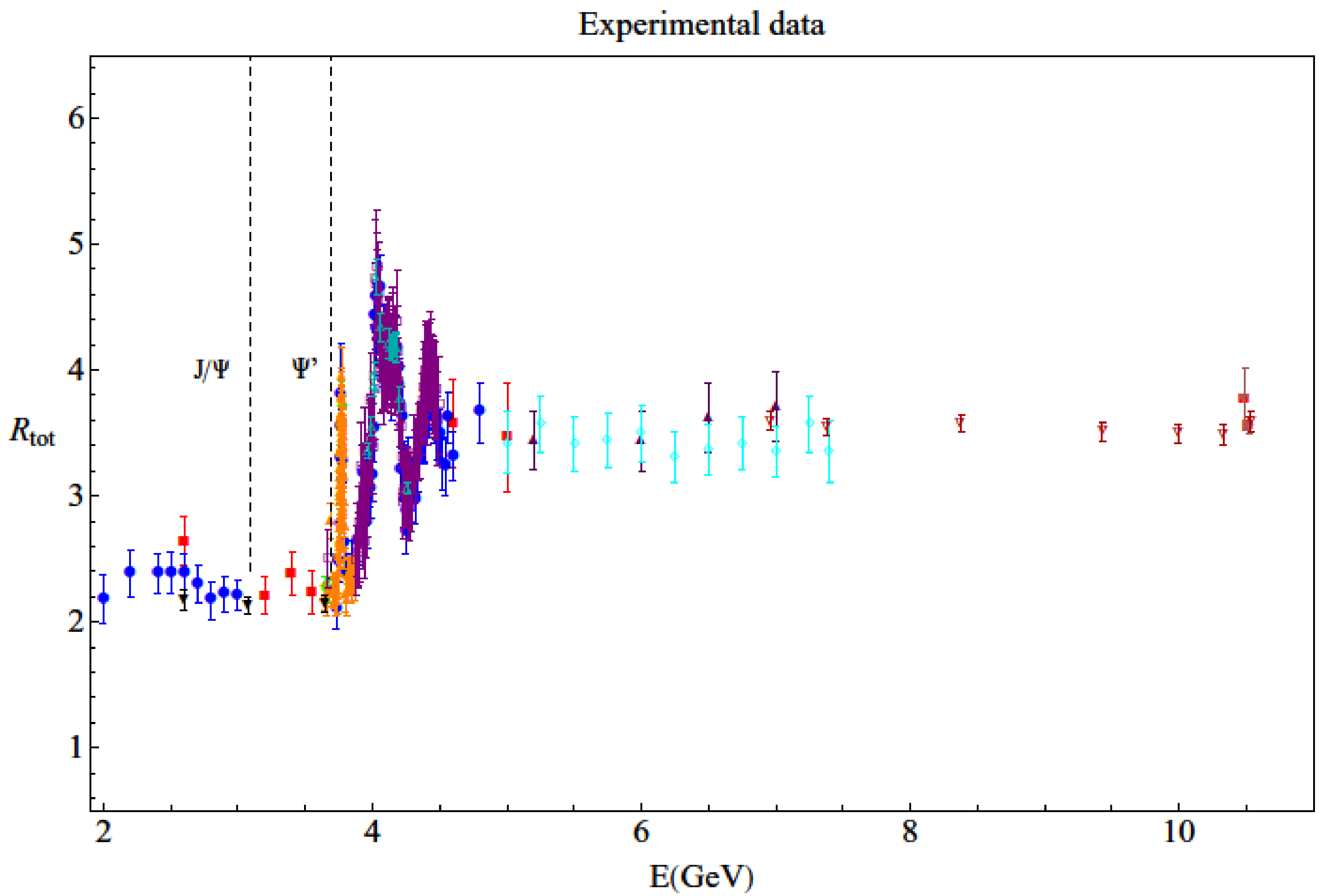
Experimental data: charm

High energy region **CLEO 2007**



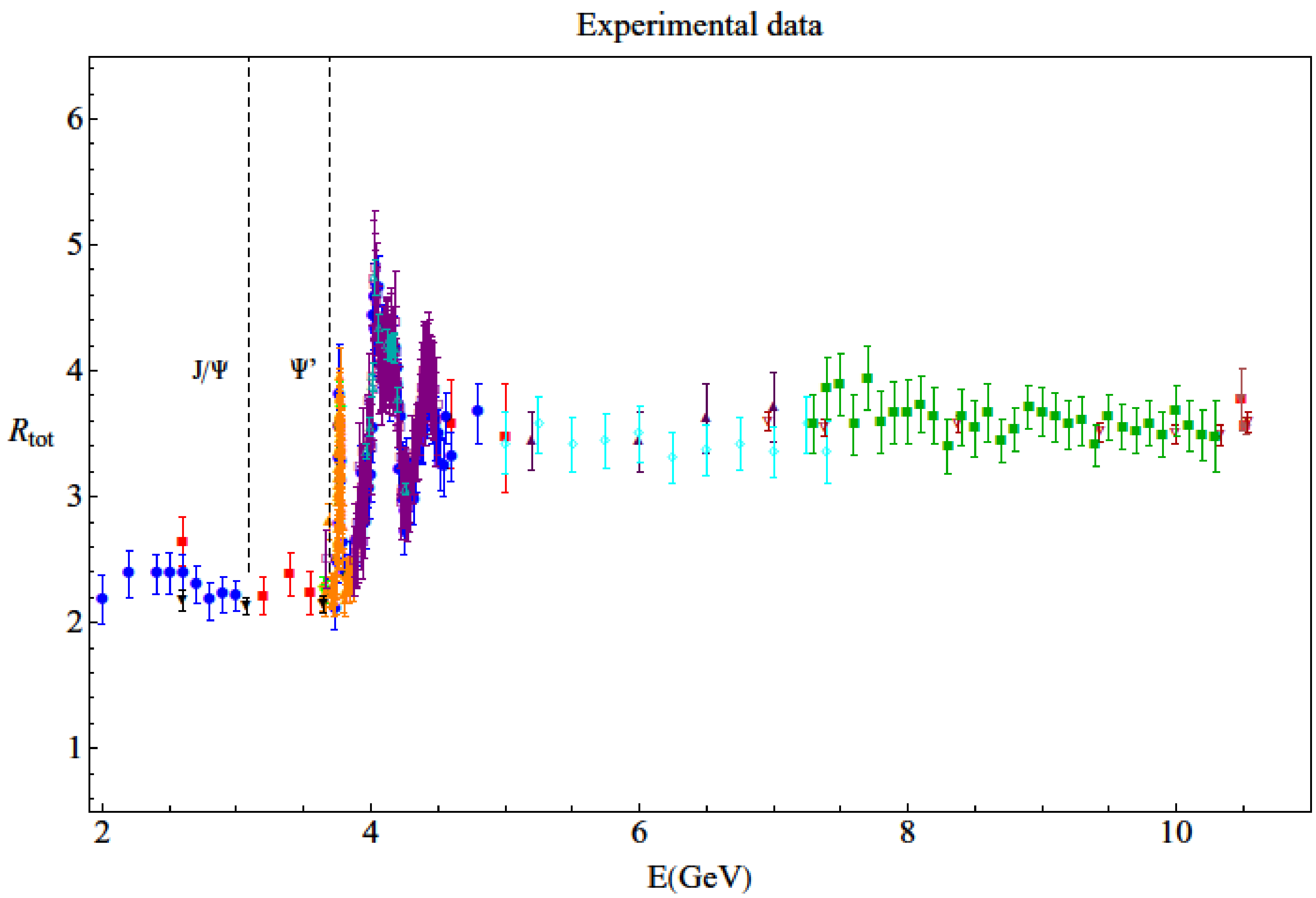
Experimental data: charm

Sub-threshold and threshold **CLEO 2009**



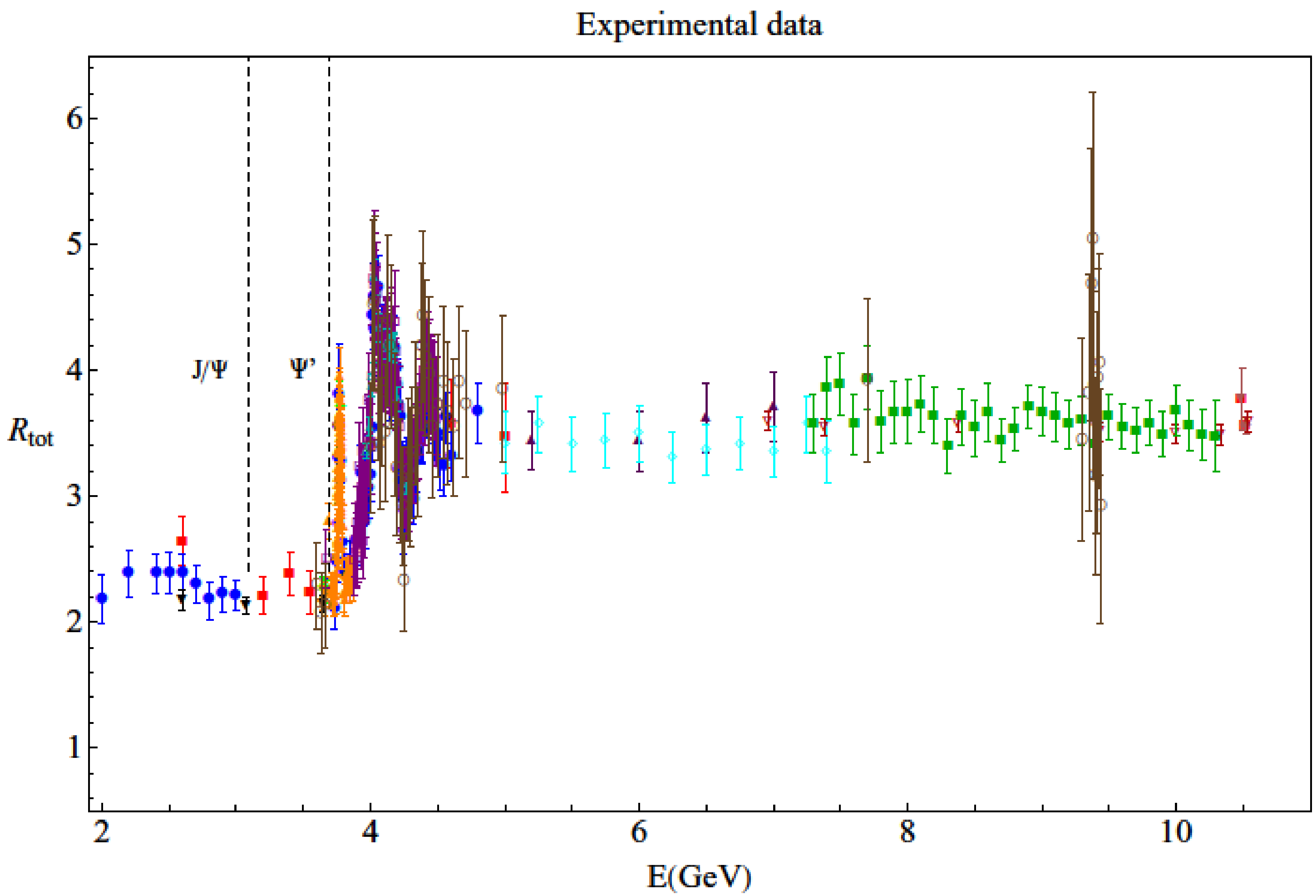
Experimental data: charm

High energy region **MD-1 1996**



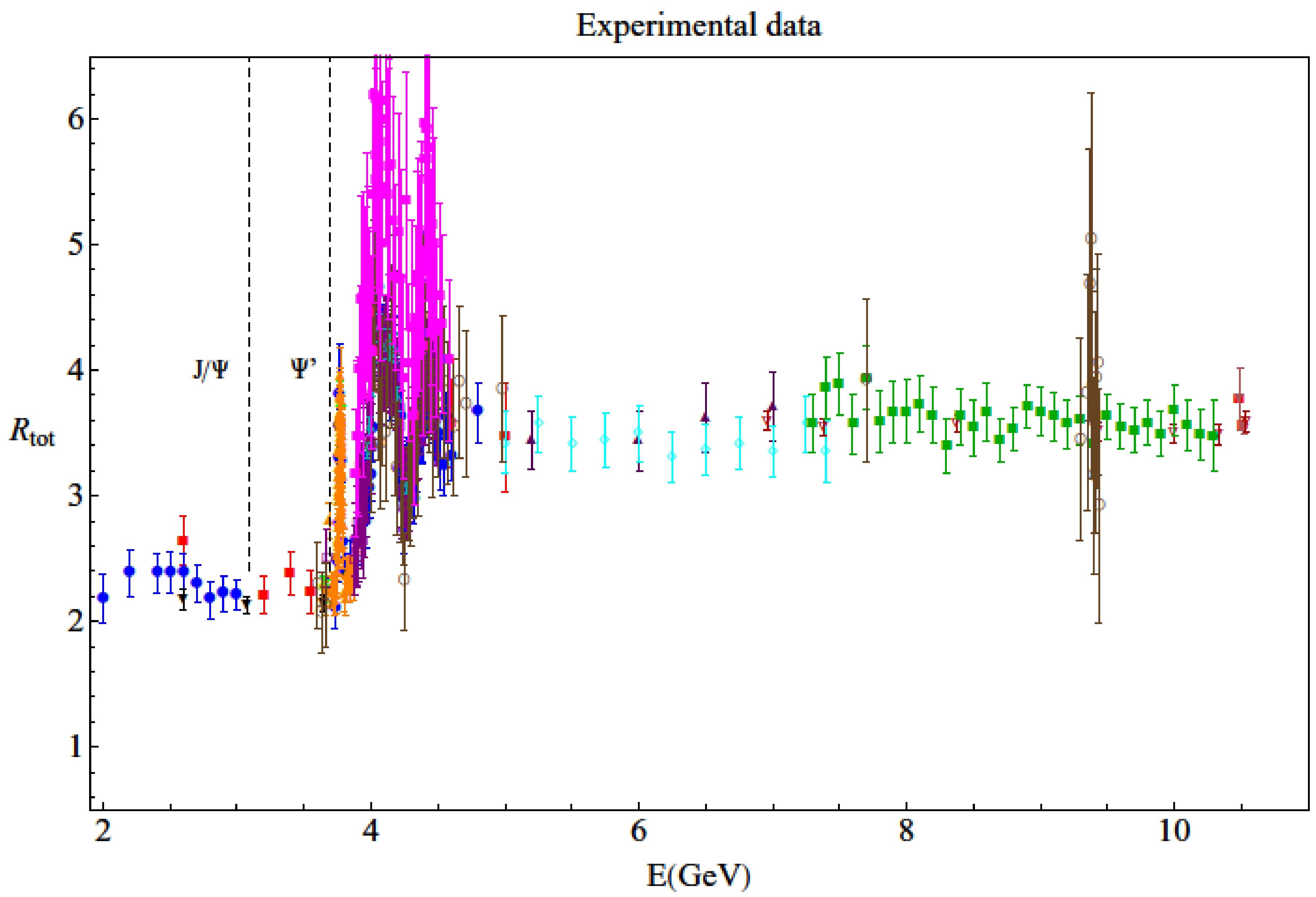
Experimental data: charm

Threshold and high energy **PLUTO 1982**



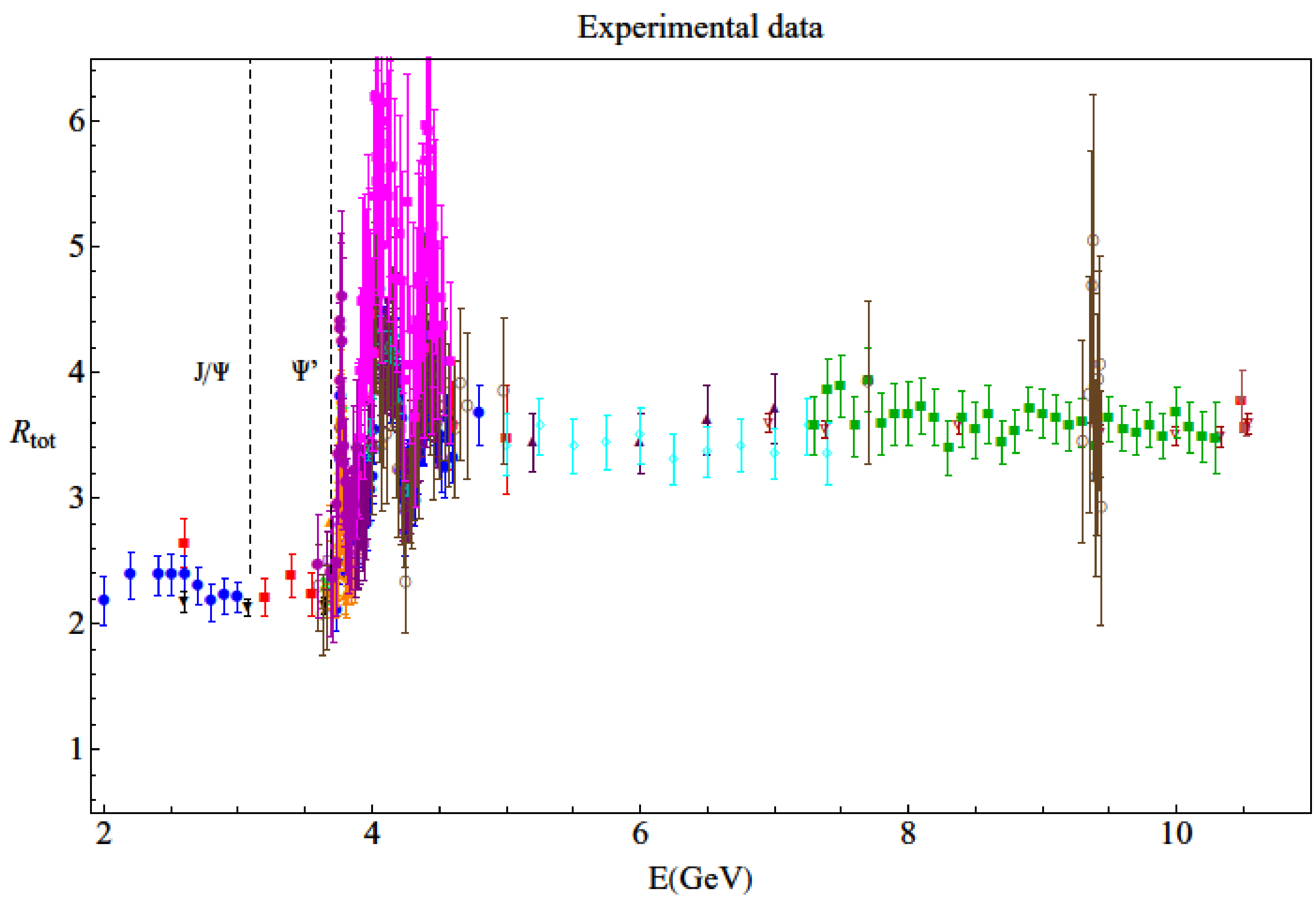
Experimental data: charm

Threshold region **MARKI 1976**



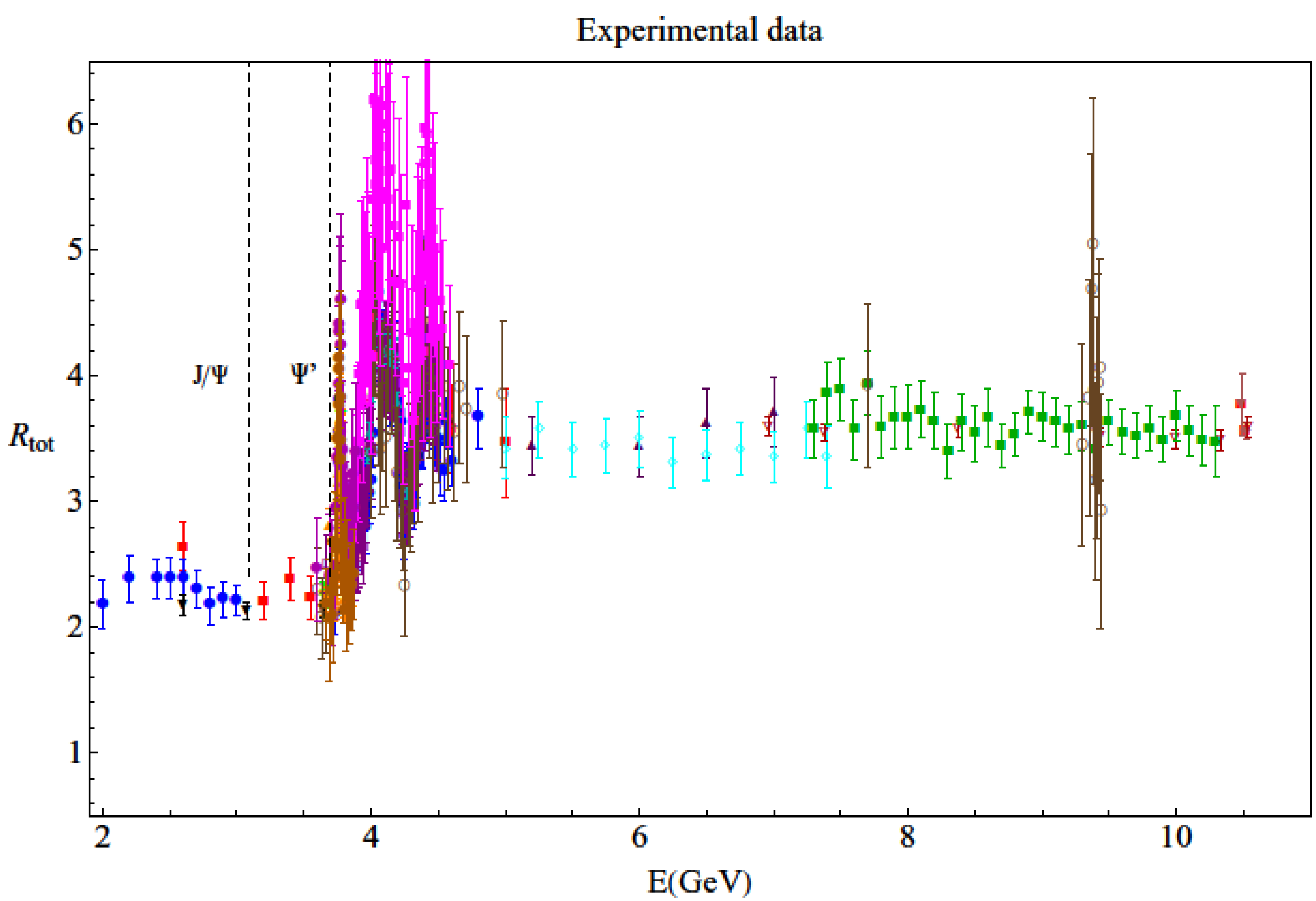
Experimental data: charm

Gap region **MARKI 1977**



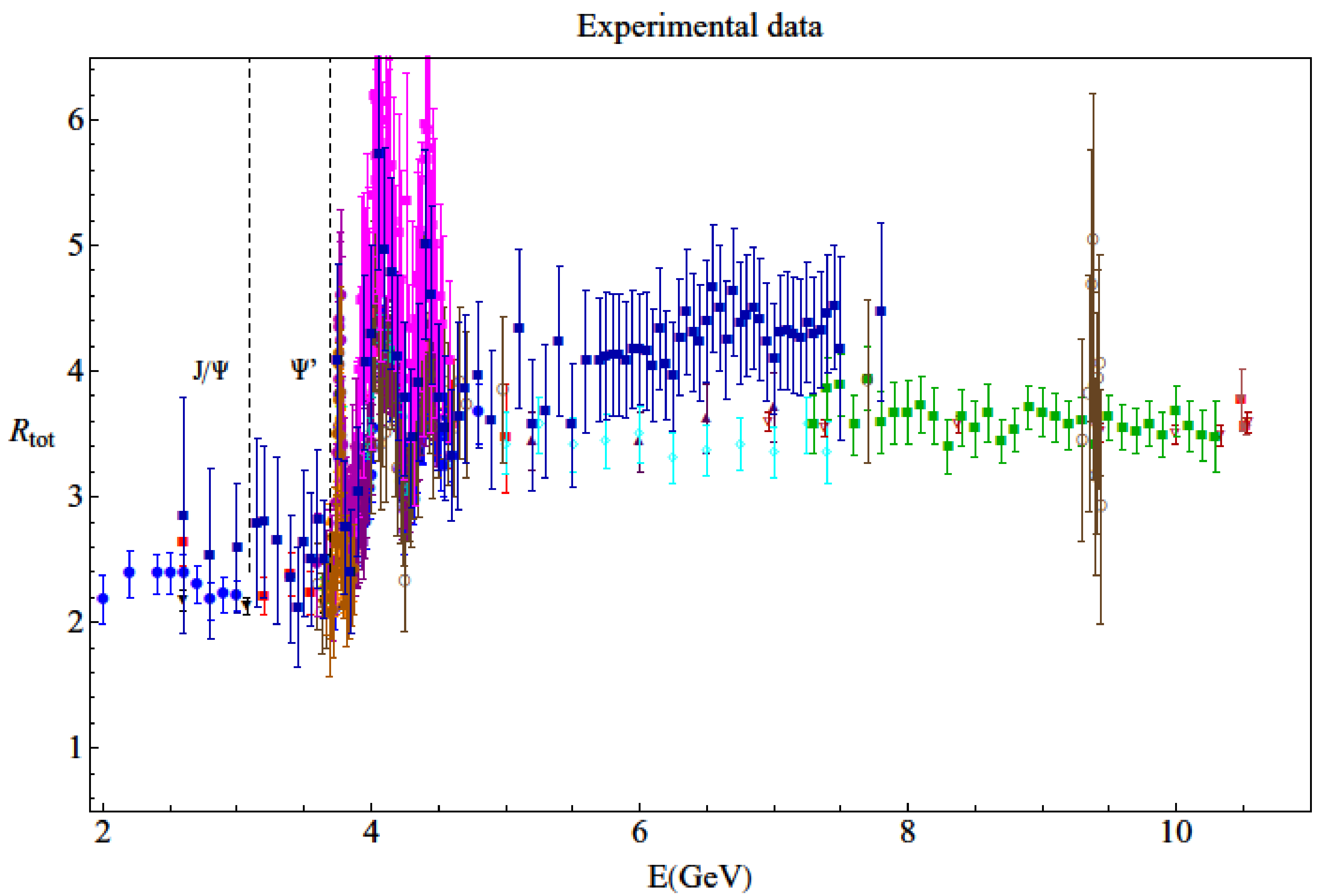
Experimental data: charm

Gap region **MARKII 1979**



Experimental data: charm

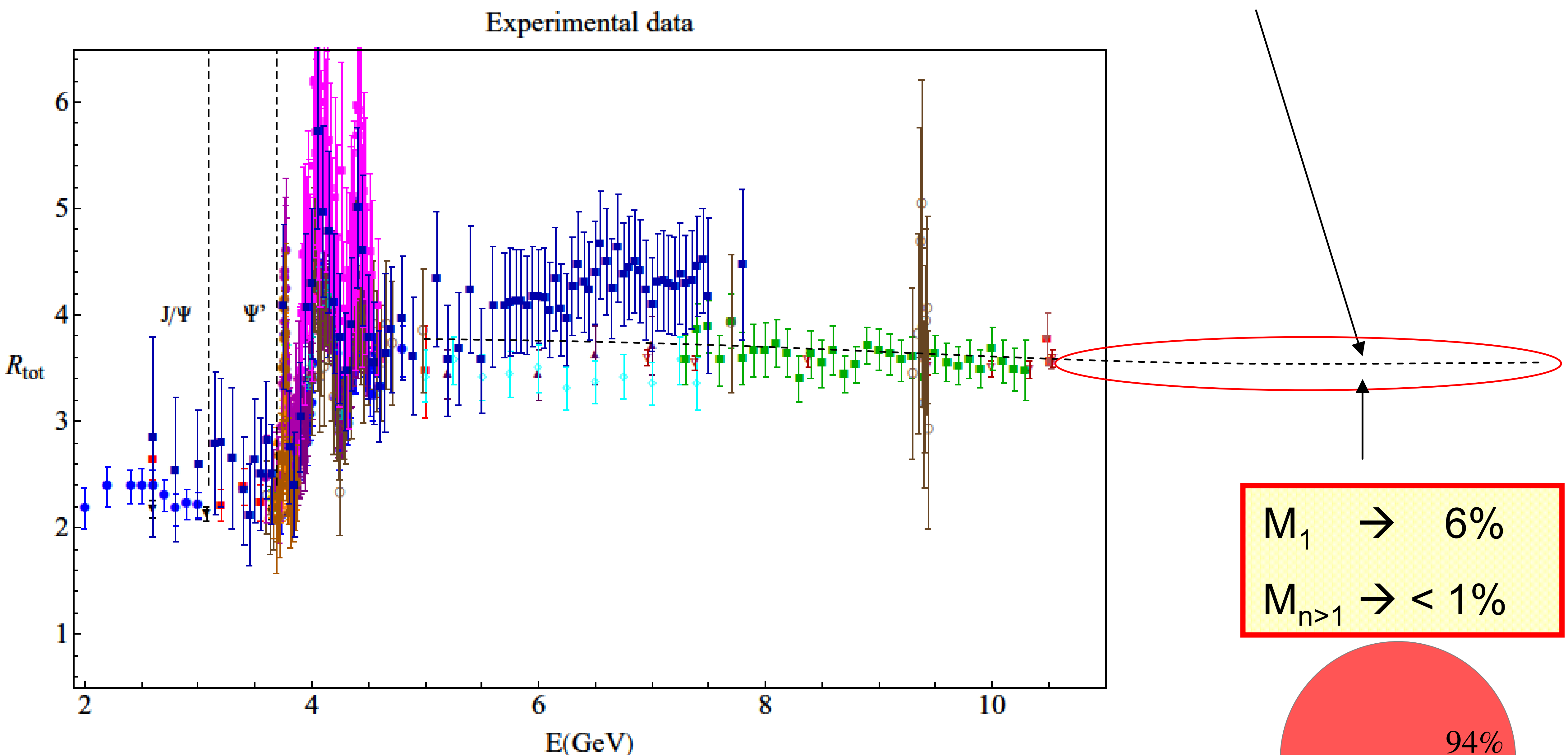
Threshold and gap regions **Mark-I 1981**



Experimental data: charm

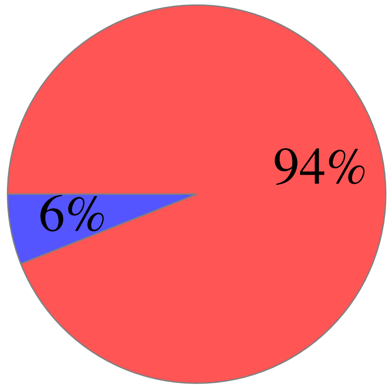
Perturbation theory

- Only where there is no data
- Assign a conservative 10% error to reduce model dependence



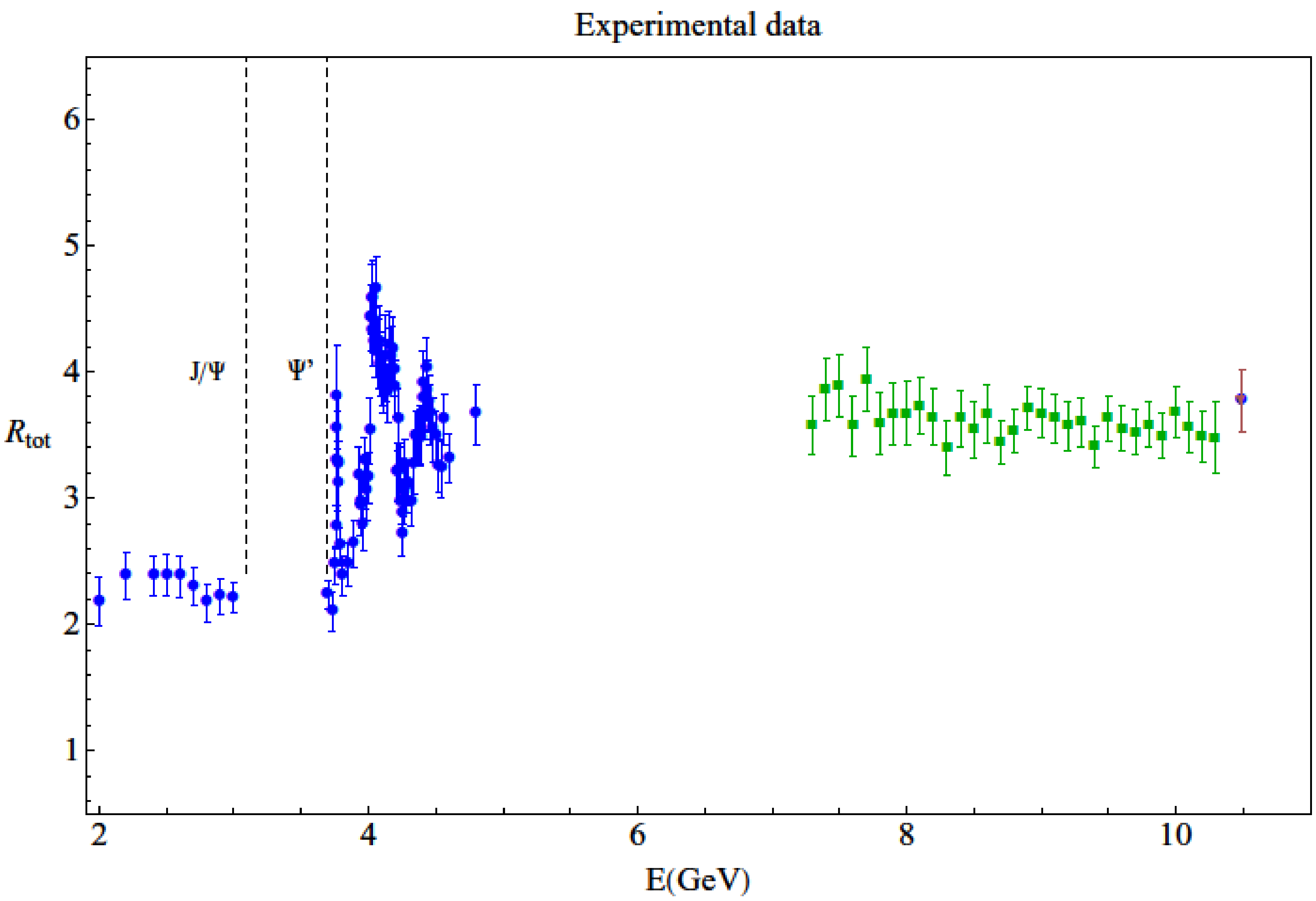
$M_1 \rightarrow 6\%$

$M_{n>1} \rightarrow < 1\%$



Experimental data: charm

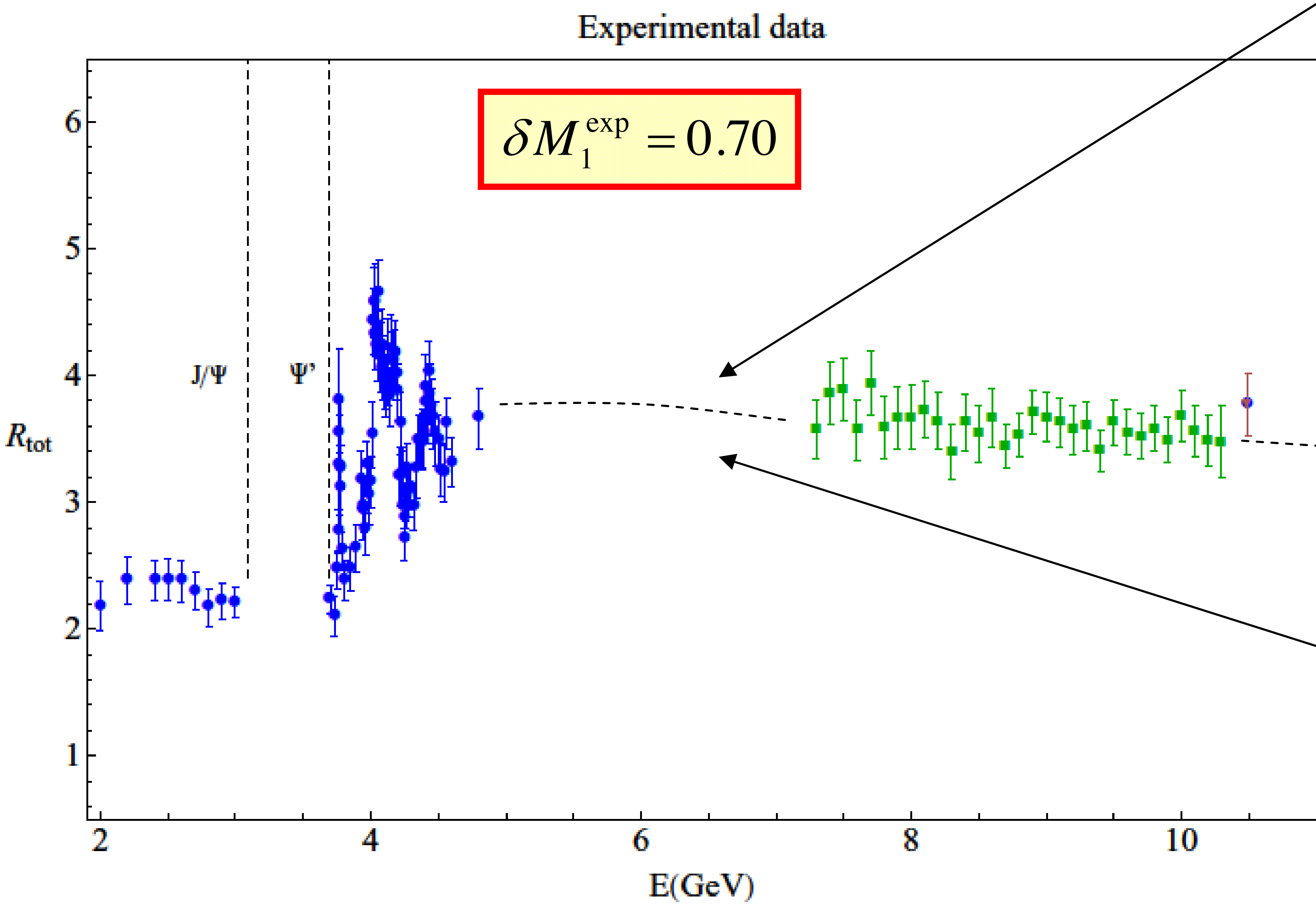
Data used in Hoang and Jamin (2004)



Experimental data: charm

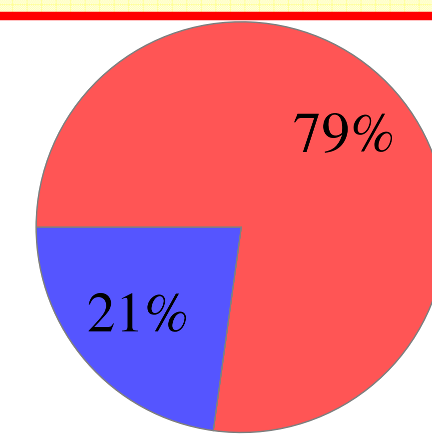
Data used in Hoang and Jamin (2004)

- Perturbation theory only in gap and region with no data
- 10% error assigned as well



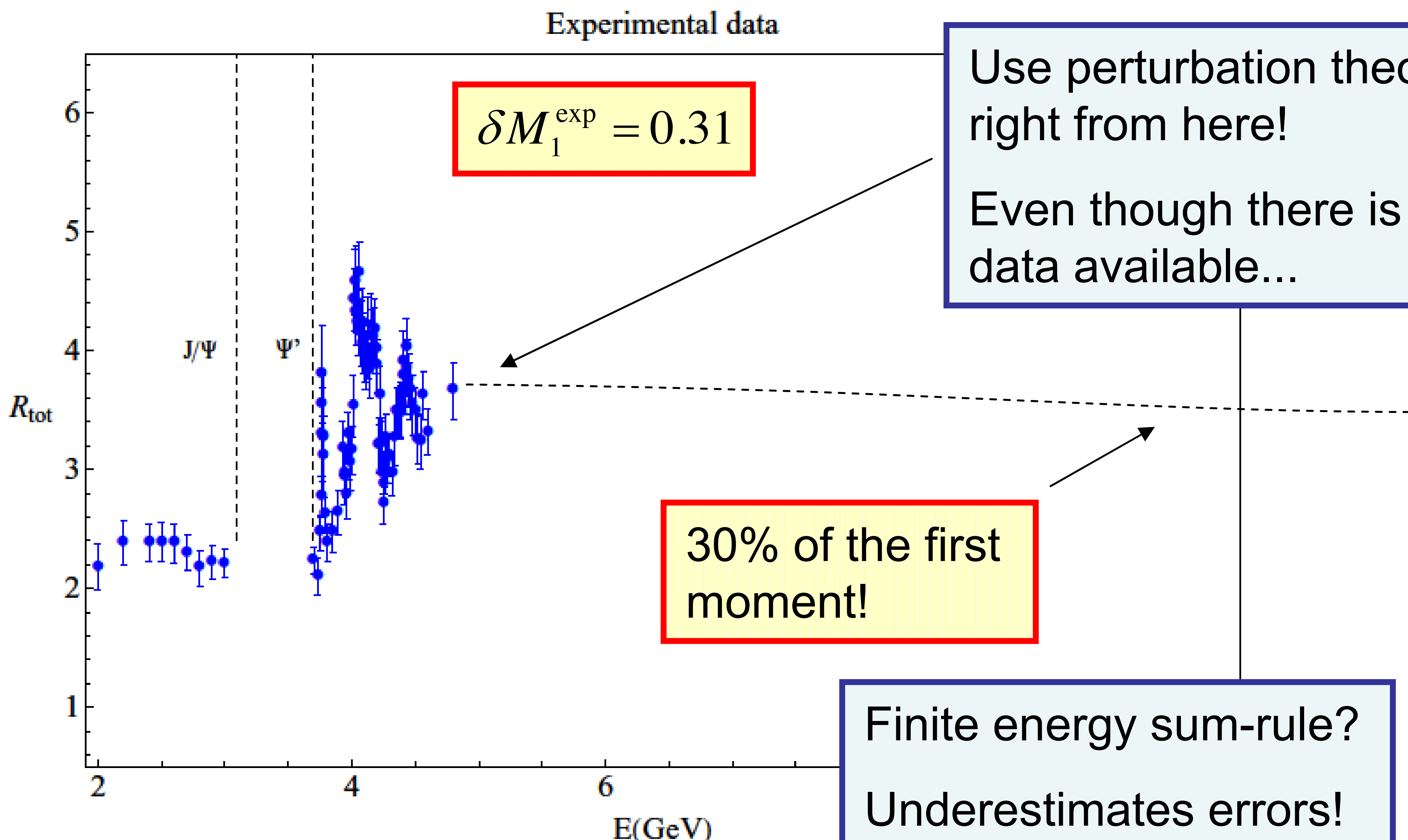
$$\delta M_1^{\exp} = 0.70$$

21% of the first moment



Experimental data: charm

Data used in Kühn et al (2001) and Boughezal et al



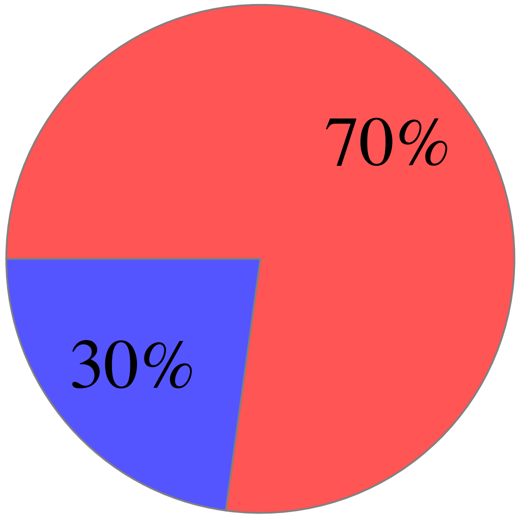
Use perturbation theory right from here!

Even though there is data available...

30% of the first moment!

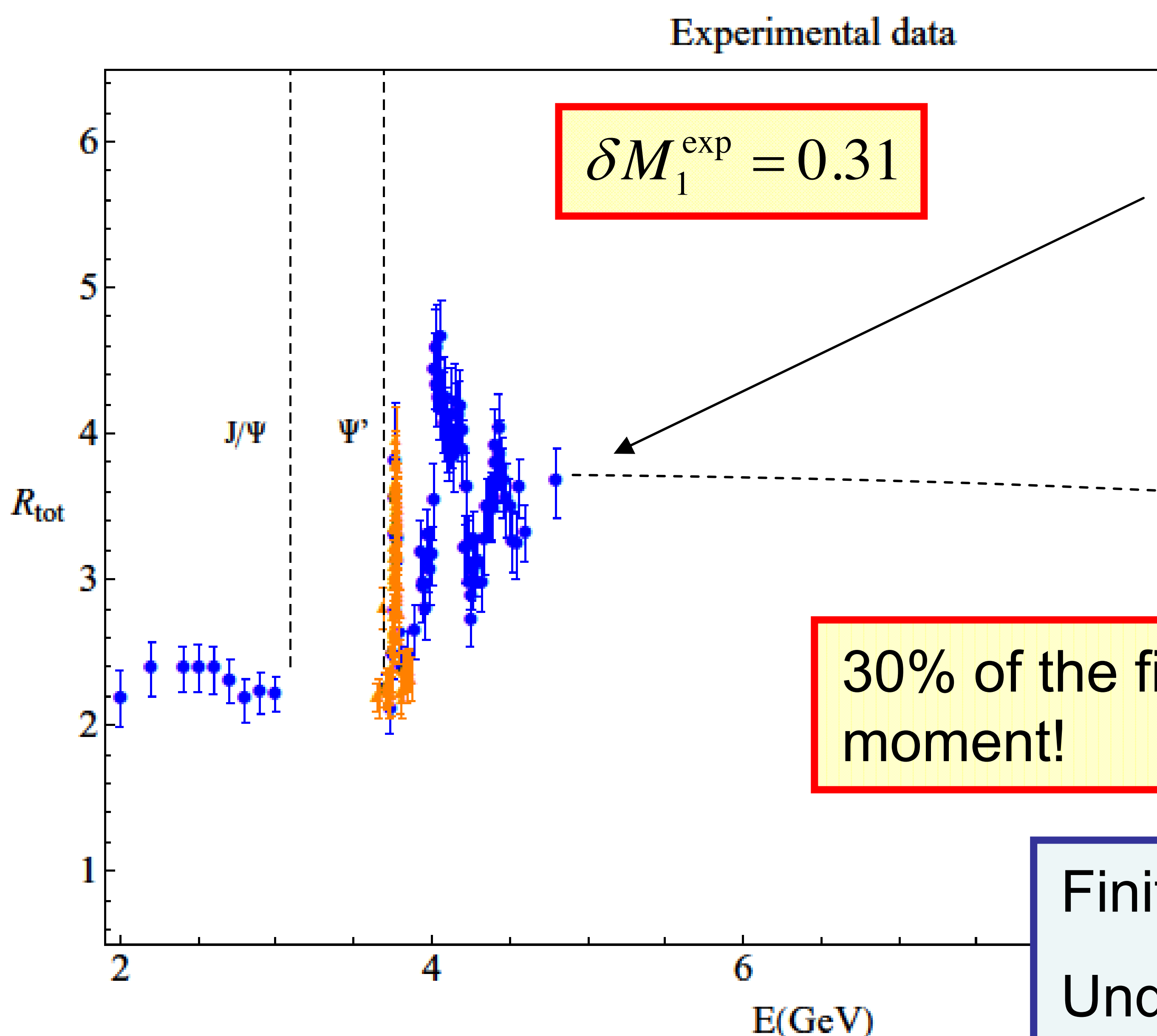
Finite energy sum-rule?

Underestimates errors! (they assing only naive theory error)



Experimental data: charm

Data used in Kuhn et al (2004, 05) and Bodenstein et al



$\delta M_1^{exp} = 0.31$

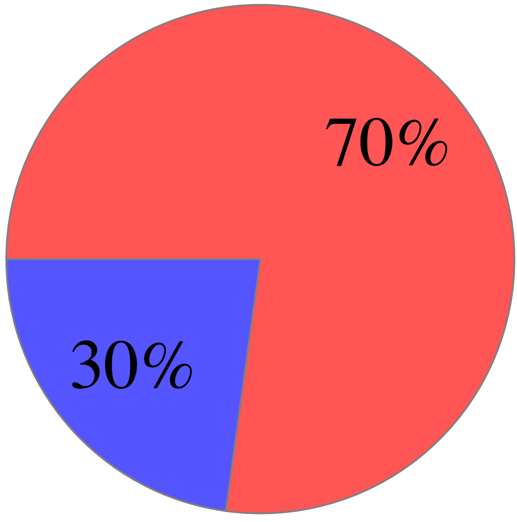
Use perturbation theory right from here!

Even though there is data available...

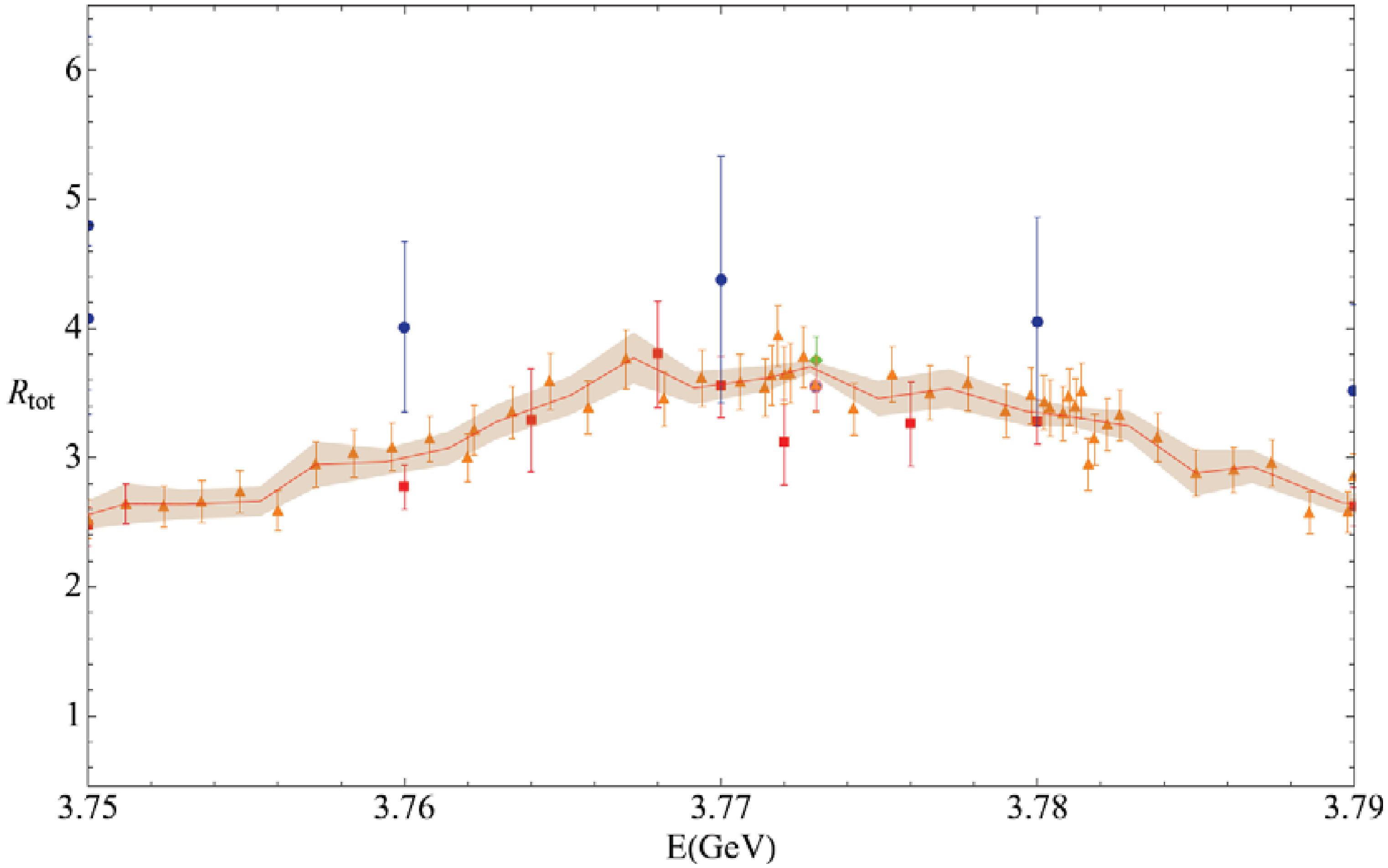
30% of the first moment!

Finite energy sum-rule?

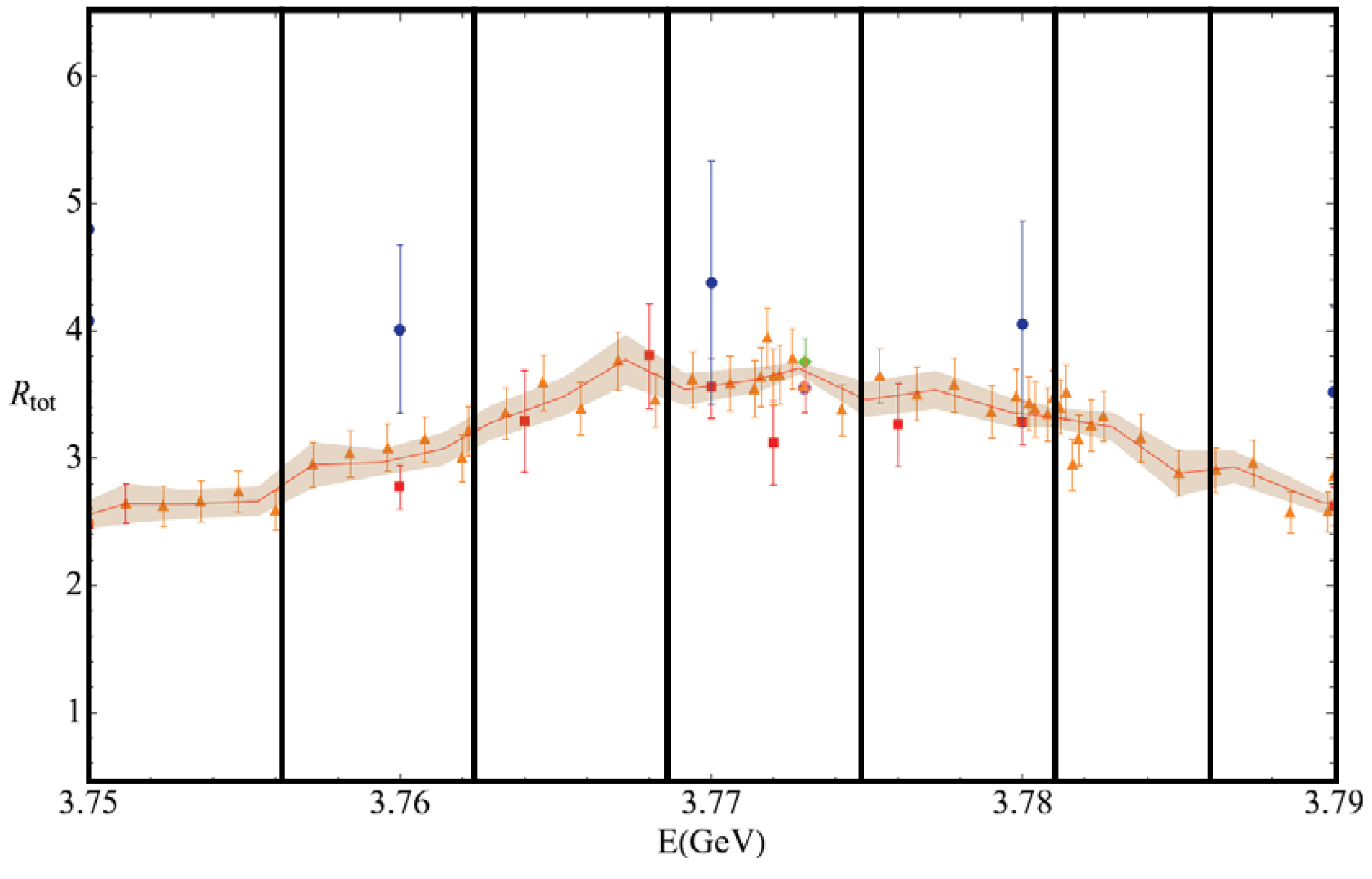
Underestimates errors! (they assing only naive theory error)



Fit procedure



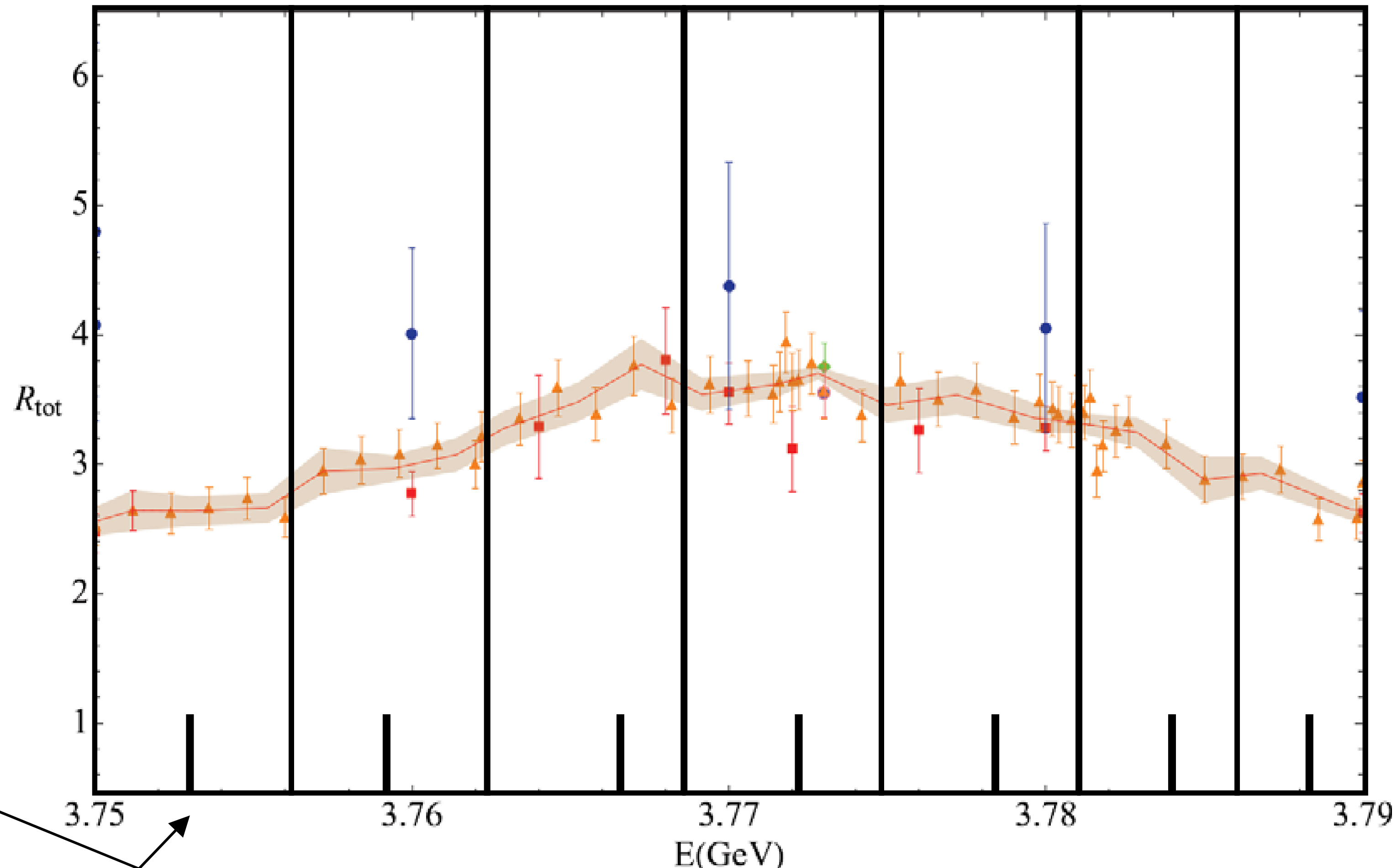
Fit procedure



1. Recluster data. Clusters not necessarily equally sized.

   Number of clusters and size of cluster according to the structure of the data

Fit procedure



Experimental energies

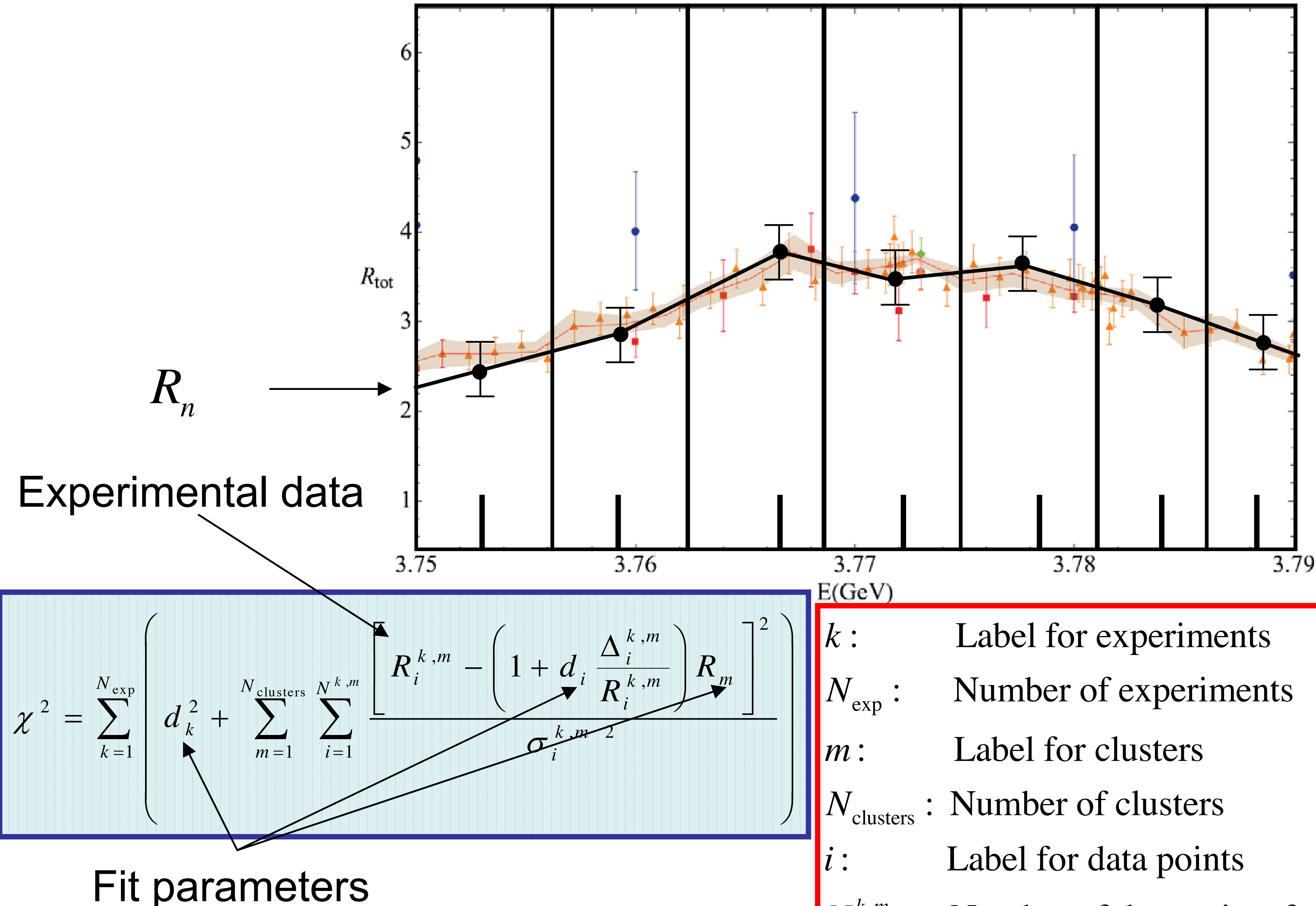
Cluster energy

$$E_m = \frac{\displaystyle\sum_{k=1}^{N_{exp}} \sum_{i=1}^{N^{k,m}} \frac{E_i^{k,m}}{\sigma_i^{k,m\,2} + \Delta_i^{k,m\,2}}}{\displaystyle\sum_{k=1}^{N_{exp}} \sum_{i=1}^{N^{k,m}} \frac{1}{\sigma_i^{k,m\,2} + \Delta_i^{k,m\,2}}}$$

- k : Label for experiments
- N_{exp} : Number of experiments
- m : Label for clusters
- $N_{clusters}$: Number of clusters
- i : Label for data points
- $N^{k,m}$: Number of data points for experiment k in cluster m

2. Calculate the energy of the cluster. One weights the energy of the data points inside the clusters with their errors.

Fit procedure

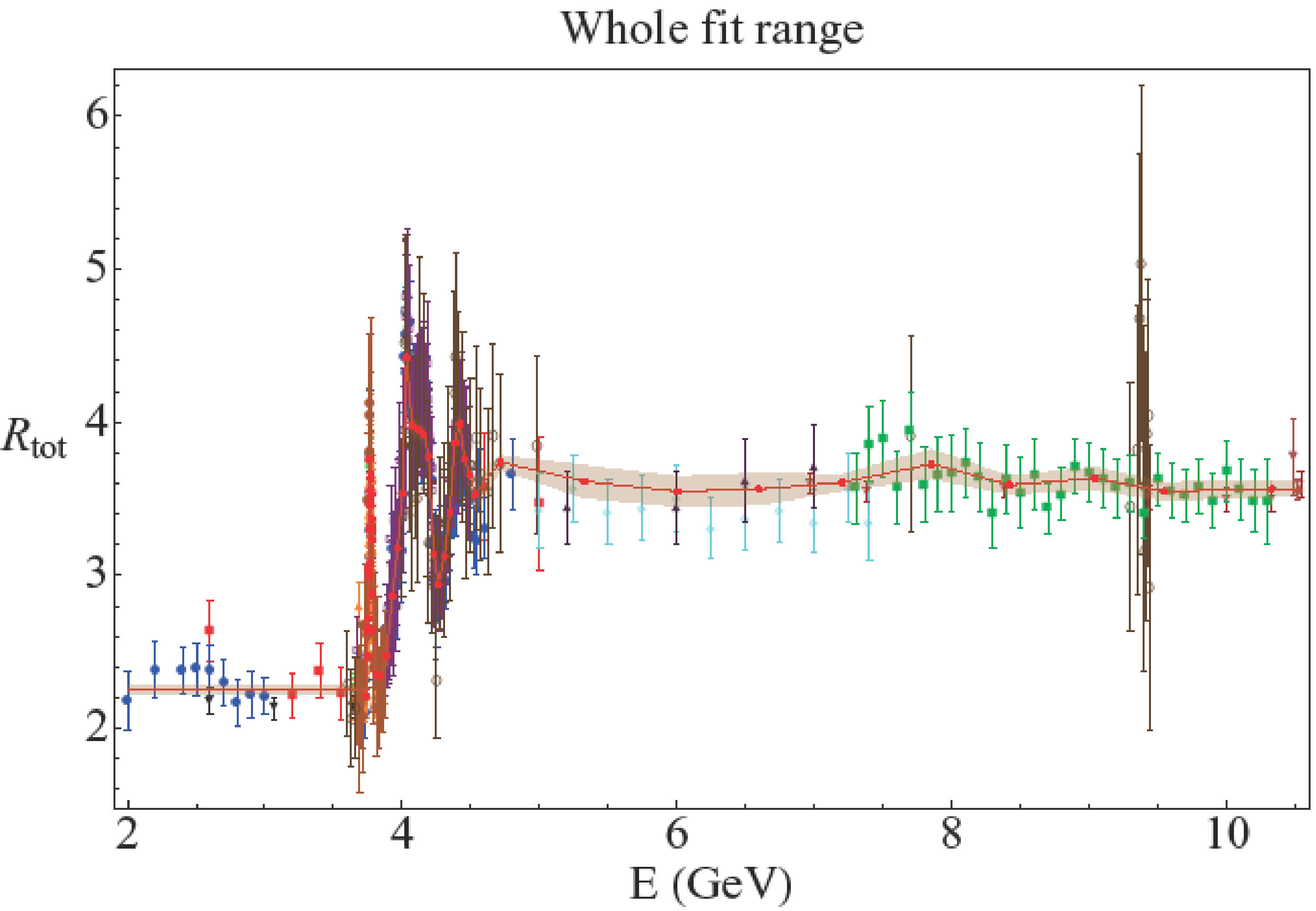


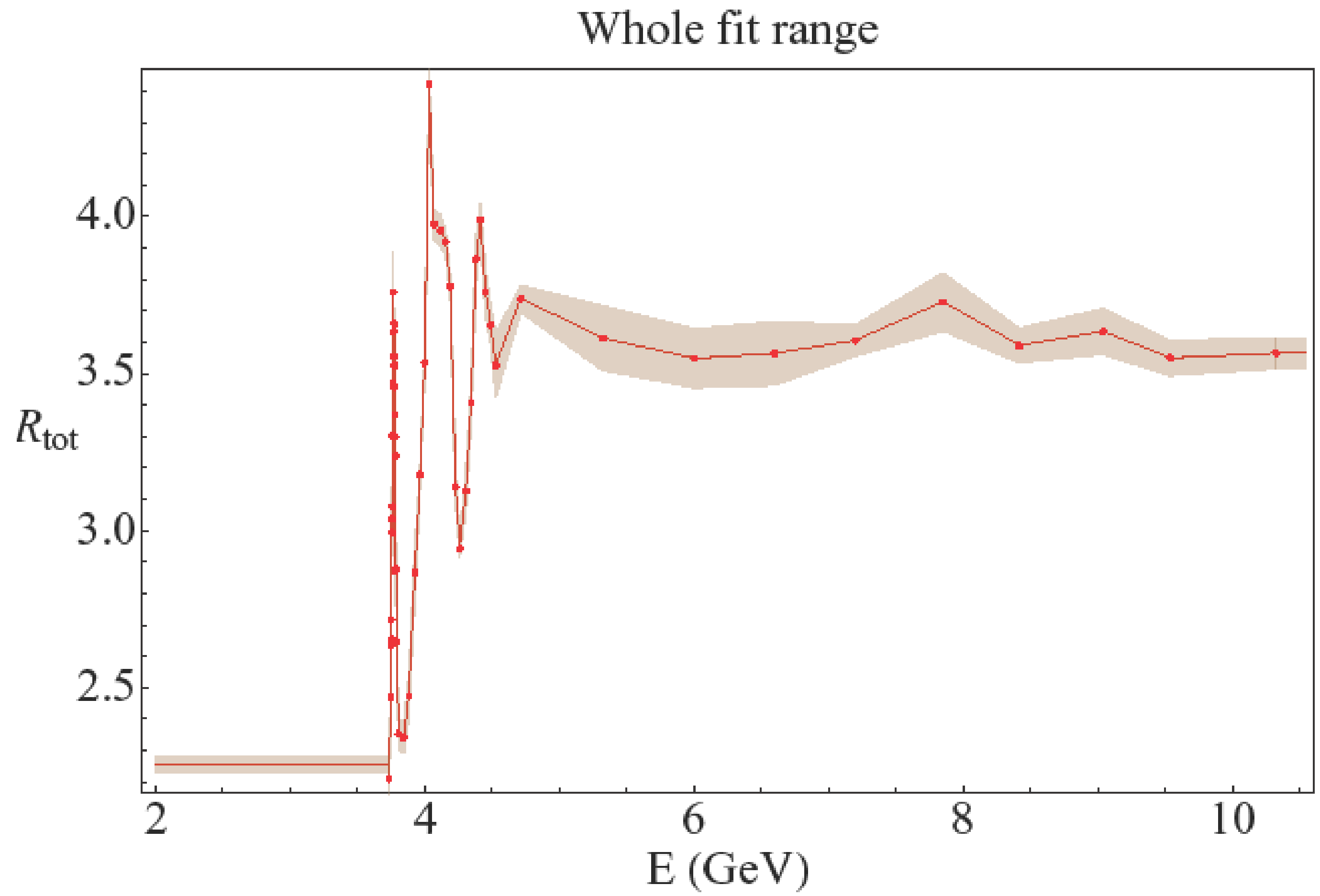
$$\chi^2 = \sum_{k=1}^{N_{\exp}} \left(d_k^2 + \sum_{m=1}^{N_{\text{clusters}}} \sum_{i=1}^{N^{k,m}} \frac{\left[R_i^{k,m} - \left(1 + d_i \frac{\Delta_i^{k,m}}{R_i^{k,m}} \right) R_m \right]^2}{\sigma_i^{k,m\,2}} \right)$$

k :	Label for experiments
$N_{\exp}$:	Number of experiments
m :	Label for clusters
N_{clusters} :	Number of clusters
i :	Label for data points
$N^{k,m}$:	Number of data points for experiment k in cluster m

3. Fit the value of R for each cluster. Data is allowed to “move” within its systematic error. The method renders errors and correlations among various clusters. One can then calculate errors and correlations for the moments.

Fit results





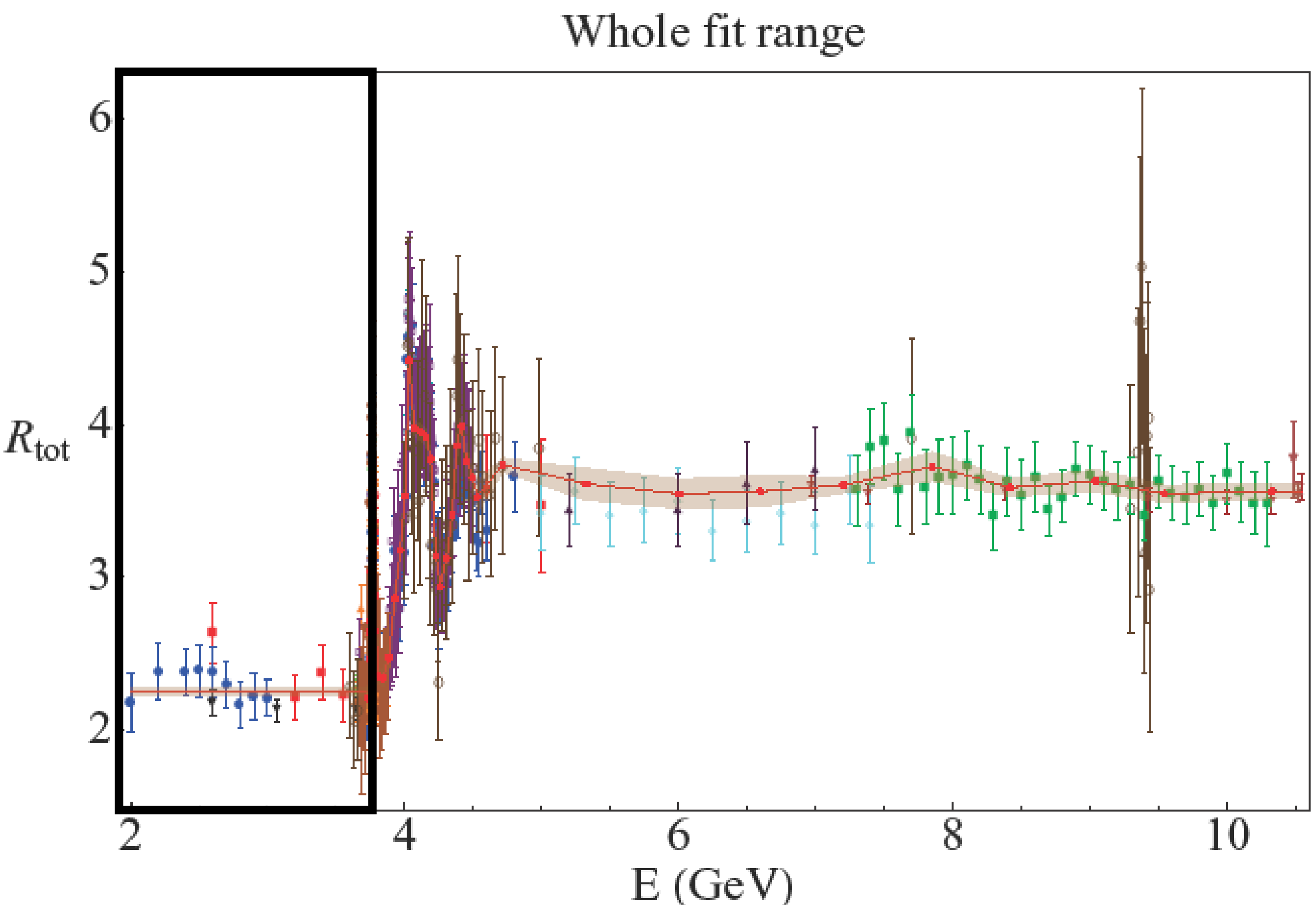
With our fit procedure we are capable of simultaneously determining the R_{uds} background and the R_{cc}

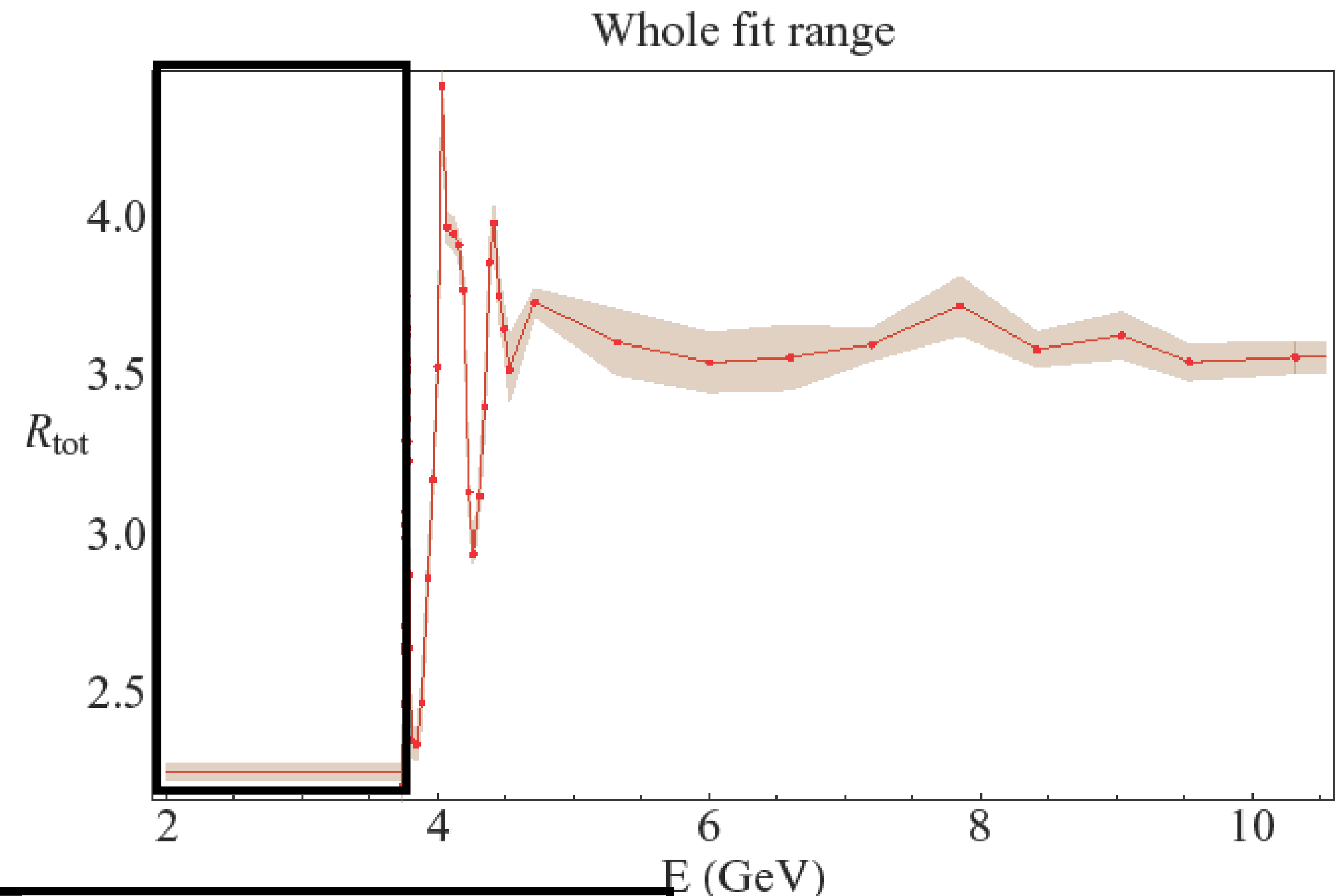
$$R_{\text{tot}} = R_{cc} + R_{uds}$$

$$\frac{\chi^2}{\text{d.o.f.}} = 1.89$$

Good quality of the fit !!!

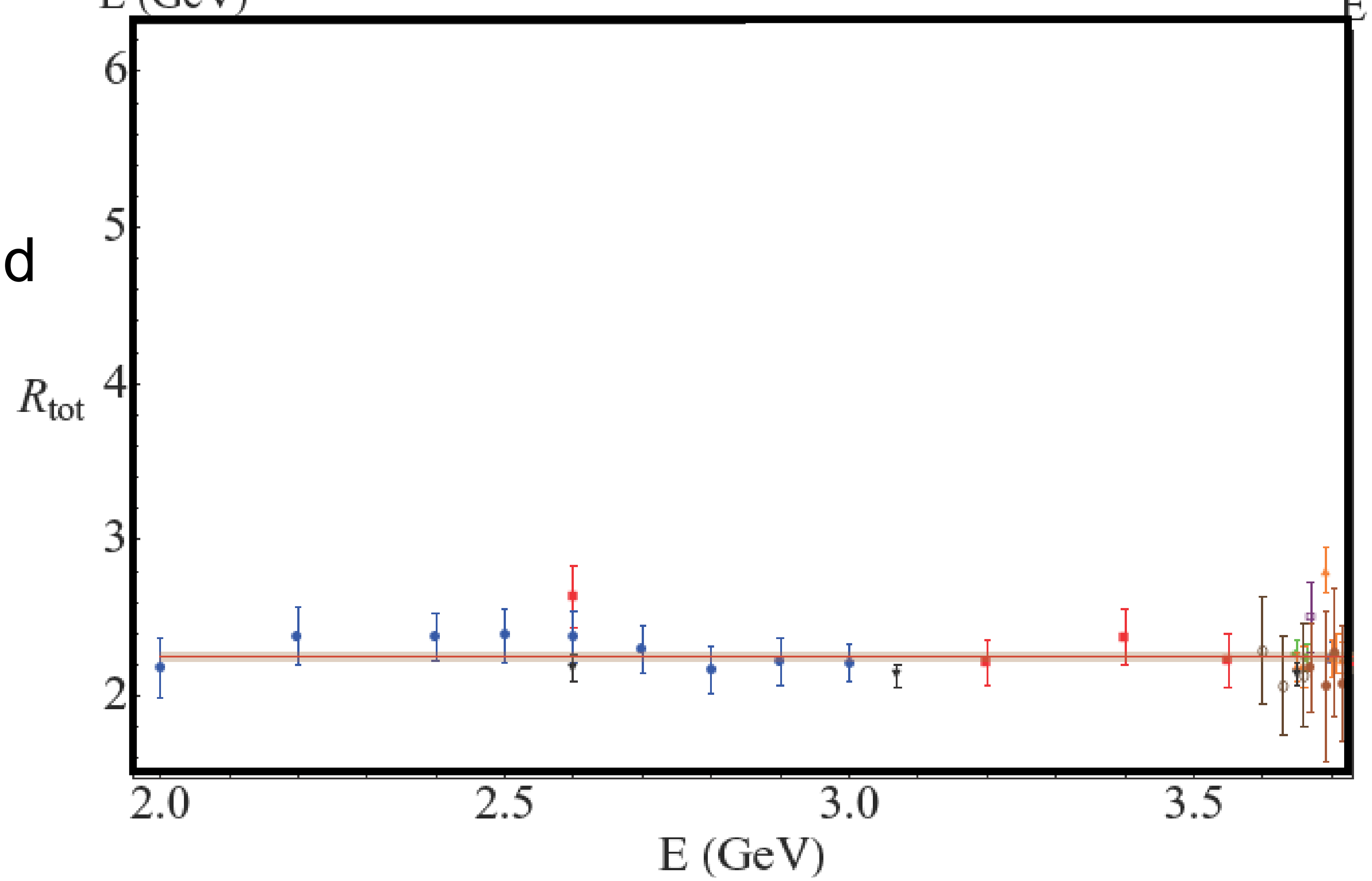
Fit results



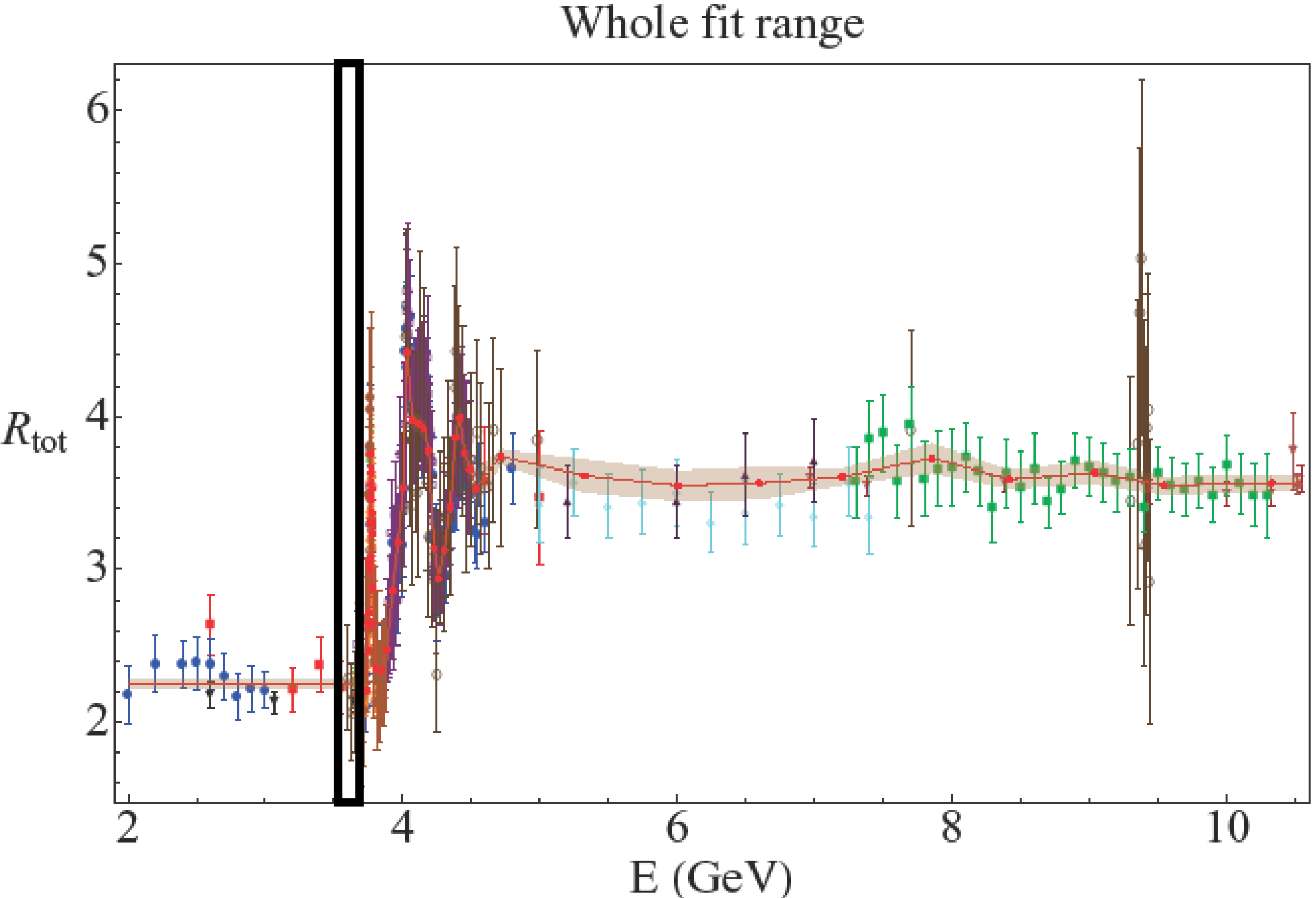


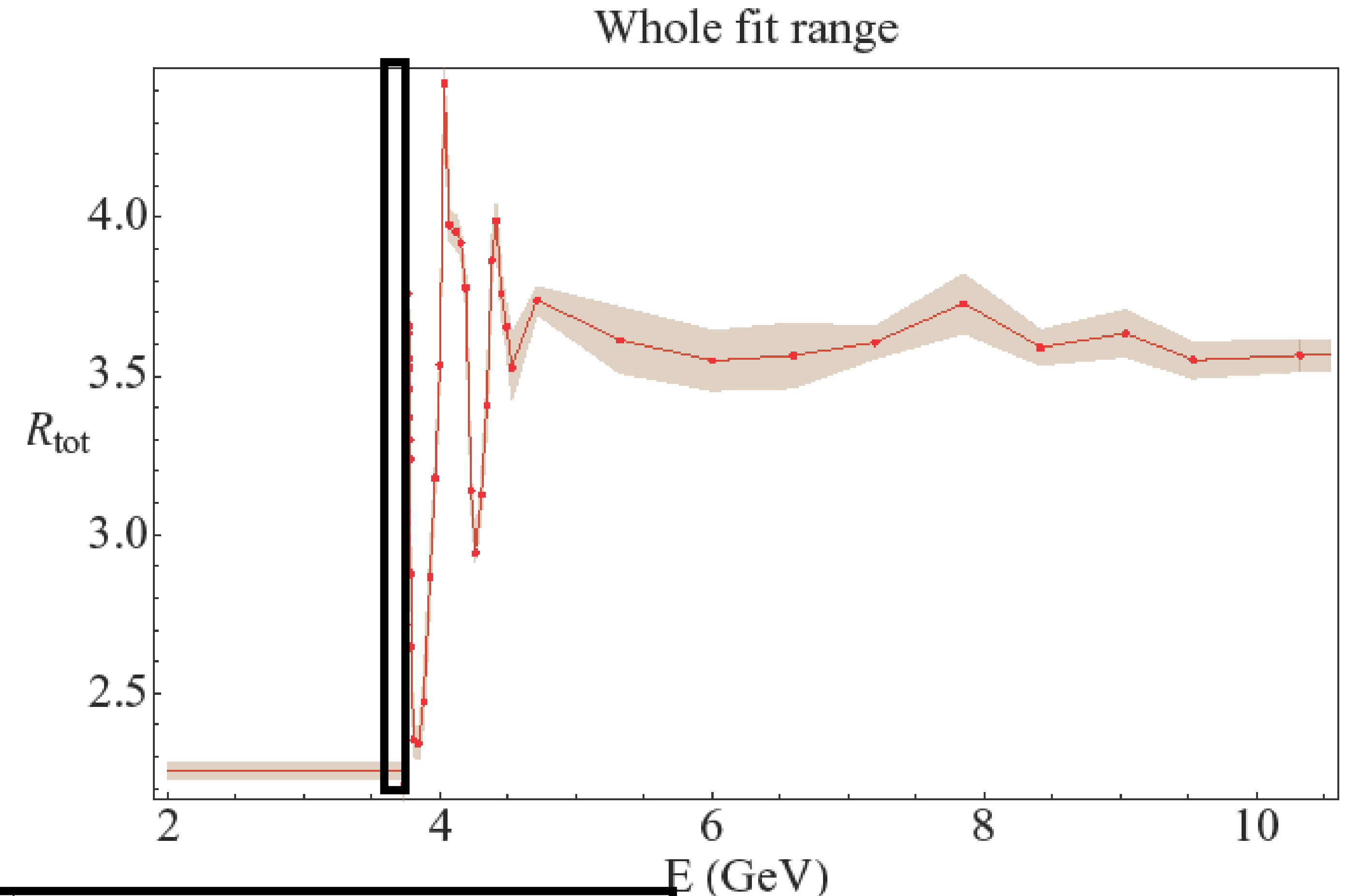
Below threshold

1 cluster



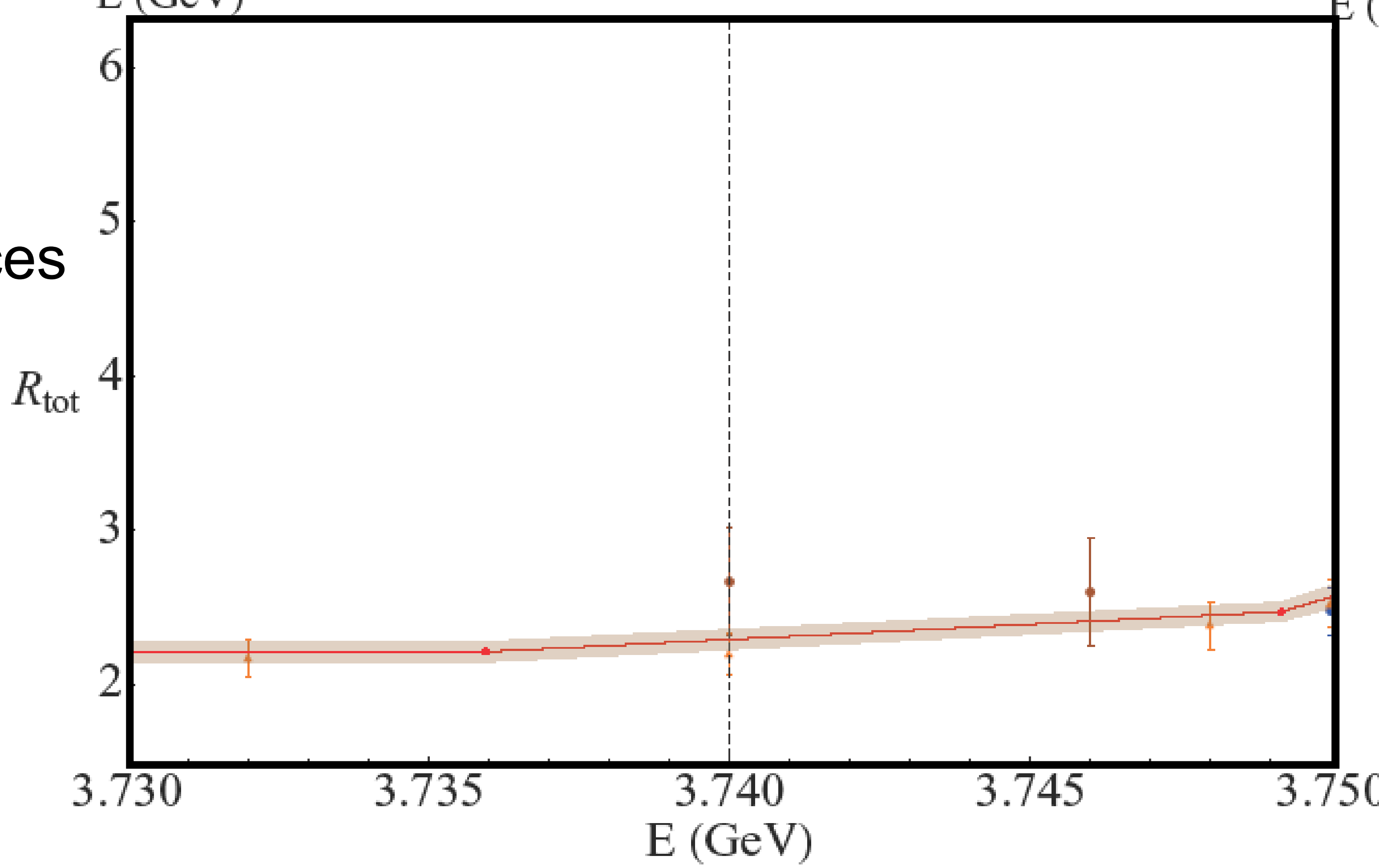
Fit results



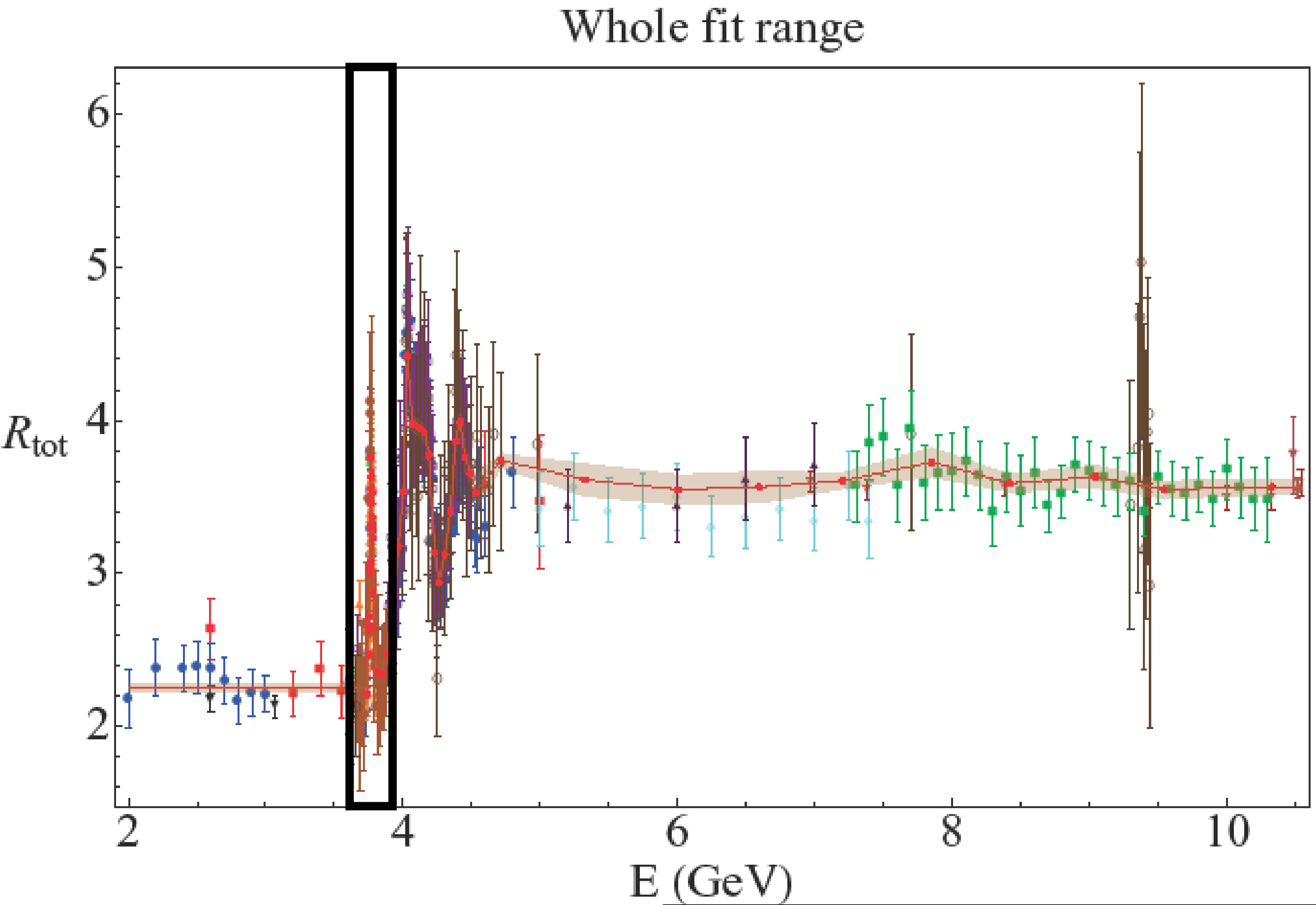


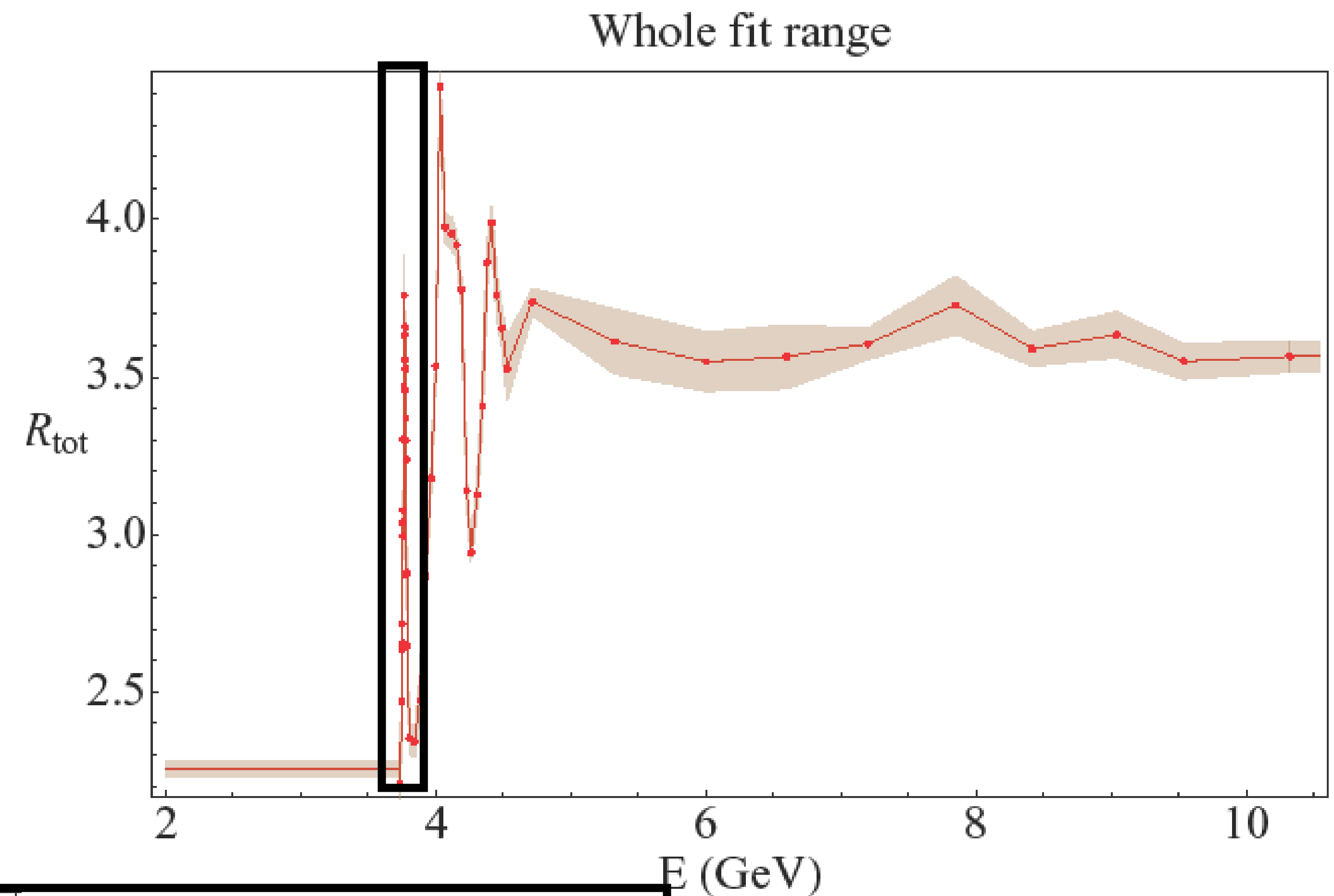
Below resonances

2 clusters



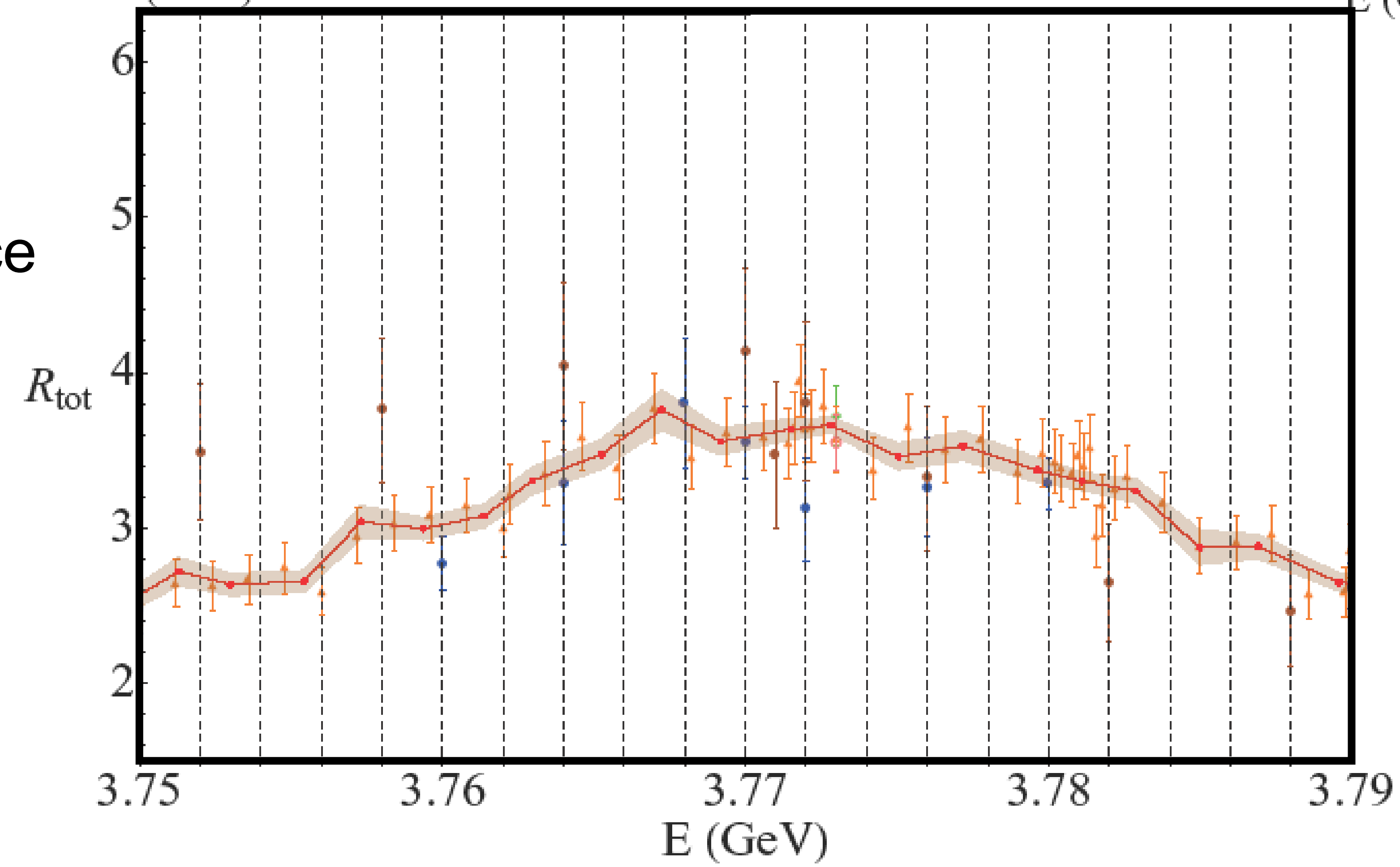
Fit results



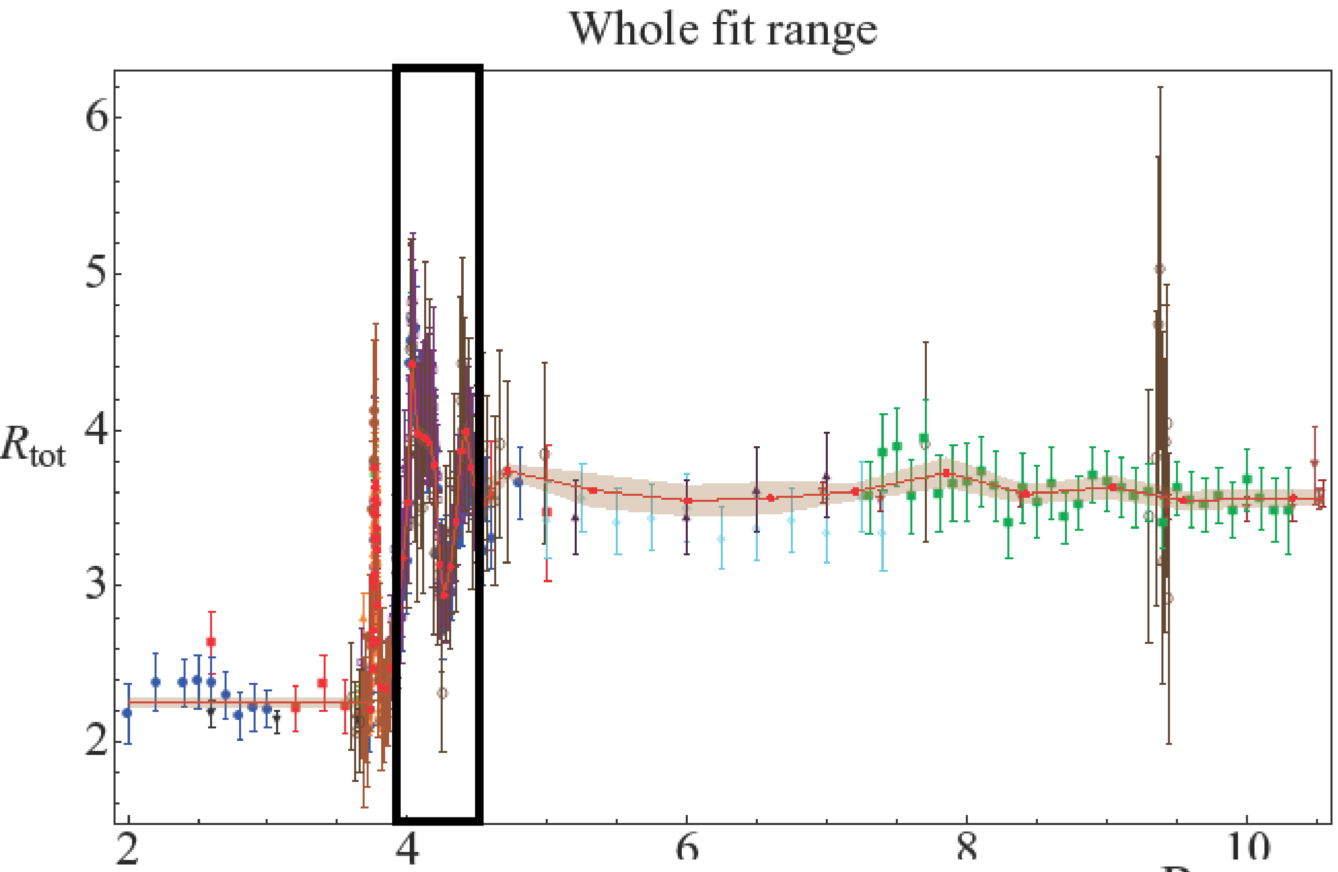


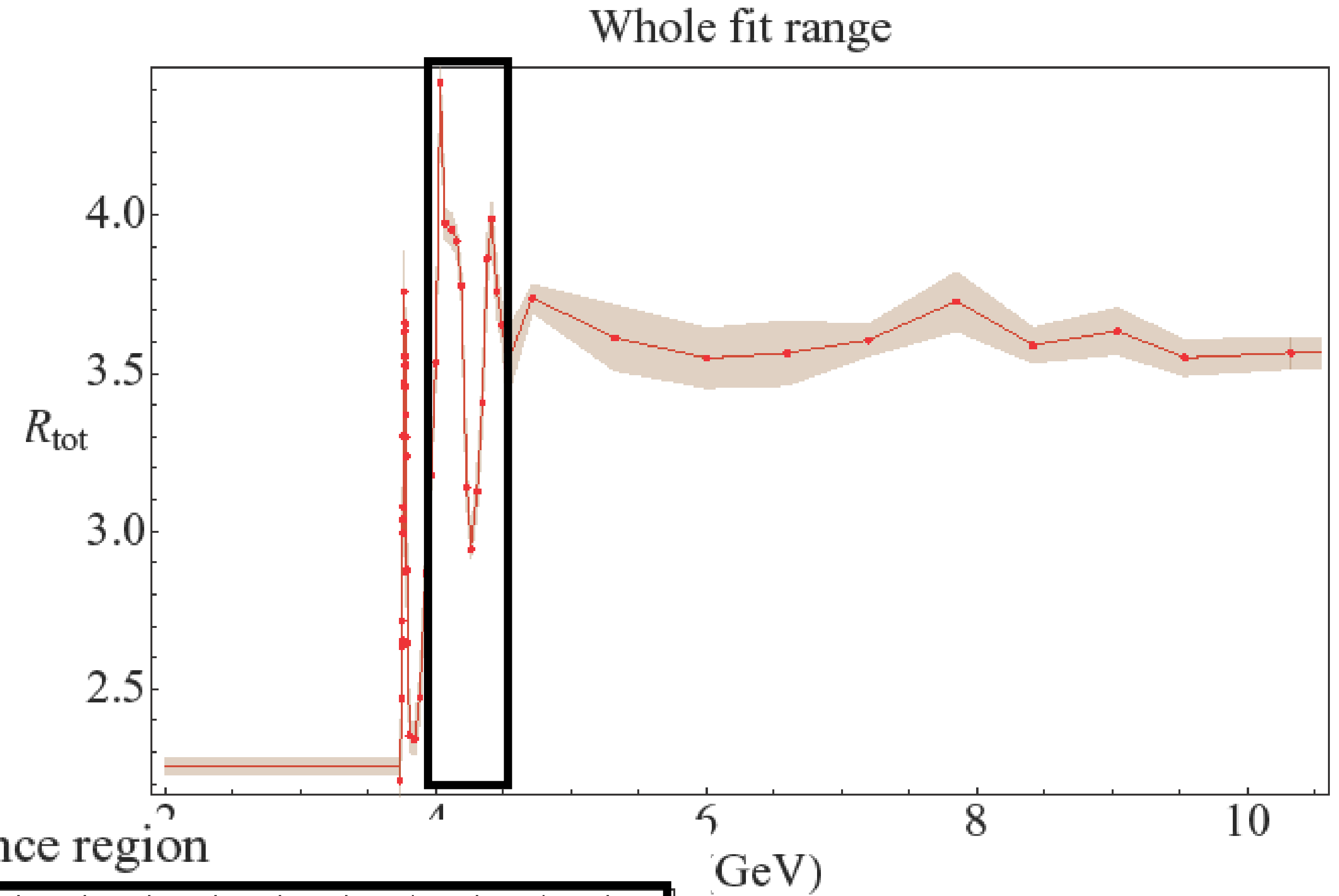
First resonance

20 clusters



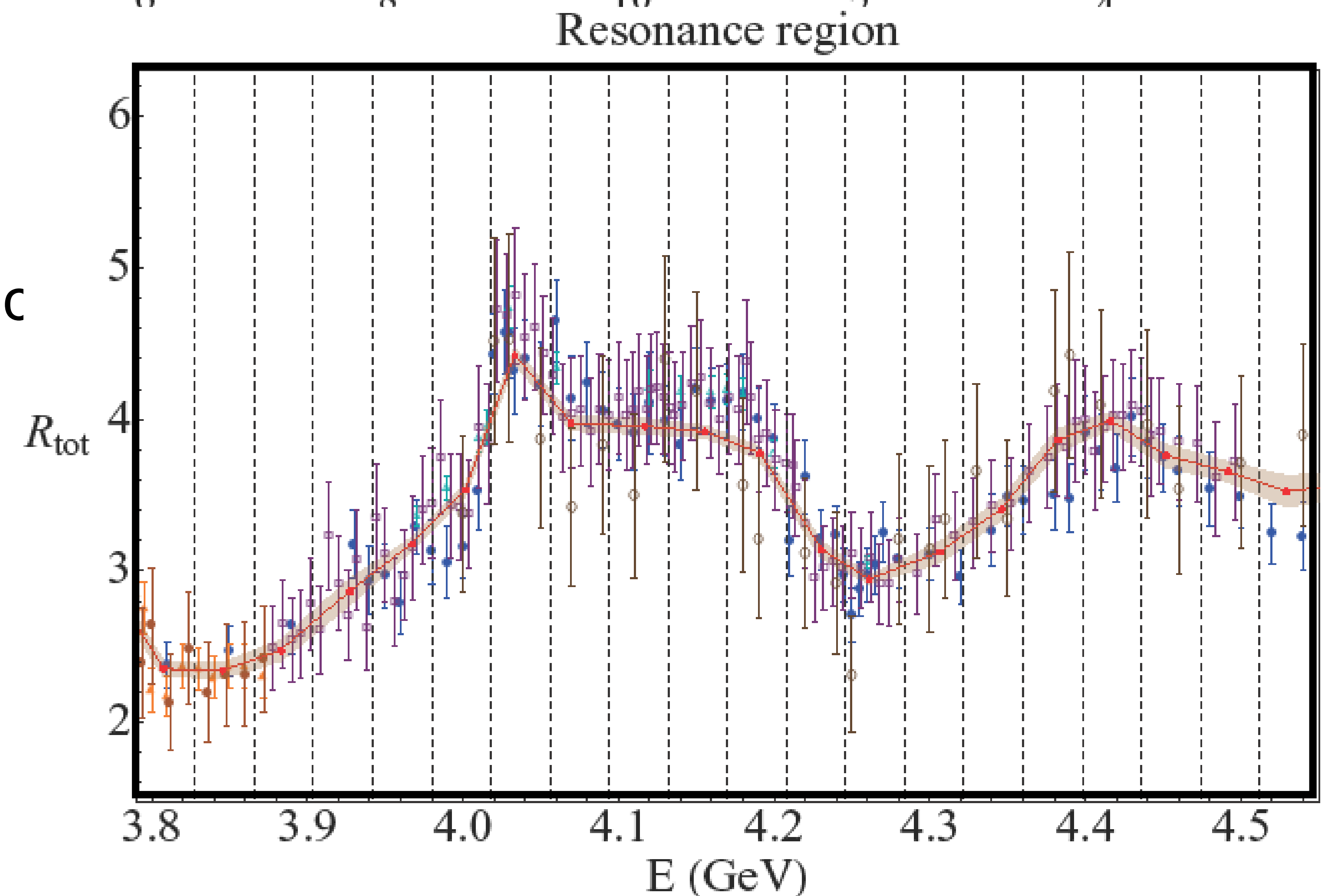
Fit results



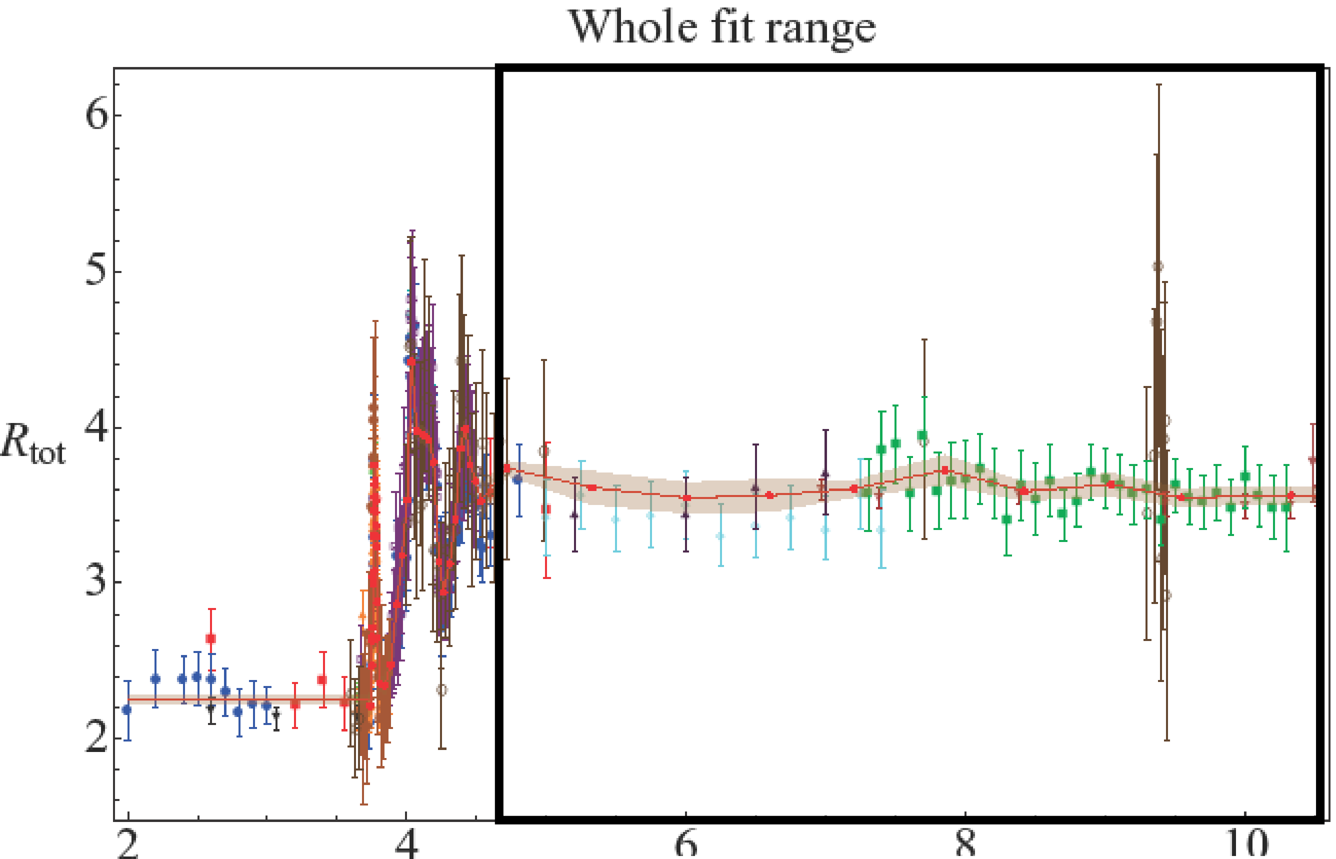


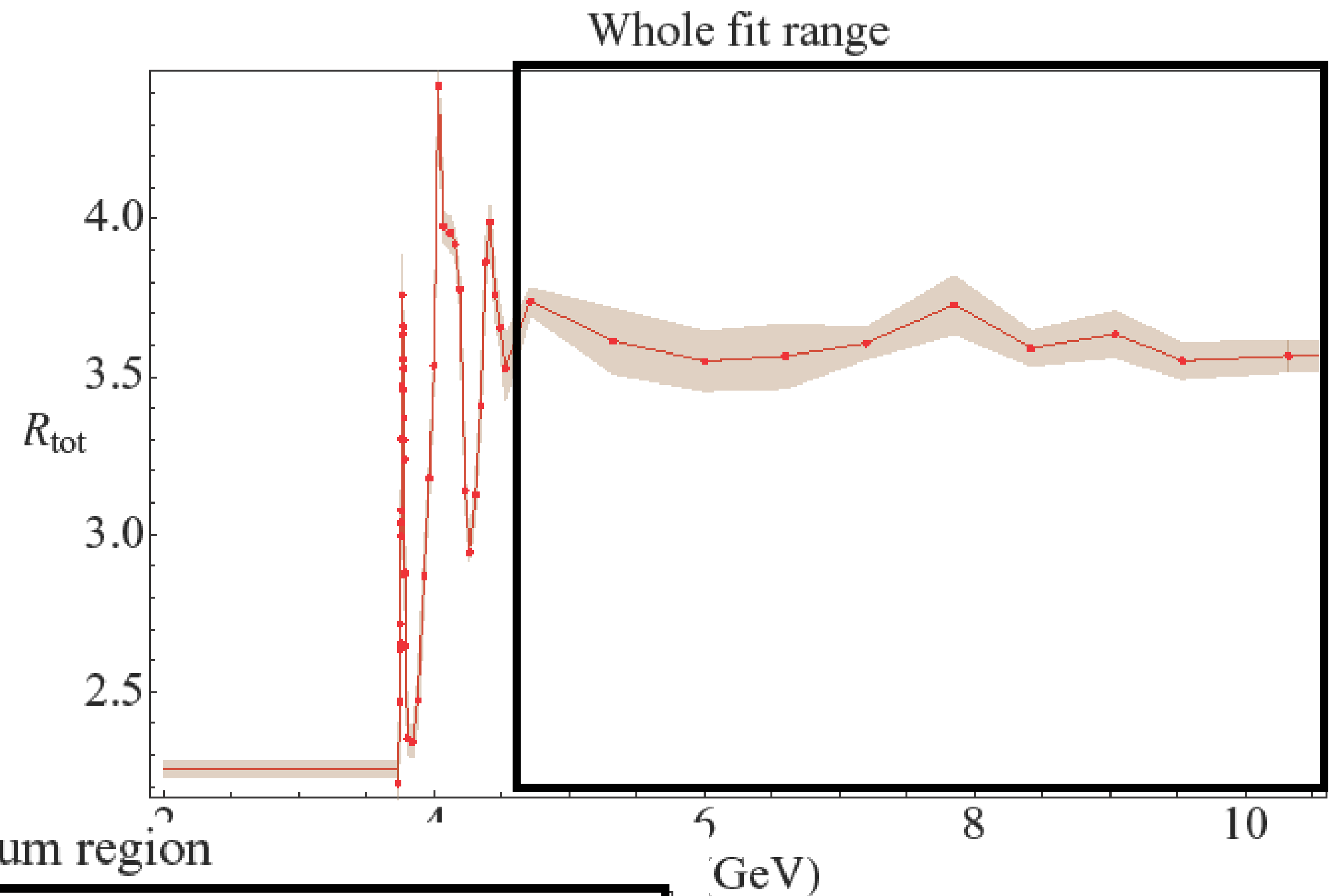
Second resonanc

20 clusters



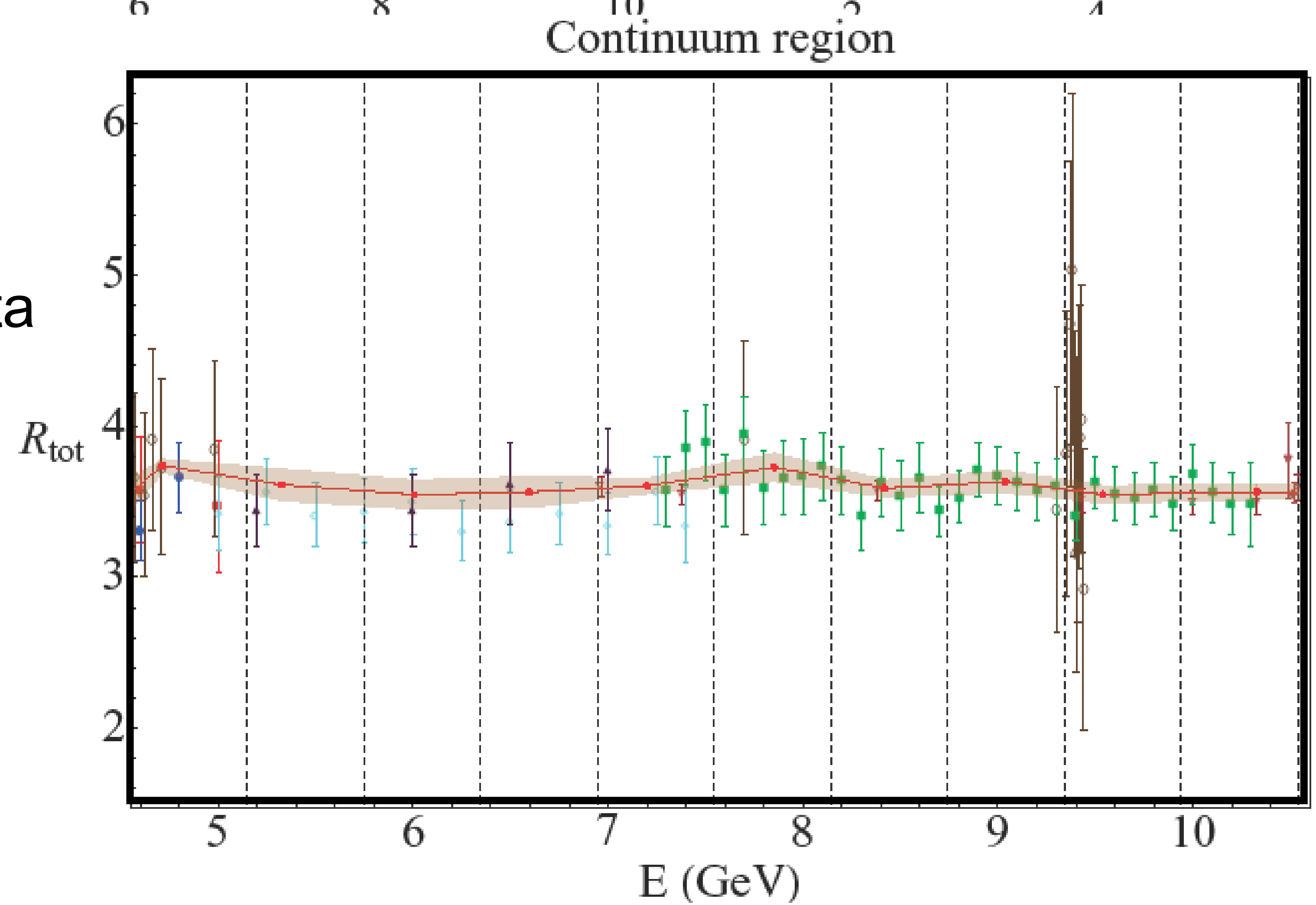
Fit results



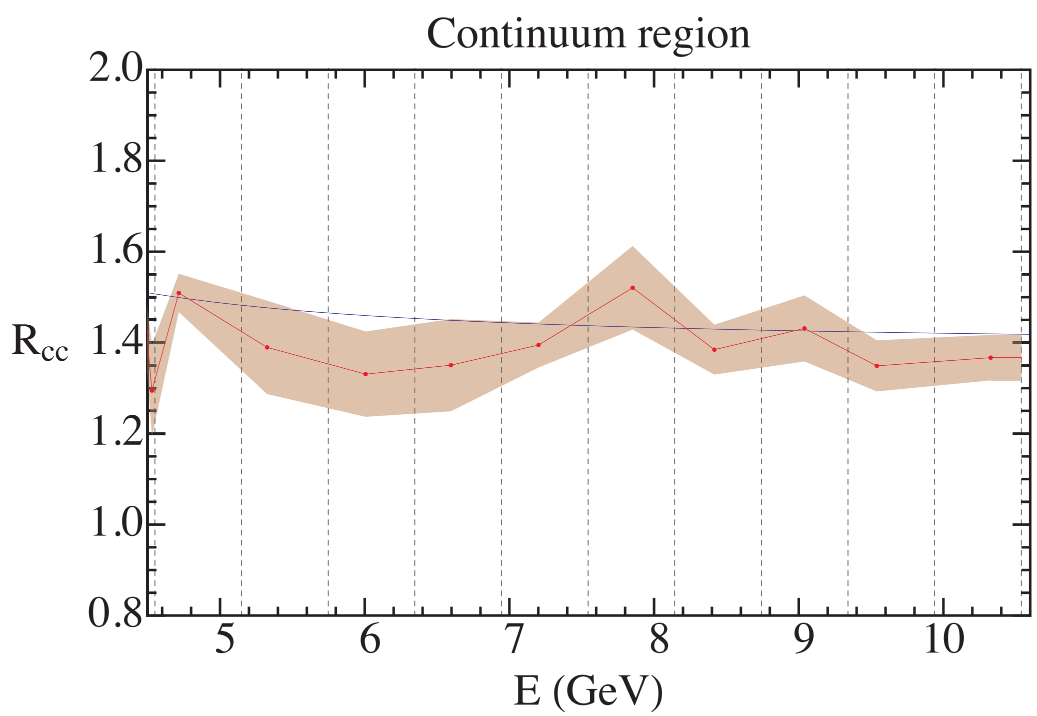


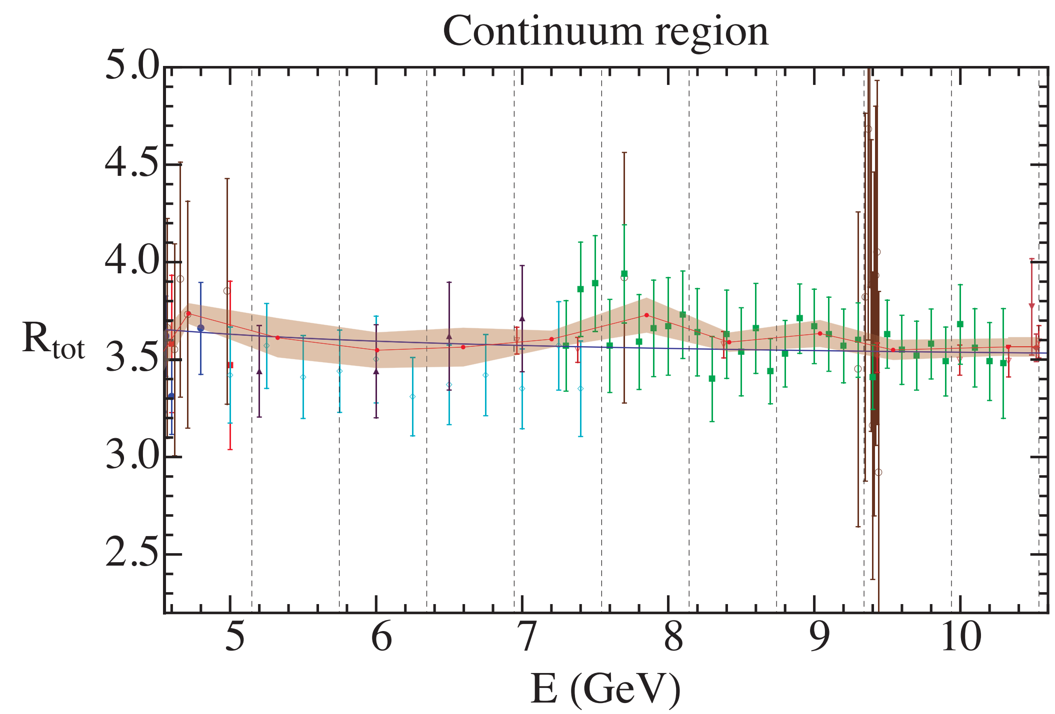
Continuum data

10 clusters



Comparison pQCD and data





$$M_{1,\mathrm{ex}}^{(4.55-10.538)\mathrm{GeV}} = 4.81 \pm 0.18$$ contribution to moment using data

$$M_{1}^{(4.55-10.538)\mathrm{GeV}} = 5.010 \pm 0.011$$ contribution to moment using pQCD

- Data seems to oscillate around pQCD at these energies
- pQCD error 0.2%, data error 4%. pQCD 4% bigger than pQCD.
- Using pQCD is OK if one assigns a more conservative error (4-10%)

Theoretical framework

Theoretical framework

$E_{\mathrm{eff}} = \frac{m_q}{\sqrt{n}}$ effective energy range (asymptotically correct)

$\frac{m_q}{\sqrt{n}} \gg \Lambda_{\mathrm{QCD}}$ For the OPE to work (otherwise bad convergence)

n = 1, 2 for charm

n = 1, 2, 3, 4 for bottom

Stay away from larger n even if things appear to be nice

Theoretical framework

$E_{\rm eff} = \frac{m_q}{\sqrt{n}}$ effective energy range (asymptotically correct)

$\frac{m_q}{\sqrt{n}} \gg \Lambda_{\rm QCD}$ For the OPE to work (otherwise bad convergence)

n = 1, 2 for charm

n = 1, 2, 3, 4 for bottom

Stay away from larger n even if things appear to be nice

OPE framework (a la ITEP)

Fairly dominated by perturbative Wilson coefficient

Include the Gluon condensate contribution → very small contribution

Similarly for pseudo-scalar correlator

Identical for bottom mass determination

Methods in perturbation theory

We use four different expansion methods, equivalent in perturbation theory, to test the convergence of the series expansion

All perturbative methods should give similar results when determining the charm mass (within theoretical uncertainties)

We use different renormalization scales for α_s denoted by μ_α and $\overline{m}_c$ denoted μ_m

Methods in perturbation theory

Fixed order expansion

$$M_n^{\rm pert} = \frac{1}{(4\overline{m}_c^2(\mu_m))^n} \sum_{i,a,b} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i C_{n,i}^{a,b} \ln^a\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_m^2}\right) \ln^b\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_\alpha^2}\right)$$

Simply compute the n-th derivative of $\Pi(q^2)$ around $q^2 = 0$

Methods in perturbation theory

Fixed order expansion

$$M_n^{\rm pert} = \frac{1}{(4\overline{m}_c^2(\mu_m))^n}\sum_{i,a,b}\left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i C_{n,i}^{a,b}\ln^a\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_m^2}\right)\ln^b\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_\alpha^2}\right)$$

take root and re-expand in α_s to the right order

Linearized expansion

$$\left(M_n^{\rm th,pert}\right)^{1/2n} = \frac{1}{2\overline{m}_c(\mu_m)}\sum_{i,a,b}\left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \tilde{C}_{n,i}^{a,b}\ln^a\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_m^2}\right)\ln^b\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_\alpha^2}\right)$$

Methods in perturbation theory

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Iterative linearized expansion

$$\overline{m}_c^{(0)} = \frac{1}{2\left(M_n^{\text{th,pert}}\right)^{1/2n}} \tilde{C}_{n,0}^{0,0}$$

$$\overline{m}_c(\mu_m) = \overline{m}_c^{(0)} \sum_{i,a,b} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \hat{C}_{n,i}^{a,b} \ln^a\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_m^2}\right) \ln^b\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_\alpha^2}\right)$$

Iterative Linearized Expansion

[Dehnadi, Hoang, VM, Zebarjad]

Start with expanded out expression and solve for the mass

$$\overline{m}_c(\mu_m) = \frac{1}{2\left(M_n^{\mathrm{th,pert}}\right)^{1/2n}} \sum_{i,a,b} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \tilde{C}_{n,i}^{a,b} \ln^a\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_m^2}\right) \ln^b\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_\alpha^2}\right)$$

mass dependence on RHS !! starts at $\mathcal{O}(\alpha_s)$

Iterative Linearized Expansion

[Dehnadi, Hoang, VM, Zebarjad]

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$$\overline{m}_c(\mu_m) = \frac{1}{2\left(M_n^{\rm th,pert}\right)^{1/2n}} \sum_{i,a,b} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \tilde{C}_{n,i}^{a,b} \ln^a\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_m^2}\right) \ln^b\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_\alpha^2}\right)$$

tree-level is simple

$$\overline{m}_c^{(0)} = \frac{1}{2\left(M_n^{\rm th,pert}\right)^{1/2n}} \tilde{C}_{n,0}^{0,0}$$

no mass depend on LHS

Iterative Linearized Expansion

[Dehnadi, Hoang, VM, Zebarjad]

Start with expanded out expression and solve for the mass

$$\overline{m}_c(\mu_m) = \frac{1}{2\Big(M_n^{\rm th,pert}\Big)^{1/2n}} \sum_{i,a,b} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \tilde{C}_{n,i}^{a,b} \ln^a\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_m^2}\right) \ln^b\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_\alpha^2}\right)$$

tree-level is simple

$$\overline{m}_c^{(0)} = \frac{1}{2\Big(M_n^{\rm th,pert}\Big)^{1/2n}} \tilde{C}_{n,0}^{0,0}$$

insert tree solution and expand to $\mathcal{O}(\alpha_s)$

1-loop also simple

$$\overline{m}_c^{(1)}(\mu_m) = \frac{1}{2\Big(M_n^{\rm th,pert}\Big)^{1/2n}} \left\{ \tilde{C}_{n,0}^{0,0} + \frac{\alpha_s(\mu_\alpha)}{\pi}\left[\tilde{C}_{n,1}^{0,0} + \tilde{C}_{n,1}^{1,0} \ln\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_m^2}\right)\right]\right\}$$

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tree-level is simple

$$\overline{m}_c^{(0)} = \frac{1}{2\left(M_n^{\rm th,pert}\right)^{1/2n}} \tilde{C}_{n,0}^{0,0}$$

insert 1-loop solution and expand to $\mathcal{O}(\alpha_s^2)$

1-loop also simple

$$\overline{m}_c^{(1)}(\mu_m) = \frac{1}{2\left(M_n^{\rm th,pert}\right)^{1/2n}} \left\{ \tilde{C}_{n,0}^{0,0} + \frac{\alpha_s(\mu_\alpha)}{\pi}\left[\tilde{C}_{n,1}^{0,0} + \tilde{C}_{n,1}^{1,0} \ln\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_m^2}\right)\right]\right\}$$

2-loop more involved

$$\overline{m}_c^{(2)}(\mu_m) = \frac{1}{2\left(M_n^{\rm th,pert}\right)^{1/2n}} \left\{ \tilde{C}_{n,0}^{0,0} + \frac{\alpha_s(\mu_\alpha)}{\pi}\left[\tilde{C}_{n,1}^{0,0} + \tilde{C}_{n,1}^{1,0} \ln\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_m^2}\right)\right] + \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^2 \left[2\,\frac{\tilde{C}_{n,1}^{1,0}\,\tilde{C}_{n,1}^{0,0}}{\tilde{C}_{n,0}^{0,0}} + 2\,\frac{(\tilde{C}_{n,1}^{1,0})^2}{\tilde{C}_{n,0}^{0,0}} \ln\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_m^2}\right) + \sum_{a,b} \tilde{C}_{n,2}^{a,b} \ln^a\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_m^2}\right) \ln^b\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_\alpha^2}\right)\right]\right\}$$

Iterative Linearized Expansion

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$$\overline{m}_c(\mu_m) = \frac{1}{2\left(M_n^{\rm th,pert}\right)^{1/2n}} \sum_{i,a,b} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \tilde{C}_{n,i}^{a,b} \ln^a\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_m^2}\right) \ln^b\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_\alpha^2}\right)$$

tree-level is simple

$$\overline{m}_c^{(0)} = \frac{1}{2\left(M_n^{\rm th,pert}\right)^{1/2n}} \tilde{C}_{n,0}^{0,0}$$

insert 2-loop solution and expand to $\mathcal{O}(\alpha_s^3)$

1-loop also simple

$$\overline{m}_c^{(1)}(\mu_m) = \frac{1}{2\left(M_n^{\rm th,pert}\right)^{1/2n}} \left\{ \tilde{C}_{n,0}^{0,0} + \frac{\alpha_s(\mu_\alpha)}{\pi}\left[\tilde{C}_{n,1}^{0,0} + \tilde{C}_{n,1}^{1,0} \ln\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_m^2}\right)\right]\right\}$$

2-loop more involved

$$\overline{m}_c^{(2)}(\mu_m) = \frac{1}{2\left(M_n^{\rm th,pert}\right)^{1/2n}} \left\{ \tilde{C}_{n,0}^{0,0} + \frac{\alpha_s(\mu_\alpha)}{\pi}\left[\tilde{C}_{n,1}^{0,0} + \tilde{C}_{n,1}^{1,0} \ln\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_m^2}\right)\right] + \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^2 \left[2\,\frac{\tilde{C}_{n,1}^{1,0}\,\tilde{C}_{n,1}^{0,0}}{\tilde{C}_{n,0}^{0,0}} + 2\,\frac{(\tilde{C}_{n,1}^{1,0})^2}{\tilde{C}_{n,0}^{0,0}} \ln\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_m^2}\right) + \sum_{a,b} \tilde{C}_{n,2}^{a,b} \ln^a\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_m^2}\right) \ln^b\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_\alpha^2}\right)\right]\right\}$$

General expression

$$\overline{m}_c(\mu_m) = \overline{m}_c^{(0)} \sum_{i,a,b} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \hat{C}_{n,i}^{a,b} \ln^a\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_m^2}\right) \ln^b\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_\alpha^2}\right)$$

Methods in perturbation theory

Fixed order expansion

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Linearized expansion

$$\left(M_n^{\rm th,pert}\right)^{1/2n} = \frac{1}{2\overline{m}_c(\mu_m)} \sum_{i,a,b} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \tilde{C}_{n,i}^{a,b} \ln^a\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_m^2}\right) \ln^b\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_\alpha^2}\right)$$

Iterative linearized expansion

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$$\overline{m}_c(\mu_m) = \overline{m}_c^{(0)} \sum_{i,a,b} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \hat{C}_{n,i}^{a,b} \ln^a\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_m^2}\right) \ln^b\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_\alpha^2}\right)$$

modified threshold vs high-energy weight

Contour improved expansion

$$M_n^{\rm c,pert} = \frac{6\pi Q_q^2}{i} \oint_{\mathcal{C}} \frac{{\rm d}s}{s^{n+1}} \Pi[\, s, \alpha_s(\mu_\alpha^c(s,\overline{m}_c^2)), \overline{m}_c(\mu_m), \mu_\alpha^c(s,\overline{m}_c^2), \mu_m\,]$$

$$(\mu_\alpha^c)^2(s,\overline{m}_c^2) = \mu_\alpha^2 \left(1 - \frac{s}{4\overline{m}_c^2(\mu_m)}\right)$$

perturbatively sensitive to $\Pi(0)$

Methods in perturbation theory

Fixed order expansion

$$M_n^{\rm pert} = \frac{1}{(4\overline{m}_c^2(\mu_m))^n} \sum_{i,a,b} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i C_{n,i}^{a,b} \ln^a\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_m^2}\right) \ln^b\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_\alpha^2}\right)$$

Linearized expansion

$$\left(M_n^{\rm th,pert}\right)^{1/2n} = \frac{1}{2\overline{m}_c(\mu_m)} \sum_{i,a,b} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \tilde{C}_{n,i}^{a,b} \ln^a\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_m^2}\right) \ln^b\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_\alpha^2}\right)$$

Iterative linearized expansion

$$\overline{m}_c^{(0)} = \frac{1}{2\left(M_n^{\rm th,pert}\right)^{1/2n}} \tilde{C}_{n,0}^{0,0}$$

$$\overline{m}_c(\mu_m) = \overline{m}_c^{(0)} \sum_{i,a,b} \left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \hat{C}_{n,i}^{a,b} \ln^a\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_m^2}\right) \ln^b\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_\alpha^2}\right)$$

Numerically solve for mass, not always has solution

Contour improved expansion

$$M_n^{\rm c,pert} = \frac{6\pi Q_q^2}{i} \oint_{\mathcal{C}} \frac{{\rm d}s}{s^{n+1}} \Pi[\, s, \alpha_s(\mu_\alpha^c(s,\overline{m}_c^2)), \overline{m}_c(\mu_m), \mu_\alpha^c(s,\overline{m}_c^2), \mu_m \,]$$

$$(\mu_\alpha^c)^2(s,\overline{m}_c^2) = \mu_\alpha^2\left(1 - \frac{s}{4\overline{m}_c^2(\mu_m)}\right)$$

Methods in perturbation theory

Fixed order expansion

$$M_n^{\rm pert} = \frac{1}{(4\overline{m}_c^2(\mu_m))^n}\sum_{i,a,b}\left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i C_{n,i}^{a,b}\,\ln^a\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_m^2}\right)\ln^b\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_\alpha^2}\right)$$

Linearized expansion

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Solve analytically for mass, always has a solution

Contour improved expansion

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μ_α- and μ_m-independent

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residual dependence on μ_α and μ_m due to truncation of series in α_s

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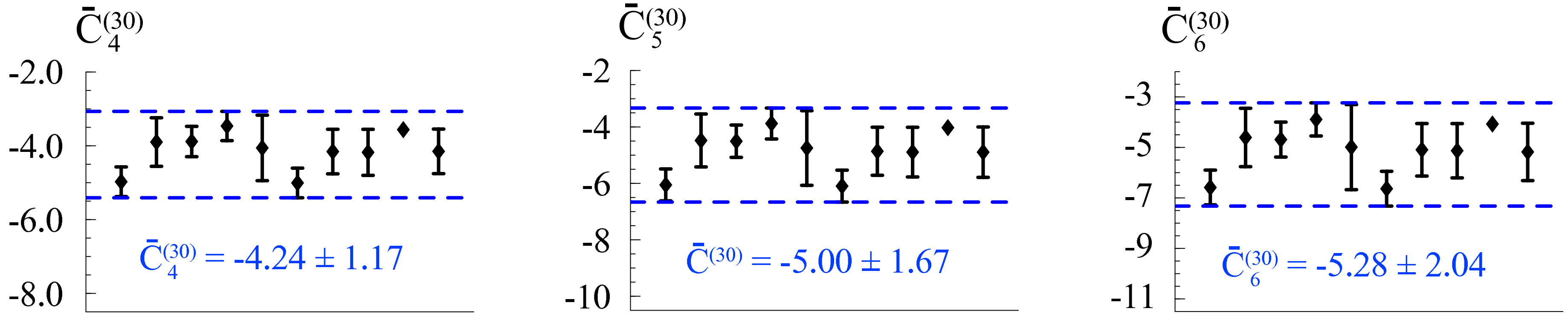
Status of computations

Moments

- For n = 1, 2, 3 the $C_n^{0,0}$ coefficients are known at $\mathcal{O}(\alpha_s^3)$
- For n ≥ 4, $C_n^{0,0}$ are known in a semi-analytic approach (Padé)
- The rest of $C_n^{a,b}$ can be deduced by RGE evolution

[Kühn et al]
[Boughezal et al]
[Maier et al]
[Hoang,VM, Zebarjad]
[Greynat et al]



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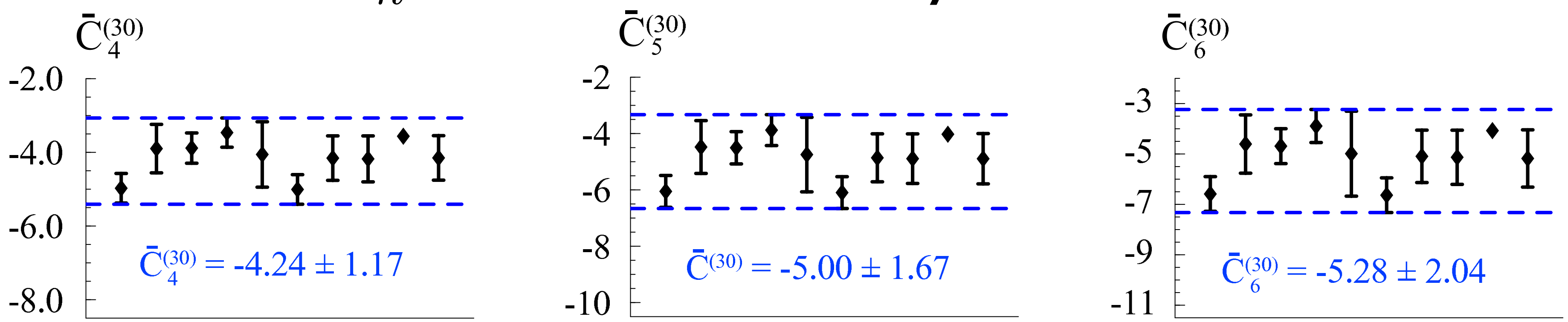
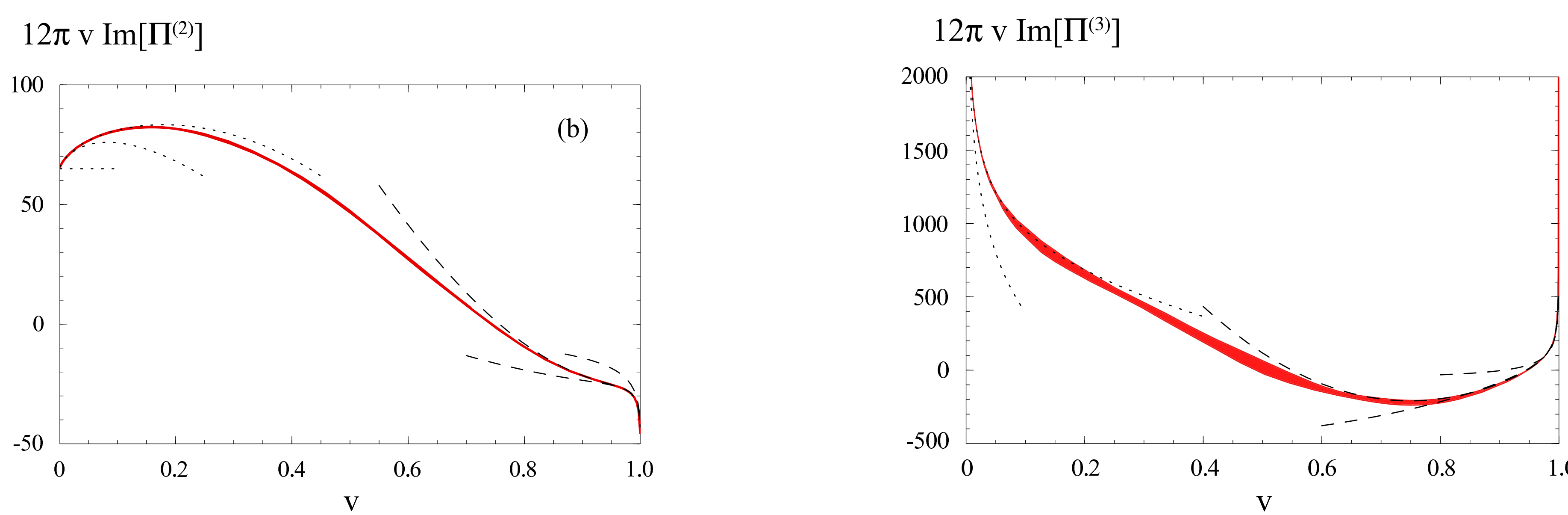


R-ratio for a massive pair of quarks

[Hoang, VM, Zebarjad]
[Greynat et al]

- Analytically known at tree and one-loop
- Known high-energy and threshold limits at two and three loops
- Semi-analytic approach (Padé) at two and three loops



Previous analyses

Use only fixed order expansion

[Chetyrkin, Kuhn, Meier, Meierhofer, Marquard, Steinhauser (2009)]

n	$m_c(3\text{ GeV})$	exp	α_s	μ	np	total
1	986	9	9	2	1	13
2	976	6	14	5	0	16
3	978	5	15	7	2	17
4	1004	3	9	31	7	33

$$\overline{m}_c(3\,\text{GeV}) = 986 \pm 13\,\text{GeV}$$

$$\overline{m}_c(\overline{m}_c) = 1279 \pm 13\,\text{GeV}$$

$$2\,\text{GeV} \le \mu_\alpha = \mu_m \le 4\,\text{GeV}$$

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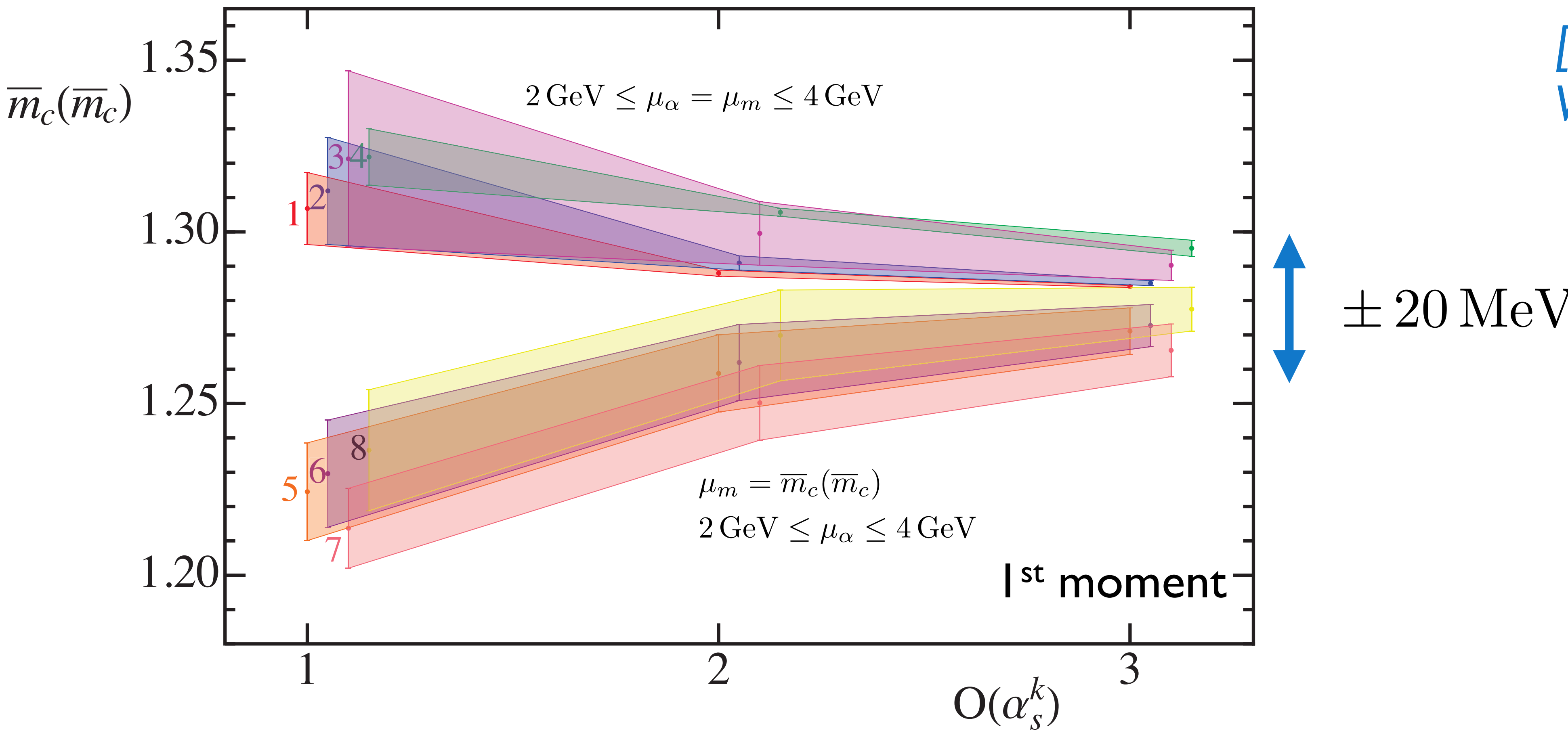
Our check on different expansions   pert uncertainty considerably larger

four methods, two variations



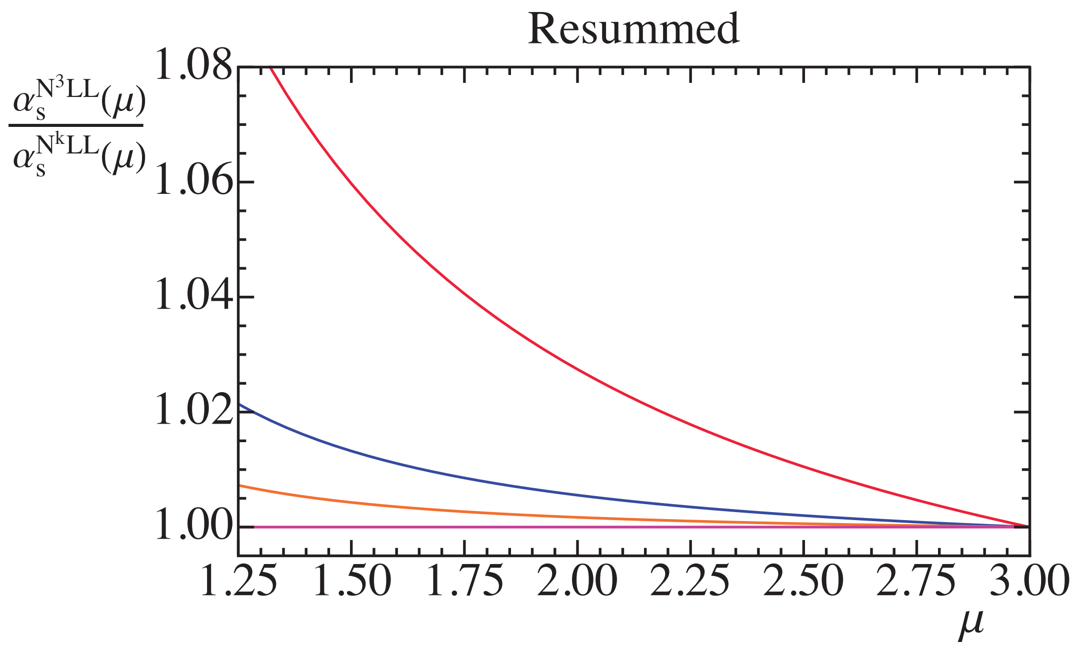
[Dehnadi, Hoang, VM, Zebarjad]

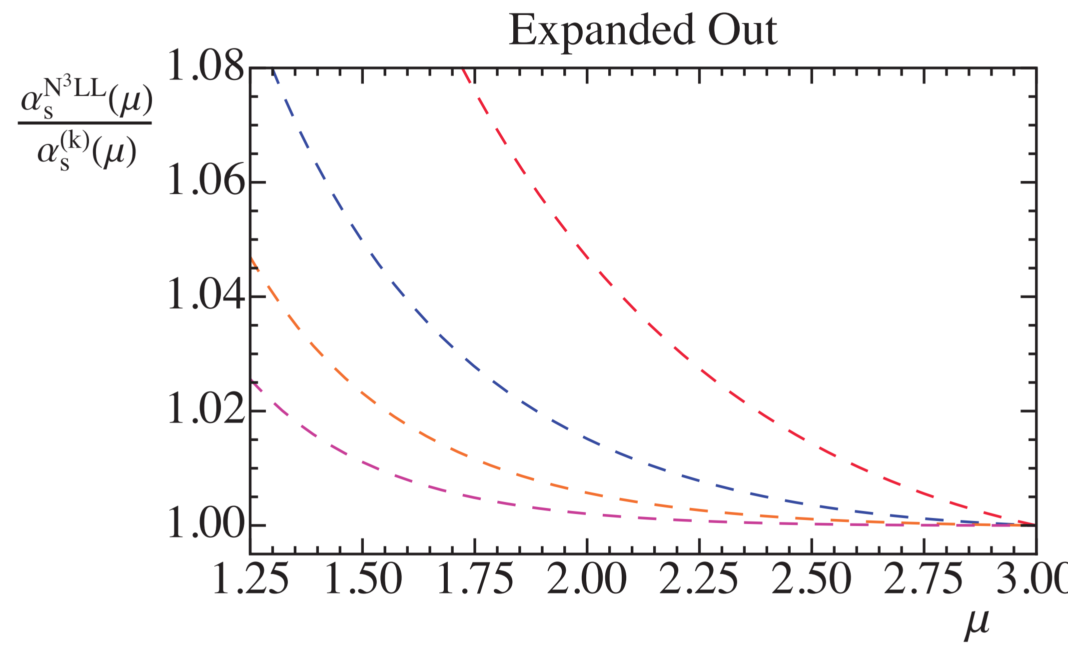
Stability of perturbation theory

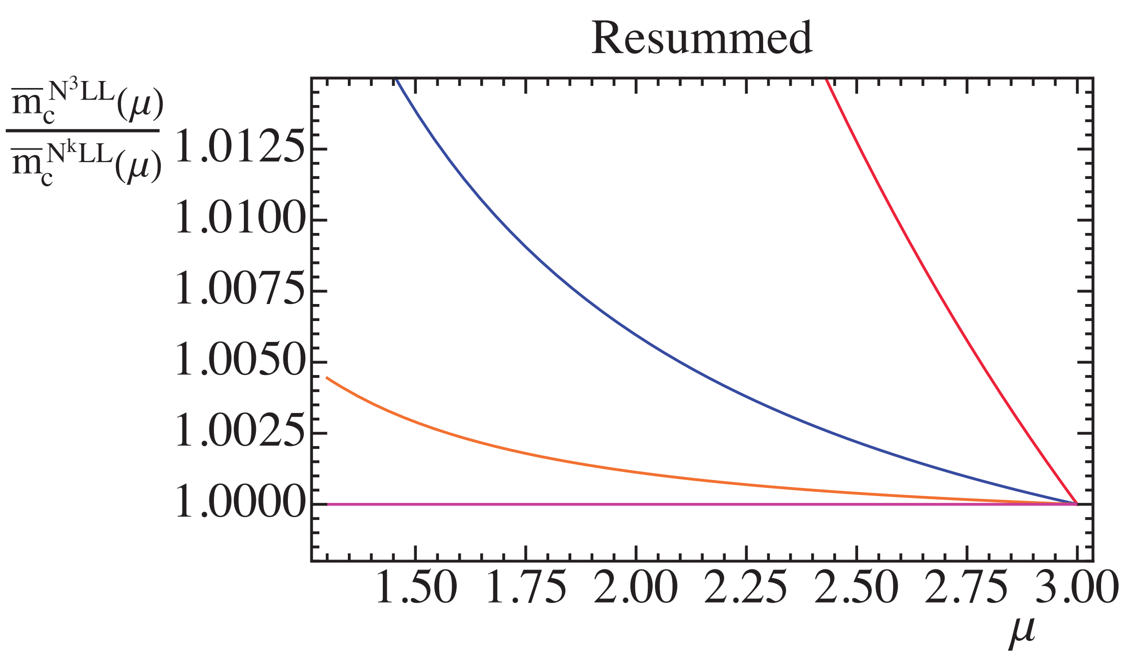
Check of pQCD @ $\mu = \overline{m}_c(\overline{m}_c)$

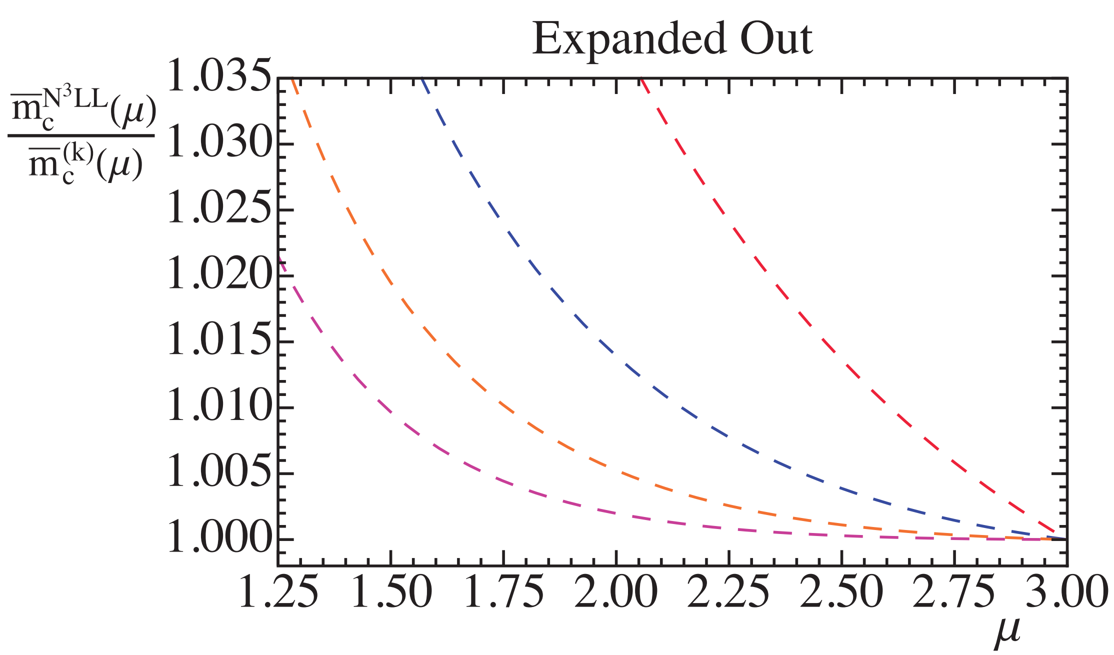
Search instabilities of pQCD

[Dehnadi, Hoang, VM, Zebarjad]







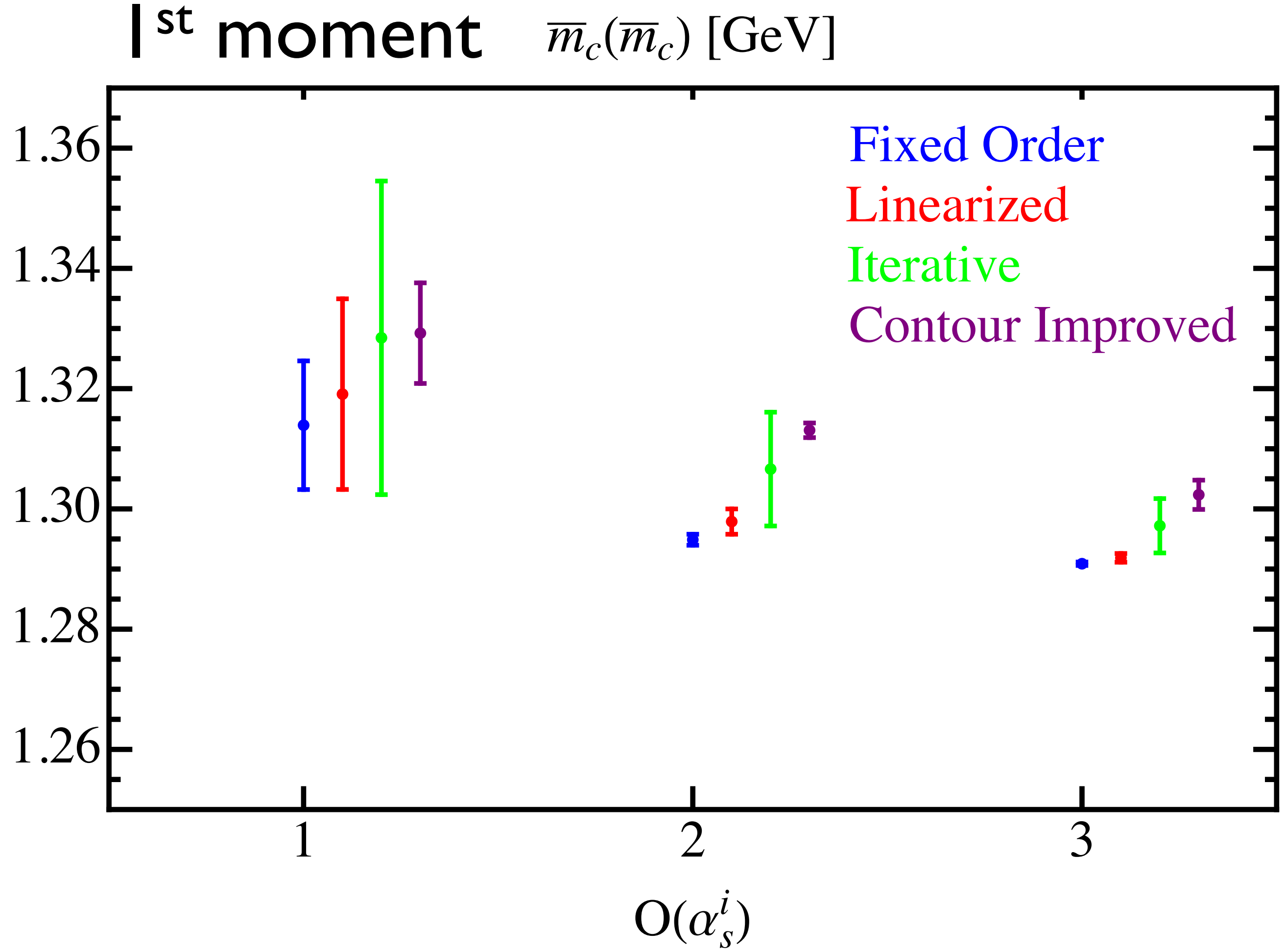


- Good convergence of pQCD observed at the charm mass scale
- No instability visible: $\mu = \overline{m}_c(\overline{m}_c)$ viable choice
- But perturbative expansion has O(10 MeV) deviations from resummed results at $\mathcal{O}(\alpha_s^3)$

Results for charm

Exploration of scale variation

1 st moment



correlated variation

$$2\,\mathrm{GeV} \leq \mu_\alpha = \mu_m \leq 4\,\mathrm{GeV}$$

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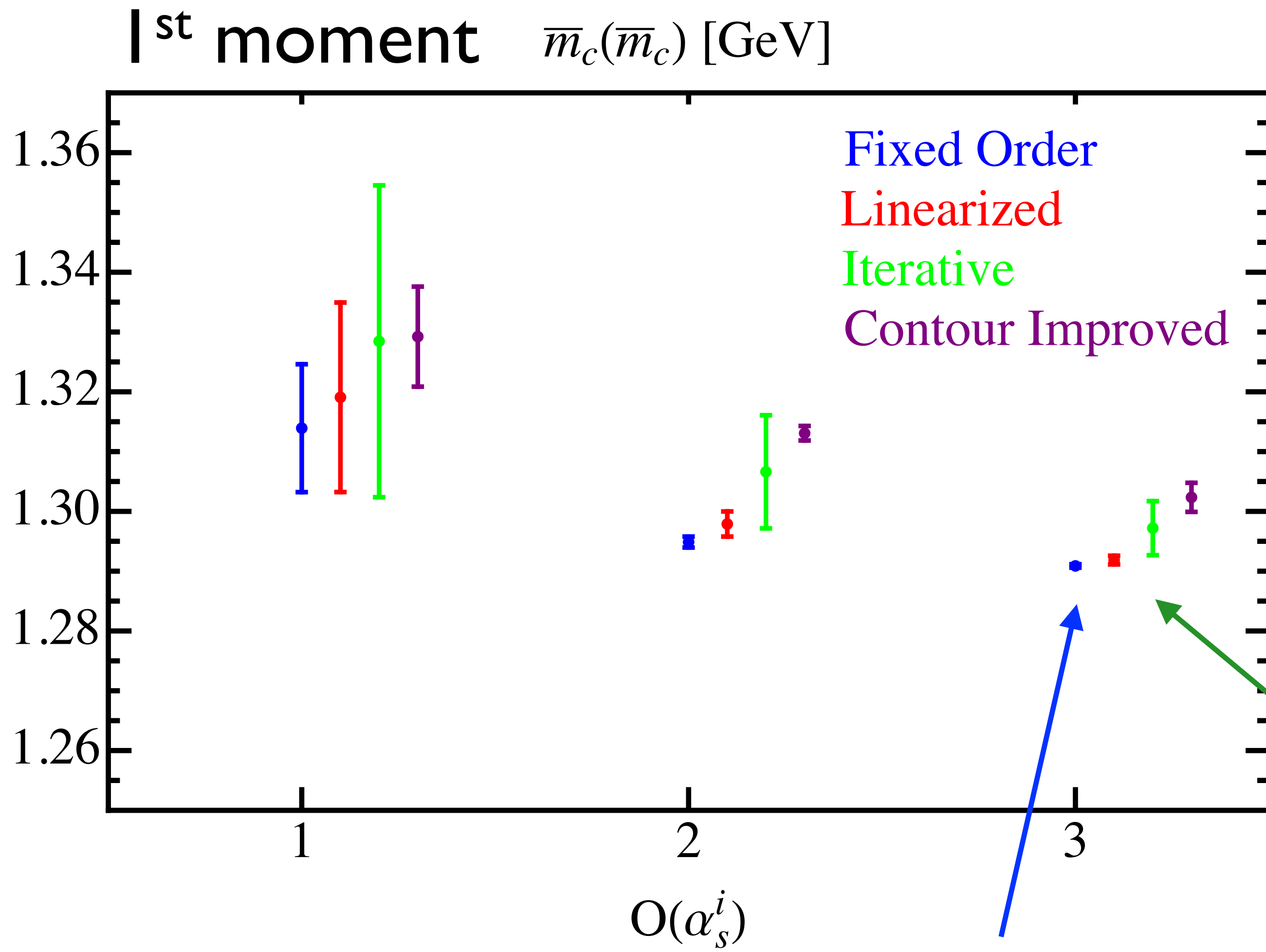

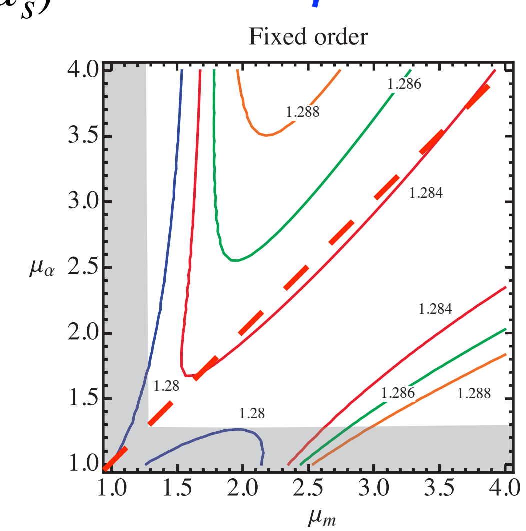

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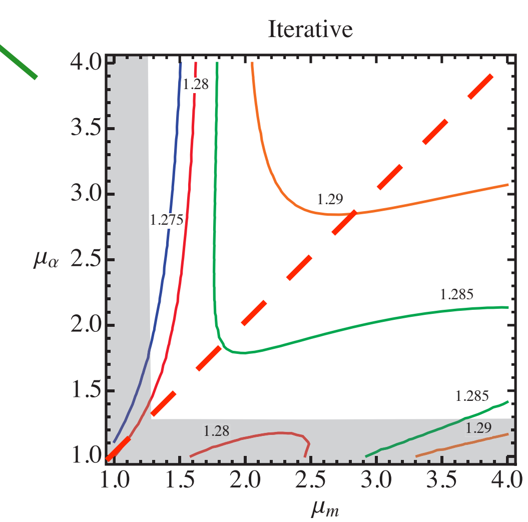

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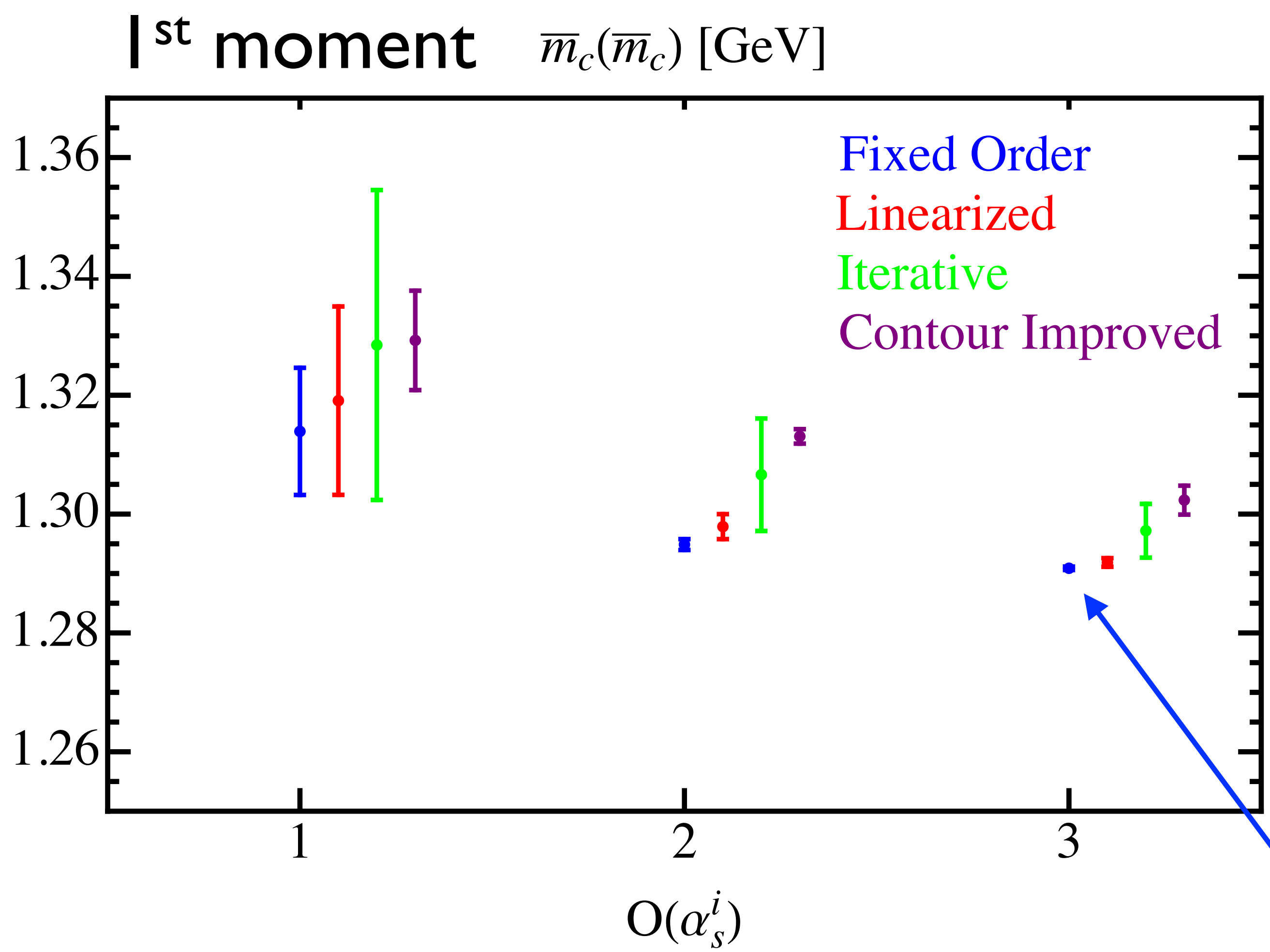


correlated variation

$$2\,\text{GeV} \leq \mu_\alpha = \mu_m \leq 4\,\text{GeV}$$

[Chetyrkin et al (2009)]

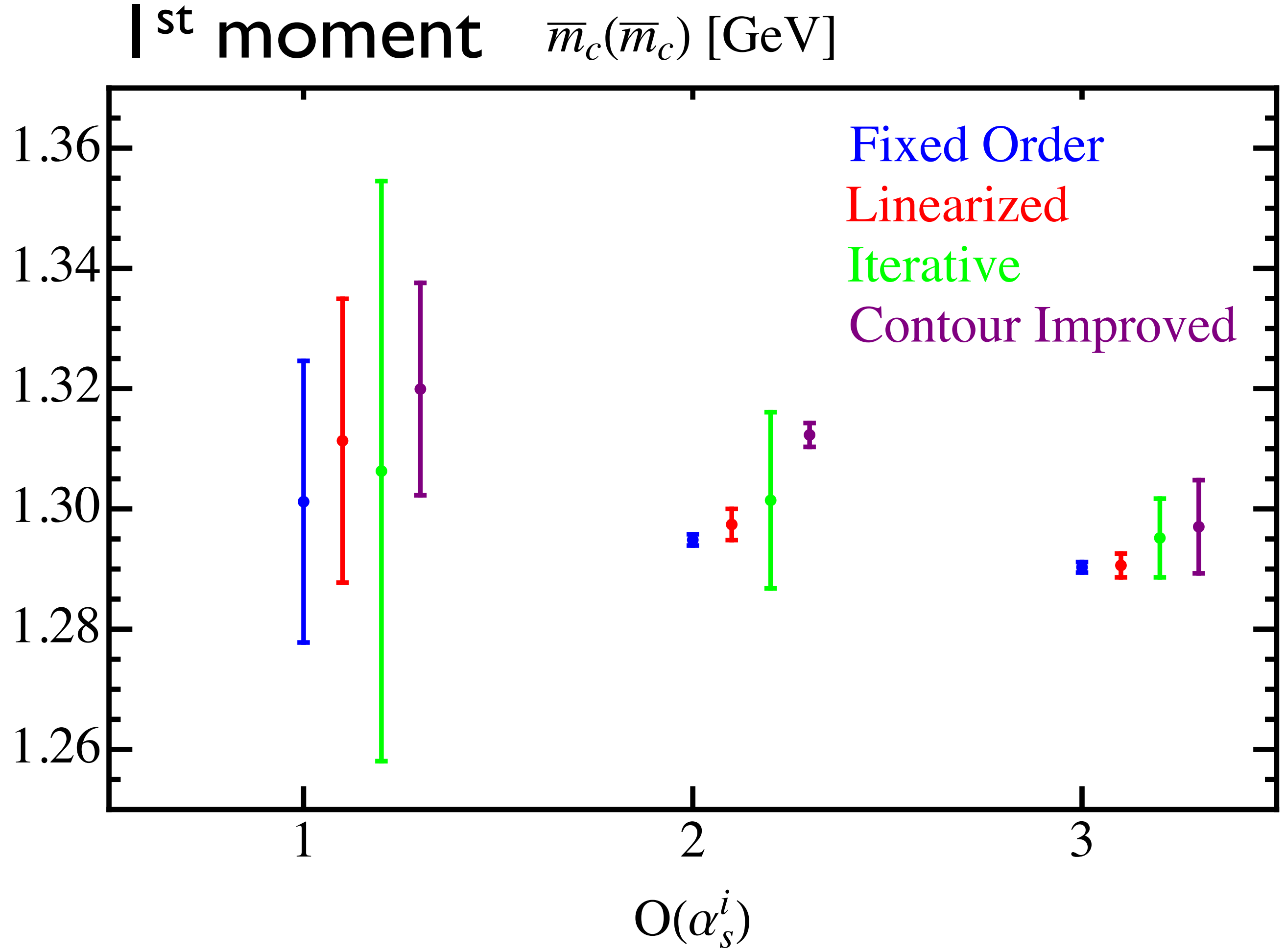
$$\overline{m}_c(\overline{m}_c) = 1279 \pm (2)_{\text{pert}} \pm (9)_{\text{exp}} \pm (9)_{\alpha_s} \pm (1)_{\langle \text{GG} \rangle}\,\text{GeV}$$

Results of different expansion methods are not consistent to each other

For one particular expansion scale variation can lead to inappropriate error estimate

Exploration of scale variation

1 st moment



correlated variation

$$\overline{m}_c(\overline{m}_c) \leq \mu_\alpha = \mu_m \leq 4\,\text{GeV}$$

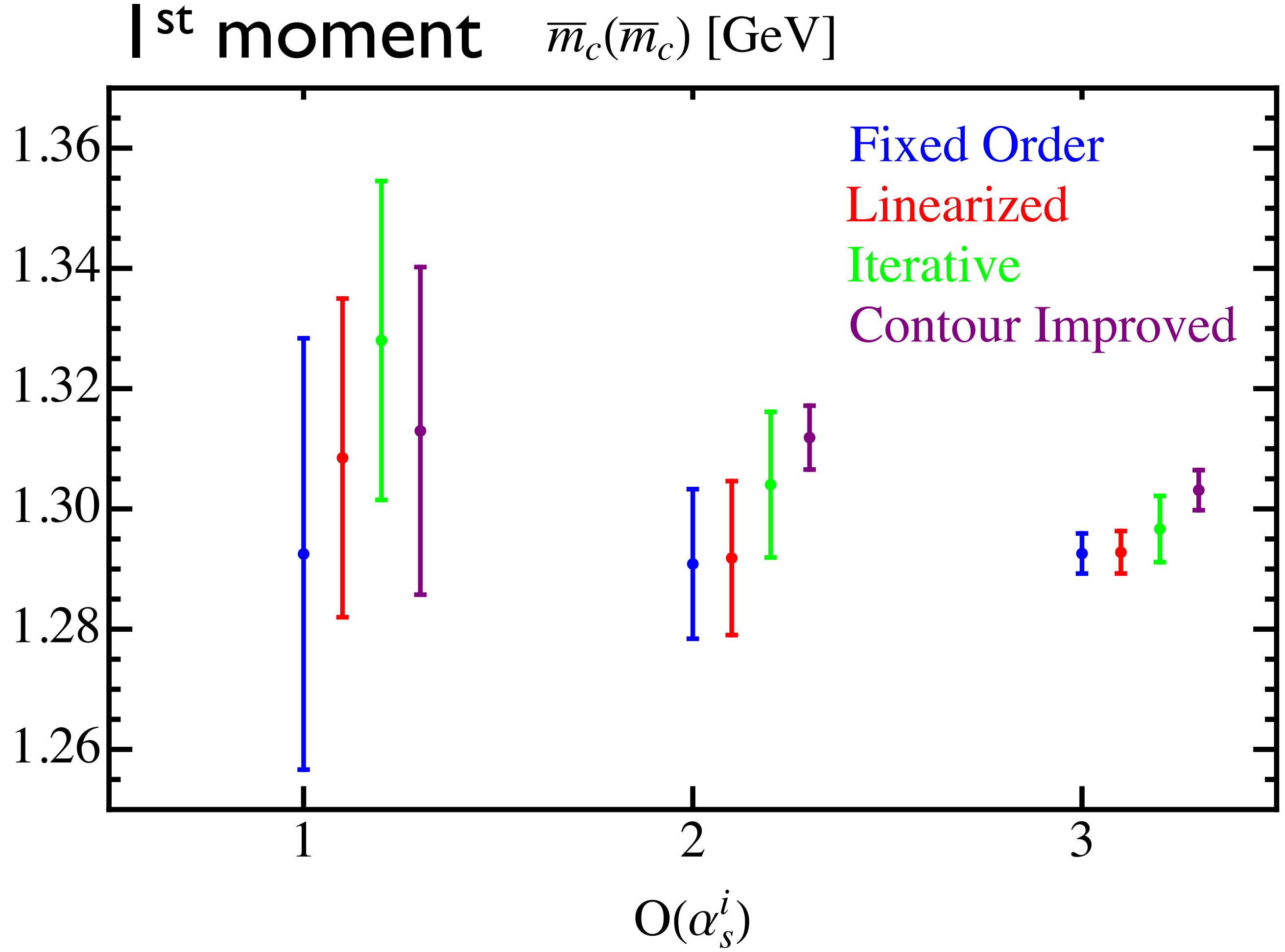
Charm mass scale should not be excluded in the perturbative extraction of the charm mass

No instabilities for renormalization scales down to $\overline{m}_c(\overline{m}_c)$

Somewhat better consistency among different expansion methods

Exploration of scale variation

1 st moment



independent variation

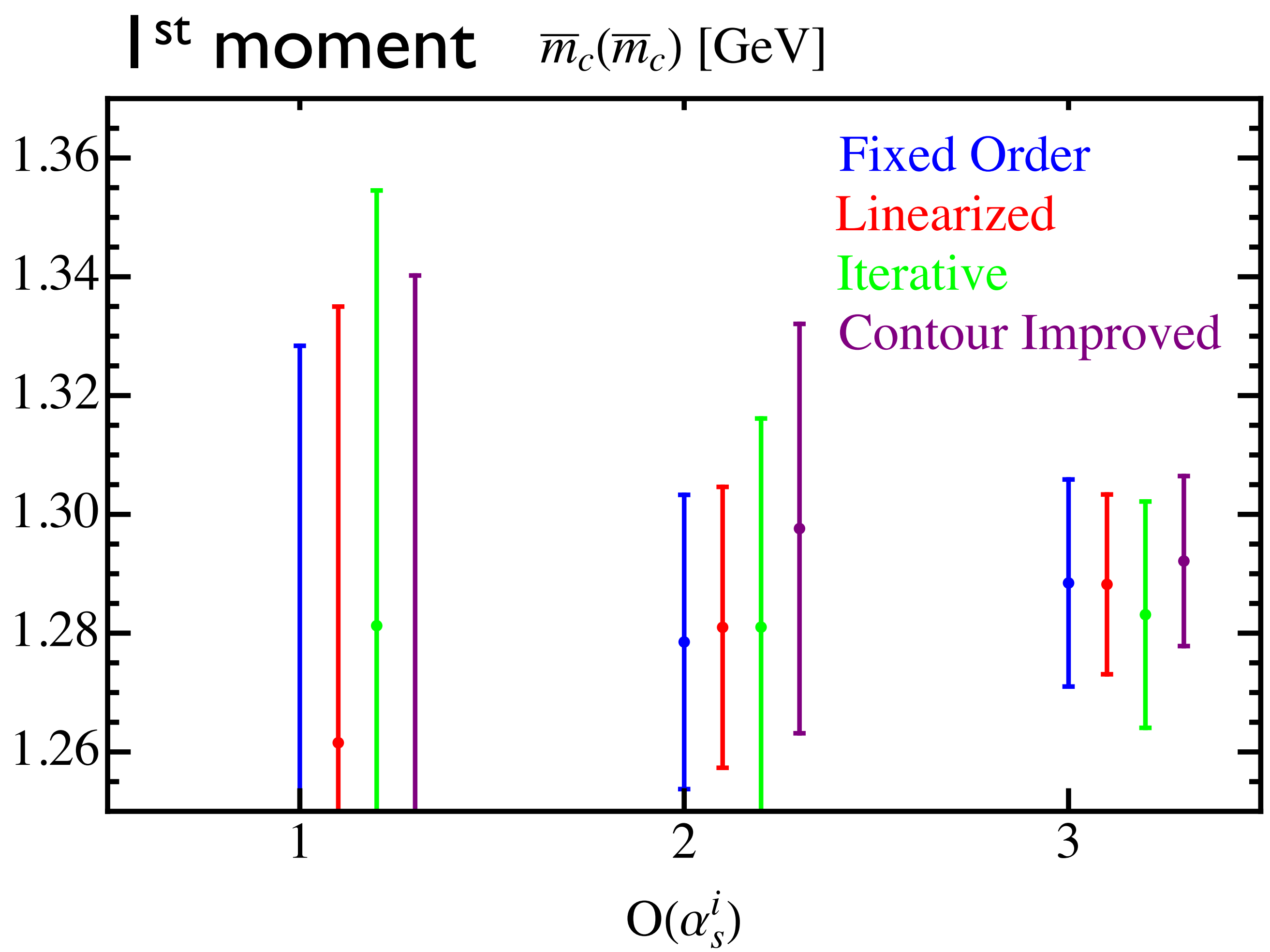
$$2\,\text{GeV} \leq \mu_\alpha, \mu_m \leq 4\,\text{GeV}$$

Correlated variation should be avoided because it may accidentally hit a contour line

Good overlap method by method, but overall not good

Somewhat better consistency among different expansion methods

Exploration of scale variation



independent variation

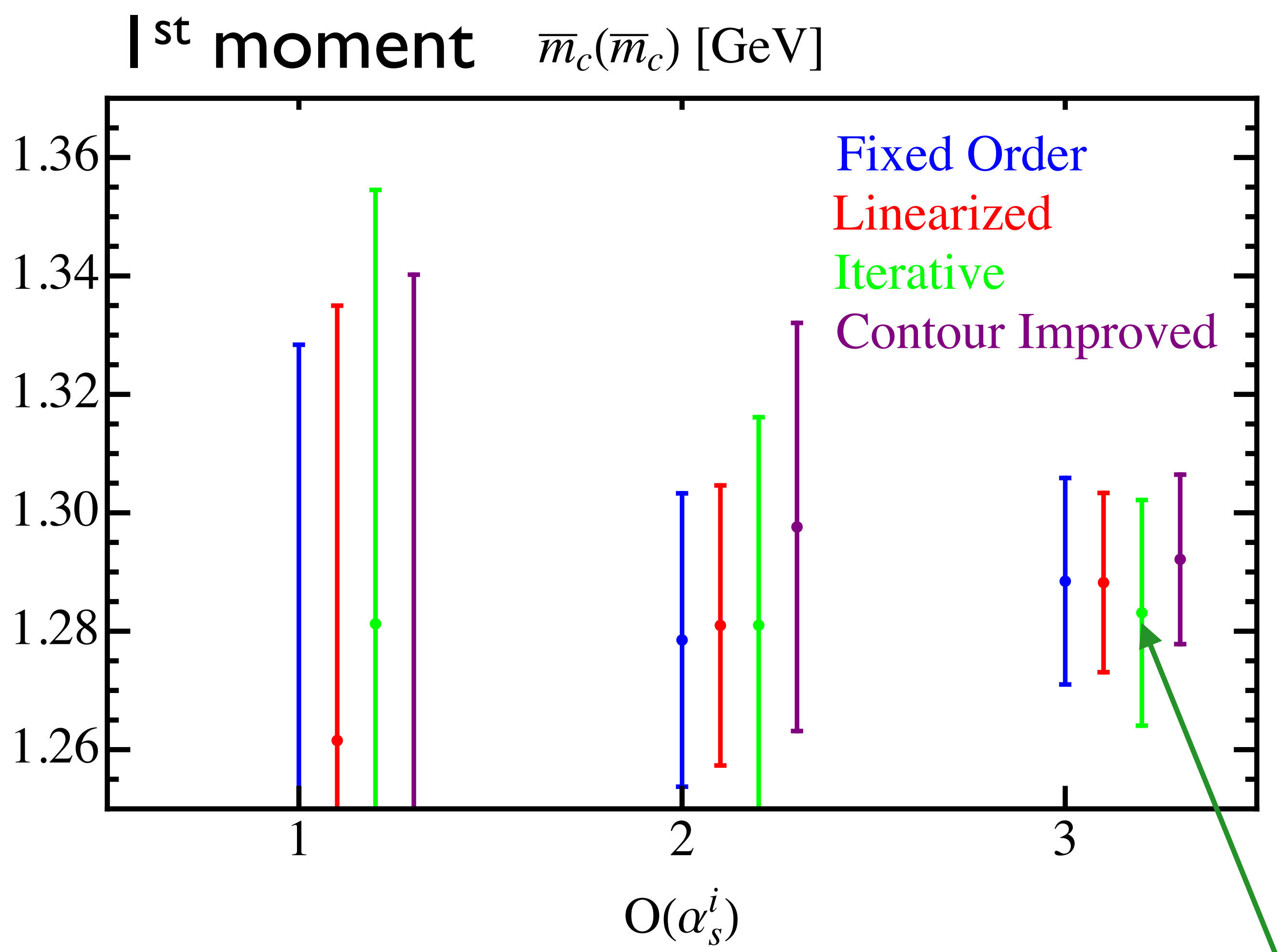
$$\overline{m}_c(\overline{m}_c) \leq \mu_\alpha = \mu_m \leq 4\,\mathrm{GeV}$$

Charm mass scale should not be excluded in the perturbative extraction of the charm mass

All expansions give similar errors $\delta m_{\rm pert} \sim 20\,\mathrm{GeV}$

Good convergence, good agreement among different methods

Exploration of scale variation



our approach

$$\overline{m}_c(\overline{m}_c) \leq \mu_\alpha = \mu_m \leq 4\,\text{GeV}$$

Charm mass scale should not be excluded in the perturbative extraction of the charm mass

Our default is iterative method

Final result [Dehnadi, Hoang, VM, Zebarjad]

using $\alpha_s(m_Z) = 0.1184 \pm 0.0021$

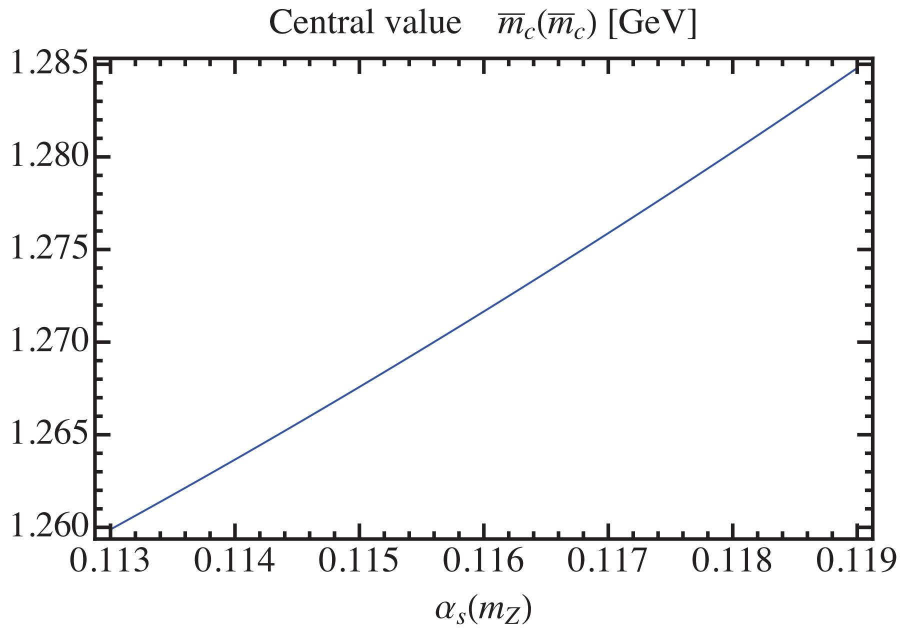
$$\overline{m}_c(\overline{m}_c) = 1.282 \pm (0.006)_{\text{stat}} \pm (0.009)_{\text{syst}} \pm (0.019)_{\text{pert}} \pm (0.010)_{\alpha_s} \pm (0.002)_{\langle GG\rangle}\,\text{GeV}$$

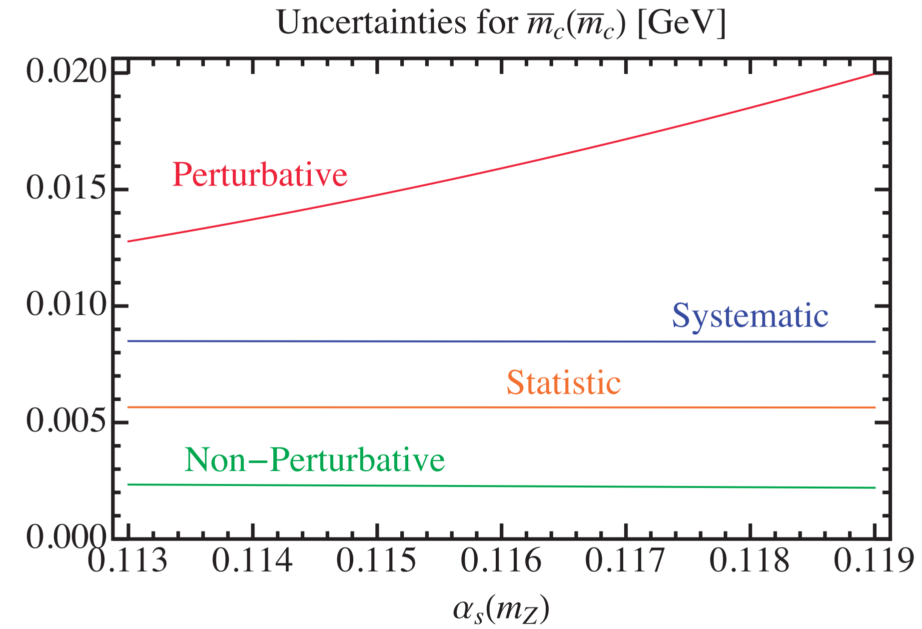
conclusions: independent variation of scales down to $\overline{m}_c(\overline{m}_c)$ so that using different expansions does not matter

Final results

[Dehnadi, Hoang, VM, Zebarjad]

Dependence of central value and errors on the value of $\alpha_s(m_Z)$





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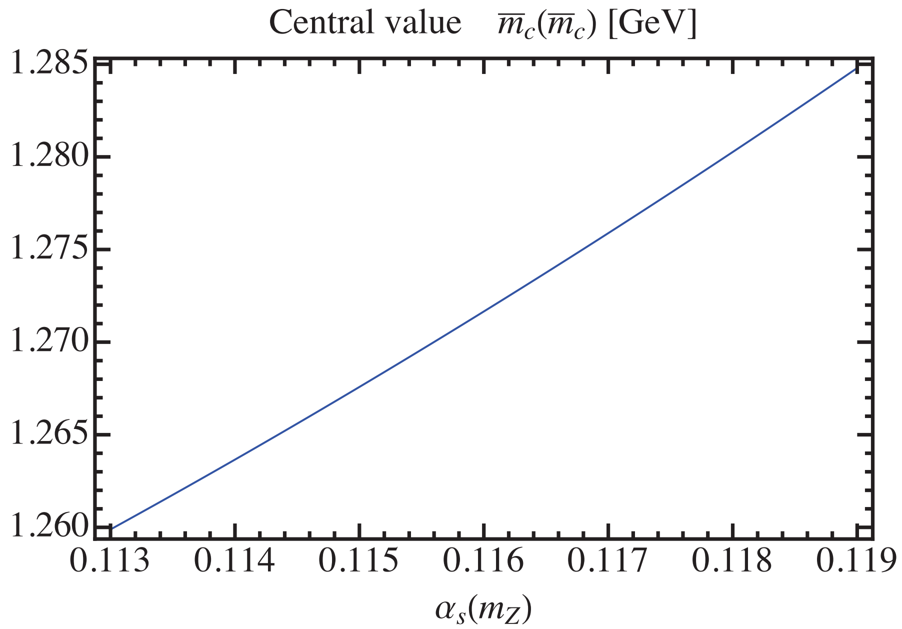
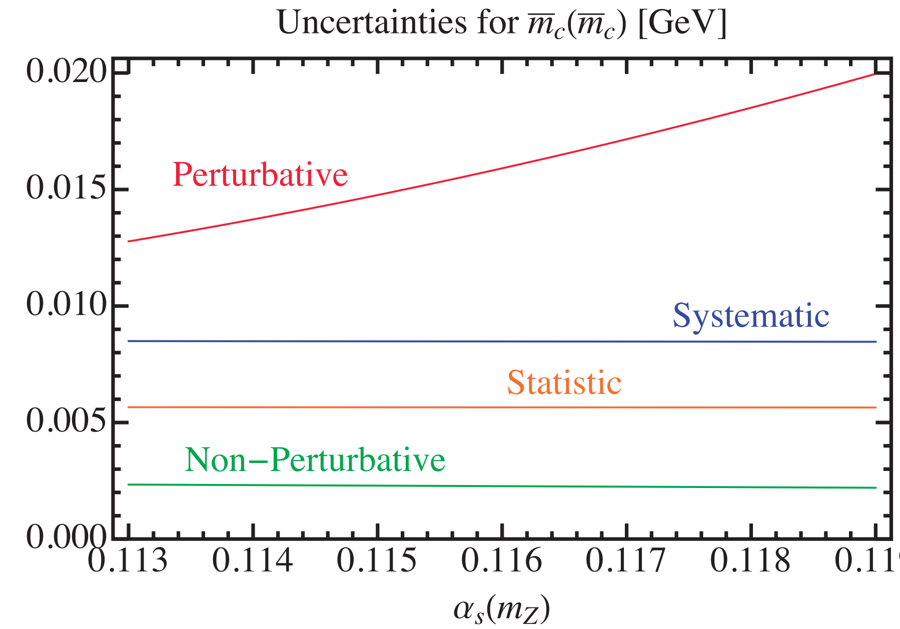




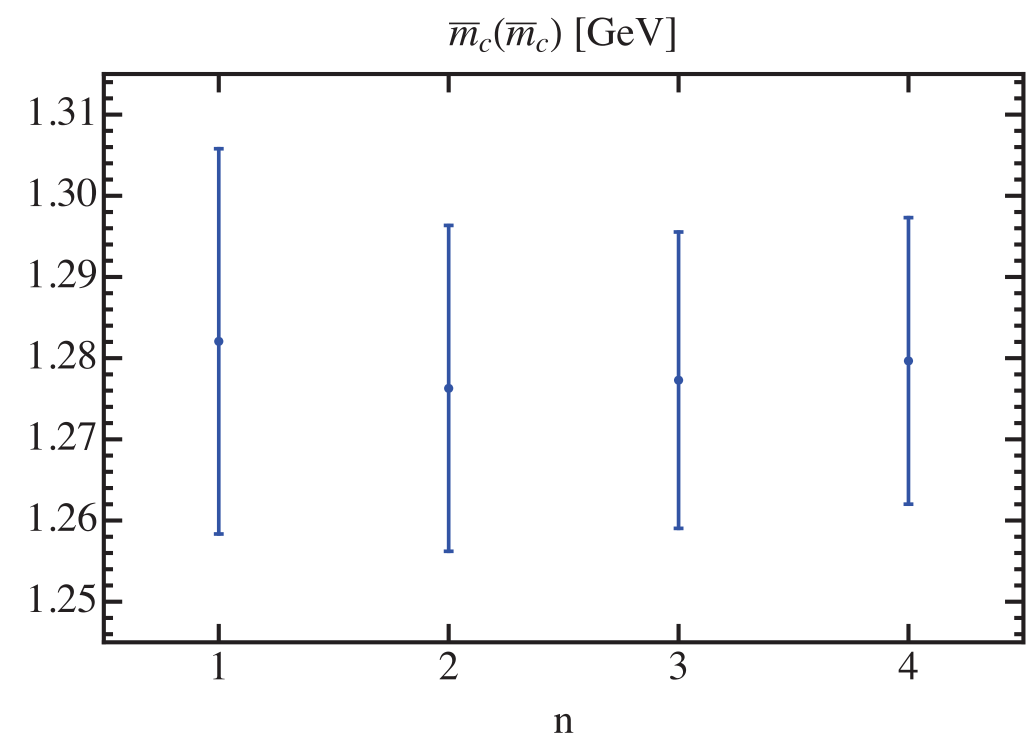
Final result for 1[st] moment

$$\overline{m}_c(\overline{m}_c) = 1.282 \pm 0.024 \text{ GeV}\,,$$

$$\overline{m}_c(3\text{ GeV}) = 0.994 \pm 0.026 \text{ GeV}\,,$$

using $\alpha_s(m_Z) = 0.1184 \pm 0.0021$

Moment dependence



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Charm mass from Pseudoscalar Correlator

Correlator and moments

$$q^2\,\Pi^P(q^2) = i\int \mathrm{d}^4x\, e^{ix\cdot q}\langle 0\big|\mathrm{T}j_5(x)j_5(0)\big|0\rangle$$

Related to longitudinal part of axial propagator

$$J_5(x) = i\,\bar{q}(x)\gamma_5 q(x)$$

Carries QCD anomalous dimension: same running as quark masa

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$$\Pi^P(q^2) = \sum_{k=0}^{\infty} M_k^P\, q^{2k} = \frac{3}{16\pi^2}\sum_{k=0}^{\infty}\frac{\bar{C}_k^P}{[4\overline{m}(\mu)^2]^k}\, q^{2k}$$

computed in pQCD at $\mathcal{O}(\alpha_s^3)$ *[Meier et al. (2008)]*

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computed in pQCD at $\mathcal{O}(\alpha_s^3)$ *[Meier et al. (2008)]*

$$\frac{\bar{C}_{n+1}^P}{[4\overline{m}_c(\mu)]^{2n}} = \bar{C}_{n+1}^{(0)}\left[\frac{R_{2n+1}}{m_\eta^{\mathrm{exp}}}\right]^{2n}$$

computed in the lattice *[Allison et al. (2008)]*

physical eta mass

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computed in the lattice *[Allison et al. (2008)]*

physical eta mass

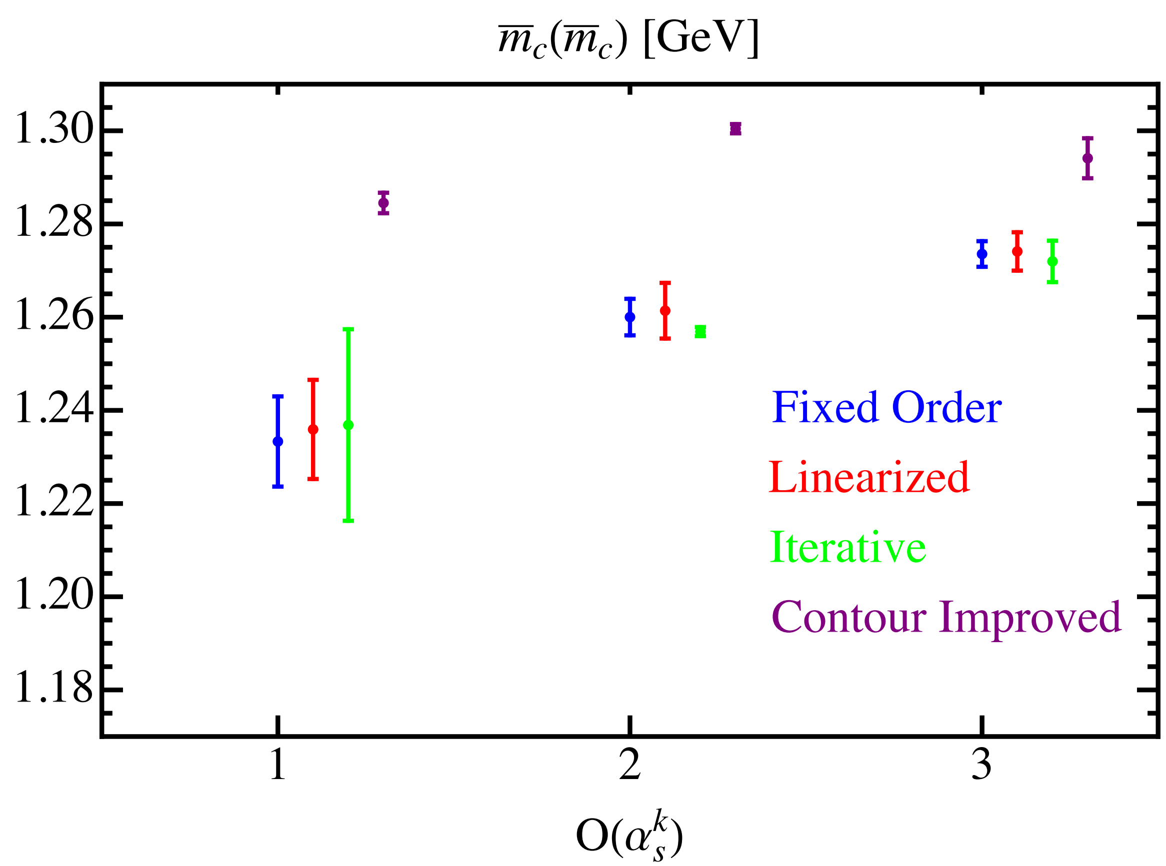
One can define, analogously to the vector correlator, the same expansions:

- Fixed order expansion
- Linearized expansion
- Iterative Linearized expansion
- Contour improved expansion

Results and comparison to another analysis

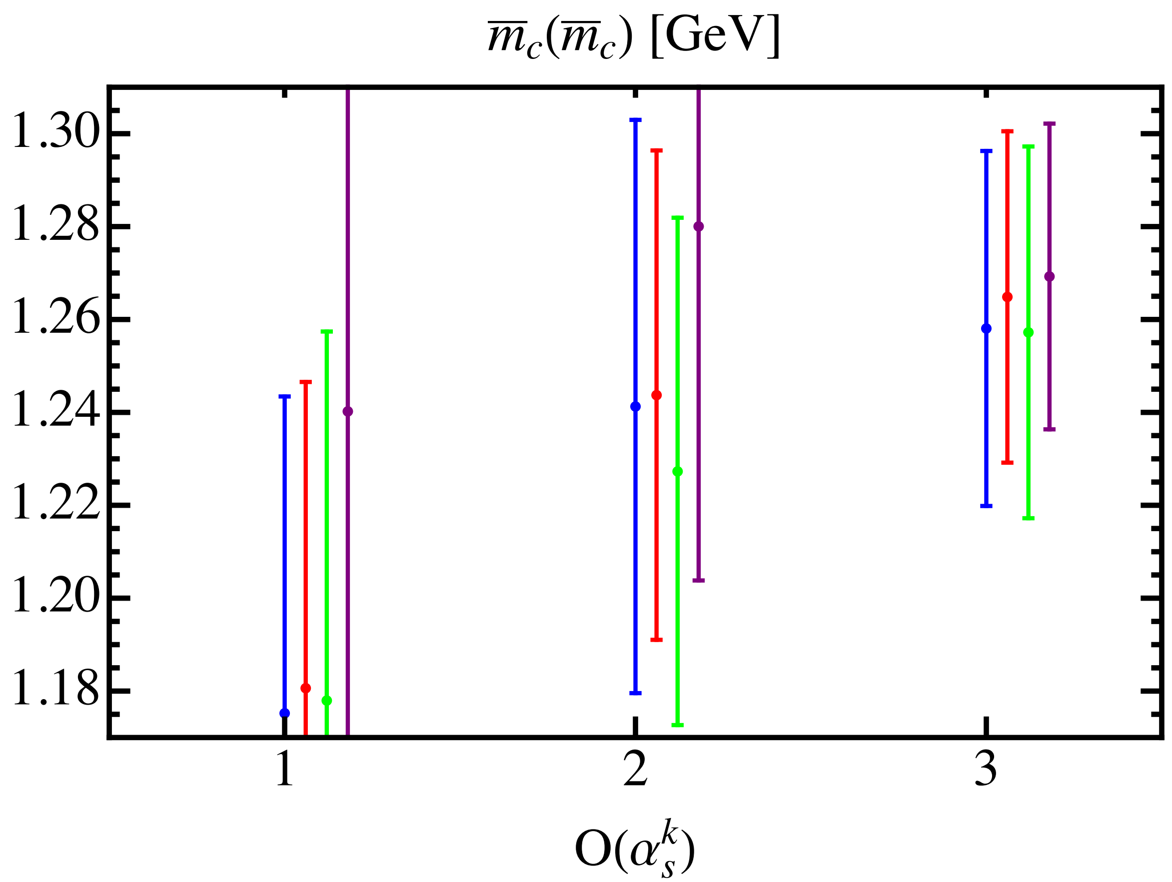
correlated

$$2\,\mathrm{GeV} \le \mu_\alpha = \mu_m \le 4\,\mathrm{GeV}$$



Independent

$$\overline{m}_c(\overline{m}_c) \le \mu_\alpha, \mu_m \le 4\,\mathrm{GeV}$$



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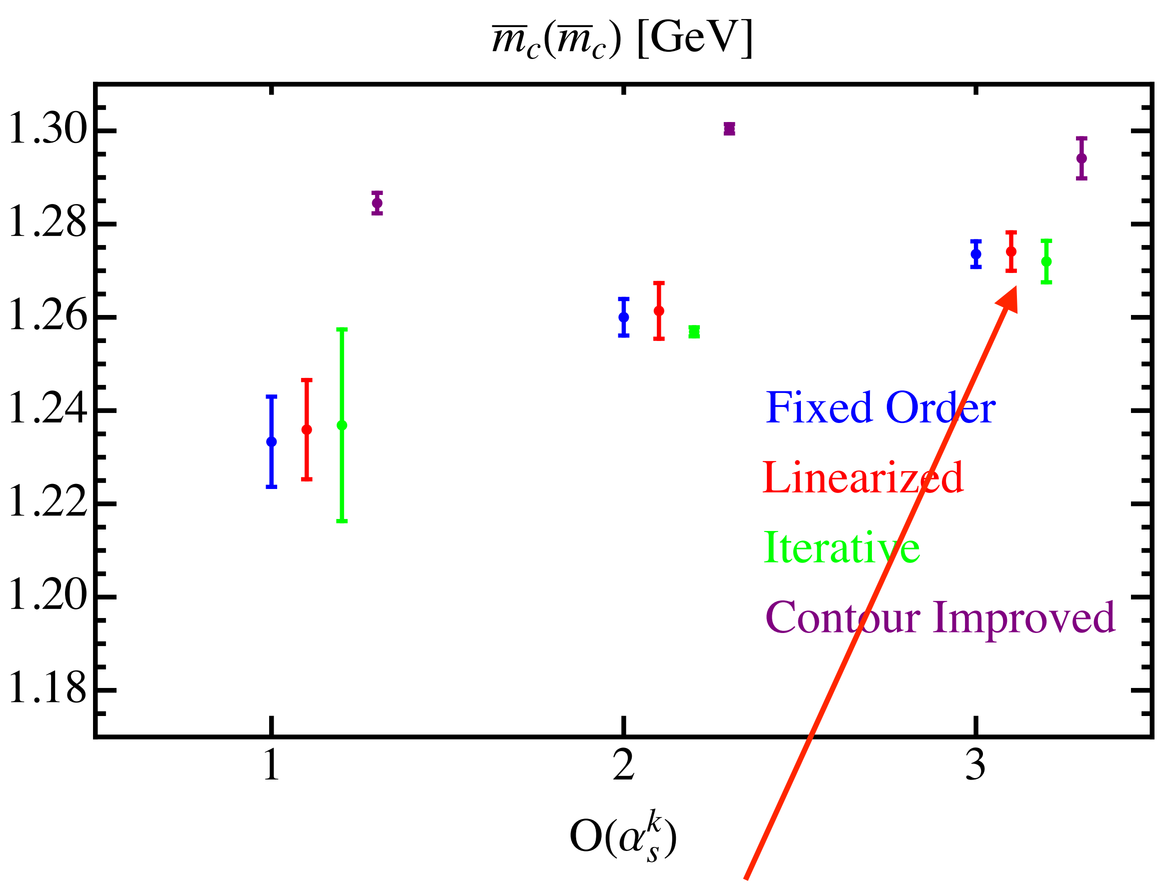

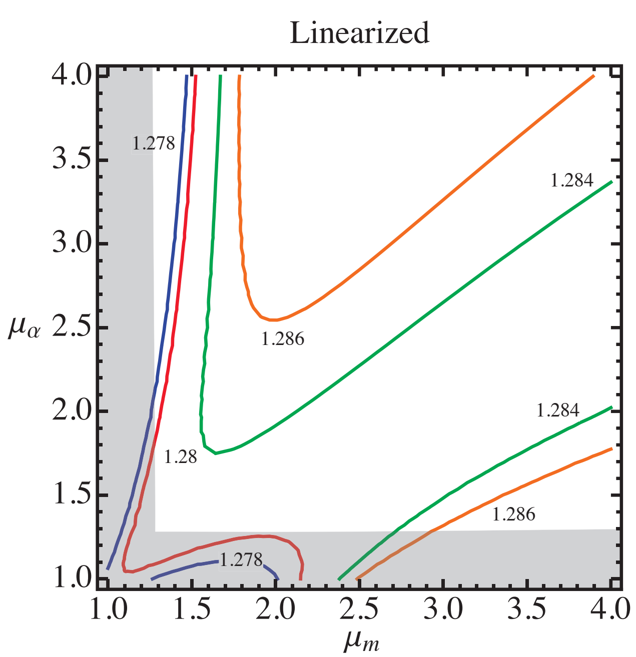

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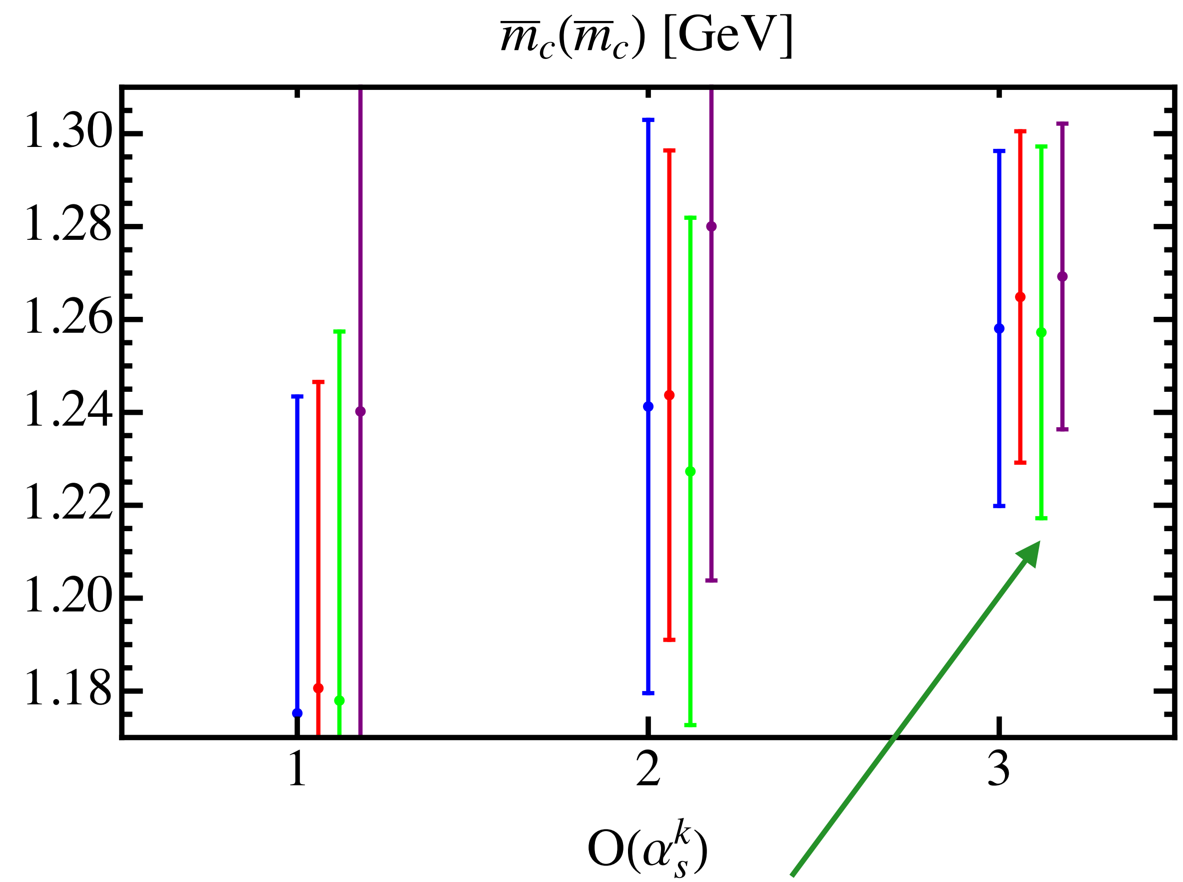
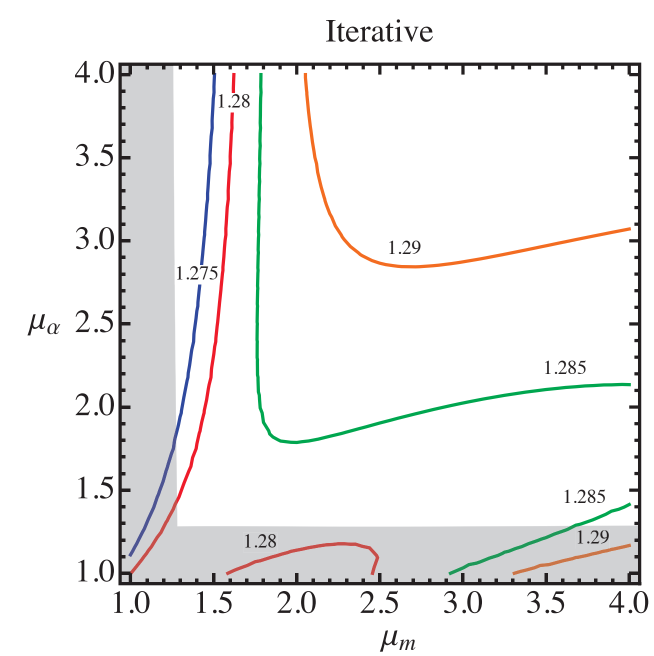
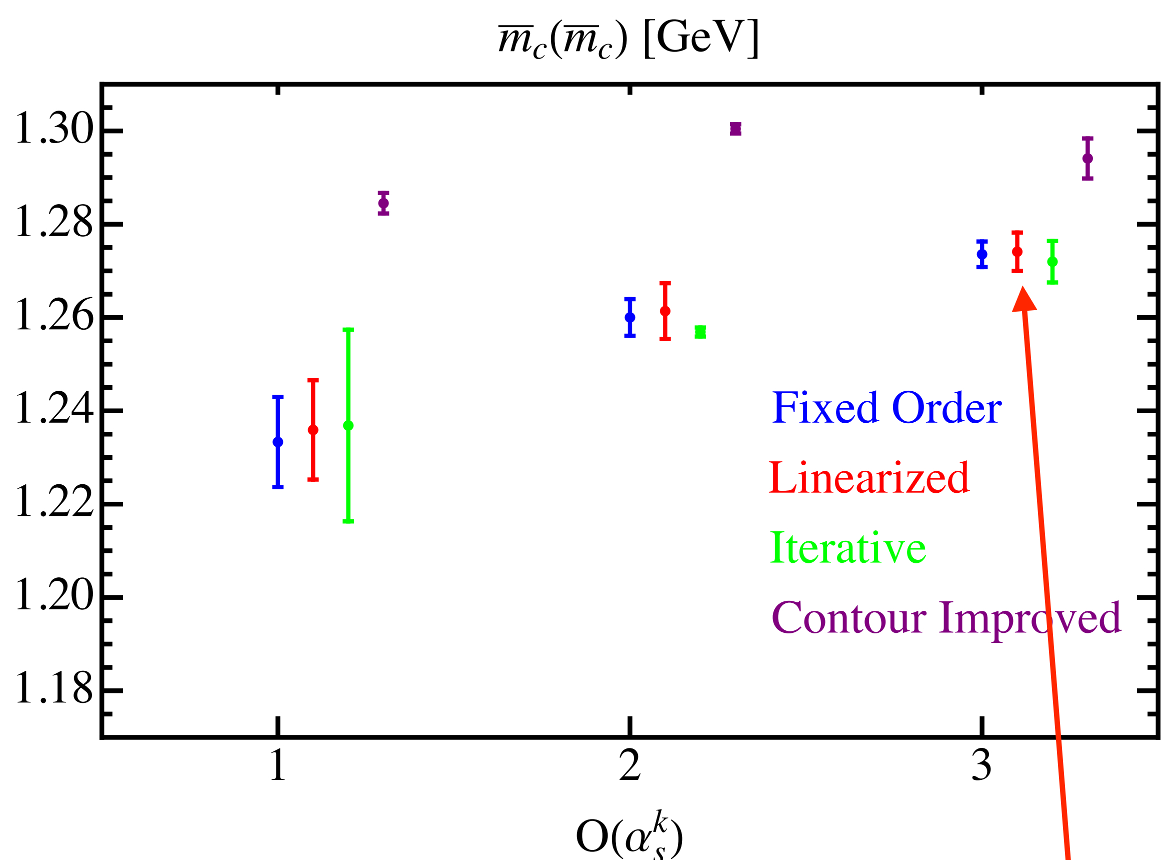




Results and comparison to another analysis

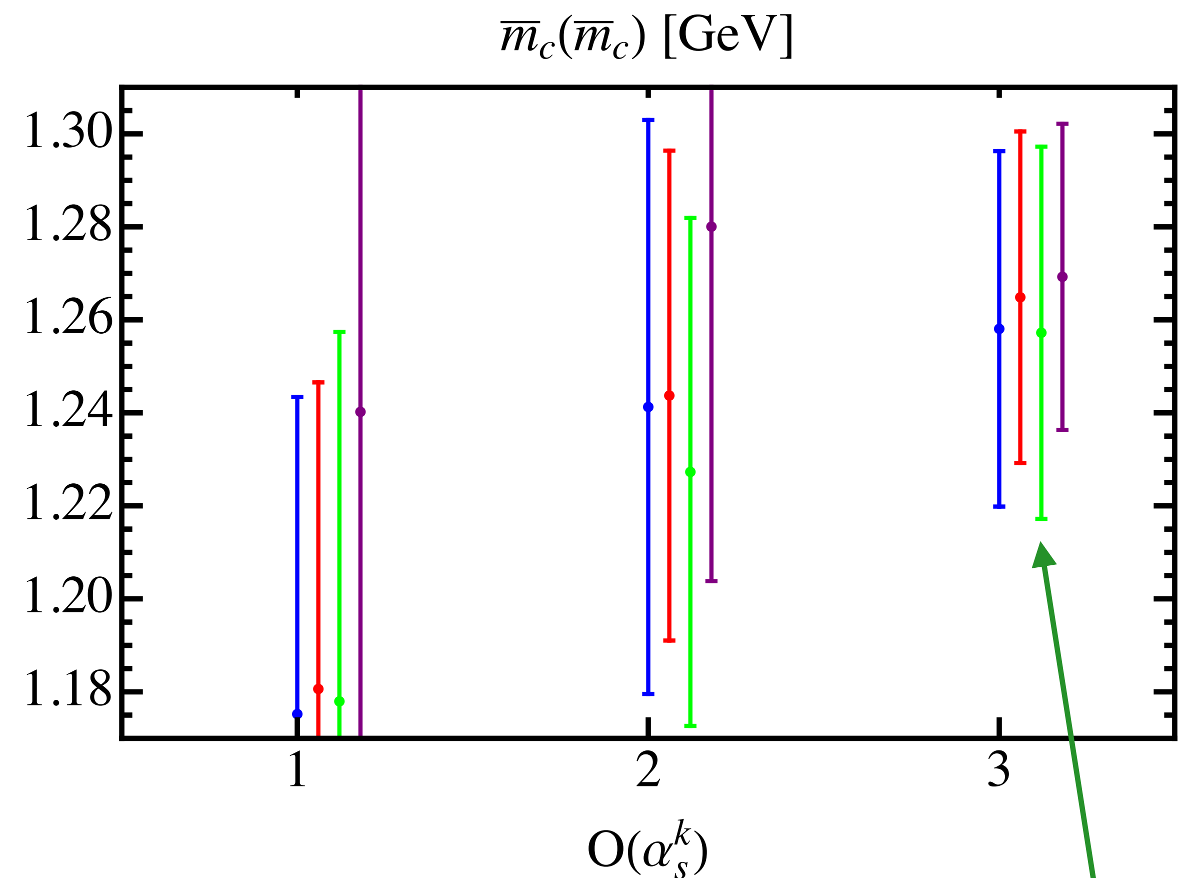
correlated

$$2\,\text{GeV} \leq \mu_\alpha = \mu_m \leq 4\,\text{GeV}$$



Independent

$$\overline{m}_c(\overline{m}_c) \leq \mu_\alpha, \mu_m \leq 4\,\text{GeV}$$



[(HPQCD collaboration) Allison et al. (2008)] [our quote]

$$\overline{m}_c(\overline{m}_c) = 1.268 \pm (0.008)_{\rm latt} \pm (0.004)_{\rm pert} \pm (0.004)_{\alpha_s} \pm (0.004)_{\langle GG\rangle}\ \text{GeV}$$

Our result (preliminary) *[Dehnadi, Hoang, VM]*

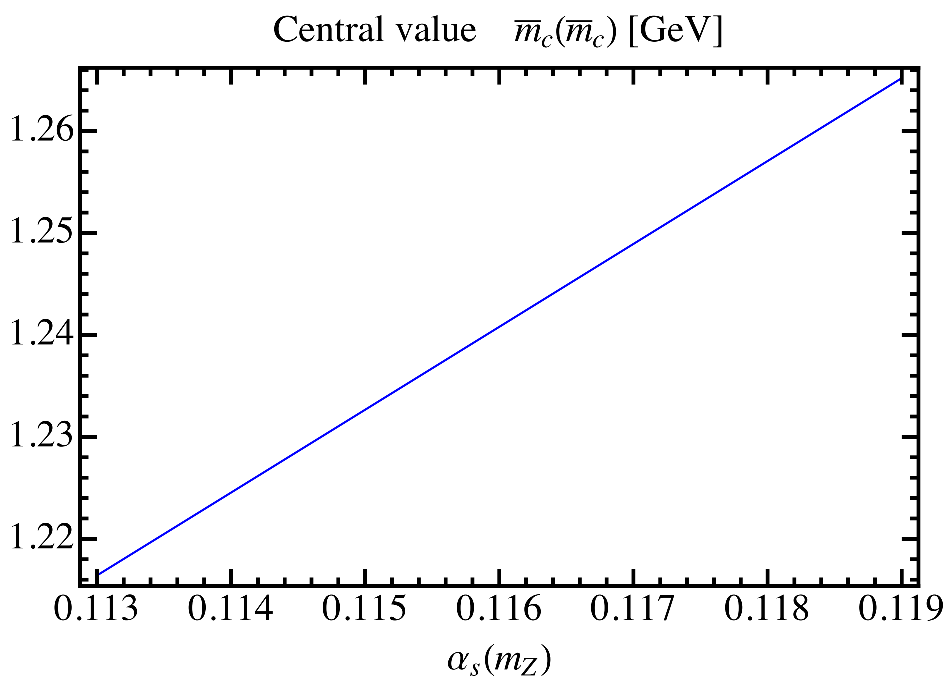
$$\overline{m}_c(\overline{m}_c) = 1.261 \pm (0.008)_{\rm latt} \pm (0.041)_{\rm pert} \pm (0.019)_{\alpha_s} \pm (0.001)_{\langle GG\rangle}\ \text{GeV}$$

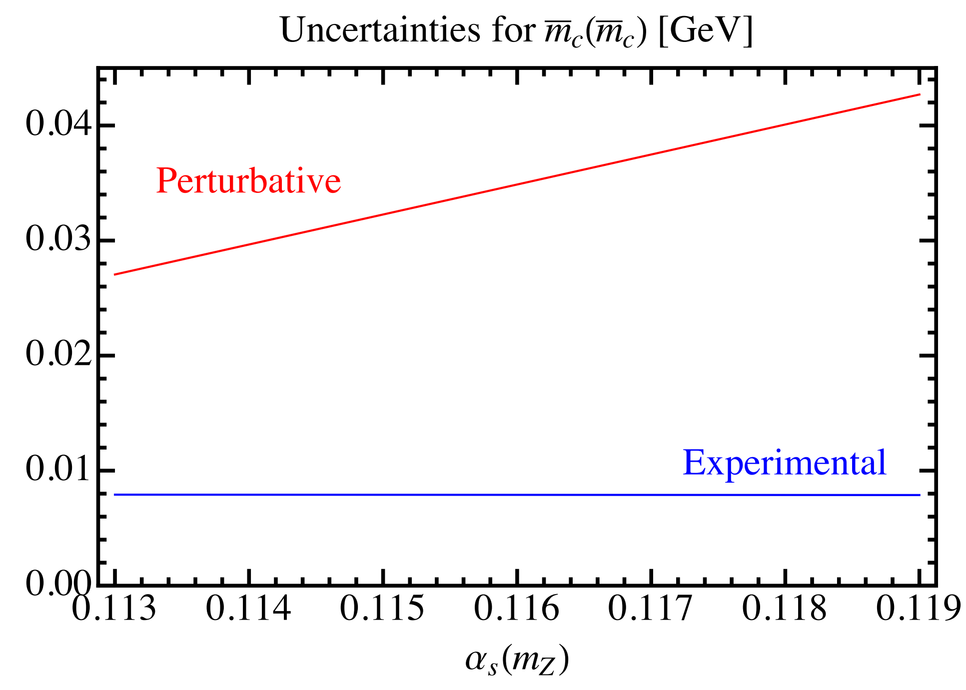
using $\alpha_s(m_Z) = 0.1184 \pm 0.0021$

Results

[Dehnadi, Hoang, VM]

Dependence of central value and errors on the value of $\alpha_s(m_Z)$





Results

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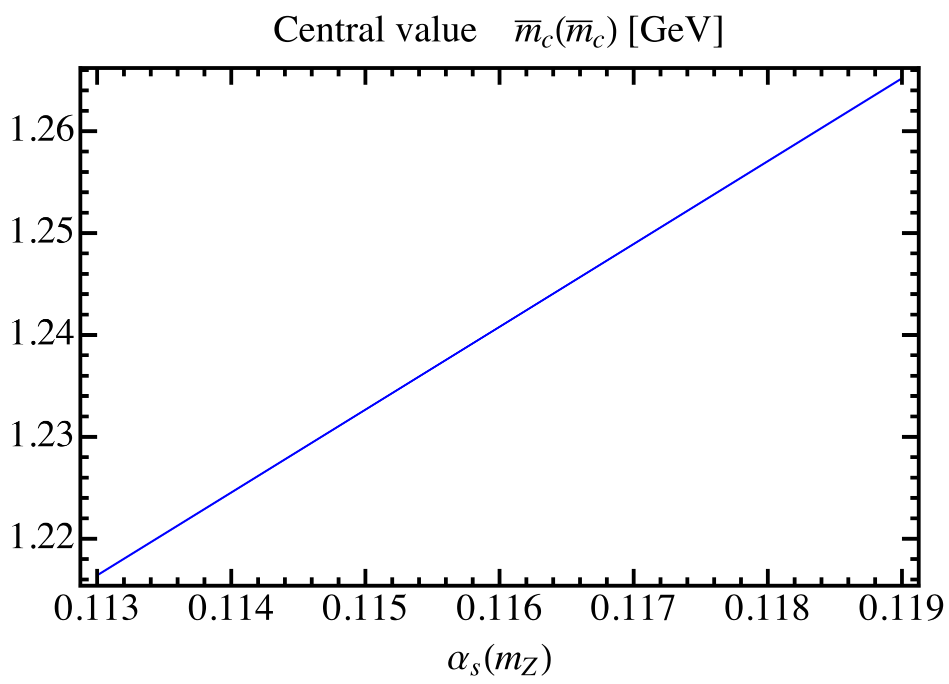
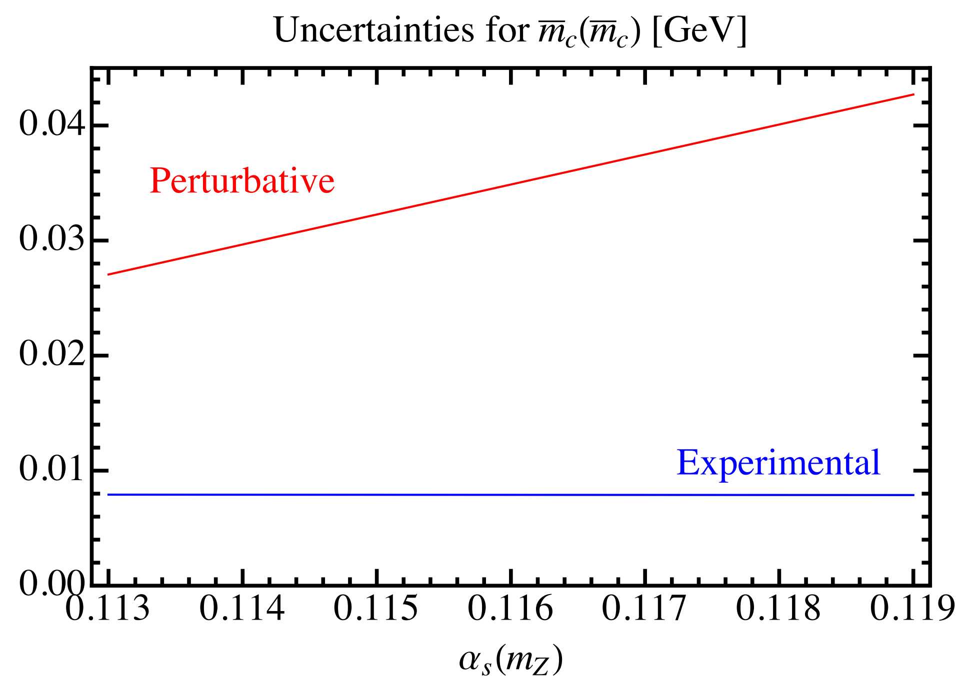


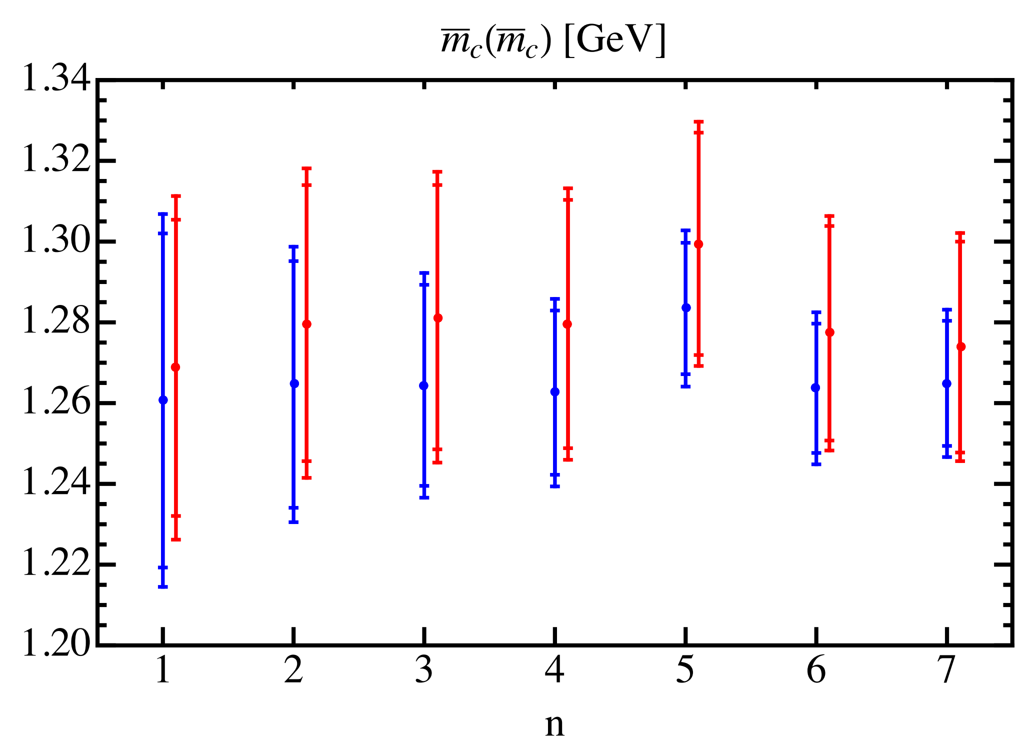


Final result for 1st moment

$$\overline{m}_c(\overline{m}_c) = 1.261 \pm 0.046 \text{ GeV}$$

using $\alpha_s(m_Z) = 0.1184 \pm 0.0021$

Order dependence



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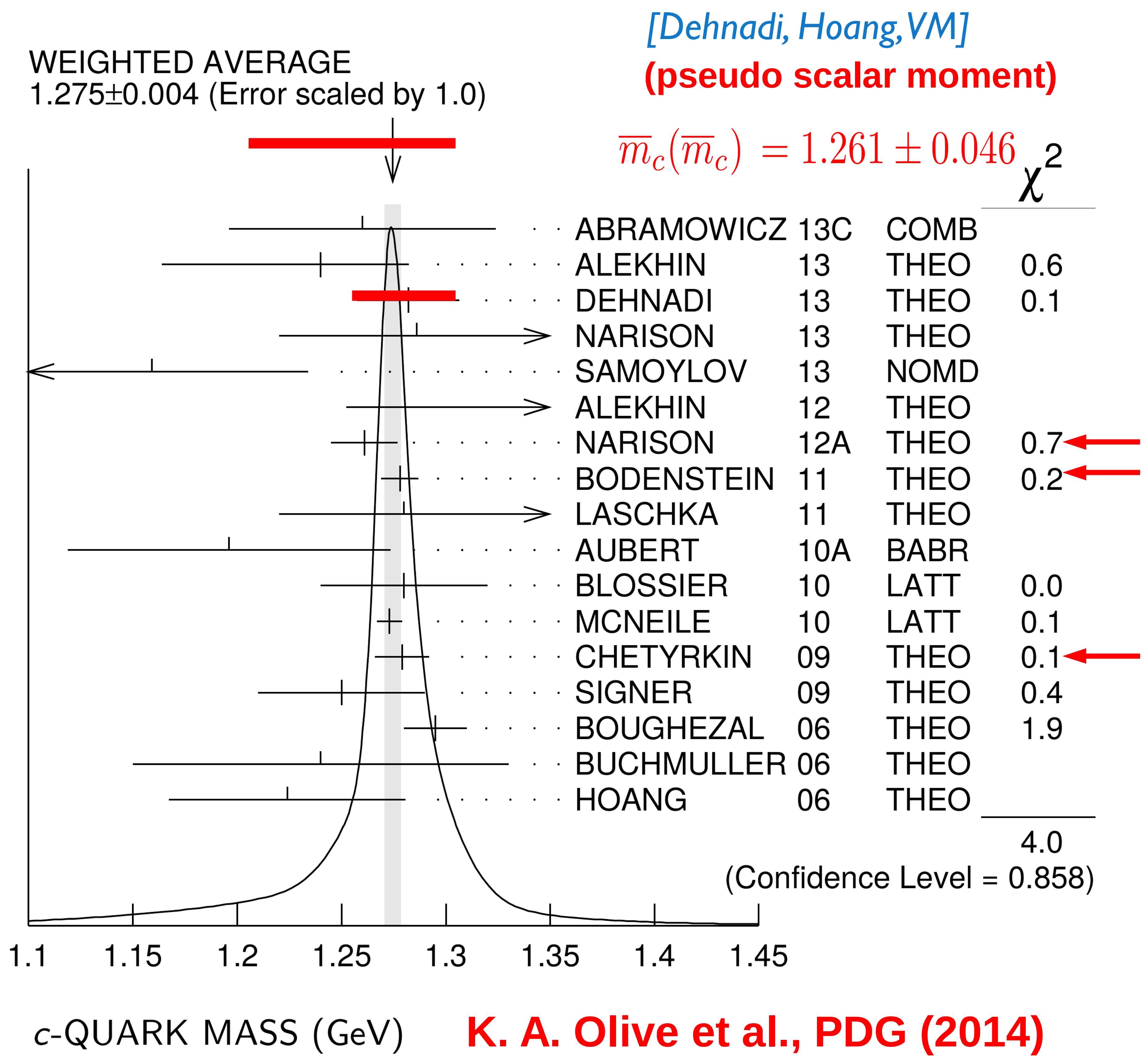
Comparison to other determinations



K. A. Olive et al., PDG (2014)

Bottom mass determination

Bottom mass determinations

[K. A. Olive et al., PDG (2014)]

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Bottom mass determinations

[K. A. Olive et al., PDG (2014)]

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Used in average

Not used in average

Bottom mass determinations

[K. A. Olive et al., PDG (2014)]

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NR Sum rules

Bottom mass determinations

[K. A. Olive et al., PDG (2014)]

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NR Sum rules

QCD Sum rules

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[K. A. Olive et al., PDG (2014)]

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NR Sum rules

QCD Sum rules

Jets

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[K. A. Olive et al., PDG (2014)]

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NR Sum rules

QCD Sum rules

Jets

Lattice QCD

Bottom mass determinations

[K. A. Olive et al., PDG (2014)]

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4.20 ± 0.04	4.67 ± 0.04	[17] BUCHMULLER	06	THEO
4.19 ± 0.06	4.66 ± 0.07	[18] PINEDA	06	THEO
4.17 ± 0.03	4.68 ± 0.03	[19] BAUER	04	THEO
4.22 ± 0.11	4.72 ± 0.12	[20,21] HOANG	04	THEO
4.19 ± 0.05	4.66 ± 0.05	[22] BORDES	03	THEO
4.20 ± 0.09	4.67 ± 0.10	[23] CORCELLA	03	THEO
4.24 ± 0.10	4.72 ± 0.11	[24] EIDEMULLER	03	THEO
4.207 ± 0.031	4.682 ± 0.035	[25] ERLER	03	THEO
4.33 ± 0.06 ± 0.10	4.82 ± 0.07 ± 0.11	[26] MAHMOOD	03	CLEO
4.190 ± 0.032	4.663 ± 0.036	[27] BRAMBILLA	02	THEO
4.346 ± 0.070	4.837 ± 0.078	[28] PENIN	02	THEO
4.164 ± 0.025	4.635 ± 0.028	[34] KUHN	07	THEO

NR Sum rules

QCD Sum rules

Jets

Lattice QCD

B decays

Bottom mass determinations

[K. A. Olive et al., PDG (2014)]

$\overline{MS}$ MASS (GeV)	1S MASS (GeV)	DOCUMENT ID		TECN
4.18 ±0.03	**OUR EVALUATION** of $\overline{MS}$ Mass. See the ideogram below.			
4.66 ±0.03	**OUR EVALUATION** of $1S$ Mass. See the ideogram below.			
4.166±0.043	4.637 ± 0.048	[1] LEE	13O	LATT
4.247±0.034	4.727 ± 0.039	[2] LUCHA	13	THEO
4.236±0.069	4.715 ± 0.077	[3] NARISON	13	THEO
4.213±0.059	4.689 ± 0.066	[4] NARISON	13A	THEO
4.171±0.009	4.642 ± 0.010	[5] BODENSTEIN	12	THEO
4.29 ±0.14	4.77 ± 0.16	[6] DIMOPOUL...	12	LATT
4.235±0.003±0.055	4.755 ± 0.003 ± 0.058	[7] HOANG	12	THEO
4.177±0.011	4.649 ± 0.012	[8] NARISON	12	THEO
$4.18\ ^{+0.05}_{-0.04}$	$4.65\ ^{+0.06}_{-0.04}$	[9] LASCHKA	11	THEO
4.186±0.044±0.015	4.659 ± 0.050 ± 0.017	[10] AUBERT	10A	BABR
4.164±0.023	4.635 ± 0.026	[11] MCNEILE	10	LATT
4.163±0.016	4.633 ± 0.018	[12] CHETYRKIN	09	THEO
5.26 ±1.2	5.85 ± 1.3	[13] ABDALLAH	08D	DLPH
4.243±0.049	4.723 ± 0.055	[14] SCHWANDA	08	BELL
4.19 ±0.40	4.66 ± 0.45	[15] ABDALLAH	06D	DLPH
4.205±0.058	4.68 ± 0.06	[16] BOUGHEZAL	06	THEO
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NR Sum rules

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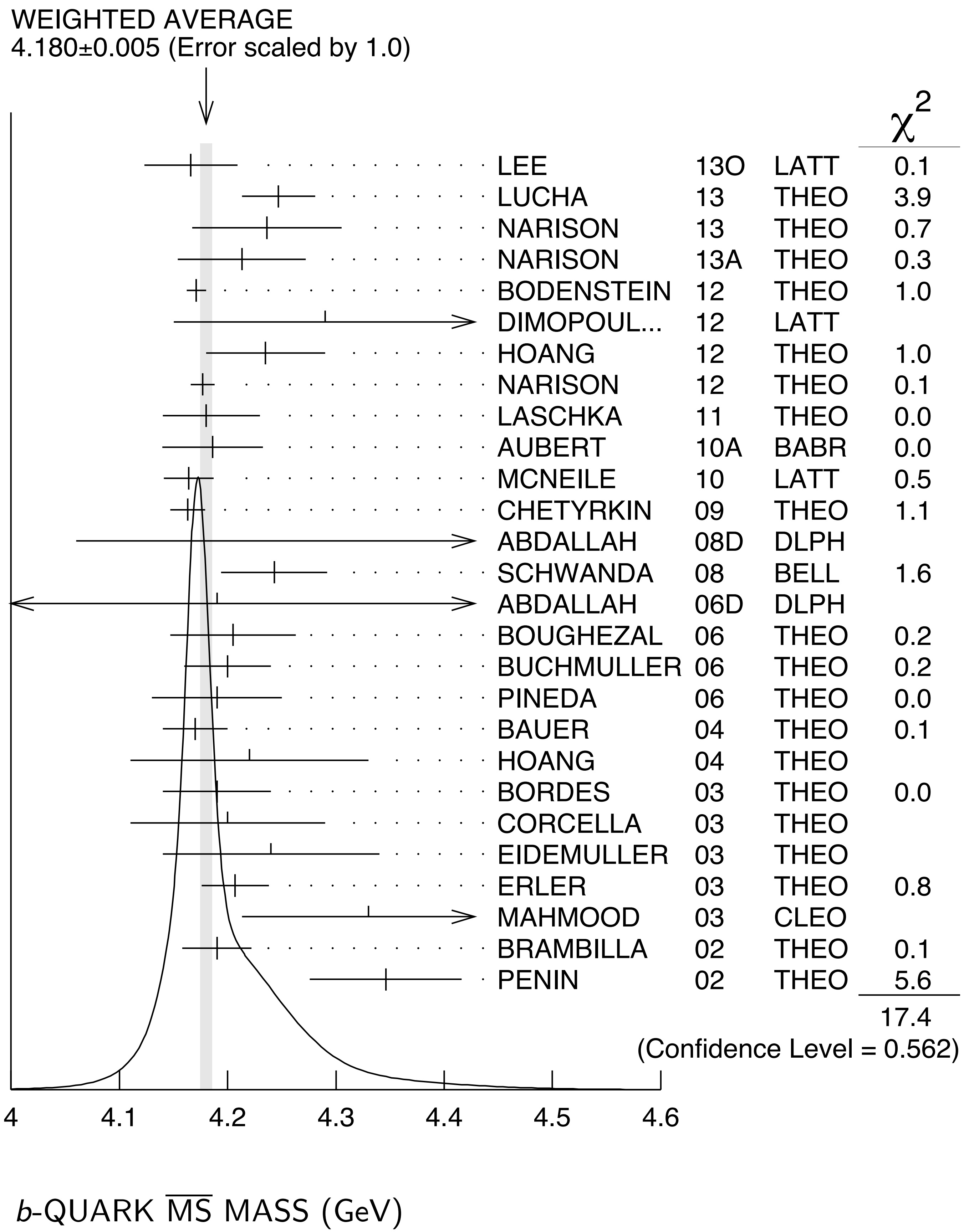
Υ spectrum

Bottom mass determinations

[K. A. Olive et al., PDG (2014)]



other recent determinations

[HPQCD Colquhoun (2014)]

$$\overline{m}_b(\overline{m}_b) = 4.196 \pm 0.023\,\mathrm{GeV}$$

relativistic QCD sum rules for large n!
uses non-relativistic lattice action !

[Penin, Zerf (2014)]

$$\overline{m}_b(\overline{m}_b) = 4.169 \pm 0.008\,\mathrm{GeV}$$

NR QCD sum rules
issues with perturbation theory

[HPQCD Donald et al (2014)]

$$\overline{m}_b(\overline{m}_b) = 4.174 \pm 0.024\,\mathrm{GeV}$$

Pseudoscalar QCD sum rules

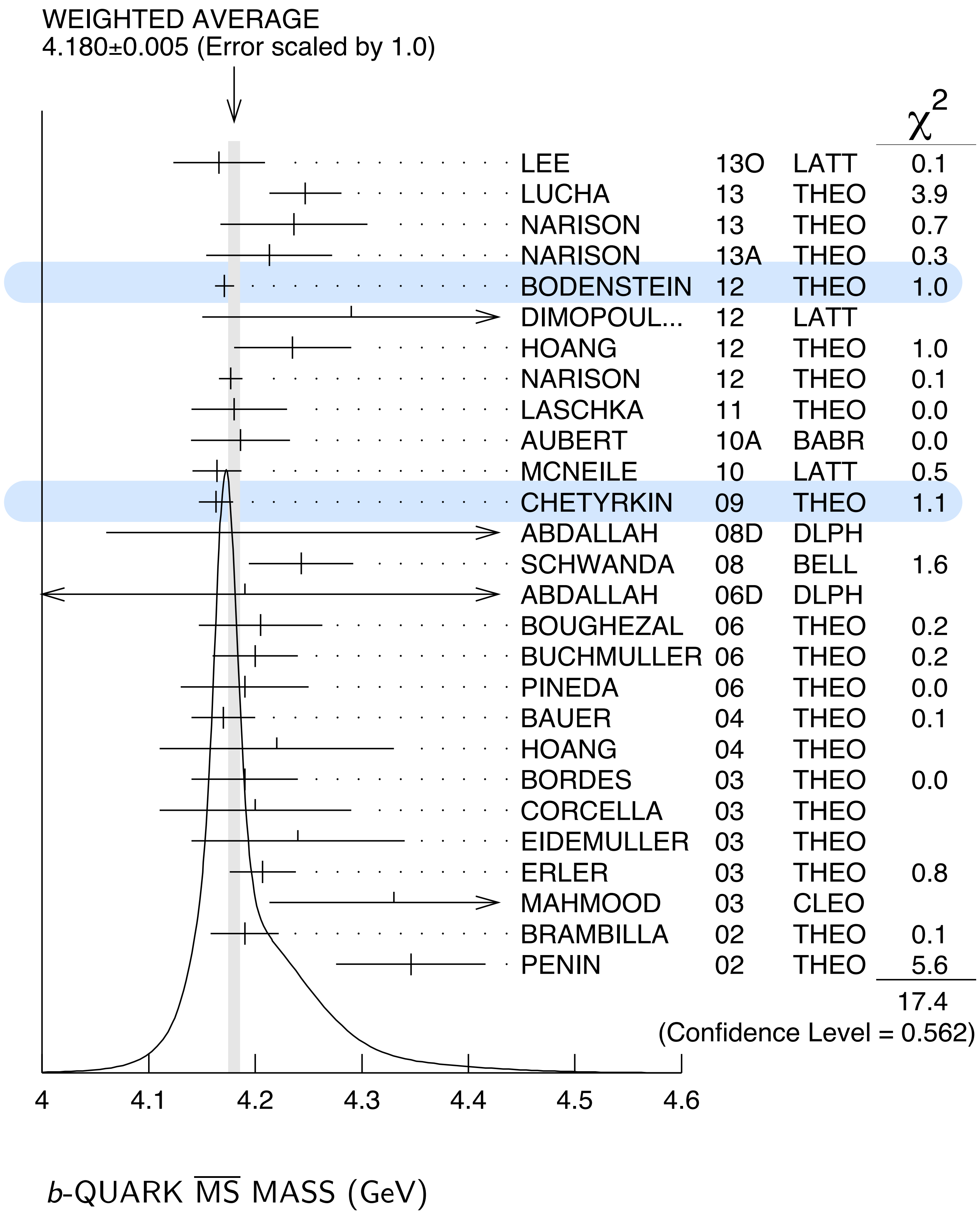
[Ayala, Cvetic, Pineda (2014)]

$$\overline{m}_b(\overline{m}_b) = 4.241 \pm 0.043\,\mathrm{GeV}$$

Υ spectrum

Bottom mass determinations

[K. A. Olive et al., PDG (2014)]



Current analyses with smallest errors

1. *[Bodenstein et al (2012)]*

$$\overline{m}_b(\overline{m}_b) = 4.171 \pm 0.009\,\text{GeV}$$

2. *[Chetyrkin et al (2012)]*

$$\overline{m}_b(\overline{m}_b) = 4.163 \pm 0.016\,\text{GeV}$$

Bottom mass determinations

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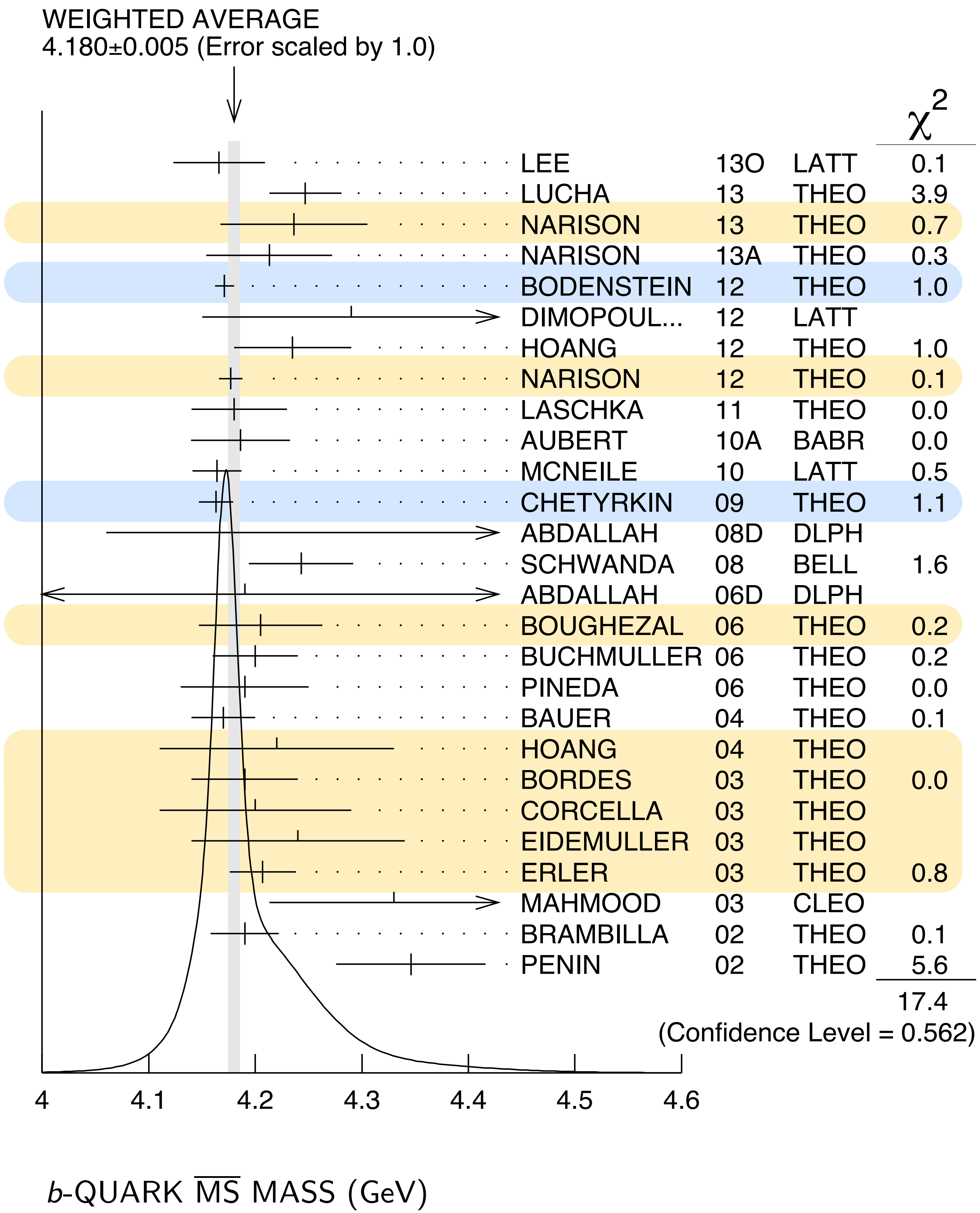

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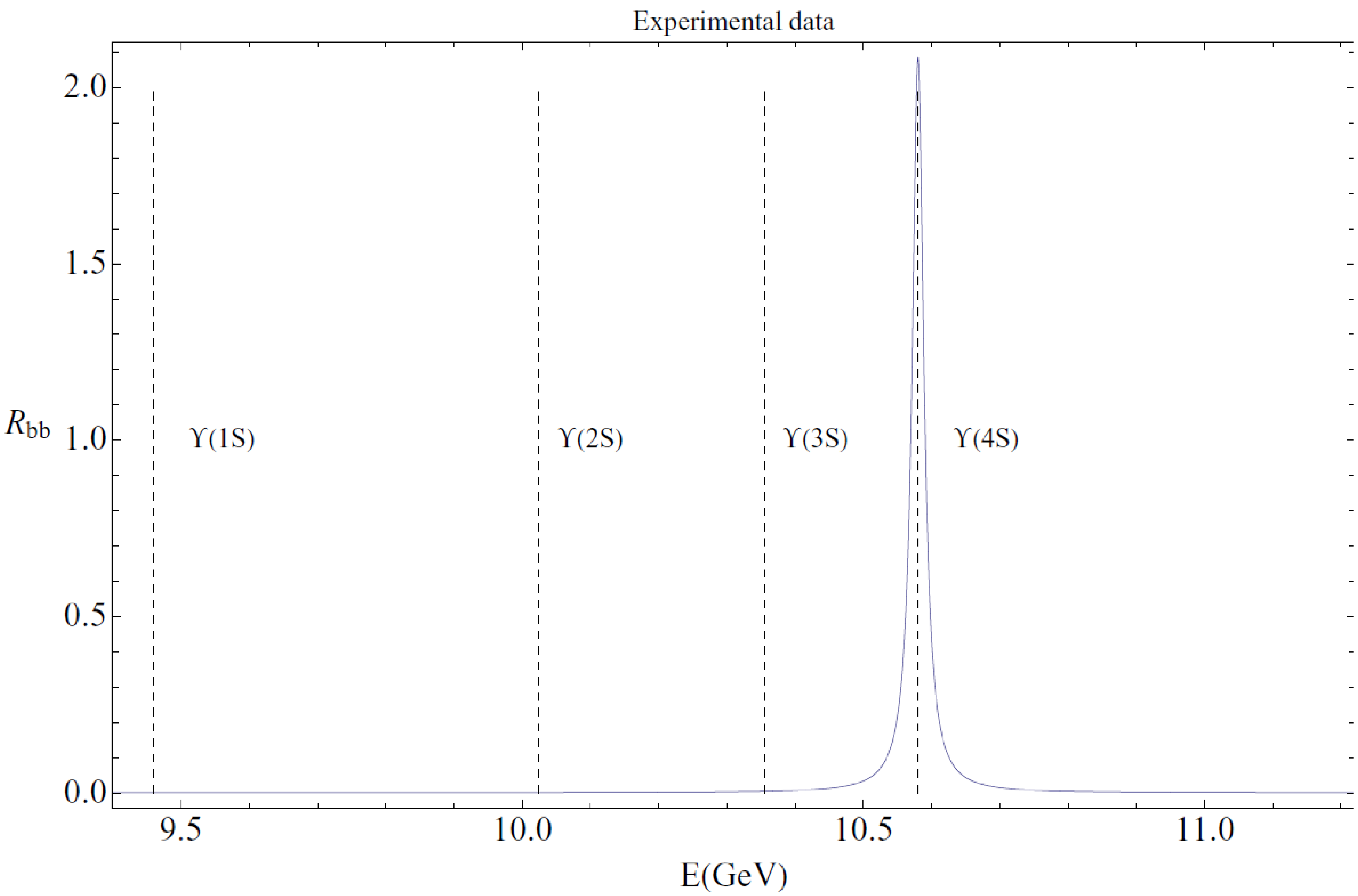
QCD sum rules (this talk)

Experimental data for bottom

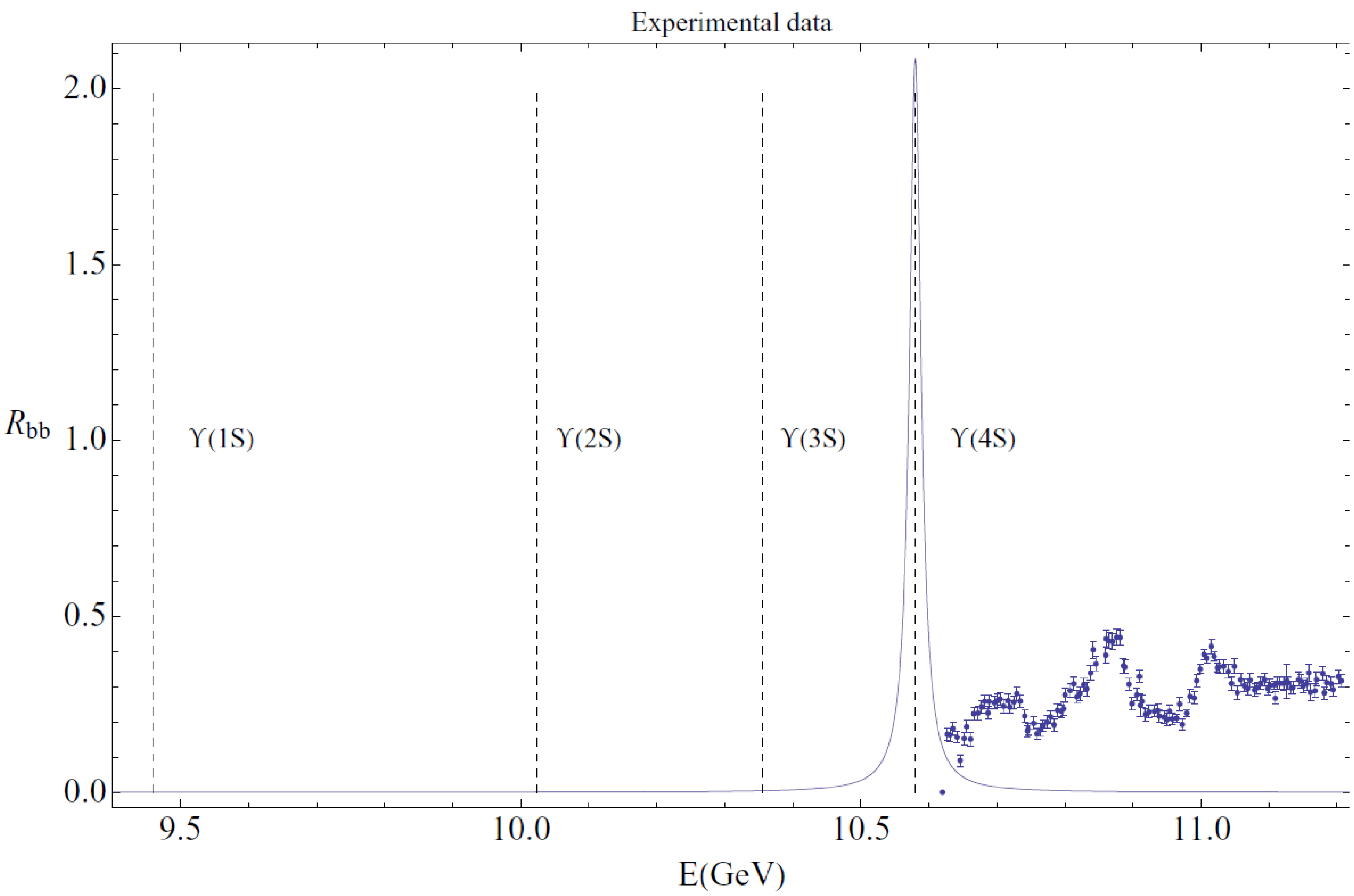
Experimental data: bottom

Narrow resonances



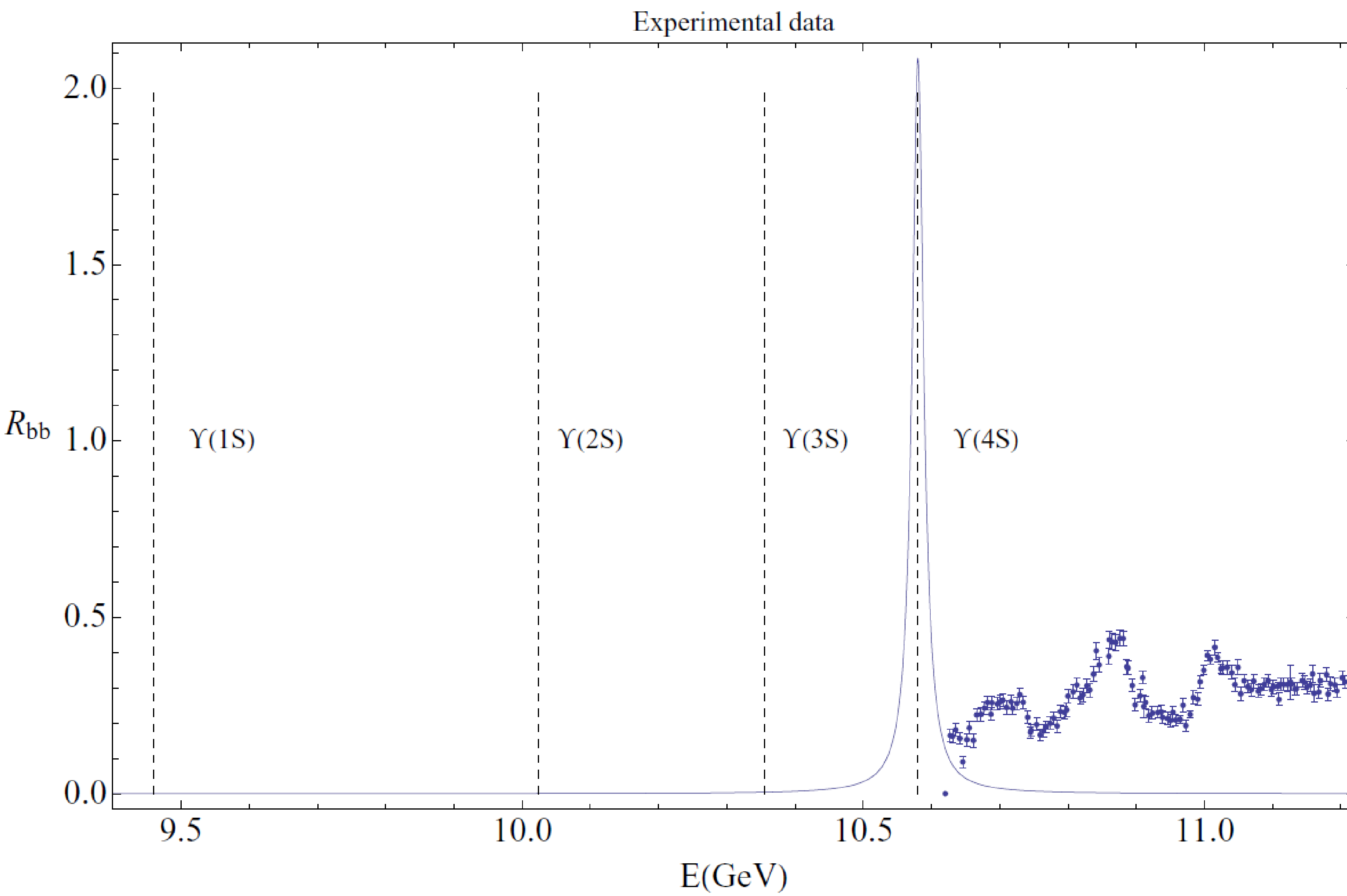
Experimental data: bottom

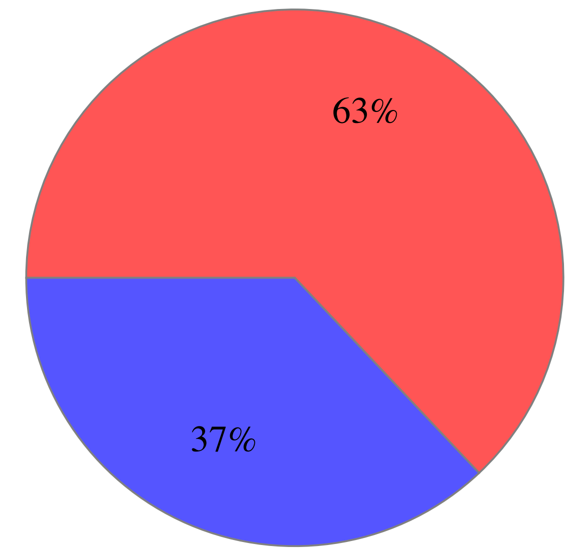
Babar data



Experimental data: bottom

Perturbation theory





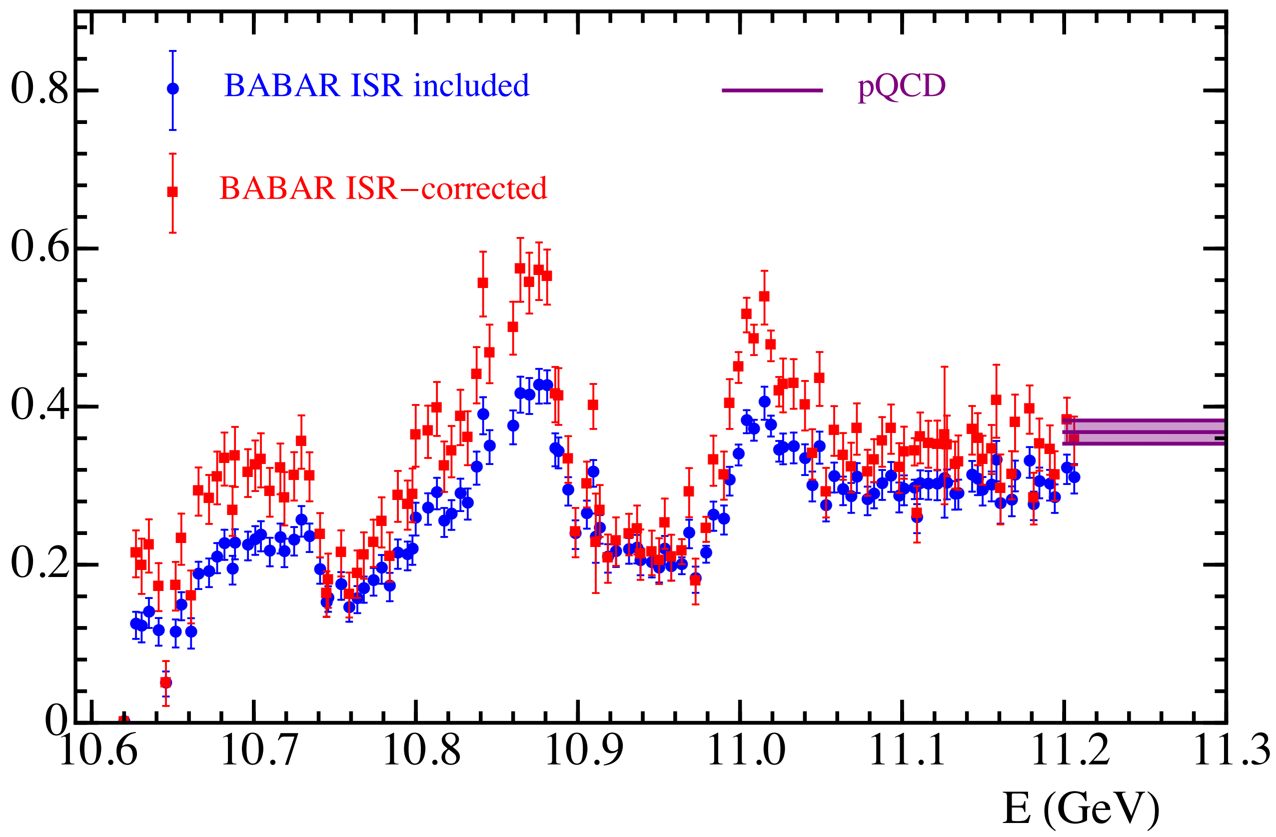
Perturbative QCD

Aren't we comparing theory to theory?

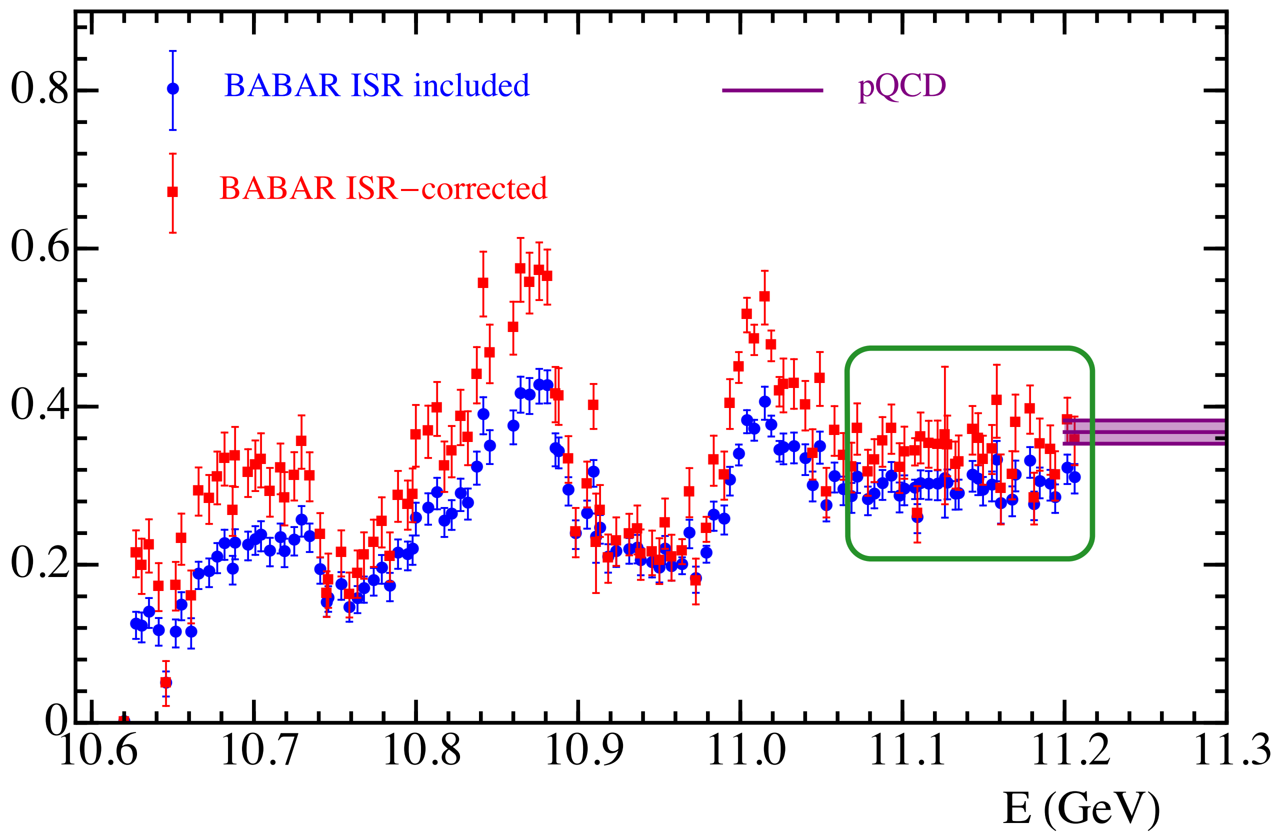
4% error gives a huge uncertainty to the first moment !!

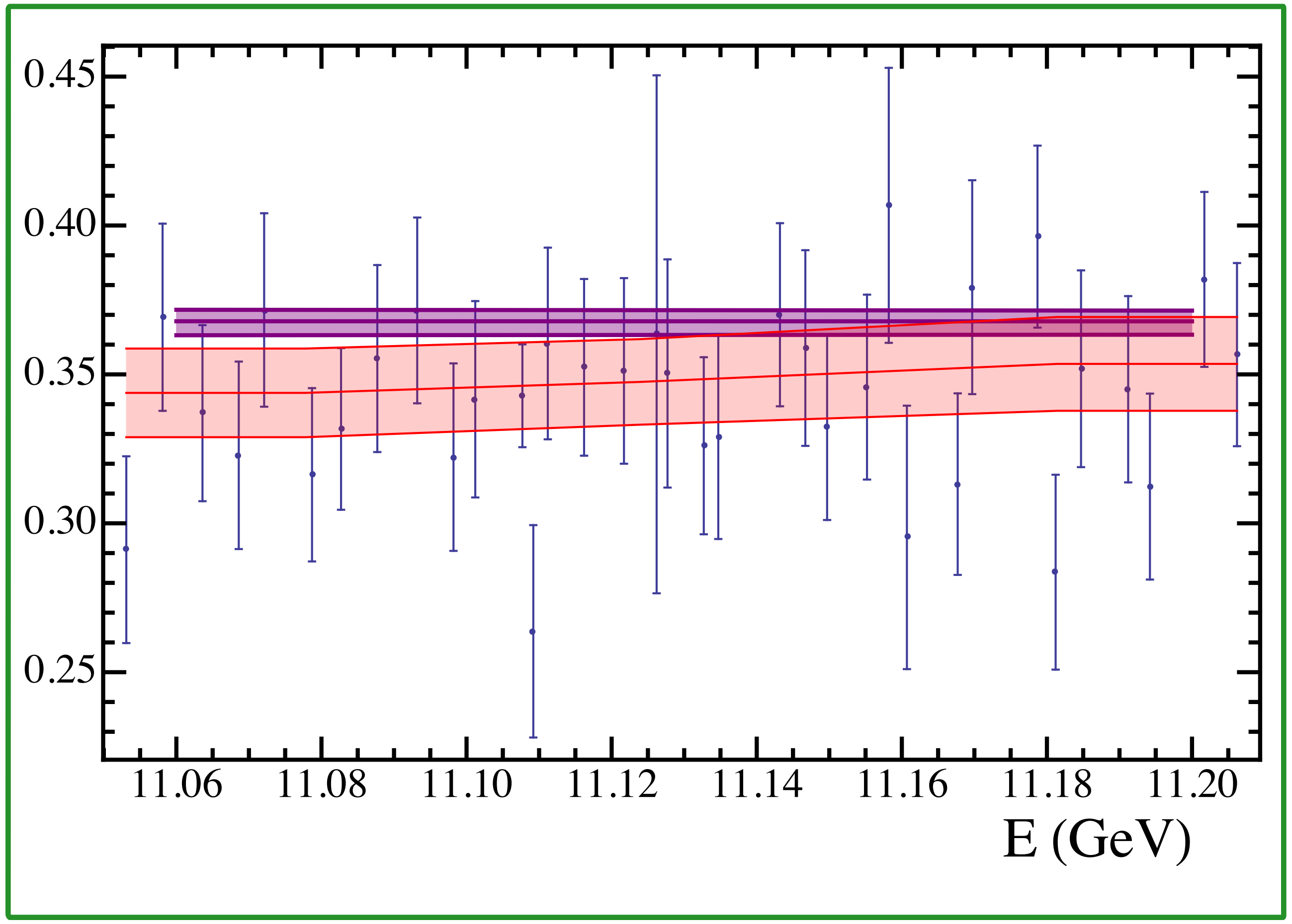
63% of the first moment from region without data !

High energy region



High energy region



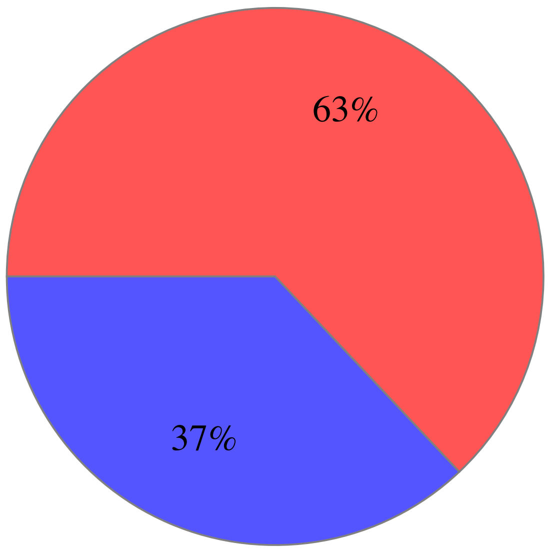


Discrepancy: (rebinned) data vs theory: 4%

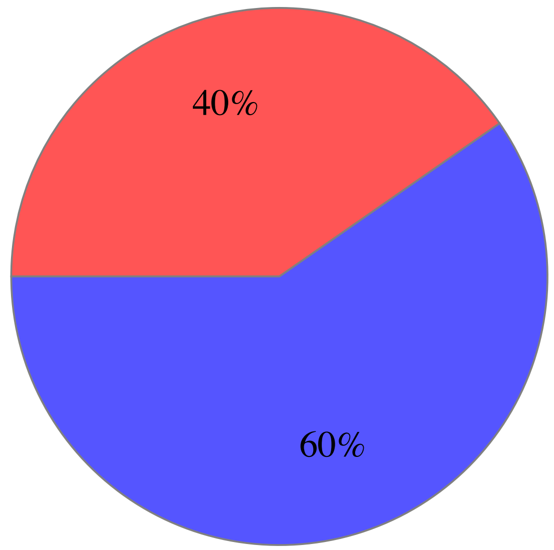
- Conservative continuum model: $R_b^{\rm model} = R_b^{\rm theory} \pm 4\%$
- Size of systematic error in rebinned data

High energy region contribution

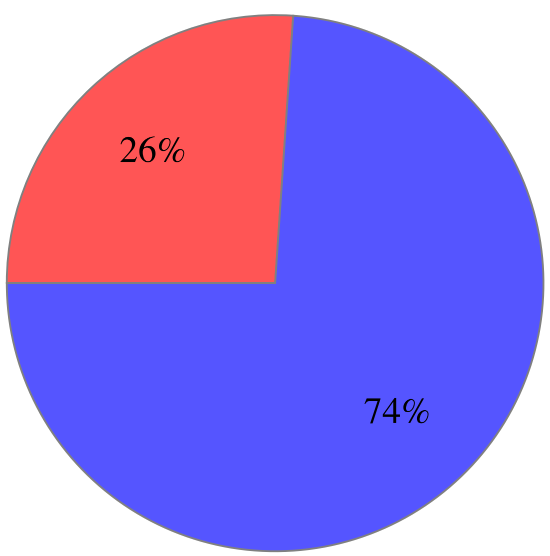
n = 1



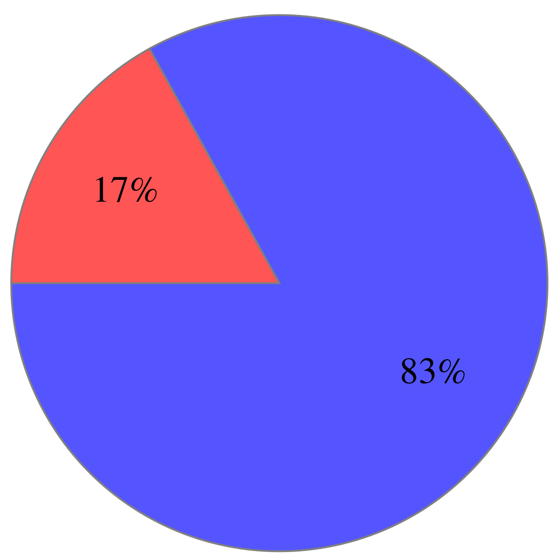
n = 2



n = 3



n = 4



Situation is less dramatic for higher moments

For n > 2 we find issues with perturbation theory

Therefore we use the 2nd moments as our default

High-energy region contributes “only” 39% of total error if 4% error assigned to theory

New experimental data in high-energy region: dramatic impact to precision!

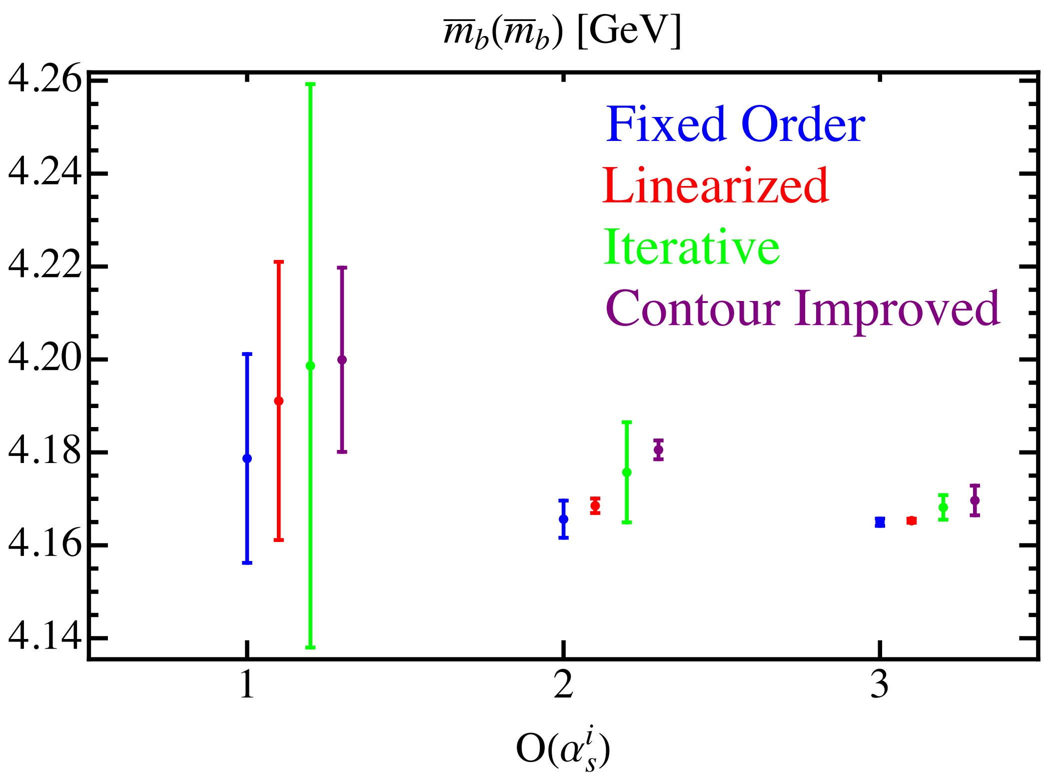
Bottom mass from Vector Correlator

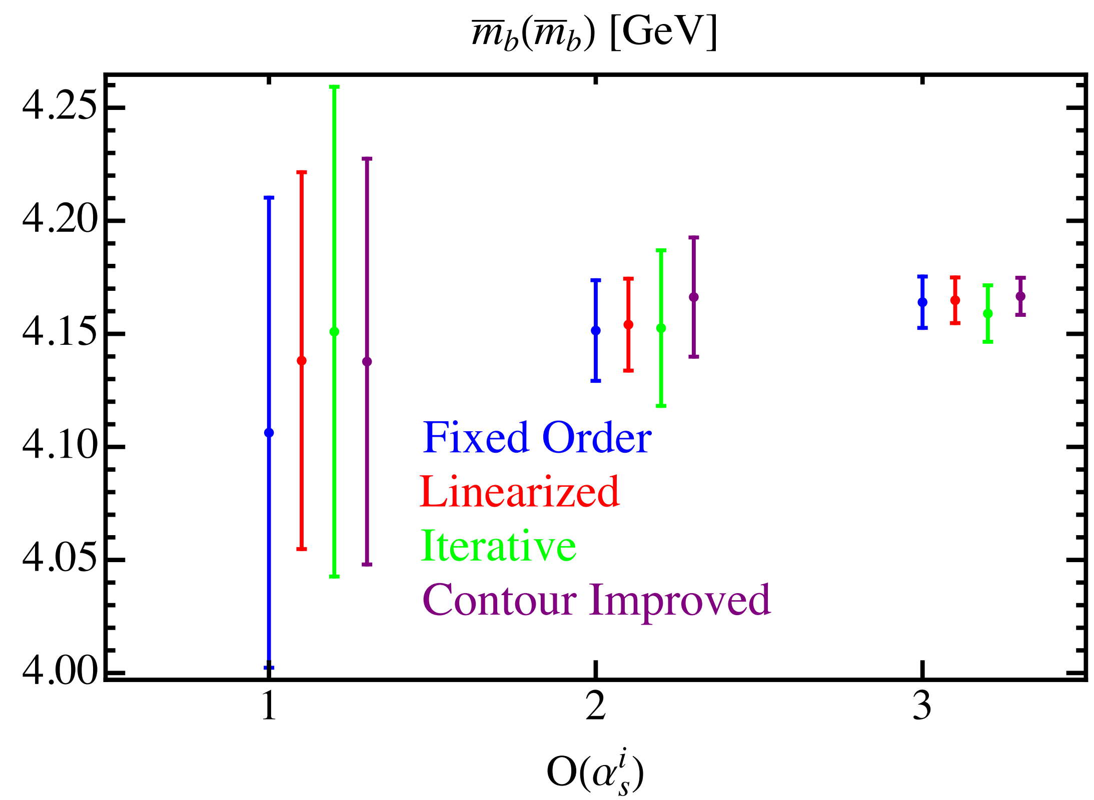
Results and comparison to another analysis

correlated **<u>2nd moment</u>** Independent

$$5\,\text{GeV} \leq \mu_\alpha = \mu_m \leq 15\,\text{GeV}$$

$$\overline{m}_b(\overline{m}_b) \leq \mu_\alpha, \mu_m \leq 15\,\text{GeV}$$

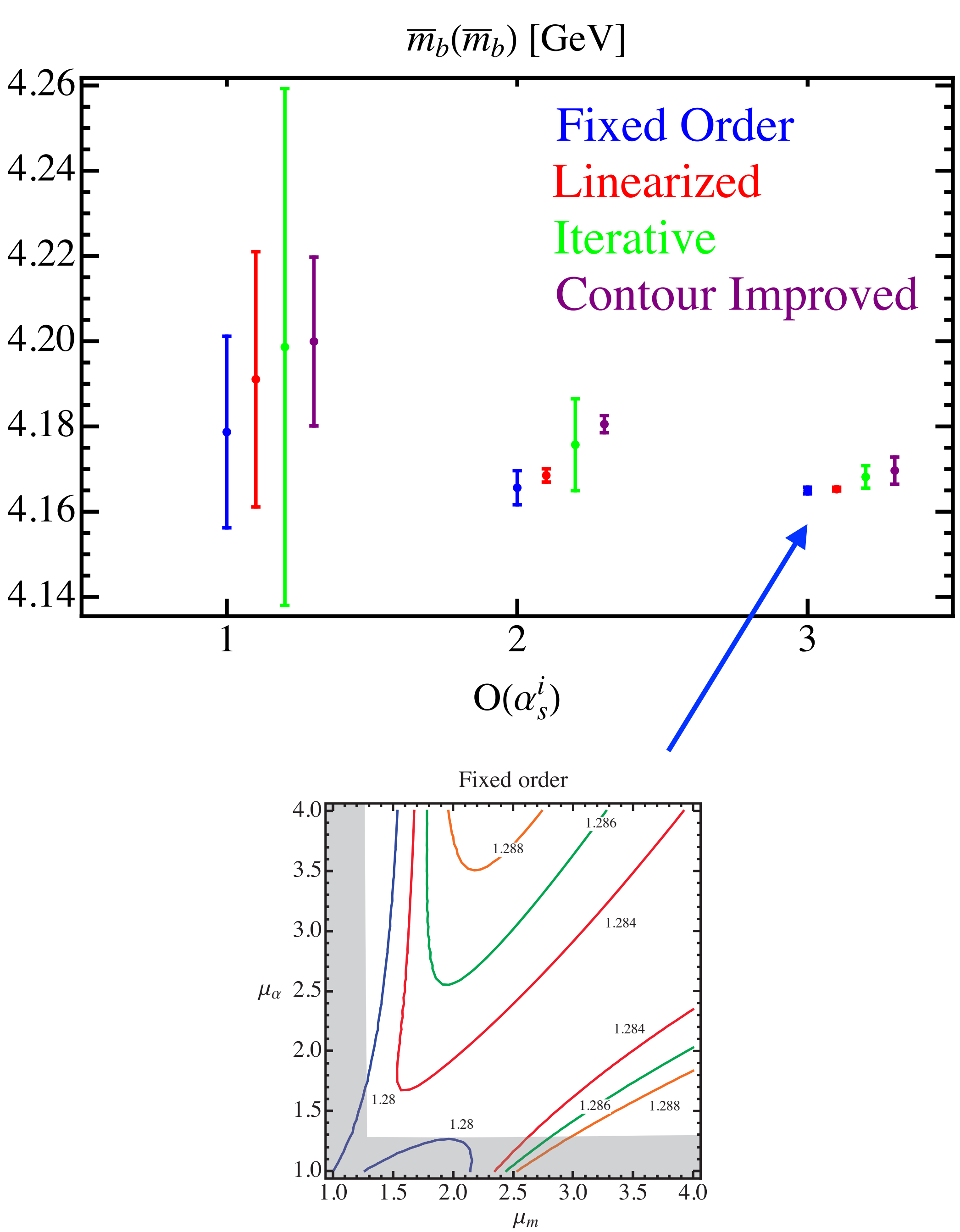


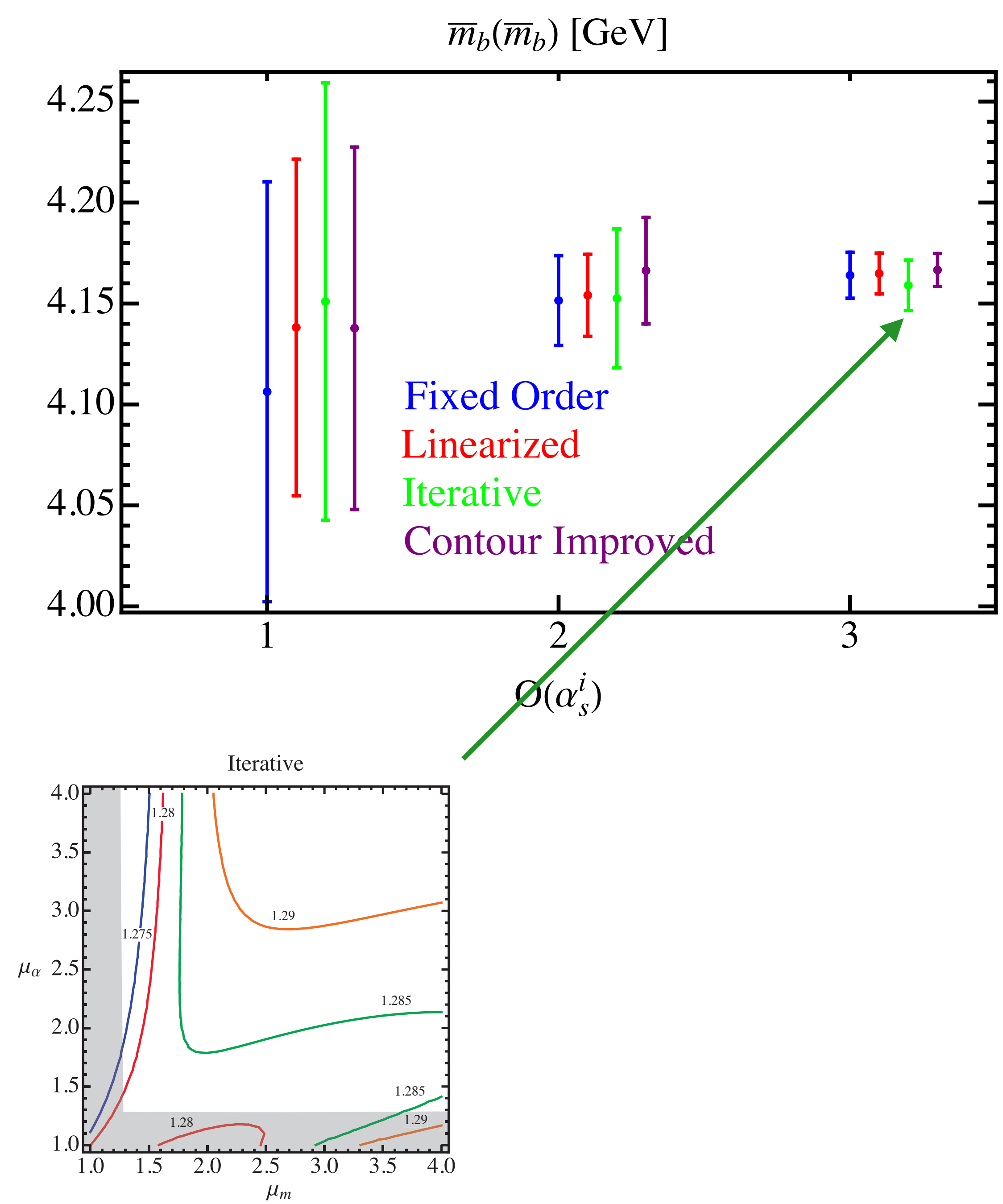


Results and comparison to another analysis

correlated   <u>2nd moment</u>   Independent

$$5\,\mathrm{GeV} \leq \mu_\alpha = \mu_m \leq 15\,\mathrm{GeV} \qquad\qquad \overline{m}_b(\overline{m}_b) \leq \mu_\alpha, \mu_m \leq 15\,\mathrm{GeV}$$





Results and comparison to another analysis

2nd moment

correlated

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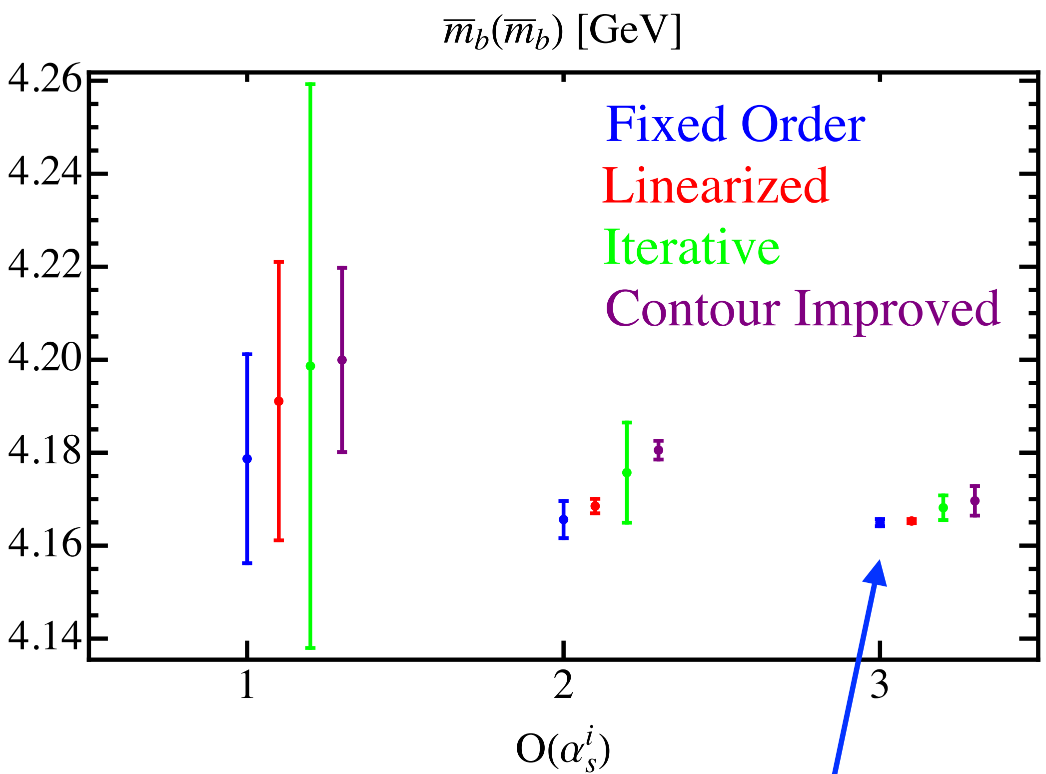
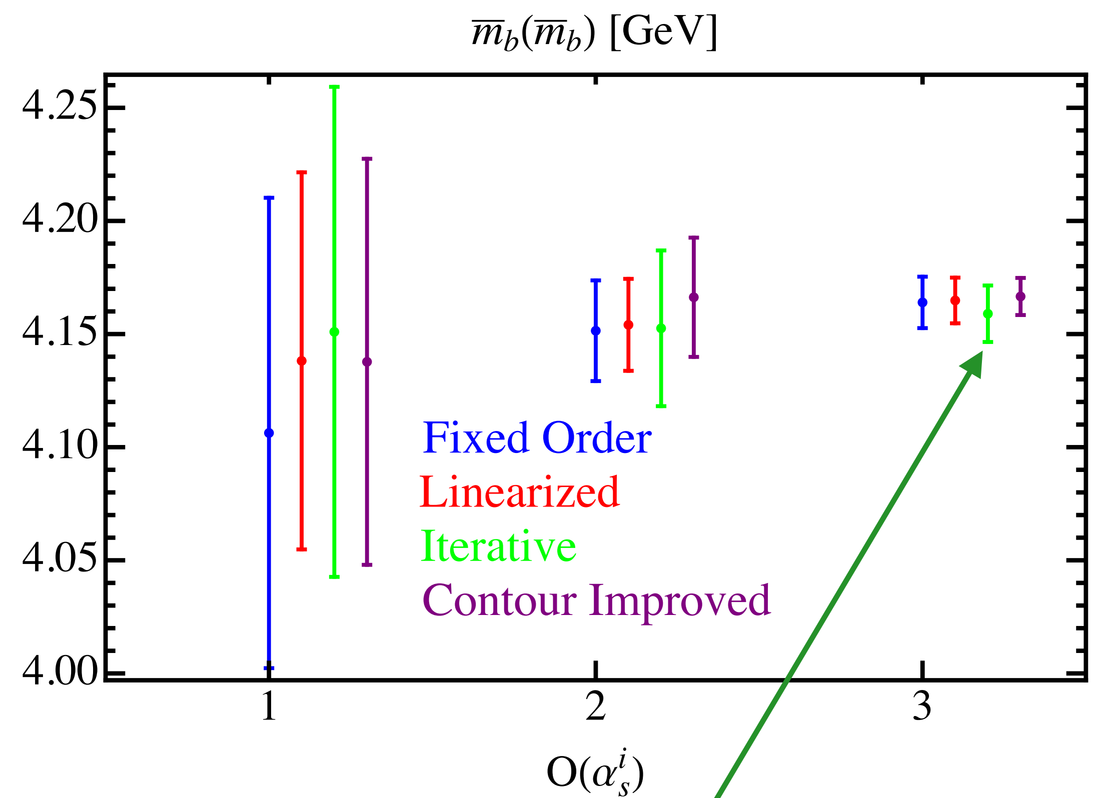


Independent

$$\overline{m}_b(\overline{m}_b) \leq \mu_\alpha, \mu_m \leq 15\,\mathrm{GeV}$$



[Chetyrkin et al (2009)]

$$\overline{m}_b(\overline{m}_b) = 4.163 \pm 0.010_{\mathrm{exp}} \pm 0.003_{\mathrm{pert}} \pm 0.012_{\alpha_s}\,\mathrm{GeV}$$

Our result (preliminary) *[Dehnadi, Hoang, VM]*

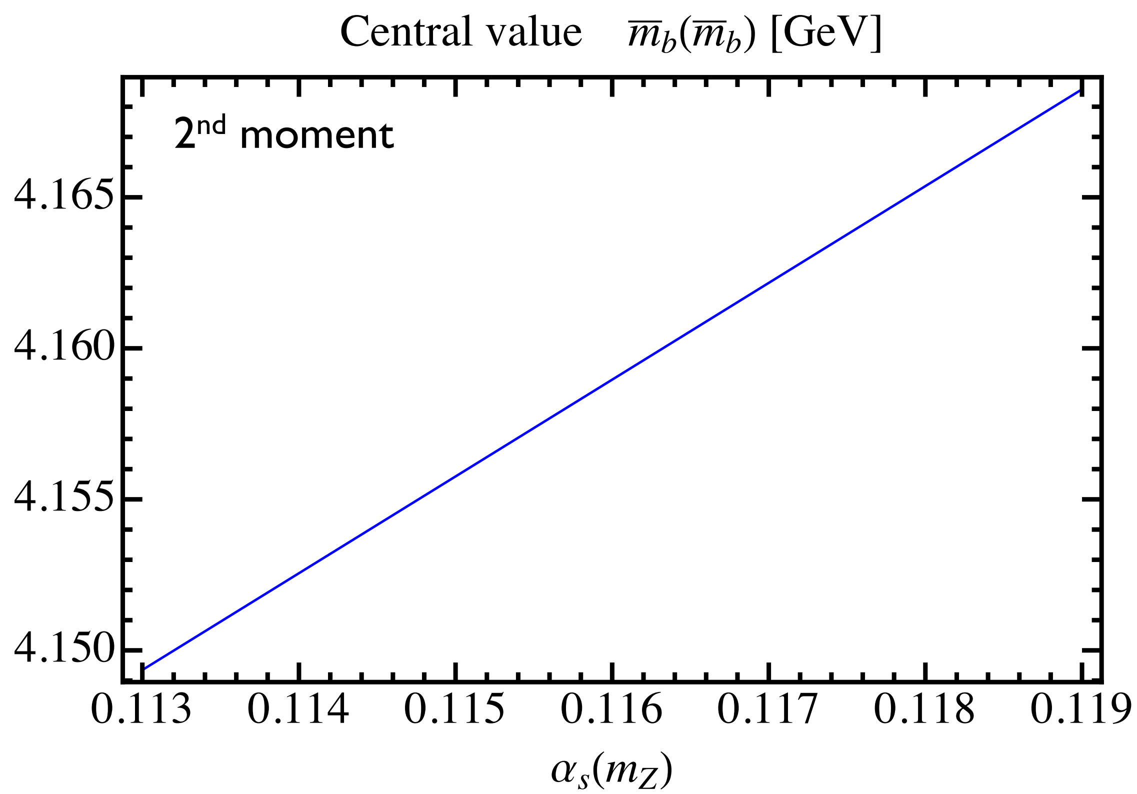
$$\overline{m}_b(\overline{m}_b) = 4.167 \pm 0.019_{\mathrm{syst}} \pm 0.004_{\mathrm{stat}} \pm 0.011_{\mathrm{pert}} \pm 0.007_{\alpha_s}\,\mathrm{GeV}$$

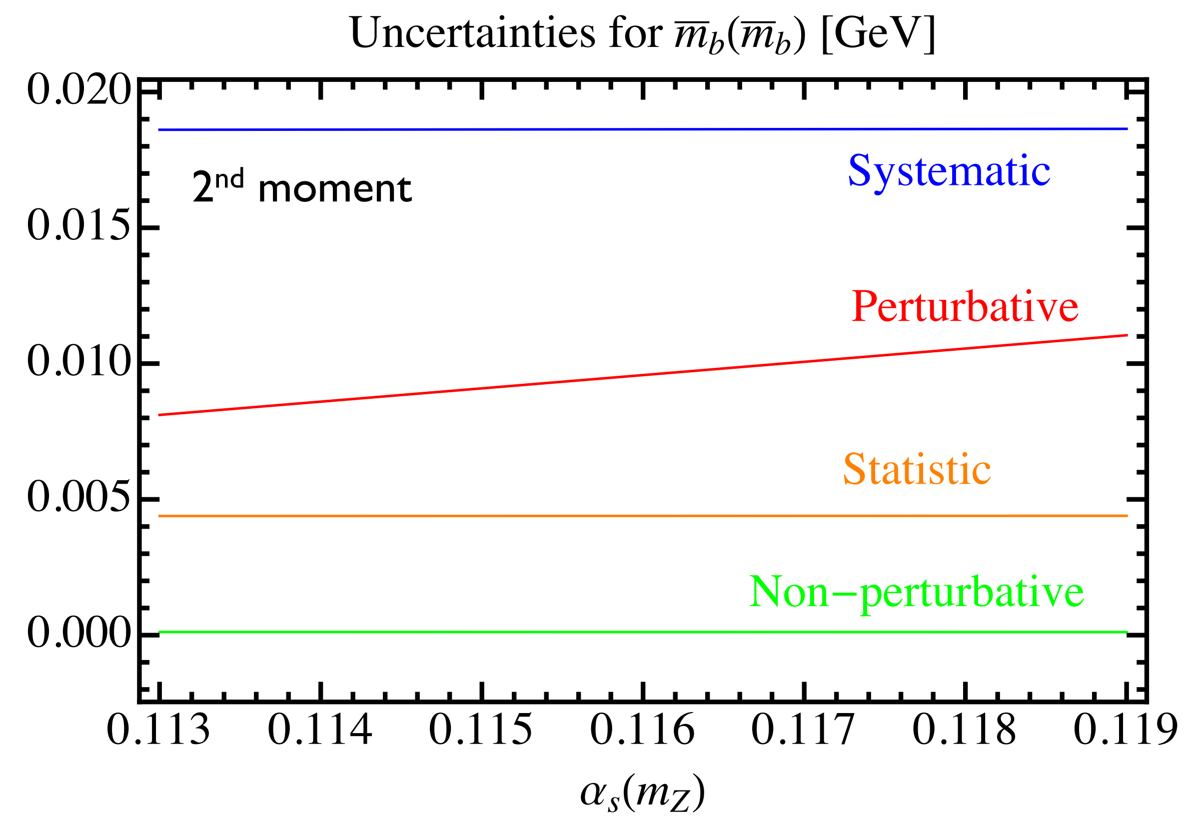
using $\alpha_s(m_Z) = 0.1184 \pm 0.0021$

Results

[Dehnadi, Hoang, VM]

Dependence of central value and errors on the value of $\alpha_s(m_Z)$

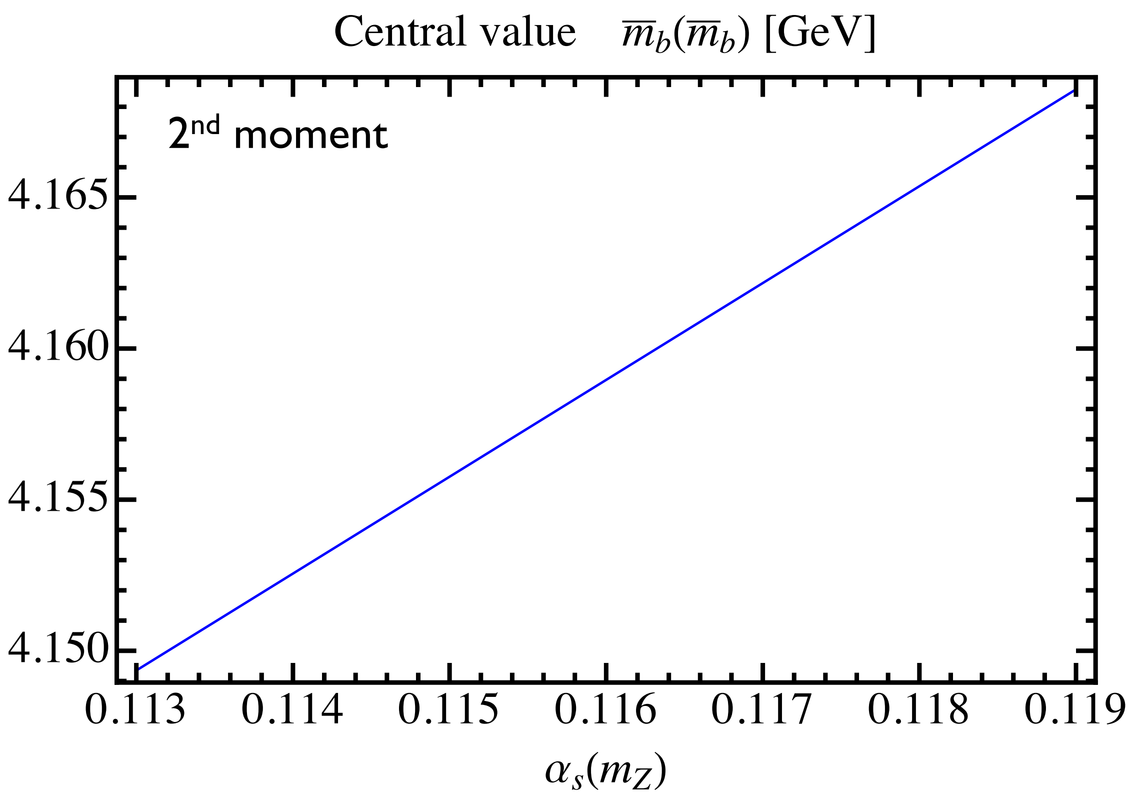


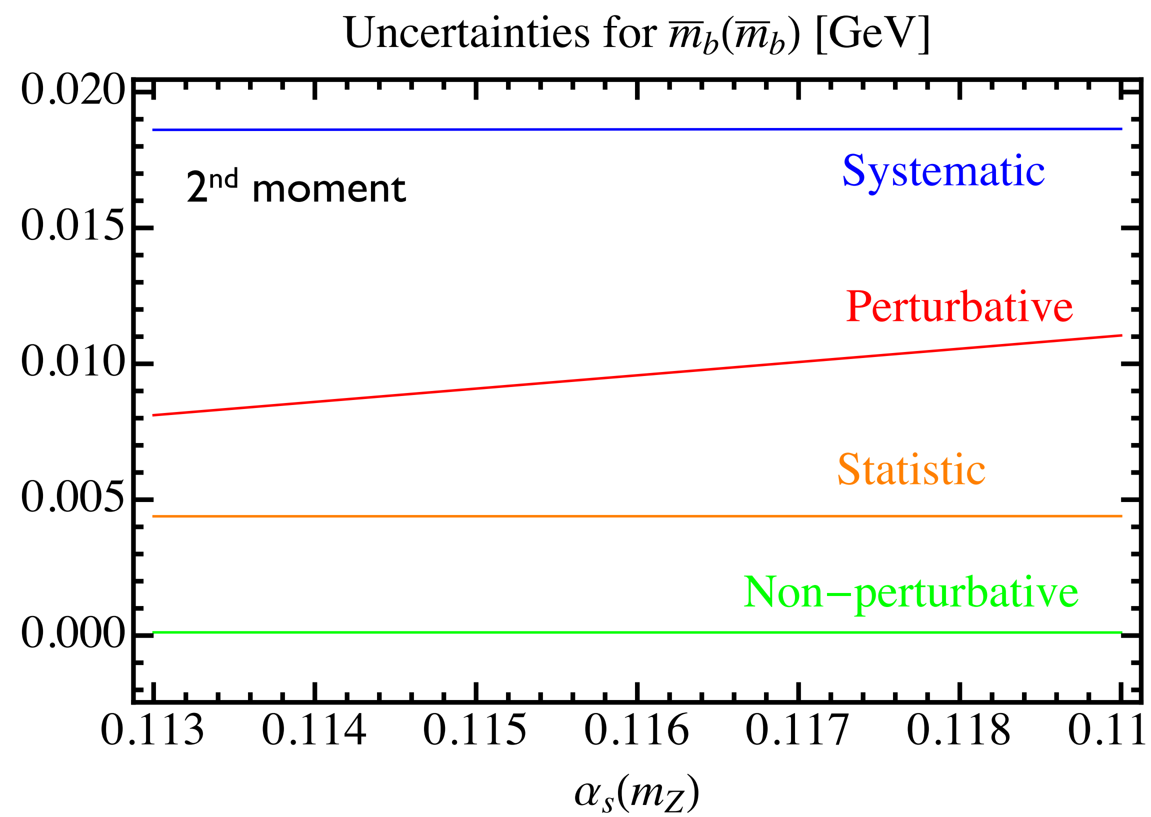


Results

[Dehnadi, Hoang, VM]

Dependence of central value and errors on the value of $\alpha_s(m_Z)$

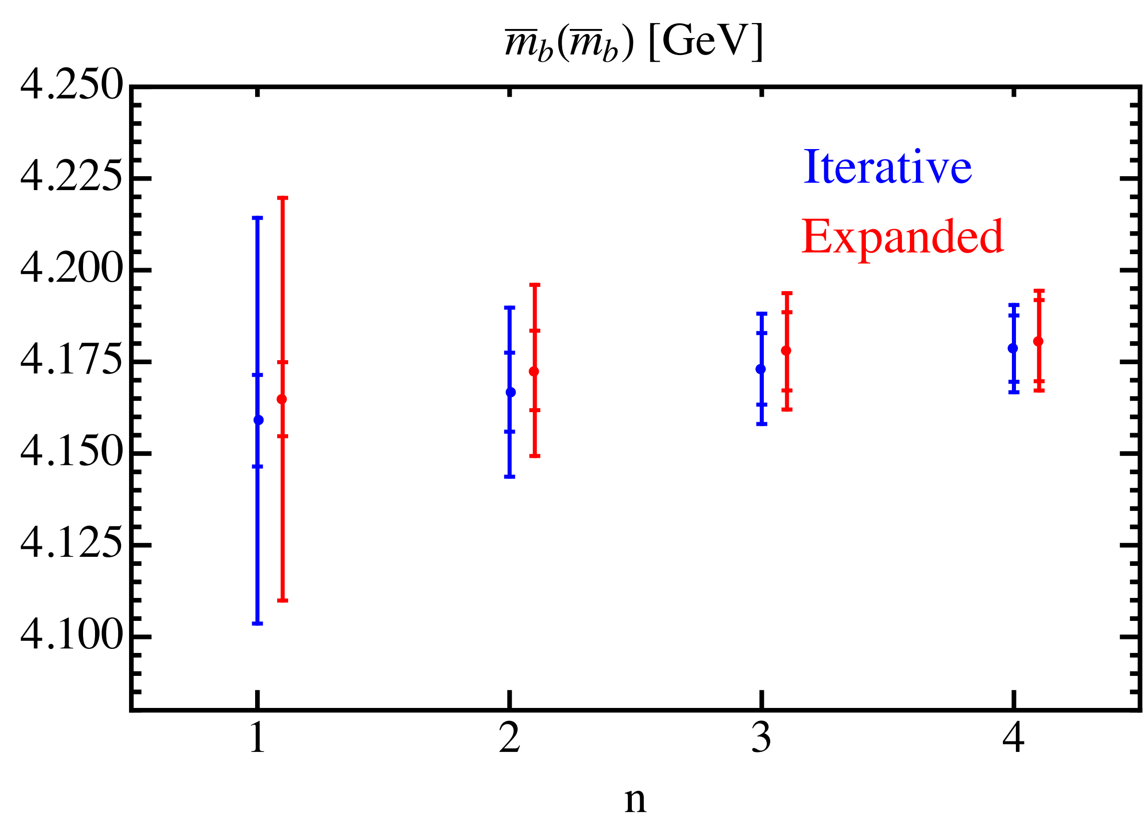




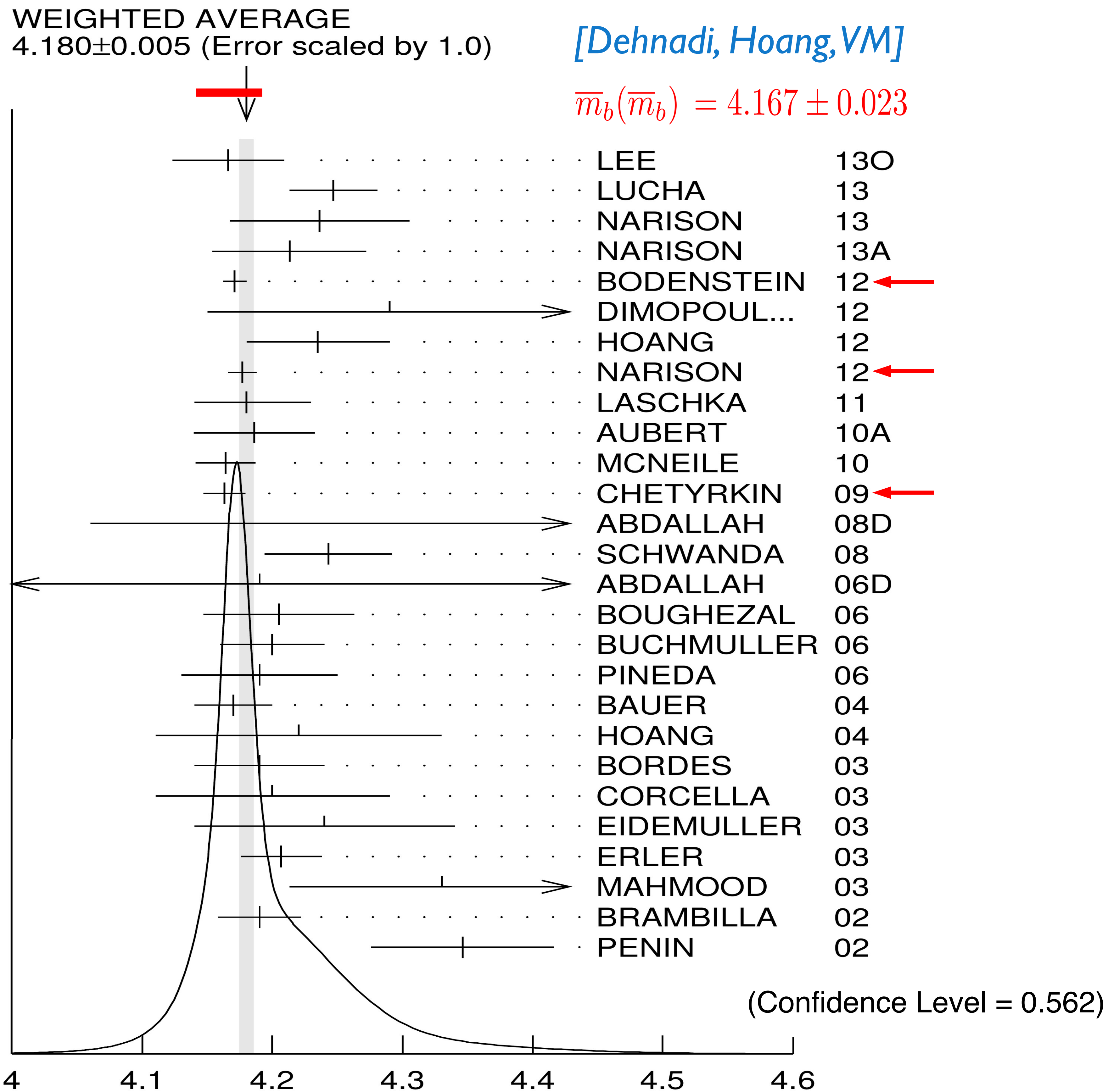
Final result for 2nd moment

$$\overline{m}_b(\overline{m}_b) = 4.167 \pm 0.023\,\mathrm{GeV}$$

using $\alpha_s(m_Z) = 0.1184 \pm 0.0021$



Comparison to other determinations



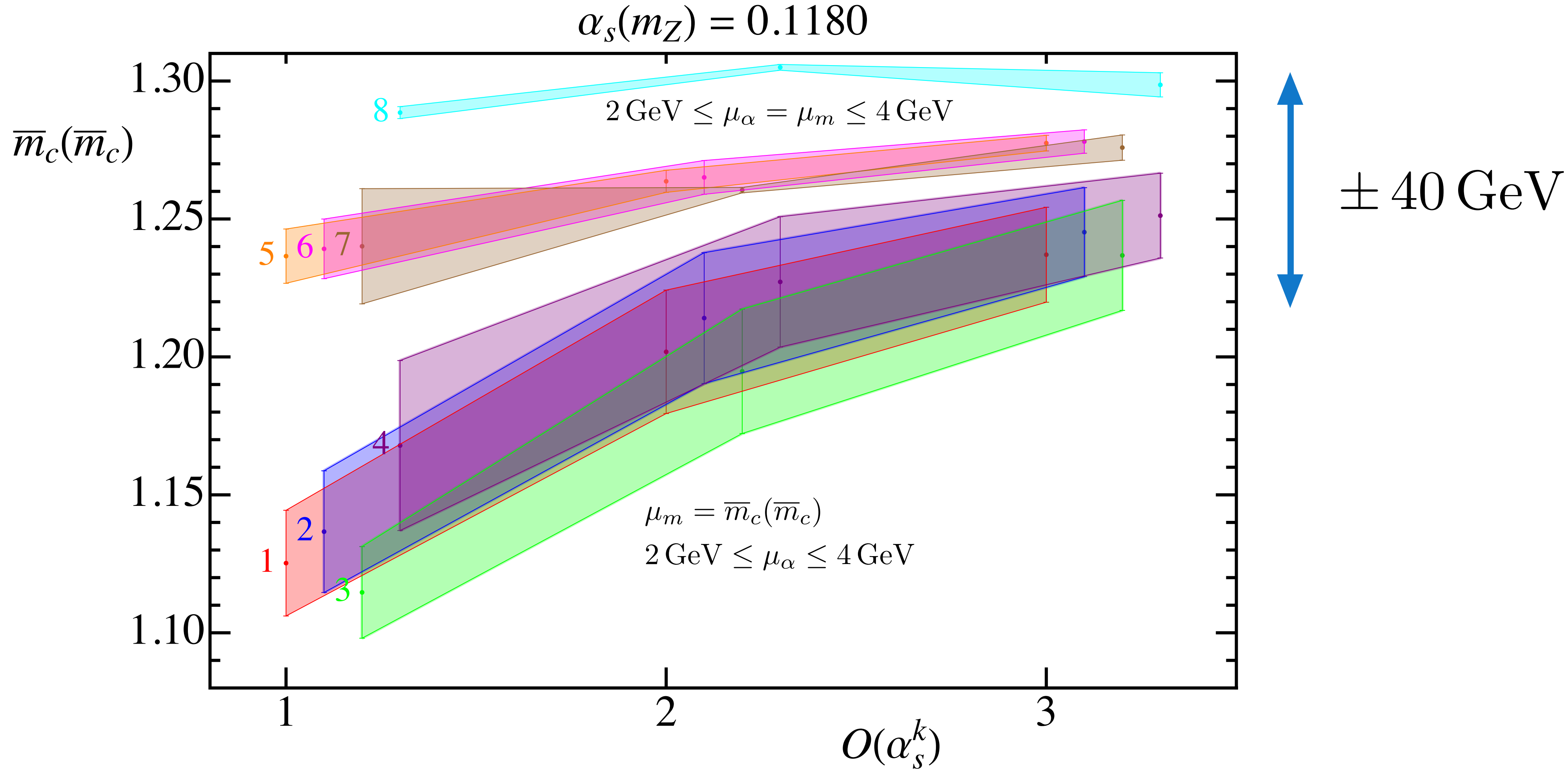
Conclusions

Conclusions & Outlook

- Double scale variation: best uncertainty estimate
- True for charm (vector and pseudo) and bottom
- Likely also true for FESR and shifted moments (future analysis)
- Pseudo-scalar determination less precise.
- Bottom: 2^{nd} moment smaller experimental error
- Comparisons with lattice, important cross check

Backup slides

Exploration of scale variation



[Dehnadi, Hoang, VM]

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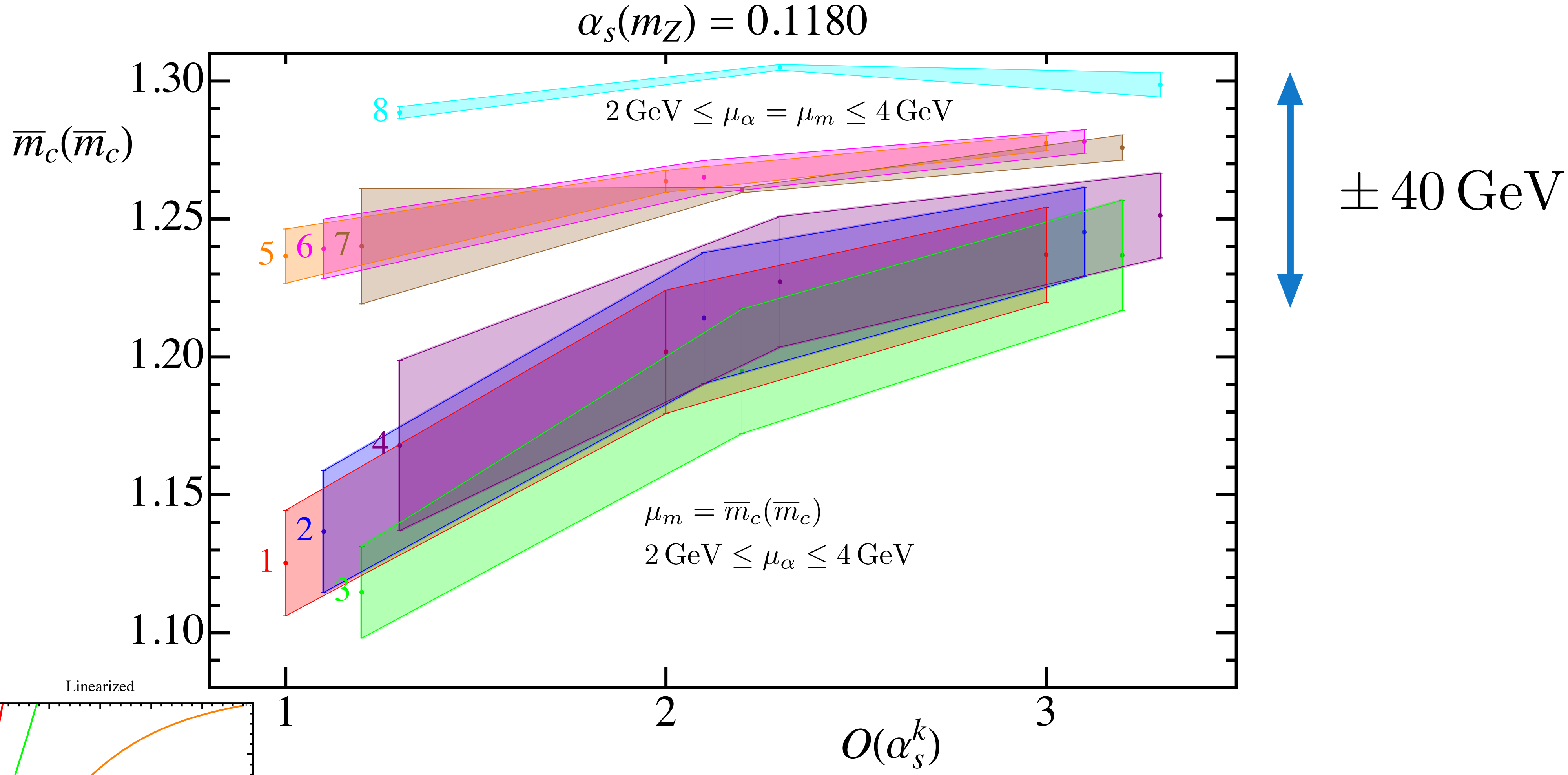

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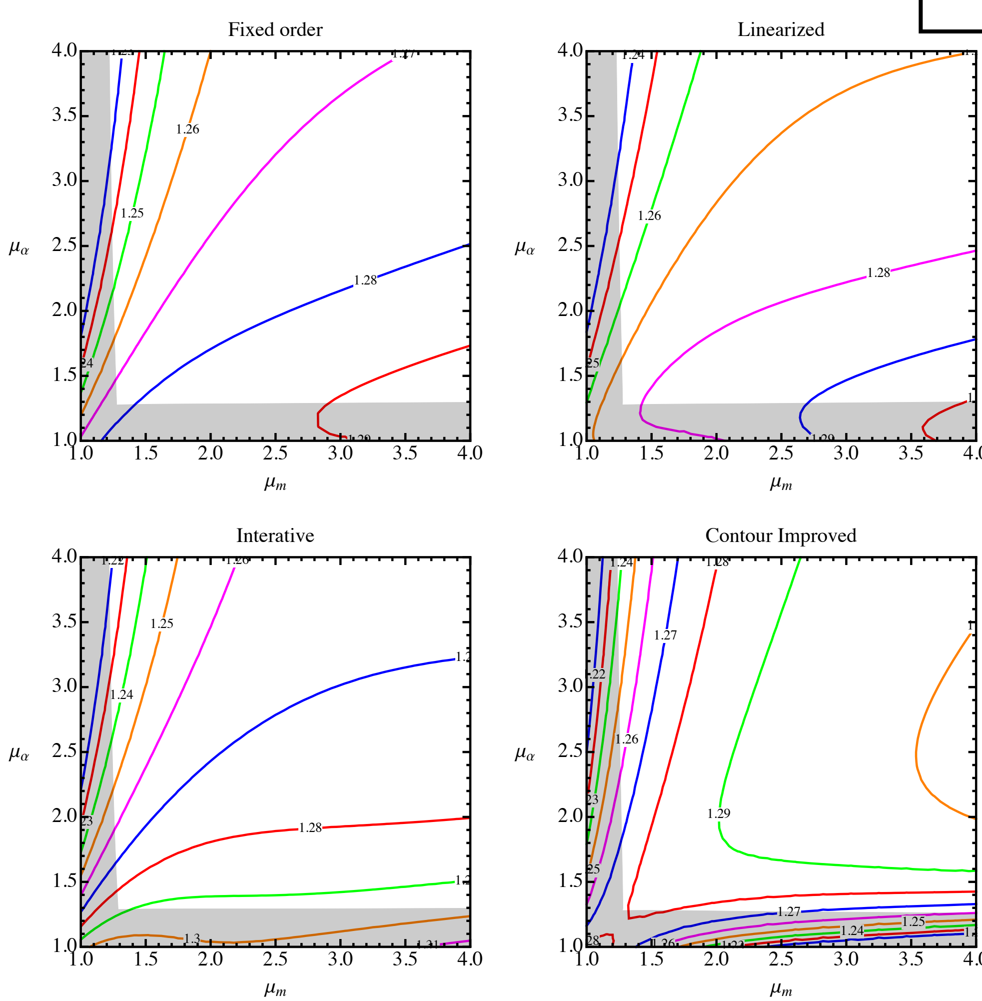

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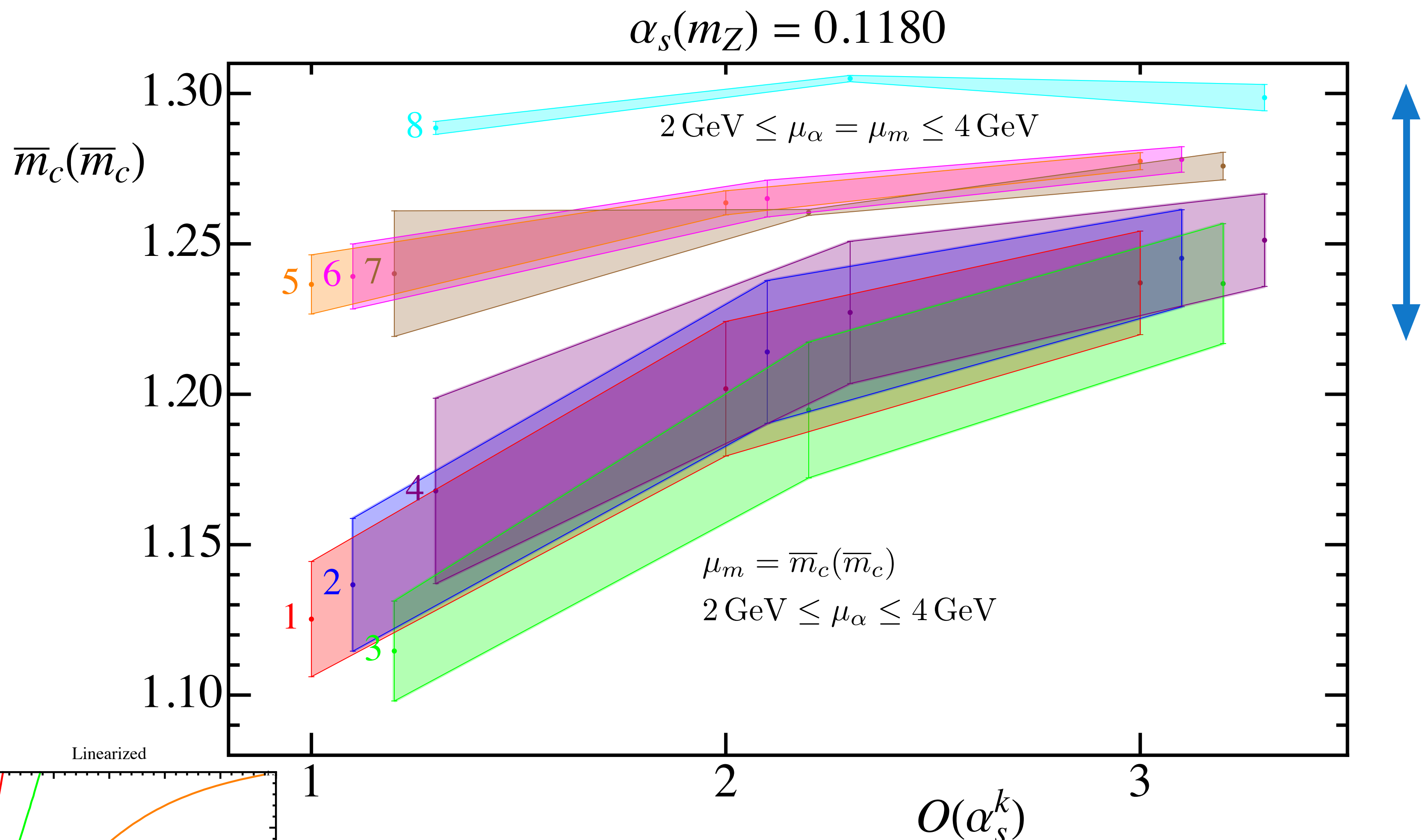
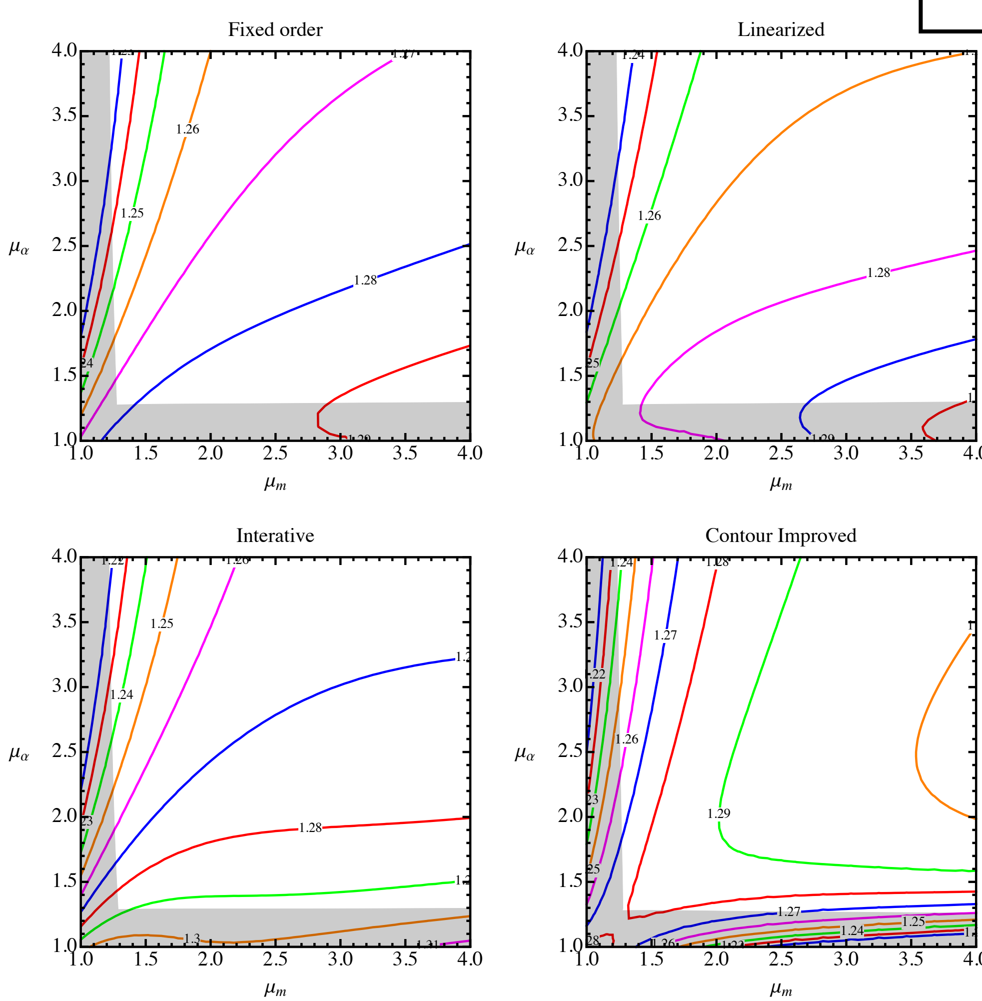
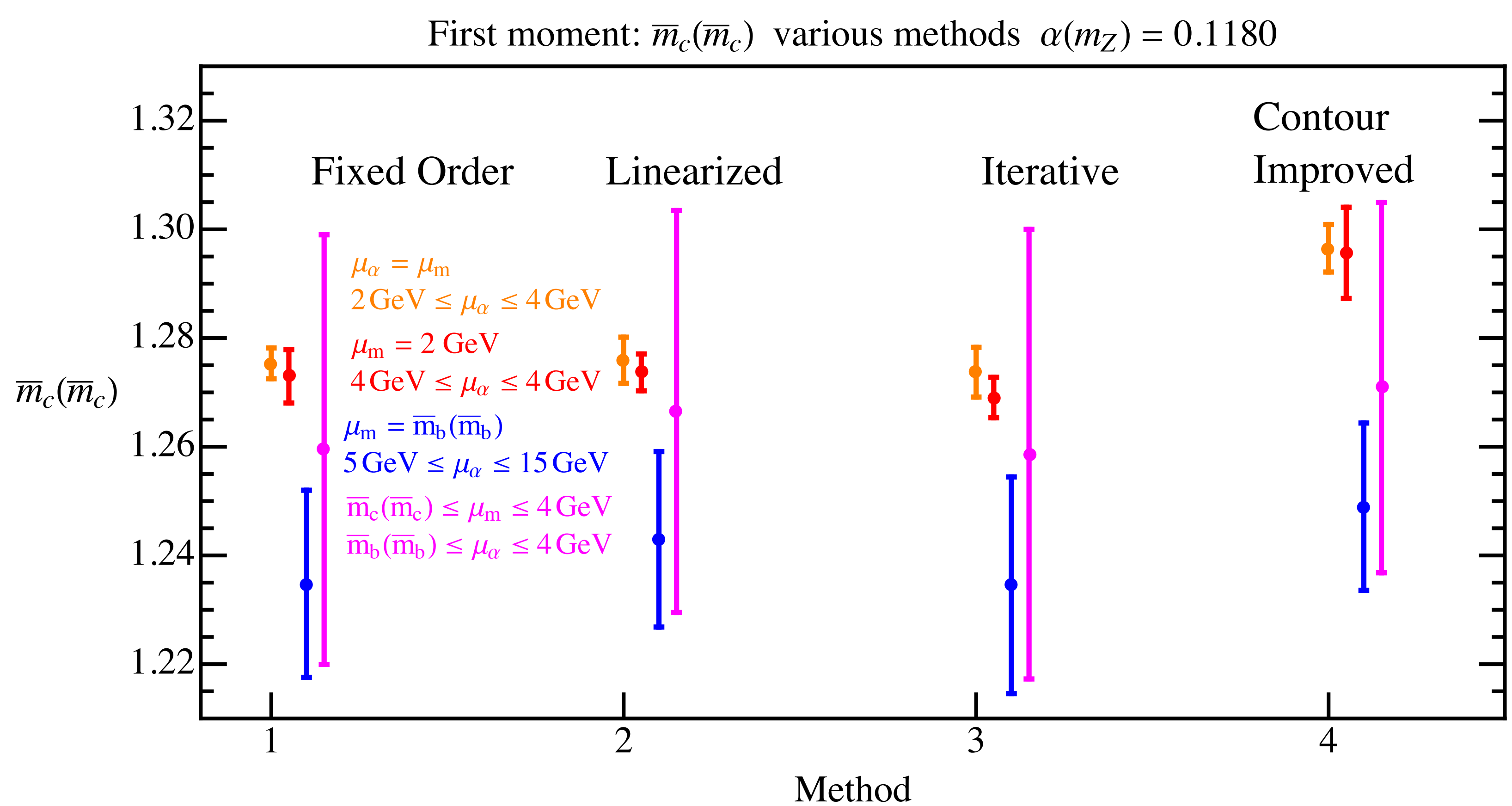


$\pm\, 40$ GeV

[Dehnadi, Hoang, VM]





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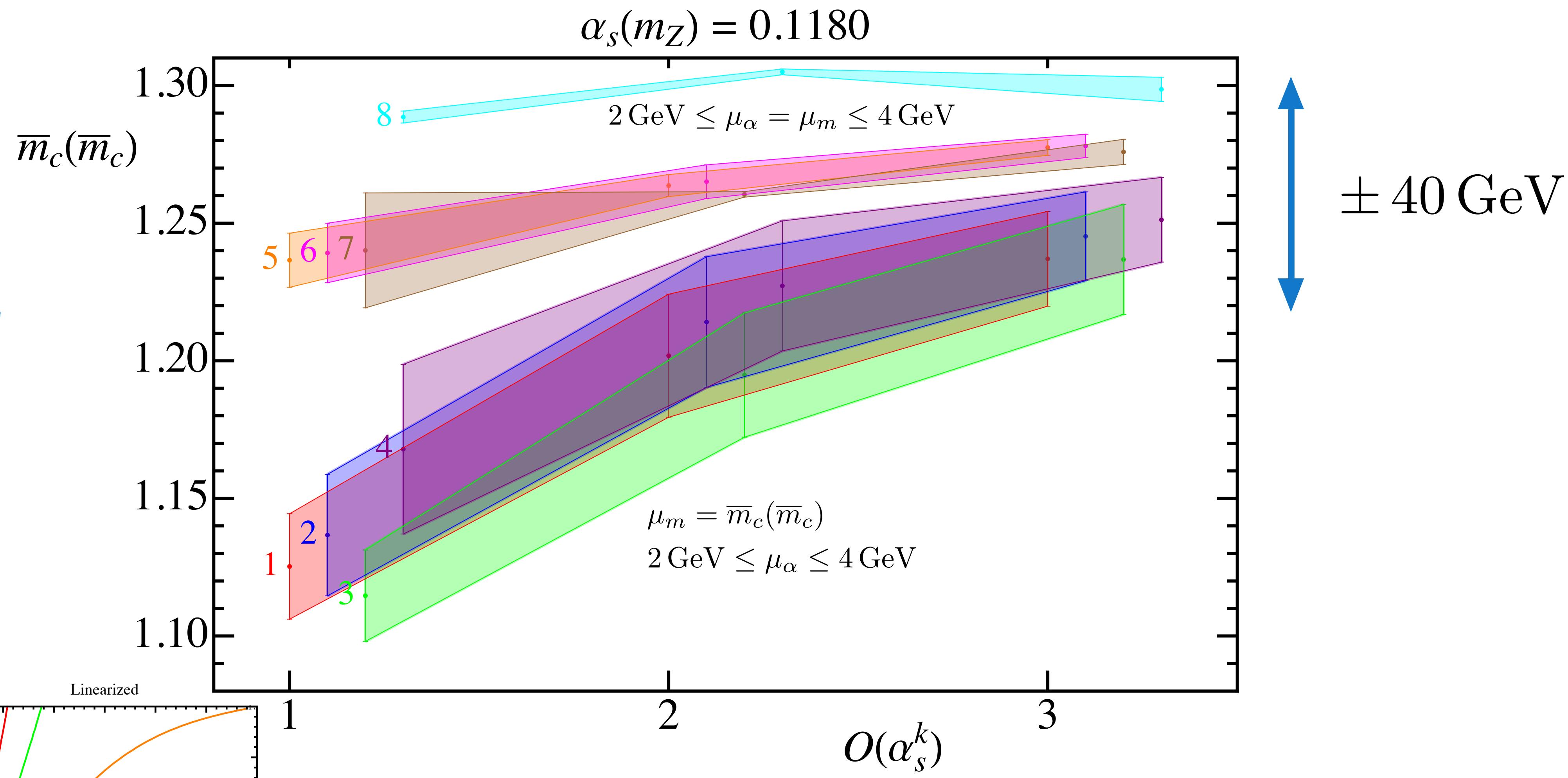
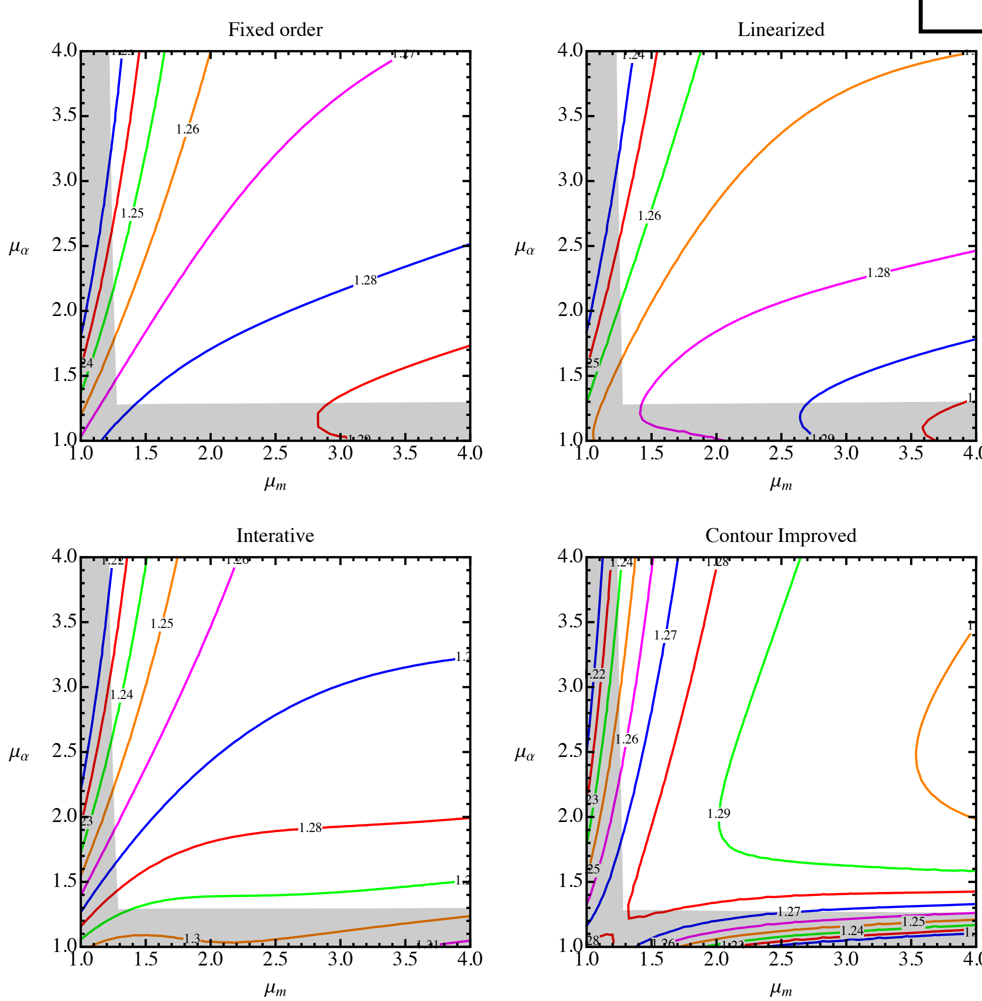
[Dehnadi, Hoang, VM]

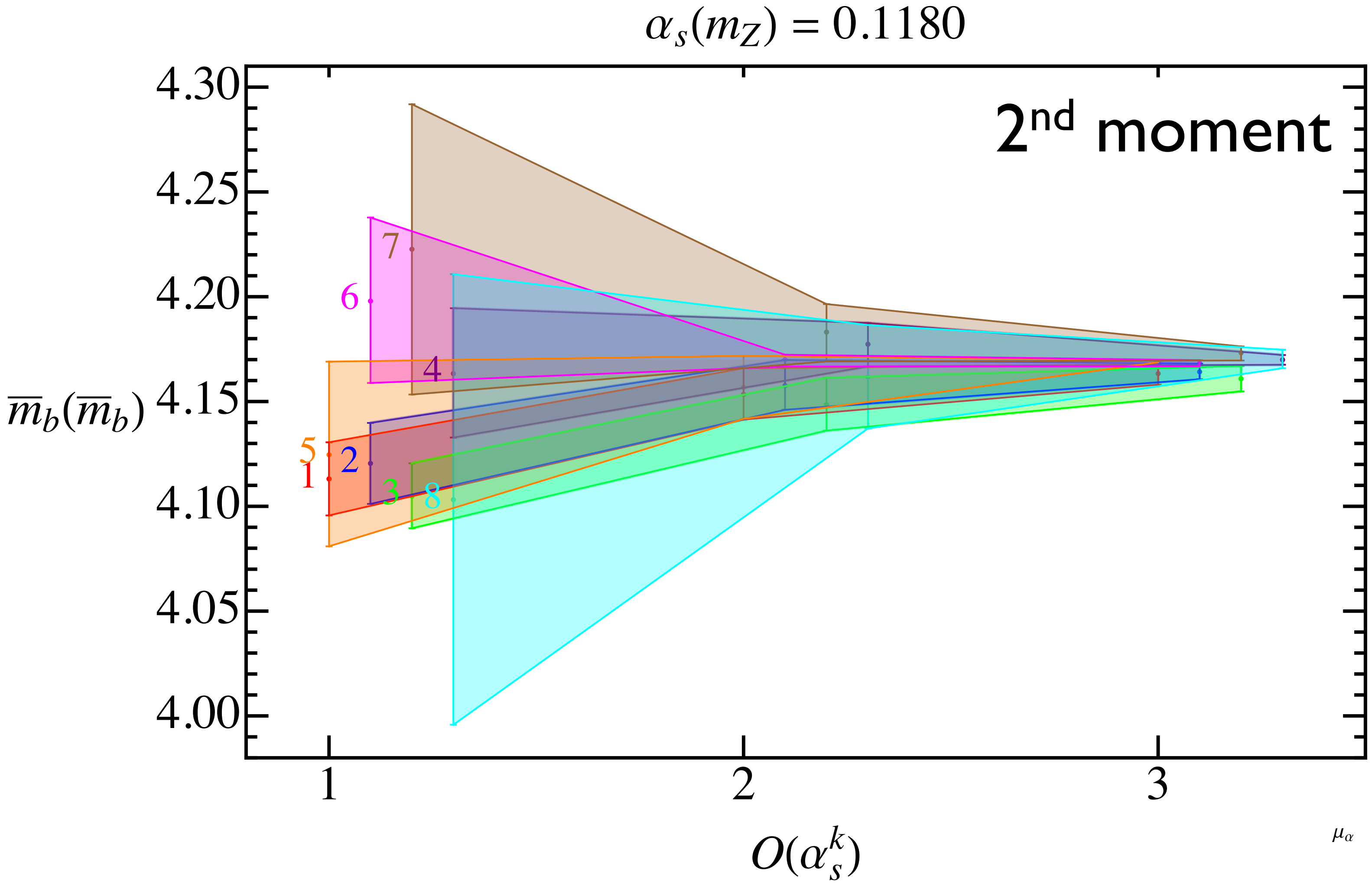




Identical conclusions:

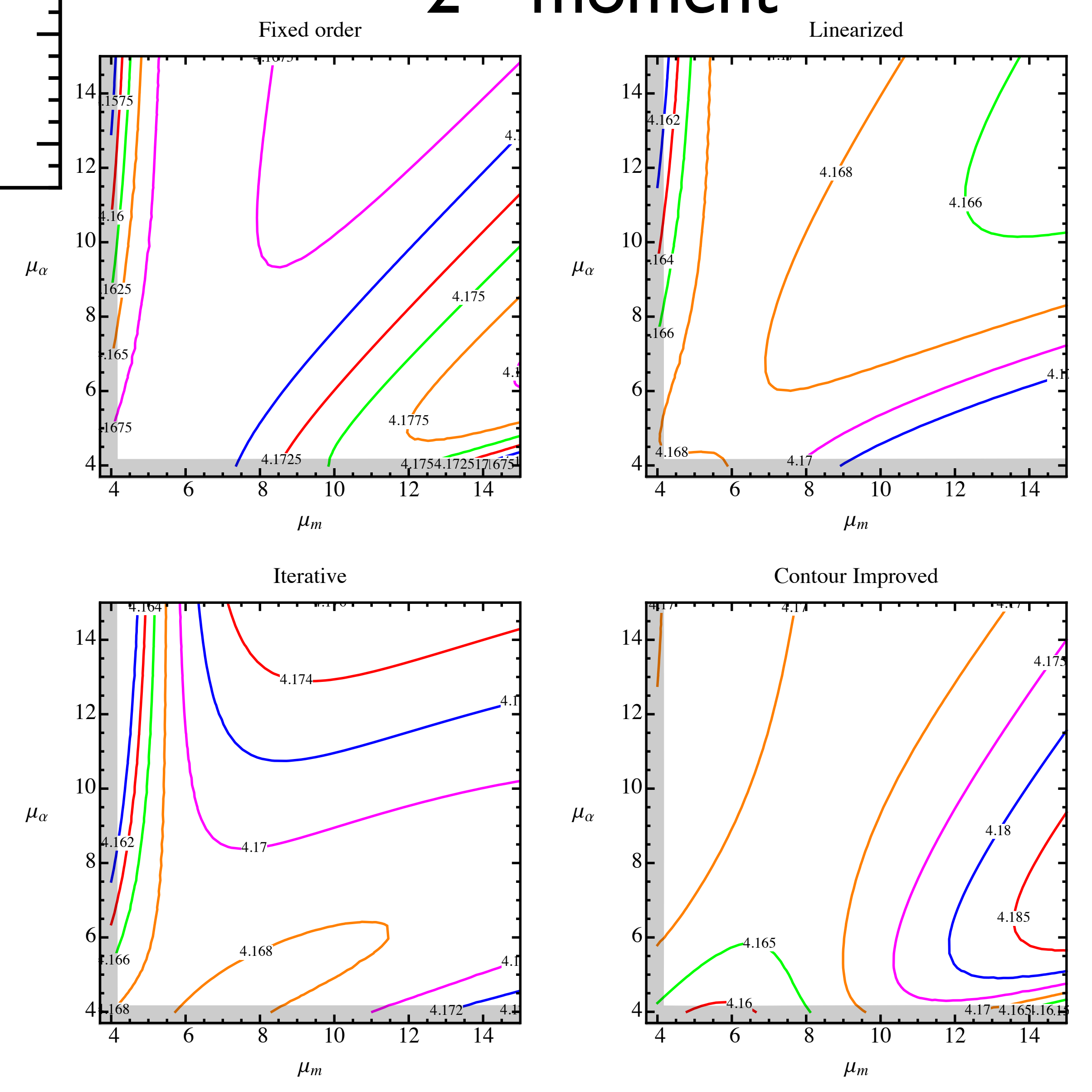
- Correlated variation underestimates error
- Sticking to a single expansion can be misleading
- Double variation is the most appropriate

Exploration of scale variation



$\pm 10\,\mathrm{MeV}$

2nd moment



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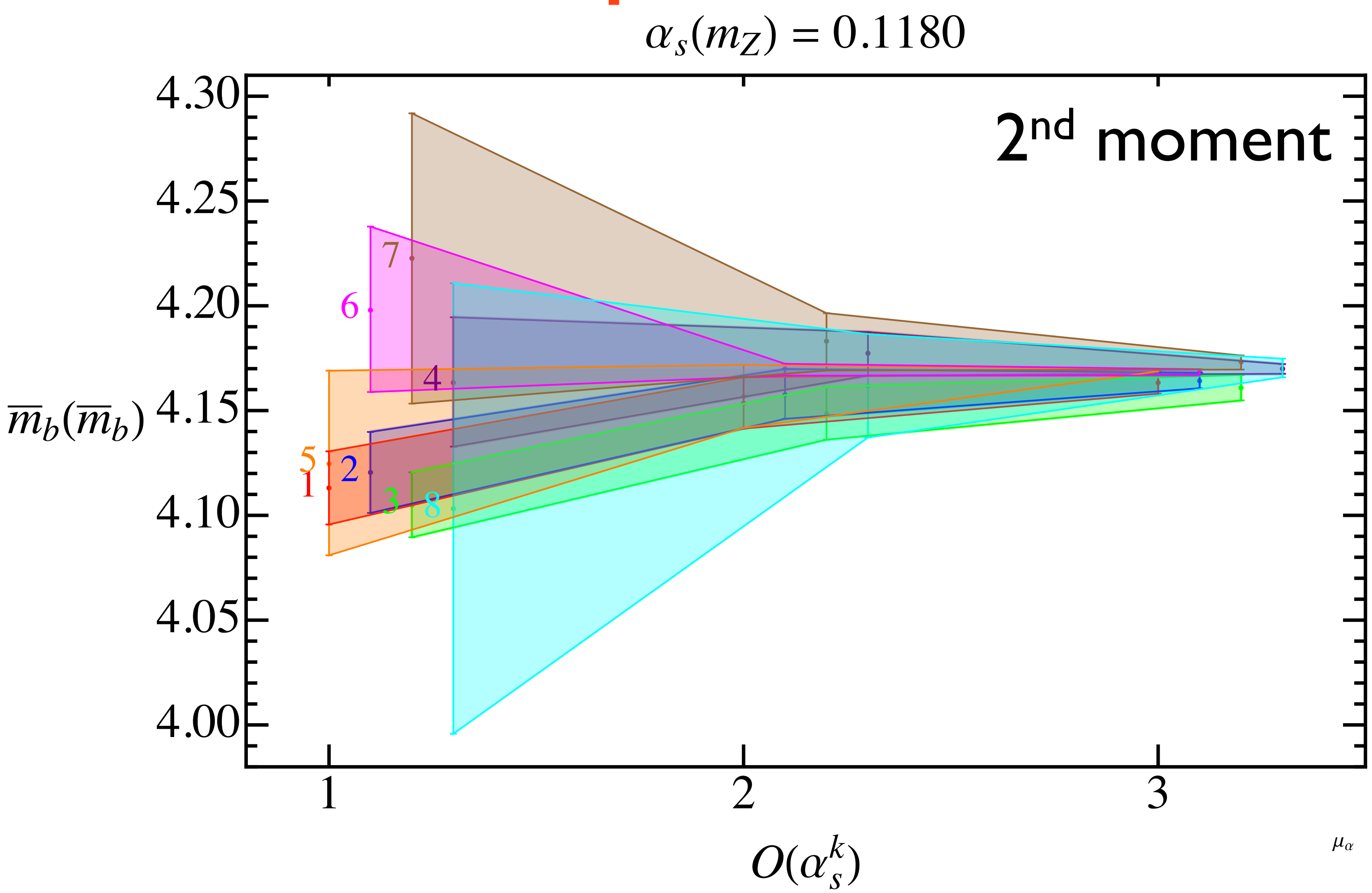
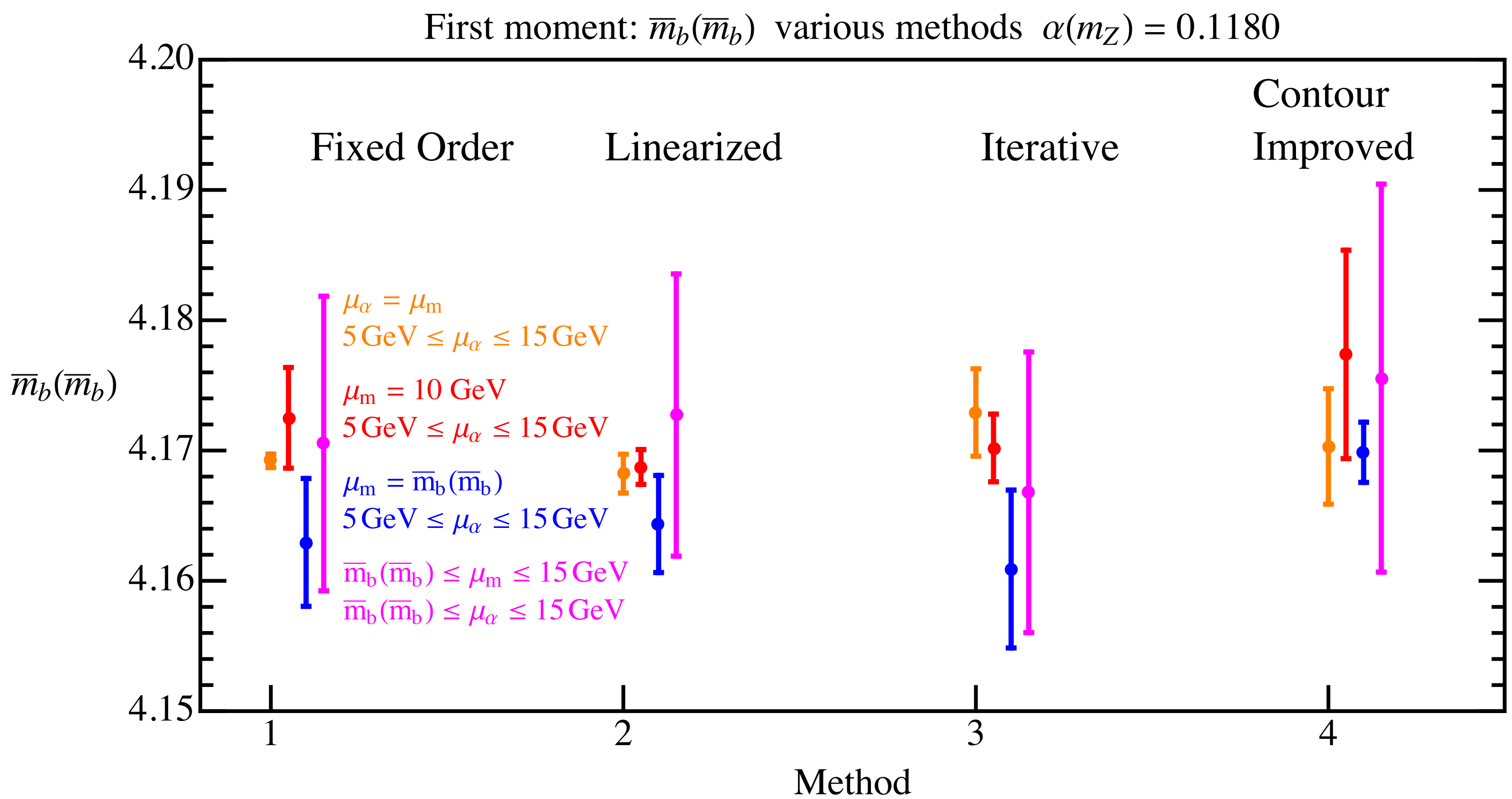
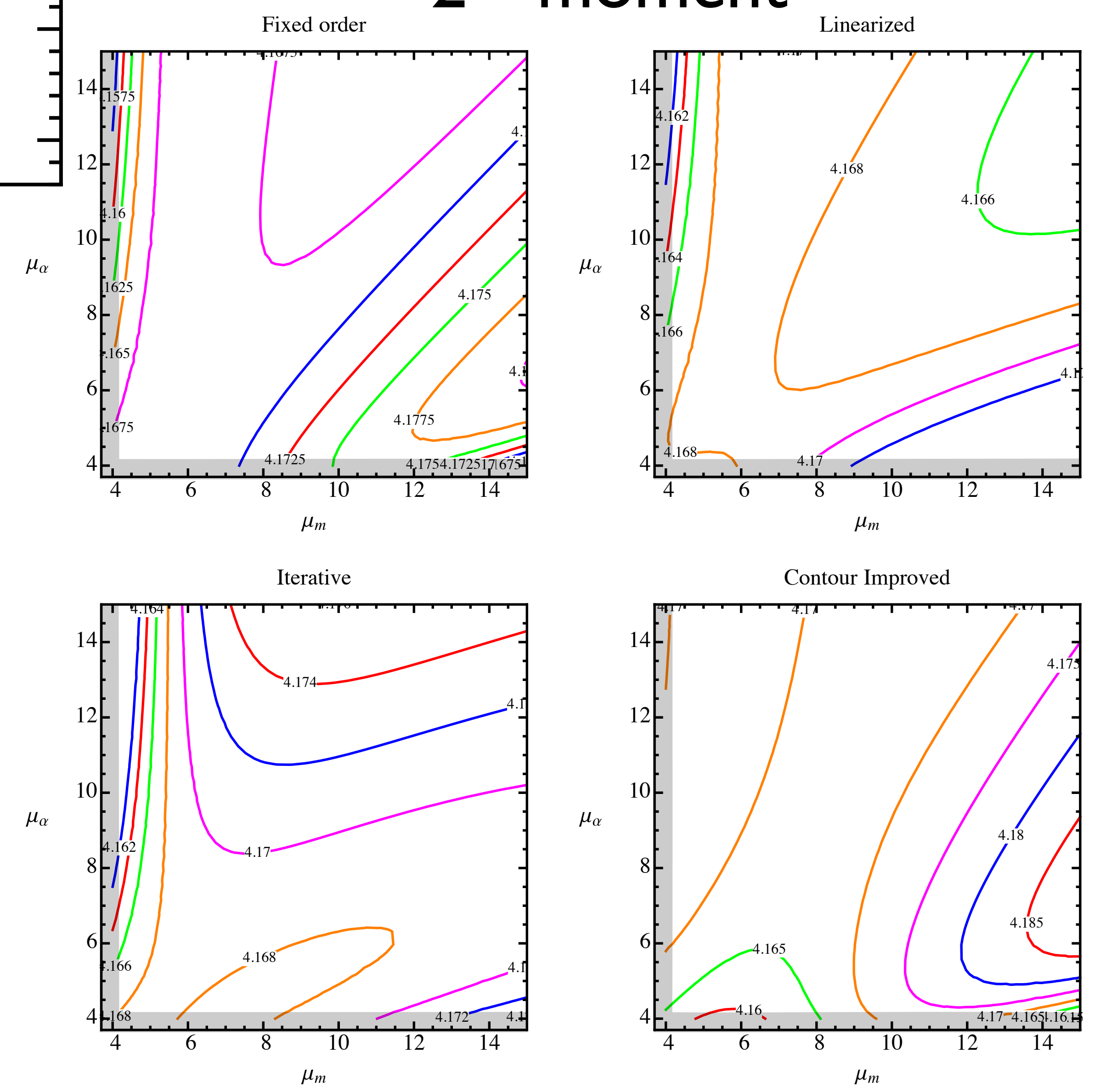


$\pm 10\,\mathrm{MeV}$



2nd moment



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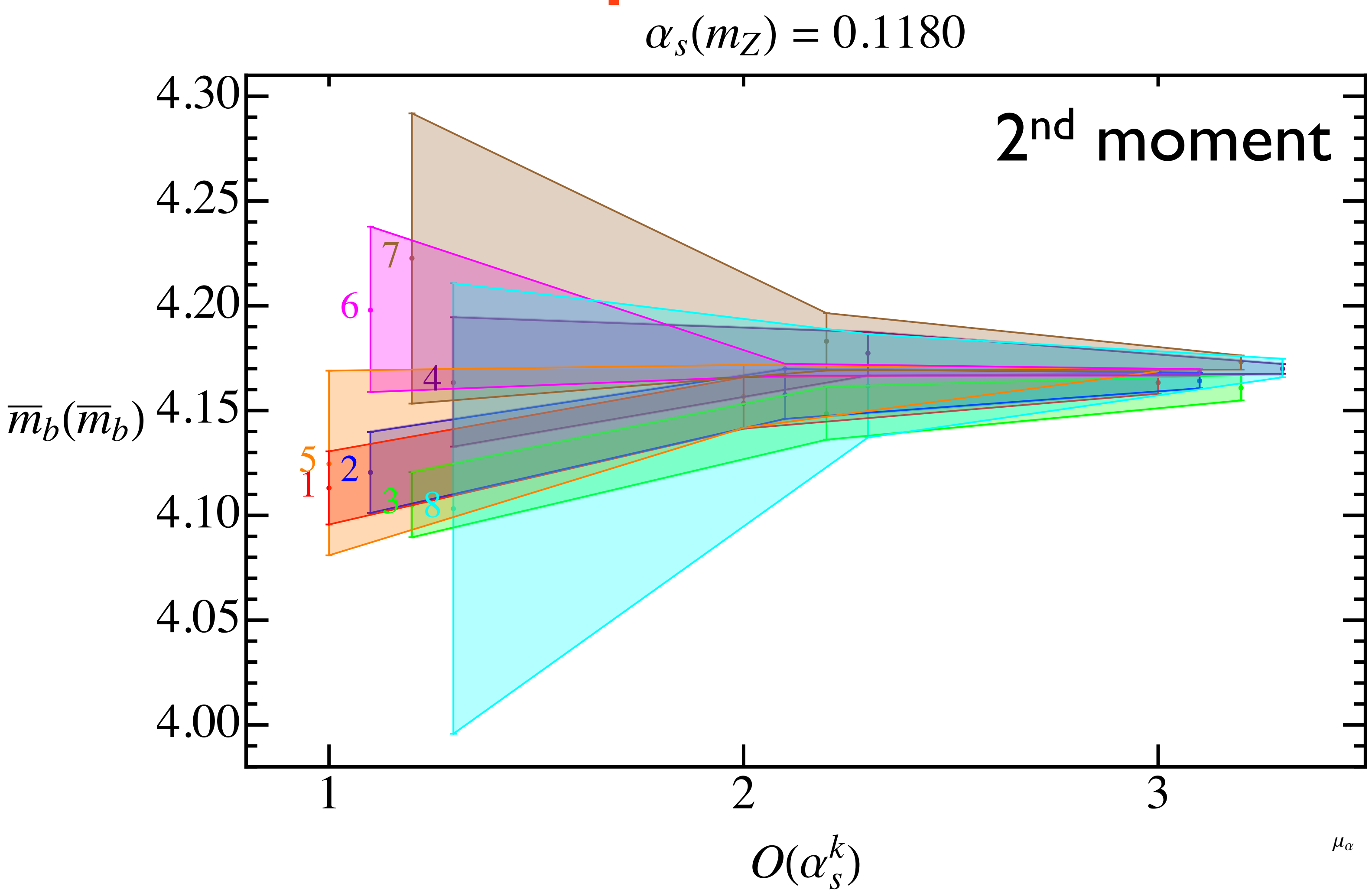

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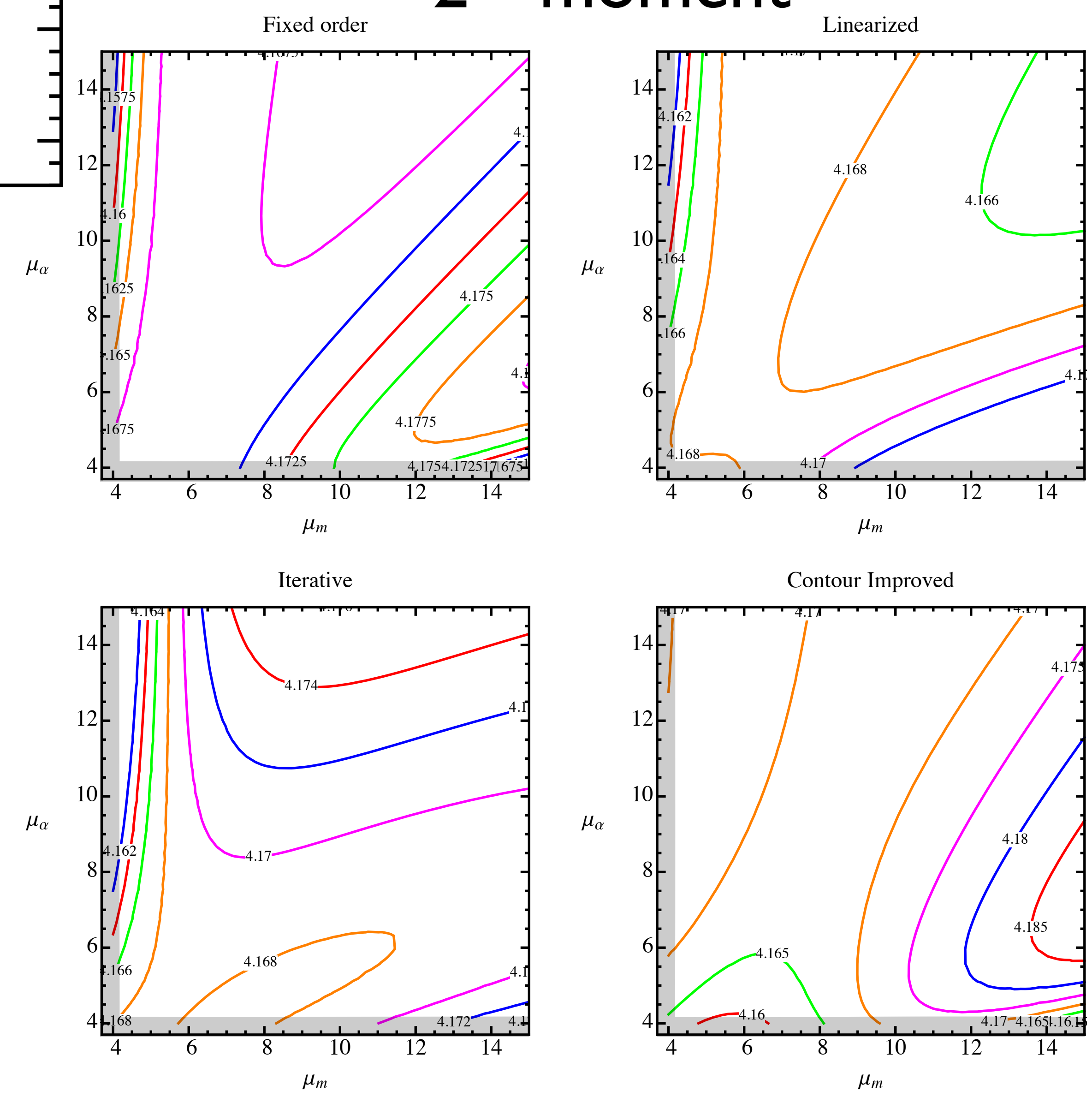
2nd moment



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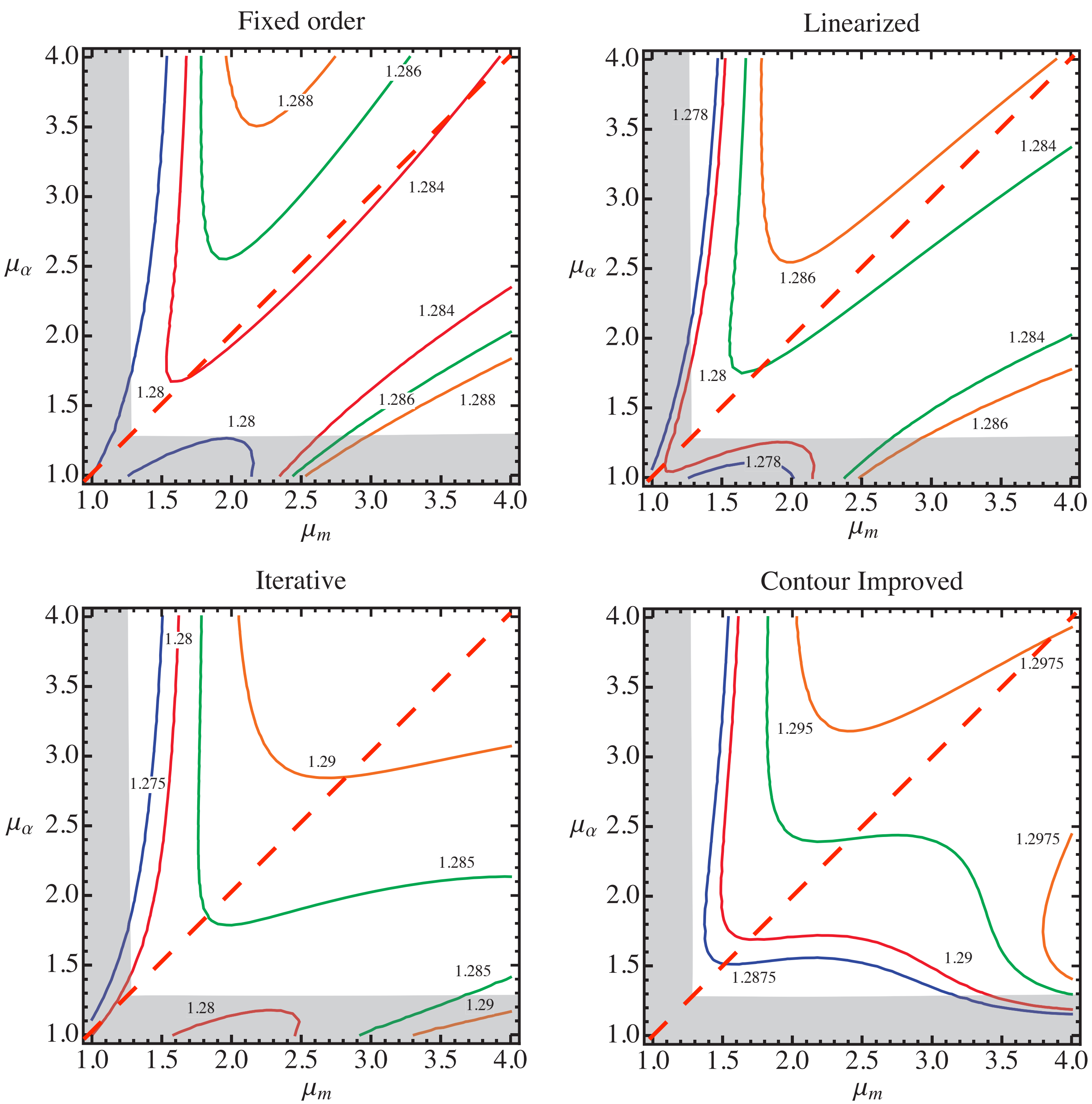
Contours in scale variations

[Dehnadi, Hoang, VM, Zebarjad]

Aims of our analysis

- Include all experimental data
- Define proper scale variation (such that different expansions give the same answer

1[st] moment at $\mathcal{O}(\alpha_s^3)$



Standard FO expansion:

Correlated $\mu_\alpha = \mu_m$ variation along a count our line

Behavior different for other types of expansions

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[Dehnadi, Hoang, VM, Zebarjad]

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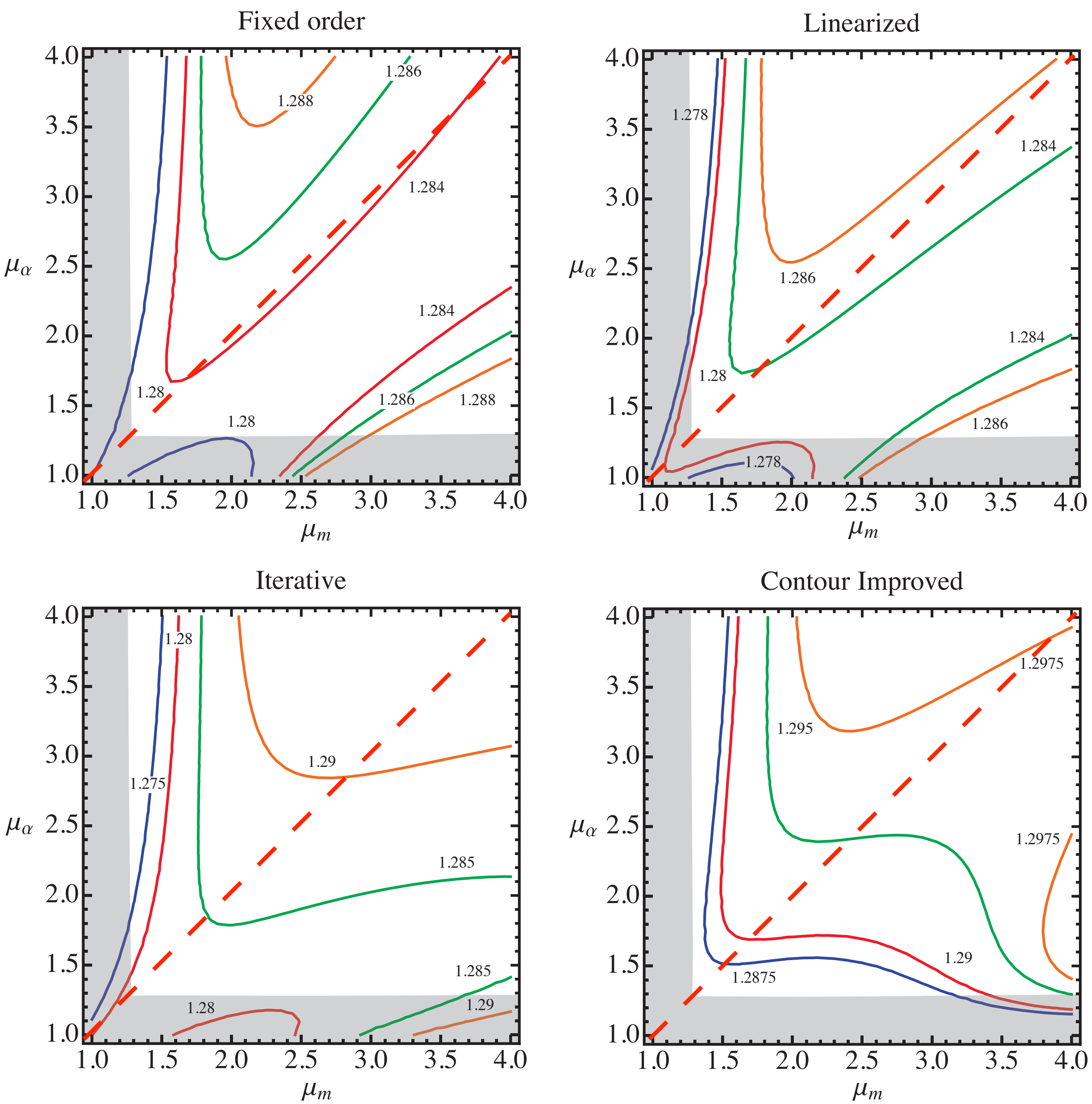
I[st] moment at $\mathcal{O}(\alpha_s^3)$



Standard FO expansion:

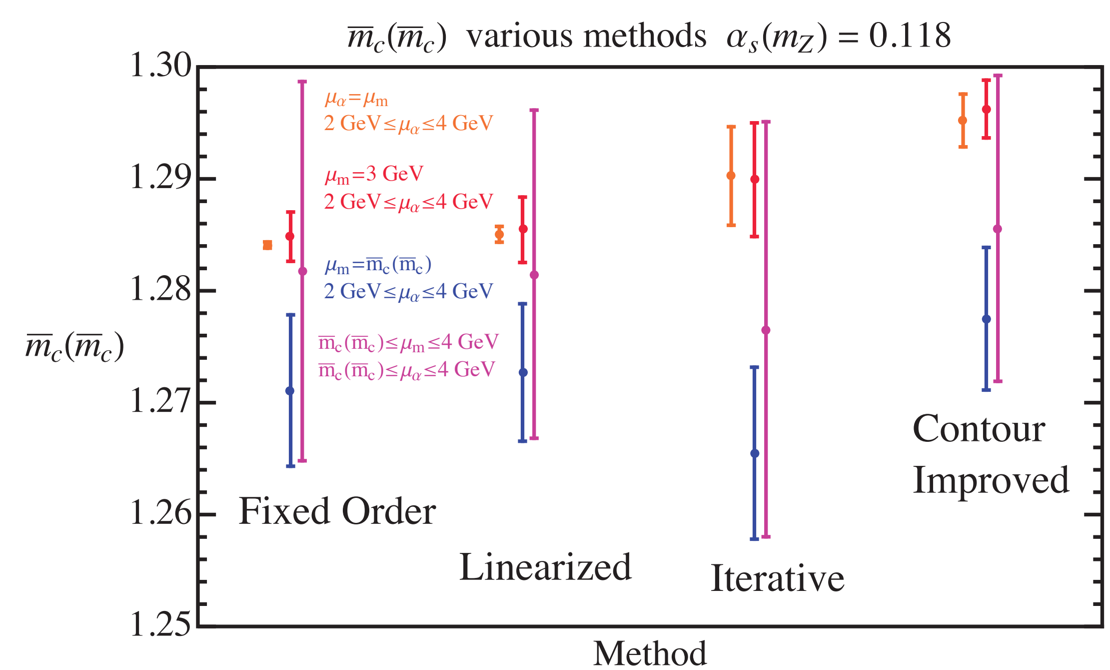
Correlated $\mu_\alpha = \mu_m$ variation along a count our line

Behavior different for other types of expansions

Our conclusions

Independent variations of μ_α and μ_m

Reasonable choice: $\overline{m}_c(\overline{m}_c) \leq \mu_\alpha, \mu_m \leq 4\,\mathrm{GeV}$



Finite Energy Sum Rules

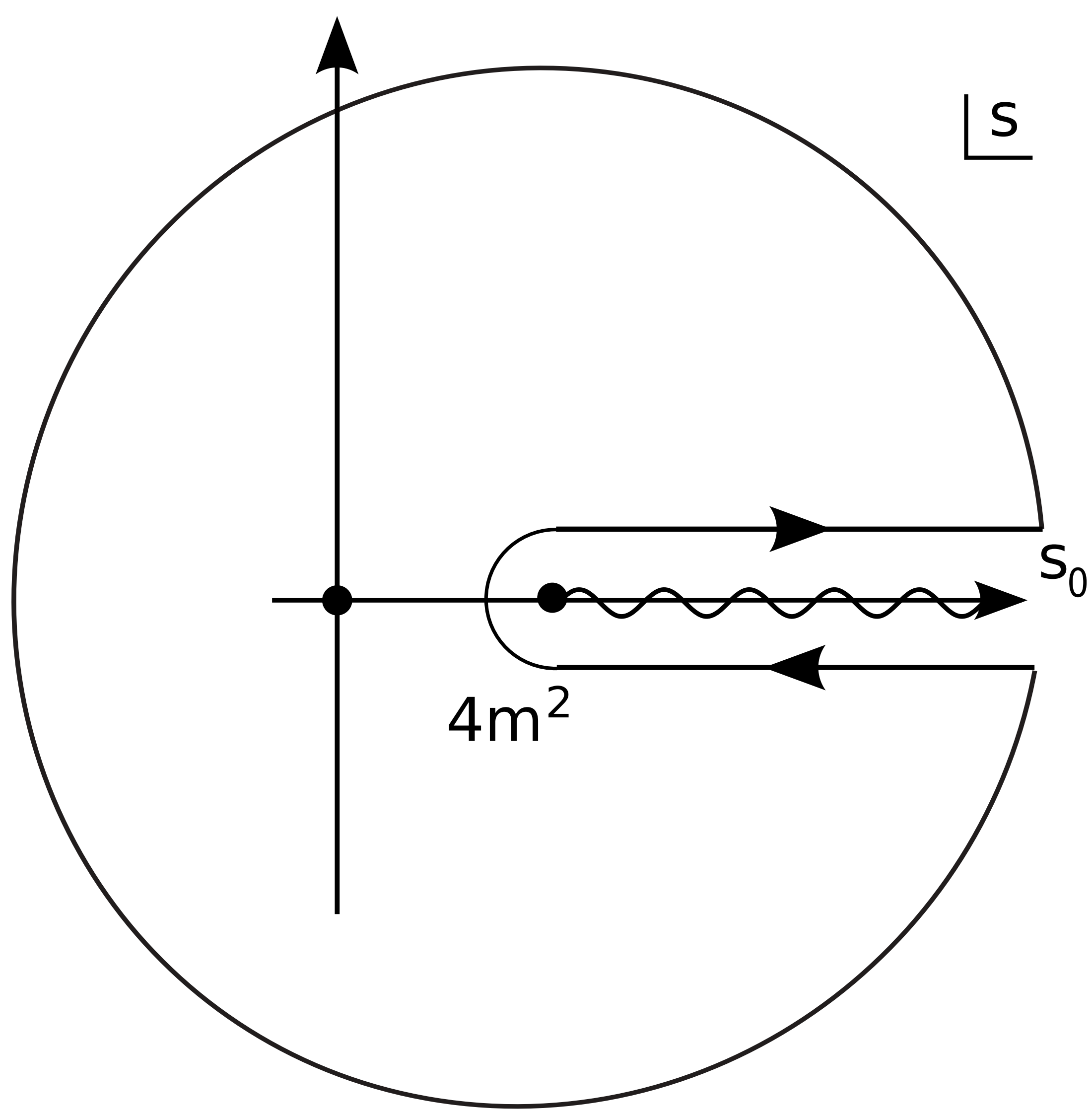
[Bodenstein et al]

$$\frac{1}{12\pi^2 Q^2}\int_0^{s_0} \mathrm{d}s\, p(s) R(s) = \frac{i}{2\pi}\oint_{\mathcal{C}} \mathrm{d}s\, p(s)\Pi(s) + \mathrm{Res}[\,p(s)\Pi(s), s=0\,]$$

Computed using experimental data

Essentially high-energy contribution, but using pQCD

Essentially "regular" moments



Finite Energy Sum Rules

[Bodenstein et al]

$$\frac{1}{12\pi^2 Q^2}\int_0^{s_0} \mathrm{d}s\, p(s) R(s) = \frac{i}{2\pi}\oint_{\mathcal{C}} \mathrm{d}s\, p(s)\Pi(s) + \mathrm{Res}[\, p(s)\Pi(s), s=0\,]$$

Computed using experimental data

Essentially high-energy contribution, but using pQCD

Essentially "regular" moments

- Motivated by tau analyses
- Cut off high-energy continuum where there is no data
- Modify weight resonances vs continuum
- Consistency requires $p(s_0) = 0$ (polynomial + pole)
- Higher order polynomials have larger condensate contributions
- In practice very similar to regular moments

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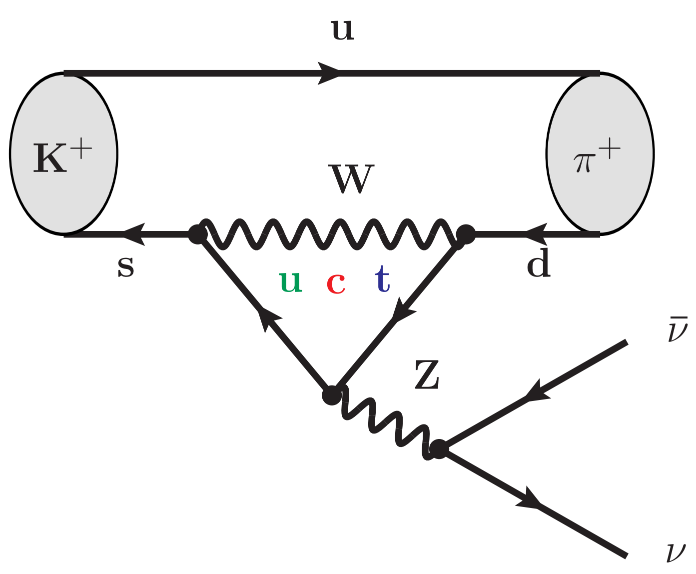
- Method use both in charm and bottom
- Theory input known at $\mathcal{O}(\alpha_s^3)$
- Conclusions drown for regular moments can be extrapolated for FESR

Why high precision?

$K^+ \to \pi^+ \nu \bar{\nu}$ NNLO QCD computations for charm distributions



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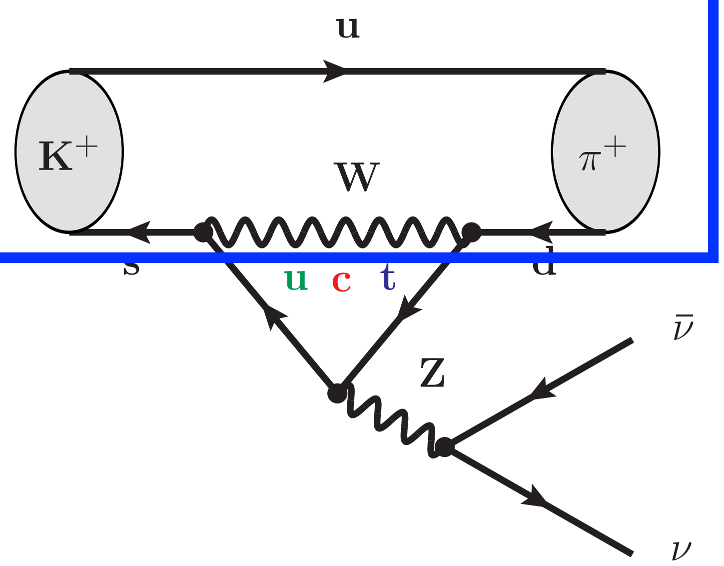
long distance effects



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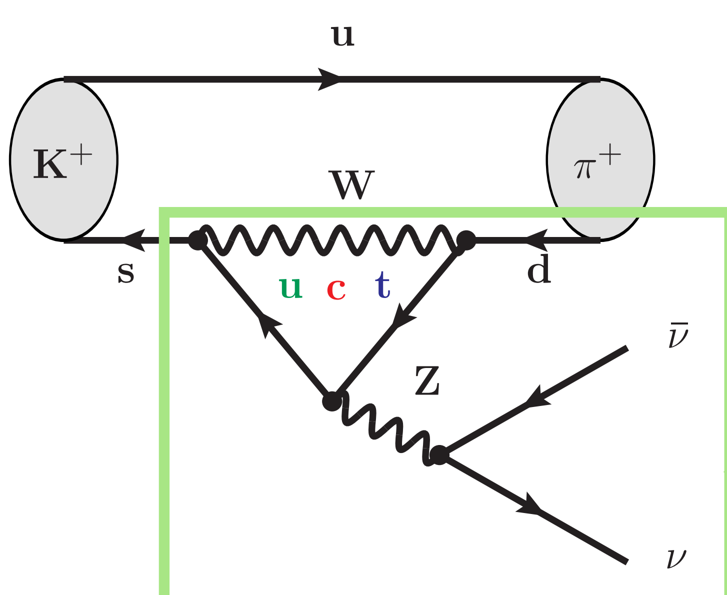
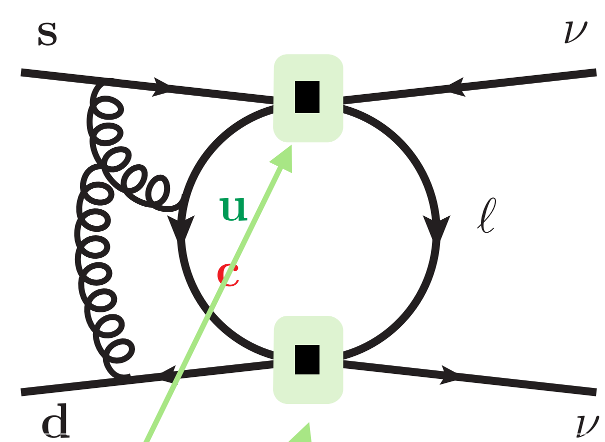
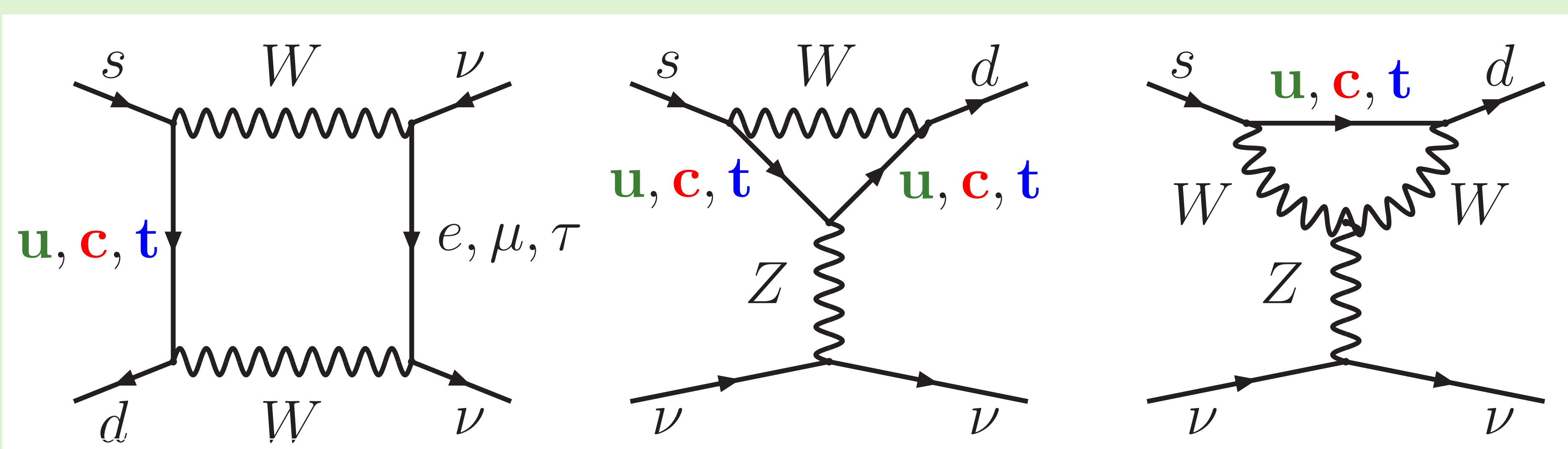


short distance



QCD corrections

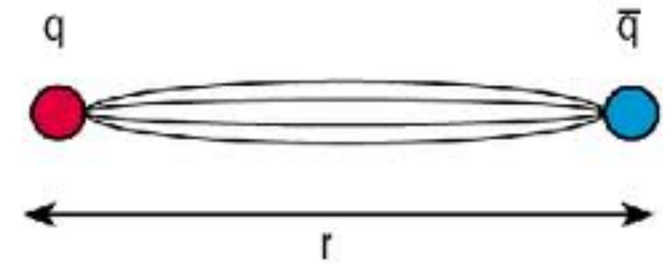
[Missiak & Gambino]



effective Hamiltonian

Theoretical remarks

Confinement: m_q not a physical observable



Parameter in QCD Lagrangian ⟶ formal definition (as for α_s)

$$\mathcal{L}_{\rm QCD} = \frac{1}{4} G^{a}_{\mu\nu} G^{\mu\nu}_{a} + \sum_f \bar{q}_f (\not{D} - m_f) q_f$$

Renormalization and scheme dependent object ⟶ $\delta m_q < \Lambda_{\rm QCD}$ possible

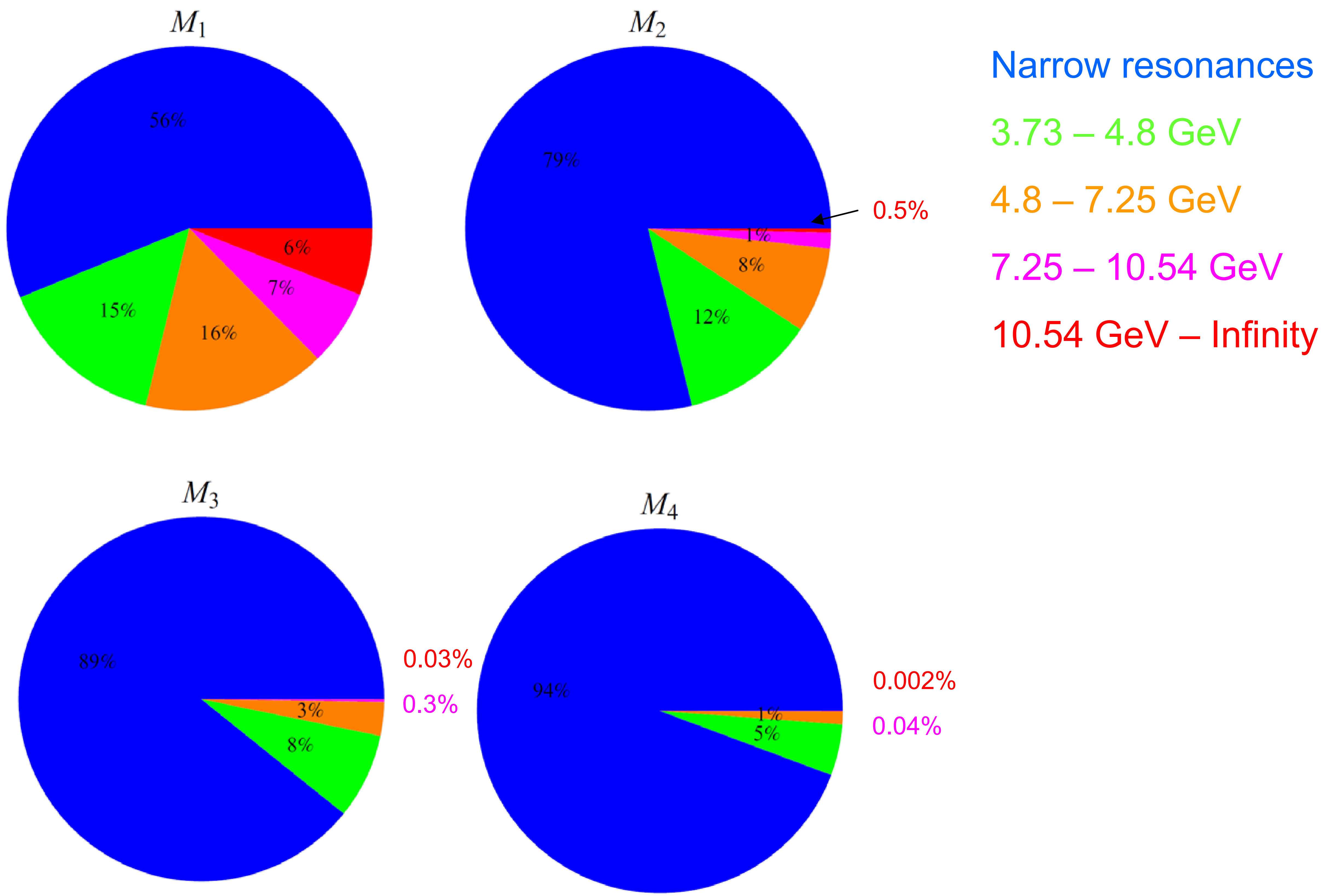
In general running mass $m(\mu)$ (RG evolution)

$$m_q^{\rm scheme\ A}(\mu) = m_q^{\rm scheme\ B}(\mu) \left[1 + \alpha_s(\mu) f_1(L) + \alpha_s(\mu)^2 f_2(L) + \ldots \right]$$

$$L = \log\left(\frac{m}{\mu}\right)$$

Only interested in short-distance schemes, which do not suffer from the $\mathcal{O}(\Lambda_{\rm QCD})$ renormalon problem inherent to the pole mass scheme

Moments budget



Methods in perturbation theory

Fixed order expansion

$$M_n^{\mathrm{pert}} = \frac{1}{(4\overline{m}_c^2(\mu_m))^n}\sum_{i,a,b}\left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i C_{n,i}^{a,b}\,\ln^a\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_m^2}\right)\ln^b\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_\alpha^2}\right)$$

Linearized expansion

$$\left(M_n^{\mathrm{th,pert}}\right)^{1/2n} = \frac{1}{2\overline{m}_c(\mu_m)}\sum_{i,a,b}\left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \tilde{C}_{n,i}^{a,b}\ln^a\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_m^2}\right)\ln^b\left(\frac{\overline{m}_c^2(\mu_m)}{\mu_\alpha^2}\right)$$

Iterative linearized expansion

$$\overline{m}_c^{(0)} = \frac{1}{2\left(M_n^{\mathrm{th,pert}}\right)^{1/2n}}\tilde{C}_{n,0}^{0,0}$$

$$\overline{m}_c(\mu_m) = \overline{m}_c^{(0)}\sum_{i,a,b}\left(\frac{\alpha_s(\mu_\alpha)}{\pi}\right)^i \hat{C}_{n,i}^{a,b}\,\ln^a\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_m^2}\right)\ln^b\left(\frac{\overline{m}_c^{(0)\,2}}{\mu_\alpha^2}\right)$$

residual dependence on μ_α due to truncation of series in α_s

Contour improved expansion

$$M_n^{\mathrm{c,pert}} = \frac{6\pi Q_q^2}{i}\oint_{\mathcal{C}}\frac{\mathrm{d}s}{s^{n+1}}\Pi[\,s,\alpha_s(\mu_\alpha^c(s,\overline{m}_c^2)),\overline{m}_c(\mu_m),\mu_\alpha^c(s,\overline{m}_c^2),\mu_m\,]$$

$$(\mu_\alpha^c)^2(s,\overline{m}_c^2) = \mu_\alpha^2\left(1-\frac{s}{4\overline{m}_c^2(\mu_m)}\right)$$

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renders correct μ_m dependence

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