Heavy-to-Light Transitions in Continuum QCD

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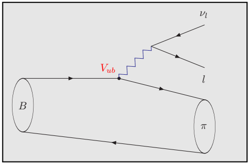
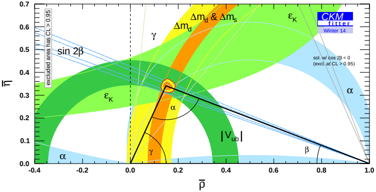
Lattice meets Continuum, Siegen 29.09.-02.10.14

Table of Contents

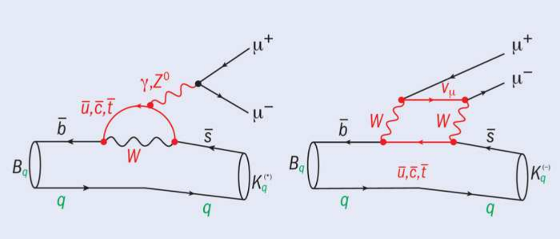
Why is this interesting?

- constraining CKM matrix by tree level decays
  - e.g. $|V_{ub}|$ via $B \to \pi l \nu$





- new physics in loop dominated decays?
  - e.g. via $B \to K^{(*)}\mu^+\mu^-$



Interesting Channels (A non complete list...)

- tree level decays

$$B \to \pi l\nu \quad B \to \rho(\omega) l\nu \quad B_s \to K l\nu$$

$$D \to \pi l\nu \quad D \to K l\nu \quad D_s \to \eta^{(\prime)} l\nu$$

$$\ldots$$

- loop decays

$$B \to K\mu^+\mu^- \quad B \to K^*\mu^+\mu^- \quad B \to K^*\gamma$$
$$\ldots$$

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- loop decays

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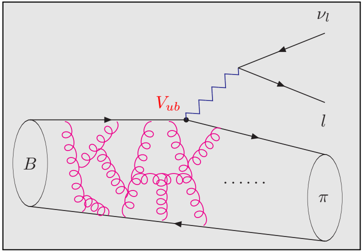
- main topic will be calculation of form factors and tree level decays

B$\to\pi$ l ν-decay as an easy example



- hadronic matrix element needed

$$\langle\pi(p)|\bar{u}\,\gamma_\mu\, b|\bar{B}(p+q)\rangle = \frac{m_B^2-m_\pi^2}{q^2} q_\mu f^0_{B\pi}(q^2) + \left(p_B+p-\frac{m_B^2-m_\pi^2}{q^2}q\right)_\mu f^+_{B\pi}(q^2)$$

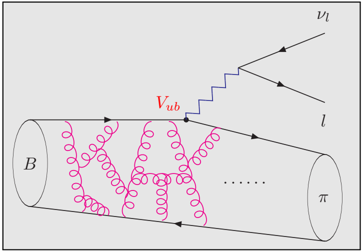
- $f^+_{B\pi}(q^2)$, $f^0_{B\pi}(q^2)$: form factors

- lepton spectrum

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2\,|V_{ub}|^2}{24\pi^3 q^4 m_B^2}(q^2-m_l^2)^2|\vec{p}_\pi| \times \left[\left(1+\frac{m_l^2}{q^2}\right)m_B^2|\vec{p}_\pi|^2\,|f^+_{B\pi}(q^2)|^2+\frac{3m_l^2}{8q^2}(m_B^2-m_\pi^2)^2|f^0_{B\pi}(q^2)|^2\right]$$

$$|\vec{p}_\pi| = \frac{1}{2m_B}\sqrt{(m_B^2+m_\pi^2-q^2)^2-4m_B^2m_\pi^2}, \quad 0\le q^2\le(m_B-m_\pi)^2\simeq 26.4\,\mathrm{GeV}^2$$

B$\rightarrow \pi$ l ν-decay as an easy example



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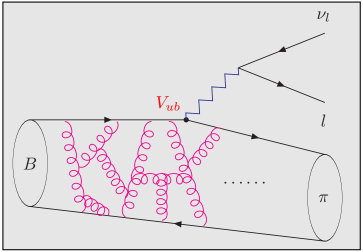

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$$0 \leq q^2 \leq (m_B-m_\pi)^2 \simeq 26.4\,\mathrm{GeV}^2$$

$B \to V$ form factors

- $V - A$ form factors (e.g. $B \to \rho l \nu$, $B \to K^* \mu^+ \mu^-$)

$$\langle V(p)|(V-A)_\mu|B(p_B)\rangle = -i\epsilon^*_\mu (m_B + m_V) A_1^V(q^2) + i(p_B + p)_\mu (\epsilon^* p_B) \frac{A_2^V(q^2)}{m_B + m_V} + iq_\mu (\epsilon^* p_B) \frac{2m_V}{q^2} \left(A_3^V(q^2) - A_0^V(q^2)\right) + \epsilon_{\mu\nu\rho\sigma} \epsilon^{*\nu} p_B^\rho p^\sigma \frac{2V^V(q^2)}{m_B + m_V}.$$

$$A_3^V(q^2) = \frac{m_B + m_V}{2m_V} A_1^V(q^2) - \frac{m_B - m_V}{2m_V} A_2^V(q^2),$$

$$A_0^V(0) = A_3^V(0),$$

$$\langle V|\partial_\mu A^\mu|B\rangle = 2m_V(\epsilon^* p_B) A_0^V(q^2)$$

- penguin form factors (e.g. $B \to K^* \mu^+ \mu^-$)

$$\langle V|\bar{\psi}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|B(p_B)\rangle = i\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu} p_B^\rho p^\sigma \, 2T_1(q^2) + T_2(q^2)\left\{\epsilon^*_\mu (m_B^2 - m_V^2) - (\epsilon^* p_B)(p_B + p)_\mu\right\} + T_3(q^2)(\epsilon^* p_B)\left\{q_\mu - \frac{q^2}{m_B^2 - m_V^2}(p_B + p)_\mu\right\}$$

$$T_1(0) = T_2(0)$$

What do we know about form factors?

- form factors factorize to all orders of α_s and leading order in $\frac{\Lambda}{m_b}$

$$\boxed{f^{\pm}_{B\pi} = C^{\pm}\,\xi_{B\pi} + \phi_B \star T^{\pm} \star \phi_\pi}$$

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$$\boxed{f^{\pm}_{B\pi} = C^{\pm}\xi_{B\pi} + \phi_B \star T^{\pm} \star \phi_\pi}$$

- hard part calculable in perturbation theory
- soft part fullfills SCET symmetry relations
  - broken by radiative corrections and power corrections
  - splitting into soft-, hard and power corrections scheme dependent
- soft part and power corrections not calculable in perturbation theory
  - optimized observables: form factor drops out at leading power

How can we calculate them in the continuum?

Light Cone Sum Rules

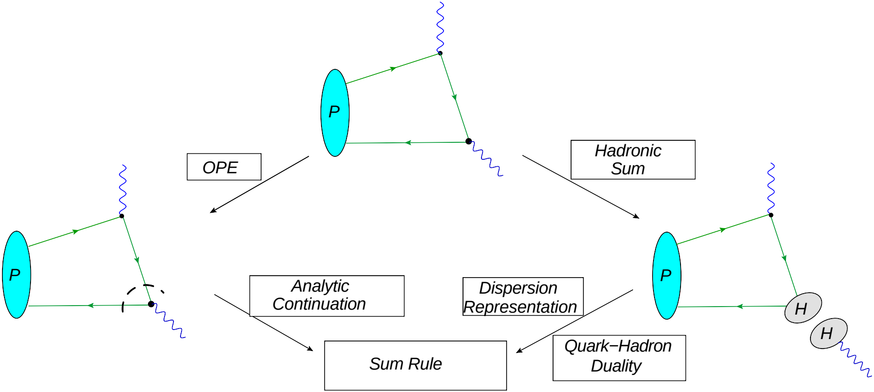
A variant: SCET sum rules

- ▷ use analyticity, operator product expansion and quark-hadron duality
- ▷ calculate soft and hard part in terms of same DAs
- ▷ no double counting
- ▷ systematic improvement possible

though limited accuracy $\sim$ 10-15 %

What are these Light Cone Sum Rules?

$$T_\mu(P,q) = i\int d^4x e^{iq\cdot x}\langle P|\,T\{j_\mu(x)j_B(0)\}|\rangle$$



$$F(Q^2) = \int_0^{s_0} ds e^{-\frac{s-m_P^2}{M^2}} \int [Dx] \sum_t \mathrm{Im}\, T_t(x_i, Q^2, \mu^2)\phi_t(x_i,\mu^2)$$

How does one get the hadronic dispersion integral?

- starting point: correlation function

$$\begin{aligned} F_\mu(p,q) &= i\int d^4x e^{iq\cdot x}\langle\pi(p)|\mathrm{T}\left\{\bar{u}(x)\gamma_\mu b(x), \bar{b}(0) i m_b \gamma_5 d(0)\right\}|0\rangle \\ &= F(q^2,(p+q)^2)\, p_\mu \;+\; \widetilde{F}(q^2,(p+q)^2)\, q_\mu \end{aligned}$$

How does one get the hadronic dispersion integral?

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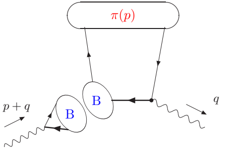
$$F_\mu(p,q) = i\int d^4x e^{iq\cdot x}\langle\pi(p)|\mathrm{T}\left\{\underbrace{\bar{u}(x)\gamma_\mu b(x), \bar{b}(0)im_b\gamma_5 d(0)}_{\sum|B(p+q)\rangle\langle B(p+q)|}\right\}|0\rangle$$

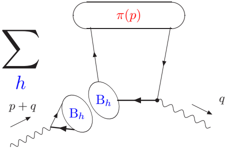
$$= F(q^2,(p+q)^2)\,p_\mu + \widetilde{F}(q^2,(p+q)^2)\,q_\mu$$

- inserting complete set of B-meson states $\to$ hadronic sum

$F(q^2,(p+q)^2) =$



$$\underbrace{}_{\dfrac{m_B^2 f_B f^+_{B\pi}(q^2)}{m_B^2 - (p+q)^2}} + \sum_h$$



$$\underbrace{}_{\int_{s_0^h}^{\infty} ds\,\dfrac{\rho^h(q^2,s)}{s-(p+q)^2}\;(*)}$$

$(*)$ subtraction terms to render dispersion integral finite not shown

Light-cone expansion

- $q^2 \ll m_b^2$, $(p+q)^2 < 0$, $|(p+q)^2| \gg \Lambda$ contraction of quark-fields

$$
\begin{aligned}
& i\int d^4x\, e^{iqx} \langle \pi(p)|\mathrm{T}\{\bar{u}(x)\gamma_\mu b(x), \bar{b}(0) i m_b \gamma_5 d(0)\}|0\rangle \\
= \quad & i m_b \int d^4x\, e^{iqx} \langle \pi(p)|\bar{u}(x)\gamma_\mu \mathrm{S}(x, m_b)\gamma_5 d(0)|0\rangle
\end{aligned}
$$

- factorization: into nonlocal matrix elements of different twist and hard scattering kernels

$$
\begin{aligned}
\langle \pi(p)|\bar{u}(x)\Gamma d(0)|0\rangle \quad &\sim \quad \varphi_\pi^{(t)}(u) \quad \text{Twist } t = 2, 3, 4, \dots \\
\langle \pi(p)|\bar{u}(x) G_{\mu\nu}(vx)\Gamma d(0)|0\rangle \quad &\sim \quad \Phi_{3\pi}^{(t)}(\alpha_i) \quad \text{Twist } t = 3, 4, \dots
\end{aligned}
$$

- general form : convolution

$$
F^{(OPE)}(q^2, (p+q)^2) = \sum_{t=2}^{4} \int Du_i \sum_{k=0,1} \left(\frac{\alpha_s}{\pi}\right)^k T_k^{(t)}(q^2, (p+q)^2, u_i)\varphi_\pi^{(t)}(u_i)
$$

What are these distribution amplitudes?

- generating functions of matrix elements of local operators

$$\langle \pi(p)|\bar{u}(x)\gamma_\mu\gamma_5 d(0)|0\rangle_{x^2=0} = \sum_n \frac{1}{n!}\langle \pi(p)|\bar{u}(\overleftarrow{D}\cdot x)^n\gamma_\mu\gamma_5 d|0\rangle$$
$$= i p_\mu f_\pi \int_0^1 du\, e^{iup\cdot x}\phi_\pi(u,\mu)$$

- conformal symmetry of massless QCD implies expansion in Gegenbauer-polynomials

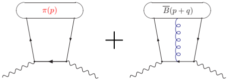
$$\phi_\pi(u,\mu) = 6u\bar{u}\left(1 + \sum_{n=2,4,\dots} a_n(\mu) C_n^{3/2}(2u-1)\right)$$

- moments $a_n(\mu)$ renormalized multiplicatively at LO
- give main parametric uncertainty of LCSRs
- lowest moments can be calculated on the lattice

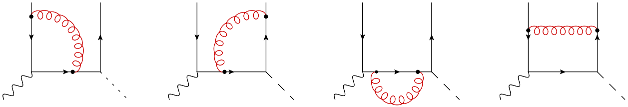
Paula's talk

What is the state of the art?

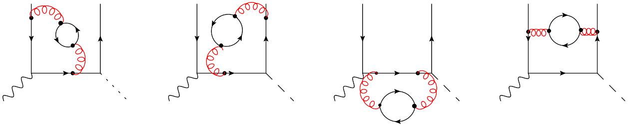
- twist-expansion at leading order up to twist 4
  - two- and three-particle contributions included



- α_s-corrections to two particle twist 2, 3 known for $B \to P, V$ form factors



- partial α_s^2-corrections to twist 2 known for $B \to P$ form factors at $q^2 = 0$ only



How can one equate OPE and hadronic result?

- Analytic continuation in $(p+q)^2$ via dispersion integral

$$F^{(OPE)}(q^2,(p+q)^2) \;=\; \frac{1}{\pi}\int_{m_b^2}^{\infty} ds\,\frac{\mathrm{Im}\,F^{(OPE)}(q^2,s)}{s\;-\;(p+q)^2}$$

(no subtraction terms shown)

- leads to following sum rule

$$\frac{2m_B^2 f_B f_{B\pi}^{+}(q^2)}{m_B^2\;-\;(p+q)^2} + \int_{s_0^h}^{\infty} ds\,\frac{\rho^h(s,q^2)}{s\;-\;(p+q)^2} \;=\; \frac{1}{\pi}\int_{m_b^2}^{\infty} ds\,\frac{\mathrm{Im}\,F^{(OPE)}(s,q^2)}{s\;-\;(p+q)^2}$$

- Borel transformation eliminates subtraction terms and suppresses higher states

$$B_{M^2}\left(\frac{1}{s-(p+q)^2}\right)^n = \frac{1}{(n-1)!}\,\frac{1}{(M^2)^{n-1}}e^{-s/M^2}, \quad B_{M^2}\left((p+q)^2\right)^n = 0$$

$\Rightarrow$ higher twist suppressed by powers of $\frac{1}{M^2}$

What is the final assumption?

- semilocal quark-hadron duality allows to approximate hadronic spectrum

$$\boxed{\int_{s_0^h}^{\infty} ds\, \rho^h(s,q^2)\, e^{-s/M^2} \simeq \frac{1}{\pi}\int_{s_0^B}^{\infty} ds\, \mathrm{Im}\, F^{(OPE)}(s,q^2)\, e^{-s/M^2}}$$

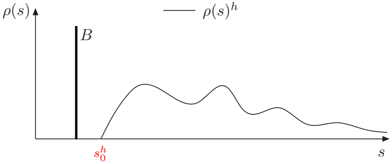
- s_0^B: duality threshold
- final expression:

sum rule

$$f_B f_{B\pi}^{+}(q^2) = \frac{e^{m_B^2/M^2}}{2m_B^2}\,\frac{1}{\pi}\int_{m_b^2}^{s_0^B} ds\, \mathrm{Im}\, F^{(OPE)}(s,q^2)\, e^{-s/M^2}$$

What are the inherent uncertainties?

- use of quark-hadron duality amounts to approximating complex hadronic spectral function



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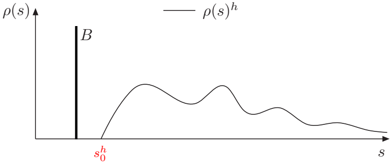

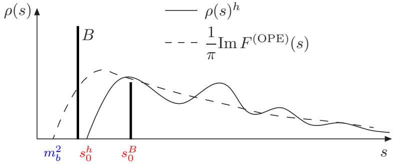

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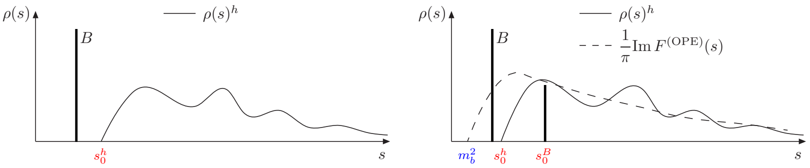

- via spectral function of OPE-result
- Borel-transformation suppresses higher states
  - sum rule not very sensitive to structure far from threshold
- approximation depends crucial on values of M^2 and s_0^B
  - no formal criteria to fix parameters
  - new ideas about combined fit to data and sum rules — Danny's talk
- uncertainty difficult to quantify — see Volodya's talk

What are the main uncertainties coming from the input?

- Gegenbauer moments of twist 2 and 3 DA

Method	$\mu = 2$ GeV	Reference
QCDSR	$0.17^{+0.14}_{-0.06}$	[Khodjamirian et al '04]
QCDSR	0.19 ± 0.05	[Ball,Braun,Lenz '06]
QCDSR,NLC	0.13 ± 0.04	[Mikhailov et al '04]
$F_{\pi\gamma\gamma^*}$,LCSR	$0.12 \pm 0.03_{\mu=2.4}$	[Schmedding '00]
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$F_{\pi\gamma\gamma^*}$,LCSR	0.3	[Bakulev et al '03]
$F_{\pi\gamma\gamma^*}$,LCSR	0.10	[Agaev '10]
$F_{\pi\gamma\gamma^*}$,LCSR,R	0.18	[Agaev '05]
F_π^{em},LCSR	$0.16 \pm 0.09 \pm 0.05$	[Khodjamirian et al '02]
F_π^{em},LCSR	0.11 ± 0.05	[Khodjamirian et al '11]
F_π^{em},LCSR,R	0.13 ± 0.02	[Agaev '05]
$F_{B\to\pi l\nu}$,LCSR	0.13 ± 0.13	[Ball, Zwicky '05]
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LQCD, quenched	$0.233 \pm 0.143^{+0.088}_{-0.038,\mu=2.67}$	[UKQCD '03]
LQCD, $N_f = 2$	0.201 ± 0.114	[QCDSF/UKQCD '06]
LQCD, $N_f = 2+1$	0.233 ± 0.088	[UKQCD '10]

Table: Different results for the Gegenbauer moments $a_2^\pi(\mu^2)$.

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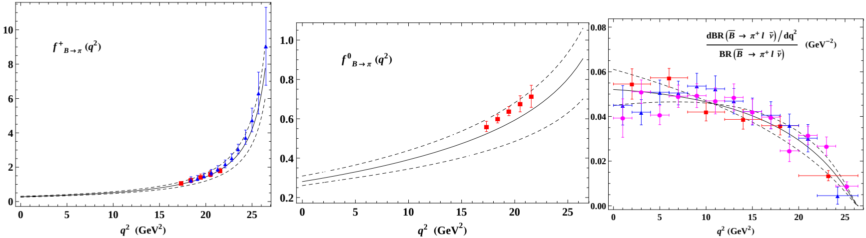
$B \to \pi$ results



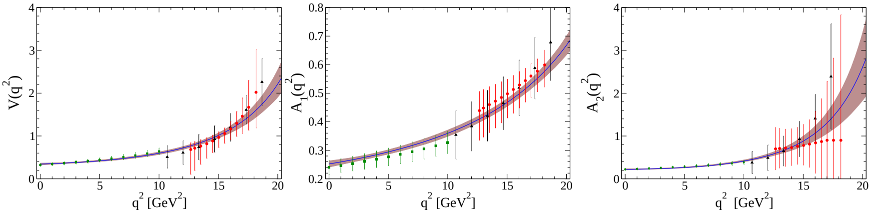
red squares: HPQCD 06, blue triangles: FNAL/MILC 09

- moments a_2, a_4 taken from fit to pion electromagnetic form factor
- results in

$$|V_{ub}| = (3.50^{+0.38}_{-0.33}|_{theor.} \pm 0.11) \times 10^{-3}$$

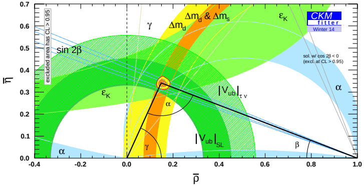
- new results with 2013 data in Danny's talk

$B \to \rho$ results



LCSR results (green squares), SPQcdR (black triangles), UKQCD (red circles)

- new Belle measurement
- $|V_{ub}|$ puzzle remains
- inclusive and $B \to \tau\nu_\tau$ still off



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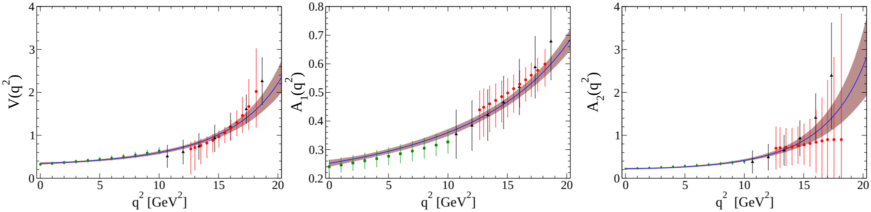

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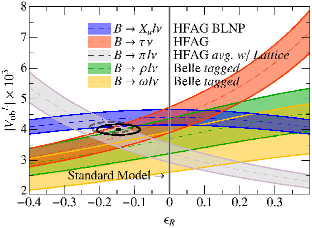
- $|V_{ub}|$ puzzle remains
- inclusive and $B \to \tau\nu_\tau$ still off



One comment on $B \to \tau \nu_\tau$

- define

$$R_{s/l}(q_1^2, q_2^2) \equiv \frac{\Delta\mathcal{B}_{B\to\pi\ell\nu_\ell}(q_1^2, q_2^2)}{\mathcal{B}(B\to\tau\nu_\tau)} \left(\frac{\tau_{B^-}}{\tau_{B^0}}\right)$$

- and get

Exp.	$\Delta\mathcal{B}(10^{-4})$	$\mathcal{B}(B\to\tau\nu_\tau)(10^{-4})$	$R_{s/l}$
BABAR	0.32 ± 0.03 $0.33 \pm 0.03 \pm 0.03$	1.76 ± 0.49	$0.20^{+0.08}_{-0.05}$
Belle	0.398 ± 0.03	$1.54^{+0.38}_{-0.37}{}^{+0.29}_{-0.31}$	$0.28^{+0.13}_{-0.07}$

QCD	$\Delta\zeta(\mathrm{ps}^{-1})$	$f_B(\mathrm{MeV})$	$R_{s/l}$
HPQCD	2.02 ± 0.55	190 ± 13	0.52 ± 0.16
FNAL/MILC	$2.21^{+0.47}_{-0.42}$	212 ± 9	0.46 ± 0.10

One comment on $B \to \tau\nu_\tau$

- define

$$R_{s/l}(q_1^2, q_2^2) \equiv \frac{\Delta\mathcal{B}_{B\to\pi\ell\nu_\ell}(q_1^2, q_2^2)}{\mathcal{B}(B \to \tau\nu_\tau)} \left(\frac{\tau_{B^-}}{\tau_{B^0}}\right)$$

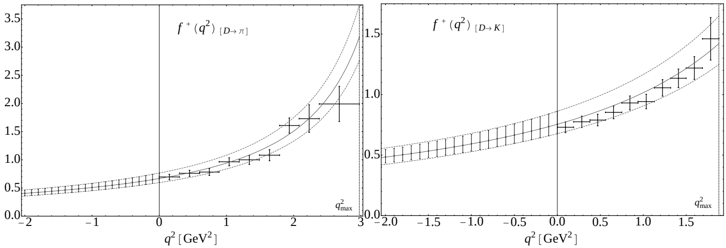
- and get

Exp.	$\Delta\mathcal{B}(10^{-4})$	$\mathcal{B}(B \to \tau\nu_\tau)(10^{-4})$	$R_{s/l}$
BABAR	0.88 ± 0.06 $0.84 \pm 0.03 \pm 0.04$	1.76 ± 0.49	$0.52^{+0.20}_{-0.12}$

QCD	$\Delta\zeta$ [Ref.]	f_B(MeV)	$R_{s/l}$
LCSR/QCDSR	$4.59^{+1.00}_{-0.85}$	210 ± 19	$0.97^{+0.28}_{-0.24}$

- $\sim 2\,\sigma$ deviation
- ratio of $B \to \pi\tau\nu_\tau$ over $B \to \pi e\nu_e$ should help

$D \to \pi, K$ results

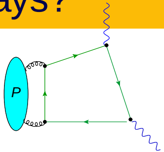


- twist 3 contribution larger than twist 2 therefore larger uncertainty
- LCSRs results in good agreement with experimental data

Cleo [08] normalized by PDG $|V_{cd}|$

$$|V_{cd}| = 0.225 \pm 0.005 \pm 0.003^{+0.016}_{-0.012} \qquad \frac{|V_{cd}|}{|V_{cs}|} = 0.236 \pm 0.006 \pm 0.003 \pm 0.013$$

What is interesting about $B, D_{(s)} \to \eta^{(\prime)}$ decays?



- additional contribution due to singlet-octet nature of η, η'
- can be described via gluon distribution amplitude

$$\phi_{Pq}^{2,1}(u,\mu^2) = 6u\bar{u}\left(1 + \sum_{n=2,4} a_n^1(\mu^2)\, C_n^{3/2}(2u-1)\right)$$

$$\phi_{Pg}^{2,1}(u,\mu^2) = 30u^2\bar{u}^2 \sum_{n=2,4} b_n^1(\mu^2)\, C_{n-1}^{5/2}(2u-1)$$

- a_n^1 and b_n^1 mix under renormalisation

$$\mu\frac{d}{d\mu}\begin{pmatrix} a_n^1 \\ b_n^1 \end{pmatrix} = \begin{pmatrix} \gamma^{qq} & \gamma^{qg} \\ \gamma^{gq} & \gamma^{gg} \end{pmatrix}\begin{pmatrix} a_n^1 \\ b_n^1 \end{pmatrix}$$

- gives significant contribution even if b_n^1 vanish at a low scale
- interesting channel to constrain b_2^1

$B, D_{(s)} \to \eta^{(\prime)}$ results

$$\frac{\Gamma\left(D_s^+ \to \eta' e^+ \nu_e\right)}{\Gamma\left(D_s^+ \to \eta e^+ \nu_e\right)} = 0.37 \pm 0.09\,(b_2^1) \pm 0.04\,(\text{rest}),$$

$$Exp: \quad 0.36 \pm 0.14$$

$$\frac{\Gamma\left(D^+ \to \eta' e^+ \nu_e\right)}{\Gamma\left(D^+ \to \eta e^+ \nu_e\right)} = 0.16 \pm 0.06\,(b_2^1) \pm 0.02\,(\text{rest}),$$

$$Exp: \quad 0.19 \pm 0.09$$

$$\frac{\Gamma\left(B \to \eta' e^+ \nu_e\right)}{\Gamma\left(B \to \eta e^+ \nu_e\right)} = 0.50 \pm 0.29\,(b_2^1) \pm 0.05\,(\text{rest}),$$

$$Exp: \quad 0.67 \pm 0.24 \pm 0.1$$

- first lattice calculation available

Set	meson	$f_0(q^2=0)$	$b\,(\text{GeV})^{-2}$	β
S	η	0.564(11)	0.127(06)	1.70(08)
($m_\pi \approx 470$ MeV)	η'	0.437(18)	0.119(23)	1.81(35)
A	η	0.542(13)	0.090(14)	2.35(36)
($m_\pi \approx 370$ MeV)	η'	0.404(25)	0.188(32)	1.13(19)
LCSRs (at M_π^{phys})	η	0.432(33)	—	—
	η'	0.520(80)	—	—

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Conclusions and outlook

- sum rules are inherently approximate
- light-cone expansion and duality approximation seem to work nicely in many different channels
- give the opportunity to combine lattice calculations for moments of distribution amplitudes with analytic expressions to understand how QCD works in these decays
- still some improvements possible
  - combined fit to sum rule results and experimental data See next talk
  - model radial excitation to improve Quark-Hadron duality approximation
  - go beyond twist 4 and NLO
- uncertainty around 10% probably remains