Form Factors for $B \to \pi\pi l\nu$

Christoph Hanhart

Forschungszentrum Jülich

In collaboration with

X.-W. Kang, B. Kubis, A. Kupść, U.-G. Meißner, F. Stollenwerk, A. Wirzba

Motivation

There is tension in the values for V_{ub}:

R. Kowalewski, T. Mannel; mini-review in RPP (PDG)

$|V_{ub}| = (4.41 \pm 0.15^{+0.15}_{-0.17})$ inclusive from $\bar{B} \to X_u l \bar{\nu}_l$

$|V_{ub}| = (3.82 \pm 0.29)$ exclusive from $\bar{B} \to \pi l \bar{\nu}_l$

$\longrightarrow$ an alternative, exclusive extraction is needed.

Candidate: $\bar{B} \to \rho l \bar{\nu}_l$

Data with sizable uncertainties by CLEO and BaBar

Relevant theoretical issues: what is measured is $\bar{B} \to \pi\pi l \bar{\nu}_l$

$\to$ How to parametrize the ρ with controlled uncertainties

$\to$ How much $f_0(500)$ (s-wave) is there?

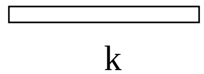
Both will be addressed here for some special kinematics

Outline

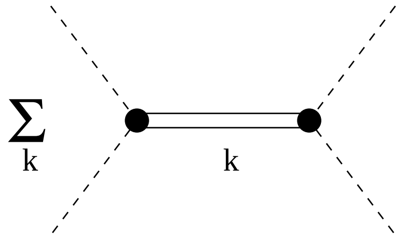
- How to treat hadronic few body systems
  - ▷ Unitarity relation
  - ▷ Remarks on dispersion theory: the Omnès function
- Illustration of universality of final state interactions
  - ▷ A model independent data analysis for $\eta \to \pi\pi\gamma$
  - ▷ Relating $\eta \to \pi\pi\gamma$ to $\eta \to \gamma\gamma^*$
- $\bar{B} \to \pi\pi l \bar{\nu}_l$
  - ▷ Inclusion of left–hand cuts
  - ▷ Results
- Outlook? — some ideas ...

Modeling hadron physics

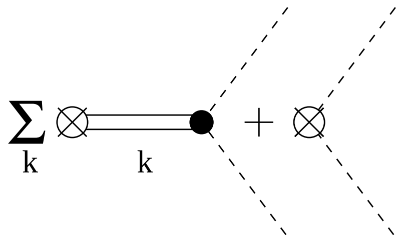
Standard treatment: sum of Breit-Wigners

Propagator: $iG_k(s) =$ 

$= i/(s - M_k^2 + iM_k\Gamma_k)$

Scattering: 

$= \sum_k i g_k^2 G_k(s)$

Production: 

$= \left(\sum_k i g_k G_k(s)\alpha_k\right) + i\beta$

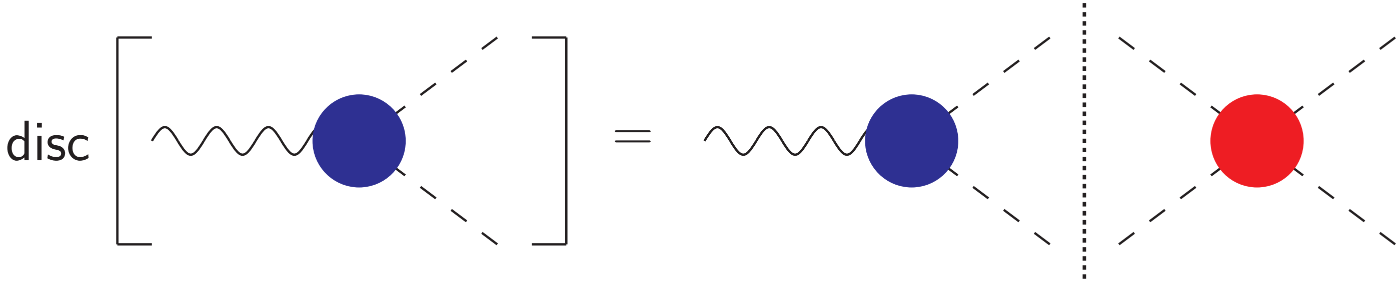
Problems:

→ Wrong threshold behavior (cured by $\Gamma = \Gamma(s)$)

→ Violates unitarity ⟶ wrong phase motion

→ Parameters reaction dependent
only pole positions and resides universal!

Unitarity relation



$$\mathrm{disc}(F)/2i = \mathrm{Im}(F) = e^{-i\delta(s)}\sin(\delta(s)) \times \theta(s-4m_\pi^2) \times F(s)$$

$\rightarrow$ Watson theorem: $F(s) = |F(s)|e^{i\delta(s)}$ and Omnès function

$$\Omega(s) = \exp\left(\frac{s}{\pi}\int_{4m_\pi^2}^{\infty} ds' \frac{\delta(s')}{s'(s'-s-i\epsilon)}\right)$$

$\Omega(s)$ is universal and fixed by $\delta(s)$

Such that we may write

$$\boxed{F(s) = P(s)\Omega(s)}$$

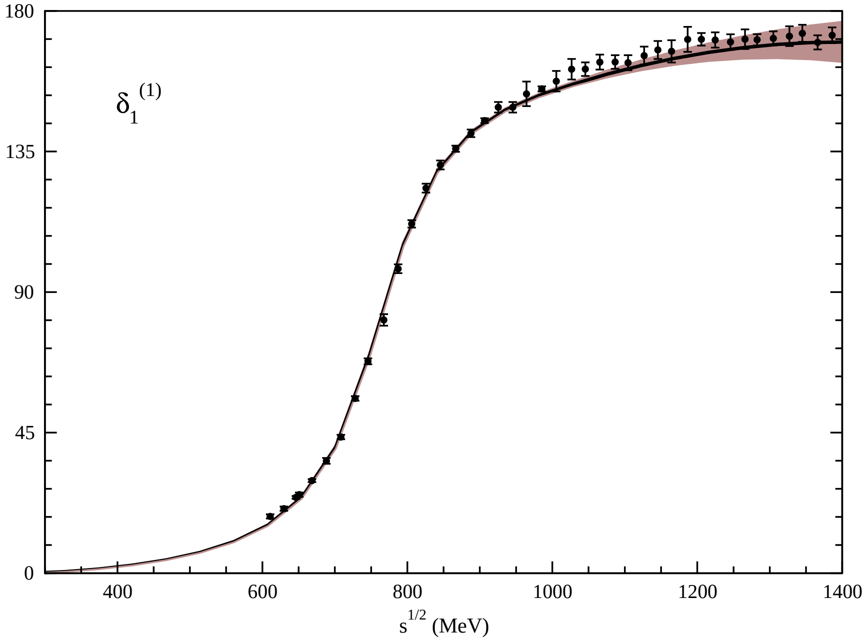
$P(s)$ reaction specific

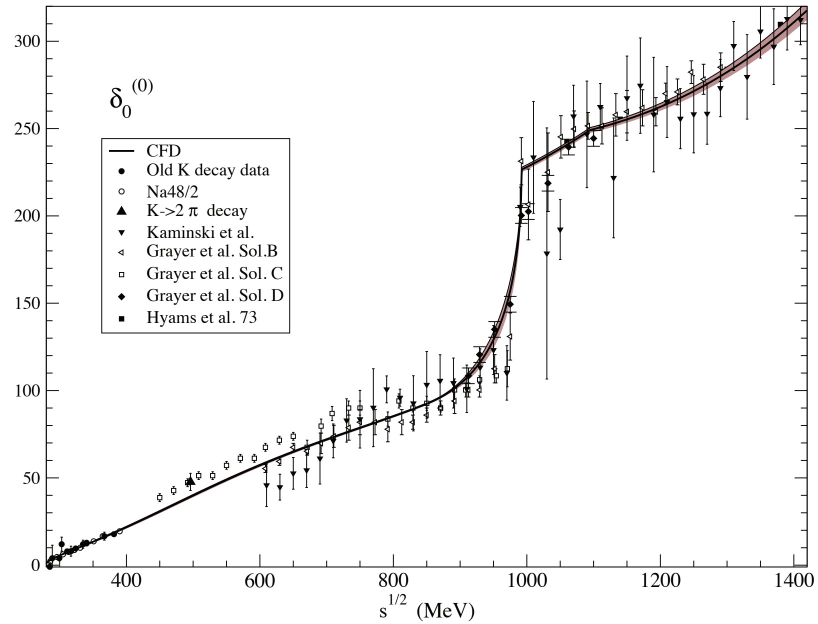
$\rightarrow$ left-hand cuts

$\rightarrow$ higher thresholds

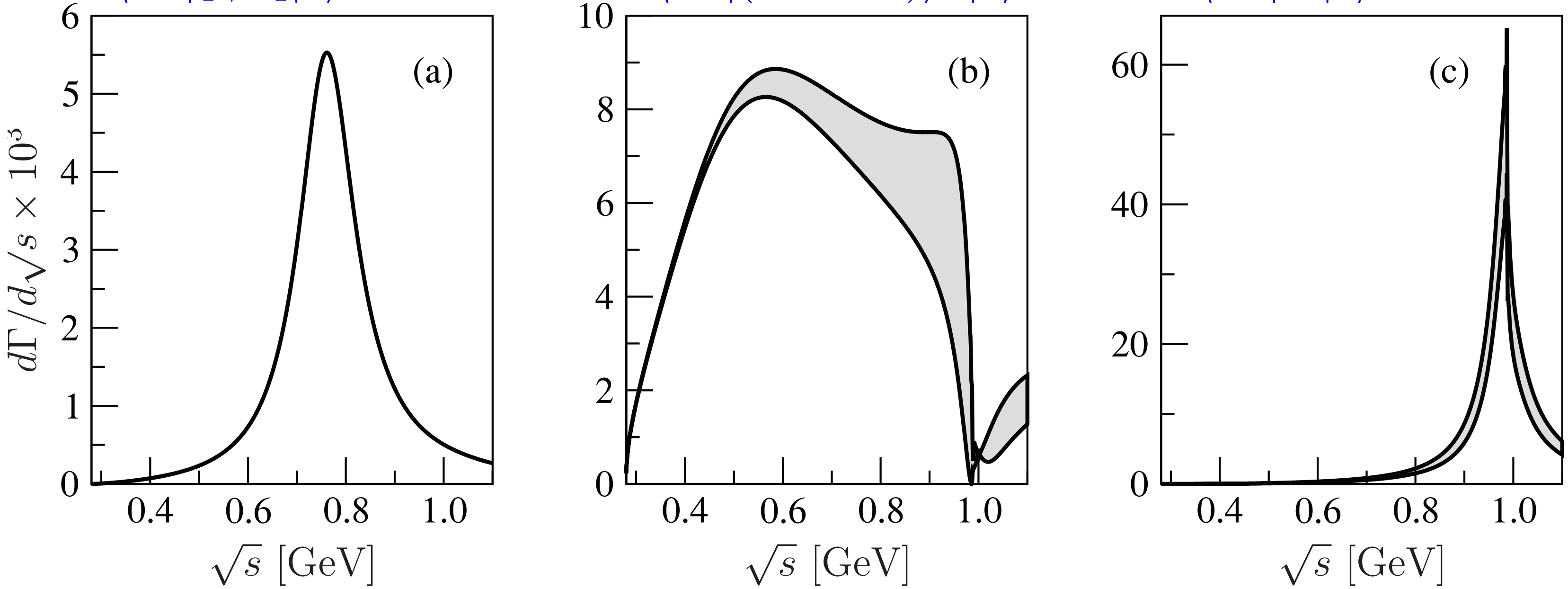
$\rightarrow$ inel. resonances

Status for $\Omega(s)$



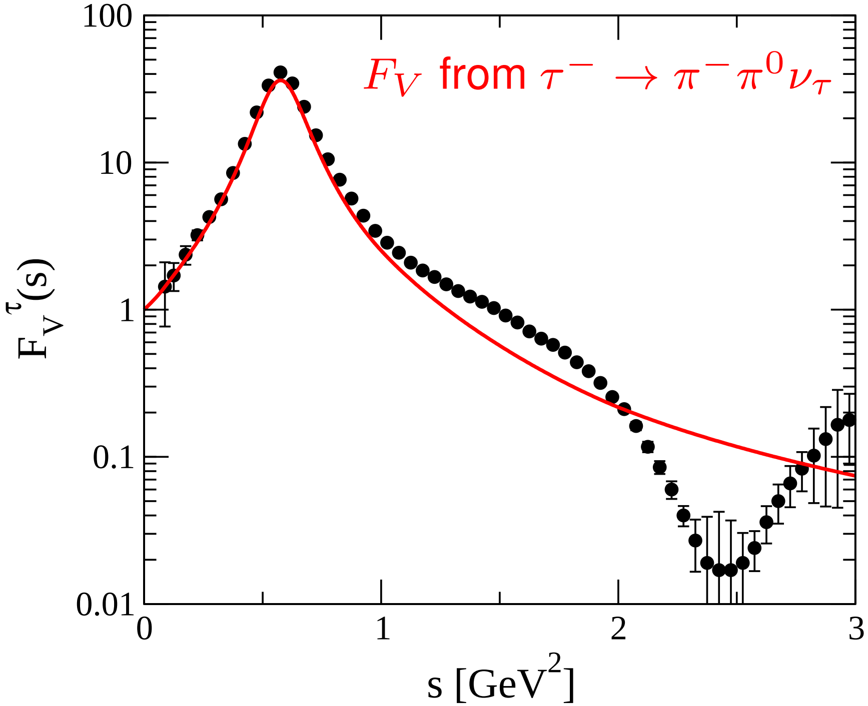


give for $\langle\pi\pi|\bar{q}\gamma^\mu q|0\rangle$ $\langle\pi\pi|(\bar{u}u+\bar{d}d)/2|0\rangle$ $\langle\pi\pi|\bar{s}s|0\rangle$



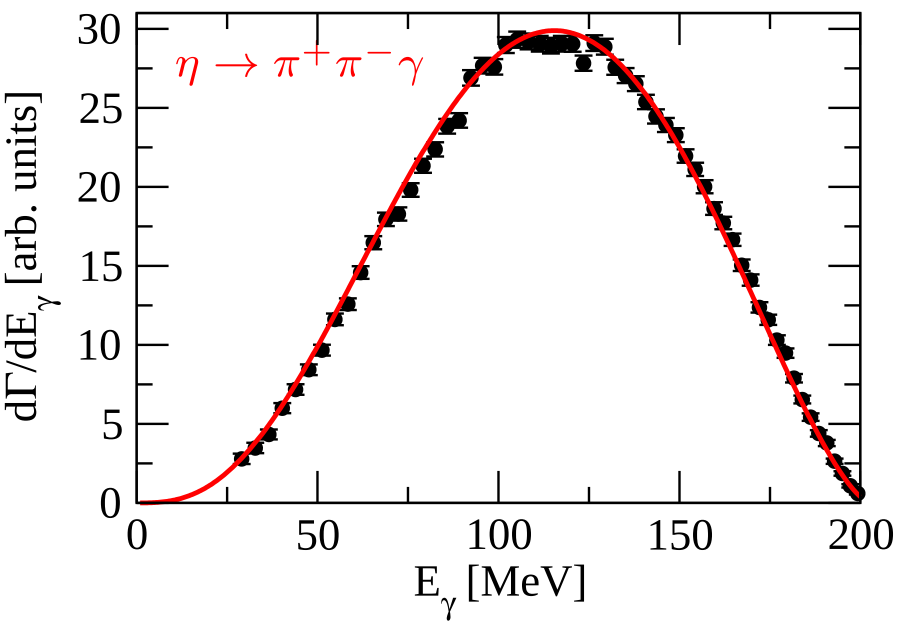
δ: Garcia-Martin et al., PRD83(2011)074004; FF's: Daub et al., JHEP01(2013)179

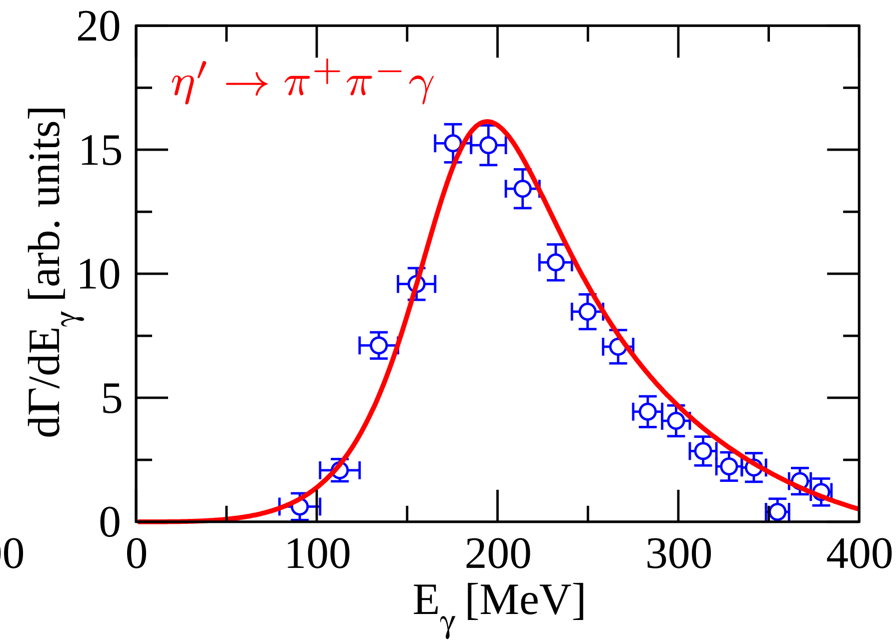
Illustration of universality of FSI



red lines: p-wave Omnès
$\times$ kinematic factors

- bulk described properly
- there are deviations
  $\longrightarrow$ $P(s)$ not constant!





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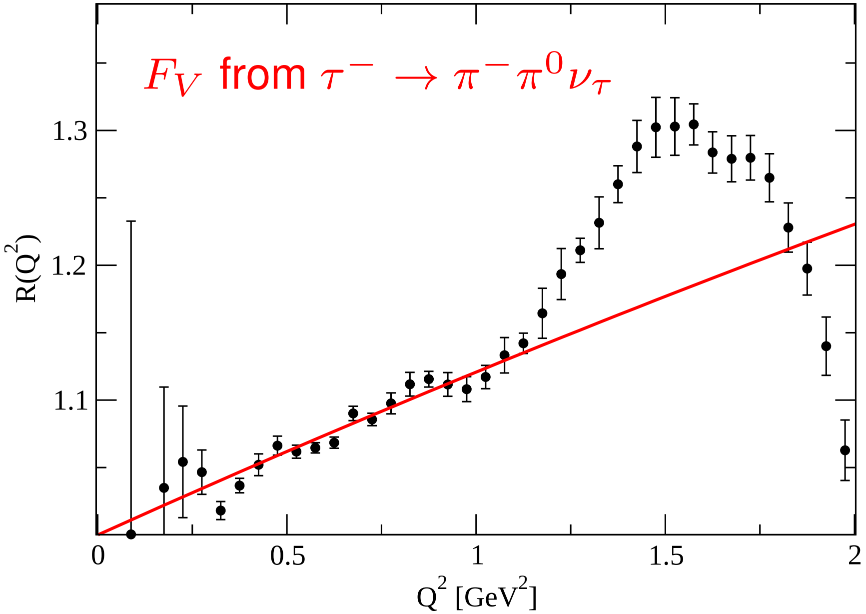
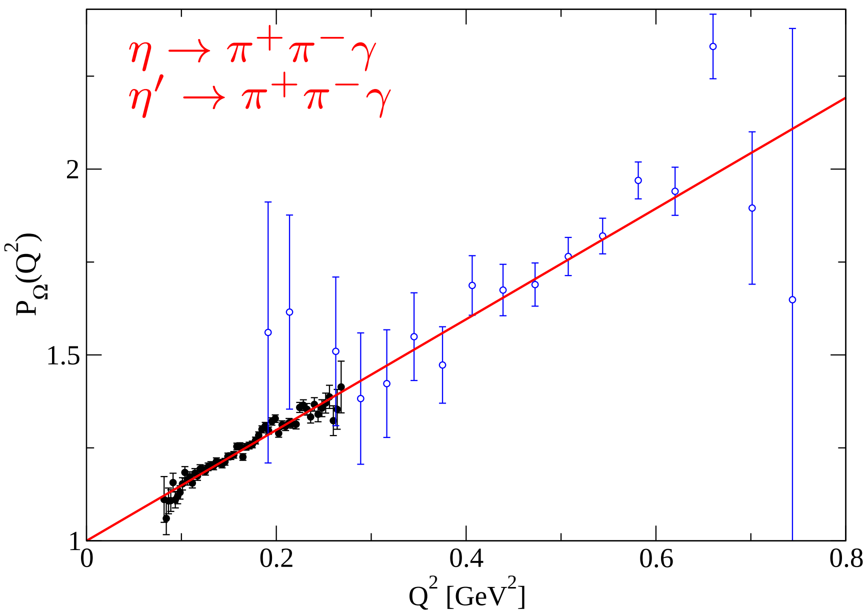




We had: $F(Q^2) = P(Q^2)\Omega(Q^2)$

We find for all 3 cases

- $P(Q^2)$ linear for $Q^2 < 1$ GeV^2
- deviations in F_V by ρ' & ρ''

$$P(Q^2) = A_0(1 + \alpha Q^2)$$

$\alpha[\gamma]$=(0.12 ± 0.01) GeV^{-2};
$\alpha[\eta\gamma]$=(1.4 ± 0.1) GeV^{-2}

$\to$ α reaction specific

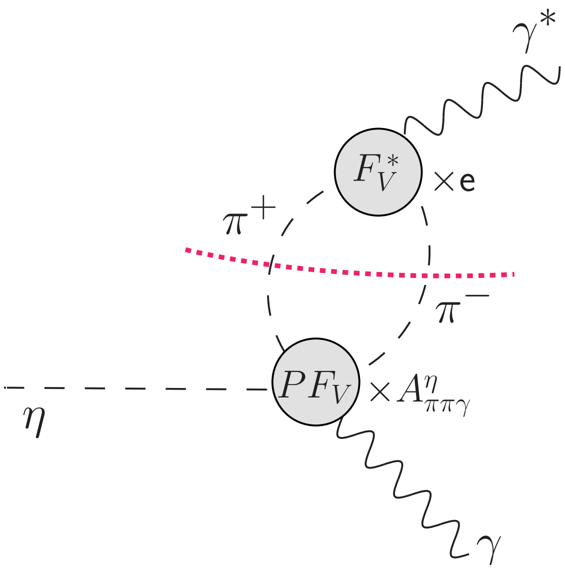
$\to$ $\alpha[\eta] = \alpha[\eta']$ understood

1-loop ChPT + large N_c

F. Stollenwerk et al., PLB **707**, 184 (2012)

Going to $\eta \to \gamma\gamma^*$

Allows for parameter free prediction for isovector part of slope



$$F_{\eta\gamma^\star\gamma}(Q^2,0) \equiv 1 + \Delta F^{(I=1)}_{\eta\gamma^\star\gamma}(Q^2,0) + \Delta F^{(I=0)}_{\eta\gamma^\star\gamma}(Q^2,0)$$

$$\Delta F^{(I=1)}_{\eta\gamma^\star\gamma} = \left(\frac{\kappa_\eta Q^2}{96\pi^2 f_\pi^2}\right) \int_{4m_\pi^2}^{\infty} ds' \, \sigma_\pi(s')^3 \, P(s') \, \frac{|F_V(s')|^2}{s' - Q^2 - i\epsilon}$$

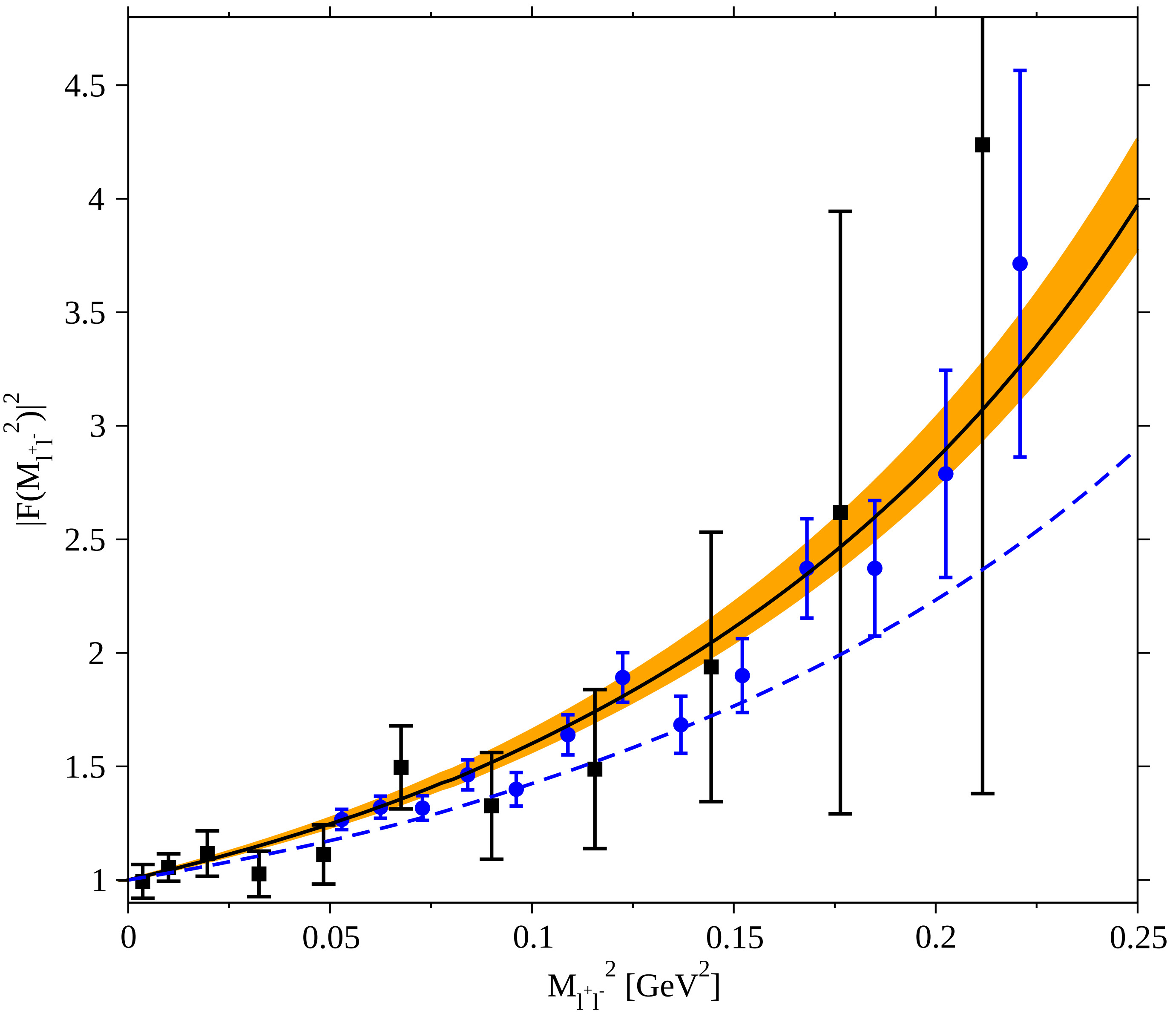
$\Delta F^{(I=0)}_{\eta\gamma^\star\gamma}$ needs to be modeled

We thus demonstrated:

- $\eta \to \gamma\gamma^*$ and $\eta \to \pi^+\pi^-\gamma$ closely connected
- FSI ($\Omega(Q^2)$) universal; driven by ρ-pole and residues
- $P(Q^2)$ (reaction dependent!) also modifies $\eta \to \gamma^*\gamma$

C. H. et al. Eur.Phys.J. C73 (2013) 2668.

Results



Successful description of data with small uncertainties

$B \to \pi\pi l\nu \; (B_{l4})$

The matrix element reads: Bijnens et al., NPB427(1994)427 (K_{l4})

$$T = \frac{G_F}{\sqrt{2}} V_{ub}^{*} \bar{\psi}_\nu(p_\nu)\gamma^\mu(1-\gamma_5)\psi_l(p_l) I_\mu \, ,$$

with the hadronic current:

$$\begin{aligned} I_\mu &= \langle \pi^+(p_+)\pi^-(p_-)|\bar{u}\gamma_\mu(1-\gamma_5)b|B^-(p_B)\rangle \\ &= -\frac{i}{m_B}(P_\mu F + Q_\mu G + L_\mu R - (H/m_B^3)\epsilon_{\mu\nu\rho\sigma}L^\nu P^\rho Q^\sigma \end{aligned}$$

with $P = p_+ + p_-$, $Q = p_+ - p_-$, $L = p_l + p_\nu$

goal: Extraction of V_{ub} with controlled uncertainty

→ Full command of the shape of the form factors

where $l_{min} = 0$ for F, R and $l_{min} = 1$ for G, H

→ A fixed normalization

Tools

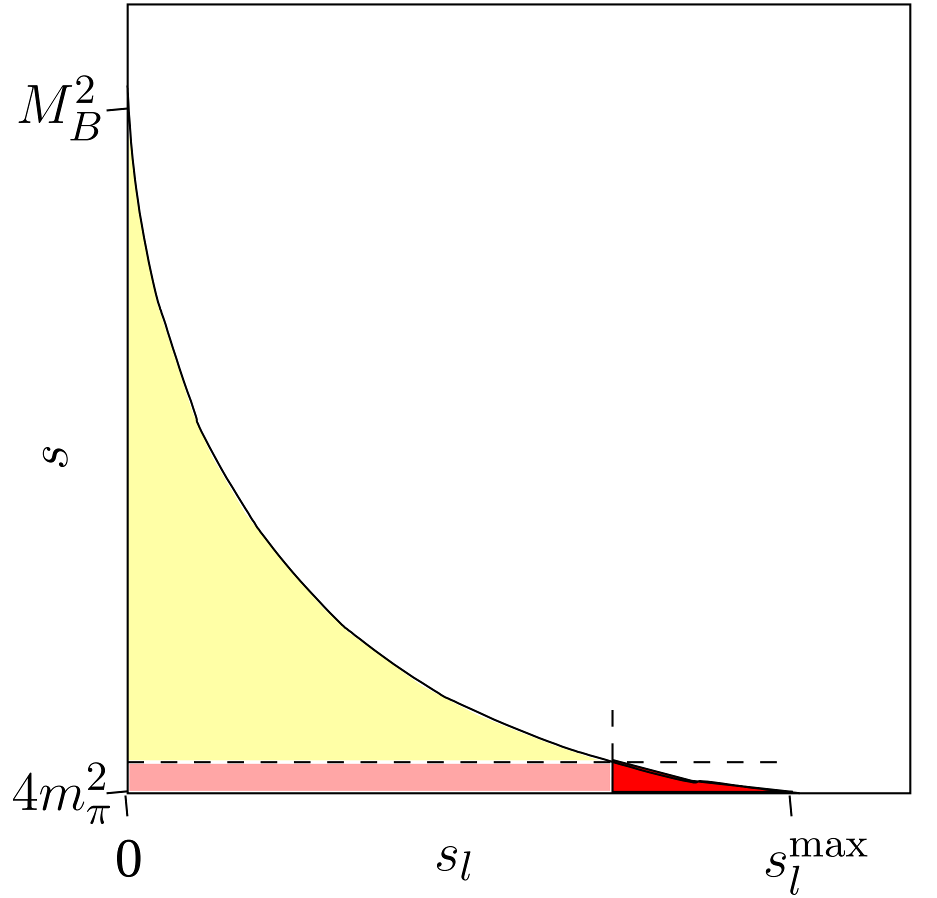
We are going to use

→ Dispersion theory
to fix shapes

$\longrightarrow s = P^2 < 1$ GeV^2

→ Heavy Meson ChPT
to fix norm

$\longrightarrow s_l = L^2 \simeq M_B^2$



adapted from Faller et al., PRD89(2014)014015

Note: dispersive approach valid for full s_l range

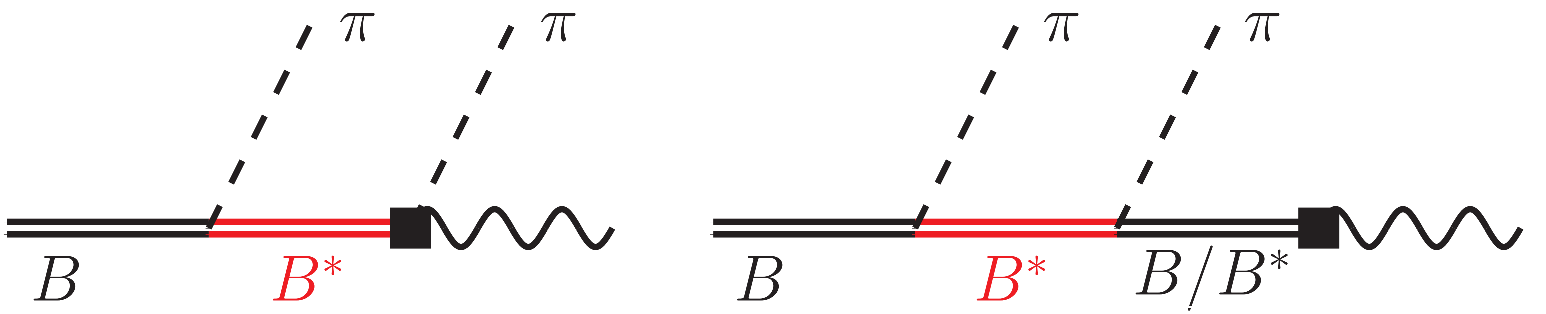
But: we can not (yet?) control s_l dependence outside red area

Yellow area needs different methods e.g., Faller et al., PRD89(2014)014015

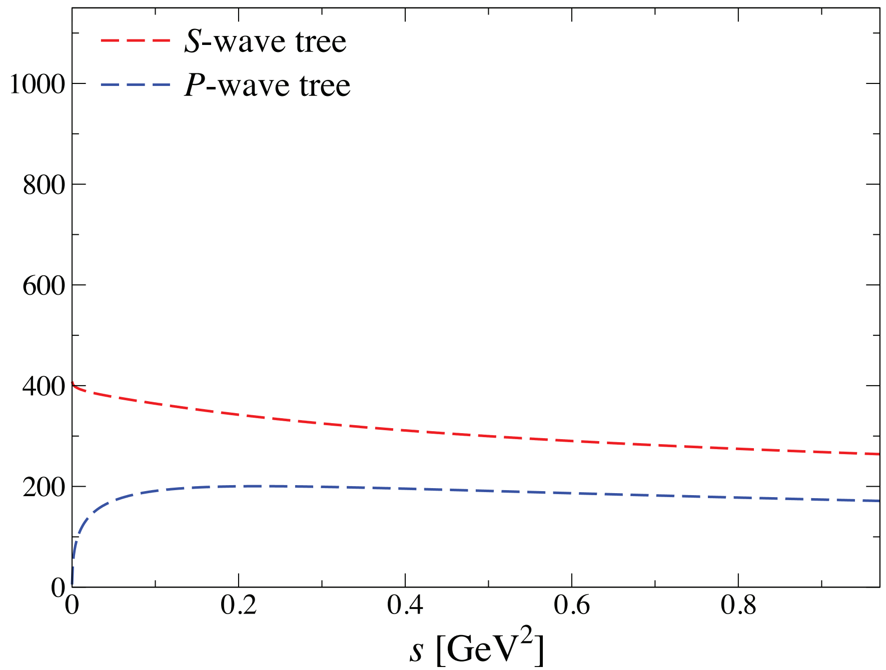
Some details ...

at leading order: $B \to \pi\pi\ell\nu$ given by B^* pole terms

determined by $g_{B^*B\pi}$ and f_B Burdman & Donoghue, PLB280(1992)287



$\longrightarrow$ Left-hand cut/ square root singularity at $s = 0$



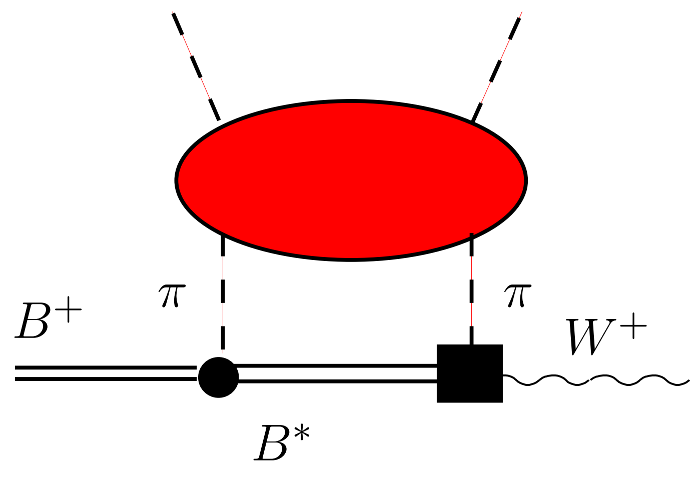
Inclusion of left-hand cut

Kang et al., PRD89(2014)053015

$$\mathrm{disc}(F(s,s_l)) = 2i\{\underbrace{F(s,s_l)}_{\text{right-hand cut}} + \underbrace{\hat{M}(s,s_l)}_{\text{left-hand cut}}\}\times\theta(s-4\,m_\pi^2)\times\sin\,\delta(s)\,e^{-i\delta(s)}$$

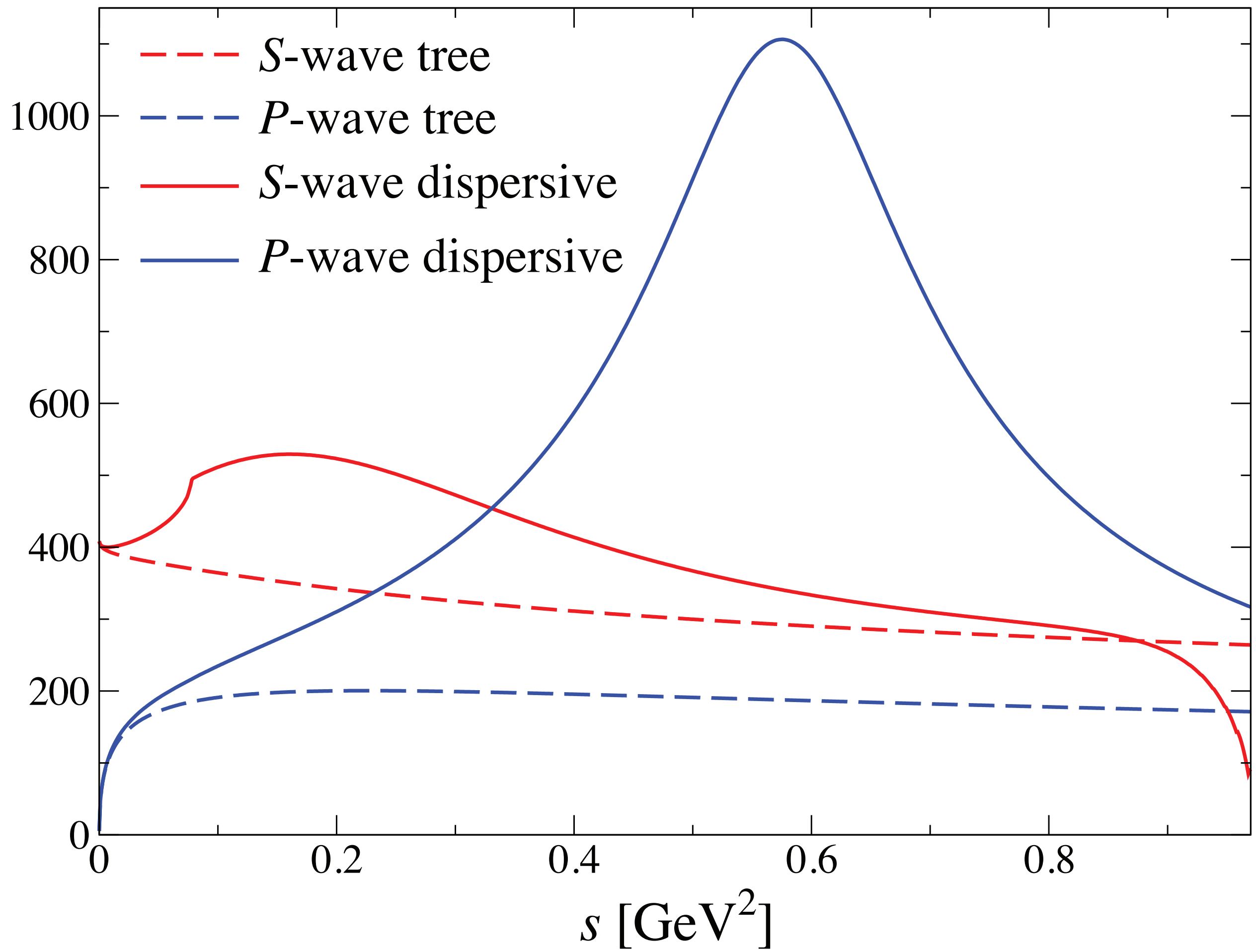
→ inhomogeneities $\hat{M}(s,s_l)$: angular averages of pole terms
We get, e.g., for P–waves

$$F(s,s_l) = \Omega_1^1(s)\Big\{a_0(s_l) + a_1(s_l)s + \frac{\cos\delta_1^1(s)\hat{M}(s,s_l)}{|\Omega_1^1(s)|} + \frac{s^2}{\pi}\mathrm{P}\int_{4m_\pi^2}^{\infty}\frac{ds'}{s'^2}\frac{\sin\delta_1^1(s')\hat{M}(s',s_l)}{|\Omega_1^1(s')|(s'-s)}\Big\}$$



→ Pole terms introduce s_l dependence.

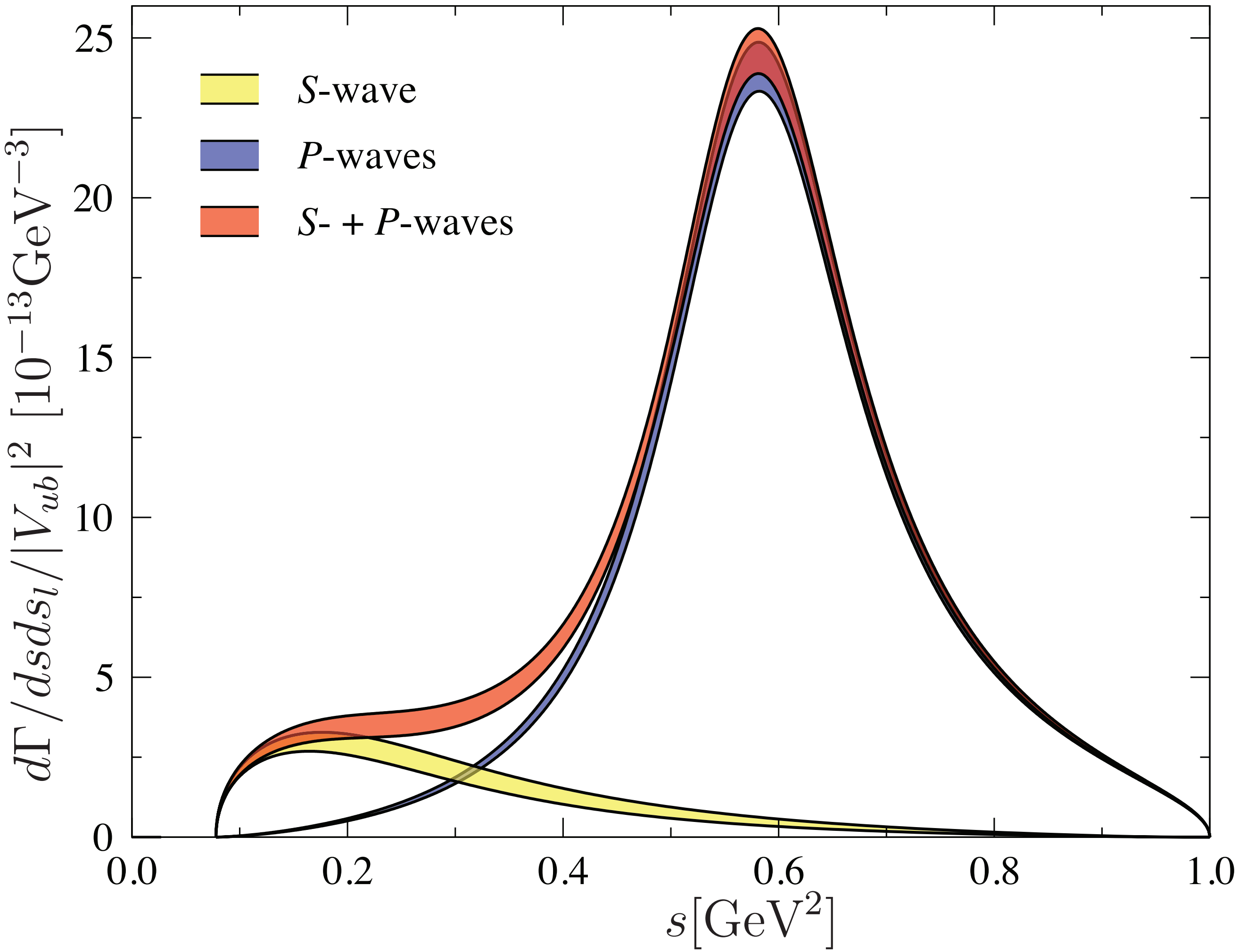
Results: form factors



→ line shapes get distorted

→ effect depends on s_l (here: $s_l = (M_B - 1\text{ GeV})^2$)

Results: B_{l4} rates



→ For $s_l \sim M_B^2$ S– and P–wave strengths fixed

→ For $s < 1$ GeV^2 shapes fixed
→ inclusion of experimental cuts straightforward

Summary

A combination of

- Heavy-Meson ChPT and
- dispersion theory

enabled us to treat $B \to \pi\pi l \bar{\nu}_l$

- model independently with
- controlled uncertainties.

Preconditions for extraction of V_{ub}

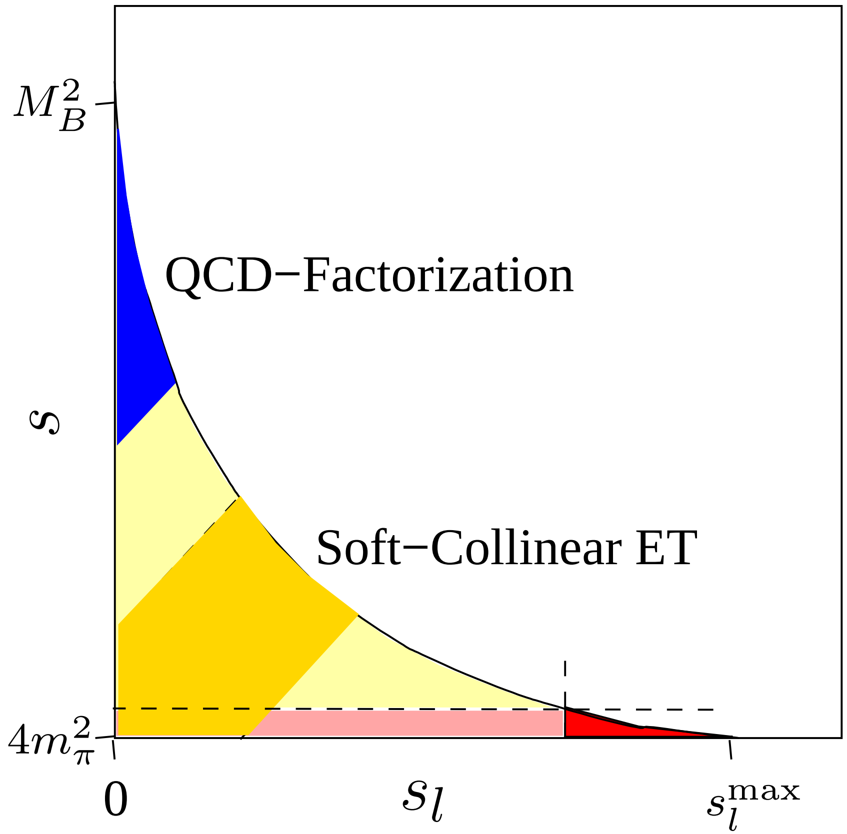
However, applicable in very limited kinematic regime only

How could this range be extended?

Are there possible synergies with lattice QCD?

Outlook - some ideas ...

Strategies I:



Test for overlap regions of the various effective theories and/or for proper interpolations

see Faller et al., PRD89(2014)014015

Use model to extend s–region:

$\to$ inelastic resonances

C.H. PLB715(2012)170

Should allow one to include a lot more data in fits

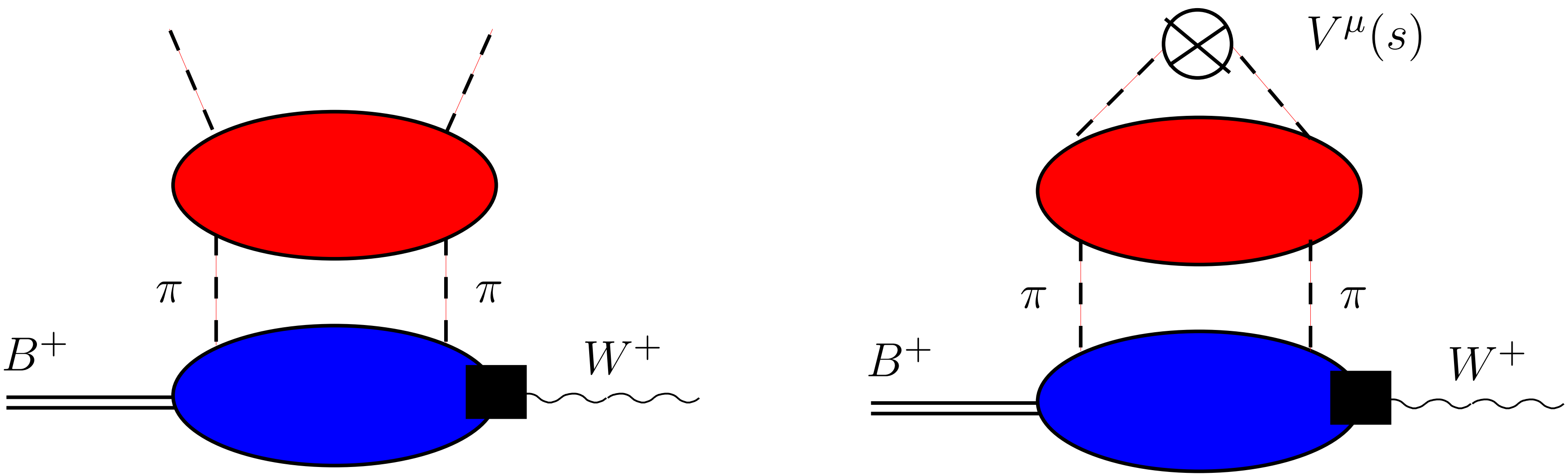
reduced uncertainty of V_{ub} extraction

Outlook - some ideas ...

Strategies II:

Dispersion theory to overcome narrow width approximation

What is measured in experiment | on the lattice



Dispersion theory provides a well defined connection!

cf. discussion on $\eta \to \pi\pi\gamma \longleftrightarrow \eta \to \gamma\gamma^*$

Outlook - some ideas ...

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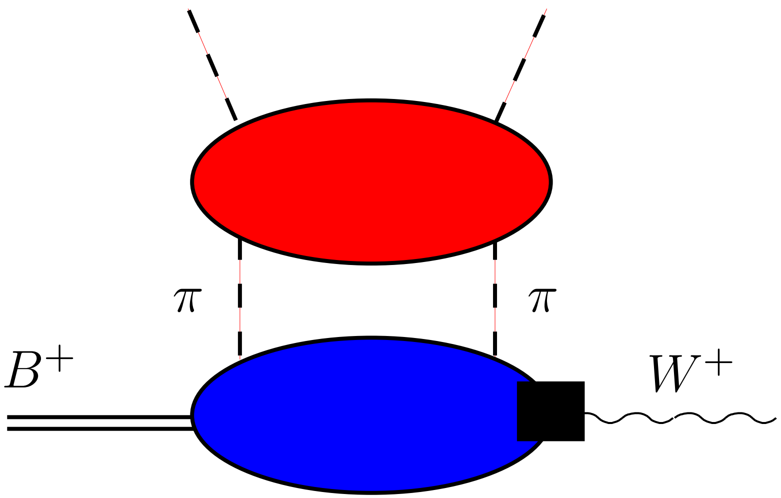
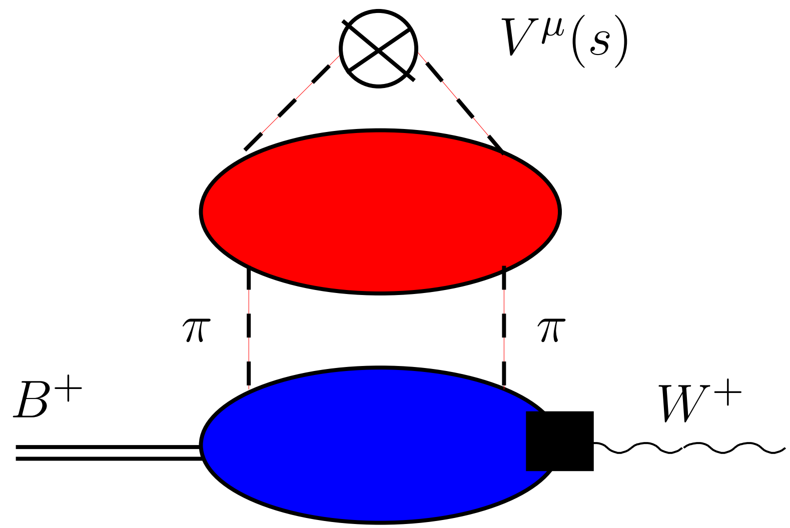
Dispersion theory to overcome narrow width approximation

What is measured in experiment



on the lattice



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THANKS A LOT FOR YOUR ATTENTION