$B \to V$ form factors on the lattice

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Lattice Meets Continuum
Siegen, 29 Sept - 2 Oct 2014

Outline

- Motivation: measurements of $b \to s$ decays

$$B \to K^* \ell^+ \ell^- \qquad B_s \to \phi \, \ell^+ \ell^-$$

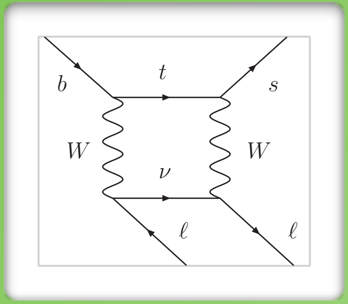
- Form factors: first unquenched calculation
- Observables
- Future: Dealing with strong decay of the vector meson
- Question: Effects of charmonium resonances at low recoil

Motivation

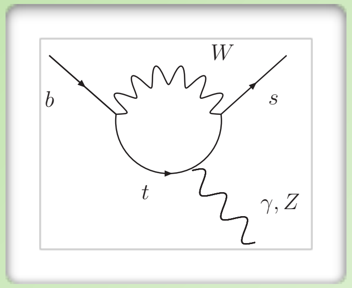
- $b \to s$ decays occur only at 1-loop level in Standard Model: Room for new physics?
- Following initial results from CDF, LHC experiments (esp LHC*b*) are making impressive measurements of rare, semileptonic decays
- There are a few tantalizing discrepancies with SM predictions
- Taken seriously, these consistently *hint* at a nonstandard contribution to the Wilson coefficient C_9
- Significant effort from theory remains to quantify and reduce SM uncertainties

Low energy description of $b \to s$ decays

$$\mathcal{H}_{\text{eff}}^{b \to s} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i (C_i \mathcal{O}_i + C_i' \mathcal{O}_i')$$



In the Standard Model, $i = 1, \ldots, 10, S, P$ with known Wilson coefficients C_i. Beyond SM, chirality-flipped operators are allowed and the $C_i^{(\prime)}$ depend on the model of new physics



Most important short-distance effects in $b \to sll$ come from 2-quark operators:

$$\mathcal{O}_9^{(\prime)} = \frac{e^2}{16\pi^2} \bar{s}\gamma^\mu P_{L(R)} b \, \bar{\ell}\gamma_\mu \ell \qquad \mathcal{O}_{10}^{(\prime)} = \frac{e^2}{16\pi^2} \bar{s}\gamma^\mu P_{L(R)} b \, \bar{\ell}\gamma_\mu \gamma_5 \ell$$

$$\mathcal{O}_7^{(\prime)} = \frac{m_b e}{16\pi^2} \bar{s}\sigma^{\mu\nu} P_{R(L)} b \, F_{\mu\nu}$$

Charmonium resonance effects arise from:

$$\mathcal{O}_1 = \bar{c}^\alpha \gamma^\mu P_L b^\beta \, \bar{s}^\beta \gamma^\mu P_L c^\alpha \qquad \mathcal{O}_2 = \bar{c}^\alpha \gamma^\mu P_L b^\alpha \, \bar{s}^\beta \gamma^\mu P_L c^\beta$$

$B \to V$ form factors

$$\langle V(k,\varepsilon)|\bar{q}\hat{\gamma}^\mu b|B(p)\rangle = \frac{2iV(q^2)}{m_B+m_V}\epsilon^{\mu\nu\rho\sigma}\varepsilon^*_\nu k_\rho p_\sigma$$

$$\langle V(k,\varepsilon)|\bar{q}\hat{\gamma}^\mu\hat{\gamma}^5 b|B(p)\rangle = 2m_V A_0(q^2)\frac{\varepsilon^*\cdot q}{q^2}q^\mu + (m_B+m_V)A_1(q^2)\left(\varepsilon^{*\mu} - \frac{\varepsilon^*\cdot q}{q^2}q^\mu\right)$$
$$- A_2(q^2)\frac{\varepsilon^*\cdot q}{m_B+m_V}\left((p+k)^\mu - \frac{m_B^2-m_V^2}{q^2}q^\mu\right)$$

$$q^\nu\langle V(k,\varepsilon)|\bar{q}\hat{\sigma}_{\mu\nu}b|B(p)\rangle = 2T_1(q^2)\epsilon_{\mu\rho\tau\sigma}\varepsilon^{*\rho}p^\tau k^\sigma$$

$$-q^\nu\langle V(k,\varepsilon)|\bar{q}\hat{\sigma}_{\mu\nu}\hat{\gamma}^5 b|B(p)\rangle = iT_2(q^2)[\varepsilon^*_\mu(m_B^2-m_V^2) - (\varepsilon^*\cdot q)(p+k)_\mu]$$
$$+ iT_3(q^2)(\varepsilon^*\cdot q)\left[q_\mu - \frac{q^2}{m_B^2-m_V^2}(p+k)_\mu\right]$$

$$A_{12}(q^2) = \frac{(m_B+m_V)^2(m_B^2-m_V^2-q^2)A_1(q^2) - \lambda A_2(q^2)}{16 m_B m_V^2(m_B+m_V)}$$

$$T_{23}(q^2) = \frac{m_B+m_V}{8m_B m_V^2}\left[\left(m_B^2+3m_V^2-q^2\right)T_2(q^2) - \frac{\lambda T_3(q^2)}{m_B^2-m_V^2}\right]$$

with $\lambda = (t_+-t)(t_--t) \qquad t=q^2 \qquad t_\pm = (m_B\pm m_V)^2$

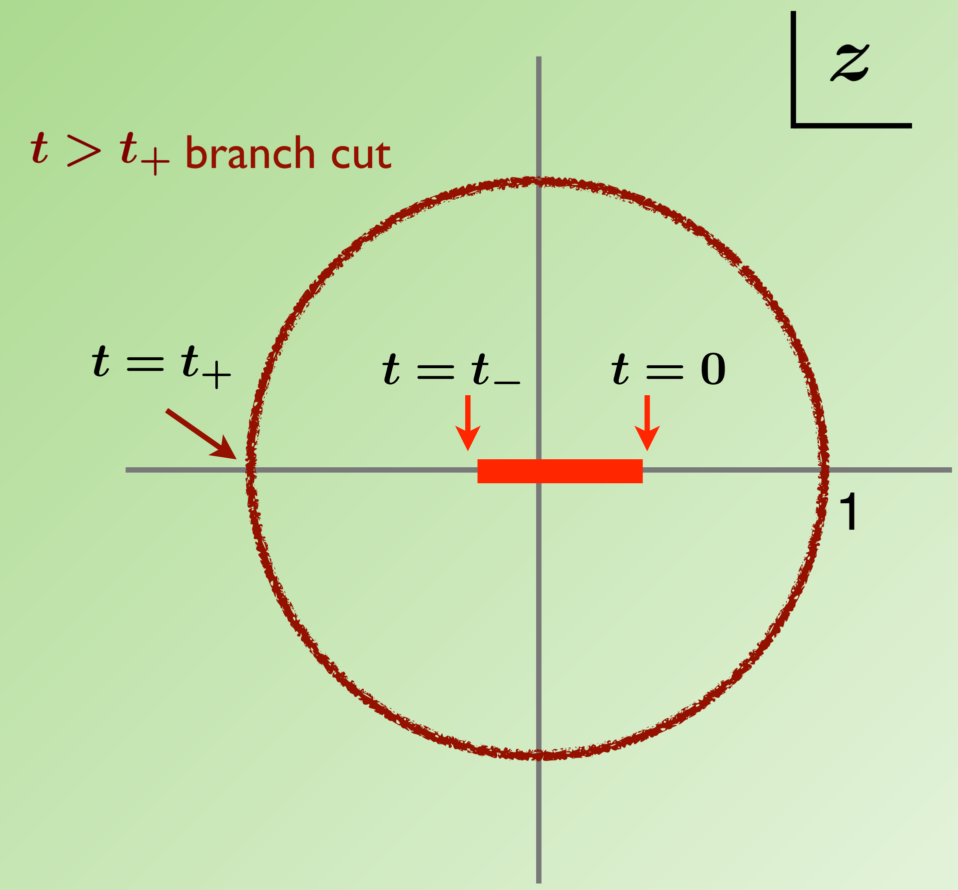
Form factor shape

Series (z) expansion

$$t = q^2 \qquad t_\pm = (m_B \pm m_F)^2$$

Choose, *e.g.* $t_0 = 12\ \mathrm{GeV}^2$

$$z = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$



Simplified series expansion

$$F(t) = \frac{1}{1 - t/m_{\mathrm{res}}^2} \sum_n a_n z^n$$

Bourrely, Caprini, Lellouch PRD **79** (2009)

Form factor shape & LQCD

$$F(t) = \frac{1}{1 - t/m_{\rm res}^2} \sum_n a_n z^n$$

As in Na, *et al.*, (HPQCD), PRD82 (2010)

discretization effects

light quark mass

strange quark mass

$$F(t) = \frac{1}{P(t;\Delta m)} [1 + b_1 (aE_V)^2 + \ldots] \sum_{n=0} a_n\, z^n [1 + c_{n1} \Delta x + c_{n1s} \Delta x_s + \ldots]$$

$$P(t;\Delta m) = 1 - \frac{t}{(m_B + \Delta m)^2} \qquad \Delta x = \frac{1}{(4\pi f)^2} (m_\pi^2 - m_{\pi,\rm phys}^2) \qquad \Delta x_s = \frac{1}{(4\pi f)^2} (m_{\eta_s}^2 - m_{\eta_s,\rm phys}^2)$$

Physical results: set *b*'s and *c*'s = 0

In our LQCD calculation:

only c_{01}, c_{01s} found to be statistically nonzero

only a_0 and a_1 determined by data

Lattice action & parameters

R Horgan, Z Liu, S Meinel, MW, Phys. Rev. D 89, 094501 (2014) [arXiv:1310.3722]

MILC lattices (2+1 asqtad staggered)
asqtad light & strange quarks
NRQCD bottom quarks

label	#	$N_x^3 \times N_t$	$am_\ell^{\rm sea}/am_s^{\rm sea}$	r_1/a	$1/a$ (GeV)
c007	2109	$20^3 \times 64$	0.007/0.05	2.625(3)	1.660(12)
c02	2052	$20^3 \times 64$	0.02/0.05	2.644(3)	1.665(12)
f0062	1910	$28^3 \times 96$	0.0062/0.031	3.699(3)	2.330(17)

ensemble	m_B (GeV)	m_{B_s} (GeV)	m_π (MeV)	m_K (MeV)	m_{η_s} (MeV)	m_ρ (MeV)	m_{K^*} (MeV)	m_ϕ (MeV)
c007	5.5439(32)	5.6233(7)	313.4(1)	563.1(1)	731.9(1)	892(28)	1045(6)	1142(3)
c02	5.5903(44)	5.6344(15)	519.2(1)	633.4(1)	730.6(1)	1050(7)	1106(4)	1162(3)
f0062	5.5785(22)	5.6629(13)	344.3(1)	589.3(2)	762.0(1)	971(7)	1035(4)	1134(2)
"physical"	5.279	5.366	140	495	686	775	892	1020

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MILC lattices (2+1 asqtad staggered)
asqtad light & strange quarks
NRQCD bottom quarks

src = 8 x
meas = 16 x

label	#	$N_x^3 \times N_t$	$am_\ell^{\text{sea}}/am_s^{\text{sea}}$	r_1/a	$1/a$ (GeV)
c007	2109	$20^3 \times 64$	0.007/0.05	2.625(3)	1.660(12)
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Operator matching

Currents using lattice fields (lattice regularization)
Heavy quarks treated using lattice NRQCD (heavy quark expansion)
Lattice calculation in low recoil regime ($|\mathbf{k}| \ll m_b$; $|\mathbf{k}| \ll 1/a$)

$$J_0^A = \bar{\psi}_q \Gamma^A \Psi_b$$

LO in $1/m_b$

$$J_1^A = -\frac{1}{2m_b} \bar{\psi}_q \Gamma^A \gamma \cdot \nabla \Psi_b$$

NLO in $1/m_b$

$$\Gamma^A \in [\gamma^\mu, \gamma^\mu\gamma^5, \sigma^{\mu\nu}, \sigma^{\mu\nu}\gamma^5]$$

Matching to continuum (V, A: conserved; T: MS-bar, μ=m_b)

$$\mathcal{J}^A = (1 + \alpha_s \rho^{(A)}) J_0^A + J_1^A - \alpha_s \zeta_{10}^{(A)} J_0^A$$

Accurate to 1-loop in α_s and NLO in $1/m_b$

Operator matching

$$\mathcal{J}^A = (1+\alpha_s\rho^{(A)})J_0^A + J_1^A - \alpha_s\zeta_{10}^{(A)}J_0^A$$

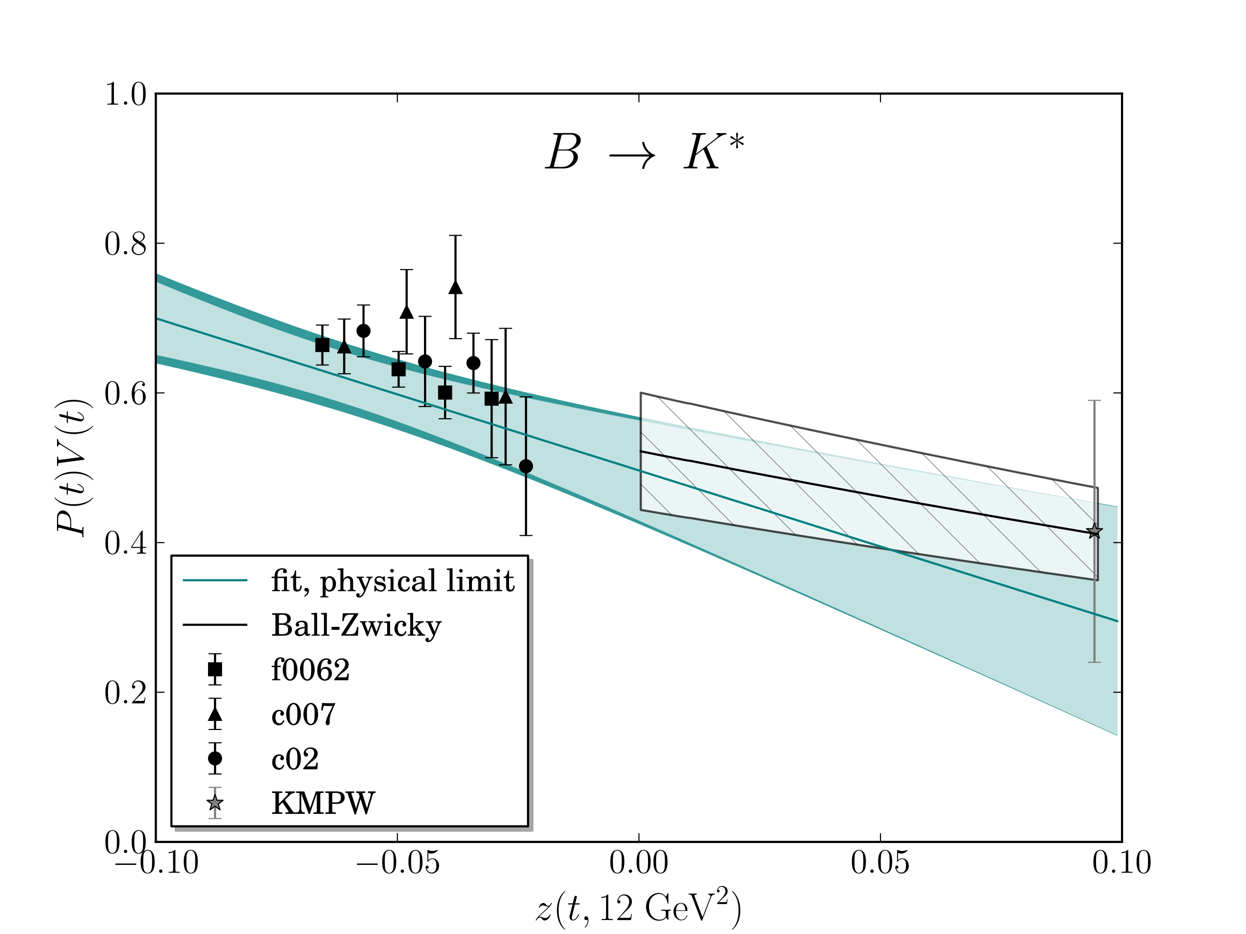
	Coarse	Fine
C_v	2.825	1.996
$\rho^{(0)}$	0.043	-0.058
$\zeta_{10}^{(0)}$	-0.166	-0.218
$\rho^{(k)}$	0.270	0.332
$\zeta_{10}^{(k)}$	0.055	0.073
$\rho^{([0\ell])}$	0.076	0.320
$\zeta_{10}^{([0\ell])}$	-0.055	-0.073
$\rho^{([k\ell])}$	0.076	0.320
$\zeta_{10}^{([k\ell])}$	-0.055	-0.073

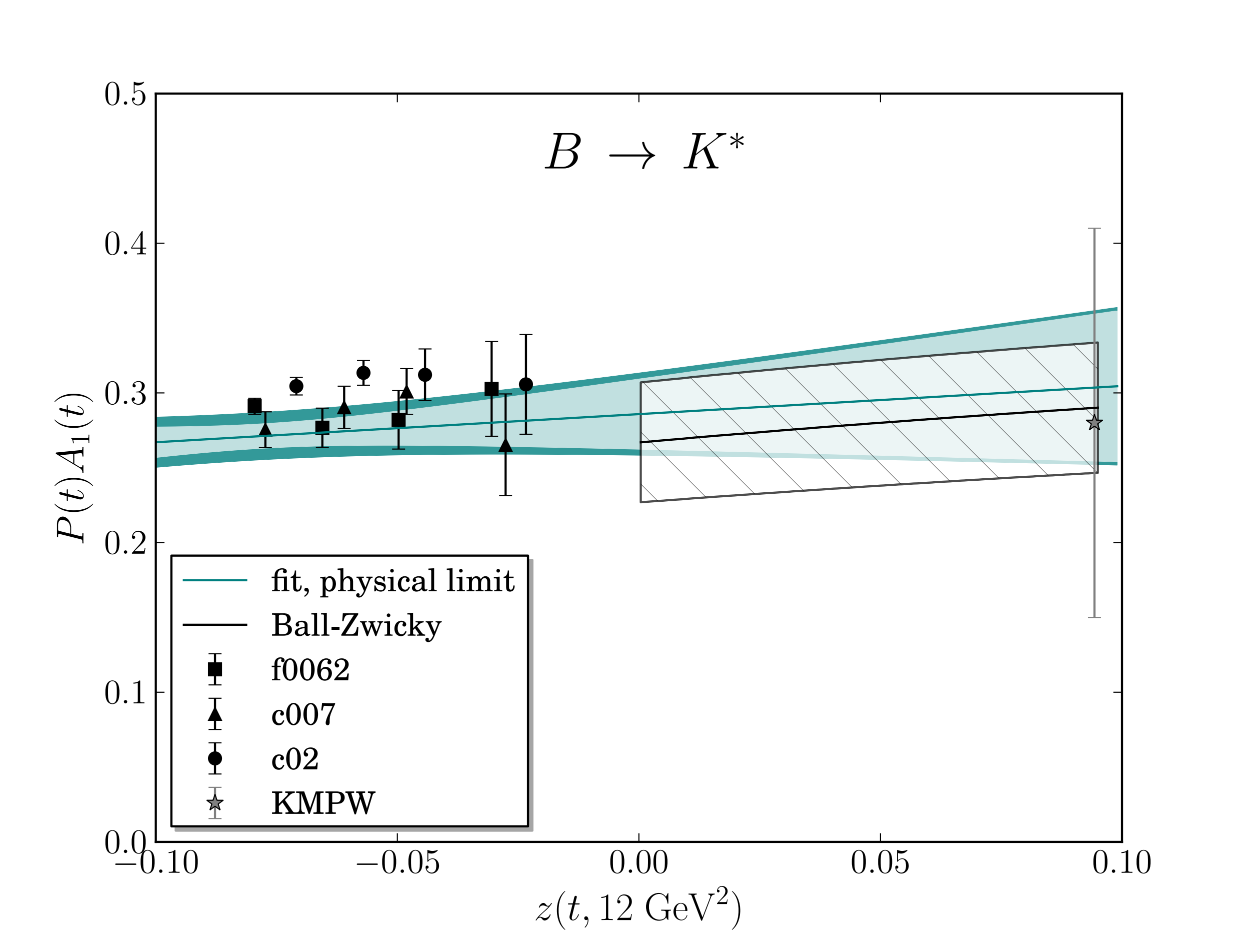
Gulez, Shigemitsu, Wingate, PRD69 (2003), PRD73 (2006); Müller, Hart, Horgan, PRD83 (2011); Müller, priv. commun.

Uncertainties:

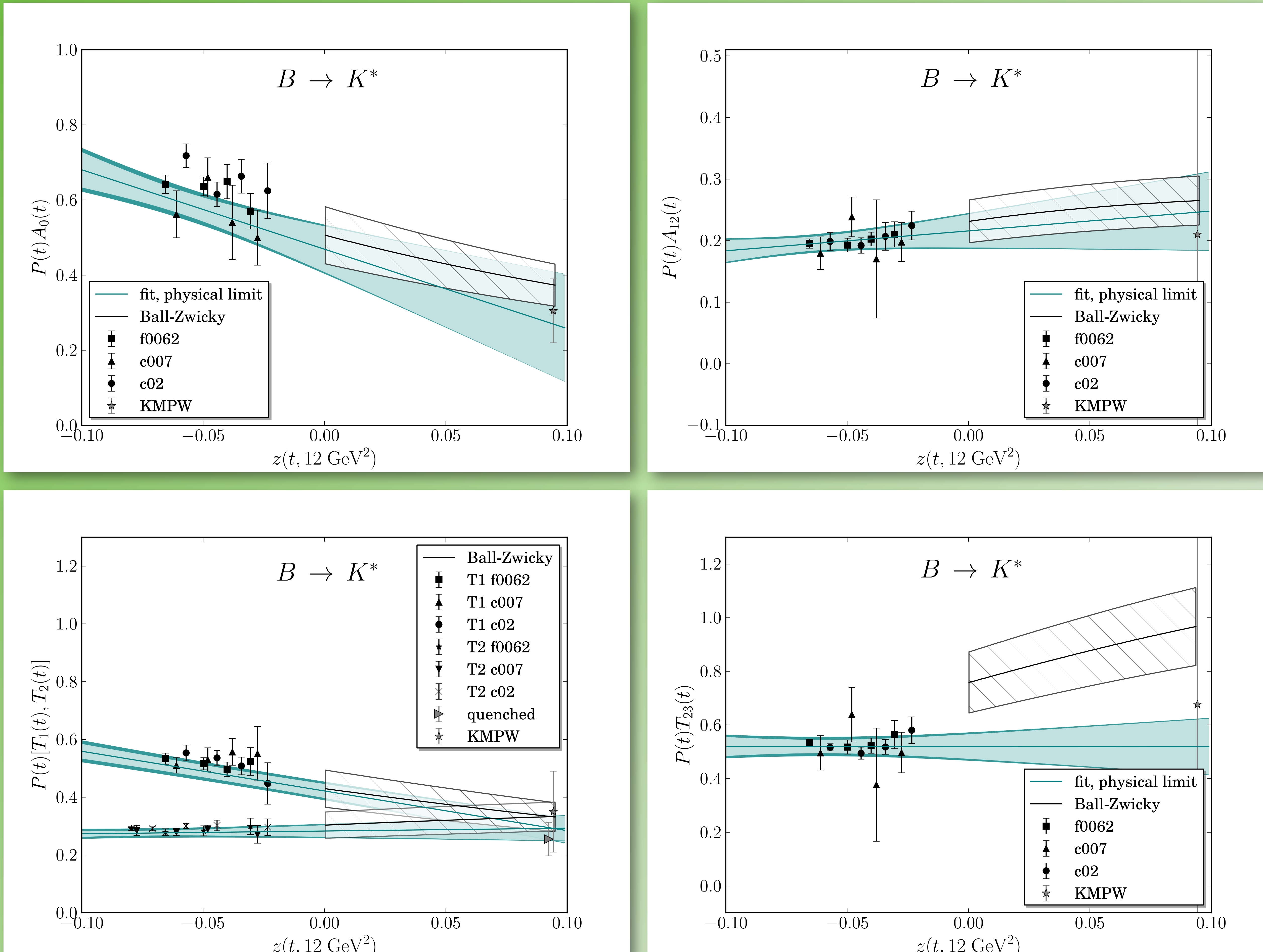
- α_s^2 — 4% : largest 1-loop contribution suppressed by α_s *biggest systematic*
- $\frac{\alpha_s\Lambda_{QCD}}{m_b}$ — 2% : largest Λ/m_b effect suppressed by α_s
- $\left(\frac{\Lambda_{QCD}}{m_b}\right)^2$ — 1% : largest Λ/m_b effect squared (& rounded up)

$B \to K^*$ form factors



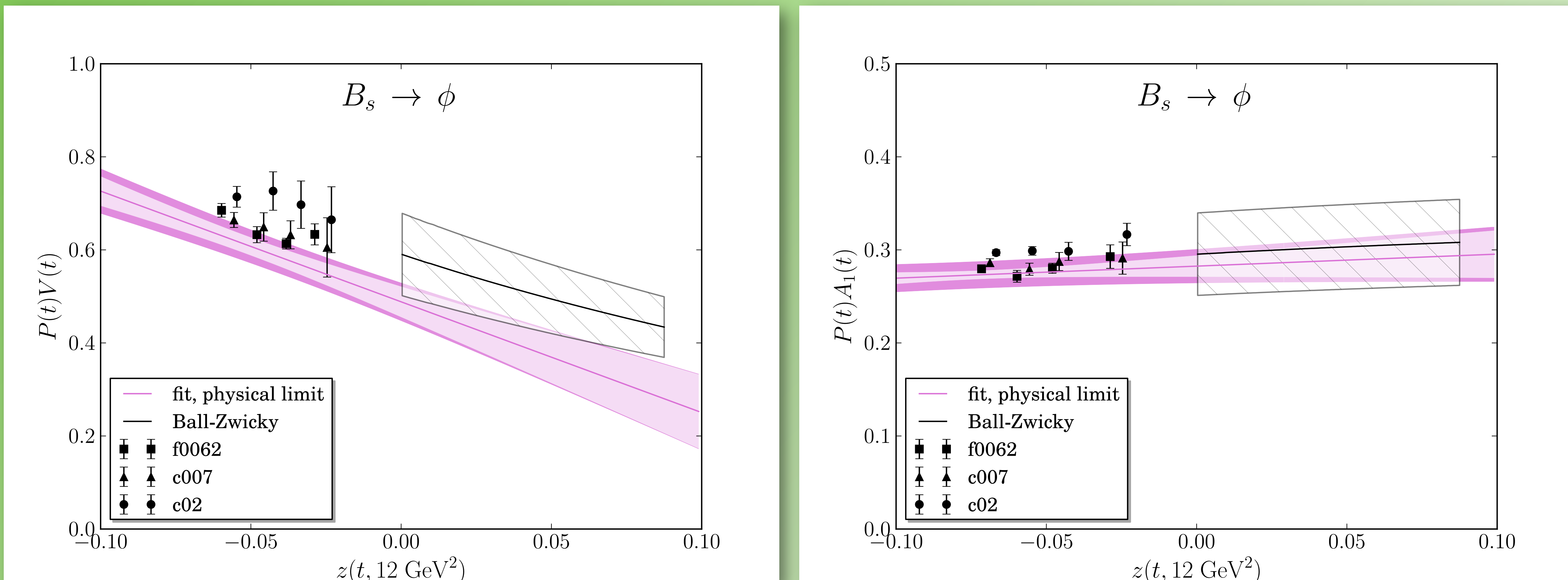


low recoil high q^2	large recoil low q^2	low recoil high q^2	large recoil low q^2



$B \to K^*$
$P(t)A_0(t)$
$z(t, 12\ \mathrm{GeV}^2)$
fit, physical limit
Ball-Zwicky
f0062
c007
c02
KMPW
$B \to K^*$
$P(t)A_{12}(t)$
$z(t, 12\ \mathrm{GeV}^2)$
fit, physical limit
Ball-Zwicky
f0062
c007
c02
KMPW
$B \to K^*$
$P(t)[T_1(t), T_2(t)]$
$z(t, 12\ \mathrm{GeV}^2)$
Ball-Zwicky
T1 f0062
T1 c007
T1 c02
T2 f0062
T2 c007
T2 c02
quenched
KMPW
$B \to K^*$
$P(t)T_{23}(t)$
$z(t, 12\ \mathrm{GeV}^2)$
fit, physical limit
Ball-Zwicky
f0062
c007
c02
KMPW

$B_s \to \varphi$ form factors

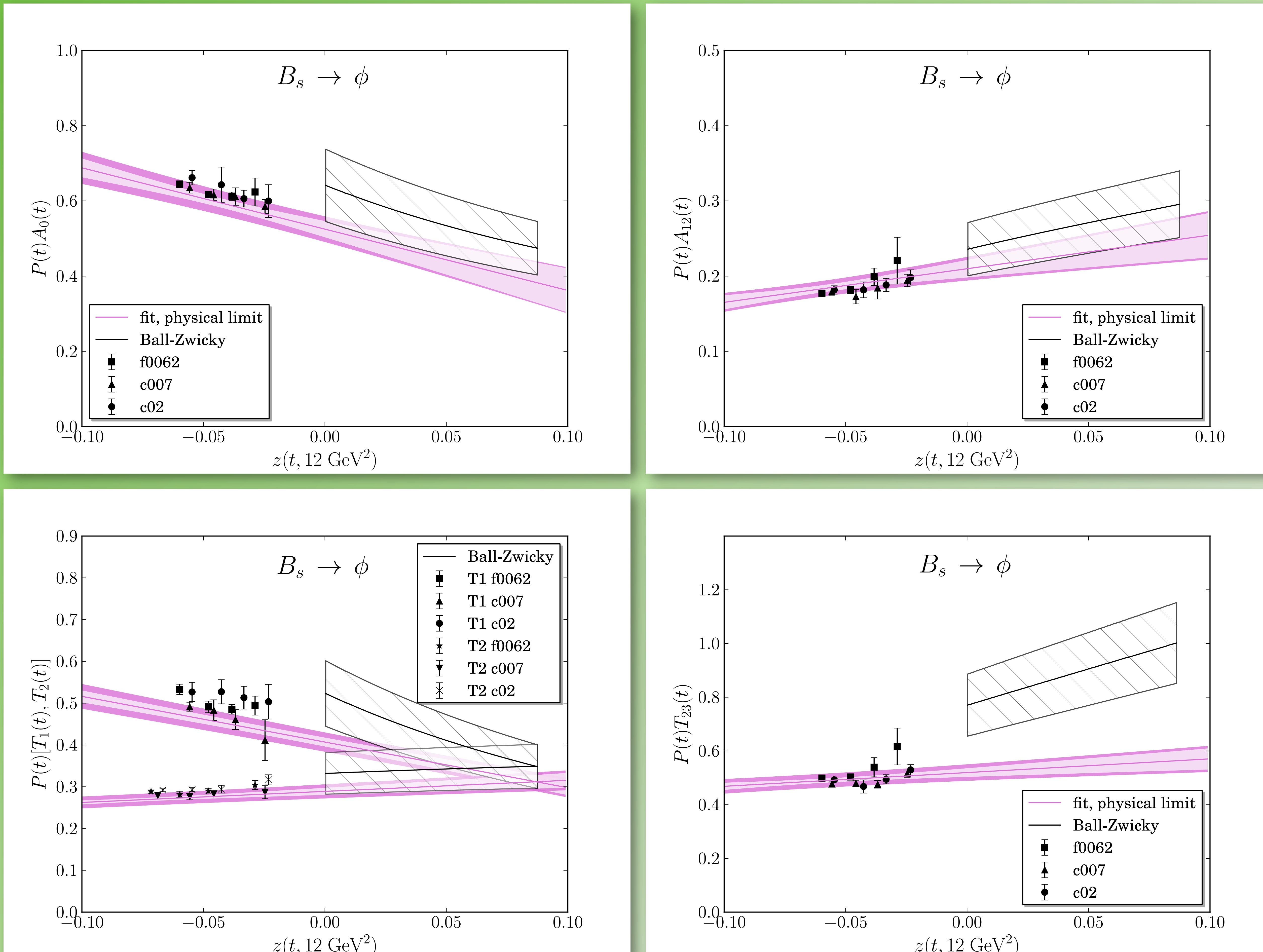


low recoil
high q^2

large recoil
low q^2

low recoil
high q^2

large recoil
low q^2



$B_s \to \phi$
$P(t)A_0(t)$
$z(t, 12\ \mathrm{GeV}^2)$
fit, physical limit
Ball-Zwicky
f0062
c007
c02
$B_s \to \phi$
$P(t)A_{12}(t)$
$z(t, 12\ \mathrm{GeV}^2)$
fit, physical limit
Ball-Zwicky
f0062
c007
c02
$B_s \to \phi$
$P(t)[T_1(t), T_2(t)]$
$z(t, 12\ \mathrm{GeV}^2)$
Ball-Zwicky
T1 f0062
T1 c007
T1 c02
T2 f0062
T2 c007
T2 c02
$B_s \to \phi$
$P(t)T_{23}(t)$
$z(t, 12\ \mathrm{GeV}^2)$
fit, physical limit
Ball-Zwicky
f0062
c007
c02

Form factor error budget

source	size
Truncation of $O(\alpha_s^2)$ terms	4%
Truncation of $O(\alpha_s \Lambda_{\mathrm{QCD}}/m_b)$ terms	2%
Truncation of $O(\Lambda_{\mathrm{QCD}}^2/m_b^2)$ terms	1%
Mistuning of m_b	$<$ 1%
Net systematic uncertainty	5%

Statistical + fitting uncertainties depend on z
Smaller than systematic unc. in some cases
Total unc. typically ≈10-20% in data range

- B to V form factors not yet at the level of rigour as other LQCD calculations, e.g. B to pseudoscalar form factors
- Must properly deal with resonant nature of vector meson (c.f. Briceño, Hansen, Walker-Loud, arXiv:1406.5965)
- Nevertheless, results are at least as reliable as other theoretical methods
- Resonance effects likely to be less for φ than for K^* — yet similar conclusions regarding branching fraction (see below)

Many observables

Angular distribution for $\bar{B} \to \bar{K}^{*0}(\to K^-\pi^+)\mu^+\mu^-$

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_l\, d\cos\theta_{K^*}\, d\phi} = \frac{9}{32\pi} I(q^2, \theta_l, \theta_{K^*}, \phi)$$

$$\begin{aligned}
I(q^2, \theta_l, \theta_{K^*}, \phi) =& I_1^s \sin^2\theta_{K^*} + I_1^c \cos^2\theta_{K^*} + (I_2^s \sin^2\theta_{K^*} + I_2^c \cos^2\theta_{K^*}) \cos 2\theta_l \\
&+ I_3 \sin^2\theta_{K^*} \sin^2\theta_l \cos 2\phi + I_4 \sin 2\theta_{K^*} \sin 2\theta_l \cos\phi \\
&+ I_5 \sin 2\theta_{K^*} \sin\theta_l \cos\phi \\
&+ (I_6^s \sin^2\theta_{K^*} + I_6^c \cos^2\theta_{K^*}) \cos\theta_l + I_7 \sin 2\theta_{K^*} \sin\theta_l \sin\phi \\
&+ I_8 \sin 2\theta_{K^*} \sin 2\theta_l \sin\phi + I_9 \sin^2\theta_{K^*} \sin^2\theta_l \sin 2\phi\,.
\end{aligned}$$

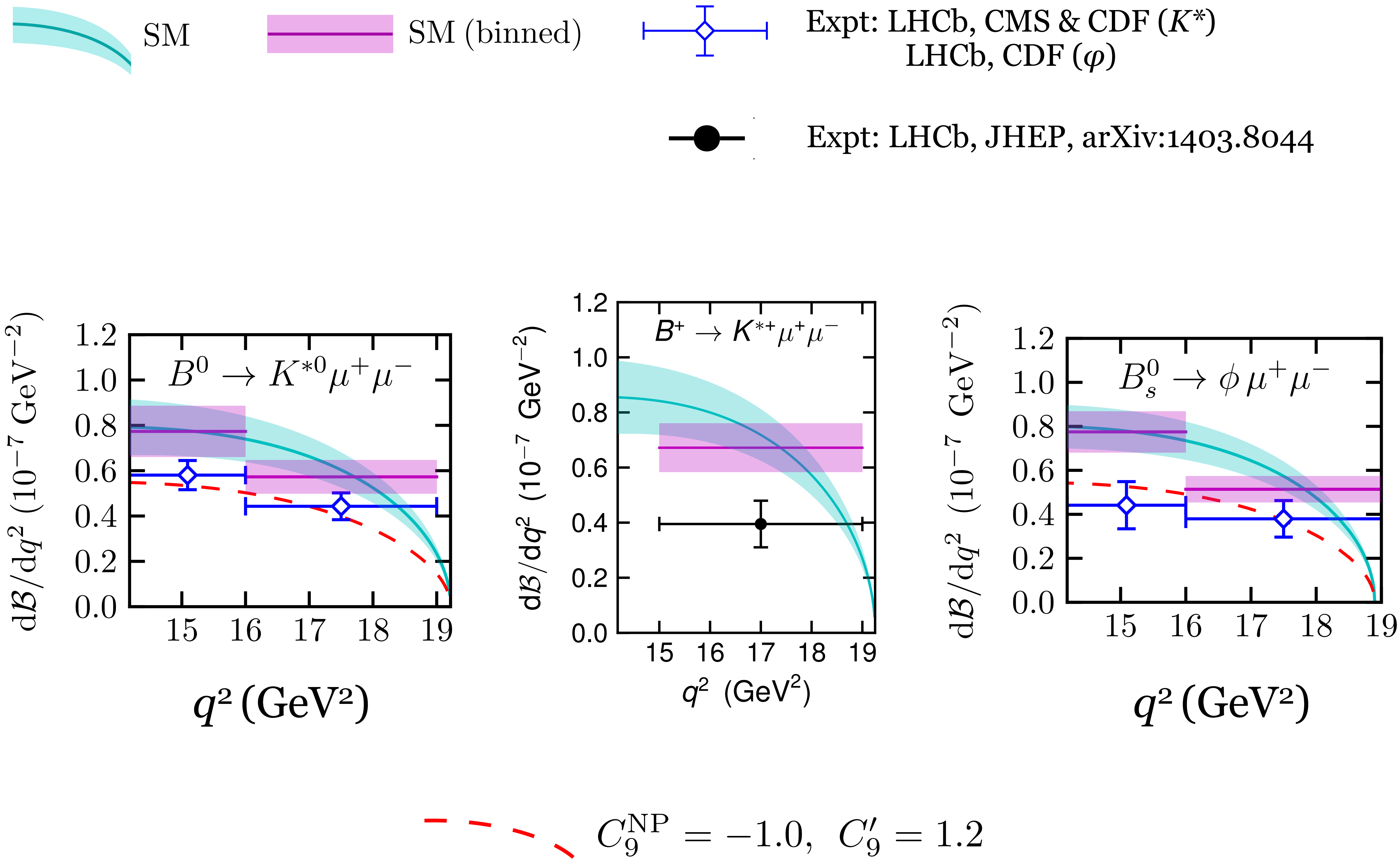
Similarly for $B \to K^{*0}(\to K^+\pi^-)\mu^+\mu^-$ with $I_{1,2,3,4,7} \to \bar{I}_{1,2,3,4,7}$ and $I_{5,6,8,9} \to -\bar{I}_{5,6,8,9}$

$$S_i = \frac{I_i + \bar{I}_i}{d(\Gamma + \bar{\Gamma})/dq^2} \qquad A_i = \frac{I_i - \bar{I}_i}{d(\Gamma + \bar{\Gamma})/dq^2}$$

$$P'_{4,5,6,8} = \frac{\langle S_{4,5,7,8} \rangle}{2\sqrt{-\langle S_2^c \rangle \langle S_2^s \rangle}} \qquad \langle\cdot\rangle \;\Rightarrow\; \text{binned in } q^2$$

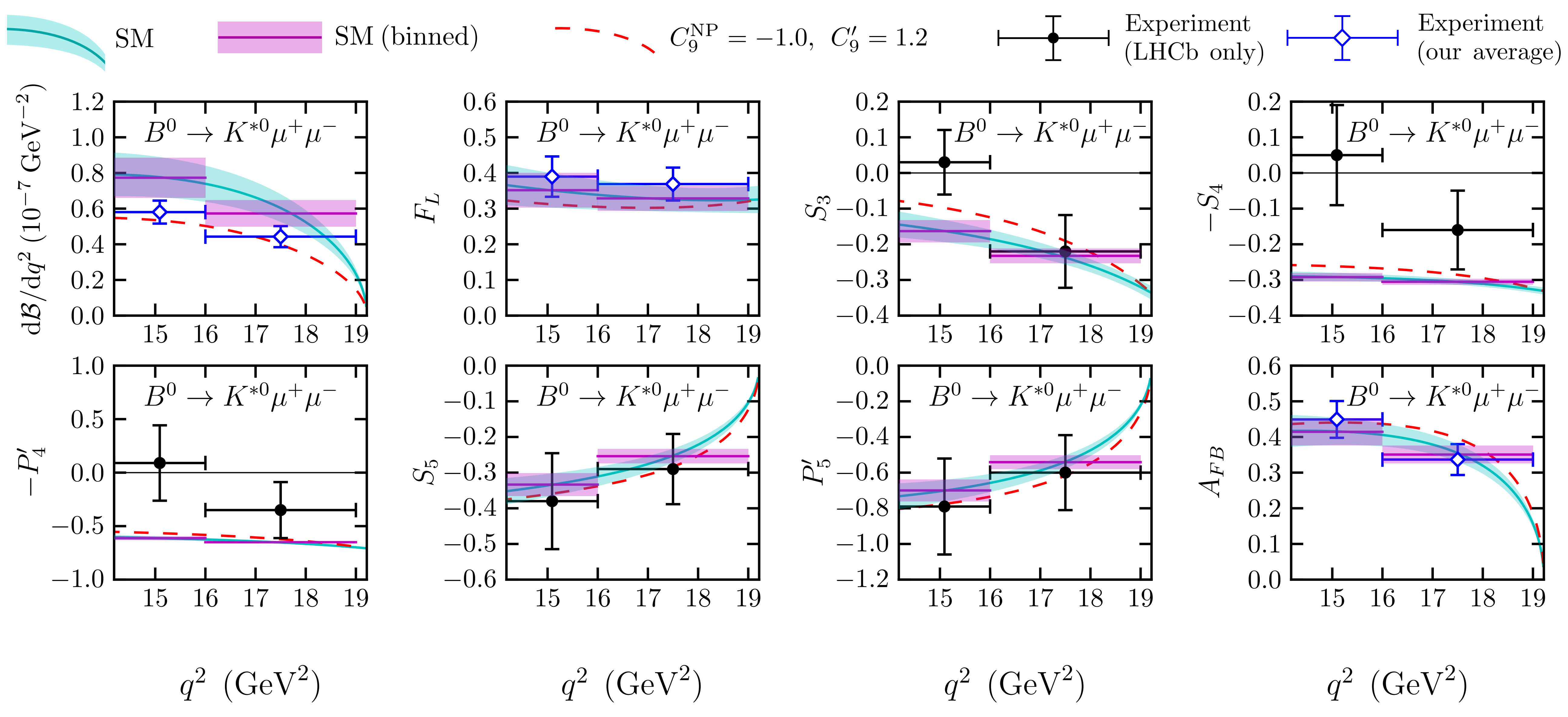
Ratios insensitive to f.f. at low q^2. Descotes-Genon, Matias, Ramon, Virto [JHEP 01 (2013) 048]

Branching fractions



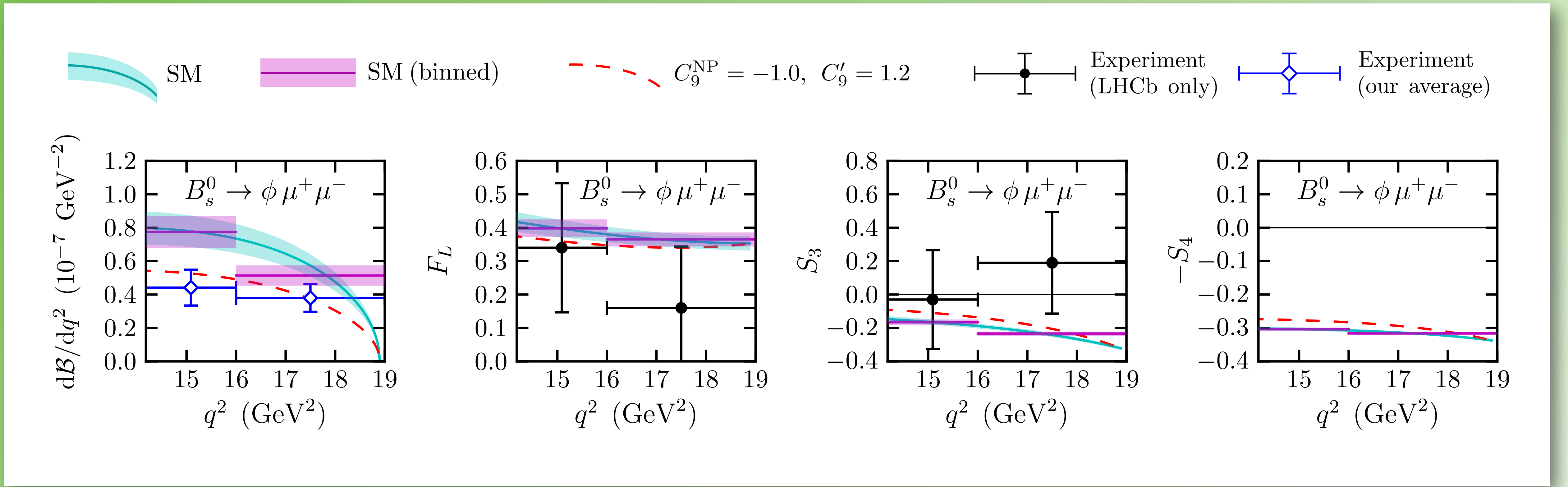
Horgan, Liu, Meinel, Wingate, PRL 112, arXiv:1310.3887; S Meinel, Paris Workshop 2014

$B \to K^* \mu^+ \mu^-$ observables



Horgan, Liu, Meinel, Wingate, PRL 112, arXiv:1310.3887

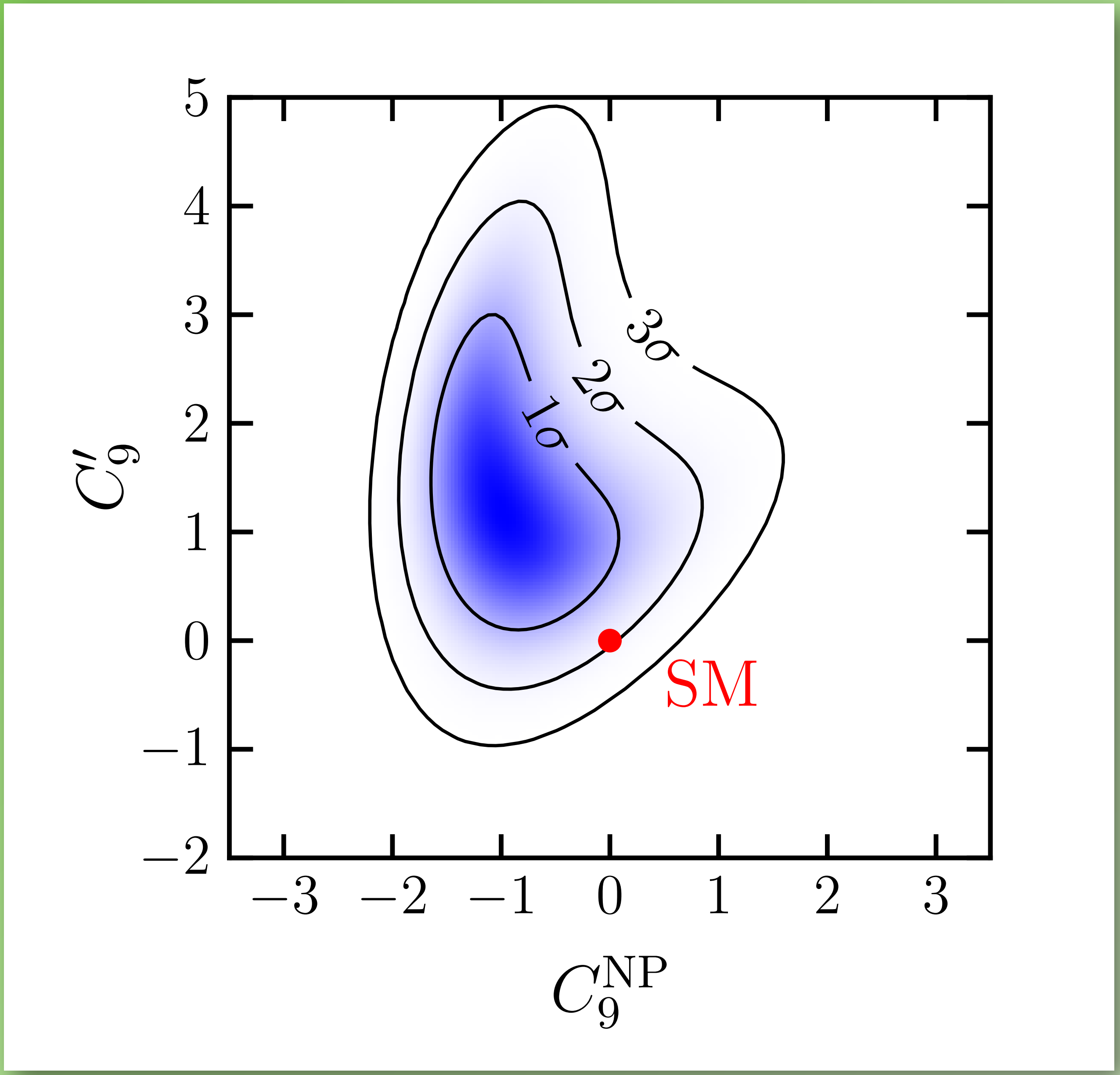
$B_s \to \varphi\, \mu^+ \mu^-$ observables



Horgan, Liu, Meinel, Wingate, PRL 112, arXiv:1310.3887

Fit to low recoil B to V data

Best fit: $C_9^{\mathrm{NP}} = -1.0 \pm 0.6$ $C_9' = 1.2 \pm 1.0$

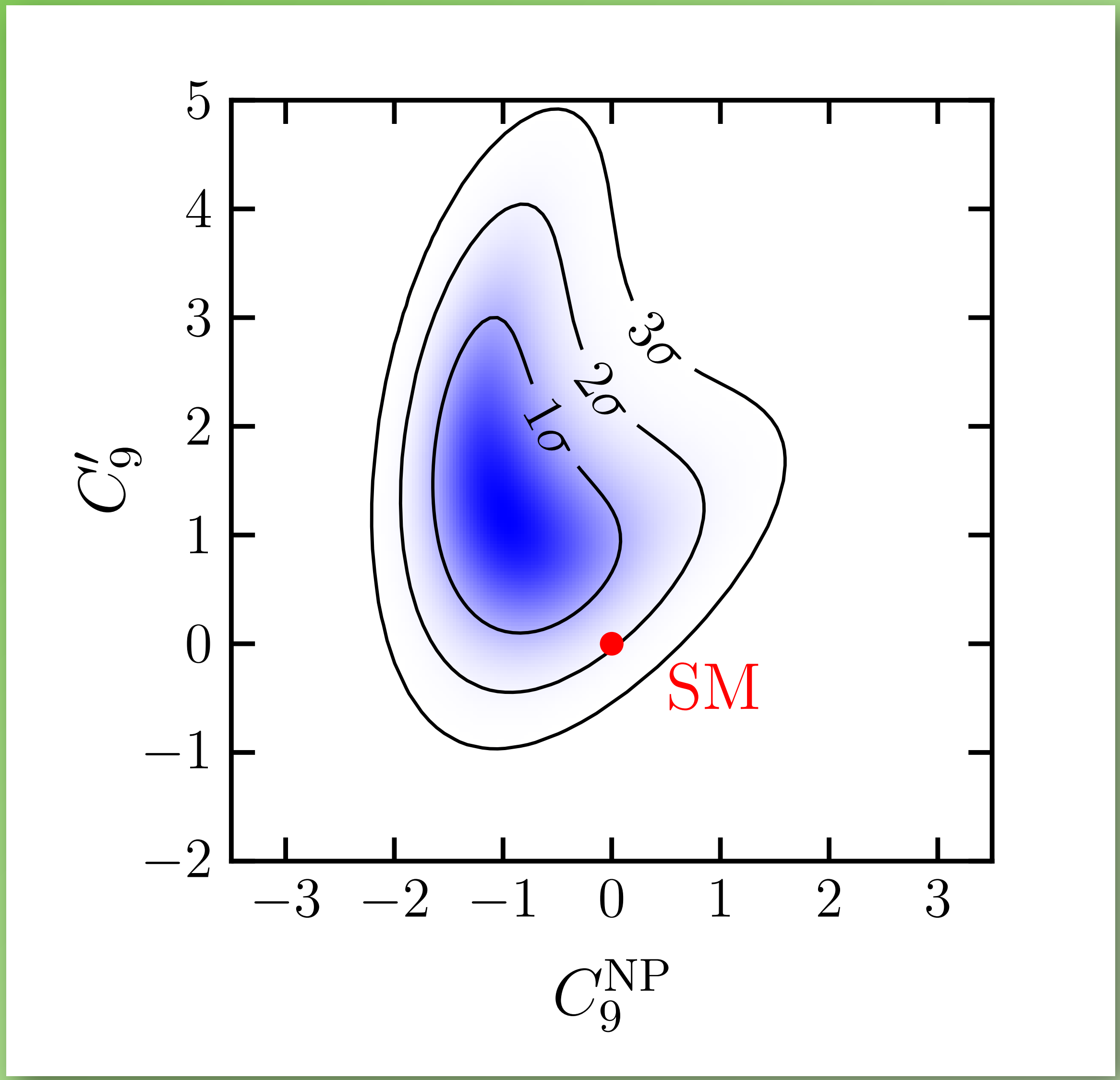


Likelihood function

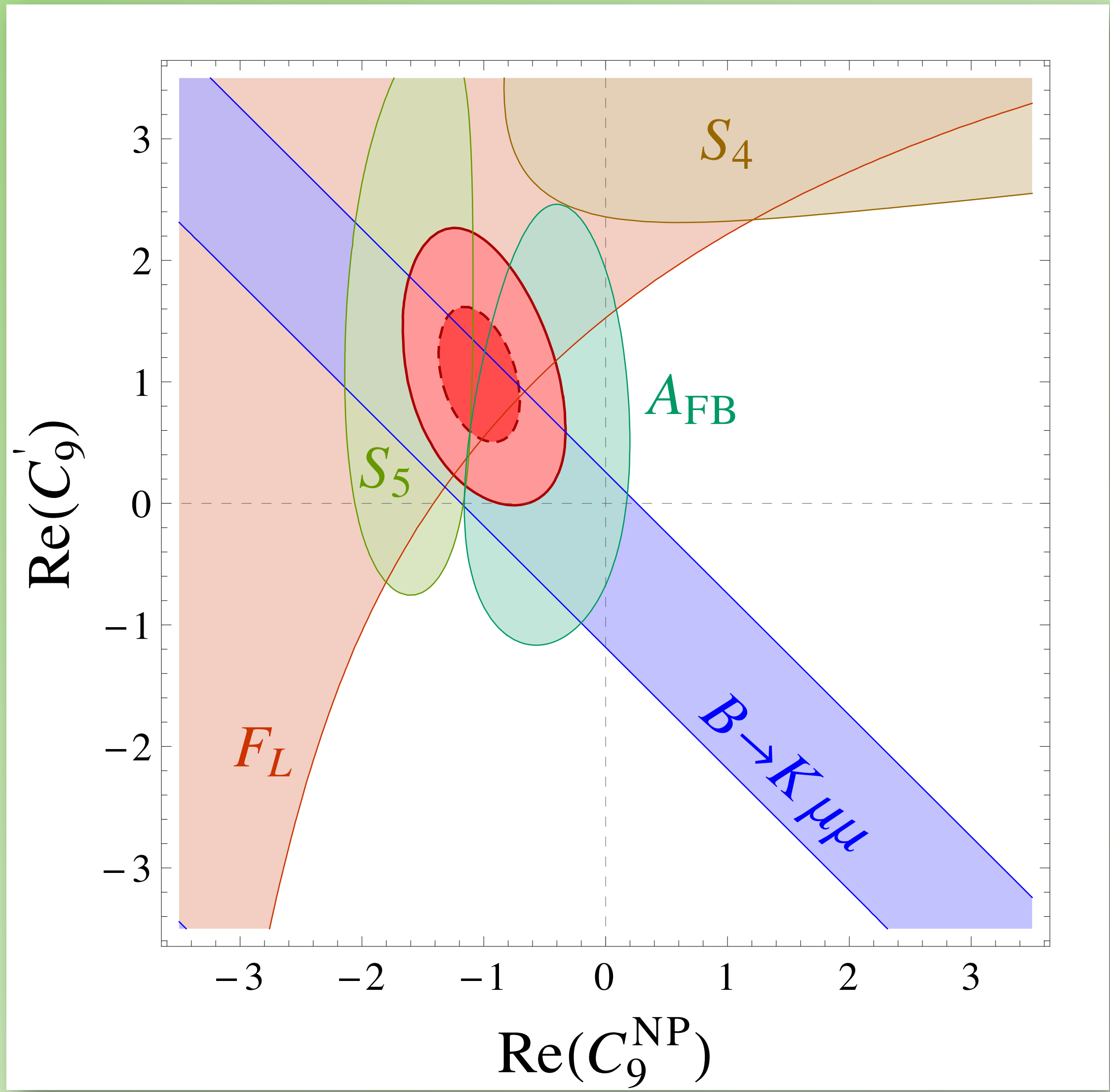
- C_9, C_9' assumed to be real
- Data in 2 highest q^2 bins
  - $B \to K^*\mu\mu$ (neutral mode): dB/dq^2, F_L, S_3, S_4, S_5, A_{FB}
  - $B_s \to \varphi\mu\mu$: dB/dq^2, F_L, S_3
- Theory correlations between observables & bins taken into account

Horgan, Liu, Meinel, Wingate, PRL 112, arXiv:1310.3887

2 complementary fits



Horgan, Liu, Meinel, Wingate,
arXiv:1310.3887



Altmannshofer & Straub,
arXiv:1308.1501

Low q^2 discrepancy

Descotes-Genon, Matias, Virto [PRD88, 074002, (2013), arXiv:1307.5683]

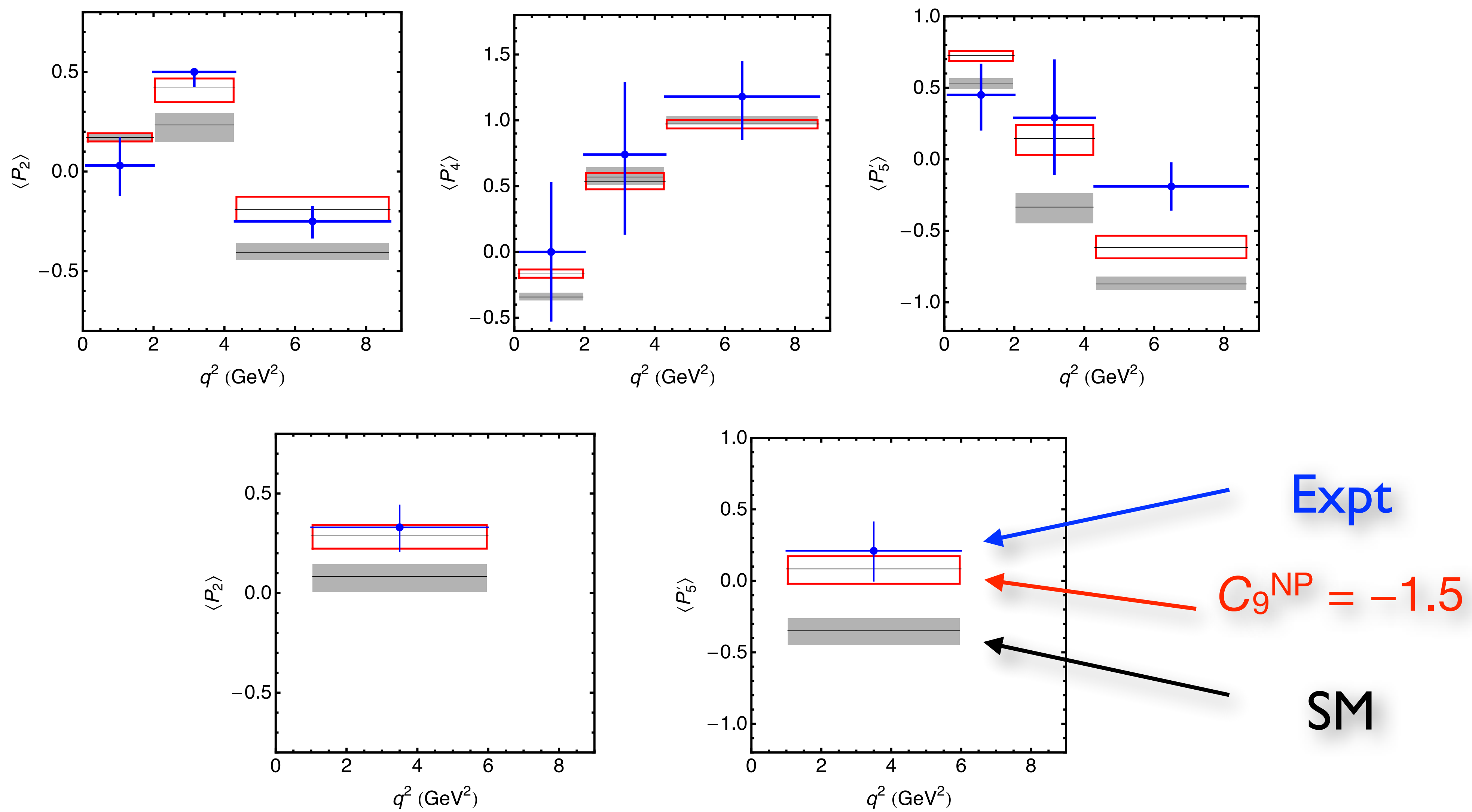
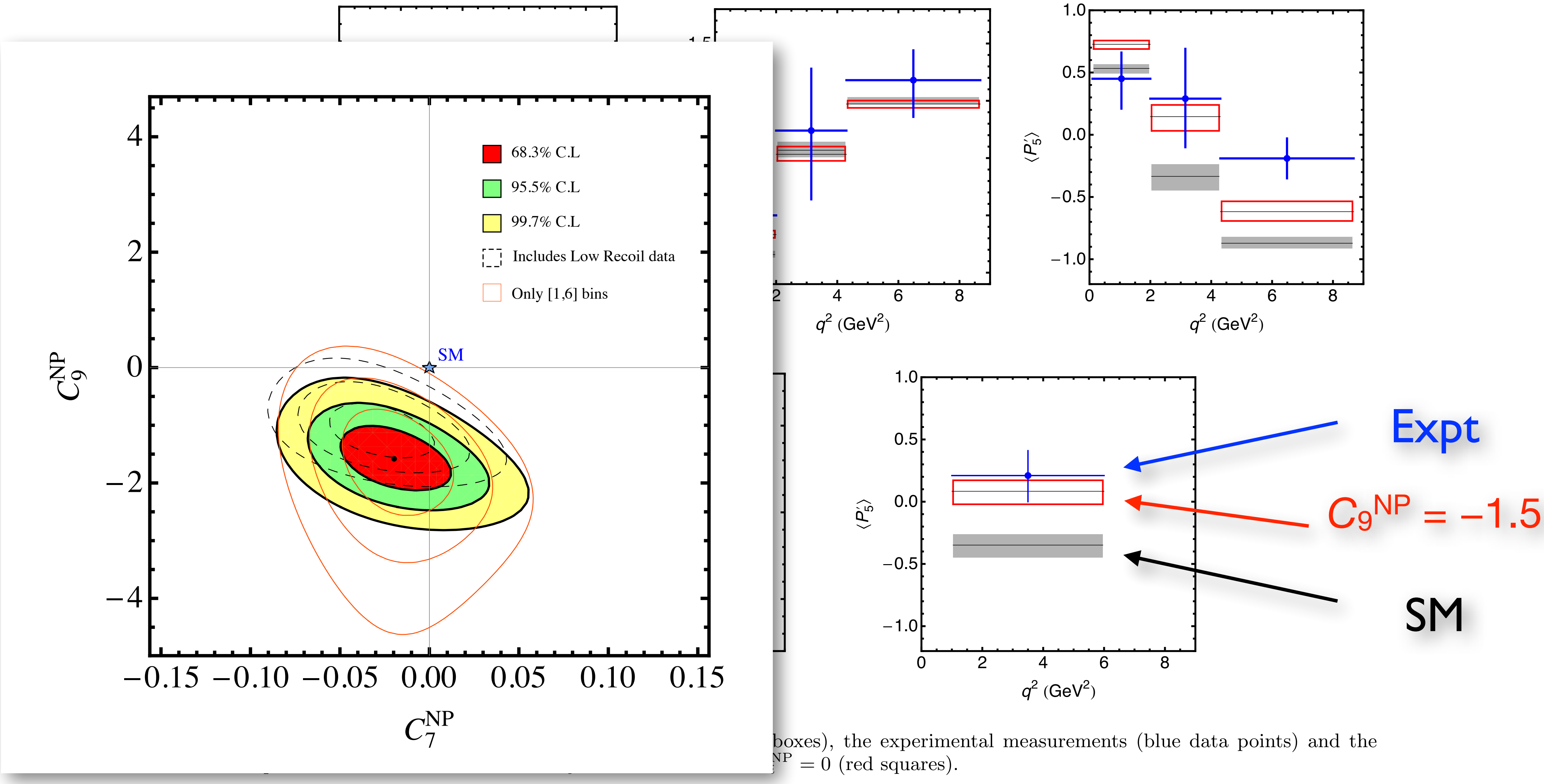


FIG. 2: Comparison between the SM predictions (gray boxes), the experimental measurements (blue data points) and the predictions for the scenario with $\mathcal{C}_9^{\rm NP} = -1.5$ and other $\mathcal{C}_i^{\rm NP} = 0$ (red squares).

Agree with negative NP contribution to C_9. They do not find $C_9' \neq 0$

Low q^2 discrepancy

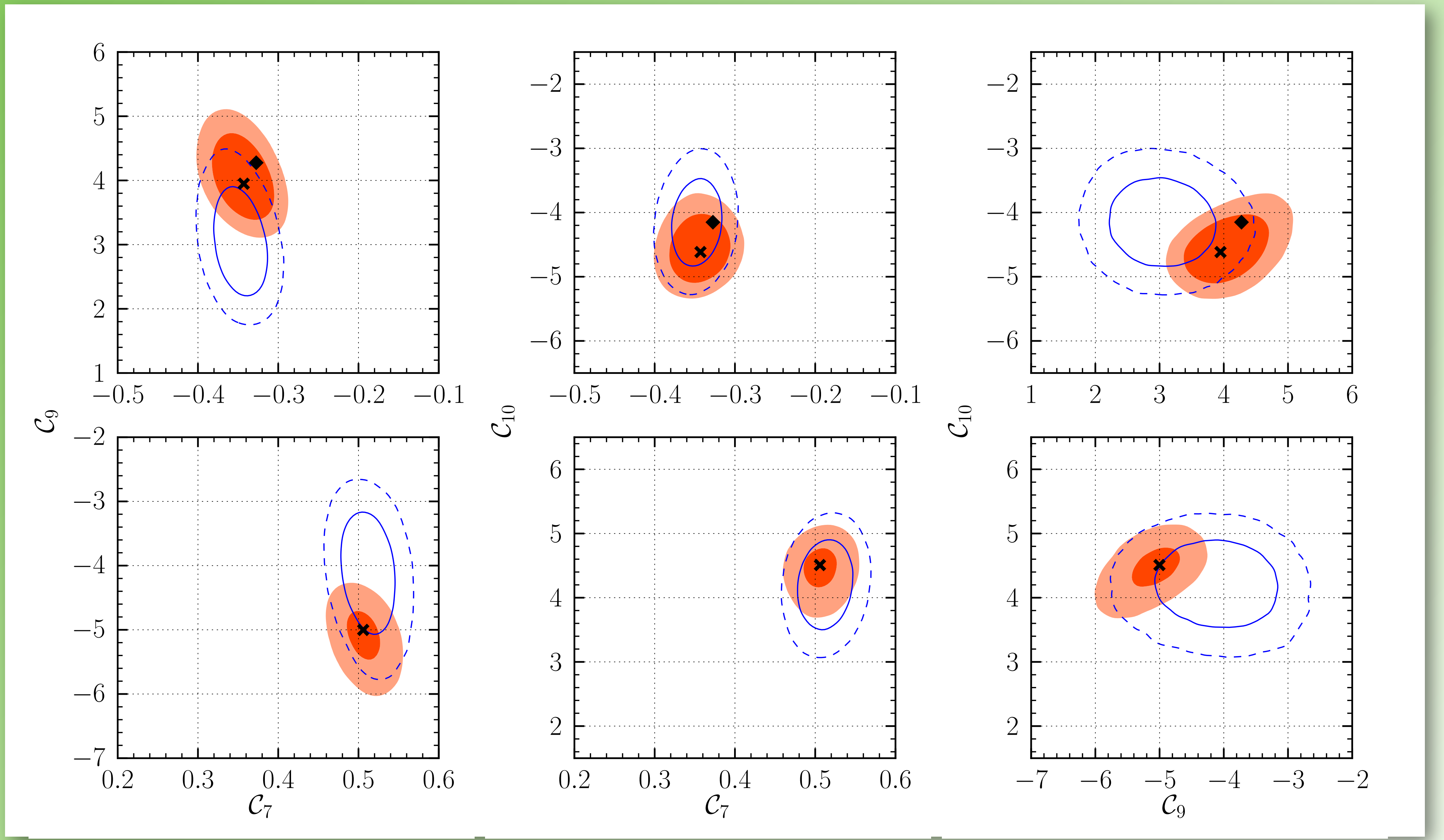
Descotes-Genon, Matias, Virto [PRD88, 074002, (2013), arXiv:1307.5683]



Agree with negative NP contribution to C_9. They do not find $C_9' \neq 0$

But another fit...

Consistent with SM (and also with negative NP contribution to C_9)



Orange: full fit. Blue: selection fit

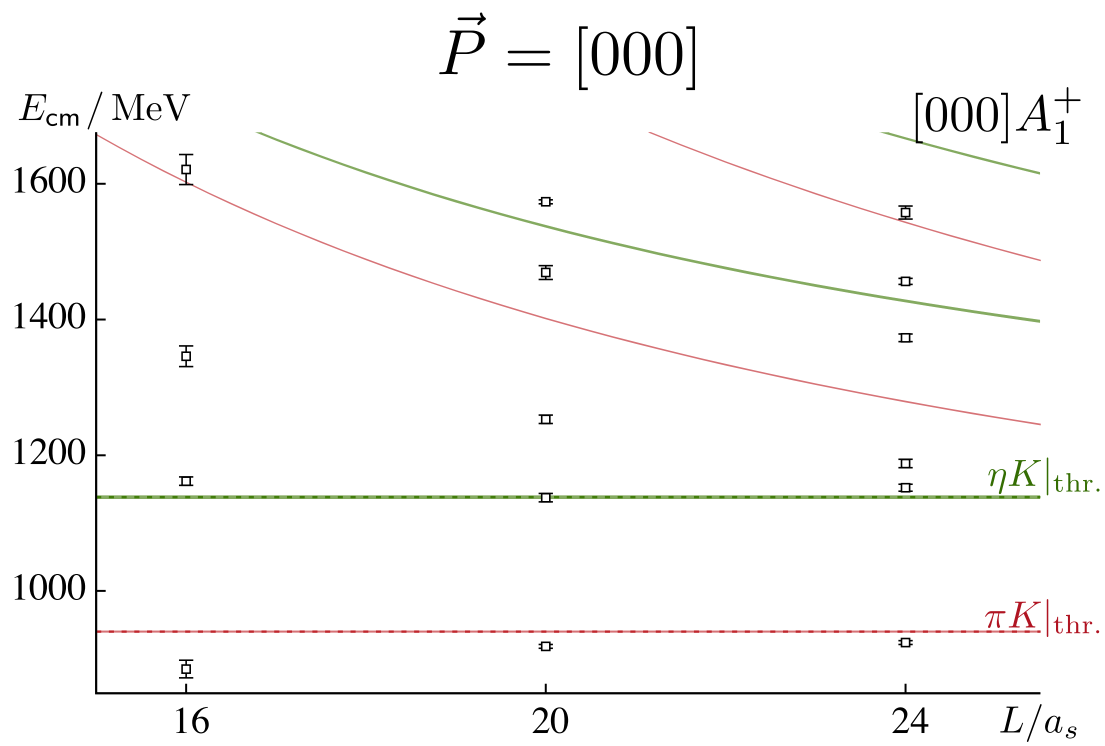
Beaujean, Bobeth, van Dyk, [Eur. Phys. J. C 74 (2014), arXiv:1310.2478]

Vector meson decay

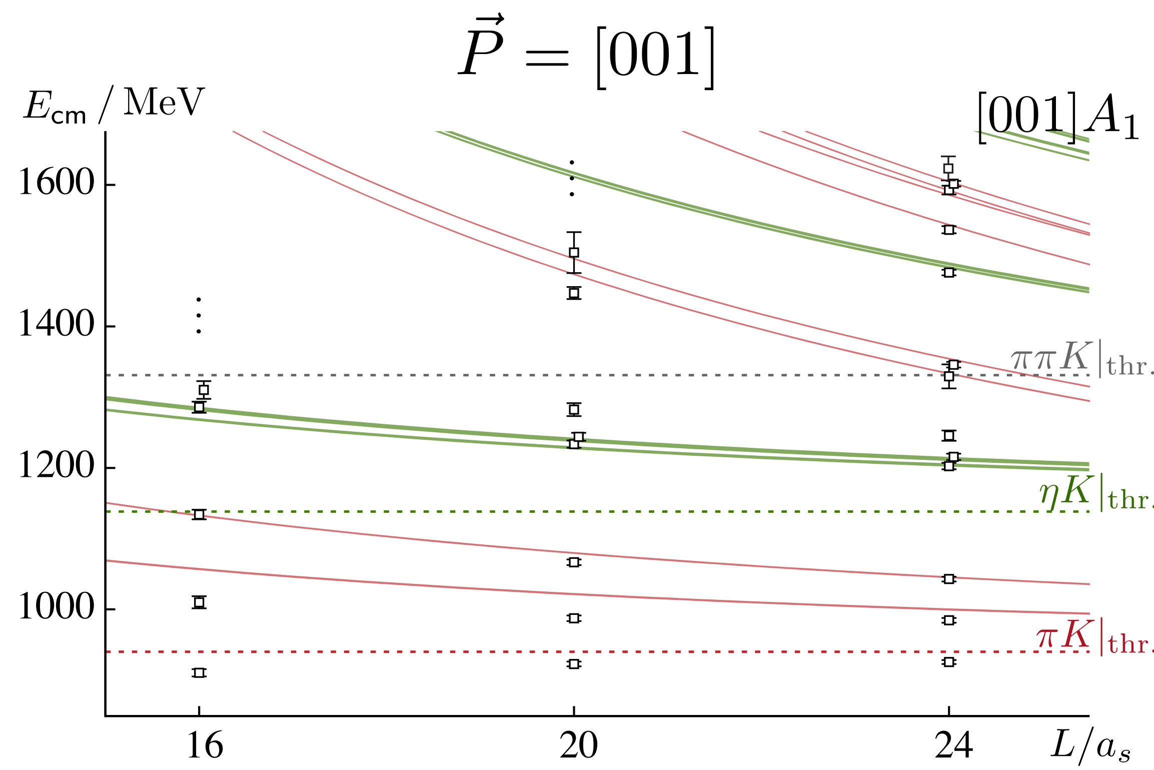
- Briceño, Hansen, Walker-Loud [arXiv:1406.5965] have extended the Lellouch-Lüscher method for $K \to \pi\pi$ to $B \to K^*$ form factors, correctly including πK and ηK states on the lattice
- Dudek, Edwards, Thomas, Wilson [arXiv:1406.4158] have begun the necessary step of numerically calculating states with quantum numbers of K^*, πK, and ηK on the lattice
- Phase shifts must be determined precisely: derivatives enter matrix element formalism
- Formalism worked out for unitary actions. No good for partial quenching or staggered.

Energy levels

Before looking $B \to K^*$ ($\to K\,\pi$) form factors, the πK and ηK systems must first be studied. This has been done by HSC on n_f=2, improved-Wilson fermion configurations.



J^P = 0+ contribute



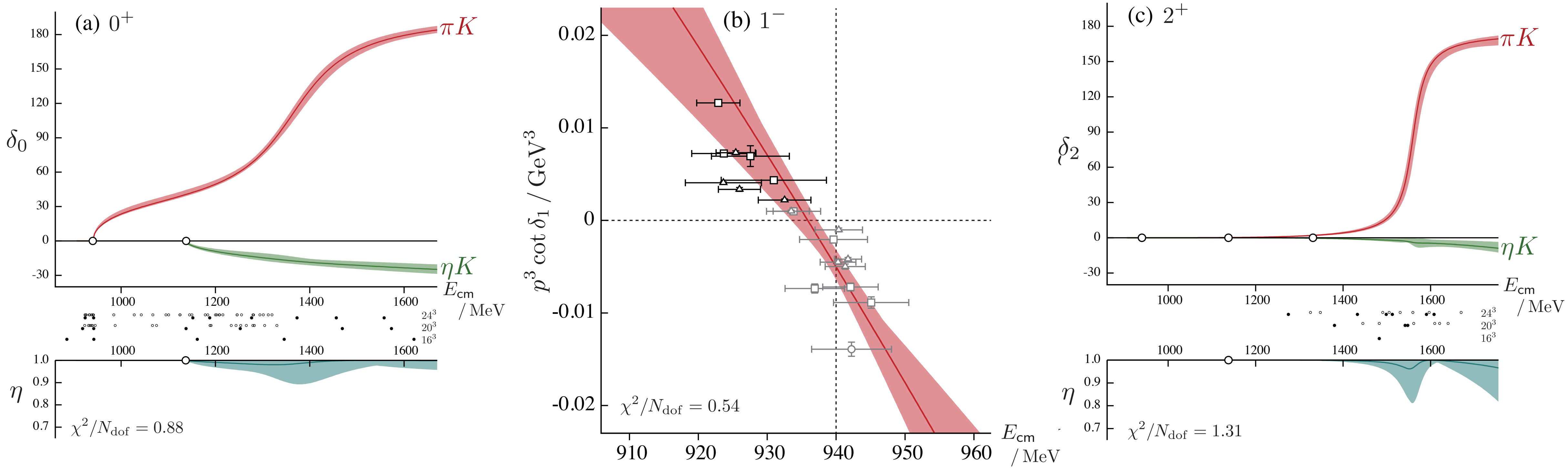
J^P = 0+, 1-, 2+ contribute

Points: Lattice QCD data.
Red curves: noninteracting πK levels (discrete momenta)
Green: noninteracting ηK levels (discrete momenta)

Dudek, Edwards, Thomas, Wilson [arXiv:1406.4158]

Phase shifts & inelasticities

$J^P = 0+$ $J^P = 1-$ $J^P = 2+$



Precise determination of phase shifts, and derivatives, are necessary ingredients to formalism for $B \to K^* (\to K\,\pi)$ form factors

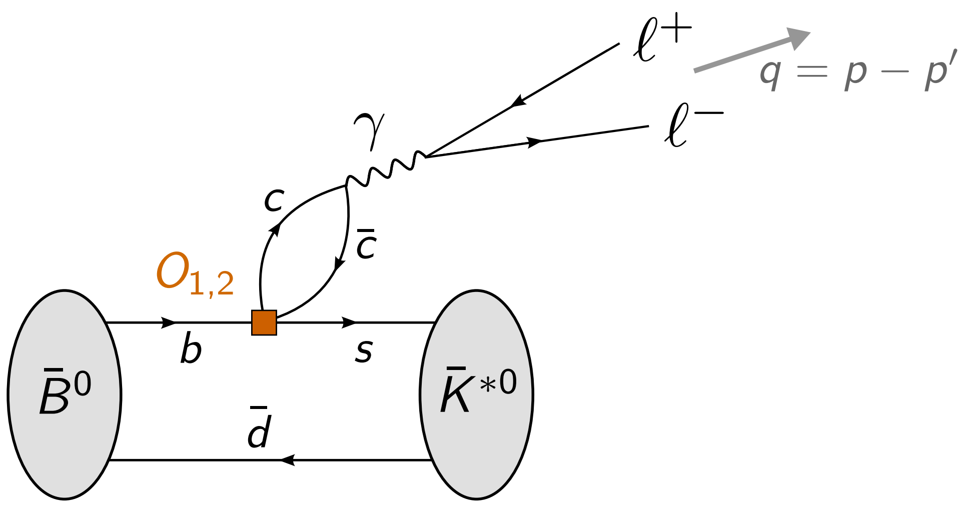
Dudek, Edwards, Thomas, Wilson [arXiv:1406.4158]

Matrix elements of nonlocal operators

$$\mathcal{M} = \frac{G_F\,\alpha}{\sqrt{2}\pi} V_{tb} V_{ts}^* \Big[(\mathcal{A}_\mu + \mathcal{T}_\mu) \bar{u}_\ell \gamma^\mu v_\ell + \mathcal{B}_\mu \bar{u}_\ell \gamma^\mu \gamma_5 v_\ell \Big]$$

$$\mathcal{A}_\mu = -\frac{2m_b}{q^2}\, q^\nu \langle \bar{K}^* |\, \bar{s}\, i\sigma_{\mu\nu} (C_7 P_R + C_7' P_L) b \,| \bar{B} \rangle + \langle \bar{K}^* |\, \bar{s} \gamma_\mu (C_9 P_L + C_9' P_R) b \,| \bar{B} \rangle$$

$$\mathcal{B}_\mu = \langle \bar{K}^* |\, \bar{s} \gamma_\mu (C_{10} P_L + C_{10}' P_R) b \,| \bar{B} \rangle$$



$$\mathcal{T}_\mu = \frac{-16 i \pi^2}{q^2} \sum_{i=1\ldots6;8} C_i \int \mathrm{d}^4x\; e^{iq\cdot x} \langle \bar{K}^* |\, \mathsf{T}\, O_i(0)\, j_\mu(x) \,| \bar{B} \rangle$$

Affects all *b* ➞ *sll* decays, regardless of initial/final hadrons

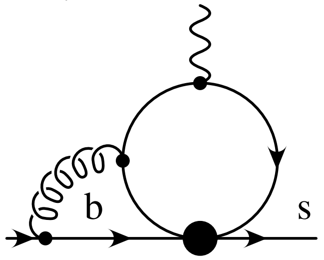
OPE at large q^2

$$\mathcal{T}_\mu = -T_7(q^2)\frac{2m_b}{q^2}\, q^\nu \langle \bar{K}^* | \bar{s}\, i\sigma_{\mu\nu} P_R b | \bar{B}\rangle + T_9(q^2)\langle \bar{K}^* | \bar{s}\gamma_\mu P_L b | \bar{B}\rangle$$

$$+O\left(\frac{\alpha_s \Lambda}{m_b}, \frac{\Lambda^2}{m_b^2}, \frac{m_c^4}{q^4}\right)$$

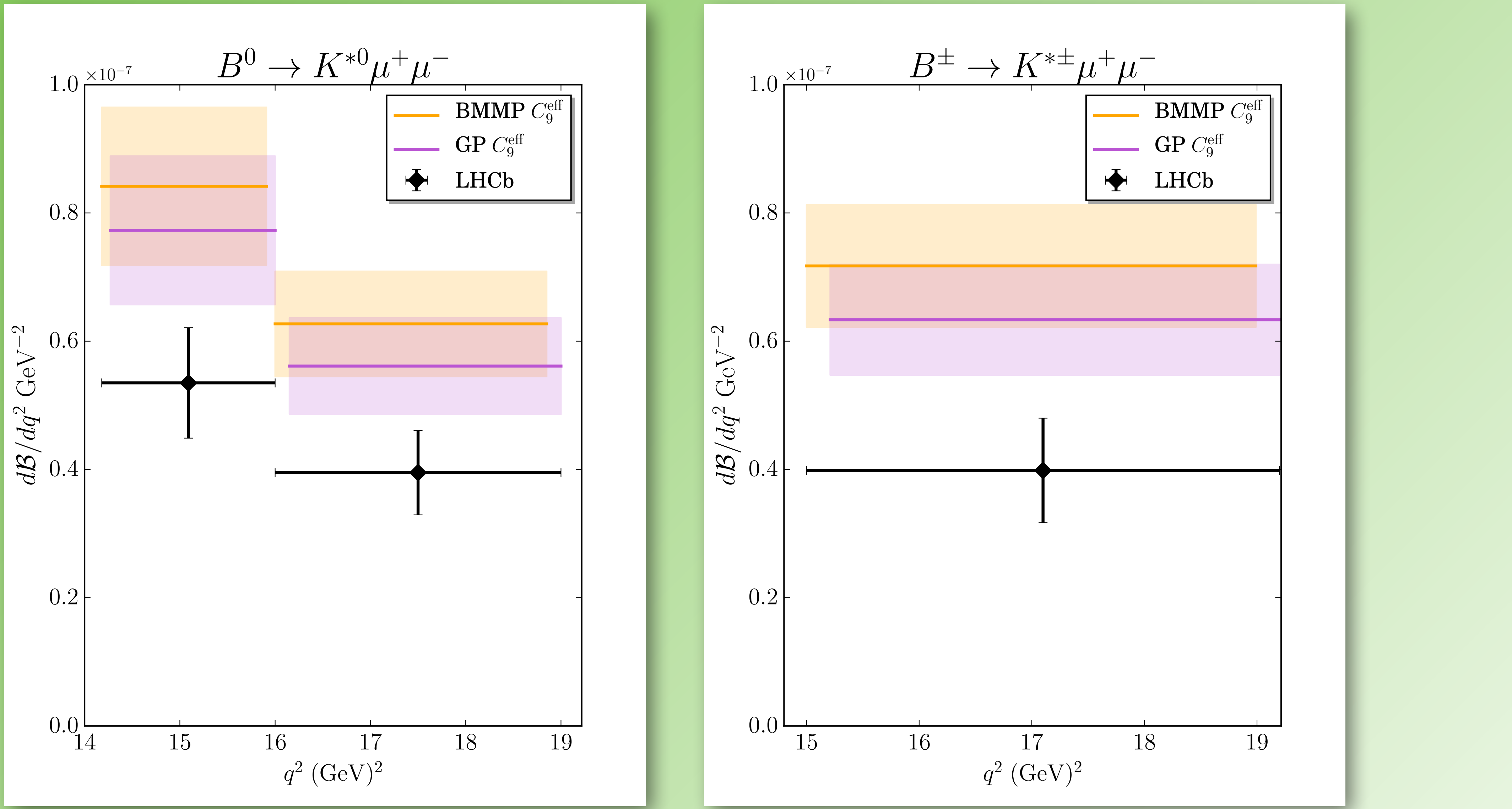
Grinstein & Pirjol, PRD 70, 114005 (2004)

- First correction in expansion (m_c^2/q^2) simply augments $C_7^{\rm eff}$ and $C_9^{\rm eff}$: Buras, Misiak, Münz, Pokorski (BMMP) ➙ Grinstein, Pirjol (GP)
- Order $\alpha_s\Lambda/m_b$ corrections calculable on lattice



- Local duality: bin observables in q^2
- Duality violations estimated to be small (~2% in model): Beylich, Buchalla, Feldmann, [Eur. Phys. J C71, 1635 (2011), arXiv:1101.5118]
- On the other hand, Lyon & Zwicky [arXiv:1406.0566] claim charmonium resonances can have a much larger effect, even on binned observables: "complete breakdown of factorization"

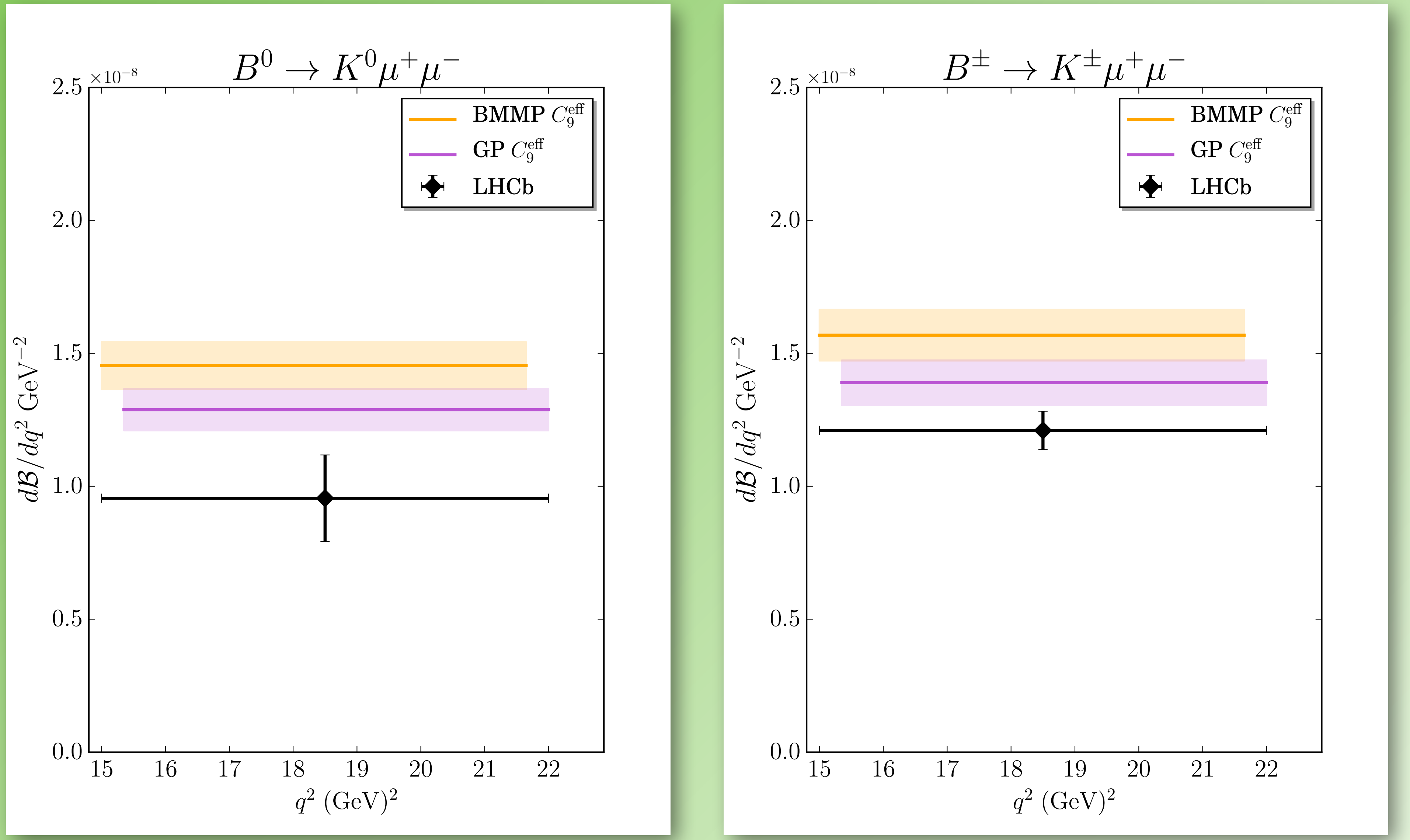
Charmonium effects from OPE: K^*



- BMMP ➙ GP correction shifts BF bin ~10%. Remaining corrections should be smaller (~1-5%, direction unknown)
- Duality violations estimated to be small (~2% in model), Beylich, Buchalla, Feldmann)
- (Calculation of non-local m.e. on lattice *very* challenging. Generalize Δm_K??)

Charmonium effects from OPE: K

Form factors from Bouchard et al. (HPQCD) [PRD 88, arXiv:1306.2384]



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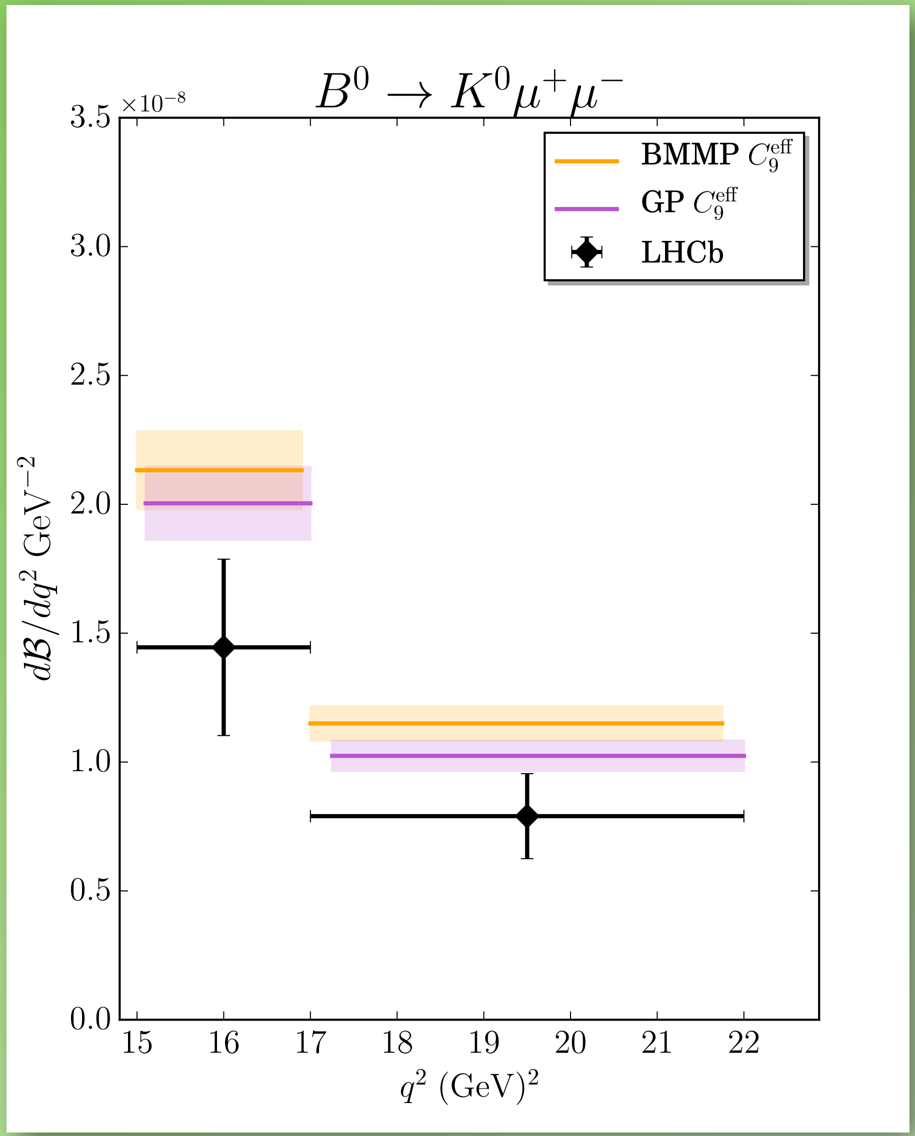
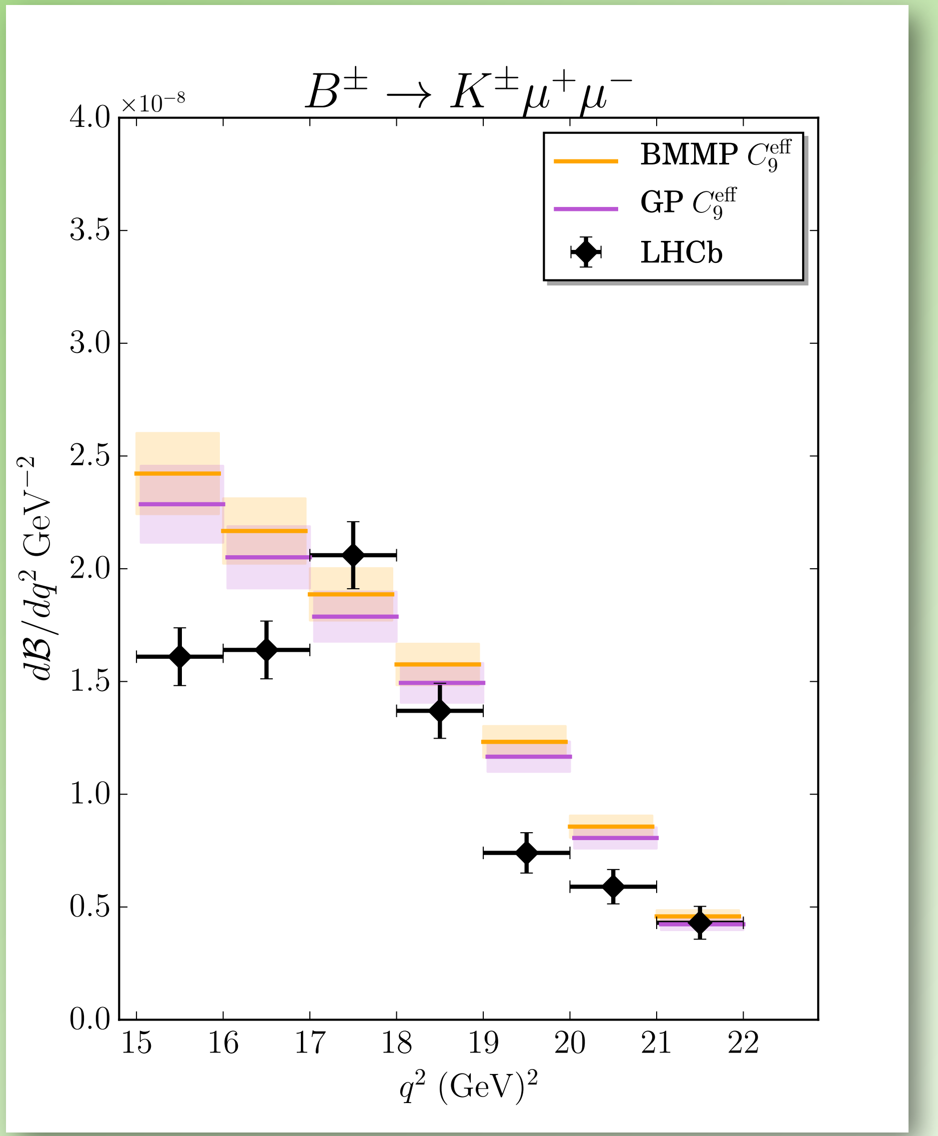
Narrower bins

Form factors from Bouchard et al. (HPQCD) [PRD 88, arXiv:1306.2384]



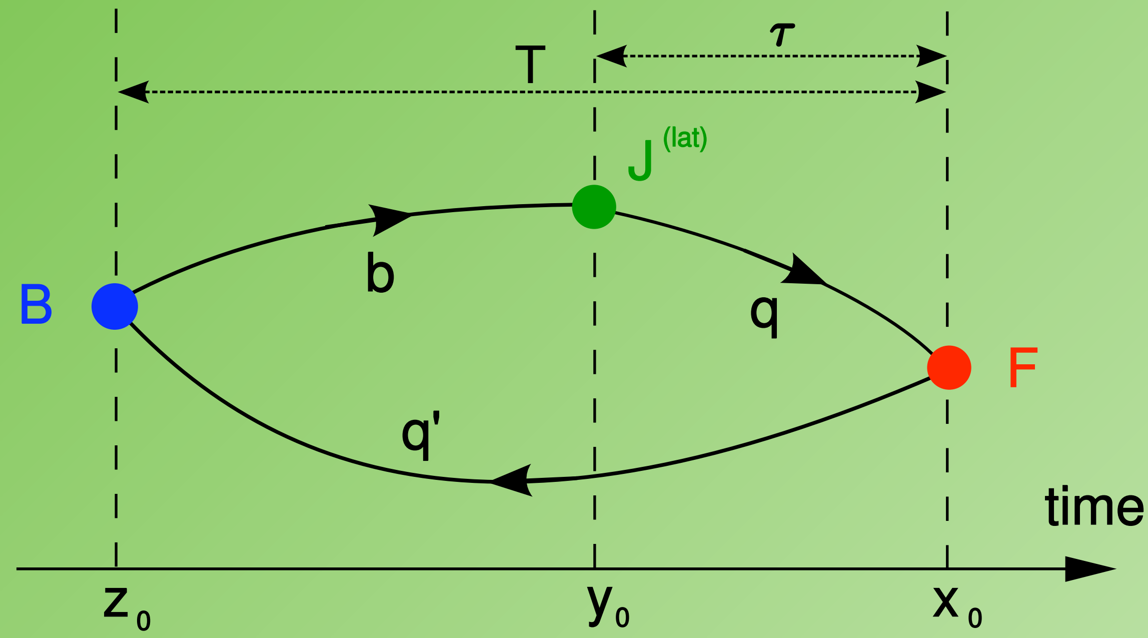


Conclusions

- Unquenched Lattice QCD calculations of $B \to K^*$ and $B_s \to \varphi$ form factors (also $B_s \to K^*$)
- Briceño, Hansen, Walker-Loud formalism for correctly treating K^* and φ will take time to implement, but in principle this can be brought under control: all form factor uncertainties quantifiable
- Experimental measurements for $B \to K^* \mu^+\mu^-$, $B_s \to \varphi\, \mu^+\mu^-$, *and* $B \to K \mu^+\mu^-$ branching fractions are low compared to present SM predictions (look forward to greater precision in $\Lambda_b \to \Lambda\, \mu^+\mu^-$)
- Is the large-q^2 OPE sufficient to account for matrix elements of nonlocal operators, at least in wide bins?
- [$B \to \rho$ still noisy even with 32K measurements.]

extra

Strange quark mass interpolation



	offspring	spectator
$B_s \to \phi$	+2-4%	+2-4%
$B \to K$	+2-4%	–8-13%
$B_s \to K^*$	–8-13%	+2-4%

$$\Delta y = \frac{1}{(4\pi f_\pi)^2}\left(m^2_{\mathrm{offspr}} - m^2_{\eta_s,\mathrm{phys}}\right)$$

$$\Delta w = \frac{1}{(4\pi f_\pi)^2}\left(m^2_{\mathrm{spect}} - m^2_{\eta_s,\mathrm{phys}}\right)$$

$$F(t;\Delta y,\Delta w) = \frac{1}{P(t)}\left[a_0\left(1 + f_{01}\Delta y + g_{01}\Delta w\right) + a_1 z\right]$$

Use results of 3-f.f. fits to include c_{01s} in final fits:

$B_s \to \phi$	$c_{01s} = f_{01} + g_{01}$
$B \to K$	$c_{01s} = f_{01}$
$B_s \to K^*$	$c_{01s} = g_{01}$

Heavy quark mass tuning

- Heavy meson masses are 5% too large
- Isgur-Wise relations (PRD42, 2388(1990)):

$$V, A_0, T_1, T_{23} \propto \sqrt{m_B}$$

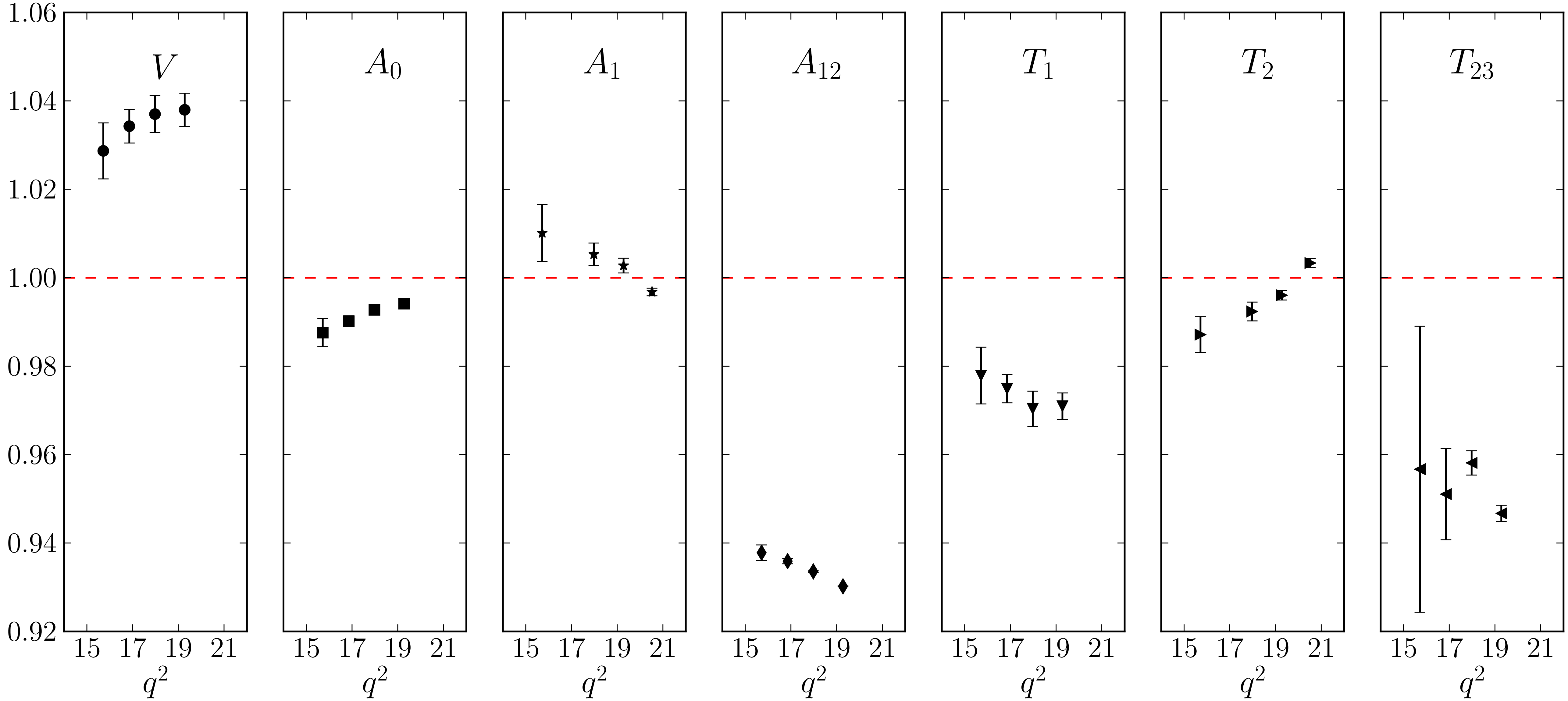
$$A_1, A_{12}, T_2 \propto \frac{1}{\sqrt{m_B}}$$

➡ Adjust central values by 2.5% in appropriate direction

➡ Remaining Λ/m_b error is less than 1%

Operator matching

[$F(q^2)$ including $1/m_b$ currents] / [$F(q^2)$ excluding $1/m_b$ currents]



1) Ratio statistically precise due to correlations

2) $1/m_b$ effect comparable size to statistical error in absolute value of f.f.