THE B MESON LIGHT-CONE DISTRIBUTION AMPLITUDE

[GUIDO BELL]

I am grateful for discussions with T. Feldmann, A. Khodjamirian, B. Lange, J. Rohrwild and C. Sachrajda

Definition

Light-cone matrix element in HQET

$$\langle 0 | \bar{q}(tn)\,[tn,0]\,\not{n}\gamma_5\,h_v(0) | \bar{B}(m_B v)\rangle = i m_B\,\tilde{f}_B(\mu)\,\tilde{\phi}_B^+(t,\mu)$$

with $n^2 = 0$, $v^2 = 1$ and $n \cdot v = 1$

Gauge-link

$$[z,0] = \mathcal{P}\,\exp\left(i g_s \int_0^1 du\; z \cdot A(uz)\right)$$

Fourier transform

$$\phi_B^+(\omega,\mu) = \int \frac{dt}{2\pi}\; e^{i\omega t}\;\tilde{\phi}_B^+(t,\mu)$$

where $\omega = n \cdot k$ is a light-cone projection of the spectator quark momentum

Motivation

Hadronic input function for exclusive energetic B decays

- QCD factorisation theorems [Beneke, Buchalla, Neubert, Sachrajda 99]
- QCD/SCET sum rules [Khodjamirian, Mannel, Offen 05; De Fazio, Feldmann, Hurth 05]

Phenomenological applications often involve inverse moments

$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega}\, \phi_B^+(\omega,\mu)$$

$$\frac{\sigma_n(\mu)}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega}\, \ln^n\left(\frac{\mu}{\omega}\right)\, \phi_B^+(\omega,\mu)$$

$\Rightarrow$ essential contribution to theoretical uncertainties in QCDF calculations

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RG evolution

Scale dependence known to one-loop order [Lange,Neubert 03]

$$\frac{d\phi_B^+(\omega,\mu)}{d\ln\mu} = -\left(\Gamma_{\rm cusp}\ \ln\left(\frac{\mu}{\omega}\right) + \gamma_+\right)\phi_B^+(\omega,\mu) + \Gamma_{\rm cusp}\int_0^\infty d\eta\ \left[\frac{\omega\,\theta(\eta-\omega)}{\eta(\eta-\omega)} + \frac{\theta(\omega-\eta)}{(\omega-\eta)}\right]_+ \phi_B^+(\eta,\mu)$$

- Sudakov-type evolution due to cusp between n^μ and v^μ Wilson lines

Complicated analytic solution [Lange, Neubert 03; Lee, Neubert 05]

$$\phi_B^+(\omega,\mu) = e^{V-2\gamma_E g}\ \frac{\Gamma(2-g)}{\Gamma(g)}\int_0^\infty \frac{d\eta}{\eta}\ \frac{\omega_<}{\omega_>}\left(\frac{\omega_>}{\mu_0}\right)^g {}_2F_1\left(1-g, 2-g; 2; \frac{\omega_<}{\omega_>}\right)\phi_B^+(\eta,\mu_0)$$

with $\omega_< = \min(\omega,\eta)$ and $\omega_> = \max(\omega,\eta)$

- RG kernels $V = V(\mu,\mu_0)$ and $g = g(\mu,\mu_0)$ are known to NLL
- valid for $0 < g < 1$ (sufficient for phenomenological applications)

Eigenfunctions of RG kernel

Continuous set of eigenfunctions [GB, Feldmann, Wang, Yip 13]

$$f_{\omega'}(\omega) = \sqrt{\frac{\omega}{\omega'}}\, J_1\left(2\sqrt{\frac{\omega}{\omega'}}\right) \qquad \gamma_{\omega'} = -\left(\Gamma_{\rm cusp} \ln\left(\frac{\mu}{\hat{\omega}'}\right) + \gamma_+\right)$$

- analogue of Gegenbauer polynomials for pion LCDA

Formal representation of RG kernel [Braun, Manashov 14]

$$\mathcal{H} = \ln(i\mu S_+) - \psi(1) - \frac{5}{4} \qquad S_+ = t^2\,\partial_t + 2\,j\,t$$

- $j = 1$ is conformal spin of light quark
- eigenfunctions: $Q_s(t) = -\dfrac{e^{is/t}}{t^2} \quad \Rightarrow \quad \left\langle e^{-i\omega t}\middle|Q_s(t)\right\rangle = \dfrac{f_{1/s}(\omega)}{s\omega}$

Dual representation

Expansion in terms of eigenfunctions

$$\phi_B^+(\omega,\mu) = \int_0^\infty \frac{d\omega'}{\omega'} \sqrt{\frac{\omega}{\omega'}}\, J_1\left(2\sqrt{\frac{\omega}{\omega'}}\right)\, \rho_B^+(\omega',\mu)$$

with inverse transformation

$$\rho_B^+(\omega',\mu) = \int_0^\infty \frac{d\omega}{\omega} \sqrt{\frac{\omega}{\omega'}}\, J_1\left(2\sqrt{\frac{\omega}{\omega'}}\right)\, \phi_B^+(\omega,\mu)$$

RG evolution of dual LCDA

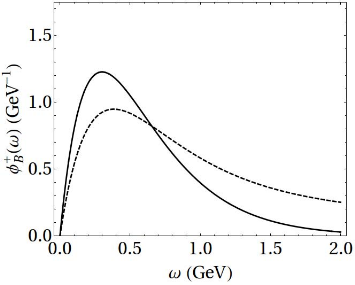
$$\boxed{\frac{d\rho_B^+(\omega',\mu)}{d\ln\mu} = -\left(\Gamma_{\rm cusp}\ln\left(\frac{\mu}{\hat{\omega}'}\right) + \gamma_+\right)\rho_B^+(\omega',\mu)} \qquad \hat{\omega}' = e^{-2\gamma_E}\omega'$$

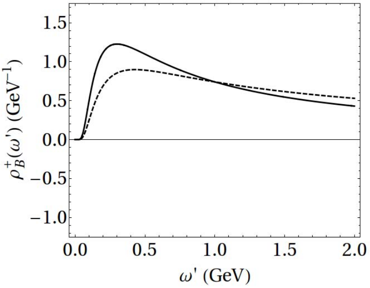
with simple multiplicative solution: $\rho_B^+(\omega',\mu) = e^V \left(\frac{\hat{\omega}'}{\mu_0}\right)^g \rho_B^+(\omega',\mu_0)$

Illustration

Exponential model

$$\phi_B^+(\omega,\mu_0) = \frac{\omega}{\omega_0^2}\; e^{-\omega/\omega_0} \qquad \Longleftrightarrow \qquad \rho_B^+(\omega',\mu_0) = \frac{1}{\omega'}\; e^{-\omega_0/\omega'}$$





solid line: $\omega_0 = 0.3$ GeV, $\mu_0 = 1$ GeV

dashed line: RG evolution with $g = 0.3$ (neglecting $e^{V-2\gamma_E g}$)

Application in QCDF

Inverse moments

$$\int_0^\infty \frac{d\omega}{\omega} \ln^n\left(\frac{\omega}{\mu}\right) \phi_B^+(\omega,\mu) \overset{n=0,1,2}{=} \int_0^\infty \frac{d\omega'}{\omega'} \ln^n\left(\frac{\hat{\omega}'}{\mu}\right) \rho_B^+(\omega',\mu)$$

Form factor relevant for $B \to \gamma\ell\nu$ with energetic photon

$$F(E_\gamma) = H(E_\gamma,\mu) \int_0^\infty \frac{d\omega'}{\omega'} j(2E_\gamma\hat{\omega}',\, \mu)\, \rho_B^+(\omega',\mu)$$

- factorisation into hard, jet and soft dynamics
- each of the ingredients has a simple multiplicative RG evolution

Operator product expansion

Positive moments can be related to local matrix elements in HQET

$$\int_0^\infty d\omega\, \omega^n \, \phi_B^+(\omega,\mu) \qquad \rightarrow \qquad \langle 0 | \bar{q}(0)\, \not{n}\gamma_5\, (in\cdot \overleftarrow{D})^n\, h_v(0) | \bar{B}(m_B v)\rangle$$

- spoiled beyond tree level – positive moments are power divergent

Introduce regularised moments [Lee, Neubert 05]

$$M_n(\Lambda_{UV},\mu) = \int_0^{\Lambda_{UV}} d\omega\, \omega^n \, \phi_B^+(\omega,\mu) = \Lambda_{UV}^n \left\{ K_0^{(n)}(\Lambda_{UV},\mu) + \frac{\bar{\Lambda}}{\Lambda_{UV}}\, K_1^{(n)}(\Lambda_{UV},\mu) + \dots \right\}$$

- representation in dual space

$$M_0(\Lambda_{UV},\mu) = \Lambda_{\rm UV} \int_0^\infty \frac{d\omega'}{\omega'}\, J_2\left(2\sqrt{\frac{\Lambda_{\rm UV}}{\omega'}}\right)\, \rho_B^+(\omega',\mu)$$

QCD-improved model-building

[Feldmann, Lange, Wang 14]

In dual space small and large ω' regions are clearly separated

- $\omega' \gg \Lambda_{QCD}$: implement OPE constraints to determine radiative tail
- $\omega' \sim \Lambda_{QCD}$: truly NP dynamics $\rightarrow$ model (connection to QCD sum rules/Lattice?)

Large ω' region

$$\rho_B^+(\omega', \mu) \simeq C_0(\omega', \mu) \frac{1}{\omega'} - \frac{2}{3} C_1(\omega', \mu) \frac{\bar{\Lambda}}{(\omega')^2} + \ldots$$

- coefficients $C_i(\omega', \mu)$ can be matched to satisfy OPE constraints
- for $\omega' \gg \mu$ use RGEs to resum large logarithms in $C_i(\omega', \mu)$

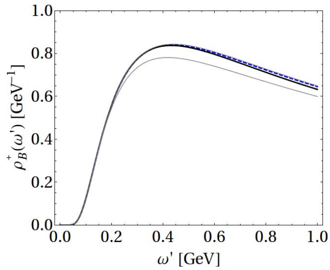
$\Rightarrow$ the whole region $\omega' \gtrsim \mu$ is determined by perturbation theory

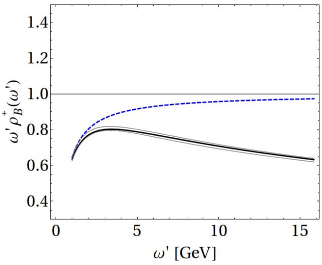
Connection to NP models

Systematic procedure to construct models that satisfy OPE constraints

- start from model for $\rho_B^+(\omega', \mu_0) \rightarrow$ good description in the low ω' region
- perform $1/\omega'$ expansion and adjust to perturbative result

Example: exponential model





Inverse moments

Split perturbative ($\omega' \geq \mu$) and non-perturbative ($\omega' \leq \mu$) contributions

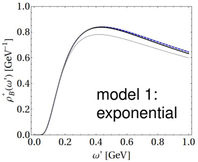
$$\int_0^\infty \frac{d\omega'}{\omega'} \ln^n\left(\frac{\hat{\omega}'}{\mu}\right) \rho_B^+(\omega',\mu) \equiv L_n^+(\mu) + L_n^-(\mu)$$

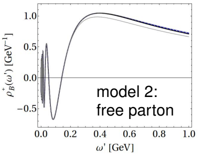
- $L_n^+(\mu)$ completely determined perturbatively (depends on HQET parameters)
- for $L_n^-(\mu)$ substitute $z = -\ln\frac{\hat{\omega}'}{\mu}$ and expand in Laguerre polynomials

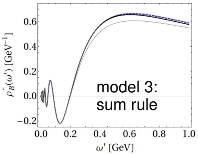
$$\rho_B^+(\hat{\mu}\, e^{-z},\mu) = \sum_{n=0}^\infty a_n(\mu)\, e^{-z}\, \mathrm{L}_n(z) \quad \Rightarrow \quad \begin{aligned} L_0^-(\mu) &= a_0(\mu) \\ L_1^-(\mu) &= a_1(\mu) - a_0(\mu) \\ L_2^-(\mu) &= 2a_2(\mu) - 4a_1(\mu) + 2a_0(\mu) \end{aligned}$$

unfortunately the truncated expansion does not close under renormalisation

Numerics







$L_n(\mu)$	total	$L_n^-(\mu)$	$L_n^+(\mu)$
L_0 (model 1)	1.67	1.58	0.086
L_0 (model 2)	1.65	1.57	0.086
L_0 (model 3)	1.21	1.12	0.086
L_1 (model 1)	-3.85	-3.93	0.074
L_1 (model 2)	-3.46	-3.54	0.074
L_1 (model 3)	-2.19	-2.27	0.074
L_2 (model 1)	11.6	11.4	0.121
L_2 (model 2)	9.03	8.91	0.121
L_2 (model 3)	5.44	5.32	0.121

$\Rightarrow$ inverse moments dominated by model-dependent contribution (independent of HQET parameters)

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Lattice QCD

Simulation of LC correlators apears to be challenging

- OPE only determines HQET parameters ($\rightarrow$ irrelevant for inverse moments)

New proposal to calculate LC correlators on the lattice [Ji 13]

- start from quasi-LCDA defined for $z^2 < 0$

$$\phi_\pi(u, P_z, \mu) \sim \int dz\, e^{izuP_z} \langle 0|\bar{q}(0)\,[0,z]\,\gamma_z\gamma_5\, q(z)|\pi(P)\rangle$$

- boost to equal-time correlator that can be calculated on the lattice

- for $P_z \rightarrow \infty$ quasi and standard LCDA are related by perturbative matching relation

$\Rightarrow$ potential to calculate B meson LCDA on the lattice?

QCD sum rules

[Braun, Ivanov, Korchemsky 03]

Start from correlation function in HQET

$$\int d^4x\, e^{-ik(v\cdot x)} \langle 0 | T\{ \bar{q}(tn)\, [tn,0]\, \not{n}\Gamma_1\, h_v(0)\, \bar{h}_v(x)\, \Gamma_2\, q(x)\} | 0\rangle \sim \frac{\tilde{f}_B(\mu)^2\, \tilde{\phi}_B^+(t,\mu)}{\bar{\Lambda}-k} + \ldots$$

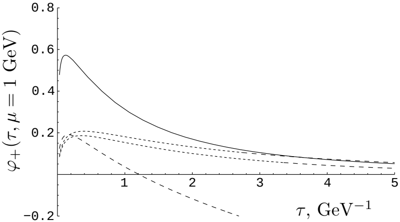
- for $|k| \sim$ few GeV $\Rightarrow$ standard two-point sum rule including NLO corrections
- NP corrections cannot be included via local condensates

$$\varphi_B^+(\tau,\mu) = \tilde{\phi}_B^+(-i\tau,\mu)$$

$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty d\tau\, \varphi_B^+(\tau,\mu)$$

solid line: perturbative contribution

dashed line: NP contribution from $\langle \bar{q}q\rangle$ and $\langle \bar{q}\sigma gGq\rangle$



Predictions

Model-dependent estimate of large distance NP contributions

- non-local quark condensate (partial resummation of OPE)
- other non-local condensates are neglected

Results for inverse moments :

- $\lambda_B(\mu = 1\text{ GeV}) = (460 \pm 110)\text{ MeV}$
- $\sigma_1(\mu = 1\text{ GeV}) = 1.4 \pm 0.4$

Questions:

- improvements of the model-dependent estimate?
- does the dual representation of the LCDA help in this context?

$B \to \gamma \ell \nu$

Hadronic tensor

$$-i \int d^4x\, e^{iqx} \langle 0 | T\{ j_{\rm em}^{\nu}(x)\, \bar{u}(0)\, \gamma^{\mu}(1-\gamma_5)\, b(0) \} | B^-(m_B v) \rangle$$

$$= \epsilon^{\mu\nu\rho\sigma}\, v^{\rho} q^{\sigma}\, F_V(E_\gamma) - i\, (g^{\mu\nu}\, v \cdot q - v^{\nu} q^{\mu})\, \tilde{F}_A(E_\gamma) + \ldots$$

Differential decay rate

$$\frac{d\Gamma}{dE_\gamma} = \frac{\alpha_{\rm em} G_F^2 |V_{ub}|^2}{6\pi^2}\, (m_B - 2E_\gamma)\, E_\gamma^3 \left\{ \left| F_V(E_\gamma) \right|^2 + \left| F_A(E_\gamma) \right|^2 \right\}$$

For $E_\gamma \sim m_B/2$ the form factors factorise

[Korchemsky, Pirjol, Yan 99; Descotes-Genon, Sachrajda 02; Lunghi, Pirjol, Wyler 02; Bosch, Hill, Lange, Neubert 03]

$$F_V(E_\gamma) = F_A(E_\gamma) = \frac{Q_u m_B f_B}{2E_\gamma \lambda_B(\mu)}\, R(E_\gamma, \mu)$$

- radiative corrections $R(E_\gamma, \mu)$ known to NLO+NLL

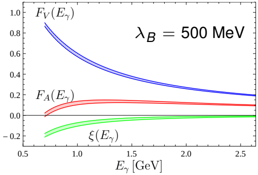
Extraction of λ_B

Including $1/m_b$ power corrections

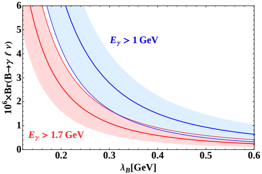
$$F_V(E_\gamma) = \frac{Q_u m_B f_B}{2E_\gamma \lambda_B(\mu)} R(E_\gamma, \mu) + \left\{ \xi(E_\gamma) + \frac{Q_b m_B f_B}{2m_b E_\gamma} + \frac{Q_u m_B f_B}{(2E_\gamma)^2} \right\}$$

$$F_A(E_\gamma) = \frac{Q_u m_B f_B}{2E_\gamma \lambda_B(\mu)} R(E_\gamma, \mu) + \left\{ \xi(E_\gamma) - \frac{Q_b m_B f_B}{2m_b E_\gamma} - \frac{Q_u m_B f_B}{(2E_\gamma)^2} + \frac{Q_\ell f_B}{E_\gamma} \right\}$$

- soft-overlap contribution to $\xi(E_\gamma)$ can be estimated using HQE and dispersion relations ($\rightarrow$ depends on B meson LCDA) [Braun, Khodjamirian 12]



- current Babar data gives lower bound $\lambda_B(\mu = 1 \text{ GeV}) > 115$ MeV good prospects at Belle II [Beneke, Rohrwild 11]



Conclusions

Recent progress in understanding formal properties of the B meson LCDA

- eigenfunctions of one-loop RG kernel
- relation to conformal symmetry
- QCD-improved model building

There still exist very few non-perturbative calculations

- any contribution will have a large impact on phenomenology