

Extrapolation and Unitarity Bounds for the $B \rightarrow \pi$ Form Factor

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based on [arxiv:1409.7816](https://arxiv.org/abs/1409.7816)

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$$\begin{aligned} \langle \pi^+(p) | \bar{u} \gamma_\mu b | \bar{B}(p+q) \rangle \\ = f_{B\pi}^+(q^2) \left[2p_\mu + \left(1 - \frac{m_B^2 - m_\pi^2}{q^2} \right) q_\mu \right] + f_{B\pi}^0(q^2) \frac{m_B^2 - m_\pi^2}{q^2} q_\mu, \end{aligned}$$

- concentrate on $f_{B\pi}^+$: only FF relevant for $B \rightarrow \pi \{e, \mu\} \nu$
- calculate FF from LCSRs
 - ▶ two-point sum rule for f_B : NLO
 - ▶ LCSR: LO for twists 2 to 4; NLO for twist 2,3 (tw-3 only asymptotic)
- parameters $\vec{\theta}$ include
 - ▶ quark masses
 - ▶ QCD condensate densities
 - ▶ coefficients of π distribution amplitudes
 - ▶ Borel parameters M^2 and \bar{M}^2
 - ▶ hadronic thresholds s_0^B and \bar{s}_0^B

Applying Bayesian statistics to the correlator(s)

Prior knowledge of input parameters

- prior $P_0(\vec{\theta})$: product of uniform and gaussian probability density functions (PDFs) for all parameters
- most importantly:
 - ▶ broad gaussian PDFs for M^2 and \overline{M}^2
 - ▶ uniform PDFs for s_0^B and \overline{s}_0^B

Update knowledge from B -meson mass

- both relevant correlators F_{LCSR} and $F_{2\text{ptSR}}$ allow determination of B -meson mass
- schematically

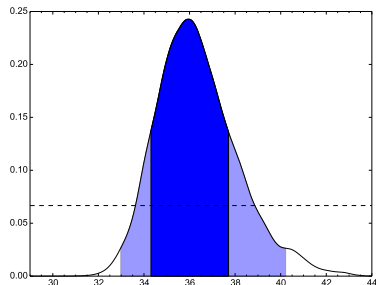
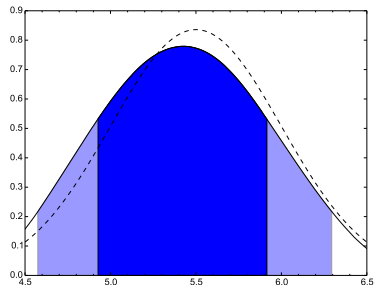
$$\frac{1}{F} \frac{\partial F}{\partial(-1/M^2)} = m_B^2$$

- define likelihood $P(m_B|\vec{\theta})$, with std. deviation $\sigma_B \equiv 1\% \cdot m_B$

Posterior of the parameter space

- compute joint posterior $P(\vec{\theta}|m_B)$
- example: 1D marginal PDFs of
 - ▶ 2ptSR Borel parameter (upper)
 - ▶ 2ptSR threshold parameter (lower)
- likelihood strongly constrains threshold

prior: dashed lines,
blue: 68% prob., light-blue: 95% prob.



Predictive distribution

compute **joint** posterior-predictive PDF $P(\vec{F}|m_B)$:

$$P(\vec{F}|m_b) = \int d\vec{\theta} \delta(\vec{F} - \vec{F}(\vec{\theta})) P(\vec{\theta}|m_B)$$

with

$$\vec{F} \equiv (f_{B\pi}^+(0), f_{B\pi}^{+'}(0), f_{B\pi}^{+''}(0), f_{B\pi}^+(10 \text{ GeV}^2), f_{B\pi}^{+'}(10 \text{ GeV}^2), f_{B\pi}^{+''}(10 \text{ GeV}^2))$$

- use $f_{B\pi}^+$ as well as its 1st and 2nd derivative w/r to q^2
- use two q^2 values: $q^2 = 0$ and $q^2 = 10 \text{ GeV}^2$
 - ▶ large distance decreases correlations
 - ▶ still within LCSR region of applicability
- obtain **joint** posterior-predictive PDF, $\sim 7\%$ uncertainty on $f_{B\pi}^+$

$$\vec{\mu}^F = (0.310, 1.55 \cdot 10^{-2}, 1.24 \cdot 10^{-3}, 0.562, 4.03 \cdot 10^{-2}, 4.71 \cdot 10^{-3})$$

$$\vec{\sigma}^F = (0.020, 0.10 \cdot 10^{-2}, 0.10 \cdot 10^{-3}, 0.032, 0.24 \cdot 10^{-2}, 0.37 \cdot 10^{-3})$$

correlation matrix available in the paper

Extrapolation

- use BCL-inspired parametrization

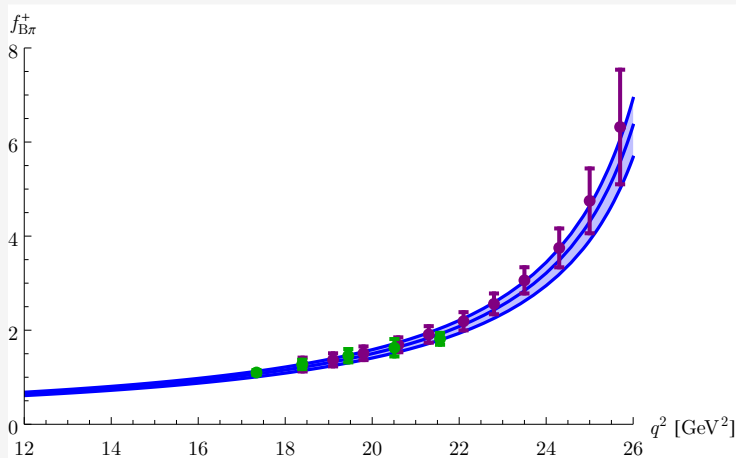
$$f_{B\pi}^+(q^2) = \frac{f_{B\pi}^+(0)}{1 - q^2/m_{B^*}^2} \times \left\{ 1 + b_1^+ [z(q^2, t_0) - z(0, t_0) - \frac{1}{3}(z(q^2, t_0)^3 - z(0, t_0)^3)] + b_2^+ [z(q^2, t_0)^2 - z(0, t_0)^2 + \frac{2}{3}(z(q^2, t_0)^3 - z(0, t_0)^3)] \right\}.$$

- z : conformal map from real-valued q^2 to unit disc in the complex plane
- fit parameters $f_{B\pi}^+(0)$, b_1^+ , b_2^+ to LCSR results

$$\vec{\mu} = (0.307, -1.31, -0.904) \quad \vec{\sigma} = (0.020, 0.42, 0.444) \quad \rho = \begin{pmatrix} 1.000 & 0.503 & -0.391 \\ 0.503 & 1.000 & -0.824 \\ -0.391 & -0.824 & 1.000 \end{pmatrix}$$

Extrapolation

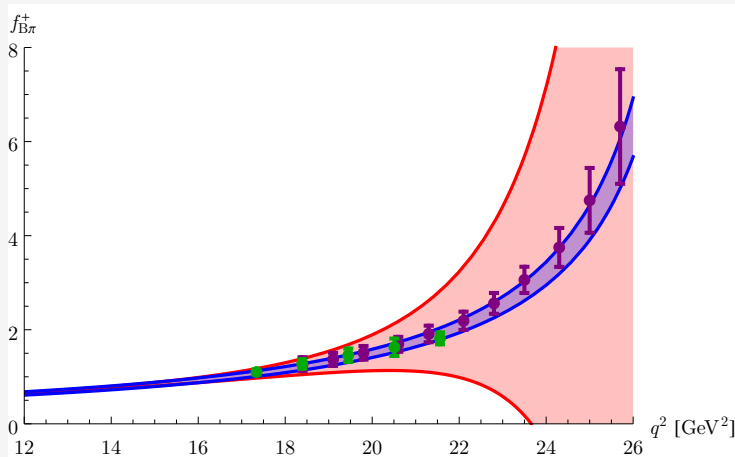
BCL and Lattice:



blue: 68% probability envelope and best-fit function,
magenta: Fermilab-MILC, green: HPQCD

Bounds from unitarity/positivity of the correlator

uses FF and 1st/2nd derivatives at $q^2 = 10 \text{ GeV}^2$ as input:

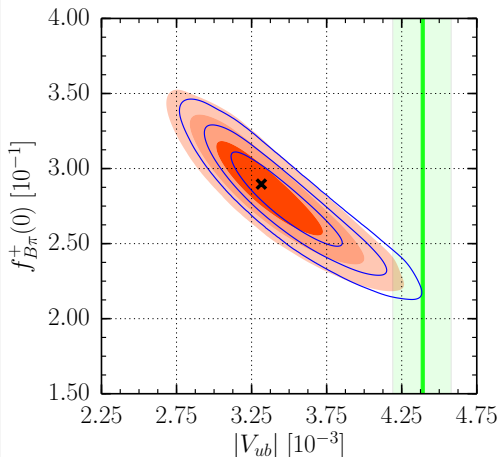


red: 68% envelope from unitarity bounds

blue: 68% probability envelope and best-fit function,

magenta: Fermilab-MILC, green: HPQCD

Determination of $|V_{ub}|$ from $\bar{B}^0 \rightarrow \pi^+ \ell \bar{\nu}_\ell$



- 2010 data: Belle+BaBar, 6 bins $q^2 \leq 12 \text{ GeV}^2$
- 2013 data: Belle+BaBar, 6 bins $q^2 \leq 12 \text{ GeV}^2$
- 2010 data vs inclusive: barely compatible @ 99% prob.
- 2013 data increases tension
- 1D marginals:
 - ▶ $|V_{ub}|^{2010} = (3.43_{-0.23}^{+0.27}) \cdot 10^{-3}$
 - ▶ $|V_{ub}|^{2013} = (3.32_{-0.22}^{+0.26}) \cdot 10^{-3}$

blue lines: 68%, 95%, 99% prob. contours for 2010 data

red area: 68%, 95%, 99% prob. contours for 2013 data

green line/area: central value/68% CL interval for GGOU/HFAG

Summary and Outlook

Summary

- reduction in theory uncertainty w/r to previous analyses
- first correlation information on $B \rightarrow P$ LCSR form factors
- improved semi model-independent bounds, become challenging to lattice data for $16 \text{ GeV}^2 \leq q^2 \leq 20 \text{ GeV}^2$
- $|V_{ub}|^{2013} = (3.32_{-0.22}^{+0.26}) \cdot 10^{-3}$: 7–8% parametric uncertainty

Outlook

- extend work to other form factors $P \rightarrow P$ form factors:
 - ▶ other transitions: $B \rightarrow K$, $B_s \rightarrow K$
 - ▶ other currents: scalar (e.g. $B \rightarrow \pi\tau\nu$), tensor (e.g. $B \rightarrow K\ell^+\ell^-$)