Tau Physics @ TeraZ

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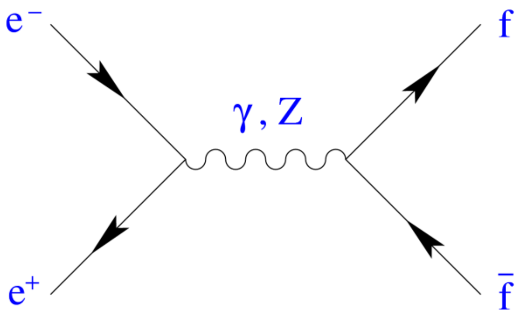
IFIC, Valencia

Sixth TLEP Workshop, CERN, 16-18 October 2013

High-Energy $\tau^+\tau^-$ Pairs



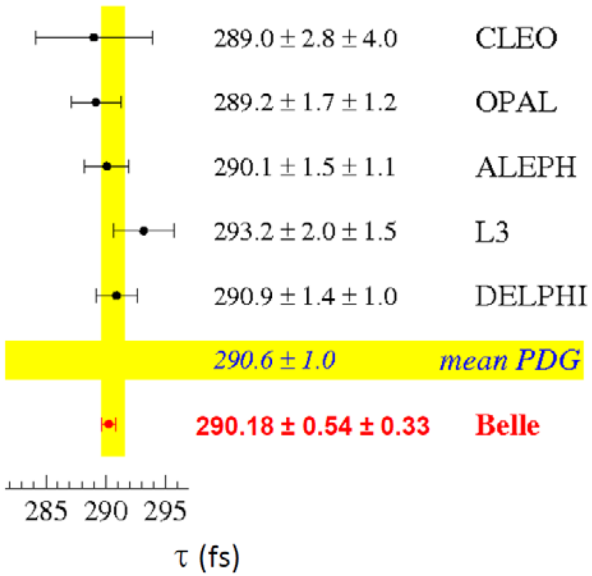
- Boosted $\tau^+\tau^-$ pairs
- Clean signature
- Manageable backgrounds

Many τ results are still dominated by LEP data

Statistics much higher at the B-Factories (backgrounds also)

Let's assume a high-statistics (10^{10}) $\tau^+\tau^-$ data sample at high energies

τ Lifetime



- Not a single lifetime measurement published by Babar/Belle
- Preliminary Babar value @ TAU 2004:
  $\tau_\tau = 289.40 \pm 0.91 \pm 0.90$ (unpublished)
- Preliminary Belle value @ TAU 2012:
  $\tau_\tau = 290.18 \pm 0.54 \pm 0.33$ (unpublished)

Charged-current universality

$$B_{\tau\to e} = \frac{B_{\tau\to\mu}}{0.972559 \pm 0.000005} = \frac{\tau_\tau}{(1632.9 \pm 0.6)\times 10^{-15}\,\mathrm{s}}$$

$$\left(B_{\tau\to\mu}/B_{\tau\to e}\right)_{\mathrm{exp}} = 0.9761 \pm 0.0028$$

Non-BF: 0.9725 ± 0.0039

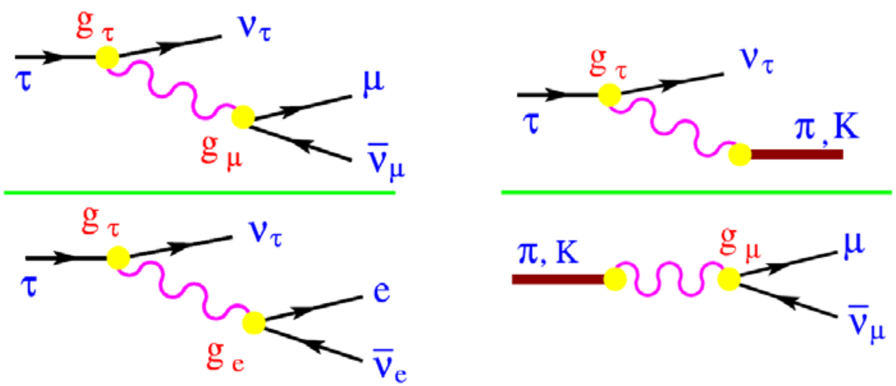
BaBar '10: 0.9796 ± 0.0039

0.4% precision

Charged Current Universality

$|g_\mu / g_e|$

$B_{\tau\to\mu}/B_{\tau\to e}$	1.0018 ± 0.0014
$B_{\pi\to\mu}/B_{\pi\to e}$	1.0021 ± 0.0016
$B_{K\to\mu}/B_{K\to e}$	0.9978 ± 0.0020
$B_{K\to\pi\mu}/B_{K\to\pi e}$	1.0010 ± 0.0025
$B_{W\to\mu}/B_{W\to e}$	0.991 ± 0.009



$|g_\tau / g_e|$

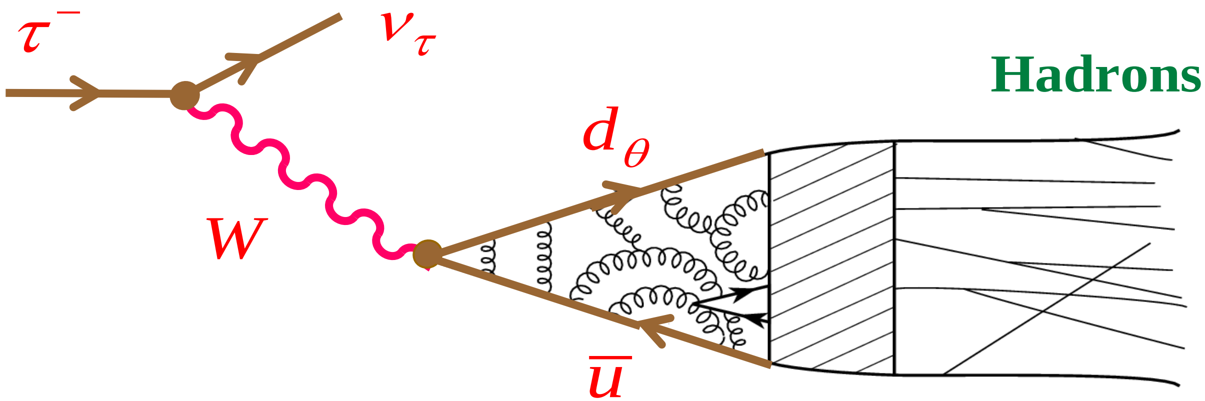
$B_{\tau\to\mu}\ \tau_\mu/\tau_\tau$	1.0024 ± 0.0021
$B_{W\to\tau}/B_{W\to e}$	1.023 ± 0.011

$|g_\tau / g_\mu|$

$B_{\tau\to e}\ \tau_\mu/\tau_\tau$	1.0006 ± 0.0021
$\Gamma_{\tau\to\pi}/\Gamma_{\pi\to\mu}$	0.9956 ± 0.0031
$\Gamma_{\tau\to K}/\Gamma_{K\to\mu}$	0.9852 ± 0.0072
$B_{W\to\tau}/B_{W\to\mu}$	1.032 ± 0.012

Precise measurement of Br(W→τ) / Br(W→μ,e) needed

HADRONIC TAU DECAY



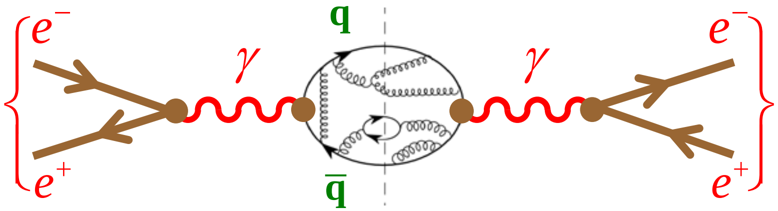
$$d_\theta = V_{ud}\, d + V_{us}\, s$$

Only lepton massive enough to decay into hadrons

$$R_\tau \equiv \frac{\Gamma\left(\tau^- \to \nu_\tau + \text{Hadrons}\right)}{\Gamma\left(\tau^- \to \nu_\tau\, e^-\, \bar{\nu}_e\right)} \approx N_C \qquad ; \qquad R_\tau = \frac{1 - B_e - B_\mu}{B_e} = 3.636 \pm 0.011$$

$$R_\tau = \frac{1}{B_e^{\text{univ}}} - 1.97256 = 3.6331 \pm 0.0088 \qquad ; \qquad R_\tau = \frac{\text{Br}\left(\tau^- \to \nu_\tau + \text{Hadrons}\right)}{B_e^{\text{univ}}} = 3.6280 \pm 0.0094$$

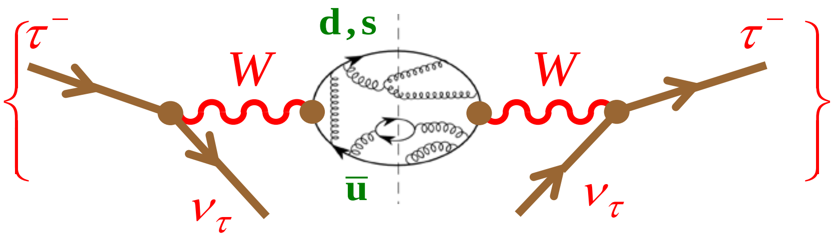
$\sigma \sim \mathrm{Im}$



$$\frac{\sigma(e^+e^-\to \mathrm{had})}{\sigma(e^+e^-\to \mu^+\mu^-)} = 12\pi\ \mathrm{Im}\,\Pi_{\mathrm{em}}(s)$$

$$\Pi^{\mu\nu}_{\mathrm{em}}(q) \equiv i\int d^4x\ e^{iqx}\,\langle 0|T[J^{\mu}_{\mathrm{em}}(x)\,J^{\nu}_{\mathrm{em}}(0)]|0\rangle = \left(-g^{\mu\nu}q^2+q^{\mu}q^{\nu}\right)\Pi_{\mathrm{em}}(q^2)$$

$\Gamma_{\tau\to\nu_\tau+\mathrm{had}} \sim \mathrm{Im}$



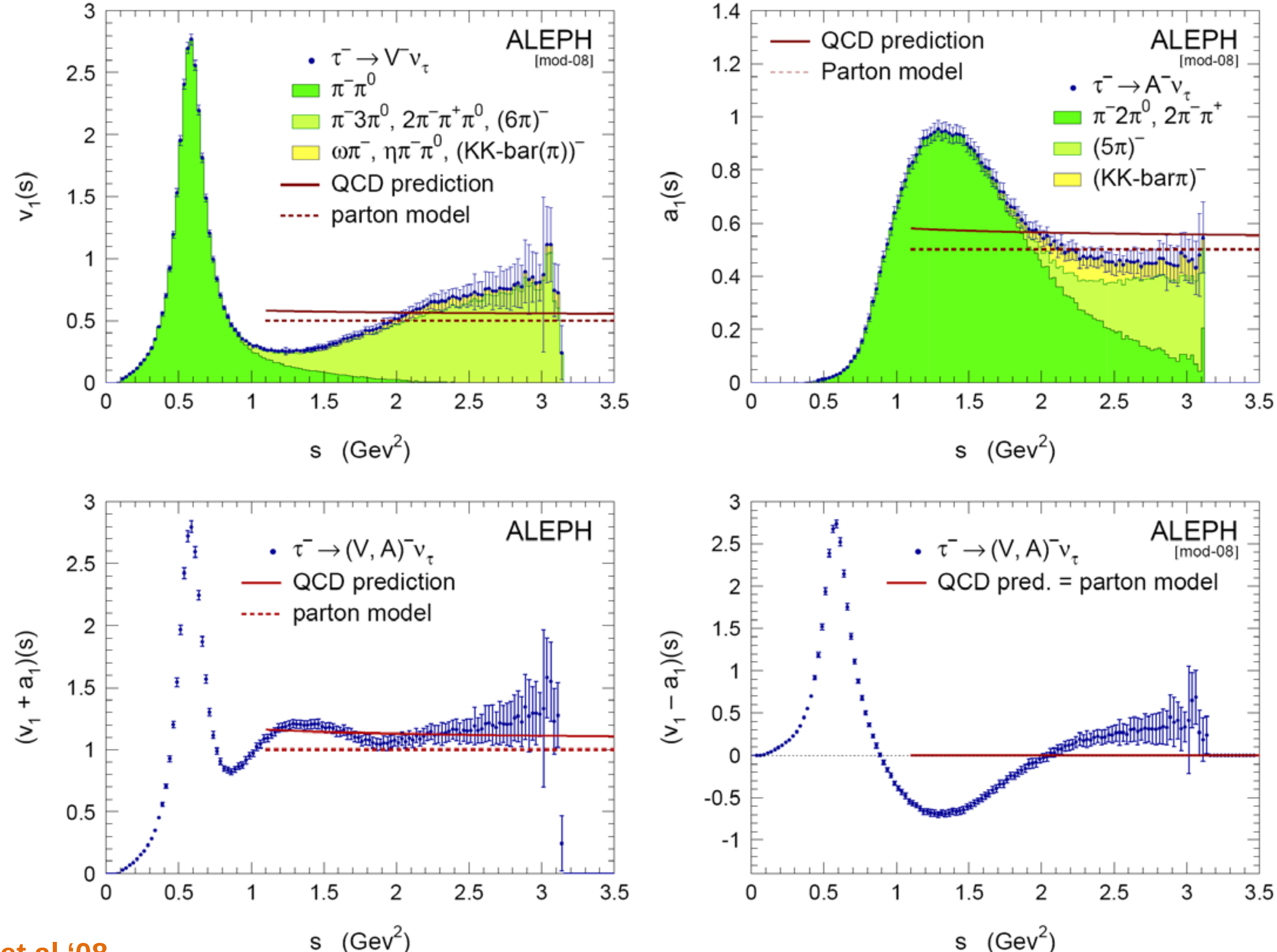
$$R_\tau \equiv \frac{\Gamma(\tau^-\to\nu_\tau+\mathrm{had})}{\Gamma(\tau^-\to\nu_\tau e^-\bar{\nu}_e)} = 12\pi\int_0^{m_\tau^2}\frac{ds}{m_\tau^2}\left(1-\frac{s}{m_\tau^2}\right)^2\left[\left(1+2\frac{s}{m_\tau^2}\right)\mathrm{Im}\,\Pi^{(1)}(s)+\mathrm{Im}\,\Pi^{(0)}(s)\right]$$

$$\Pi^{(J)}(s) \equiv |V_{ud}|^2\left[\Pi^{(J)}_{ud,V}(s)+\Pi^{(J)}_{ud,A}(s)\right] + |V_{us}|^2\left[\Pi^{(J)}_{us,V}(s)+\Pi^{(J)}_{us,A}(s)\right]$$

$$\Pi^{\mu\nu}_{ij,J}(q) \equiv i\int d^4x\ e^{iqx}\,\langle 0|T[J^{\mu}_{ij}(x)\,J^{\nu}_{ij}(0)^\dagger]|0\rangle = \left(-g^{\mu\nu}q^2+q^{\mu}q^{\nu}\right)\Pi^{(1)}_{ij,J}(q^2)+q^{\mu}q^{\nu}\,\Pi^{(0)}_{ij,J}(q^2)$$

SPECTRAL FUNCTIONS

$$v_1(s) = 2\pi \, \mathrm{Im}\, \Pi^{(0+1)}_{ud,V}(s) \qquad a_1(s) = 2\pi \, \mathrm{Im}\, \Pi^{(0+1)}_{ud,A}(s)$$



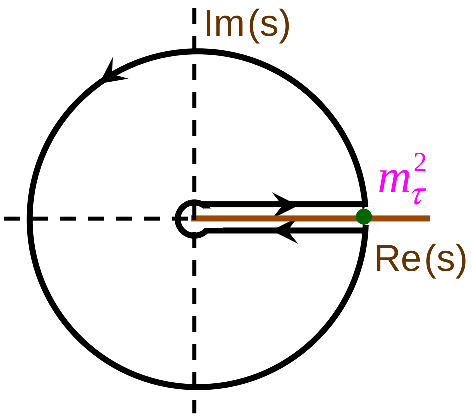
Better data needed

Davier et al '08

QCD Prediction of R_τ

Braaten-Narison-Pich'92

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau + \text{had})}{\Gamma(\tau^- \to \nu_\tau e^- \bar{\nu}_e)} = 12\pi \int_0^1 dx\,(1-x)^2 \left[(1+2x)\,\mathrm{Im}\,\Pi^{(1)}(x m_\tau^2) + \mathrm{Im}\,\Pi^{(0)}(x m_\tau^2)\right]$$



$$R_\tau = 6\pi i \oint_{|x|=1} dx\,(1-x)^2 \left[(1+2x)\,\Pi^{(0+1)}(x m_\tau^2) - 2x\,\Pi^{(0)}(x m_\tau^2)\right]$$

$$\Pi^{(J)}(s) = \sum_{D=2n} \frac{C_D^{(J)}(s,\mu)\,\langle O_D(\mu)\rangle}{(-s)^{D/2}}$$

OPE

$$R_\tau = N_C\, S_{\mathrm{EW}} \left(1 + \delta_{\mathrm{P}} + \delta_{\mathrm{NP}}\right) = R_{\tau,V} + R_{\tau,A} + R_{\tau,S}$$

$S_{\mathrm{EW}} = 1.0201\,(3)$; $\delta_{\mathrm{NP}} = -0.0059 \pm 0.0014$

Marciano-Sirlin, Braaten-Li, Erler — Fitted from data (Davier et al)

$$\delta_{\mathrm{P}} = a_\tau + 5.20\, a_\tau^2 + 26\, a_\tau^3 + 127\, a_\tau^4 + \ldots \approx 20\% \quad ; \quad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

Baikov-Chetyrkin-Kühn

Perturbative $(m_q=0)$

$$-s\,\frac{d}{ds}\,\Pi^{(0+1)}(s) = \frac{1}{4\pi^2}\sum_{n=0} K_n \left(\frac{\alpha_s(-s)}{\pi}\right)^n$$

$$K_0 = K_1 = 1 \quad , \quad K_2 = 1.63982 \quad , \quad K_3 = 6.37101 \quad , \quad K_4 = 49.07570$$

Baikov-Chetyrkin-Kühn '08

$$\delta_P = \sum_{n=1} K_n\, A^{(n)}(\alpha_s) = a_\tau + 5.20\, a_\tau^2 + 26\, a_\tau^3 + 127\, a_\tau^4 + \cdots$$

Le Diberder- Pich '92 — CIPT — FOPT

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}(1-2x+2x^3-x^4)\left(\frac{\alpha_s(-s)}{\pi}\right)^n = a_\tau^n + \cdots \quad ; \quad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

Power Corrections

$$\Pi^{(0+1)}_{\rm OPE}(s) \approx \frac{1}{4\pi^2}\sum_{n\ge 2}\frac{C_{2n}\langle O_{2n}\rangle}{(-s)^n}$$

Braaten-Narison-Pich '92

$$C_4\langle O_4\rangle \approx \frac{2\pi}{3}\langle 0|\alpha_s G^{\mu\nu}G_{\mu\nu}|0\rangle$$

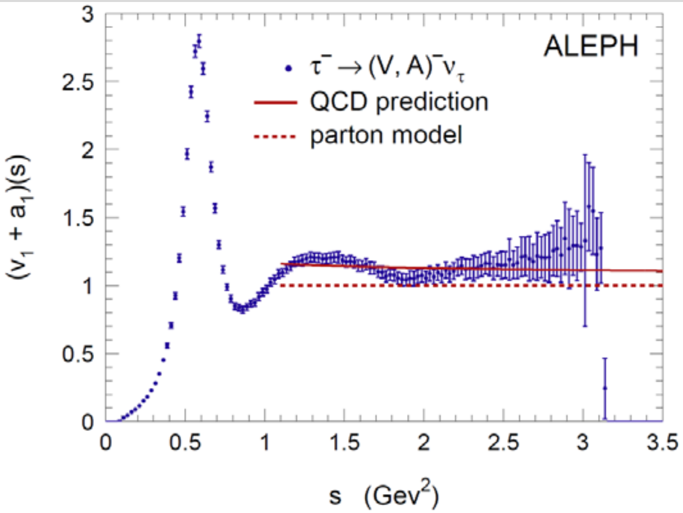
$$\delta_{\rm NP} \approx \frac{-1}{2\pi i}\oint_{|x|=1} dx\;(1-3x^2+2x^3)\sum_{n\ge 2}\frac{C_{2n}\langle O_{2n}\rangle}{(-x m_\tau^2)^n} = -3\,\frac{C_6\langle O_6\rangle}{m_\tau^6} - 2\,\frac{C_8\langle O_8\rangle}{m_\tau^8}$$

Suppressed by m_τ^6 [additional chiral suppression in $C_6\langle O_6\rangle^{V+A}$]

Spectral Function Distribution

- **Moments:**

$$R_\tau^{kl}(s_0) \equiv \int_0^{s_0} ds \left(1 - \frac{s}{s_0}\right)^k \left(\frac{s}{m_\tau^2}\right)^l \frac{dR_\tau}{ds}$$



Sensitivity to power corrections (k,l)

The non-perturbative contribution to R_τ can be obtained from the invariant-mass distribution of the final hadrons

$$\delta_{\mathrm{NP}} = -0.0059 \pm 0.0014$$ **Davier et al. (ALEPH data)**

- **Duality Violations: (non-pinched moments)**

$$\oint_{x=1} dx\ \Pi(x\, m_\tau^2)$$

$$\mathrm{Im}\,\Pi(s) \sim \exp(-\delta - \gamma s)\ \sin(\alpha + \beta s)$$

$$\delta_{\mathrm{NP}} = -0.003 \pm 0.012$$ **Boito et al. (OPAL data)**

Recent $\alpha_s(m_\tau)$ Analyses

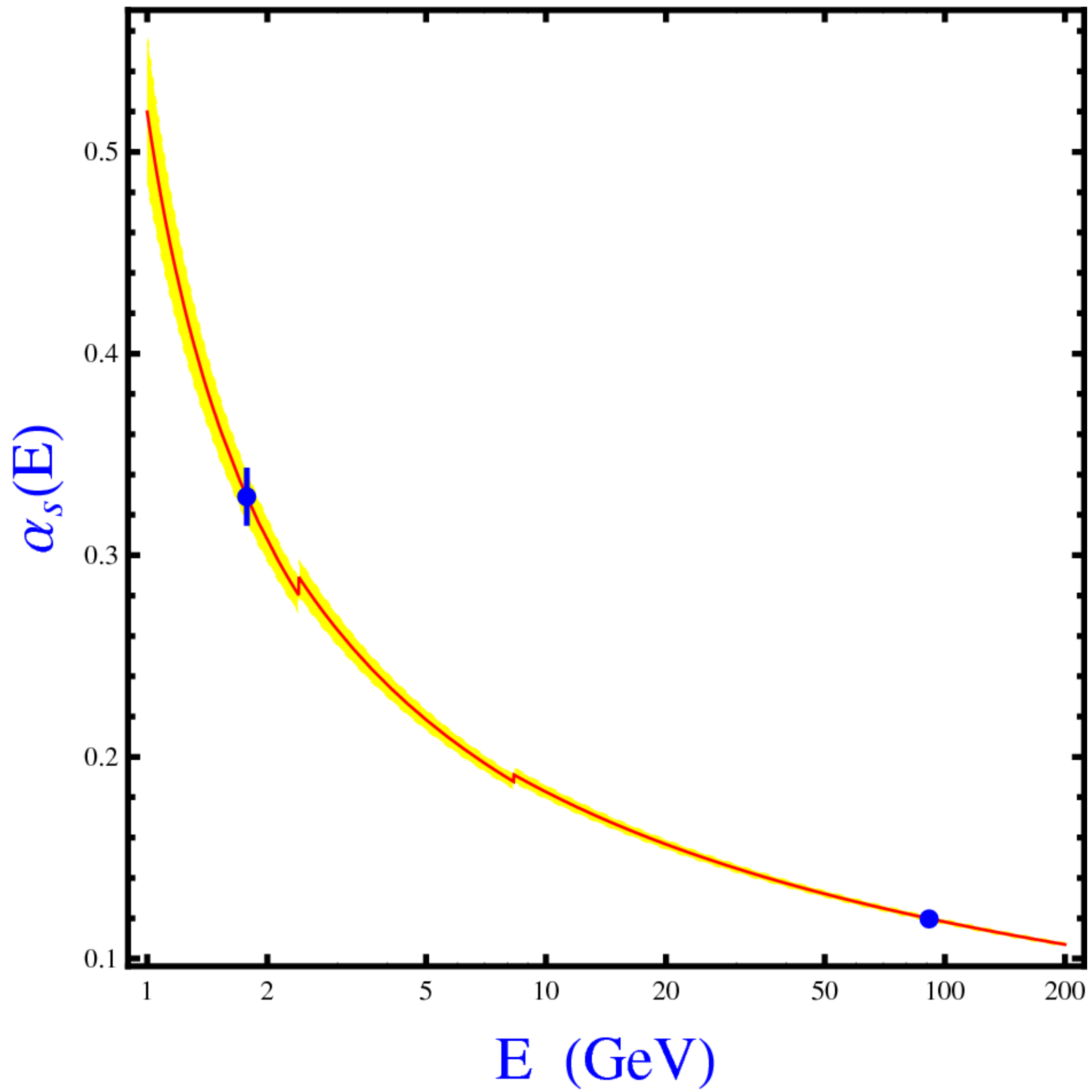
Reference	Method	δ_{NP}	δ_P	$\alpha_s(m_\tau)$	$\alpha_s(m_Z)$
Baikov et al	CIPT, FOPT		0.1998 (43)	**0.332 (16)**	**0.1202 (19)**
Davier et al	CIPT	– 0.0059 (14)	0.2066 (70)	**0.344 (09)**	**0.1212 (11)**
Beneke-Jamin	BSR + FOPT	– 0.007 (3)	0.2042 (50)	**0.316 (06)**	**0.1180 (08)**
Maltman-Yavin	PWM + CIPT	+ 0.012 (18)	–	**0.321 (13)**	**0.1187 (16)**
Menke	CIPT, FOPT		0.2042 (50)	**0.342 (11)**	**0.1213 (12)**
Narison	CIPT, FOPT		–	**0.324 (08)**	**0.1192 (10)**
Caprini-Fischer	BSR + CIPT		0.2037 (54)	**0.322 (16)**	–
Abbas et al	IFOPT		0.2037 (54)	**0.338 (10)**	
Cvetič et al	β_{exp} + CIPT		0.2040 (40)	**0.341 (08)**	**0.1211 (10)**
Boito et al 2012	CIPT, DV	– 0.002 (12)	–	**0.347 (25)**	**0.1216 (27)**
	FOPT, DV	– 0.004 (12)		**0.325 (18)**	**0.1191 (22)**
Pich	CIPT	– 0.0059 (14)	0.1995 (33)	**0.339 (13)**	**0.1210 (15)**
	FOPT			**0.318 (14)**	**0.1185 (18)**
My Average	**CIPT, FOPT**		**0.1995 (33)**	**0.329 (13)**	**0.1198 (15)**

CIPT: Contour-improved perturbation theory
FOPT: Fixed-order perturbation theory
BSR: Borel summation of renormalon series
IFOPT Improved FOPT

β_{exp}: Expansion in derivatives of α_s (β function)
PWM: Pinched-weight moments
CIPTm: Modified CIPT (conformal mapping)
DV: Duality violation (OPAL only)

Present Status

A. Pich, arXiv:1303.2262



$$\alpha_s(m_\tau^2) = 0.329 \pm 0.013$$

$$\Downarrow$$

$$\alpha_s(M_Z^2) = 0.1198 \pm 0.0015$$

$$\alpha_s(M_Z^2)_{Z\,\mathrm{width}} = 0.1197 \pm 0.0028$$

The most precise test of Asymptotic Freedom

$$\alpha_s^\tau(M_Z^2) - \alpha_s^Z(M_Z^2) = 0.0001 \pm 0.0015_\tau \pm 0.0028_Z$$

V_{us} Determination

Gámiz-Jamin-Pich-Prades-Schwab

$$|V_{us}|^2 = \frac{R^{00}_{\tau,S}}{\frac{R^{00}_{\tau,V+A}}{|V_{ud}|^2} - \delta R^{00}_{\tau,\mathrm{th}}}$$

$$\delta R^{kl}_\tau \approx 24 \, \frac{m_s^2(m_\tau^2)}{m_\tau^2} \, \Delta_{kl}(\alpha_s)$$

$$\delta R^{00}_{\tau,\mathrm{th}} \equiv \underbrace{0.1544\,(37)}_{J=0} + \underbrace{0.086\,(32)}_{m_s(2\ \mathrm{GeV}) = 94\,(6)\ \mathrm{MeV}} = 0.240\,(32)$$

$R^{00}_{\tau,S} = 0.1612\ (28)$

$R^{00}_{\tau,V+A} = 3.4671\ (84)$

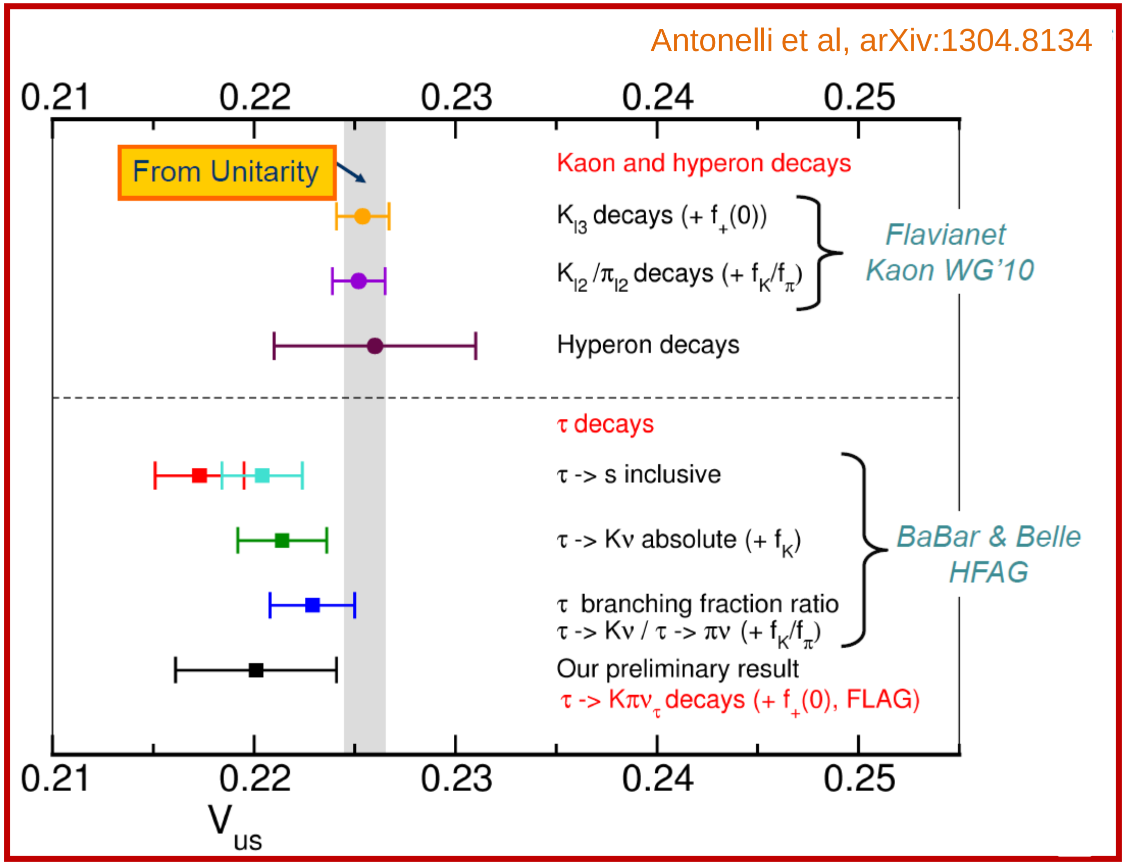
$|V_{ud}| = 0.97425\ (22)$

⟹ $|V_{us}| = 0.2173 \pm 0.0020_{\mathrm{exp}} \pm 0.0010_{\mathrm{th}}$

K_{l3}: $|V_{us}| = 0.2238 \pm 0.0011$ $\quad [\, f_+(0) = 0.967 \pm 0.004 \,]$

The τ could give the most precise V_{us} determination

V_{us} from τ



HFAG'12

Branching fraction	HFAG Winter 2012 fit	
$\Gamma_{10} = K^- \nu_\tau$	$(0.6955 \pm 0.0096) \cdot 10^{-2}$	⇨ $(0.713 \pm 0.003)\ .\ 10^{-2}$
$\Gamma_{16} = K^- \pi^0 \nu_\tau$	$(0.4322 \pm 0.0149) \cdot 10^{-2}$	⇨ $(0.4473 \pm 0.0244)\ .\ 10^{-2}$
$\Gamma_{23} = K^- 2\pi^0 \nu_\tau$ (ex. K^0)	$(0.0630 \pm 0.0222) \cdot 10^{-2}$	
$\Gamma_{28} = K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	$(0.0419 \pm 0.0218) \cdot 10^{-2}$	
$\Gamma_{35} = \pi^- \overline{K}^0 \nu_\tau$	$(0.8206 \pm 0.0182) \cdot 10^{-2}$	⇨ $(0.8627 \pm 0.0353)\ .\ 10^{-2}$
$\Gamma_{110} = X_s^- \nu_\tau$	$(2.8746 \pm 0.0498) \cdot 10^{-2}$	⇨ $(2.9496 \pm 0.0571)\ .\ 10^{-2}$

Slight underestimate of τ branching ratios ?

Use modes measured in K decays

Larger R_S

R_S = 0.1663 (34)

Larger V_{us}

$|V_{us}|$ = 0.2207 (25)

Better data needed!

Remaining Problems

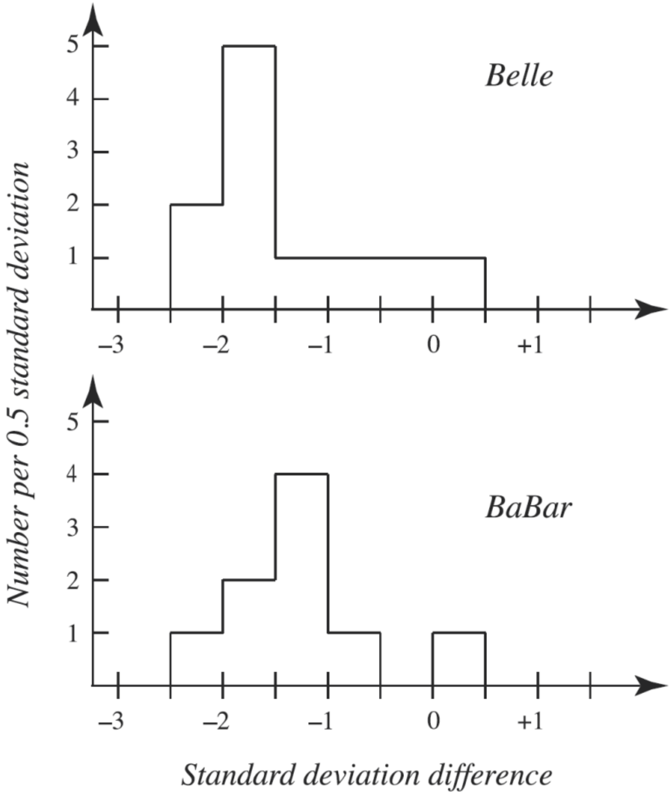
PDG 2012:

"Eighteen of the 20 *B-factory* branching fraction measurements are smaller than the non-*B-factory values. The average normalized difference between* the two sets of measurements is -1.30 (-1.41 for the 11 Belle measurements and -1.24 for the 9 BaBar measurements)"

Missing modes ?

Modes measured by the two BF experiments

Mode	BaBar – Belle Normalized Difference (#σ)
$\pi^-\pi^+\pi^-\nu_\tau$ (ex. K^0)	$+1.4$
$K^-\pi^+\pi^-\nu_\tau$ (ex. K^0)	-2.9
$K^-K^+\pi^-\nu_\tau$	-2.9
$K^-K^+K^-\nu_\tau$	-5.4
$\eta K^-\nu_\tau$	-1.0
$\phi K^-\nu_\tau$	-1.3



PHYSICS OUTLOOK

Many more interesting topics

- Tests of QCD and the Electroweak Theory
- Looking for Signals of New Phenomena
- Superb Tool for New Physics Searches

High-statistics τ data samples at high energies would provide many useful informations

Systematic errors & backgrounds need to be studied

Backup Slides

Perturbative Uncertainty on $\alpha_s(m_\tau)$

$$-s\,\frac{d}{ds}\,\Pi^{(0+1)}(s) = \frac{1}{4\pi^2}\sum_{n=0} K_n\; a(-s)^n$$

$$\delta_{\rm P} = \underbrace{\sum_{n=1} K_n\, A^{(n)}(\alpha_s)}_{\text{CIPT}} = \underbrace{\sum_{n=0} r_n\, a_\tau^n}_{\text{FOPT}}$$

$$r_n = K_n + g_n$$

$$A^{(n)}(\alpha_s) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}\,(1-2x+2x^3-x^4)\, a_\tau(-x\, m_\tau^2)^n = a_\tau^n + \cdots \qquad ; \qquad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

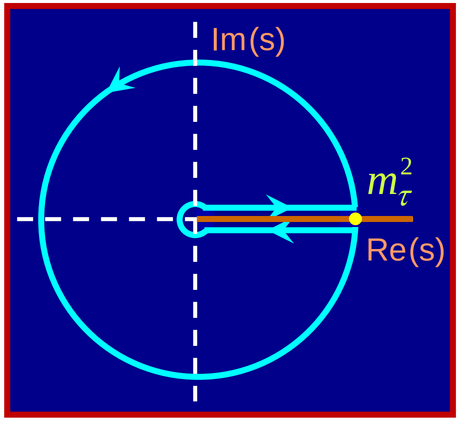
n	1	2	3	4	5
K_n	1	1.6398	6.3710	49.0757	
g_n	0	3.5625	19.9949	78.0029	307.78
r_n	1	5.2023	26.3659	127.079	

The dominant corrections come from the contour integration

Le Diberder- Pich 1992

Large running of a_s along the circle $s = m_\tau^2\, e^{i\varphi}$, $\varphi \in [0\,,2\pi]$

$$A^{(n)}(a_\tau) \equiv \frac{1}{2\pi i}\oint_{|x|=1}\frac{dx}{x}(1-2x+2x^3-x^4)\,a_\tau(-x m_\tau^2)^n = a_\tau^n + \cdots \qquad ; \qquad a_\tau \equiv \alpha_s(m_\tau)/\pi$$



$$A^{(1)}(a_\tau) = a_\tau - \frac{19}{24}\beta_1 a_\tau^2 + \left[\beta_1^2\left(\frac{265}{288}-\frac{\pi^2}{12}\right)-\frac{19}{24}\beta_2\right] a_\tau^3 + \cdots$$

$$a(-s) \simeq \frac{a_\tau}{1-\frac{\beta_1}{2}a_\tau \log\left(-s/m_\tau^2\right)} = \frac{a_\tau}{1-i\frac{\beta_1}{2}a_\tau\phi} = a_\tau\sum_n\left(i\frac{\beta_1}{2}a_\tau\phi\right)^n \qquad ; \qquad \phi\in[0,2\pi]$$

FOPT expansion only convergent if $\alpha_\tau < 0.14$ (0.11) [at 1 (3) loops]

Experimentally $\alpha_\tau \approx 0.11$ ➡ **FOPT should not be used** (divergent series)

FOPT suffers a large renormalization-scale dependence **(Le Diberder- Pich , Menke)**

The difference between FOPT and CIPT grows at higher orders

Renormalons

$$D(s) \equiv -s \frac{d}{ds} \Pi^{(0+1)}(s) = \frac{1}{4\pi^2} \sum_{n=0} K_n \; a(-s)^n$$

Asymptotic series

Borel Summation:

$$B(t) \equiv \sum_{n=0} K_{n+1} \frac{t^n}{n!} \quad \Longrightarrow \quad D(s) = \frac{1}{4\pi^2}\left\{1 + \int_0^\infty dt \; e^{-t/a(-s)} \; B(t)\right\}$$

However, $B(t)$ **has pole singularities at**

- $u \equiv -\beta_1 t/2 = +n \qquad (n \geq 2) \qquad$ **Infrared Renormalons**
- $u \equiv -\beta_1 t/2 = -n \qquad (n \geq 1) \qquad$ **Ultraviolet Renormalons**

IR - n Renormalon $\Longrightarrow$ **Ambiguity:** $\delta D(s) \sim \left(\frac{\Lambda^2}{-s}\right)^n$

Renormalon Hypothesis: Asymptotics already reached

Modelling a better behaved FOPT

(Beneke – Jamin)

- **Large higher-order K_n corrections could cancel the g_n ones**

  Happens in the "large-β_0" approximation (UV renormalon chain)

- D = 4 corrections very suppressed in R_τ

  ➡ **n = 2 IR renormalons can do the job** **($K_n \approx - g_n$)**

- No sign of renormalon behaviour in known coefficients

  ➡ **n = -1,2,3 renormalons + linear polynomial**

  5 unknown constants fitted to K_n $(2 \le n \le 5)$. $K_5 = 283$ assumed

- **Borel summation:** large renormalon contributions. Smaller α

Nice model of higher orders. But too many different possibilities …

(Descotes-Genon – Malaescu)