PRECISION CALCULATIONS FOR TLEP

selected examples, mainly from QCD, not a review

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SFB TR9 Computational Theoretical Particle Physics

Tera Z: M_Z

aim $\delta M_Z = 0.1$ MeV   (LEP: 2.1 MeV)

present theory errors (QED! line shape!)

ISR: 0.1 MeV; lepton pairs: 0.3 MeV; parametrization: 0.1 MeV

[stated in TLEP-paper]

Tera Z: Γ_Z

aim $\delta\Gamma_Z = 0.1$ MeV   (LEP: 2495.2 ± 2.3 MeV)

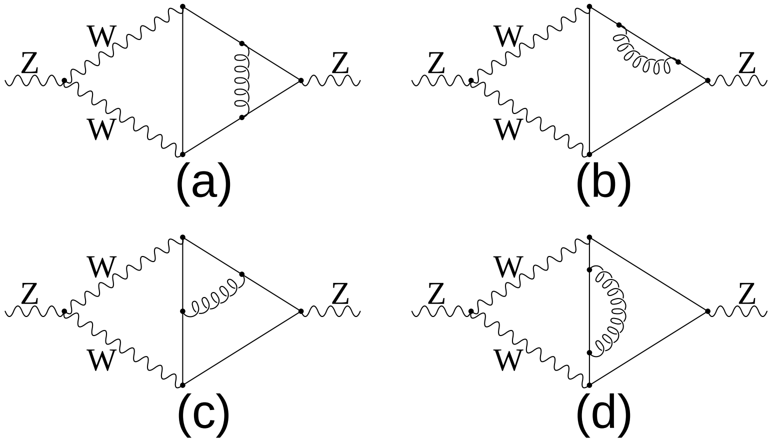
present theory error: 0.2 MeV from ?

[stated in TLEP-paper]

closer look on QCD and mixed EW $\otimes$ QCD

Mixed electroweak and QCD: light quarks (u,d,c,s)

terms of $O(\alpha\alpha_s)$, Czarnecki, JK; hep-ph/9608366



$$\Delta\Gamma \equiv \Gamma(\text{two loop EW} \star \text{QCD}) - \Gamma_{\text{Born}}\delta^{\text{NLO}}_{\text{EW}}\delta^{\text{NLO}}_{\text{QCD}} = -0.55 \text{ MeV}$$

three loop: reduction by $\# \cdot \frac{\alpha_s}{\pi} = \#0.04$

should not exceed 5!

corrections of $O(\alpha_w\alpha_s^2)$ (three loop)

difficult, but feasible!

Tera Z: $\Gamma(Z \to b\bar{b}) \equiv \Gamma_b$

aim: $\delta R_b \equiv \frac{\delta\Gamma_b}{\Gamma_{had}} = 2 - 5 \times 10^{-5}$ (LEP: $R_b = 0.21629 \pm 0.00066$)

2×10^{-5} corresponds to 0.030 MeV!

corrections specific for $b\bar{b}$:

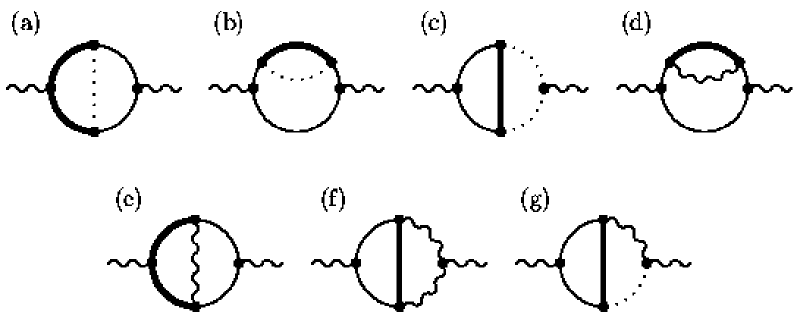
m_t^2-enhancement: order $G_F m_t^2$ and $G_F m_t^2 \alpha_s$

$\Delta\Gamma = \frac{G_F M^3}{16\pi^3} G_f m_t^2 (1 - \frac{2}{3} s_w^2)(1 - \frac{\pi^2 - 3}{3} \frac{\alpha_s}{\pi})$ Chetyrkin, Kwiatkowski, Steinhauser, 1993

Complete $\alpha_w \alpha_s$ result:

$$\Gamma_b - \Gamma_q = (-5.69 - 0.79 \quad O(\alpha)$$

$$+0.50 + 0.06 \quad O(\alpha\alpha_s))$$ MeV



dressed with gluons

separated into m_t^2-enhanced and rest

(Harlander, Seidensticker, Steinhauser

hep-ph/9712228)

motivates the evaluation of m_t^2-enhanced corrections of $O(G_F m_t^2 \alpha_s^2)$

(Chetyrkin, Steinhauser, hep-ph/990480)

$\delta\Gamma_b(G_F m_t^2 \alpha_s^2) \approx 0.1$MeV   (non-singlet)

(absent in Z-fitter, G-fitter!)

many top-induced corrections become significantly smaller, if m_t is expressed in $\overline{MS}$ convention

$$\bar{m}_t(\bar{m}_t) = m_{pole}\left(1 - 1.33\left(\frac{\alpha_s}{\pi}\right) - 7.50\left(\frac{\alpha_s}{\pi}\right)^2 - 78.9\left(\frac{\alpha_s}{\pi}\right)^3 + \#\alpha_s^4\right)$$

(Karlsruhe, 1999)   in progress

$$= (173.20 - 7.97 - 1.55 - 0.56)\,\text{GeV}$$

$$= \left(163.47 \pm 0.83|_{m_t} \pm 0.18|_{\alpha_s} \pm ?|_{th}\right)\,\text{GeV}$$

top scan $\Rightarrow$ m(potential subtracted)

$\delta m_t \sim 20 - 30$ MeV

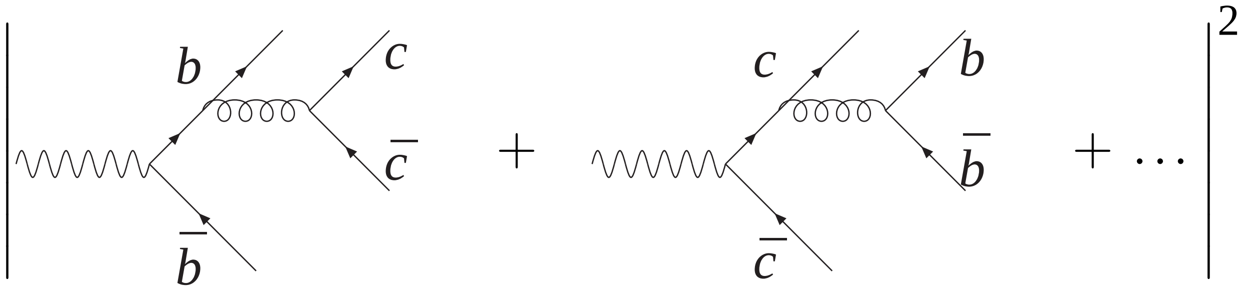
Tera Z: $\Gamma_b(Z \to b\bar{b})$

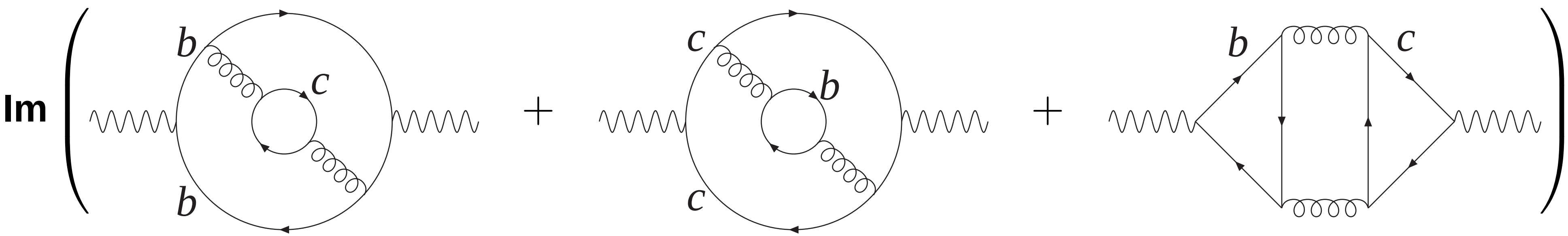
Can we isolate the $Zb\bar{b}$-vertex?

$R_b = 0.21629 \pm 0.00066$ (LEP); $3\% \mathrel{\widehat{=}} 1.65$ MeV

TLEP: $2-5\times 10^{-5} \mathrel{\widehat{=}} 50-120$ keV

conceptual problem: singlet-terms





mixed contributions, “singlet”

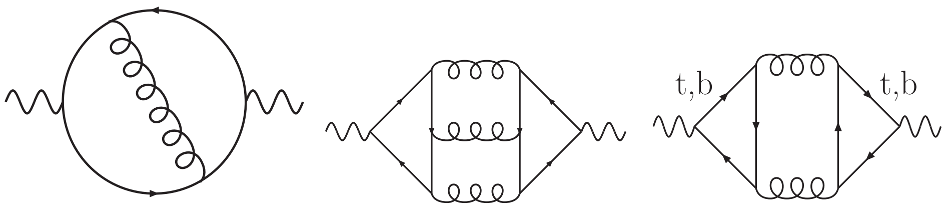
$$\Gamma^{\text{singlet}}_{b\bar{b}c\bar{c}} = \left(\frac{G_F M_Z^3}{8\sqrt{2}\pi}\right) 0.31 \left(\frac{\alpha_s}{\pi}\right)^2 \approx 340 \text{ keV}$$

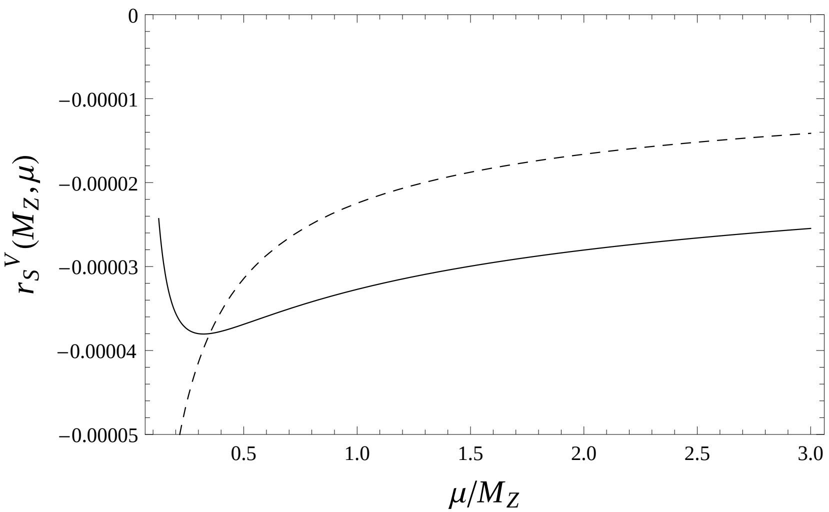
(total hadronic rate more robust!)

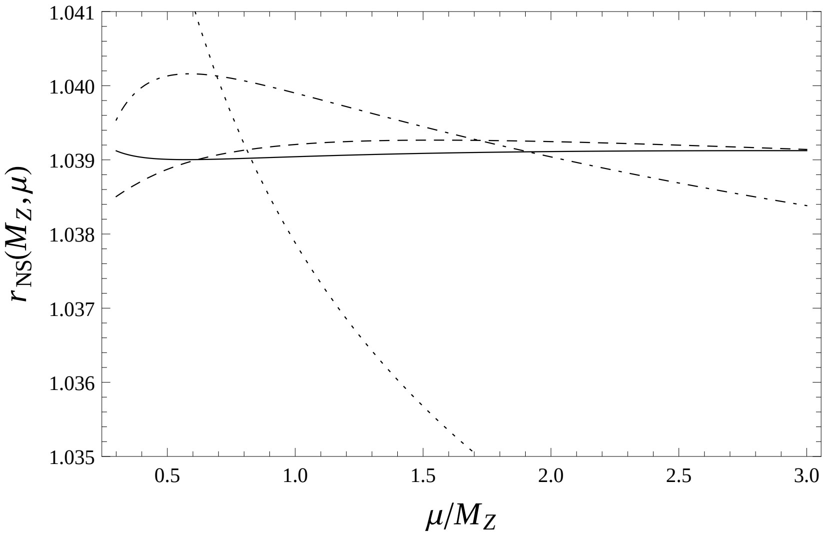
Tera Z: Γ_{had} and $\Gamma_{had}/\Gamma_{lept}$ corrections known to $O(\alpha_s^4)$, N^3LO

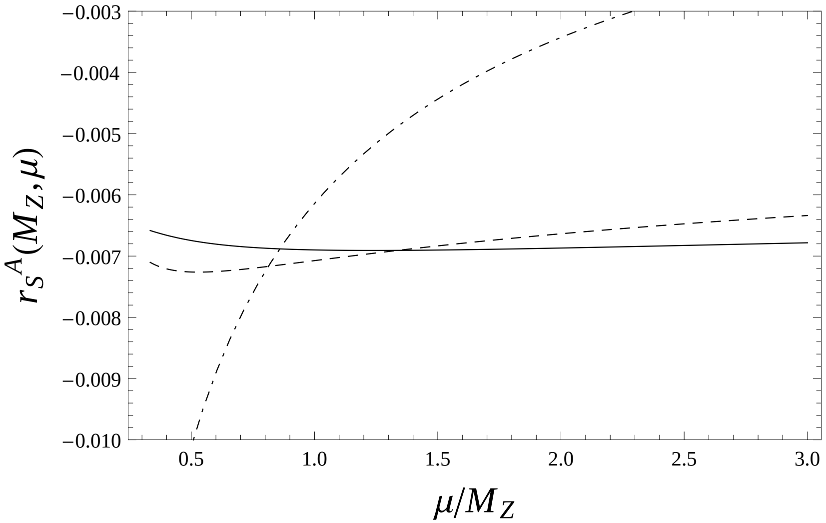
(Baikov, Chetyrkin, JK, Rittinger, arxiv: 0801.1821, 1201.5804)

non-singlet & singlet, vector & axial correlators









- theory uncertainty from $M_Z/3 < \mu < 3M_Z$

$$\left.\begin{aligned} \Rightarrow \quad \delta\Gamma_{NS} &= 101\text{keV}; \\ \delta\Gamma_S^V &= 2.7\text{keV}; \\ \delta\Gamma_S^A &= 42\text{keV}; \end{aligned}\right\} \quad \begin{gathered} \Sigma = 145.7\text{keV} \\ (\text{corresponds to } \delta\alpha_s \sim 3 \times 10^{-4}) \end{gathered}$$

TLEP: $\delta\Gamma_{had} \widehat{=} 100$ keV

- similar analysis of $W \to$ had only affected by non-singlet corrections!

- b-mass corrections under control: $m_b^2\alpha_s^4;\ m_b^4\alpha_s^3;\ \ldots$

- one more loop?

$\alpha_s^2(1979),\ \alpha_s^3(1991),\ \alpha_s^4(2008),\ \alpha_s^5(\,?\,),$

guesses on α_s^5 based on . . . .

lattice?

M_W

LEP: $\delta M_W \simeq 30$ MeV; TLEP: $\delta M_W \simeq 0.5 - 1$ MeV

Theory

$$M_W^2 = f(G_F, M_Z, m_t, \Delta\alpha, \ldots) = \frac{M_Z^2}{2(1-\delta\rho)} \left(1 + \sqrt{1 - \frac{4\pi\alpha(1-\delta\rho)}{\sqrt{2}G_F M_Z^2} \left(\frac{1}{1-\Delta\alpha} + \ldots\right)}\right);$$

m_t-dependence through $\delta\rho_t$

$$\delta M_W \approx M_W \frac{1}{2} \frac{\cos^2\theta_w}{\cos^2\theta_w - \sin^2\theta_w} \delta\rho \approx 5.7 \times 10^4 \, \delta\rho \text{ [MeV]}$$

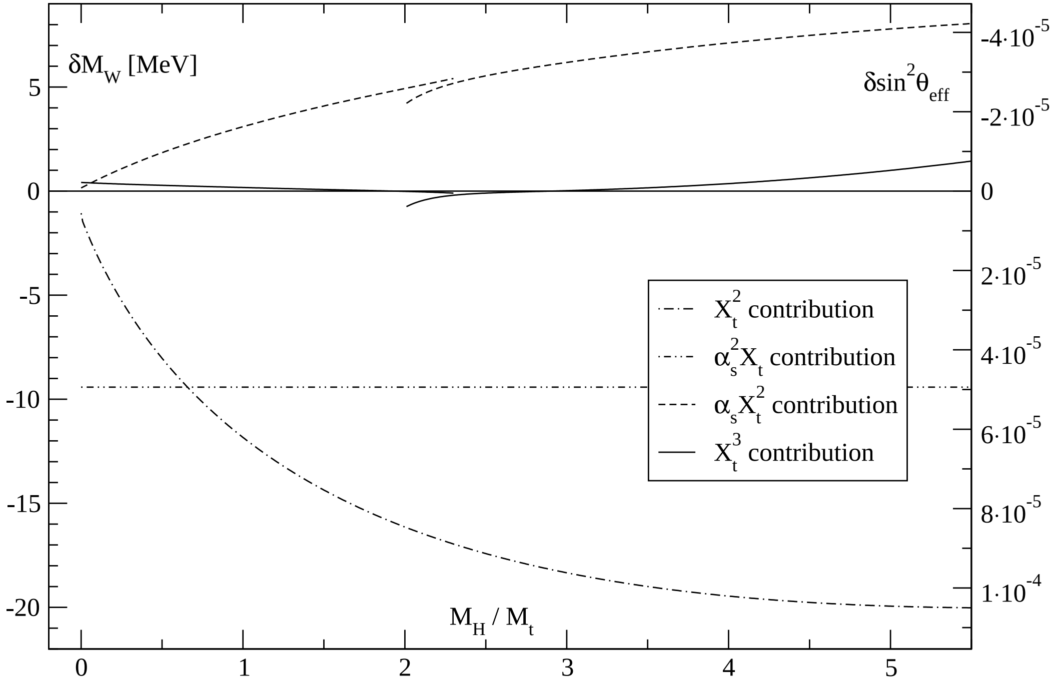
$$\delta\rho_t = 3X_t \left(1 - 2.8599\left(\frac{\alpha_s}{\pi}\right) - 14.594\left(\frac{\alpha_s}{\pi}\right)^2 - 93.1\left(\frac{\alpha_s}{\pi}\right)^3\right)$$

↓ ↓

$\delta M_W = 9.5$ MeV $\qquad \delta M_W = 2.1$ MeV

α_s^3: 4 loop (Chetyrkin, JK, Maierhöfer, Sturm, 2006)

mixed QCD $\star$ electroweak



three loop $(X_t \equiv G_F m_t^2)$

X_t^3	(purely weak)	$\Rightarrow$ 200eV
$\alpha_s X_t^2$	(mixed)	$\Rightarrow$ 2.5MeV
$\alpha_s^2 X_t$	(QCD three loop)	$\Rightarrow -9.5$MeV
$\alpha_s^3 X_t$	(QCD four loop)	$\Rightarrow$ 2.1MeV

the future

individual uncalculated higher orders below 0.5 MeV

examples:

$\alpha_s^2 X_t^2$ presumably feasible (4 loop tadpoles)

$\alpha_s^4 X_t$ 5 loop tadpoles?

dominant contribution from $m_t(pole) \Rightarrow \bar{m}_t$

crucial input: $\boxed{m_t}$

$\delta M_W \approx 6 \times 10^{-3} \delta m_t$

$\delta m_t = 1$ GeV $\Rightarrow \delta M_W \approx 6$ MeV (status)

conversely:

TLEP: $\delta M_W = 0.5$ MeV requires $\delta m_t = 100$ MeV

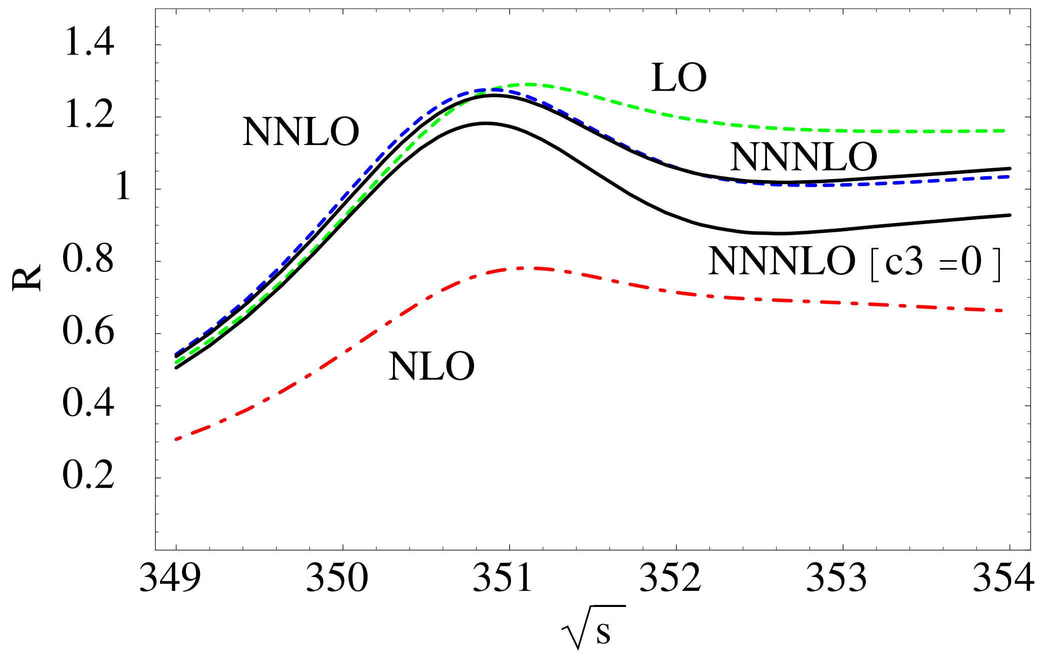
TLEP: $\delta m_t = 10 - 20$ MeV

based on bold extrapolation of ILC study (ILC: 35 MeV, no theory error)

σ_{tot} known to NNLO

momentum distribution etc: LO only

σ_{tot} in N^3LO under construction (Beneke, Steinhauser, ...)



(Beneke, Kiyo, Schuller)

robust location of threshold

extraction of λ_{Yuk} requires normalization!

important ingredient: $\bar{m}_t(\bar{m}_t) \Leftrightarrow m_{pole}$

example: $m_{pole} = 173.20 \pm 0.87$ GeV

$$\begin{aligned} \bar{m}_t(\bar{m}_t) &= m_{pole}\left(1 - 1.33\left(\frac{\alpha_s}{\pi}\right) - 7.50\left(\frac{\alpha_s}{\pi}\right)^2 - 78.9\left(\frac{\alpha_s}{\pi}\right)^3 \ldots\right) \\ &= (173.20 - 7.97 - 1.55 - 0.56)\,\text{GeV} \\ &= \left(163.47 \pm 0.83|_{m_t} \pm 0.18|_{\alpha_s} \pm ?|_{th}\right)\text{GeV} \end{aligned}$$

4 loop term under construction

H

example: $H \to b\bar{b}$ dominant decay mode, all branching ratios are affected!

TLEP: $\sigma_{HZ} \times Br(H \to b\bar{b})$: aim 0.2%

Higgs WG, arXiv:1307.1347

$$\frac{\delta\Gamma_b}{\Gamma_b} = \mp 2.3\%|_{\alpha_s} \pm 3.2\%|_{m_b} \pm 2.0\%|_{th} \quad \Rightarrow \; 7.5\%$$

$\alpha_s = 0.119 \pm 0.002, \; m_b|_{pole} = 4.49 \pm 0.06$ GeV

status: $\Gamma_b = \frac{G_F M_H}{4\sqrt{2}\pi} m_b^2(M_H) R^S(M_H)$

$$\begin{aligned} R^S(M_H) &= 1 + 5.667\left(\frac{\alpha_s}{\pi}\right) + 29.147\left(\frac{\alpha_s}{\pi}\right)^2 + 41.758\left(\frac{\alpha_s}{\pi}\right)^3 - 825.7\left(\frac{\alpha_s}{\pi}\right)^4 \\ &= 1 + 0.19551 + 0.03469 + 0.00171 + 0.00117 \\ &= 1.2307 \end{aligned}$$

(Chetyrkin, Baikov, JK, 2006)

Theory uncertainty ($M_H/3 < \mu < 3M_H$) : 5‰ (four loop) reduced to 1.5‰ (five loop) present uncertainties:

$m_b(10\text{GeV}) = 3610 - \frac{\alpha_s - 0.1189}{0.002} 12 \pm 11$ MeV (Karlsruhe, arXiv:0907.2110)

$$\left(\begin{array}{ll} \text{Bodenstein+Dominguez:} & 3623(9)\text{ MeV} \\ \text{HPQCD} & 3617(25)\text{ MeV} \end{array} \right)$$

α_s ($\Rightarrow$ m_b-determination; running to M_H; R^S)

running from 10 GeV to M_H depends on

anomalous mass dimension, β-function and α_s

$m_b(M_H) = 2759 \pm 8|_{m_b} \pm 27|_{\alpha_s}$ MeV

γ_4 (five loop): Baikov + Chetyrkin, 2012

β_4 under construction

$$\frac{\delta m_b^2(M_H)}{m_b^2(M_H)} = -1.4 \times 10^{-4}\ (b_4 = 0) \quad | \quad -4.3 \times 10^{-4}\ (b_4 = 100) \quad | \quad -7.3 \times 10^{-4}\ (b_4 = 200)$$

to be compared with $\delta\Gamma/\Gamma = 2.0 \times 10^{-4}$ (TLEP)

$$\delta\Gamma_b/\Gamma_b = \pm 8.3\times 10^{-3}|_{m_b} - 13\times 10^{-3}|_{\alpha_s,\text{running}} + 4\times 10^{-3}|_{\alpha_s;R^S} = 17\times 10^{-3} \text{ (now)}$$

perspectives: (assume $\delta\alpha_s = 2\times 10^{-4}$)

$\delta m_b(10\text{GeV})/m_b \sim 10^{-3}$ conceivable (dominated by $\delta\Gamma(\Upsilon\to e^+e^-)$)

$$\Rightarrow \frac{\delta\Gamma_b}{\Gamma_b} = \pm 2\times 10^{-3}|_{m_b} \pm 1.3\times 10^{-3}|_{\alpha_s,\text{running}} \pm 1\times 10^{-3}|_{\text{theory}}$$

similarly: Γ_c

$$\begin{aligned} \delta m_c(3\text{ GeV})/m_c(3\text{ GeV}) &= 13\text{ MeV}/986\text{ MeV} \quad \text{(now)} \\ &= 5\text{ MeV}/986\text{ MeV} \quad \text{(conceivable)} \end{aligned}$$

$$\begin{aligned} m_c(M_H) &= (609 \pm 8|_{m_c} \pm 9|_{\alpha_s})\text{ MeV} \quad \text{(now)} \\ & \qquad \pm 3\text{ MeV} \quad \text{(conceivable)} \\ \Rightarrow \frac{\delta\Gamma_c}{\Gamma_c} &= \pm 5.5\times 10^{-2} \quad \text{(now)} \\ &= \pm 1\times 10^{-2} \quad \text{(conceivable)} \end{aligned}$$

Starting from order α_s^3 the separation of $H\to gg$ and $H\to b\bar{b}$

is no longer unambiguously possible. (Chetyrkin, Steinhauser, 1997)

SUMMARY

- theory predictions do not (yet?) fulfill TLEP requirements,

- missing corrections are presumably feasible (QCD),

- important experimental input from low-energy e^+e^- annihilation:

  m_b, m_c, $\Delta\alpha$, $(\alpha_s?)$,

- m_b determiantion $\Rightarrow \Gamma(H \to b\bar{b})$

  usage of $m_b(pole)$ is strongly disfavoured compared to $\bar{m}_b(10\text{ GeV})$