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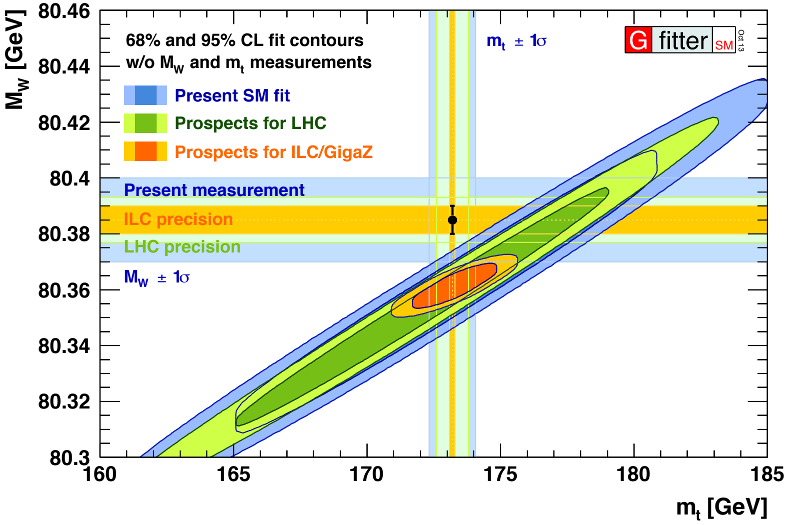
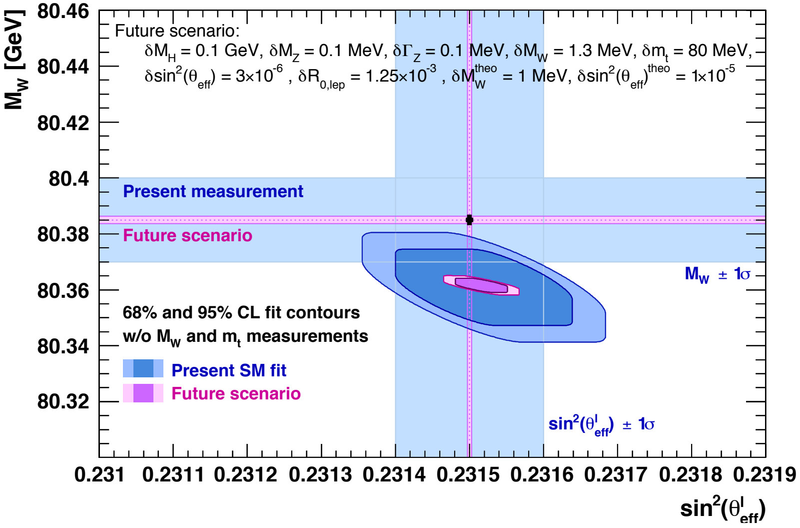
6th TLEP workshop, CERN,
17th October, 2013

Future collider prospects of the ElectroWeak fit of the SM





Outline

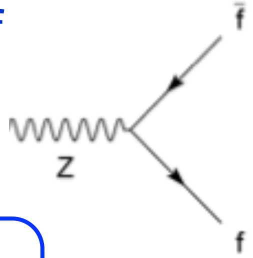
This presentation:

- ✓ Brief introduction to key (future) observables
- ✓ Prospects for LHC-300, ILC/GigaZ, TLEP
- ✓ Outlook

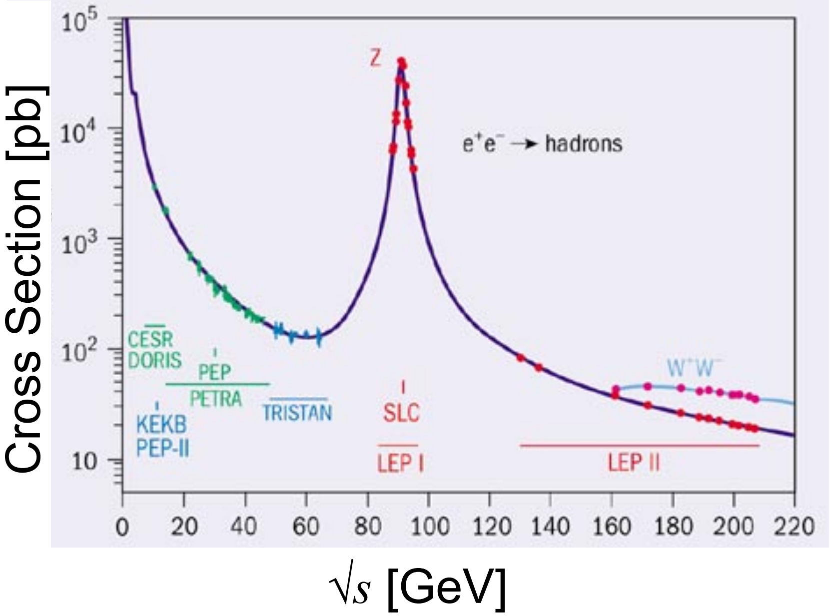
(Results presented here based on Gfitter software.)

The predictive power of the SM

- As the Z boson couples to all fermions, it is ideal to measure & study both the electroweak and strong interactions.
- Tree level relations for Z→$f\bar{f}$
  - $i\bar{f}\gamma^\mu\left(g_{V,f} - g_{A,f}\gamma_5\right) f Z_\mu$



- Prediction EWSB at tree-level: $\boxed{\dfrac{M_W^2}{M_Z^2 c_W^2} = 1}$



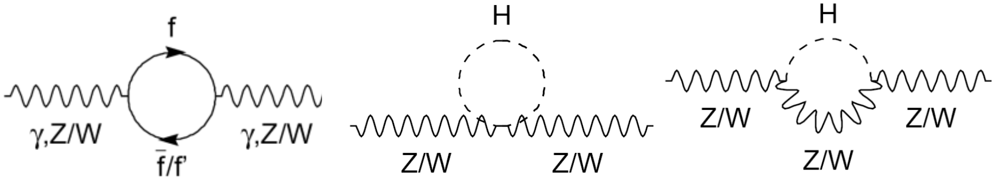
- The impact of loop corrections
  - Absorbed into EW form factors: ρ, κ, Δr
  - Effective couplings at the Z-pole
  - Quadraticly dependent on m_t, *logarithmic* dependence on M_H

$$g_{V,f} = \sqrt{\rho_Z^f}\left(I_3^f - 2Q^f \sin^2\theta_{\mathrm{eff}}^f\right)$$

$$g_{A,f} = \sqrt{\rho_Z^f} I_3^f$$

$$\sin^2\theta_{\mathrm{eff}}^f = \kappa_Z^f \sin^2\theta_W$$

$$M_W^2 = \frac{M_Z^2}{2}\left(1 + \sqrt{1 - \frac{\sqrt{8}\pi\alpha(1+\Delta r)}{G_F M_Z^2}}\right)$$



After the Higgs: the Electroweak Fit of the SM

Unique situation:

- *For first time SM is fully over-constrained.*
- *And for first time electroweak observables can be unambiguously predicted at loop level.*
- *Powerful predictions of key observables now possible, much better than w/o M_H*

Paradigm shift for EW fit.
From (Higgs) mass predictions to precision tests for:

- Self-consistency of the Standard Model
- Possible contributions from BSM models
- Improved accuracies set benchmark for new direct measurements!

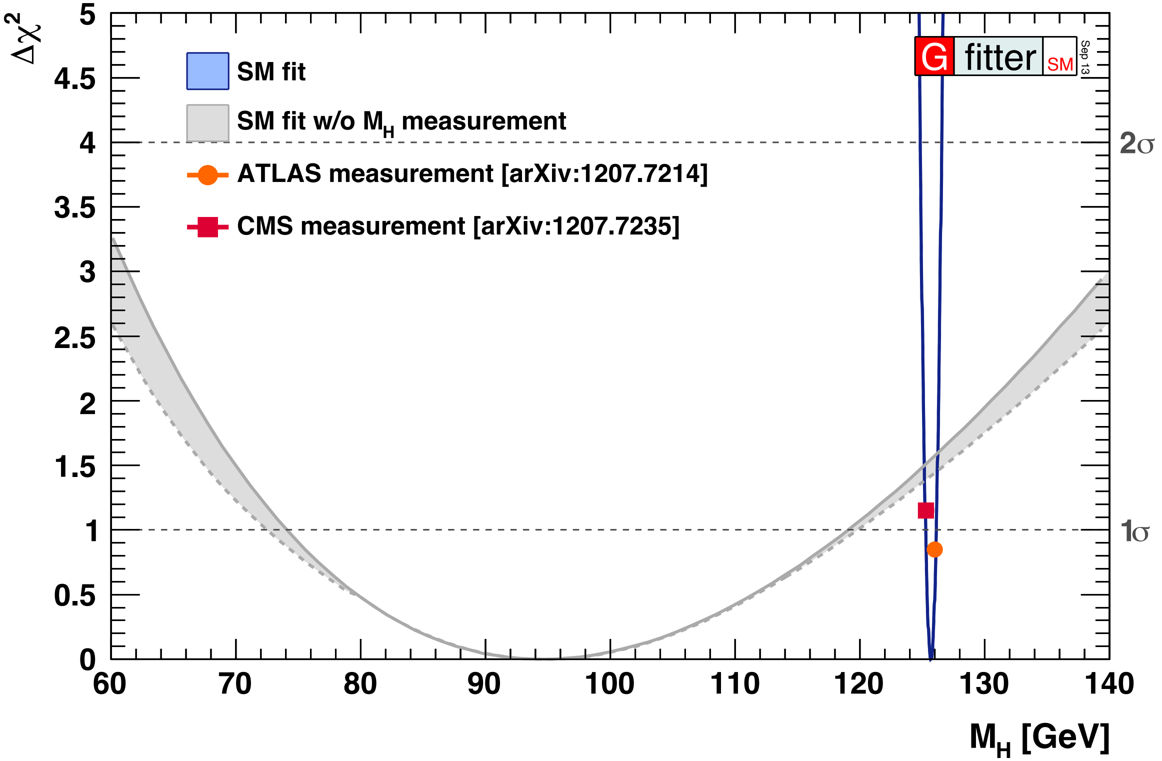
Electroweak Fit – Experimental inputs

- Latest experimental inputs:
  - Z-pole observables: from LEP / SLC
    [ADLO+SLD, Phys. Rept. 427, 257 (2006)]
  - M_W and Γ_W from LEP/Tevatron
    [arXiv:1204.0042, arXiv:1302.3415]
  - m_{top} latest avg from Tevatron
    [arXiv:1305.3929]
  - m_c, m_b world averages (PDG)
    [PDG, J. Phys. G33,1 (2006)]
  - $\Delta\alpha_{had}^{(5)}(M_Z^2)$ including α_S dependency
    [Davier et al., EPJC 71, 1515 (2011)]
  - M_H from LHC
    [arXiv:1207.7214, arXiv:1207.7235]

- 7 (+2) free fit parameters:
  - M_H, M_Z, $\alpha_S(M_Z^2)$, $\Delta\alpha_{had}^{(5)}(M_Z^2)$, m_t, m_c, m_b
  - 2 theory nuisance parameters
    - δM_W (4 MeV), $\delta\sin^2\theta^l_{eff}$ (4.7×10^{-5})

M_H [GeV]°	125.7 ± 0.4	LHC
M_W [GeV]	80.385 ± 0.015	Tevatron
Γ_W [GeV]	2.085 ± 0.042	
M_Z [GeV]	91.1875 ± 0.0021	
Γ_Z [GeV]	2.4952 ± 0.0023	
$\sigma^0_{\rm had}$ [nb]	41.540 ± 0.037	LEP
R^0_ℓ	20.767 ± 0.025	
$A^{0,\ell}_{\rm FB}$	0.0171 ± 0.0010	
A_ℓ $(\star)$	0.1499 ± 0.0018	SLC
$\sin^2\theta^\ell_{\rm eff}(Q_{\rm FB})$	0.2324 ± 0.0012	
A_c	0.670 ± 0.027	SLC
A_b	0.923 ± 0.020	
$A^{0,c}_{\rm FB}$	0.0707 ± 0.0035	LEP
$A^{0,b}_{\rm FB}$	0.0992 ± 0.0016	
R^0_c	0.1721 ± 0.0030	
R^0_b	0.21629 ± 0.00066	
$\overline{m}_c$ [GeV]	$1.27^{+0.07}_{-0.11}$	
$\overline{m}_b$ [GeV]	$4.20^{+0.17}_{-0.07}$	
m_t [GeV]	173.20 ± 0.87	Tevatron
$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ $(\dagger\triangle)$	2756 ± 10	

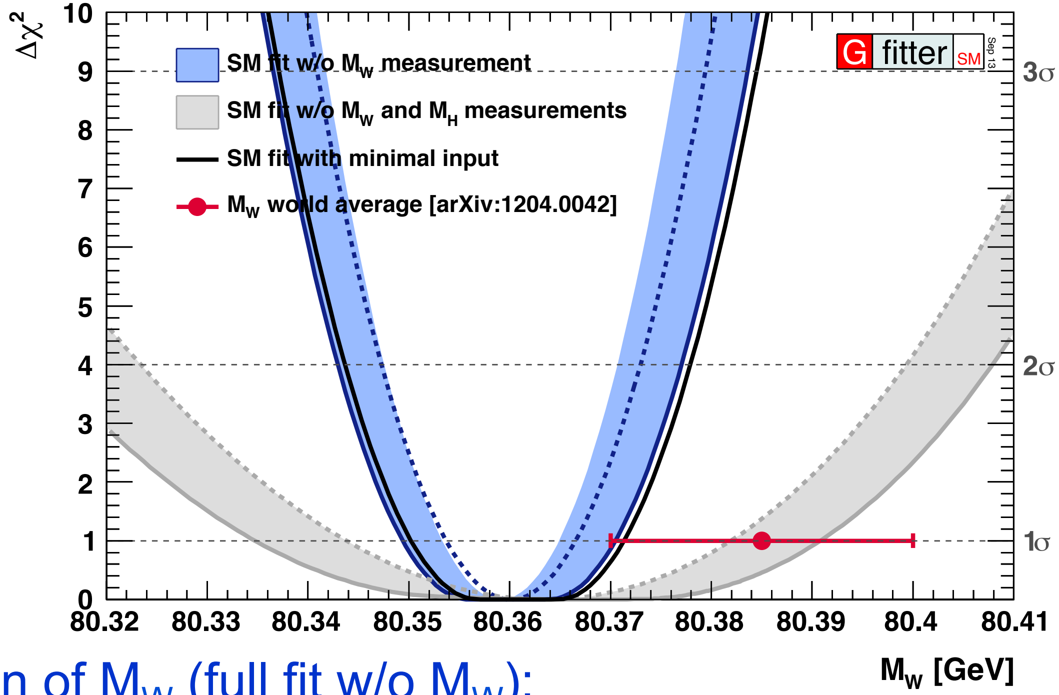
After the Higgs: status of the SM



- Overall consistency of the Standard Model fit is very good.
  - M_H consistent at 1.3σ with indirect prediction from EW fit.
    - Higgs mass prediction: 94^{+25}_{-22} GeV. (Measurement: 126 GeV.)
  - p-Value of global electroweak fit of SM: 18^{+2} % (pseudo-experiments)

Indirect determination of W mass

- Scan of $\Delta\chi^2$ profile versus M_W
  - Also shown: SM fit with minimal inputs: M_Z, G_F, $\Delta\alpha_{had}^{(5)}(M_Z)$, $\alpha_s(M_Z)$, M_H, and fermion masses
  - Good consistency between total fit and SM w/ minimal inputs



- M_H measurement allows for precise constraint on M_W
  - Agreement at 1.4σ
- Fit result for indirect determination of M_W (full fit w/o M_W):

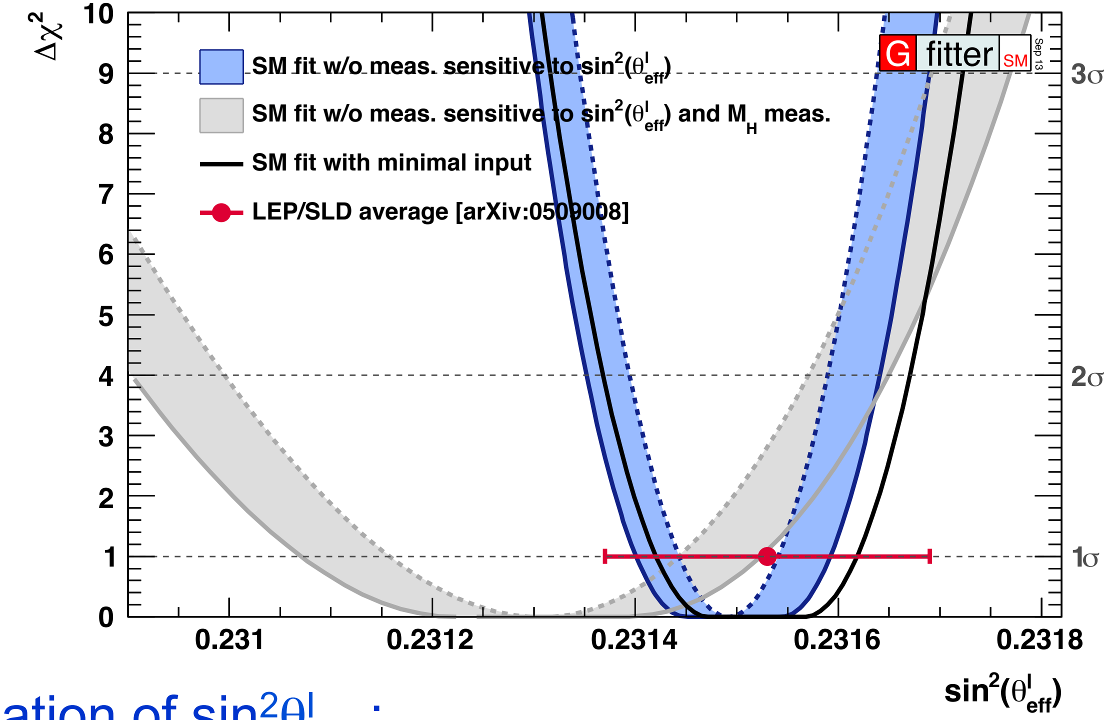
$$\begin{aligned} M_W &= 80.3593 \pm \boxed{0.0056_{m_t}} \pm 0.0026_{M_Z} \pm 0.0018_{\Delta\alpha_{\rm had}} \\ &\quad \pm 0.0017_{\alpha_S} \pm 0.0002_{M_H} \pm \boxed{0.0040_{\rm theo}} \\ &= 80.359 \pm 0.011_{\rm tot}\ , \end{aligned}$$

- More precise estimate of M_W than the direct measurements!
  - Uncertainty on world average measurement: 15 MeV

Indirect effective weak mixing angle

- Right: scan of $\Delta\chi^2$ profile versus $\sin^2\theta^l_{\text{eff}}$
  - All sensitive measurements removed from the SM fit.
  - Also shown: SM fit with minimal inputs

- M_H measurement allows for very precise constraint on $\sin^2\theta^l_{\text{eff}}$



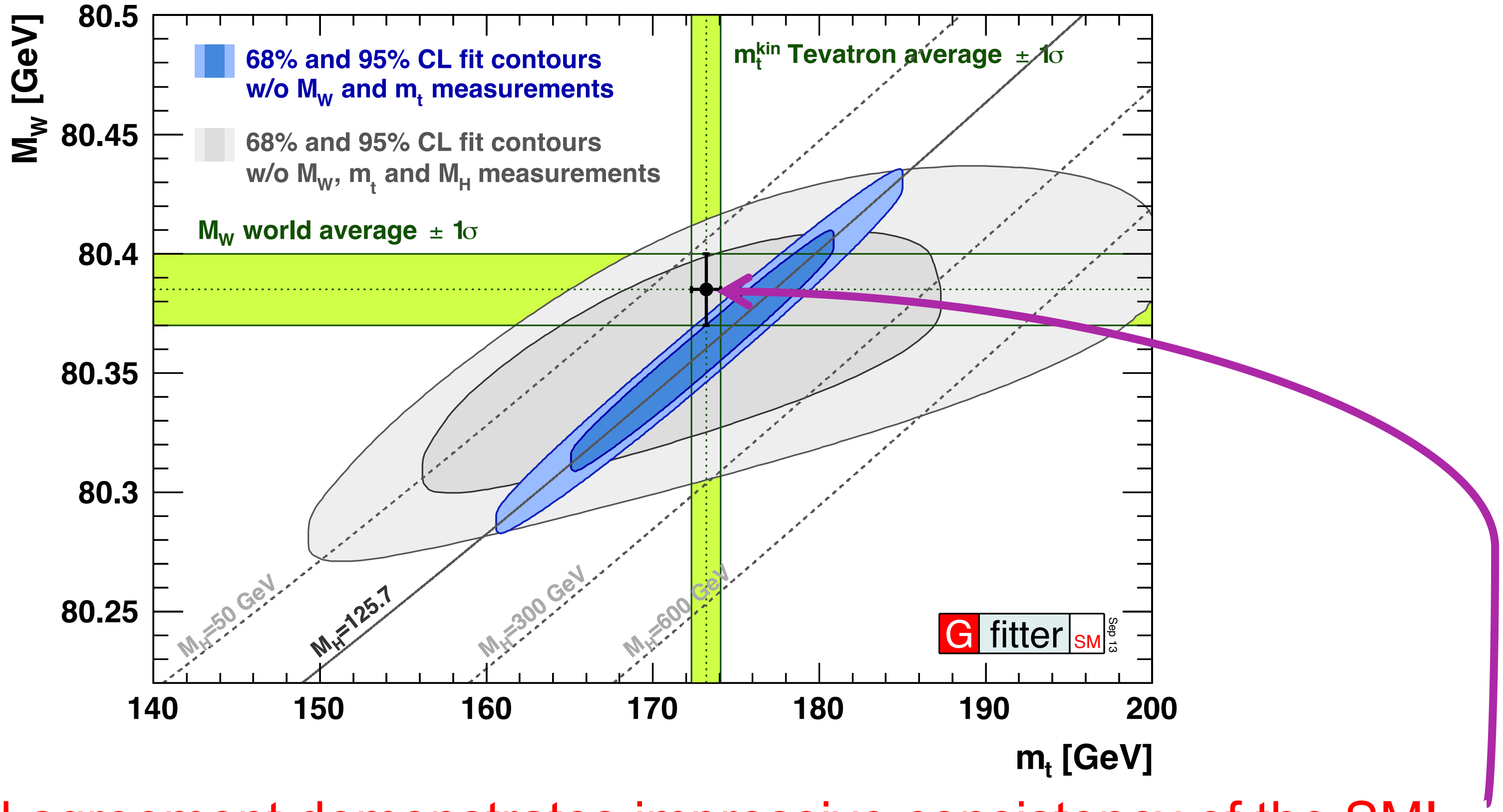
- Fit result for indirect determination of $\sin^2\theta^l_{\text{eff}}$:

$$\begin{aligned}\sin^2\theta^\ell_{\text{eff}} &= 0.231496 \pm \boxed{0.000030_{m_t}} \pm 0.000015_{M_Z} \pm \boxed{0.000035_{\Delta\alpha_{\text{had}}}} \\ &\qquad \pm 0.000010_{\alpha_S} \pm 0.000002_{M_H} \pm \boxed{0.000047_{\text{theo}}} , \\ &= 0.23150 \pm 0.00010_{\text{tot}} ,\end{aligned}$$

- More precise than direct determination (from LEP/SLD) !
  - Uncertainty on LEP/SLD average: 1.6x10^{-4}

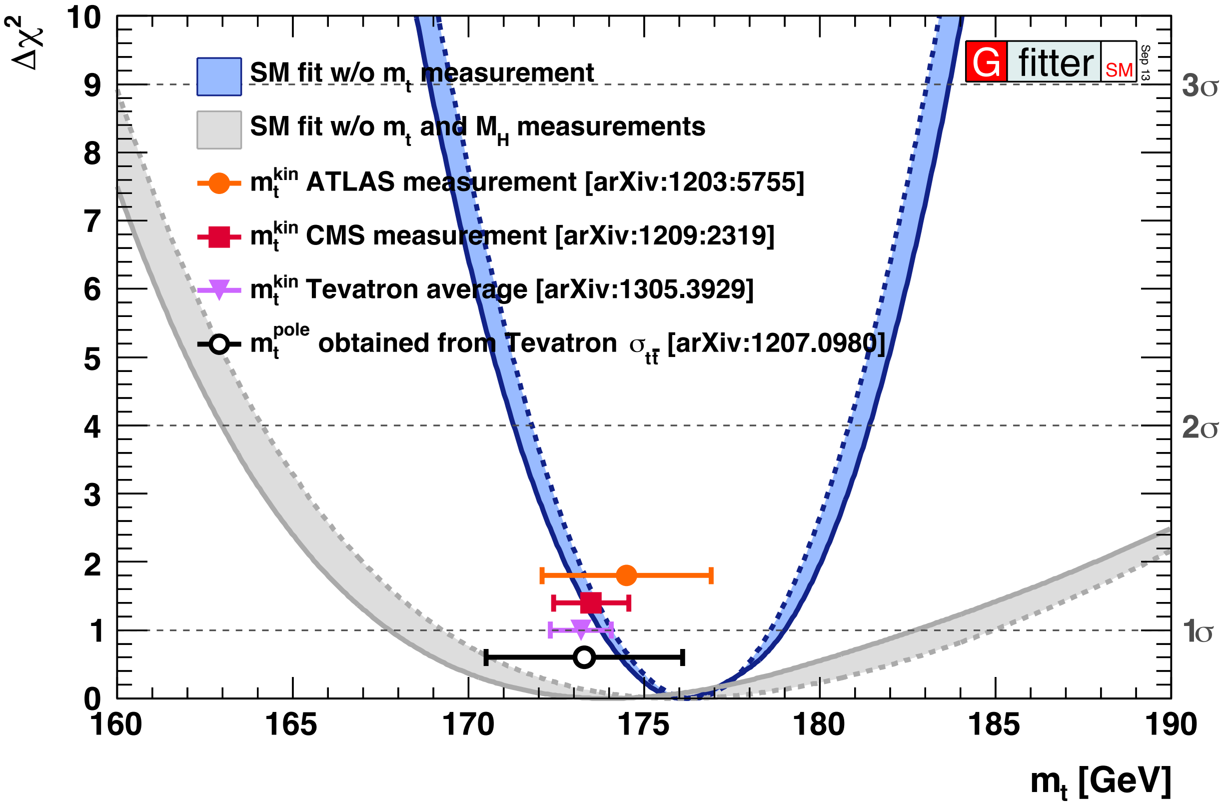
State of the SM: W versus top mass

- Scan of M_W vs m_t, with the direct measurements excluded from the fit.
- Results from Higgs measurement significantly reduces allowed indirect parameter space → corners the SM!



- Observed agreement demonstrates impressive consistency of the SM!

Indirect determination of top mass

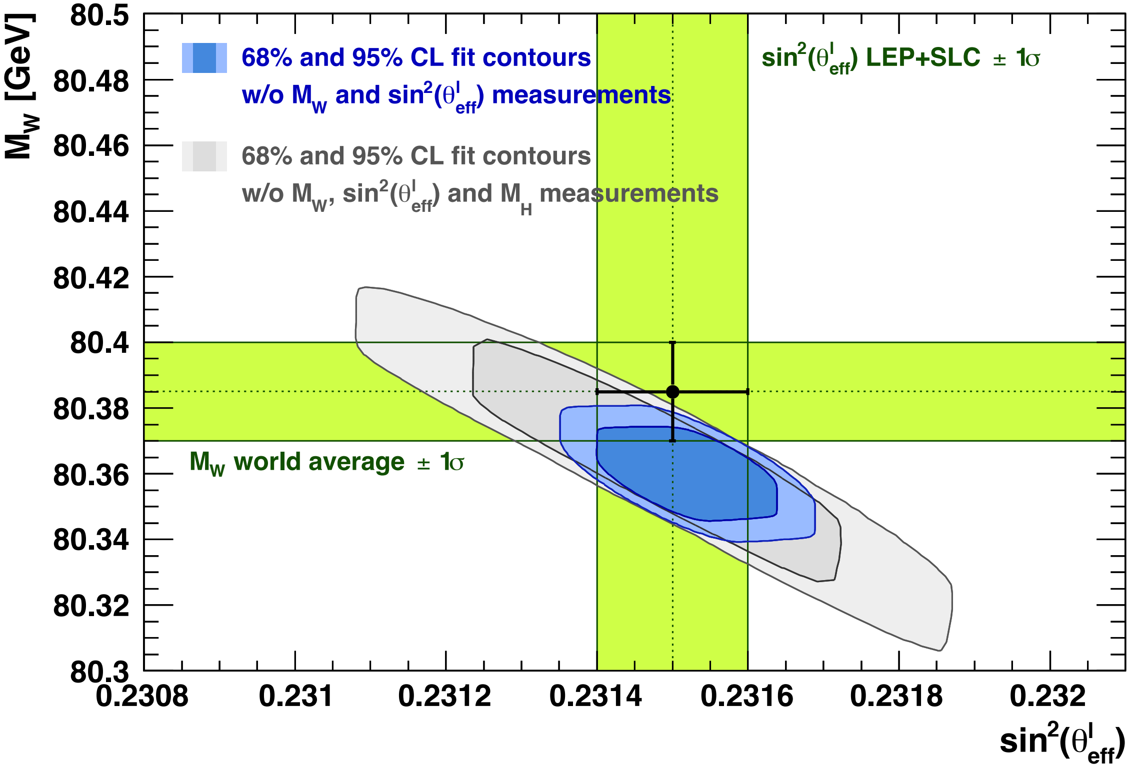


- Shown: scan of $\Delta\chi^2$ profile versus m_t (without m_t measurement)
  - M_H measurement allows for significant better constraint of m_t
  - Indirect determination consistent with direct measurements
    - Remember: fully obtained from loop corrections!
- Indirect result: $m_t = 176.1^{+2.9}_{-2.4}$ GeV

Tevatron: 173.2 ± 0.9 GeV
LHC: 173.3 ± 1.0 GeV

State of the SM: W mass versus $\sin^2\theta^l_{eff}$

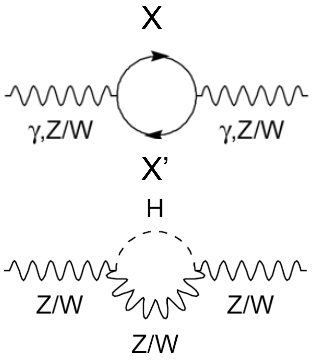
- Scan of M_W vs $\sin^2\theta^l_{eff}$, with direct measurements excluded from the fit.
- Again, significant reduction allowed indirect parameter space from Higgs mass measurement.



- M_W and $\sin^2\theta^l_{eff}$ have become *the* sensitive probes of new physics!
  - Both are 'tree-level' SM predictions.

Constraints on Oblique Corrections

- If energy scale of NP is high, BSM physics appears dominantly through vacuum polarization corrections
  - Aka, "oblique corrections"
- Oblique corrections reabsorbed into electroweak form factors
  - $\Delta\rho$, $\Delta\kappa$, Δr parameters, appearing in: M_W^2, $\sin^2\theta_{eff}$, G_F, α, etc.
- Electroweak fit sensitive to BSM physics through oblique corrections
  - Similar to sensitivity to top and Higgs loop corrections.



- Oblique corrections from New Physics described through STU parametrization

[Peskin and Takeuchi, Phys. Rev. D46, 1 (1991)]

$$\boxed{O_{meas} = O_{SM,REF}(m_H,m_t) + c_S S + c_T T + c_U U}$$

S :	New Physics contributions to neutral currents
T :	Difference between neutral and charged current processes – sensitive to weak isospin violation
U :	(+S) New Physics contributions to charged currents. U only sensitive to W mass and width, usually very small in NP models (often: U=0)

- Also implemented: extended parameters (VWX), correction to Z→bb couplings.

[Burgess et al., Phys. Lett. B326, 276 (1994)]
[Burgess et al., Phys. Rev. D49, 6115 (1994)]

Fit results for S, T, U

- S,T,U obtained from fit to the EW observables
- SM: M_H = 126 GeV, m_t = 173 GeV
  - This defines (S,T,U) = (0,0,0)
- SM: S, T depend logarithmically on M_H
- Fit result:

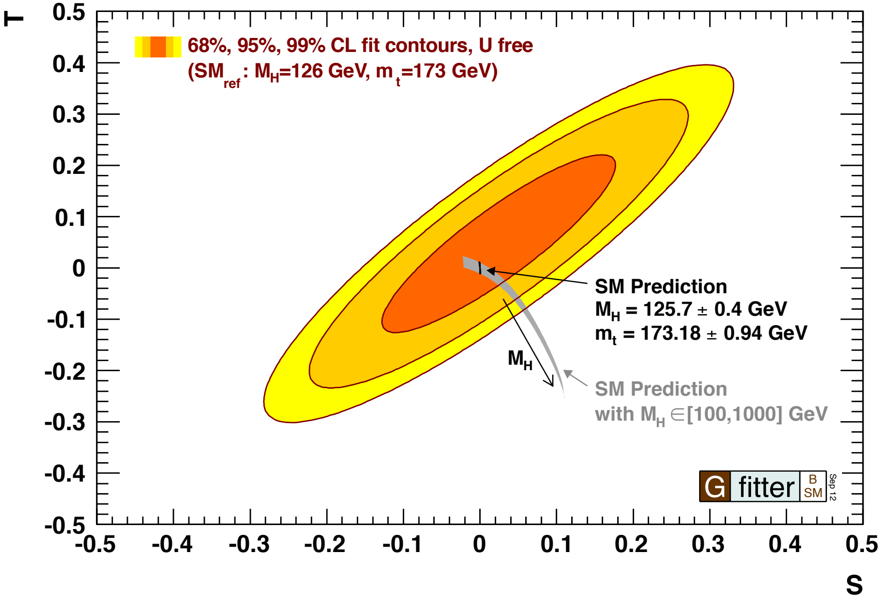
$S = 0.03 \pm 0.10$

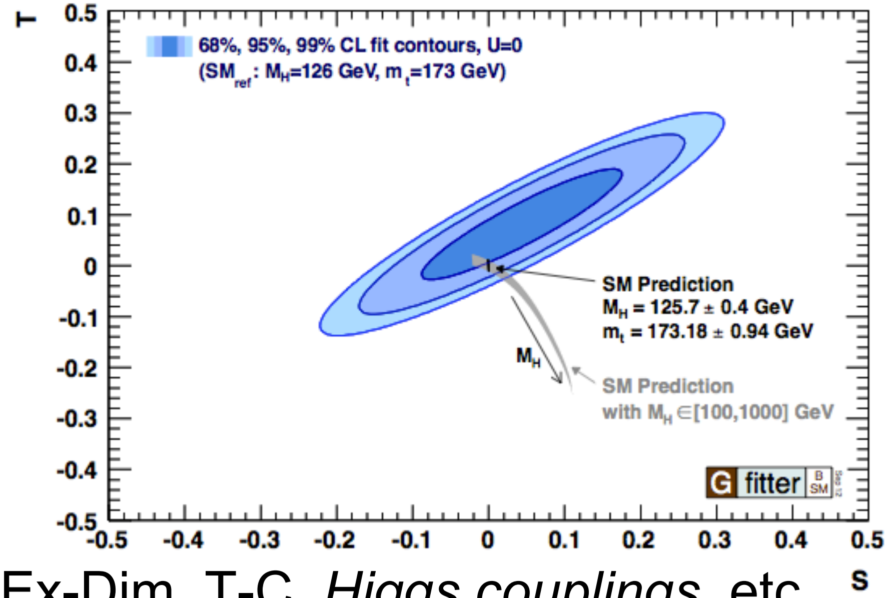
$T = 0.05 \pm 0.12$

$U = 0.03 \pm 0.10$

	S	*T*	*U*
S	1	+0.89	-0.54
T		1	-0.80
U			1

- Stronger constraints from fit with U=0.
- Also available for Z→bb correction.
- No indication for new physics.
- Can now use this to constrain 4th gen, Ex-Dim, T-C, *Higgs couplings*, etc.





Future prospects for the SM fit

Prospects scenarios: LHC, ILC/GigaZ, TLEP

Prospects of EW fit tested for three scenarios:

1. LHC Run-2+3
2. ILC with GigaZ (*)
3. Future scenario (= TLEP-like)

(*) GigaZ:

- Operation of ILC at lower energies like Z-pole or WW threshold.
  - Allows to perform precision measurements of EW sector of the SM.
- At Z-pole, several billion Z's can be studied within 1-2 months (days).
- Physics of LEP1 and SLC can be revisited with few days of data.

In following studies:
Central values of input measurements adjusted to M_H = 126 GeV.

- *(Except where indicated.)*

Prospects of EW fit

All three scenarios:

- Low-energy data results to improve $\Delta\alpha_{had}$:
  - ISR-based (BABAR), KLOE-II, VEPP-2000 (at energy below cc resonance), and BESIII e^+e^- cross-section measurements, in particular around cc resonance.
  - Plus: improved α_s, improvements in theory: $\Delta\alpha_{had}$: $10^{-4} \rightarrow 5 \cdot 10^{-5}$
- Assuming ~25% of today's theoretical uncertainties on M_W and $\sin^2\theta^l_{eff}$
  - Implies three-loop EW calculations!
  - δM_W (4→1 MeV), $\delta\sin^2\theta^l_{eff}$ ($4.7\times10^{-5} \rightarrow 1\times10^{-5}$)
  - (Theoretical uncertainty estimates from recent Snowmass report)

Experimental inputs – Present uncertainties

	Experimental input [$\pm 1\sigma$]
Parameter	Present
M_H [GeV]	0.4
M_W [MeV]	15
M_Z [MeV]	2.1
m_t [GeV]	0.9
Γ_Z [MeV]	2.3
$\sin^2\theta^{\ell}_{\rm eff}$ [$\cdot 10^{-5}$]	16
R^0_l [$\cdot 10^{-3}$]	25
$\Delta\alpha^5_{\rm had}(M_Z^2)$ [$\cdot 10^{-5}$]	10
$\alpha_S(M_Z^2)$ [$\cdot 10^{-4}$]	–
$\delta_{\rm th} M_W$ [MeV]	4
$\delta_{\rm th} \sin^2\theta^{\ell}_{\rm eff}$ [$\cdot 10^{-5}$]	4.7

Present scenario, two fit setups:

- *Present fit*: current full EW fit.
- *Present uncertainties*: central values of input measurements adjusted to M_H = 126 GeV, and
- EW fit with minimal inputs
  - E.g. all asymmetry measurements replaced by $\sin^2\theta^l_{\rm eff}$

Experimental inputs – Predicted uncertainties

	Experimental input $[\pm 1\sigma]$		
Parameter	Present		LHC
M_H [GeV]	0.4	⇨	< 0.1
M_W [MeV]	15	⇨	8
M_Z [MeV]	2.1		2.1
m_t [GeV]	0.9	⇨	0.6
Γ_Z [MeV]	2.3		2.3
$\sin^2\theta^\ell_{\rm eff}$ $[\cdot 10^{-5}]$	16		16
R^0_l $[\cdot 10^{-3}]$	25		25
$\Delta\alpha^5_{\rm had}(M_Z^2)$ $[\cdot 10^{-5}]$	10	⇨	4.7
$\alpha_S(M_Z^2)$ $[\cdot 10^{-4}]$	–		–
$\delta_{\rm th} M_W$ [MeV]	4	⇨	1
$\delta_{\rm th} \sin^2\theta^\ell_{\rm eff}$ $[\cdot 10^{-5}]$	4.7	⇨	1

LHC scenario:

- Run 2+3, i.e. 300/fb of data.
- Numbers inspired by recent LHC upgrade studies.
- Possibly optimistic scenario, but not impossible.

- Final W and top mass measurements, combination with LEP and Tevatron.

Experimental inputs – Predicted uncertainties

Parameter	Experimental input $[\pm 1\sigma]$		
	Present	LHC	ILC/GigaZ
M_H [GeV]	0.4	⇨ < 0.1	< 0.1
M_W [MeV]	15	⇨ 8	⇨ 5
M_Z [MeV]	2.1	2.1	2.1
m_t [GeV]	0.9	⇨ 0.6	0.1
Γ_Z [MeV]	2.3	2.3	⇨ 0.8
$\sin^2\theta^{\ell}_{\rm eff}$ $[\cdot 10^{-5}]$	16	16	⇨ 1.3
R^0_l $[\cdot 10^{-3}]$	25	25	⇨ 4
$\Delta\alpha^5_{\rm had}(M_Z^2)$ $[\cdot 10^{-5}]$	10	⇨ 4.7	4.7
$\alpha_S(M_Z^2)$ $[\cdot 10^{-4}]$	–	–	–
$\delta_{\rm th} M_W$ [MeV]	4	⇨ 1	1
$\delta_{\rm th}\sin^2\theta^{\ell}_{\rm eff}$ $[\cdot 10^{-5}]$	4.7	⇨ 1	1

ILC scenario:

- Prospects from ILC TDR (Vol-2).

- M_W : WW threshold scan + kinematic reconstruction.
- m_t : ttbar production threshold scan.
- $\delta A^{0,f}_{LR}$: $10^{-3} \rightarrow 10^{-4}$
- High statistics on Z pole

- Improvement in Higgs mass over LHC has negligible impact on fit results.
- Possible improvement in Γ_Z, but has small impact on fit.

Experimental inputs – Predicted uncertainties

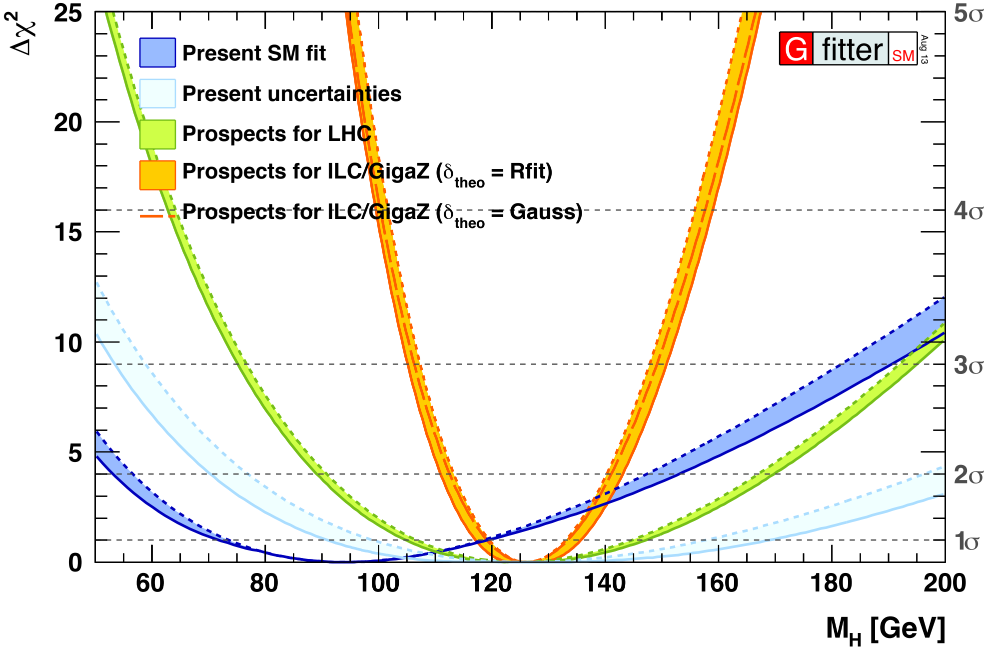
	Experimental input $[\pm 1\sigma]$			
Parameter	Present	LHC	ILC/GigaZ	TLEP
M_H [GeV]	0.4	⇨ < 0.1	< 0.1	< 0.1
M_W [MeV]	15	⇨ 8	⇨ 5	⇨ 1.3
M_Z [MeV]	2.1	2.1	2.1	⇨ 0.1
m_t [GeV]	0.9	⇨ 0.6	0.1	0.08
Γ_Z [MeV]	2.3	2.3	⇨ 0.8	⇨ 0.1
$\sin^2\theta^\ell_{\mathrm{eff}}$ $[\cdot 10^{-5}]$	16	16	⇨ 1.3	⇨ 0.3
R^0_l $[\cdot 10^{-3}]$	25	25	⇨ 4	⇨ 1.3
$\Delta\alpha^5_{\mathrm{had}}(M^2_Z)$ $[\cdot 10^{-5}]$	10	⇨ 4.7	4.7	4.7
$\alpha_S(M^2_Z)$ $[\cdot 10^{-4}]$	–	–	–	–
$\delta_{\mathrm{th}} M_W$ [MeV]	4	⇨ 1	1	1
$\delta_{\mathrm{th}} \sin^2\theta^\ell_{\mathrm{eff}}$ $[\cdot 10^{-5}]$	4.7	⇨ 1	1	1

TLEP scenario:

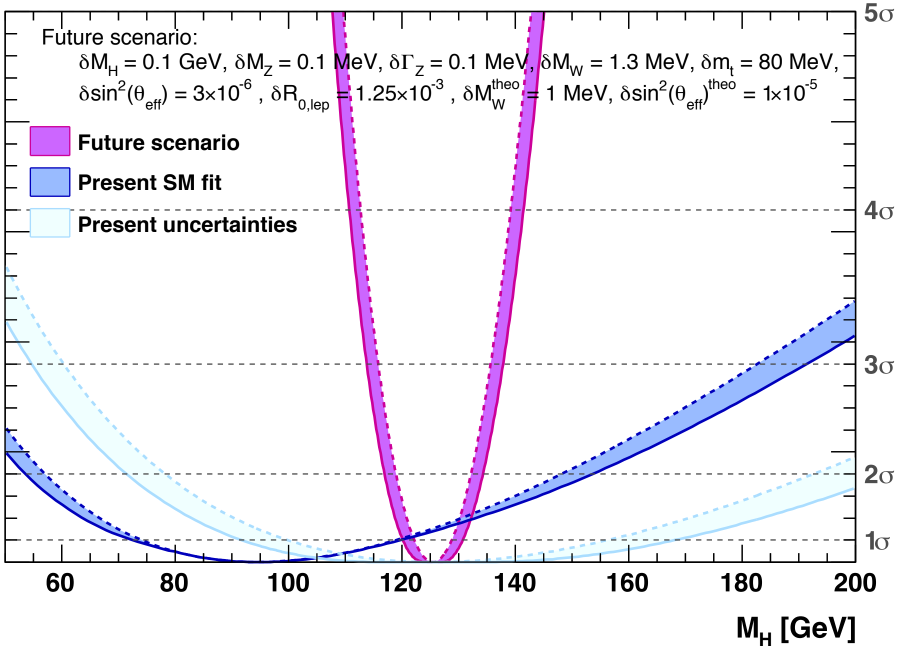
- *Preliminary estimates*
- Clearly not the same level of understanding as LHC or ILC.
- Uncertainties may turn out completely different.
  - From arXiv:1308.6176,
  - and Snowmass report.
  - Of these two, we take most conservative estimate.
- Note: top mass dominated by theoretical uncertainty.
- Higher statistics
- From beam energy precision: improved M_Z and Γ_Z

Prospects of the EW fit: Higgs mass (126 GeV)

Present / LHC / ILC

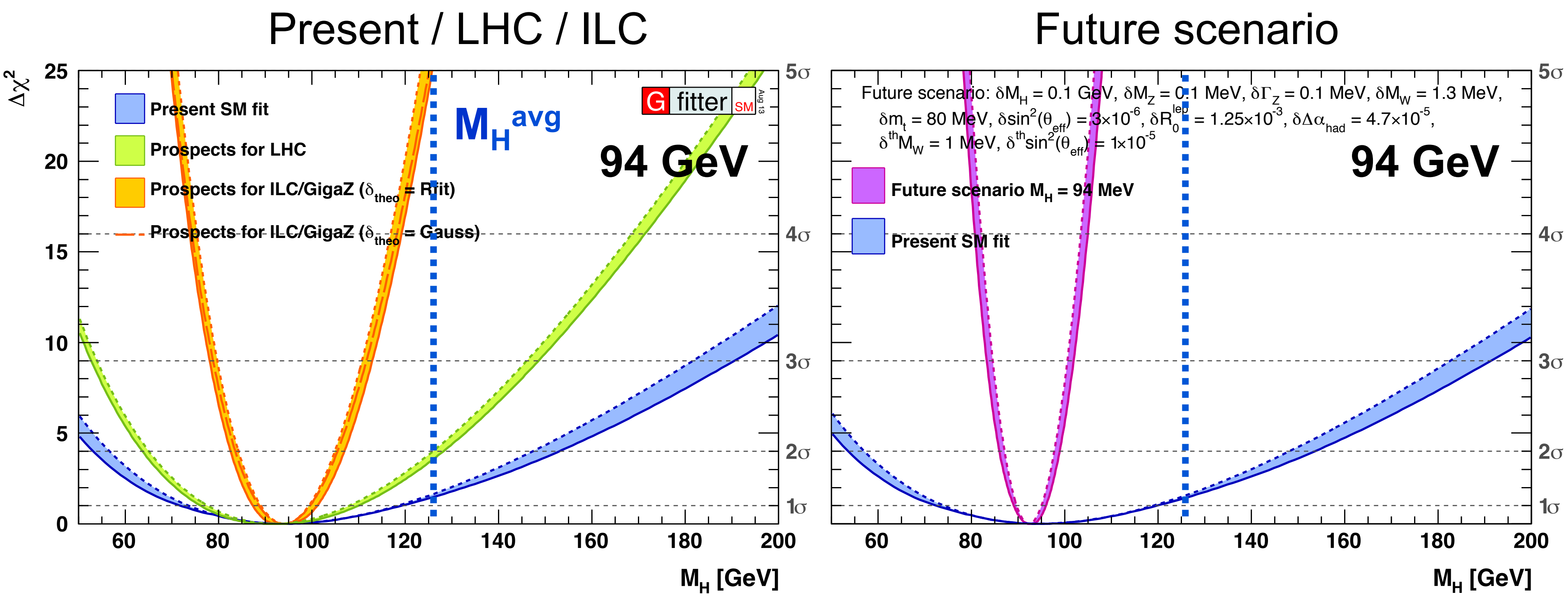


Future scenario



- Logarithmic dependency on M_H → cannot compete with direct M_H meas.
- Indirect prediction M_H dominated by theory uncertainties.
  - ILC with (without) theory errors: $M_H = 126^{+10}_{-9}$ (±7) GeV
  - ILC with present-day theory uncertainties: $M_H = 126^{+20}_{-17}$ GeV
  - TLEP with (without) theory errors: $M_H = 126 \pm 5$ (±3) GeV

Prospects of the EW fit: Higgs mass (94 GeV)

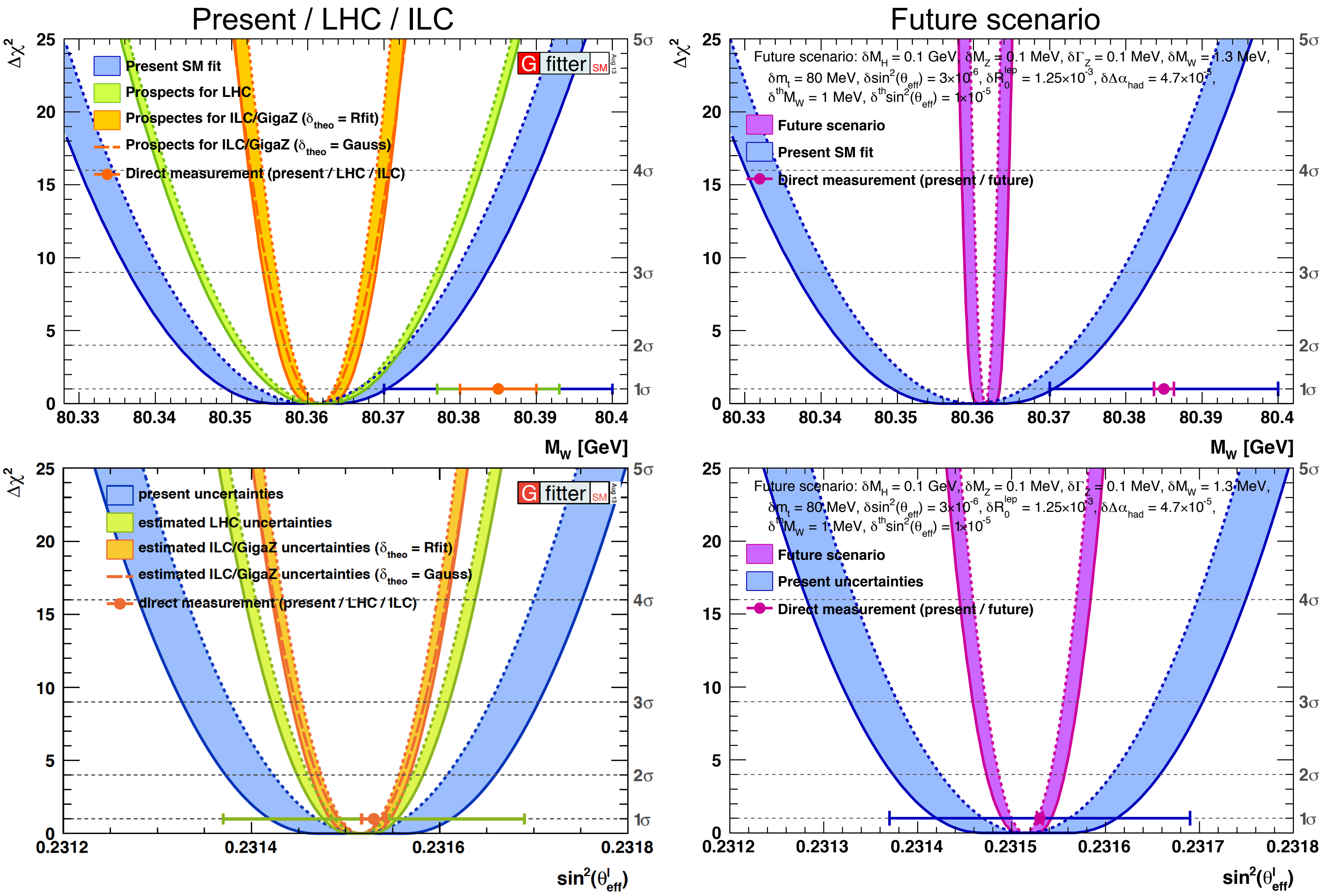


Present / LHC / ILC

Future scenario

- If EWP-data central values are unchanged, i.e. they keep favoring low value of Higgs mass (94 GeV), >5σ discrepancy with measured Higgs mass.
  - In both ILC and TLEP scenarios.

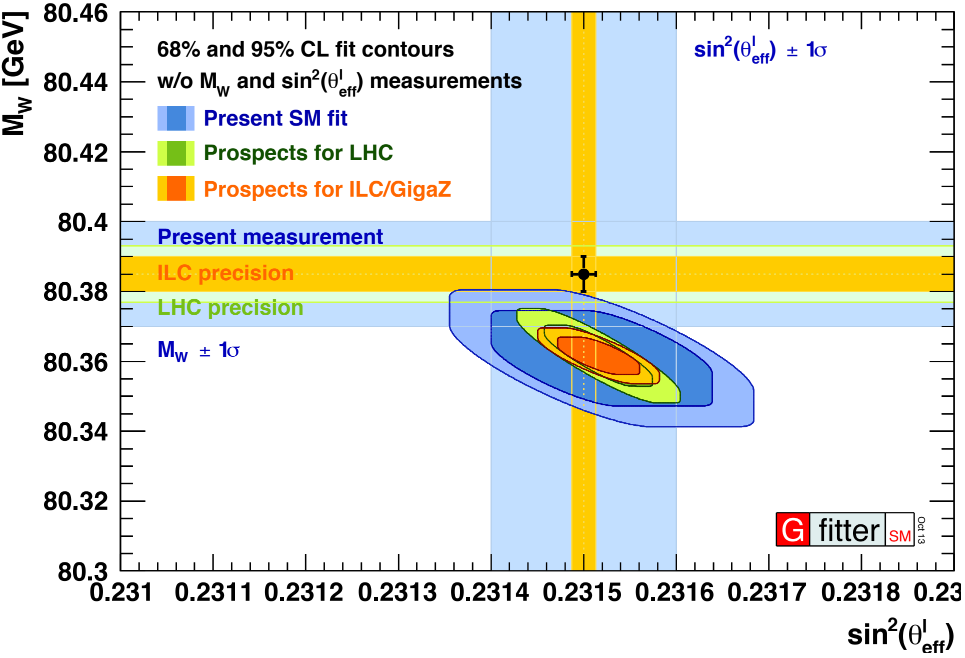
Prospects of the EW fit: W mass and $\sin^2\theta^l_{eff}$


Present / LHC / ILC

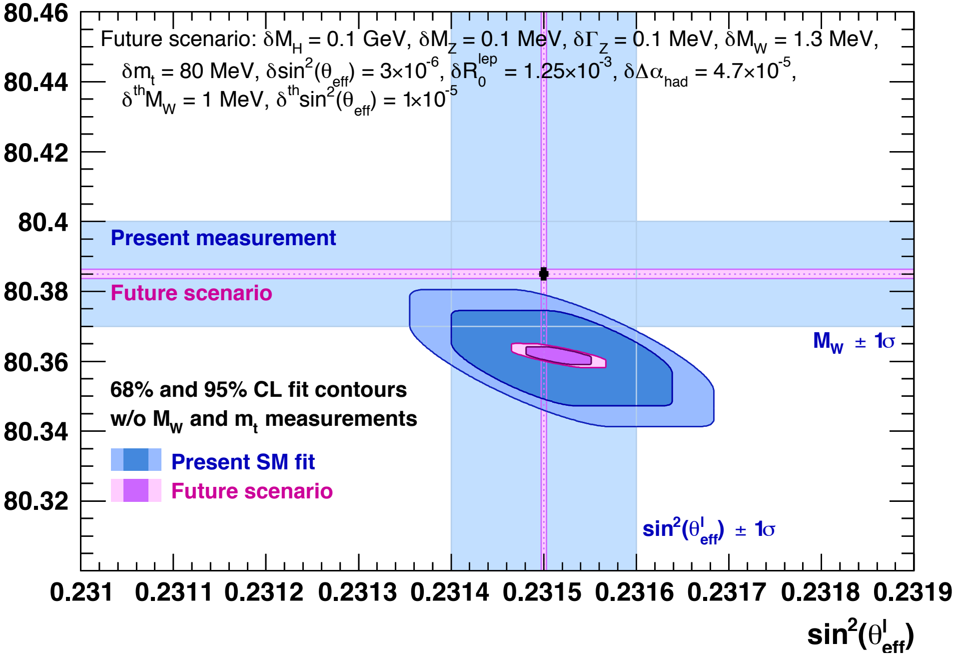
Future scenario

Prospects of the EW fit: W mass versus $\sin^2\theta^l_{eff}$

Present / LHC / ILC



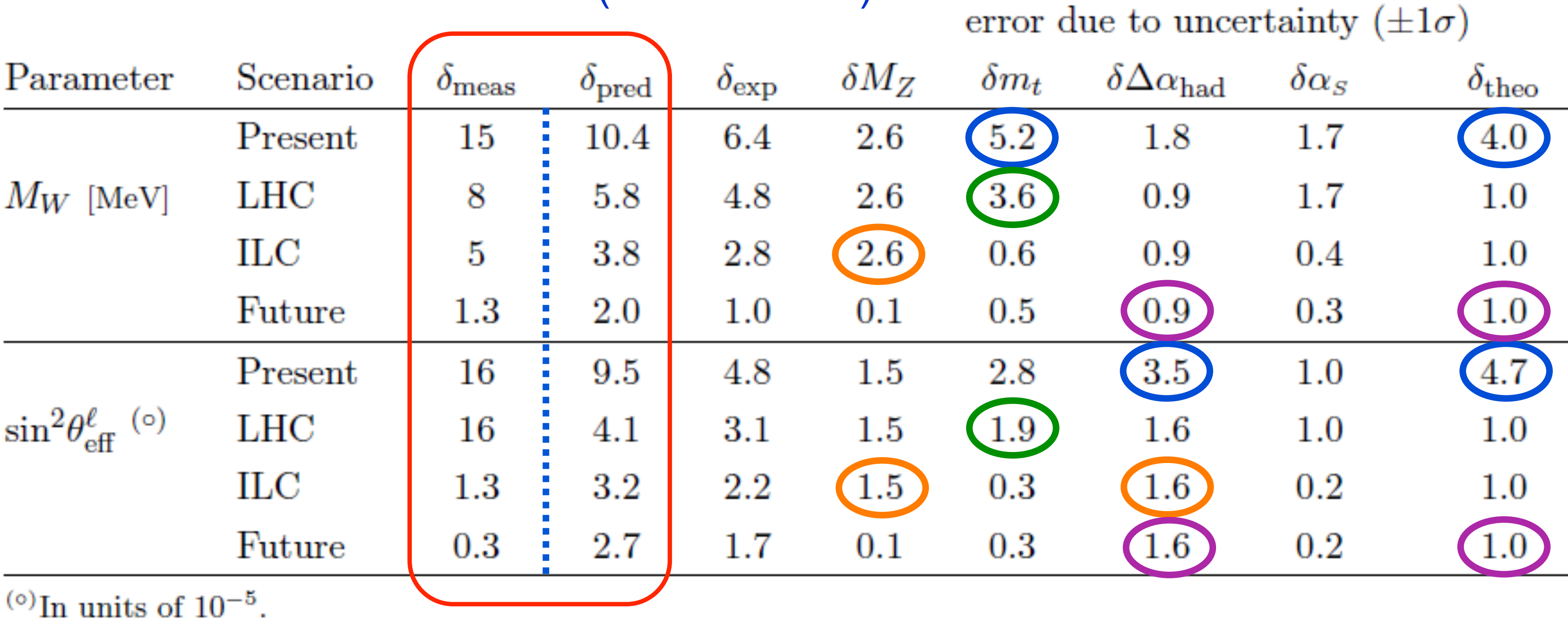
Future scenario



- Huge reduction of uncertainty on indirect determinations of m_W, and $\sin^2\theta^l_{eff}$, by a factor of ≳3 (≳4-5) at ILC (TLEP).
- Assuming central values of M_W and $\sin^2\theta^l_{eff}$ do not change, a deviation between the SM prediction and the direct measurements would be prominently visible, at both ILC and TLEP.
  - But also in LHC-300 scenario, from improved theory uncertainties.

Confrontation of measurement and prediction

- Breakdown of individual contributions to errors of M_W and $\sin^2\theta^l_{eff}$
- Parametric uncertainties (not the full fit).



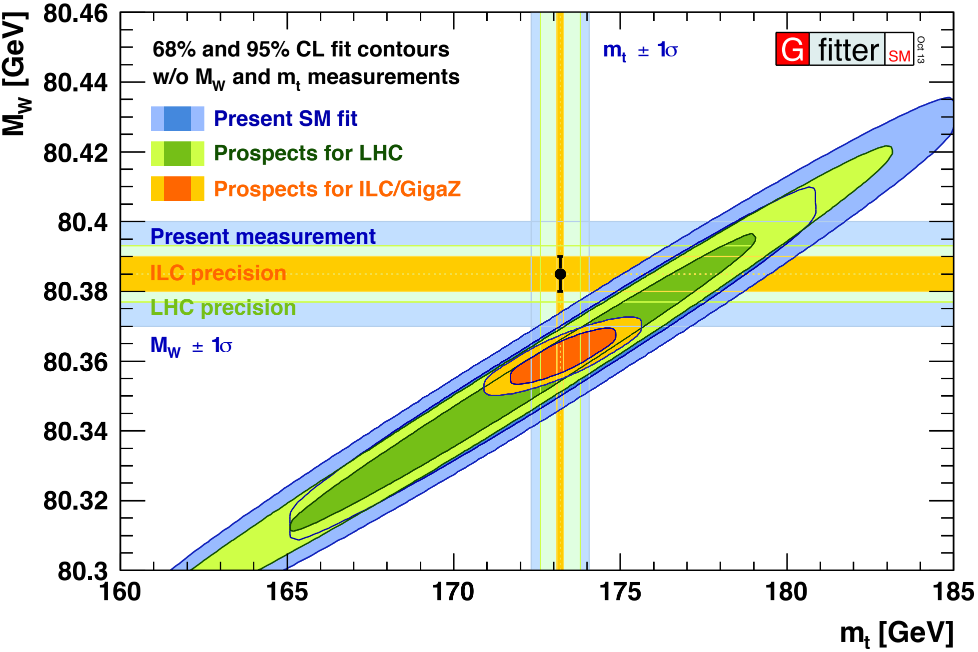
Parameter	Scenario	δ_{meas}	δ_{pred}	error due to uncertainty ($\pm 1\sigma$)					
				δ_{exp}	δM_Z	δm_t	$\delta\Delta\alpha_{had}$	$\delta\alpha_S$	δ_{theo}
M_W [MeV]	Present	15	10.4	6.4	2.6	5.2	1.8	1.7	4.0
	LHC	8	5.8	4.8	2.6	3.6	0.9	1.7	1.0
	ILC	5	3.8	2.8	2.6	0.6	0.9	0.4	1.0
	Future	1.3	2.0	1.0	0.1	0.5	0.9	0.3	1.0
$\sin^2\theta^\ell_{eff}$ $^{(\circ)}$	Present	16	9.5	4.8	1.5	2.8	3.5	1.0	4.7
	LHC	16	4.1	3.1	1.5	1.9	1.6	1.0	1.0
	ILC	1.3	3.2	2.2	1.5	0.3	1.6	0.2	1.0
	Future	0.3	2.7	1.7	0.1	0.3	1.6	0.2	1.0

$^{(\circ)}$In units of 10^{-5}.

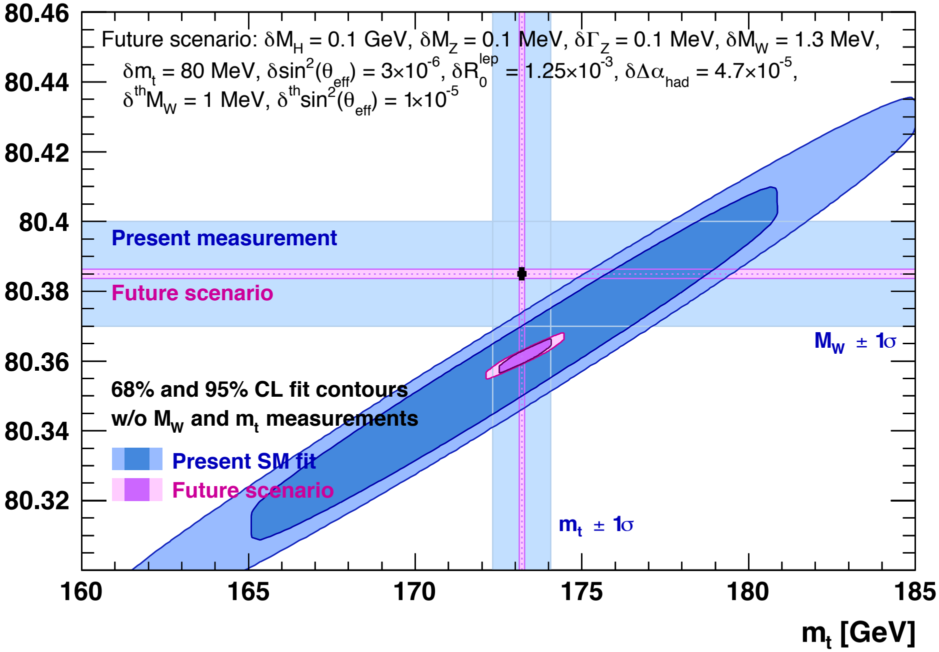
- M_W and $\sin^2\theta^l_{eff}$ are sensitive probes of new physics! In all scenarios.
- At ILC/GigaZ, precision of M_Z will become important again.
- At TLEP ('Future'), limited by external inputs: theory errors and $\Delta\alpha_{had}$

Prospects of the EW fit: W versus top mass

Present / LHC / ILC

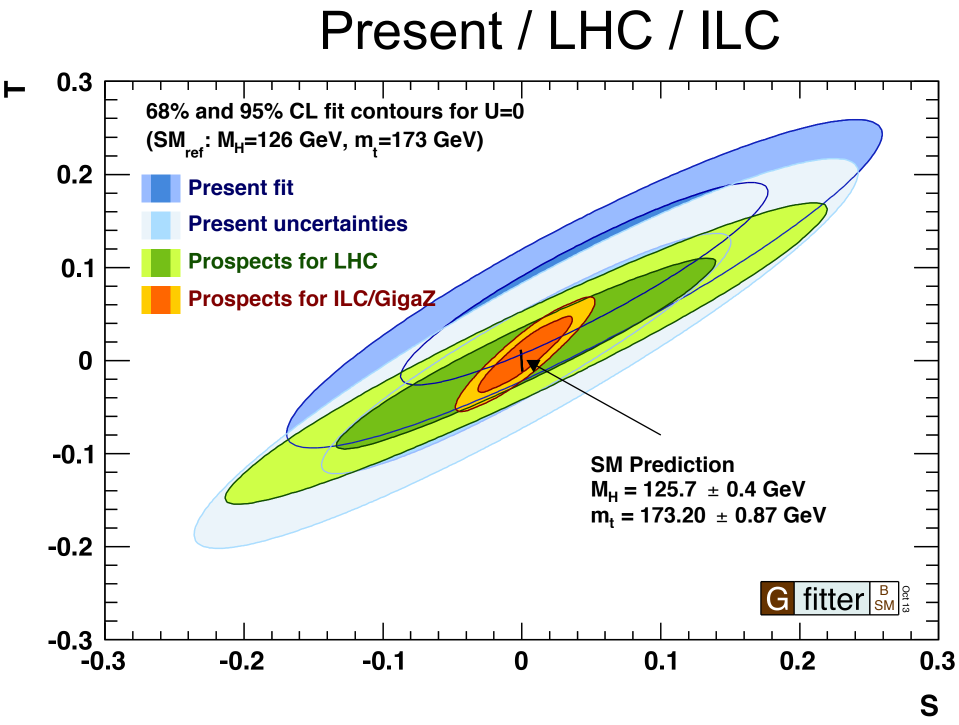


Future scenario

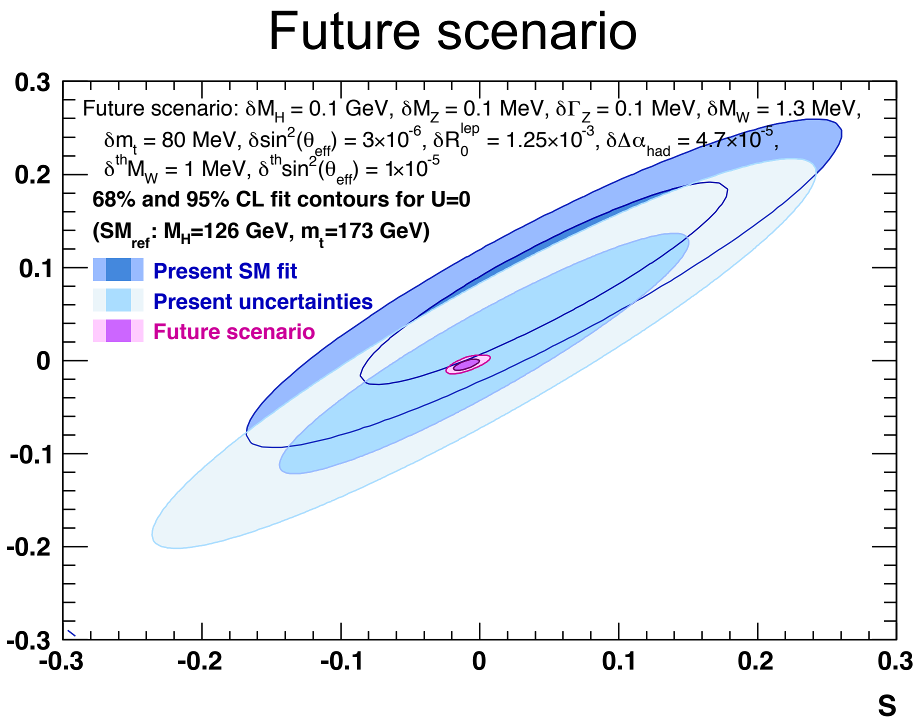


- Huge reduction of uncertainty on indirect determinations of m_t and m_W by a factor of ≳3 (≳5) at ILC (TLEP).
- Assuming central values of m_t and M_W do not change, a deviation between the SM prediction and the direct measurements would be prominently visible.

Prospects of EW fit: S versus T



Present / LHC / ILC

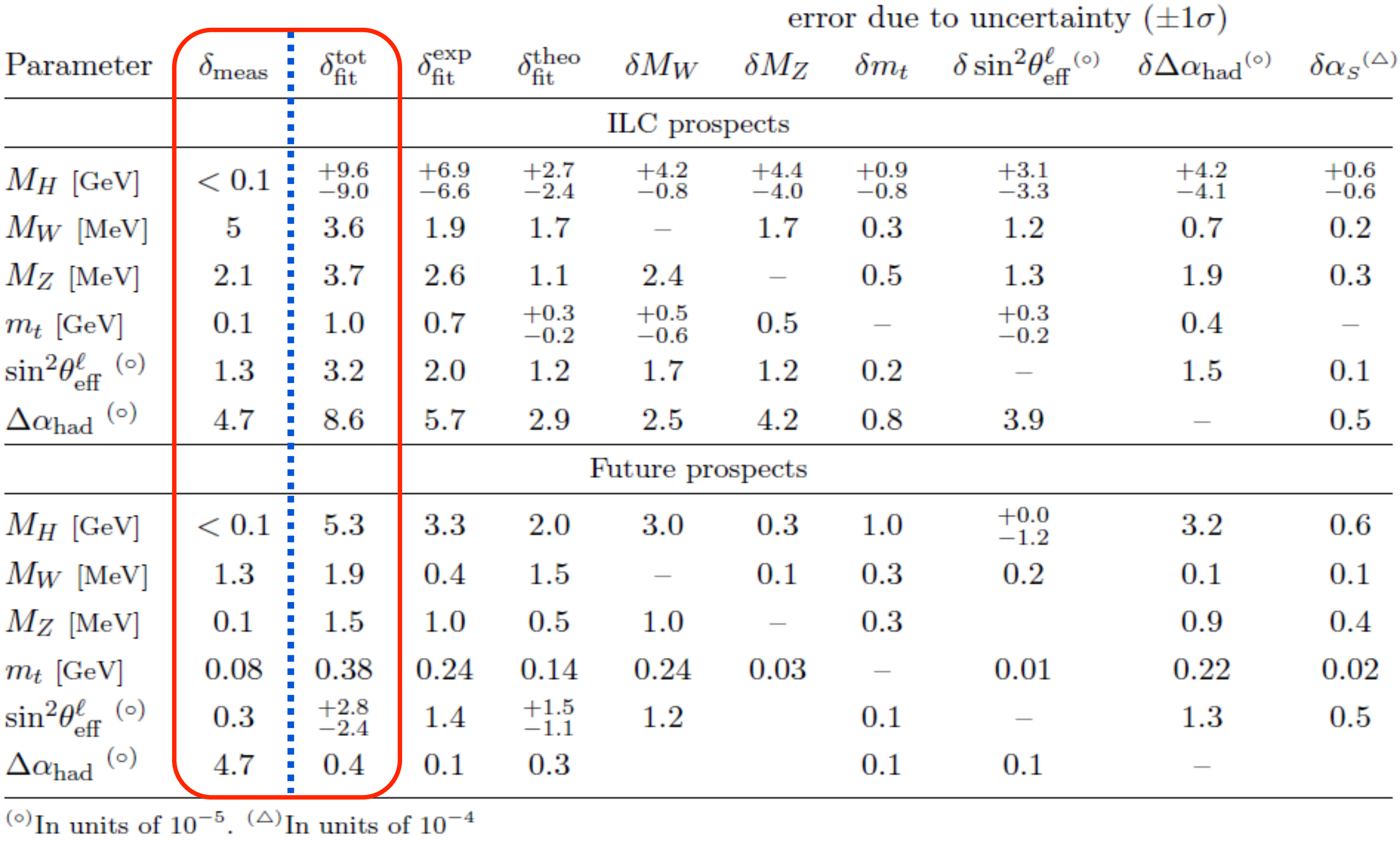


Future scenario

- For STU parameters, improvement of factor of ≳4 (≳10) is possible at ILC (TLEP).

- Again, at both ILC and TLEP a deviation between the SM predictions and direct measurements would be prominently visible.

Predicted uncertainties from EW fit



					error due to uncertainty ($\pm 1\sigma$)					
Parameter	δ_{meas}	$\delta_{\text{fit}}^{\text{tot}}$	$\delta_{\text{fit}}^{\text{exp}}$	$\delta_{\text{fit}}^{\text{theo}}$	δM_W	δM_Z	δm_t	$\delta \sin^2\theta^{\ell}_{\text{eff}}{}^{(\circ)}$	$\delta\Delta\alpha_{\text{had}}{}^{(\circ)}$	$\delta\alpha_S{}^{(\triangle)}$
					ILC prospects					
M_H [GeV]	< 0.1	$^{+9.6}_{-9.0}$	$^{+6.9}_{-6.6}$	$^{+2.7}_{-2.4}$	$^{+4.2}_{-0.8}$	$^{+4.4}_{-4.0}$	$^{+0.9}_{-0.8}$	$^{+3.1}_{-3.3}$	$^{+4.2}_{-4.1}$	$^{+0.6}_{-0.6}$
M_W [MeV]	5	3.6	1.9	1.7	–	1.7	0.3	1.2	0.7	0.2
M_Z [MeV]	2.1	3.7	2.6	1.1	2.4	–	0.5	1.3	1.9	0.3
m_t [GeV]	0.1	1.0	0.7	$^{+0.3}_{-0.2}$	$^{+0.5}_{-0.6}$	0.5	–	$^{+0.3}_{-0.2}$	0.4	–
$\sin^2\theta^{\ell}_{\text{eff}}$ $^{(\circ)}$	1.3	3.2	2.0	1.2	1.7	1.2	0.2	–	1.5	0.1
$\Delta\alpha_{\text{had}}$ $^{(\circ)}$	4.7	8.6	5.7	2.9	2.5	4.2	0.8	3.9	–	0.5
					Future prospects					
M_H [GeV]	< 0.1	5.3	3.3	2.0	3.0	0.3	1.0	$^{+0.0}_{-1.2}$	3.2	0.6
M_W [MeV]	1.3	1.9	0.4	1.5	–	0.1	0.3	0.2	0.1	0.1
M_Z [MeV]	0.1	1.5	1.0	0.5	1.0	–	0.3		0.9	0.4
m_t [GeV]	0.08	0.38	0.24	0.14	0.24	0.03	–	0.01	0.22	0.02
$\sin^2\theta^{\ell}_{\text{eff}}$ $^{(\circ)}$	0.3	$^{+2.8}_{-2.4}$	1.4	$^{+1.5}_{-1.1}$	1.2		0.1	–	1.3	0.5
$\Delta\alpha_{\text{had}}$ $^{(\circ)}$	4.7	0.4	0.1	0.3			0.1	0.1	–	

$^{(\circ)}$In units of 10^{-5}. $^{(\triangle)}$In units of 10^{-4}

- Breakdown of uncertainties derived from EW fit. (Note: *correlated* errors.)
- Compared to parametric breakdown: reduced experimental, but increased theory errors. Slightly smaller total errors.

Summary of predictions

	Experimental input [$\pm 1\sigma$]			Indirect determination [$\pm 1\sigma_{\rm exp} \pm 1\sigma_{\rm theo}$]		
Parameter	LHC	ILC/GigaZ	Future	LHC	ILC/GigaZ	Future
M_H [GeV]	< 0.1	< 0.1	< 0.1	$^{+20}_{-18}$ $^{+3}_{-2}$	$^{+6.9}_{-6.6}$ $^{+2.7}_{-2.4}$	± 3.3 ± 2.0
M_W [MeV]	8	5	1.3	± 4.8 ± 1.0	± 1.9 ± 1.7	± 0.4 ± 1.5
M_Z [MeV]	2.1	2.1	0.1	± 6.9 ± 0.8	± 2.6 ± 1.1	± 1.0 ± 0.5
m_t [GeV]	0.6	0.1	0.08	± 1.4 ± 0.2	± 0.7 $^{+0.3}_{-0.2}$	± 0.24 ± 0.14
Γ_Z [MeV]	2.3	0.8	0.1	± 6.9 ± 0.8	± 2.6 ± 1.1	± 0.4 ± 0.1
$\sin^2\theta^{\ell}_{\rm eff}$ [$\cdot 10^{-5}$]	16	1.3	0.3	± 2.7 ± 1.1	± 2.0 ± 1.2	± 1.4 $^{+1.5}_{-1.1}$
R^0_l [$\cdot 10^{-3}$]	25	4	1.3	–	–	–
$\Delta\alpha^5_{\rm had}(M_Z^2)$ [$\cdot 10^{-5}$]	4.7	4.7	4.7	± 36 ± 4	± 5.7 ± 2.9	± 0.1 ± 0.3
$\alpha_s(M_Z^2)$ [$\cdot 10^{-4}$]	–	–	–	± 27 ± 1	$^{+6.8}_{-6.3}$ $^{+0.3}_{-0.2}$	± 3.8 ± 0.1
$\delta_{\rm th} M_W$ [MeV]	1	1	1	–	–	–
$\delta_{\rm th} \sin^2\theta^{\ell}_{\rm eff}$ [$\cdot 10^{-5}$]	1	1	1	–	–	–
$S\vert_{U=0}$	–	–	–	± 0.09	± 0.02	< 0.01
$T\vert_{U=0}$	–	–	–	± 0.06	± 0.02	< 0.01

Outlook

- Including M_H measurement, M_W, $\sin^2\theta^l_{eff}$ have become sensitive probes of new physics.
- Prospects: including new data electroweak fits remain very interesting in coming years!
  - Significant increase in predictive power of the fit obtained in all three scenarios studied.

- *ILC/GigaZ and TLEP provide excellent new physics sensitivity.*

- Assuming good control over systematic effects, predictions for M_W, $\sin^2\theta^l_{eff}$, STU are improved with a factor of ≳3 (≳5) at ILC (TLEP).
- Predicted uncertainties on M_W, $\sin^2\theta^l_{eff}$ dominated by:
  - ILC: δM_Z
  - TLEP: external inputs: δ(theory), $\delta\Delta\alpha_{had}$

Thanks!

Backup

G fitter

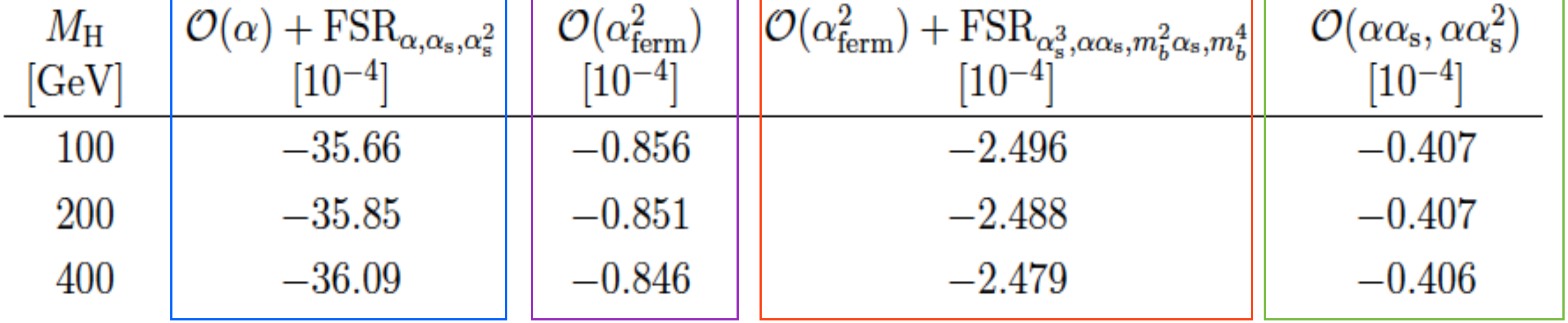
A Generic Fitter Project for HEP Model Testing

Backup

New R^0_b calculation

[A. Freitas et al., JHEP 1208, 050 (2012)]

- The branching ratio R^0_b: partial decay width of Z→bb to Z→qq
- Freitas et al: full EW 2-loop calculation of Z→bb
- Contribution of same terms as in the calculation of $\sin^2\theta^{bb}_{eff}$ → cross-check of two results found good agreement
- Two-loop EW corrections now much smaller than experimental uncertainty (6.6×10^{-4})



M_H [GeV]	1-loop EW and QCD correction to FSR $\mathcal{O}(\alpha)+\mathrm{FSR}_{\alpha,\alpha_s,\alpha_s^2}$ [10^{-4}]	2-loop EW correction $\mathcal{O}(\alpha^2_{\mathrm{ferm}})$ [10^{-4}]	2-loop EW and 2+3-loop QCD correction to FSR $\mathcal{O}(\alpha^2_{\mathrm{ferm}})+\mathrm{FSR}_{\alpha_s^3,\alpha\alpha_s,m_b^2\alpha_s,m_b^4}$ [10^{-4}]	1+2-loop QCD correction to gauge boson self-energies $\mathcal{O}(\alpha\alpha_s,\alpha\alpha_s^2)$ [10^{-4}]
100	−35.66	−0.856	−2.496	−0.407
200	−35.85	−0.851	−2.488	−0.407
400	−36.09	−0.846	−2.479	−0.406

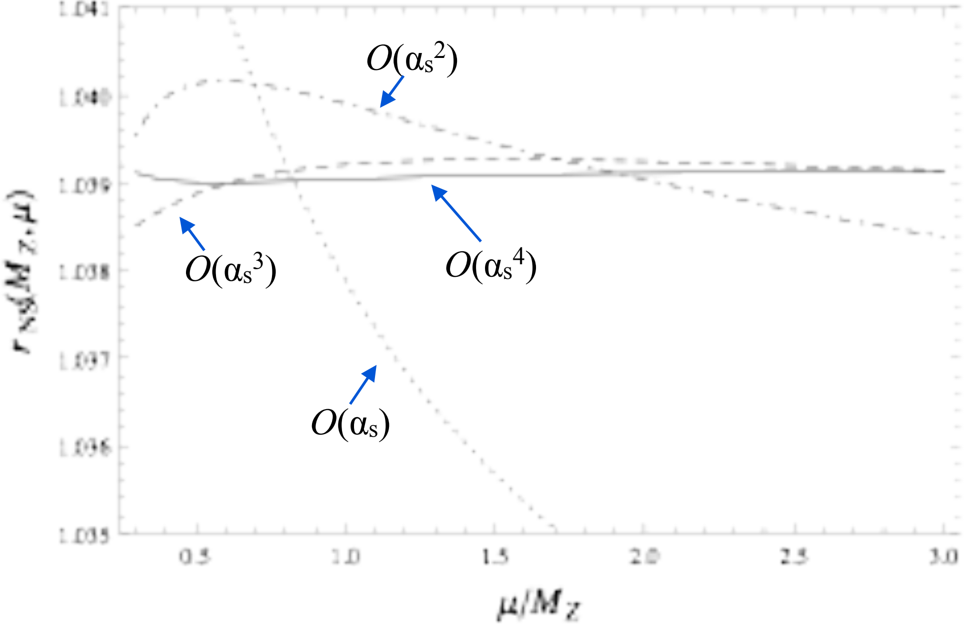
Radiator Functions

- Partial widths are defined inclusively: contain both QCD and QED contributions.
- Corrections expressed as so-called radiator functions $R_{A,f}$ and $R_{V,f}$

$$\Gamma_{f\bar{f}} = N_c^f \frac{G_F M_Z^3}{6\sqrt{2}\pi} \left(|g_{A,f}|^2 R_{A,f} + |g_{V,f}|^2 R_{V,f} \right)^2$$

[D. Bardin, G. Passarino, “The Standard Model in the Making”, Clarendon Press (1999)]

- High sensitivity to the strong coupling α_s
- Recently, full four-loop calculation of QCD Adler function became available (N^3LO)
- Much-reduced scale dependence!
- Theoretical uncertainty of 0.1 MeV, compared with experimental uncertainty of 2.0 MeV.

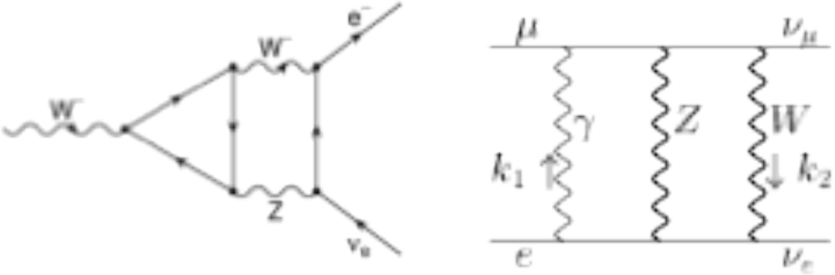


[P. Baikov et al., Phys. Rev. Lett. 108, 222003 (2012)]
[P. Baikov et al Phys. Rev. Lett. 104, 132004 (2010)]

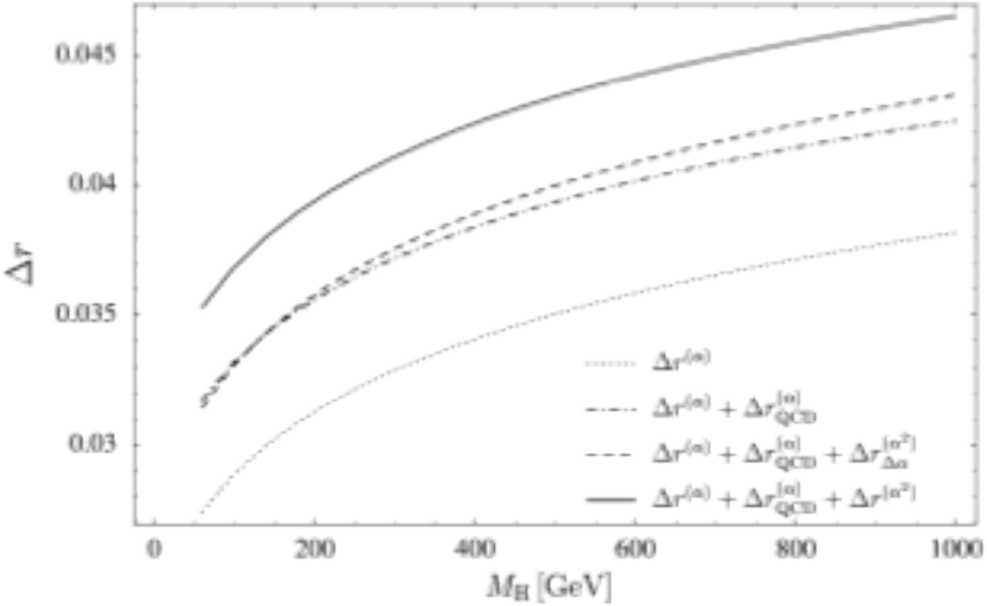
Calculation of M_W

- Full EW one- and two-loop calculation of fermionic and bosonic contributions.
- One- and two-loop QCD corrections and leading terms of higher order corrections.
- Results for Δr include terms of order $O(\alpha)$, $O(\alpha\alpha_s)$, $O(\alpha\alpha_s^2)$, $O(\alpha^2_{ferm})$, $O(\alpha^2_{bos})$, $O(\alpha^2\alpha_s m_t^4)$, $O(\alpha^3 m_t^6)$
- Uncertainty estimate:
  - Missing terms of order $\mathbf{O(\alpha^2\alpha_s)}$: about 3 MeV (from $O(\alpha^2\alpha_s m_t^4)$)
  - Electroweak three-loop correction $\mathbf{O(\alpha^3)}$: < 2 MeV
  - Three-loop QCD corrections $\mathbf{O(\alpha_s^3)}$: < 2 MeV
- Total: $\delta M_W \approx 4$ MeV

[M Awramik et al., Phys. Rev. D69, 053006 (2004)]
[M Awramik et al., Phys. Rev. Lett. 89, 241801 (2002)]



[A Freitas et al., Phys. Lett. B495, 338 (2000)]



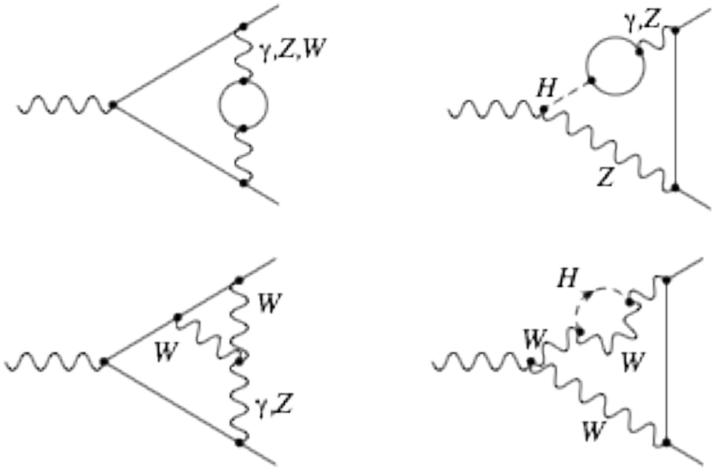
Calculation of $\sin^2(\theta^l_{eff})$

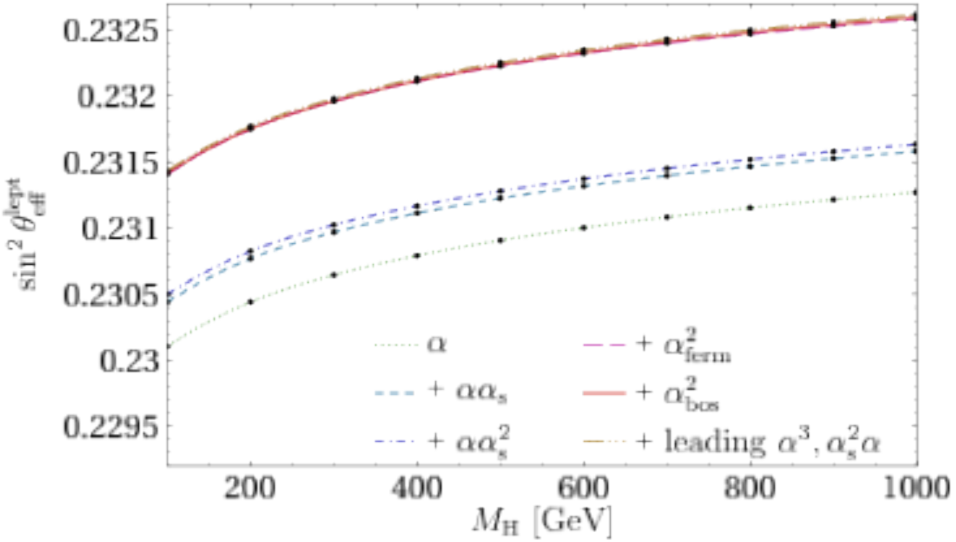
[M Awramik et al, Phys. Rev. Lett. 93, 201805 (2004)]
[M Awramik et al., JHEP 11, 048 (2006)]

- Effective mixing angle:

$$\sin^2\theta^{\rm lept}_{\rm eff} = \left(1 - M_W^2/M_Z^2\right)(1+\Delta\kappa)$$

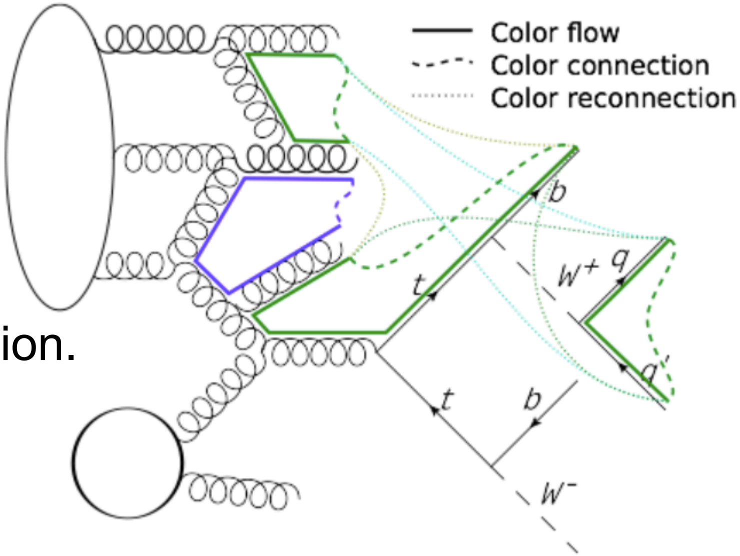
- Two-loop EW and QCD correction to $\Delta\kappa$ known, leading terms of higher order QCD corrections.
- Fermionic two-loop correction about 10^{-3}, whereas bosonic one 10^{-5}.
- Uncertainty estimate obtained with different methods, geometric progression, leading to total of: $\delta\sin^2(\theta^l_{eff}) = 4.7\text{x}10^{-5}$





Uncertainty in Top mass definition

- Difficult to define a pole mass for heavy, unstable and colored particle.
  - Single top decays before hadronizing. To have colorless final states, additional quarks needed.
  - *Non-perturb.* color-reconnection effects in fragmentation → biases in simulation.
  - 'Renormalon' ambiguity in top mass definition.
    - For pole mass, not for MS-bar scheme.
  - Impact of finite top width effects.

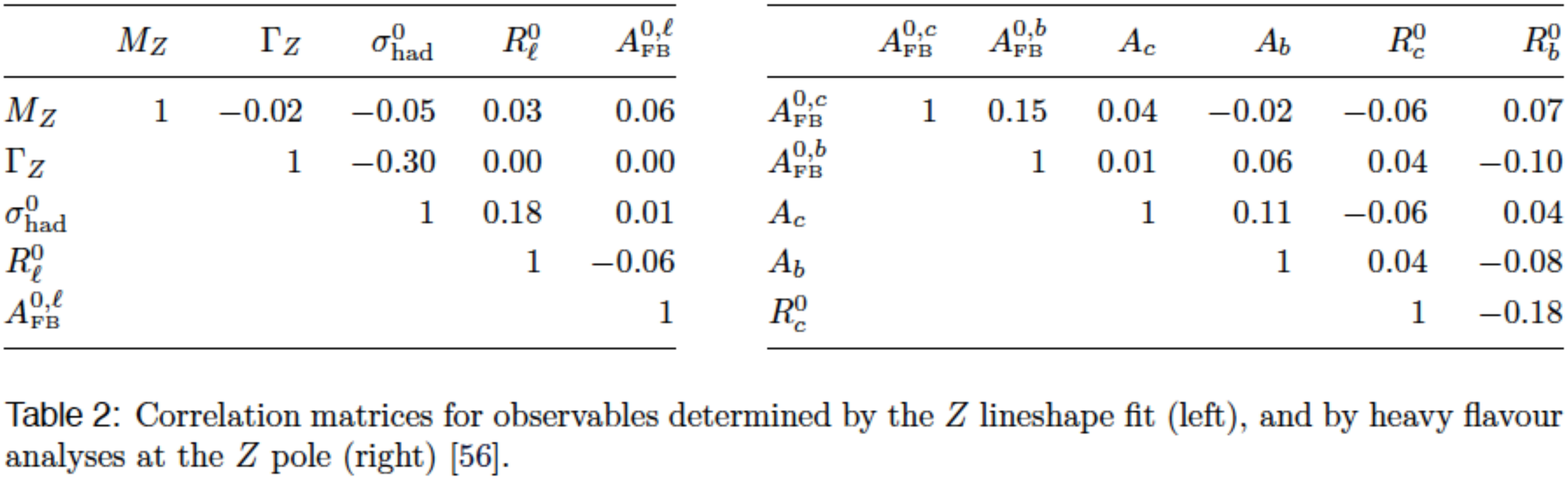


- Result: $m_t^{exp} \neq m_t^{pole}$, and event-dependent.
- The top mass extracted in hadron collisions is not well defined below a precision of $O(\Gamma_t)$ ~ 1 GeV

- Hard to estimate additional theo. uncertainties. With 0.5 GeV on m_t:
  - $M_H = 90^{+34}_{-21}$ GeV, $M_W = 80.359 \pm 0.013$ GeV, $\sin^2\theta^l_{eff} = 0.23148 \pm 0.00010$.
  - Only small deterioration in precision.

Input correlations of the EW fit

- Input correlation coefficients between Z pole measurements



	M_Z	Γ_Z	σ^0_{had}	R^0_ℓ	$A^{0,\ell}_{\mathrm{FB}}$
M_Z	1	-0.02	-0.05	0.03	0.06
Γ_Z		1	-0.30	0.00	0.00
σ^0_{had}			1	0.18	0.01
R^0_ℓ				1	-0.06
$A^{0,\ell}_{\mathrm{FB}}$					1

	$A^{0,c}_{\mathrm{FB}}$	$A^{0,b}_{\mathrm{FB}}$	A_c	A_b	R^0_c	R^0_b
$A^{0,c}_{\mathrm{FB}}$	1	0.15	0.04	-0.02	-0.06	0.07
$A^{0,b}_{\mathrm{FB}}$		1	0.01	0.06	0.04	-0.10
A_c			1	0.11	-0.06	0.04
A_b				1	0.04	-0.08
R^0_c					1	-0.18

Table 2: Correlation matrices for observables determined by the Z lineshape fit (left), and by heavy flavour analyses at the Z pole (right) [56].

Measurements at the Z-pole (1/2)

- Total cross-section of Z→f$\bar{f}$
  - Expressed in terms of partial decay width of initial and final width:

$$\boxed{\sigma_{f\bar{f}}^{Z} = \sigma_{f\bar{f}}^{0} \frac{s\Gamma_Z^2}{(s-M_Z^2)^2 + s^2\Gamma_Z^2/M_Z^2} \frac{1}{R_{\mathrm{QED}}} \quad \text{with} \quad \sigma_{f\bar{f}}^{0} = \frac{12\pi}{M_Z^2}\frac{\Gamma_{ee}\Gamma_{f\bar{f}}}{\Gamma_Z^2}}$$

Corrected for QED radiation

  - Full width: $\Gamma_Z = \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + \Gamma_{\mathrm{had}} + \Gamma_{\mathrm{inv}}$
  - (Correlated set of measurements.)

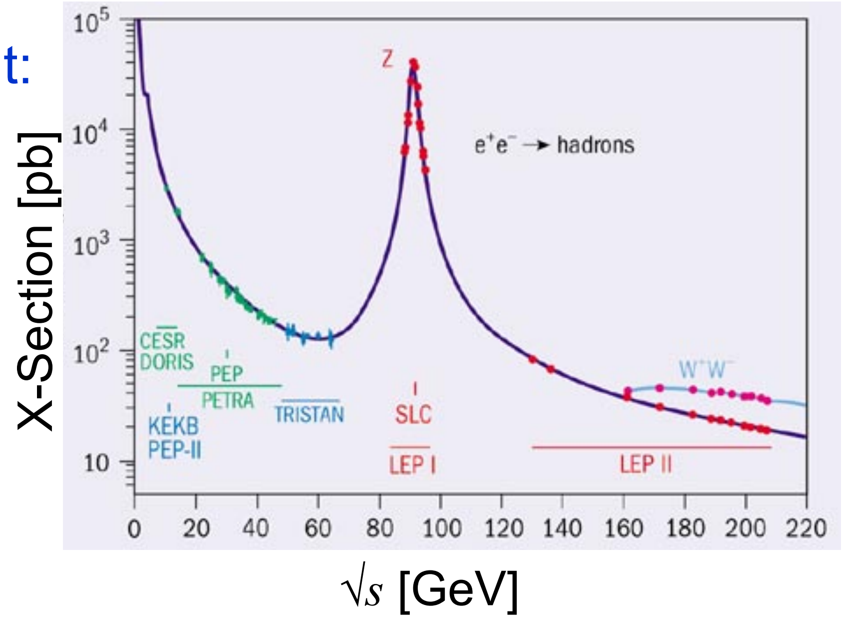
- Set of input (width) parameters to EW fit:
  - Z mass and width: $\boxed{M_Z}$, $\boxed{\Gamma_Z}$
  - Hadronic pole cross section:

$$\boxed{\sigma_{\mathrm{had}}^{0}} = 12\pi/M_Z^2 \cdot \Gamma_{ee}\Gamma_{\mathrm{had}}/\Gamma_Z^2$$

  - Three leptonic ratios (lepton univ.):

$$\boxed{R_\ell^0} = R_e^0 = \Gamma_{\mathrm{had}}/\Gamma_{ee}\ (= R_\mu^0 = R_\tau^0)$$

  - Hadronic-width ratios: $\boxed{R_b^0}$, $\boxed{R_c^0}$



Measurements at the Z-pole (2/2)

Definition of Asymmetry

- Distinguish vector and axial-vector couplings of the Z

$$A_f = \frac{g_{L,f}^2 - g_{R,f}^2}{g_{L,f}^2 + g_{R,f}^2} = \frac{2 g_{V,f}\, g_{A,f}}{g_{V,f}^2 + g_{A,f}^2}$$

- Directly related to: $\sin^2\theta_{\mathrm{eff}}^{f\bar{f}} = \frac{1}{4Q_f}\left(1 + \mathcal{R}e\left(\frac{g_{V,f}}{g_{A,f}}\right)\right)$

Observables

- In case of no beam polarisation (LEP) use final state angular distribution to define *forward/backward asymmetry:*

$$A_{FB}^f = \frac{N_F^f - N_B^f}{N_F^f + N_B^f} \qquad A_{FB}^{0,f} = \frac{3}{4} A_e A_f$$

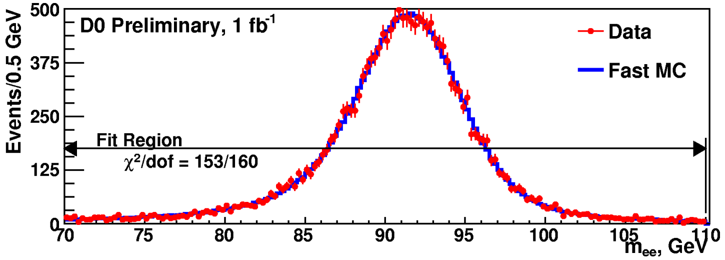
- Polarised beams (SLC), define *left/right asymmetry:*

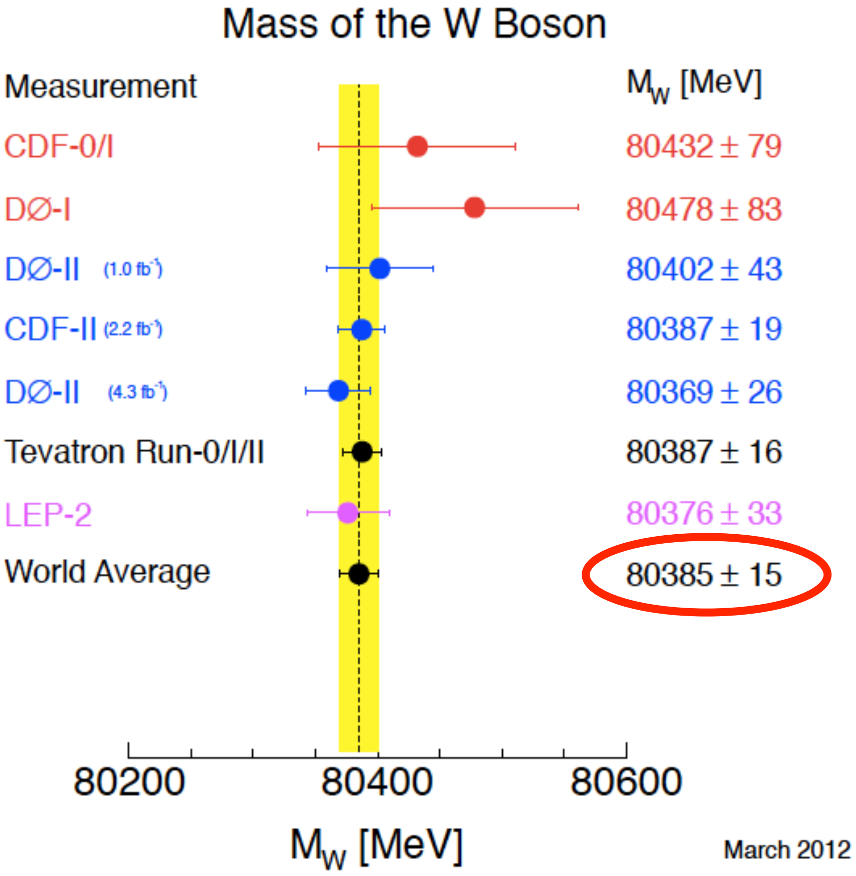
$$A_{LR}^f = \frac{N_L^f - N_R^f}{N_L^f + N_R^f} \frac{1}{\langle |P|_e \rangle} \qquad A_{LR}^0 = A_e$$

- Measurements: $\boxed{A_{FB}^{0,\ell}}$ $\boxed{A_{FB}^{0,c}}$ $\boxed{A_{FB}^{0,b}}$ $\boxed{A_\ell}$ $\boxed{A_c}$ $\boxed{A_b}$

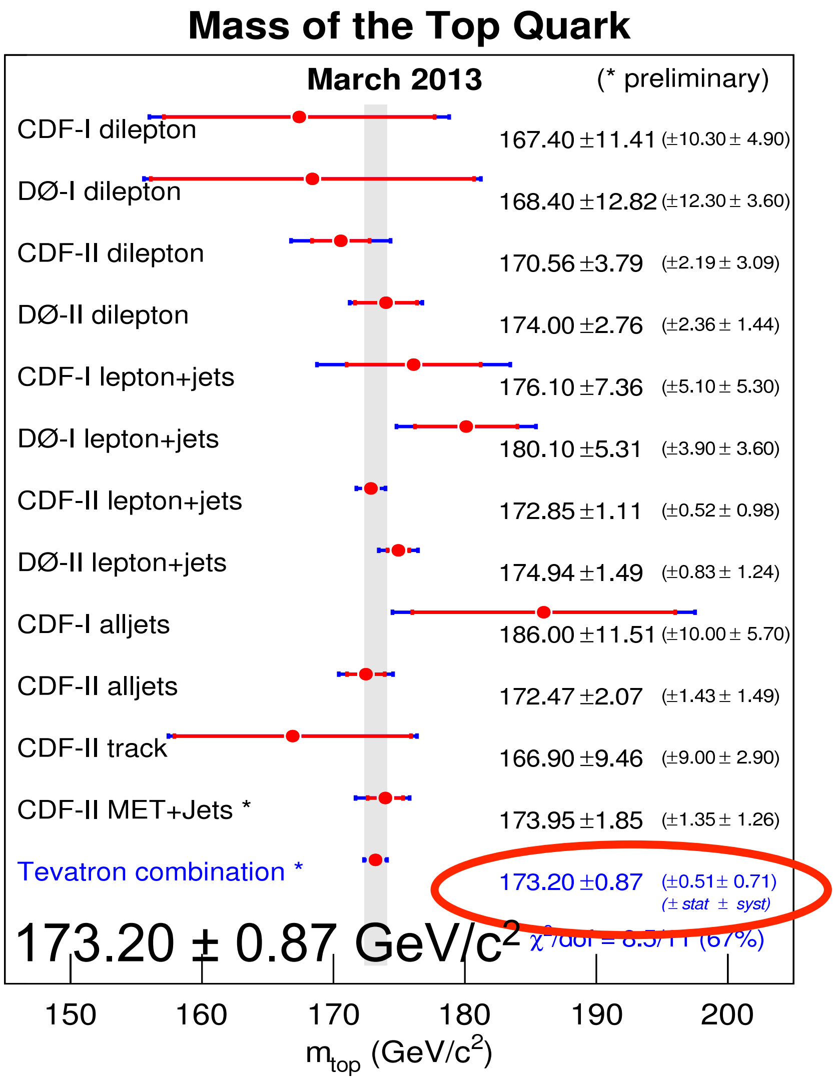
Latest averages for M_W and m_{top}

Latest Tevatron result from: arXiv:1204.0042





Tevatron result from: arXiv:1305.3929



(LHC average: 173.29 ± 0.95 GeV/c^2)

The electromagnetic coupling

- The EW fit requires precise knowledge of α(M_Z) – better than 1% level
  - Enters various places: hadr. radiator functions, predictions of M_W and $\sin^2\theta^f_{eff}$
- Conventionally parametrized as (α(0) = fine structure constant) :

$$\alpha(s) = \frac{\alpha(0)}{1 - \Delta\alpha(s)}$$

- Evolution with renormalization scale:

$$\Delta\alpha(s) = \Delta\alpha_{\rm lep}(s) + \Delta\alpha^{(5)}_{\rm had}(s) + \Delta\alpha_{\rm top}(s)$$

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- Leptonic term known up to *four* loops (for $q^2 \gg m_l^2$)
- Top quark contribution known up to 2 loops, *small: -0.7×10^{-4}*

[C.Sturm, arXiv: 1305.0581]

[M. Steinhauser, PLB 429, 158 (1998)]

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- Hadronic contribution (from the 5 light quarks) completely dominates overall uncertainty on $\alpha(M_Z)$.

- Difficult to calculate, cannot be obtained from pQCD alone.
  - Analysis of low-energy e^+e^- data
  - Usage of pQCD if lack of data

$$\boxed{\Delta\alpha^{(5)}_{had}(M_Z) = (274.9 \pm 1.0) \cdot 10^{-4}}$$

- Similar analysis to evaluation of hadronic contribution to $(g-2)_\mu$

[M. Davier et al., Eur. Phys. J. C71, 1515 (2011)]

Theoretical inputs

- Radiative corrections are important!
  - E.g. consider tree-level EW unification relation:
    - This predicts: $M_W = (79.964 \pm 0.005)$ GeV
    - Experiment: $M_W = (80.385 \pm 0.015)$ GeV

$$M_W^2\Big|_{\text{tree-level}} = \frac{M_Z^2}{2} \cdot \left(1 + \sqrt{1 - \frac{\sqrt{8}\pi\alpha}{G_F M_Z^2}}\right)$$

- Without loop corrections: shift of 400 MeV, 27σ discrepancy!

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- In EW fit with Gfitter we use state-of-the-art calculations:
  - M_W Mass of the W boson [M. Awramik et al., Phys. Rev. D69, 053006 (2004)]
  - $\sin^2\theta^f_{\text{eff}}$ Effective weak mixing angle [M. Awramik et al., JHEP 11, 048 (2006), M. Awramik et al., Nucl.Phys.B813:174-187 (2009)]
    - Full two-loop + leading beyond-two-loop form factor corrections
  - Γ_{had} QCD Adler functions at N^3LO [P. A. Baikov et al., PRL108, 222003 (2012)]
    - N^3LO prediction of the hadronic cross section
  - R_b Partial width of Z→bb [Freitas et al., JHEP08, 050 (2012)] ⟵ **Update! EW 2-loop calc.**

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  - R_b Partial width of Z→bb [Freitas et al., JHEP08, 050 (2012)] ← **Update! EW 2-loop calc.**
- Two nuisance parameters in EW fit for theoretical uncertainties:
  - *δM_W (4 MeV), $\delta\sin^2\theta^l_{\text{eff}}$ ($4.7\text{x}10^{-5}$)*

Updated calculation of R_b

- The branching ratio R^0_b = partial decay width of Z→bb to Z→qq
- We use calculation with full EW 2-loop corrections of Z→bb
  - From A. Freitas etal, JHEP 1208 (2012) 050, Erratum. 1305 (2013) 074.

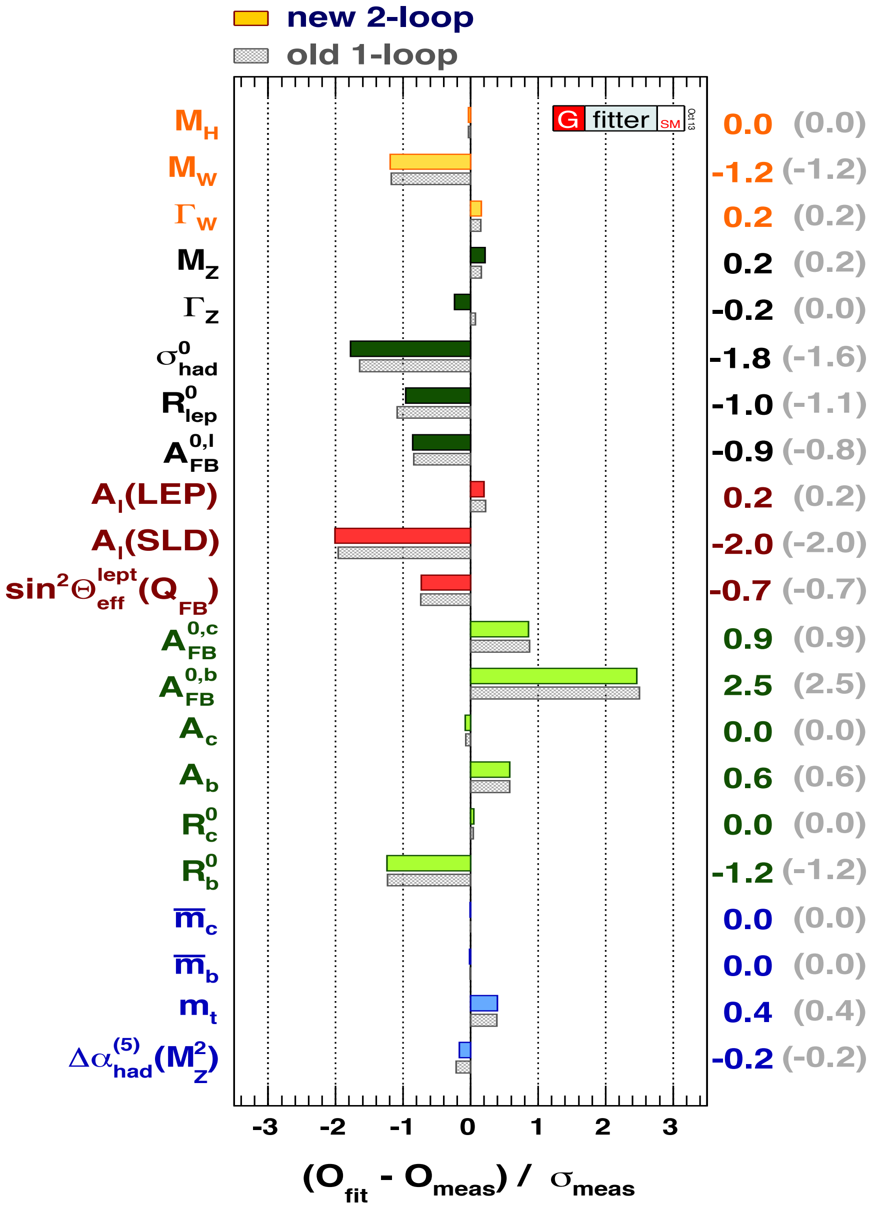
Recently a mistake was found in this calculation.

- ***Old:*** Two-loop corrections to R^0_b comparable to experimental uncertainty (6.6×10^{-4})
  - Moved theoretical prediction by 1.5σ
    - Much more than the originally estimated theory uncertainty!
- ***New:*** bug in calculation of R^0_b has been corrected, resulting in a sizable reduction of the size of the two-loop correction.

- All results shown here and on Gfitter homepage use the corrected R^0_b calculation.

Full EW 2-loop calculations

- Recent paper by A. Freitas, arXiv:1310.2256.
- Contains full two-loop fermionic EW corrections to the Z-boson width and production rate.
- Only small impact on EW fit results compared with 1-loop results.



Plot inspired by Eberhardt et al. [arXiv:1209.1101]

Electroweak Fit – SM Fit Results

- From the Gfitter Group, EPJC 72, 2205 (2012)
- Left: full fit incl. M_H
- Middle: not incl. M_H
- Right: fit incl M_H, not the row

Parameter	Input value	Free in fit	Fit Result	Fit without M_H measurements	Fit without exp. input in line
M_H [GeV]$^{\circ}$	$125.7^{+0.4}_{-0.4}$	yes	$125.7^{+0.4}_{-0.4}$	94.7^{+25}_{-22}	94.7^{+25}_{-22}
M_W [GeV]	80.385 ± 0.015	–	$80.367^{+0.006}_{-0.007}$	$80.367^{+0.006}_{-0.007}$	80.360 ± 0.011
Γ_W [GeV]	2.085 ± 0.042	–	2.091 ± 0.001	2.091 ± 0.001	2.091 ± 0.001
M_Z [GeV]	91.1875 ± 0.0021	yes	91.1878 ± 0.0021	91.1878 ± 0.0021	91.1978 ± 0.0114
Γ_Z [GeV]	2.4952 ± 0.0023	–	2.4954 ± 0.0014	2.4954 ± 0.0014	2.4950 ± 0.0017
σ^0_{had} [nb]	41.540 ± 0.037	–	41.479 ± 0.014	41.479 ± 0.014	41.471 ± 0.015
R^0_ℓ	20.767 ± 0.025	–	20.740 ± 0.017	20.740 ± 0.017	20.715 ± 0.026
$A^{0,\ell}_{\text{FB}}$	0.0171 ± 0.0010	–	$0.01626^{+0.0001}_{-0.0002}$	$0.01626^{+0.0001}_{-0.0002}$	0.01624 ± 0.0002
A_ℓ $^{(\star)}$	0.1499 ± 0.0018	–	0.1472 ± 0.0007	0.1472 ± 0.0007	–
$\sin^2\theta^\ell_{\text{eff}}(Q_{\text{FB}})$	0.2324 ± 0.0012	–	$0.23149^{+0.00010}_{-0.00008}$	$0.23149^{+0.00010}_{-0.00008}$	0.23150 ± 0.00009
A_c	0.670 ± 0.027	–	$0.6679^{+0.00034}_{-0.00028}$	$0.6679^{+0.00034}_{-0.00028}$	0.6680 ± 0.00031
A_b	0.923 ± 0.020	–	$0.93464^{+0.00005}_{-0.00007}$	$0.93464^{+0.00005}_{-0.00007}$	0.93463 ± 0.00006
$A^{0,c}_{\text{FB}}$	0.0707 ± 0.0035	–	0.0738 ± 0.0004	0.0738 ± 0.0004	0.0737 ± 0.0004
$A^{0,b}_{\text{FB}}$	0.0992 ± 0.0016	–	0.1032 ± 0.0005	0.1032 ± 0.0005	0.1034 ± 0.0003
R^0_c	0.1721 ± 0.0030	–	0.17223 ± 0.00006	0.17223 ± 0.00006	0.17223 ± 0.00006
R^0_b	0.21629 ± 0.00066	–	0.21548 ± 0.00005	0.21548 ± 0.00005	0.21547 ± 0.00005
$\overline{m}_c$ [GeV]	$1.27^{+0.07}_{-0.11}$	yes	$1.27^{+0.07}_{-0.11}$	$1.27^{+0.07}_{-0.11}$	–
$\overline{m}_b$ [GeV]	$4.20^{+0.17}_{-0.07}$	yes	$4.20^{+0.17}_{-0.07}$	$4.20^{+0.17}_{-0.07}$	–
m_t [GeV]	173.20 ± 0.87	yes	173.53 ± 0.82	173.53 ± 0.82	$176.11^{+2.88}_{-2.35}$
$\Delta\alpha^{(5)}_{\text{had}}(M^2_Z)$ $^{(\dagger\triangle)}$	2757 ± 10	yes	2755 ± 11	2755 ± 11	2718^{+49}_{-43}
$\alpha_s(M^2_Z)$	–	yes	$0.1190^{+0.0028}_{-0.0027}$	$0.1190^{+0.0028}_{-0.0027}$	0.1190 ± 0.0027
$\delta_{\text{th}} M_W$ [MeV]	$[-4, 4]_{\text{theo}}$	yes	4	4	–
$\delta_{\text{th}} \sin^2\theta^\ell_{\text{eff}}$ $^{(\dagger)}$	$[-4.7, 4.7]_{\text{theo}}$	yes	-0.6	-0.5	–