The $g_2$ Spin Structure Function

Melissa Cummings
The College of William and Mary
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Inclusive Electron Scattering

$N(e,e')X$

$p = (E,k)$
$q = (\nu,q)$

$p' = (E',k')$

$q = \bar{q} = \bar{k} - \bar{k}'$

$Q^2 = -q^2 = 4EE'\sin^2 \frac{\theta}{2}$

$x = \frac{Q^2}{2M\nu}$

To describe scattering from a nucleon requires structure functions:

$$\frac{d^2\sigma}{d\Omega dE'} = \sigma_{Mott}\left[\frac{1}{\nu}F_2(x,Q^2) + \frac{2}{M}F_1(x,Q^2)\tan^2\frac{\theta}{2}\right].$$

Inclusive inelastic unpolarized cross section
Inclusive Electron Scattering

\[ N(e, e') X \]

\[ p = (E, k) \]
\[ p' = (E', k') \]
\[ q = (\nu, q) \]
\[ \theta \]
\[ P = (M, 0) \]

Inclusive polarized cross sections

\[
\frac{d^2 \sigma}{dE'd\Omega}(\downarrow\uparrow - \uparrow\downarrow) = \frac{4\alpha^2}{MQ^2} \frac{E'}{\nu E} \left[ (E + E' \cos \theta) g_1(x, Q^2) - \frac{Q^2}{\nu} g_2(x, Q^2) \right]
\]

\[
\frac{d^2 \sigma}{dE'd\Omega}(\downarrow\Rightarrow - \uparrow\Rightarrow) = \frac{4\alpha^2}{MQ^2} \frac{\sin \theta}{\nu^2 E} \left[ \nu g_1(x, Q^2) + 2E g_2(x, Q^2) \right]
\]

\[ \vec{q} = \vec{k} - \vec{k}' \]

\[ Q^2 = -q^2 = 4EE' \sin^2 \frac{\theta}{2} \]

\[ x = \frac{Q^2}{2M\nu} \]

\[ g_1, g_2 \text{ are related to the spin distribution} \]
Quark-Parton Model

• Bjorken Scaling Limit:

\[ Q^2 \rightarrow \infty \]
\[ \nu \rightarrow \infty \]

such that:
\[ \omega = \frac{1}{x} = \frac{2M\nu}{Q^2} \]

is fixed

• Structure functions can be written in terms of quark distribution functions:

\[ F_1(x) = \frac{1}{2} \sum_f z_f^2[q_f(x) + \bar{q}_f(x)] \]
\[ F_2(x) = 2xF_1(x) \]
\[ g_1(x) = \frac{1}{2} \sum_f z_f^2[q_f(x) - \bar{q}_f(x)] \]

No simple interpretation for \( g_2 \)

\( g_2 \) includes contributions from quark gluon interactions
What is $g_2$?

(Parton Model Description)

\[ g_2(x, Q^2) = g_2^{WW}(x, Q^2) + \bar{g}_2(x, Q^2) \]
What is $g_2$?

$$g_2(x, Q^2) = g^{WW}_2(x, Q^2) + \bar{g}_2(x, Q^2)$$

Wandzura–Wilczek Relation:

$$g^{WW}_2(x, Q^2) = -g_1(x, Q^2) + \int_x^1 \frac{dy}{y} g_1(y, Q^2)$$

Leading twist-2 term
What is $g_2$?

$$g_2(x, Q^2) = g_2^{WW}(x, Q^2) + \bar{g}_2(x, Q^2)$$

$$\int_x^1 \frac{\partial}{\partial y} \left[ \frac{m_q}{M} h_T(y, Q^2) + \zeta(y, Q^2) \right] \frac{dy}{y}$$

$h_T$: Arises from quark transverse polarization distribution

$\zeta$: Arises from quark-gluon interactions (twist-3)
Measurements of $g_2$ and its Moments

• Measurements of $g_2$ require a transversely polarized target – more difficult experimentally

• 0th moment (no $x$-weighting): Burkhardt-Cottingham Sum Rule
  • Valid at all $Q^2$

$$
\int_0^1 g_2(x, Q^2)\,dx = 0
$$

• 2nd moment ($x^2$ weighting):
  • High $Q^2$ – $d_2$, twist-3 color polarizability, test of lattice QCD
  • Low $Q^2$ – spin polarizabilities, test of χPT
Measurements of $g_2$ and its Moments

CEBAF
- High intensity electron accelerator based on CW SRF technology
- $E_{\text{max}} = 6 \text{ GeV}$
- $I_{\text{max}} = 200 \mu\text{A}$
- $\text{Pol}_{\text{max}} = 85%$

**Recently upgraded to 12 GeV**
Measurements of $g_2$ and its Moments

• Prior to measurements at JLab, first dedicated experiment was SLAC E155x

• $g_2$ Measurements on the neutron at JLab:
  • E97-103: $W>2$ GeV, $Q^2 \approx 1$ GeV$^2$, $x \approx 0.2$, study higher twist (published)
  • E99-117: $W>2$ GeV, high $Q^2$ (3-5 GeV$^2$) (published)
  • E94-010: moments at low $Q^2$ (0.1-1 GeV$^2$) (published)
  • E97-110: moments at very low $Q^2$ (0.02-0.3 GeV$^2$) (analysis)
  • E01-012: moments at intermediate $Q^2$ (1-4 GeV$^2$) (submitted)
  • E06-014: moments at high $Q^2$ (2-6 GeV$^2$) (analysis)

• $g_2$ Measurements on the proton at JLab:
  • RSS: moments at intermediate $Q^2$ (1-2 Gev$^2$) (published)
  • SANE: moments at high $Q^2$ (2-6 GeV$^2$) (analysis)
  • E08-027 ($g_2^p$): moments at very low $Q^2$ (0.02-0.2 GeV$^2$) (analysis)
$\Gamma_2 = \int_0^1 g_2(x, Q^2) dx = 0$

Brown: SLAC E155x
Red: Hall C RSS
Black: Hall A E94-010
Green: Hall A E97-110 (preliminary)
Blue: Hall A E01-012

BC Sum = Measured + Low x + Elastic

Measured: open circles
Low x: unmeasured low-x part of the integral – assume leading twist behavior
Elastic: obtained from well known Form Factors
2\textsuperscript{nd} Moment: Spin Polarizabilities

- Generalized spin polarizabilities $\gamma_0$ and $\delta_{LT}$ are a benchmark test of $\chi$PT
  
  - Difficulty is how to include the nucleon resonance contributions

- $\gamma_0$ is sensitive to resonances, $\delta_{LT}$ is not

- Neutron results for $\gamma_0$

\begin{equation}
\gamma_0 (Q^2) = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 \left[ g_1(x, Q^2) - \frac{4M^2}{Q^2} x^2 g_2(x, Q^2) \right] dx
\end{equation}
2nd Moment: Spin Polarizabilities

- Neutron results for $\delta_{LT}$
- $\delta_{LT}$ is seen as a more suitable testing ground – insensitive to $\Delta$-resonance
- Data is in significant disagreement with $\chi$PT calculations

$$\delta_{LT} (Q^2) = \frac{16\alpha M^2}{Q^6} \int_0^{x_0} x^2 \left[ g_1 (x, Q^2) + g_2 (x, Q^2) \right] dx$$

(V. Sulkosky)
$d_2(Q^2) = \int_0^1 dx x^2 \left( 2g_1(x, Q^2) + 3g_2(x, Q^2) \right)$

$= 3 \int_0^1 dx x^2 \left( g_2(x, Q^2) - g_{WW}(x, Q^2) \right)$

- Doesn’t contain any twist-2 contributions
- Only contributions from measured region
- High precision data at large $Q^2$ is necessary for a benchmark test of Lattice QCD predictions
$g_2^p$ Experiment at JLab (E08-027)

- Will provide the first measurement of $g_2$ for the proton at low to moderate $Q^2$
- Will provide insight on several outstanding physics puzzles:
  - BC sum rule
    - Discrepancy suggested for high-$Q^2$ data
  - $\delta_{LT}$ polarizability
    - $\chi$PT calculations do not match data
  - Finite size effects:
    - Hydrogen hyperfine splitting: proton structure contributes to uncertainty
    - Proton charge radius: proton polarizability contributes to uncertainty
- Data was taken in Hall A in 2012 – analysis is currently underway
$g_2^p$ Collaboration

**Spokespeople**
Alexandre Camsonne
Jian-Ping Chen
Don Crabb
Karl Slifer

**Post Docs**
Kalyan Allada
Elena Long
James Maxwell
Vince Sulkosky
Jixie Zhang

**Graduate Students**
Tobias Badman
Melissa Cummings
Chao Gu
Min Huang
Jie Liu
Pengjia Zhu
Ryan Zielinski

**JLab Target Group**
**Hall A Collaboration**

**Advisor**
Todd Averett
Finite Size Effects

• Hyperfine Splitting of Hydrogen
  • Splitting is defined in terms of Fermi Energy $E_f$
    \[ \Delta E = (1 + \delta) E_F \]
    Where:
    \[ \delta = 1 + (\delta_{QED} + \delta_R + \delta_{small}) + \Delta_S \]

• Proton Charge Radius

  • Results from $\mu$P disagrees with eP scattering result by $\sim 7\sigma$

  • Main uncertainties arise from proton polarizability and differing values of the Zemach radius

Correction for proton structure – contains contribution from $g_2$
(dominated by $g_2$ at low $Q^2$)
Experimental Technique

\[
\frac{d^2\sigma}{dE'd\Omega} (\downarrow\uparrow - \uparrow\downarrow) = \frac{4\alpha^2}{MQ^2\nu E} \left( (E + E' \cos \theta)g_1(x, Q^2) - \frac{Q^2}{\nu}g_2(x, Q^2) \right)
\]

\[
\frac{d^2\sigma}{dE'd\Omega} (\downarrow\Rightarrow - \uparrow\Rightarrow) = \frac{4\alpha^2 \sin \theta}{MQ^2\nu^2 E} \left[ \nu g_1(x, Q^2) + 2E g_2(x, Q^2) \right]
\]

\(\Delta\sigma_{||}\) measured during EG4 experiment in Hall B: will extract \(g_1^p\) at low \(Q^2\)

\(\Delta\sigma_{\perp}\) obtained from \(g_2^p\) experiment
Experimental Setup

- Large scale installation in Hall A
  - DNP NH$_3$ target with 2.5/5 T magnetic field (longitudinal and transverse configurations)
  - New beamline diagnostics for low current (<100 nA) running
- Chicane and septum magnets
- Local dump
Polarized NH$_3$ Target

Dynamic Nuclear Polarization

Target Polarization Results for 5T Field Setting

Run #

Average Polarization:
5T: ~70%
2.5T: ~15%

courtesy of T. Badman

M. Cummings  DIS2014  04/30/14
Detector Stack

High Efficiency (>99%) for gas Cherenkov and lead glass calorimeters

Gas Cherenkov

VDCs

Lead Glass Calorimeters

Scintillators

Legend:
- 2.2 GeV, 2.5T, 90deg
- 1.7 GeV, 2.5T, 90deg
- 1.2 GeV, 2.5T, 90deg
- 1.2 GeV, 2.5T, 90deg (short cell)
- 2.2 GeV, 5.0T, 90deg
- 2.2 GeV, 5.0T, 0deg
- 3.3 GeV, 5.0T, 90deg
Kinematic Coverage

First data on $g_2$ for proton at low $Q^2$

$W < 2 \text{ GeV}$

$0.02 < Q^2 < 0.2 \text{ GeV}^2$

<table>
<thead>
<tr>
<th>Beam Energy (GeV)</th>
<th>Target Field (T)</th>
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<tr>
<td>2.2</td>
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<tr>
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<tr>
<td>2.2</td>
<td>5.0</td>
</tr>
<tr>
<td>3.3</td>
<td>5.0</td>
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</table>
Status of Analysis

Completed

- Run DB
- HRS Optics
  - Field measurement analysis
  - VDC $t_0$ calibration
  - Simulation Package
  - Optics with target field (LHRS)
- Detector Calibrations/Efficiency Studies
  - Gas Cherenkov
  - Lead Glass Calorimeters
  - Scintillator trigger efficiencies
- Scalers
  - BCM calibration
  - Helicity decoding
  - Dead time calculations
- Target Polarization Analysis
- BPM Calibrations

In Progress

- Raster Size Calibrations
- Packing Fraction/Dilution Analysis
- Elastic Analysis
- Yields/Radiative Corrections
Preliminary Results

Asymmetry

\[ A_\perp = \left( \frac{1}{P_b P_t} \right) \frac{Y_+ - Y_-}{Y_+ + Y_-} \]

\[ Y_\pm = \frac{N_\pm}{Q_\pm LT_\pm} \]
Summary of $g_2^p$

• $g_2^p$ experiment will provide first precision measurement for proton at low $Q^2$
  $0.02 < Q^2 < 0.2$ GeV$^2$

• Will provide insight on several outstanding physics puzzles
  • BC Sum Rule: Violation suggested for proton at large $Q^2$ (SLAC E155x)
  • Longitudinal-transverse spin polarizability: benchmark test of $\chi$PT, discrepancy seen for neutron data
  • Hydrogen hyperfine splitting: correction for proton structure contributes to uncertainty
  • Proton charge radius: contributions to uncertainty include proton polarizability
Future Experiments

- Upcoming measurements at JLab in the 12 GeV era
  - Hall A
    - E12-06-122: $A1n$ in valence quark region (8.8 and 6.6 GeV)
  - Hall B
    - E12-06-109: longitudinal spin structure of the nucleon
  - Hall C
    - E12-06-110: $A1n$ in valence quark region (11 GeV)
    - E12-06-121: $g_2^n$ and $d_2^n$ at high $Q^2$
Backup
Finite Size Effects

Hyperfine Splitting of Hydrogen:

Splitting expressed in terms of Fermi Energy $E_F$:

$$\Delta_E = (1 + \delta) E_F$$

Where:

$$\delta = 1 + (\delta_{QED} + \delta_R + \delta_{small}) + \Delta_S$$

$$\Delta_S = \Delta_Z + \Delta_{pol}$$

$$\Delta_{pol} = \frac{\alpha m_e}{\pi g_p m_p} (\Delta_1 + \Delta_2)$$

Dominated by low $Q^2 g_2^p$
\[ \Delta_s \text{ depends on ground state and excited properties:} \]

\[ \Delta_s = \Delta_Z + \Delta_{pol} \]

- **Determined from elastic scattering:**
  \[ \Delta_Z = -2\alpha m_e r_Z (1 + \delta_{rad}^Z) \]

- **Involves the Pauli form factor and } g_1 \text{ structure function}
  \[ \Delta_2 = -24 m_p^2 \int_0^{x_{th}} \frac{dQ^2}{Q^4} B_2(Q^2) \]
  \[ B_2(Q^2) = \int_0^{x_{th}} dx \beta_2(\tau) g_2(x, Q^2) \]
  \[ \beta_2(\tau) = 1 + 2\tau - 2\sqrt{\tau(\tau + 1)} \]
  \[ \tau = \nu^2 / Q^2 \quad x_{th} = \text{pion production threshold} \]

- **Involves contributions where the proton is excited:**
  \[ \Delta_{pol} = \frac{\alpha m_e}{\pi g_p m_p} \left( \Delta_1 + \Delta_2 \right) \]

- **Depends only on the } g_2 \text{ structure function}
Finite Size Effects

Proton Charge Radius:

- Proton charge radius from \( \mu P \) disagrees with eP scattering result by \(~7\sigma\)

\[
\begin{align*}
\langle R_p \rangle &= 0.84184 \pm 0.00067 \text{ fm} & \text{Lamb shift in muonic hydrogen} \\
\langle R_p \rangle &= 0.897 \pm 0.018 \text{ fm} & \text{World analysis of eP scattering} \\
\langle R_p \rangle &= 0.8768 \pm 0.0069 \text{ fm} & \text{CODATA world average}
\end{align*}
\]

- Main uncertainties arise from the proton polarizability and different value of the Zemach radius
Systematic Error Budget for Polarized Cross Section Difference

<table>
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<tr>
<th>Source</th>
<th>%</th>
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<tbody>
<tr>
<td>Cross Section</td>
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<tr>
<td>$P_b P_t$</td>
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<td>Radiative Corrections</td>
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<td>Parallel Contribution</td>
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<tr>
<td><strong>Total</strong></td>
<td>7-9</td>
</tr>
</tbody>
</table>
Error Budget

Experimental Observables:

\[ A_{\text{raw}} = \frac{N^+}{LT+Q^+} - \frac{N^-}{LT-Q^-} \]

\[ \frac{A^\text{exp}}{fP_tP_b} = \frac{A_{\text{raw}}}{\sim1} \]

<table>
<thead>
<tr>
<th>Source</th>
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<tbody>
<tr>
<td>Target Polarization</td>
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<tr>
<td>Beam Polarization</td>
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<tr>
<td>Dilution Factor/Packing Fraction</td>
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</tbody>
</table>
Error Budget

Experimental Observables:

\[
\sigma^\text{raw}_0 = \frac{d\sigma^\text{raw}}{d\Omega dE'} = \frac{p_s N}{N_{\text{in}} \rho L T \epsilon_{\text{det}}} \frac{1}{\Delta \Omega \Delta E' \Delta Z}
\]

\[
\sigma^\text{exp}_0 = \sigma^\text{raw}_0 - \sigma^\text{unpol}
\]

<table>
<thead>
<tr>
<th>Source</th>
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<td>Acceptance/Optics</td>
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<td>Density</td>
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<td>Beam Charge</td>
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<td>Position &amp; Angle Determination</td>
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<td>Detector Efficiencies</td>
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<td>Background (pions)</td>
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<tr>
<td>Radiative Corrections</td>
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