The g_2 Spin Structure Function

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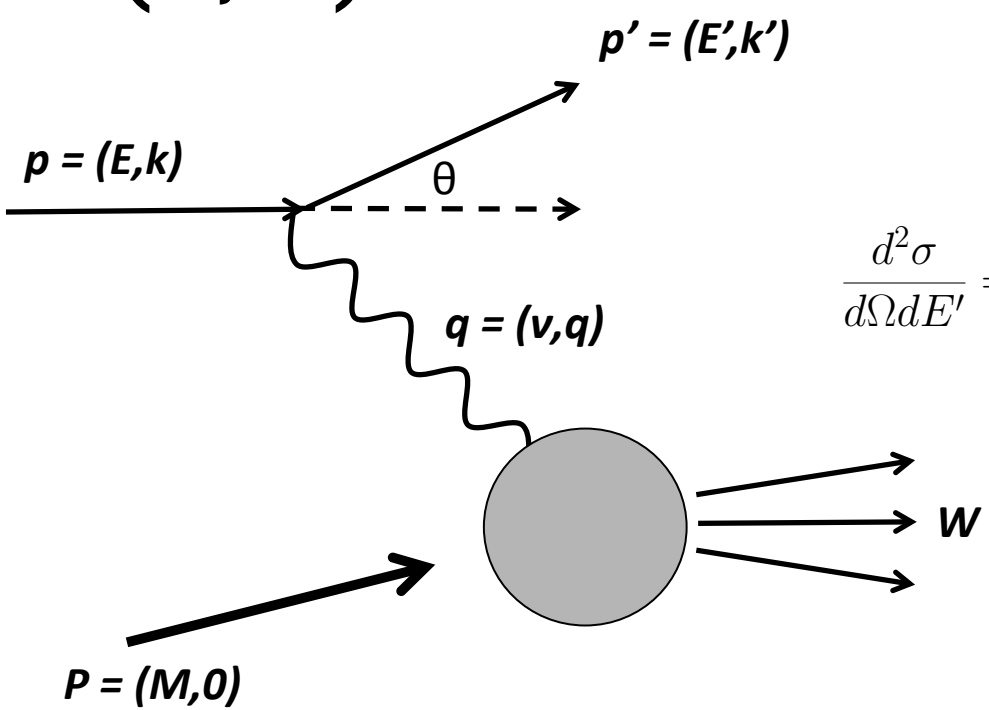
Jefferson Lab

Inclusive Electron Scattering

N(e,e')X



To describe scattering from a nucleon requires structure functions:

$$\frac{d^2\sigma}{d\Omega dE'} = \sigma_{Mott}[\frac{1}{\nu}F_2(x,Q^2) + \frac{2}{M}F_1(x,Q^2)\tan^2\frac{\theta}{2}]$$

Inclusive inelastic
unpolarized cross section

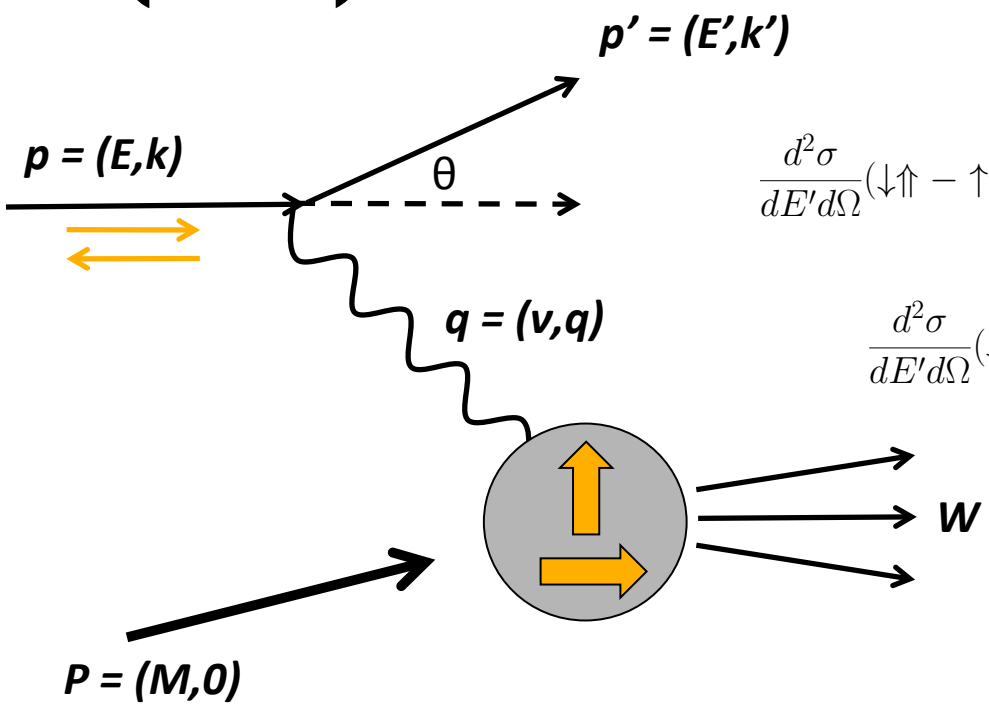
$$\vec{q} = \vec{k} - \vec{k'}$$

$$Q^2 = -q^2 = 4EE'\sin^2\frac{\theta}{2}$$

$$x = \frac{Q^2}{2M\nu}$$

Inclusive Electron Scattering

N(e,e')X



*Inclusive **polarized** cross sections*

$$\frac{d^2\sigma}{dE'd\Omega}(\downarrow\Uparrow - \uparrow\Uparrow) = \frac{4\alpha^2}{MQ^2}\frac{E'}{\nu E}\left[(E + E'\cos\theta)g_1(x,Q^2) - \frac{Q^2}{\nu}g_2(x,Q^2)\right]$$

$$\frac{d^2\sigma}{dE'd\Omega}(\downarrow\Rightarrow - \uparrow\Rightarrow) = \frac{4\alpha^2\sin\theta}{MQ^2}\frac{E'^2}{\nu^2 E}\left[\nu g_1(x,Q^2) + 2E g_2(x,Q^2)\right]$$

g_1,g_2 are related to the spin distribution

$$\vec{q} = \vec{k} - \vec{k'}$$

$$Q^2 = -q^2 = 4EE'\sin^2\frac{\theta}{2}$$

$$x = \frac{Q^2}{2M\nu}$$

Quark-Parton Model

- Bjorken Scaling Limit:

$$Q^2 \to \infty \qquad \nu \to \infty$$

such that: $\omega = \frac{1}{x} = \frac{2M\nu}{Q^2}$ is fixed

- Structure functions can be written in terms of quark distribution functions:

$$F_1(x) = \frac{1}{2}\sum_f z_f^2[q_f(x) + \bar{q}_f(x)]$$

$$F_2(x) = 2xF_1(x)$$

$$g_1(x) = \frac{1}{2}\sum_f z_f^2[q_f(x) - \bar{q}_f(x)]$$

No simple interpretation for g_2

g_2 includes contributions from quark gluon interactions

What is g_2?

(Parton Model Description)

$$g_2(x, Q^2) = g_2^{WW}(x, Q^2) + \bar{g}_2(x, Q^2)$$

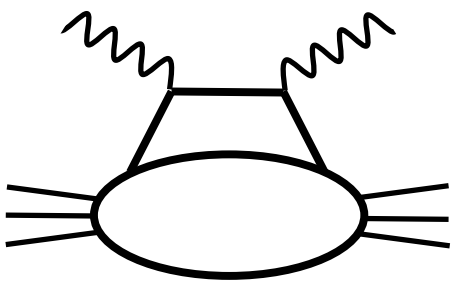
What is g_2?

$$g_2(x,Q^2) = \boxed{g_2^{WW}(x,Q^2)} + \bar{g}_2(x,Q^2)$$

Wandzura–Wilczek Relation:

$$g_2^{WW}(x,Q^2) = -g_1(x,Q^2) + \int_x^1 \frac{dy}{y} g_1(y,Q^2)$$

Leading twist-2 term



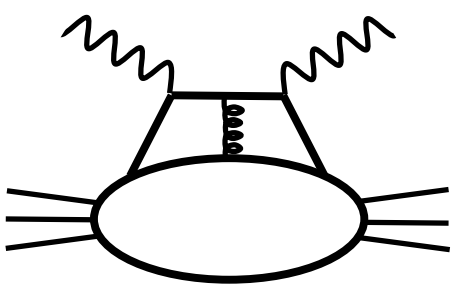
What is g_2?

$$g_2(x,Q^2) = g_2^{WW}(x,Q^2) + \bar{g}_2(x,Q^2)$$

$$\int_x^1 \frac{\partial}{\partial y}\left[\frac{m_q}{M} h_T(y,Q^2) + \zeta(y,Q^2)\right]\frac{dy}{y}$$

h_T: Arises from quark transverse polarization distribution

ζ: Arises from quark-gluon interactions (twist-3)



Measurements of g_2 and its Moments

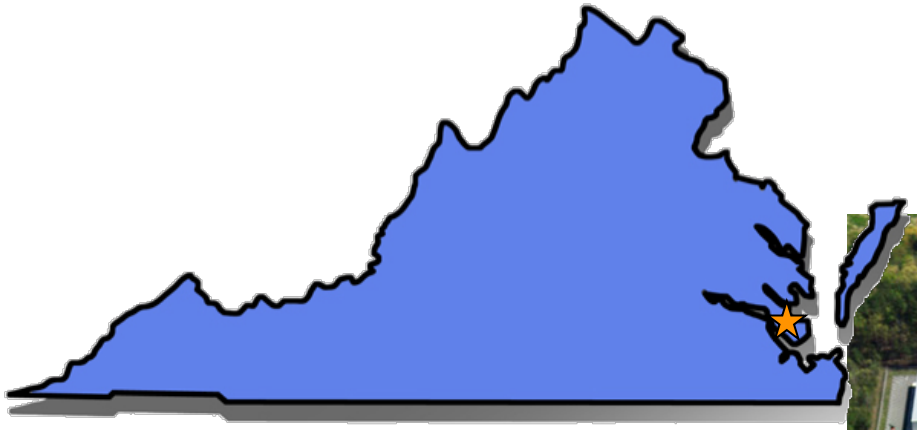
- Measurements of g_2 require a transversely polarized target – more difficult experimentally

- 0th moment (no x-weighting): Burkhardt-Cottingham Sum Rule
  - Valid at all Q^2

$$\int_0^1 g_2(x, Q^2)dx = 0$$

- 2nd moment (x^2 weighting):
  - High Q^2 – d_2, twist-3 color polarizability, test of lattice QCD
  - Low Q^2 – spin polarizabilities, test of χPT

Measurements of g_2 and its Moments



Jefferson Lab

CEBAF

- High intensity electron accelerator based on CW SRF technology
- E_{max} = 6 GeV
- I_{max} = 200 μA
- Pol_{max} = 85%

Recently upgraded to 12 GeV

Measurements of g_2 and its Moments

- Prior to measurements at JLab, first dedicated experiment was SLAC E155x

- g_2 Measurements on the **neutron** at JLab:
  - E97-103: W>2 GeV, $Q^2 \approx 1$ GeV^2, $x \approx 0.2$, study higher twist (published)
  - E99-117: W>2 GeV, high Q^2 (3-5 GeV^2) (published)
  - E94-010: moments at low Q^2 (0.1-1 GeV^2) (published)
  - E97-110: moments at very low Q^2 (0.02-0.3 GeV^2) (analysis)
  - E01-012: moments at intermediate Q^2 (1-4 GeV^2) (submitted)
  - E06-014: moments at high Q^2 (2-6 GeV^2) (analysis)

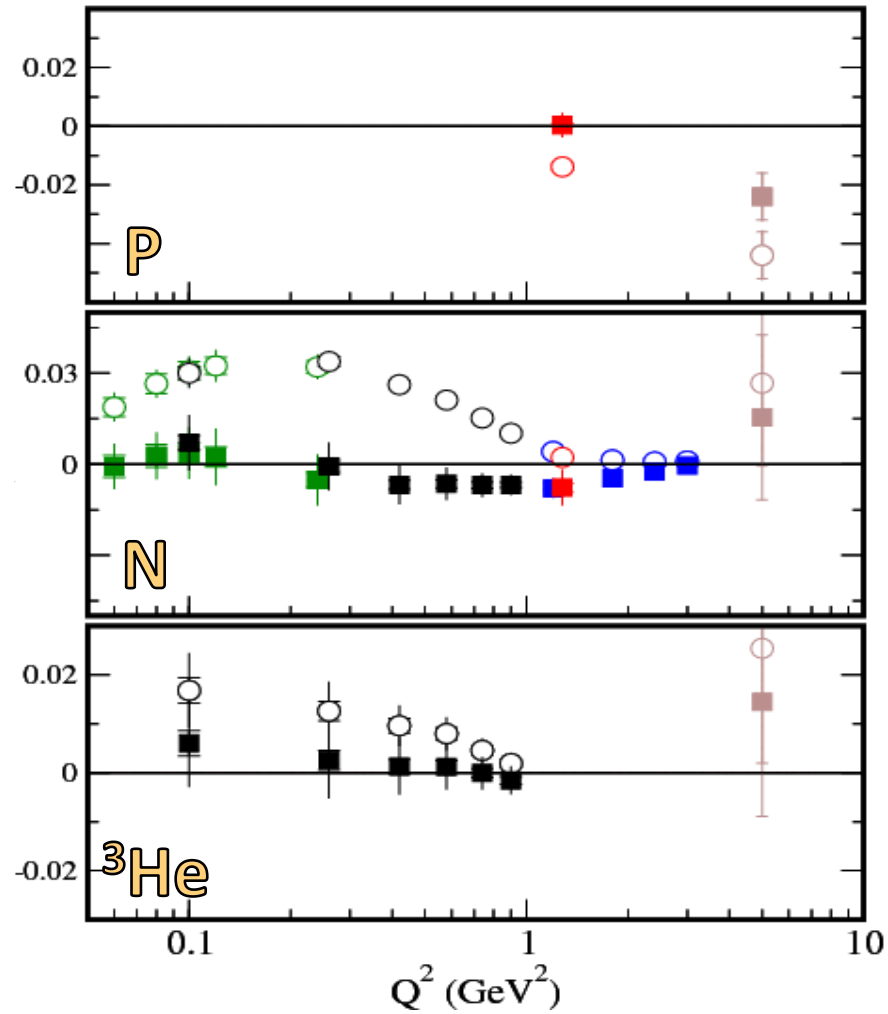
- g_2 Measurements on the **proton** at JLab:
  - RSS: moments at intermediate Q^2 (1-2 Gev^2) (published)
  - SANE: moments at high Q^2 (2-6 GeV^2) (analysis)
  - E08-027 (g_2^p): moments at very low Q^2 (0.02-0.2 GeV^2) (analysis)

0th Moment of g_2

$$\Gamma_2 = \int_0^1 g_2(x, Q^2)dx = 0$$

Brown: SLAC E155x
Red: Hall C RSS
Black: Hall A E94-010
Green: Hall A E97-110 (preliminary)
Blue: Hall A E01-012

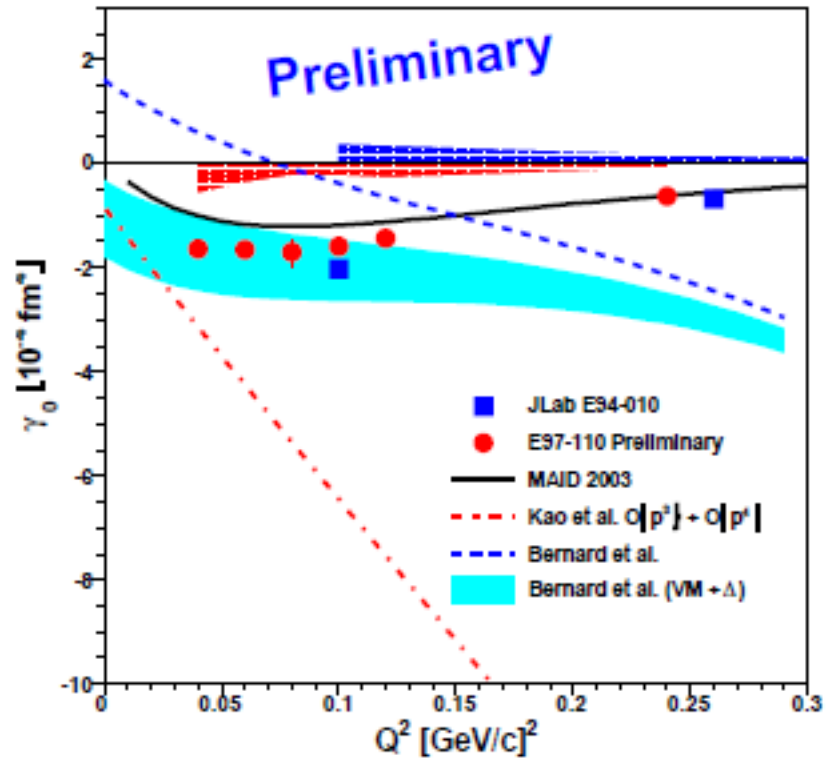
BC Sum = Measured + Low x + Elastic
Measured: open circles
Low x: unmeasured low-*x* part of the integral – assume leading twist behavior
Elastic: obtained from well known Form Factors



2nd Moment: Spin Polarizabilities

(V. Sulkosky)

- Generalized spin polarizabilities γ_0 and δ_{LT} are a benchmark test of χPT
  - Difficulty is how to include the nucleon resonance contributions
  - γ_0 is sensitive to resonances, δ_{LT} is not
- Neutron results for γ_0

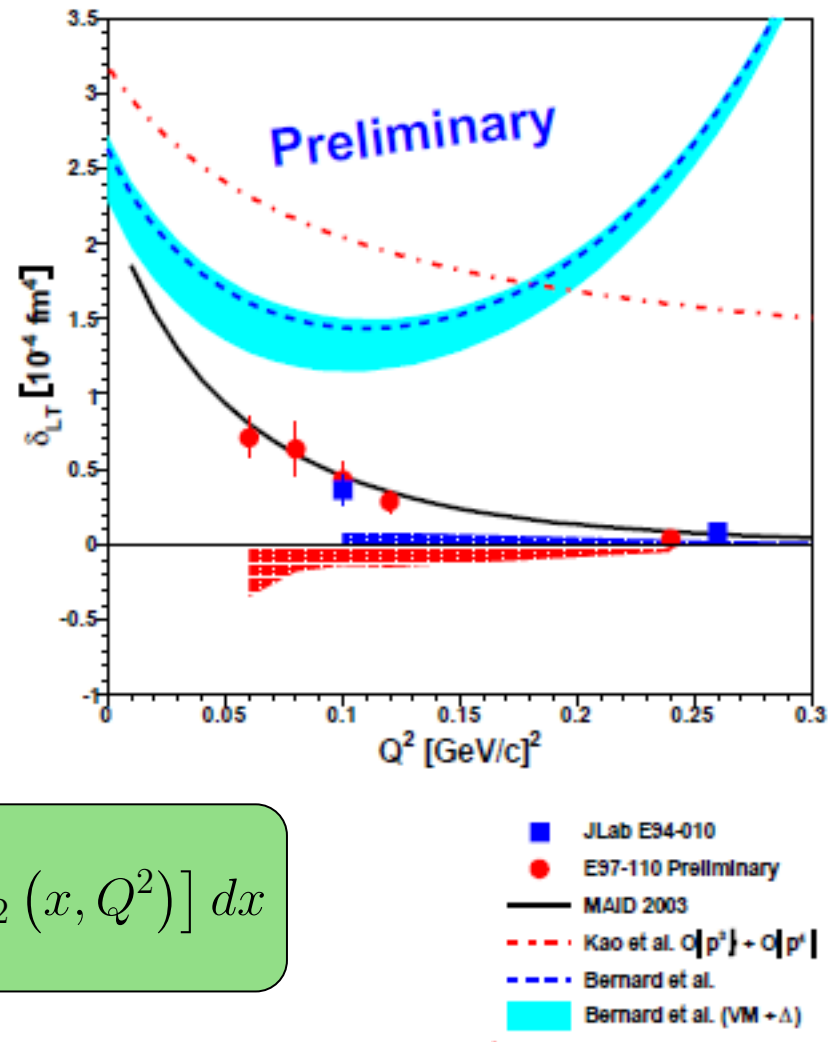


$$\gamma_0\left(Q^2\right) = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2\left[g_1\left(x,Q^2\right) - \frac{4M^2}{Q^2}x^2 g_2\left(x,Q^2\right)\right]dx$$

2nd Moment: Spin Polarizabilities

(V. Sulkosky)

- Neutron results for δ_{LT}
- δ_{LT} is seen as a more suitable testing ground – insensitive to Δ-resonance
- Data is in significant disagreement with χPT calculations



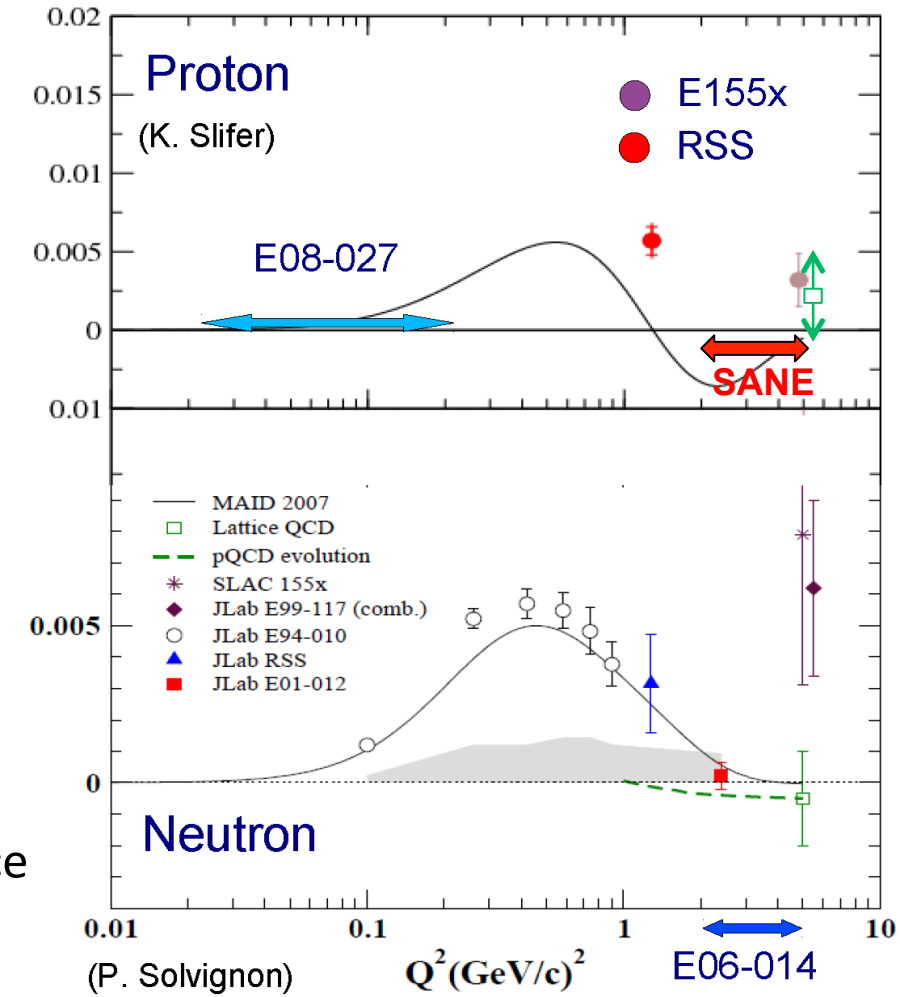
$$\delta_{LT}\left(Q^2\right) = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} x^2\left[g_1\left(x,Q^2\right)+g_2\left(x,Q^2\right)\right]dx$$

d_2 & Higher Twist

$$d_2(Q^2) = \int_0^1 dx x^2 \left(2g_1(x,Q^2) + 3g_2(x,Q^2)\right)$$

$$= 3\int_0^1 dx x^2 \left(g_2(x,Q^2) - g_2^{WW}(x,Q^2)\right)$$

- Doesn't contain any twist-2 contributions
- Only contributions from measured region
- High precision data at large Q^2 is necessary for a benchmark test of Lattice QCD predictions



g_2^p Experiment at JLab (E08-027)

- Will provide the first measurement of g_2 for the proton at low to moderate Q^2

- Will provide insight on several outstanding physics puzzles:
  - BC sum rule
    - Discrepancy suggested for high-Q^2 data
  - δ_{LT} polarizability
    - χPT calculations do not match data
  - Finite size effects:
    - Hydrogen hyperfine splitting: proton structure contributes to uncertainty
    - Proton charge radius: proton polarizability contributes to uncertainty

- Data was taken in Hall A in 2012 – analysis is currently underway

g_2^p Collaboration

Spokespeople
Alexandre Camsonne
Jian-Ping Chen
Don Crabb
Karl Slifer

Post Docs
Kalyan Allada
Elena Long
James Maxwell
Vince Sulkosky
Jixie Zhang

Advisor
Todd Averett

Graduate Students
Tobias Badman
Melissa Cummings
Chao Gu
Min Huang
Jie Liu
Pengjia Zhu
Ryan Zielinski

JLab Target Group

Hall A Collaboration

Finite Size Effects

- Hyperfine Splitting of Hydrogen
  - Splitting is defined in terms of Fermi Energy E_f

$$\Delta_E = (1+\delta)\, E_F$$

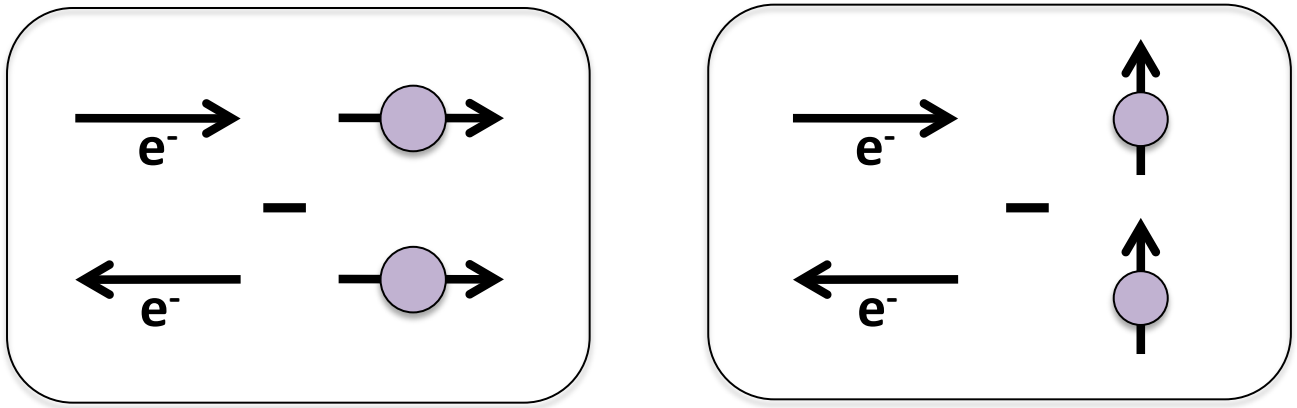
Where:

$$\delta = 1 + (\delta_{QED} + \delta_R + \delta_{small}) + \Delta_S$$

Correction for proton structure – contains contribution from g_2 (dominated by g_2 at low Q^2)

- Proton Charge Radius
  - Results from μP disagrees with eP scattering result by ~7σ
  - Main uncertainties arise from proton polarizability and differing values of the Zemach radius

Experimental Technique



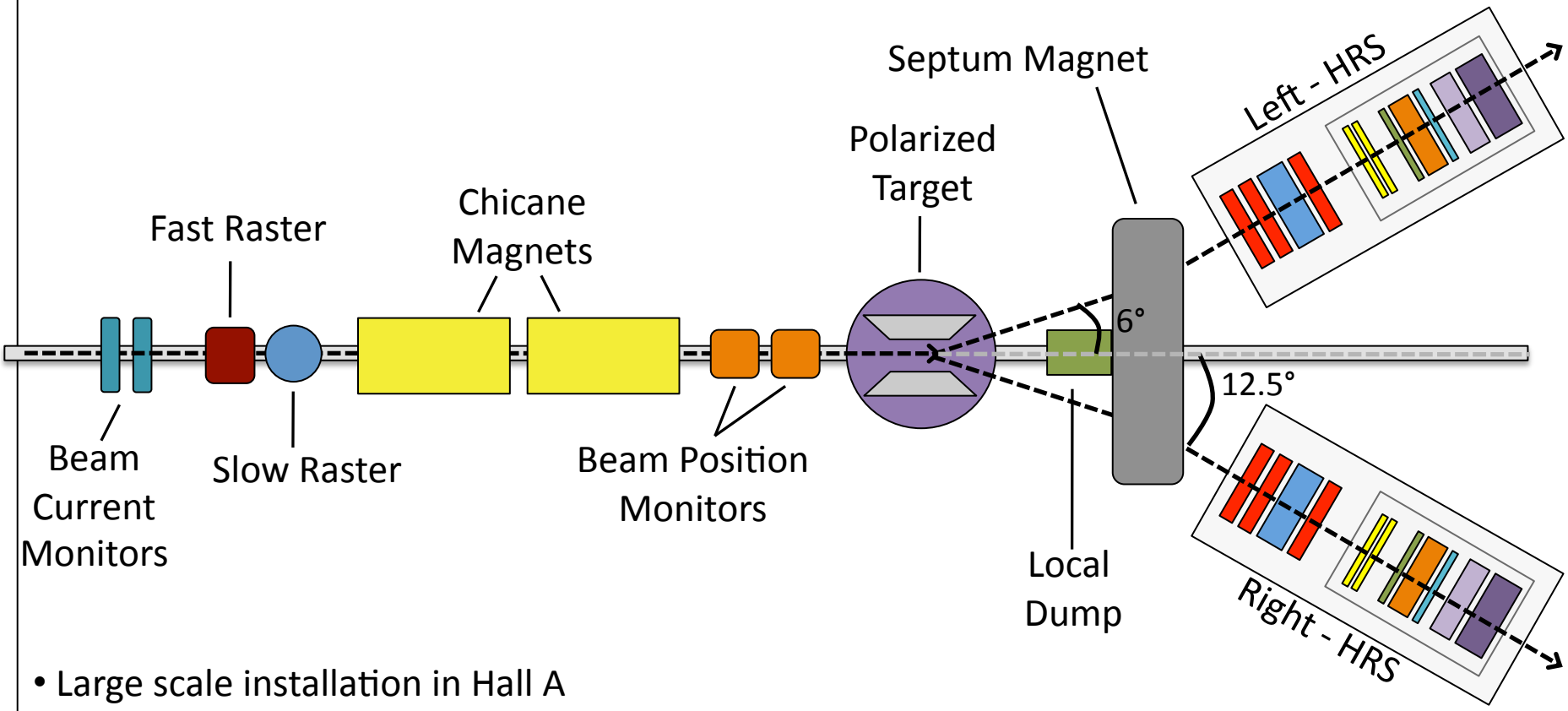
$$\frac{d^2\sigma}{dE'd\Omega}(\downarrow\Uparrow - \uparrow\Uparrow) = \frac{4\alpha^2}{MQ^2}\frac{E'}{\nu E}\left[(E + E'\cos\theta)g_1(x,Q^2) - \frac{Q^2}{\nu}g_2(x,Q^2)\right]$$

$\Delta\sigma_{||}$ measured during EG4 experiment in Hall B: will extract g_1^p at low Q^2

$$\frac{d^2\sigma}{dE'd\Omega}(\downarrow\Rightarrow - \uparrow\Rightarrow) = \frac{4\alpha^2\sin\theta}{MQ^2}\frac{E'^2}{\nu^2 E}\left[\nu g_1(x,Q^2) + 2Eg_2(x,Q^2)\right]$$

$\Delta\sigma_{\perp}$ obtained from g_2^p experiment

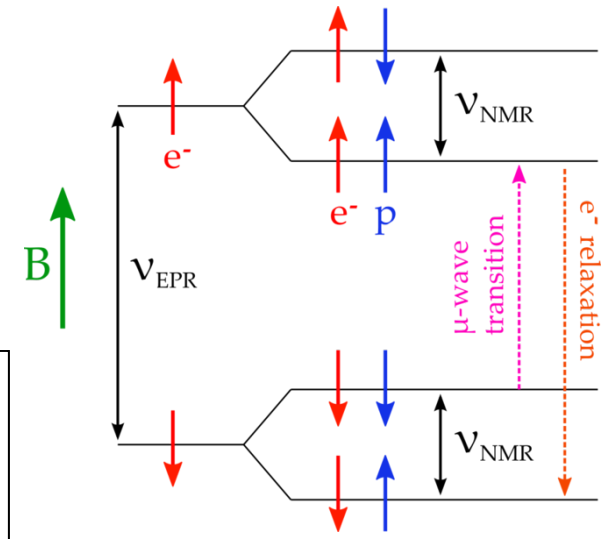
Experimental Setup



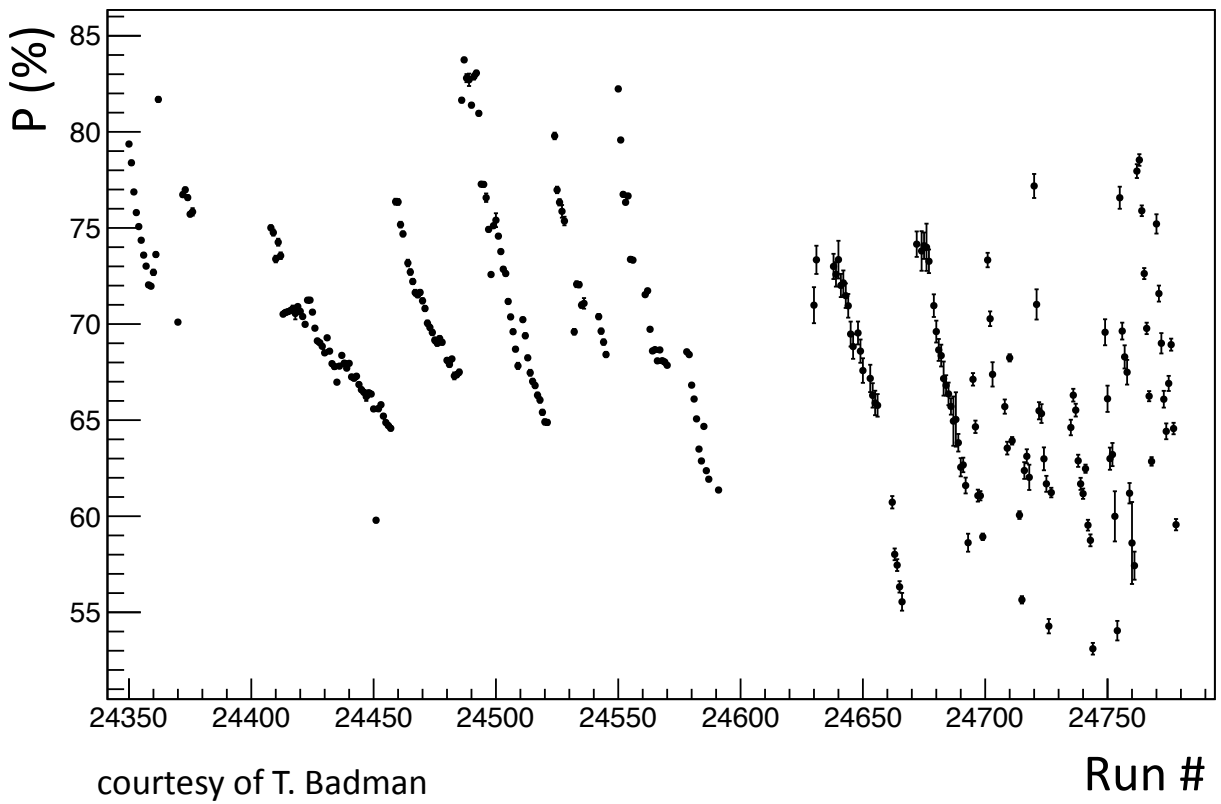
- Large scale installation in Hall A
  - DNP NH_3 target with 2.5/5 T magnetic field (longitudinal and transverse configurations)
  - New beamline diagnostics for low current (<100 nA) running
  - Chicane and septum magnets
  - Local dump

Polarized NH_3 Target

Dynamic Nuclear Polarization



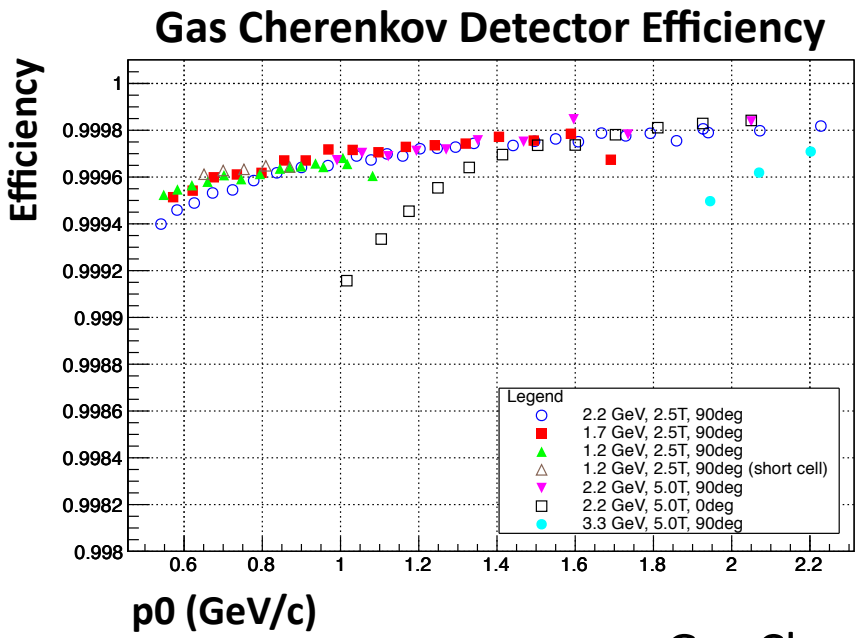
Target Polarization Results for 5T Field Setting



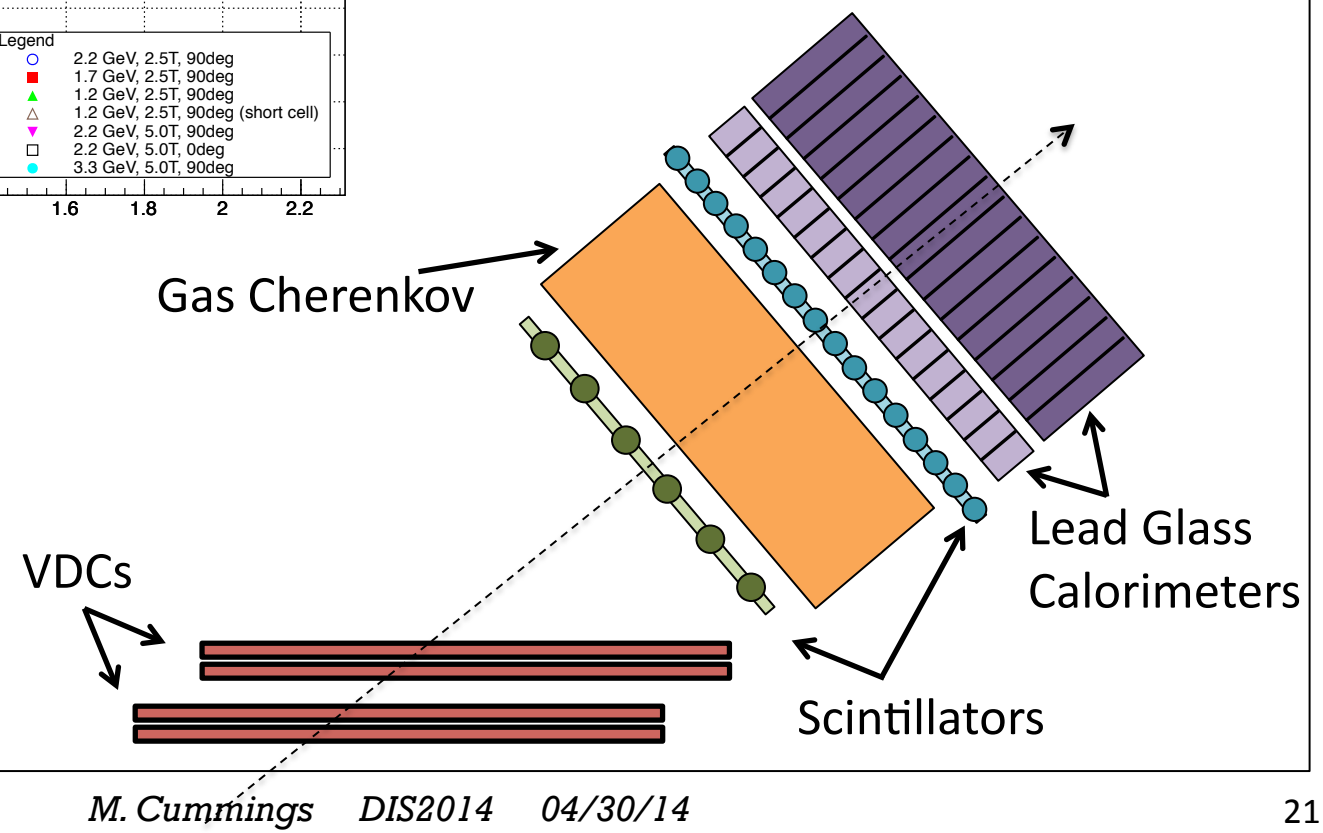
courtesy of T. Badman

Average Polarization:
5T: ~70%
2.5T: ~15%

Detector Stack

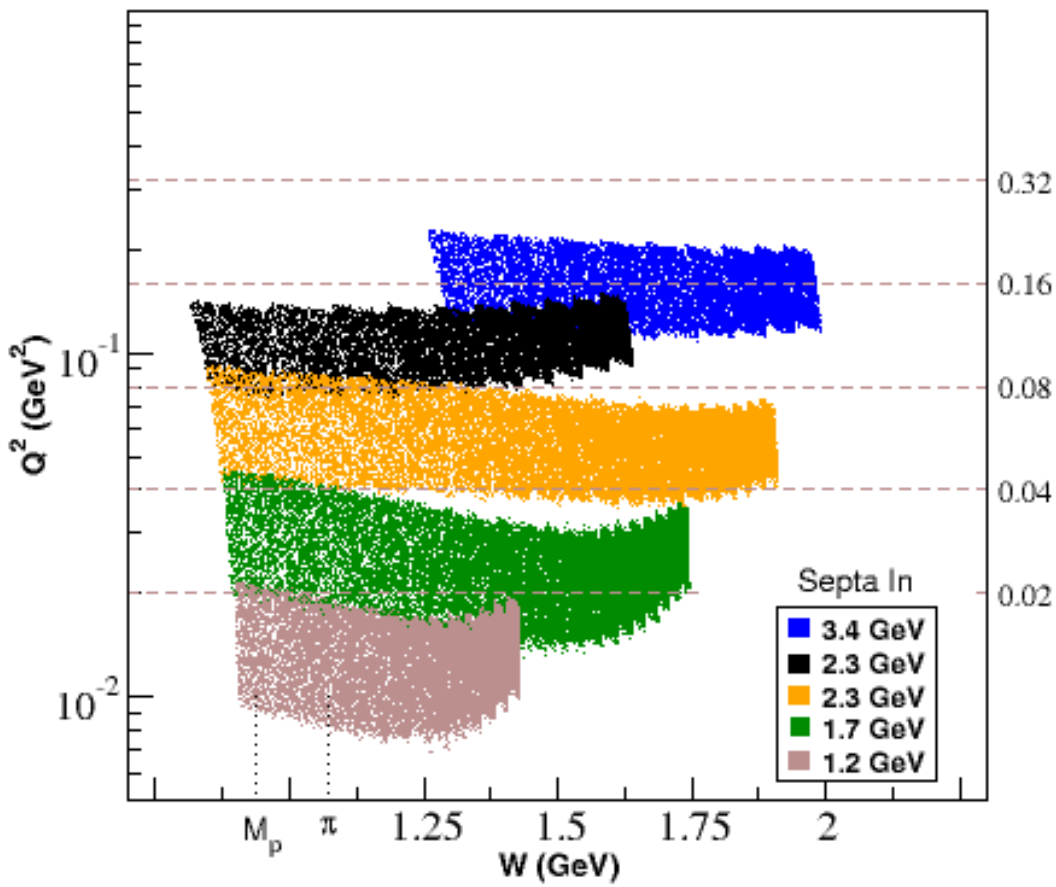


High Efficiency (>99%) for gas Cherenkov and lead glass calorimeters



Kinematic Coverage

First data on g_2 for proton at low Q^2



$W < 2$ GeV

$0.02 < Q^2 < 0.2$ GeV^2

Beam Energy (GeV)	Target Field (T)
2.2	2.5
1.7	2.5
1.1	2.5
2.2	5.0
3.3	5.0

Status of Analysis

Completed

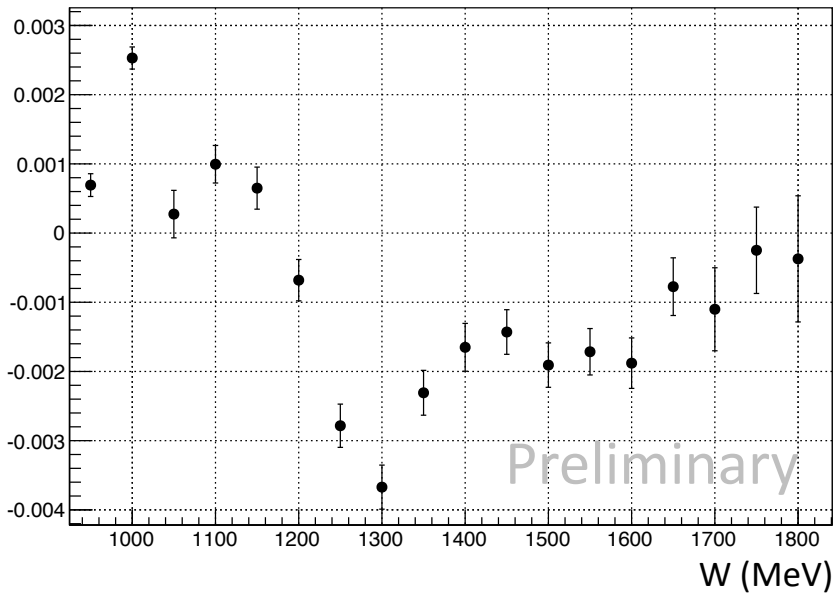
- Run DB
- HRS Optics
  - Field measurement analysis
  - VDC t_0 calibration
  - Simulation Package
  - Optics with target field (LHRS)
- Detector Calibrations/Efficiency Studies
  - Gas Cherenkov
  - Lead Glass Calorimeters
  - Scintillator trigger efficiencies
- Scalers
  - BCM calibration
  - Helicity decoding
  - Dead time calculations
- Target Polarization Analysis
- BPM Calibrations

In Progress

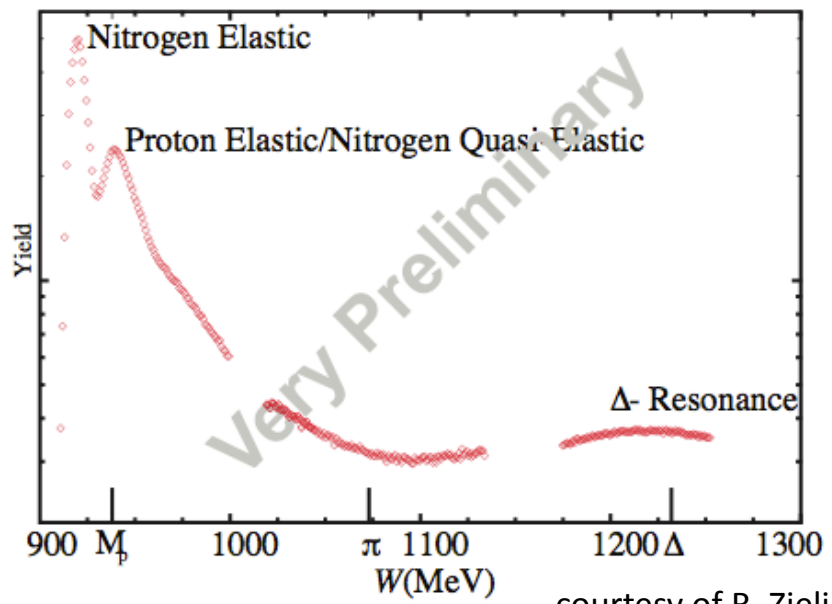
- Raster Size Calibrations
- Packing Fraction/Dilution Analysis
- Elastic Analysis
- Yields/Radiative Corrections

Preliminary Results

Asymmetry



Yield



courtesy of R. Zielinski

$$A_{\perp} = \left(\frac{1}{P_b P_t}\right)\frac{Y_+ - Y_-}{Y_+ + Y_-}$$

$$Y_{\pm} = \frac{N_{\pm}}{Q_{\pm} L T_{\pm}}$$

Summary of g_2^p

- g_2^p experiment will provide first precision measurement for proton at low Q^2
  $0.02 < Q^2 < 0.2$ GeV^2

- Will provide insight on several outstanding physics puzzles
  - BC Sum Rule: Violation suggested for proton at large Q^2 (SLAC E155x)
  - Longitudinal-transverse spin polarizability: benchmark test of χPT, discrepancy seen for neutron data
  - Hydrogen hyperfine splitting: correction for proton structure contributes to uncertainty
  - Proton charge radius: contributions to uncertainty include proton polarizability

Future Experiments

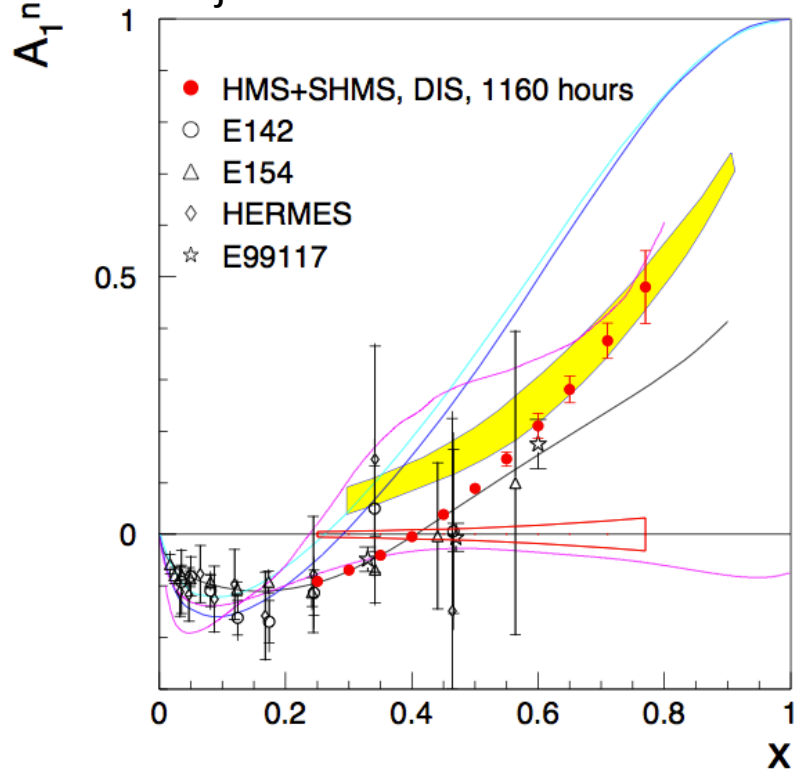
- Upcoming measurements at JLab in the 12 GeV era

- Hall A
  - E12-06-122: *A1n* in valence quark region (8.8 and 6.6 GeV)

- Hall B
  - E12-06-109: longitudinal spin structure of the nucleon

- Hall C
  - E12-06-110: *A1n* in valence quark region (11 GeV)
  - E12-06-121: g_2^n and d_2^n at high Q^2

Projected Results for E12-06-110



Backup

Finite Size Effects

Hyperfine Splitting of Hydrogen:

Splitting expressed in terms of Fermi Energy E_F:

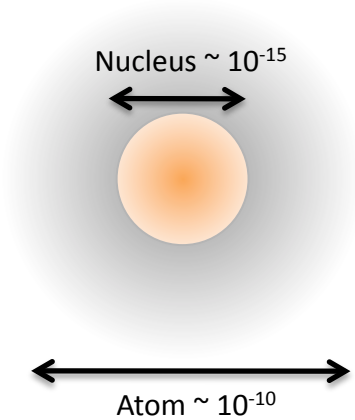
$$\Delta_E = (1+\delta)\, E_F$$

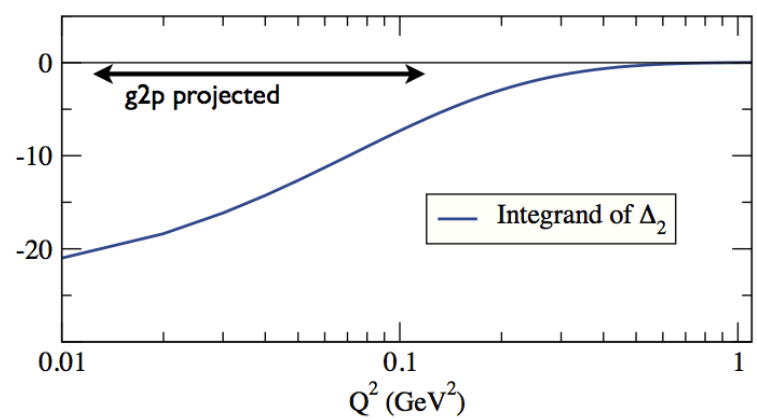
Where:

$$\delta = 1 + (\delta_{QED} + \delta_R + \delta_{small}) + \Delta_S$$

$$\Delta_S = \Delta_Z + \Delta_{pol}$$

$$\Delta_{pol} = \frac{\alpha m_e}{\pi g_p m_p}(\Delta_1 + \Delta_2)$$





Dominated by low Q^2 g_2^p

Finite Size Effects

Δ_S depends on ground state and excited properties:

$$\Delta_S = \Delta_Z + \Delta_{pol}$$

Determined from elastic scattering:

$$\Delta_Z = -2\alpha m_e r_Z (1 + \delta_Z^{rad})$$

Involves contributions where the proton is excited:

$$\Delta_{pol} = \frac{\alpha m_e}{\pi g_p m_p}(\Delta_1 + \Delta_2)$$

Depends only on the g_2 structure function

Involves the Pauli form factor and g_1 structure function

$$\Delta_2 = -24 m_p^2 \int_0^\infty \frac{dQ^2}{Q^4} B_2(Q^2)$$

$$B_2(Q^2) = \int_0^{x_{th}} dx \beta_2(\tau) g_2(x, Q^2)$$

$$\beta_2(\tau) = 1 + 2\tau - 2\sqrt{\tau(\tau+1)}$$

$$\tau = \nu^2/Q^2 \qquad x^{th} = \text{pion production threshold}$$

Finite Size Effects

Proton Charge Radius:

- Proton charge radius from μP disagrees with eP scattering result by ~7σ

$\langle R_p \rangle = 0.84184 \pm 0.00067$ fm	Lamb shift in muonic hydrogen
$\langle R_p \rangle = 0.897 \pm 0.018$ fm	World analysis of eP scattering
$\langle R_p \rangle = 0.8768 \pm 0.0069$ fm	CODATA world average

- Main uncertainties arise from the proton polarizability and different value of the Zemach radius

Error Budget

Systematic Error Budget for Polarized Cross Section Difference

Source	%
Cross Section	5-7
$P_b P_t$	4-5
Radiative Corrections	3
Parallel Contribution	< 1
Total	**7-9**

Error Budget

Experimental Observables:

$$A_{raw} = \frac{\frac{N^+}{LT^+Q^+} - \frac{N^-}{LT^-Q^-}}{\frac{N^+}{LT^+Q^+} + \frac{N^-}{LT^-Q^-}} \longrightarrow A_\perp^{exp} = \frac{A_\perp^{raw}}{f P_t P_b}$$

Source	%
Target Polarization	3-4
Beam Polarization	2-3
Dilution Factor/Packing Fraction	~1

Error Budget

Experimental Observables:

$$\sigma_0^{raw} = \frac{d\sigma^{raw}}{d\Omega dE'} = \frac{ps_1 N}{N_{in}\rho L T \epsilon_{det}} \frac{1}{\Delta\Omega\Delta E'\Delta Z}$$

$$\sigma_0^{exp} = \sigma_0^{raw} - \sigma^{unpol}$$

Source	%
Acceptance/Optics	~3
Dilution Factor/Packing Fraction	~1
Density	2-3
Beam Charge	1-2
Position & Angle Determination	2-4
Detector Efficiencies	~1
Background (pions)	< 1
Radiative Corrections	1-4