Timelike Compton Scattering  
with a linearly polarized photon beam

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Processes

- Universality of GPDs,
- Meson production - additional difficulties,
So, in addition to spacelike DVCS ... 

Figure: Deeply Virtual Compton Scattering (DVCS): $l N \rightarrow l' N' \gamma$
we can also study timelike DVCS

![Diagram of Timelike Compton Scattering (TCS): $\gamma N \rightarrow l^+ l^- N'$](image)

**Figure**: Timelike Compton Scattering (TCS): $\gamma N \rightarrow l^+ l^- N'$

Why TCS:
- universality of the GPDs
- another source for GPDs (special sensitivity on real part of GPD $H$),
- spacelike-timelike crossing,
General Compton Scattering:

\[ \gamma^*(q_{in}) N(p) \rightarrow \gamma^*(q_{out}) N'(p') \]

variables, describing the processes of interest in this generalized Bjorken limit, are the scaling variable \( \xi \) and skewness \( \eta > 0 \):

\[ \xi = -\frac{q_{out}^2 + q_{in}^2}{q_{out}^2 - q_{in}^2} \eta, \quad \eta = \frac{q_{out}^2 - q_{in}^2}{(p + p') \cdot (q_{in} + q_{out})}. \]

- **DDVCS**: \( q_{in}^2 < 0, \quad q_{out}^2 > 0, \quad \eta \neq \xi \)
- **DVCS**: \( q_{in}^2 < 0, \quad q_{out}^2 = 0, \quad \eta = \xi > 0 \)
- **TCS**: \( q_{in}^2 = 0, \quad q_{out}^2 > 0, \quad \eta = -\xi > 0 \)
Coefficient functions and Compton Form Factors

CFFs are the GPD dependent quantities which enter the amplitudes. They are defined through relations:

\[
A^{\mu\nu} (\xi, \eta, t) = -e^2 \frac{1}{(P + P')^+} \bar{u}(P') \left[ g^{\mu\nu} \left( \mathcal{H}(\xi, \eta, t) \gamma^+ + \mathcal{E}(\xi, \eta, t) \frac{i\sigma^+ \Delta}{2M} \right) 
+ i\epsilon^{\mu\nu} \left( \tilde{\mathcal{H}}(\xi, \eta, t) \gamma^+ \gamma^5 + \tilde{\mathcal{E}}(\xi, \eta, t) \frac{\Delta^+ \gamma^5}{2M} \right) \right] u(P),
\]

where:

\[
\mathcal{H}(\xi, \eta, t) = + \int_{-1}^{1} dx \left( \sum_q T^q (x, \xi, \eta) H^q (x, \eta, t) + T^g (x, \xi, \eta) H^g (x, \eta, t) \right)
\]

\[
\tilde{\mathcal{H}}(\xi, \eta, t) = - \int_{-1}^{1} dx \left( \sum_q \tilde{T}^q (x, \xi, \eta) \tilde{H}^q (x, \eta, t) + \tilde{T}^g (x, \xi, \eta) \tilde{H}^g (x, \eta, t) \right).
\]
Coefficient functions

Renormalized coefficient functions for DVCS are given by

\[
T^q(x) = \left[ C^q_0(x) + C^q_1(x) + \ln \left( \frac{Q^2}{\mu_F^2} \right) \cdot C^q_{\text{coll}}(x) \right] - (x \to -x),
\]

\[
T^g(x) = \left[ C^g_1(x) + \ln \left( \frac{Q^2}{\mu_F^2} \right) \cdot C^g_{\text{coll}}(x) \right] + (x \to -x),
\]

\[
\tilde{T}^q(x) = \left[ \tilde{C}^q_0(x) + \tilde{C}^q_1(x) + \ln \left( \frac{Q^2}{\mu_F^2} \right) \cdot \tilde{C}^q_{\text{coll}}(x) \right] + (x \to -x),
\]

\[
\tilde{T}^g(x) = \left[ \tilde{C}^g_1(x) + \ln \left( \frac{Q^2}{\mu_F^2} \right) \cdot \tilde{C}^g_{\text{coll}}(x) \right] - (x \to -x).
\]

The results for DVCS and TCS cases are simply related:

\[
TCS_T(x, \eta) = \pm \left(^{DVCS}T(x, \xi = \eta) + i\pi \cdot C_{\text{coll}}(x, \xi = \eta) \right)^*,
\]


where + (−) sign corresponds to vector (axial) case.
In our analysis we use two GPD models based on double distribution:

\[ F_i(x, \xi, t) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(\beta + \xi \alpha - x) f_i(\beta, \alpha, t) + D_i^F \left( \frac{x}{\xi}, t \right) \Theta(\xi^2 - x^2), \]

The DD \( f_i \) reads

\[ f_i(\beta, \alpha, t) = g_i(\beta, t) h_i(\beta) \frac{\Gamma(2n_i + 2)}{2^{2n_i+1} \Gamma^2(n_i + 1)} \frac{[(1 - |\beta|)^2 - \alpha^2]^{n_i}}{(1 - |\beta|)^{2n_i+1}}, \]

where:

\[ h_g(\beta) = |\beta| \, g(|\beta|), \quad \tilde{h}_g(\beta) = \beta \, \Delta g(|\beta|), \]
\[ h_{sea}^q(\beta) = q_{sea}(|\beta|) \text{sign}(\beta), \quad \tilde{h}_{sea}^q(\beta) = \Delta q_{sea}(|\beta|), \]
\[ h_{val}^q(\beta) = q_{val}(\beta) \Theta(\beta), \quad \tilde{h}_{val}^q(\beta) = \Delta q_{val}(\beta) \Theta(\beta). \]

\( D_i^F \) denotes the Polyakov-Weiss D-term. In our estimates we will use parametrizations obtained by a fit to the chiral soliton model.
**Factorized:**

based on MSTW08 PDFs with simple factorizing ansatz for $t$-dependence

\[
g_u(\beta, t) = \frac{1}{2} F_1^u(t), \quad F_1^u(t) = 2F_1^p(t) + F_1^n(t),
\]
\[
g_d(\beta, t) = F_1^d(t), \quad F_1^d(t) = F_1^p(t) + 2F_1^n(t),
\]
\[
g_s(\beta, t) = g_g(\beta, t) = F_D(t), \quad F_D(t) = (1 - t/M_V^2)^{-2},
\]

with $M_V = 0.84 \, \text{GeV}$, $F_1^p$ and $F_1^n$ are electromagnetic Dirac form factors of the proton and neutron. We use that model to construct only $\mathcal{H}$.

**Goloskokov-Kroll:**

based on CTEQ6m PDFs, and

\[
g_{i}(\beta, t) = e^{b_i t} |\beta|^{-\alpha'_i t}
\]

and simple parametrization of the sea quarks:

\[
H_{\text{sea}}^u = H_{\text{sea}}^d = \kappa_s H_{\text{sea}}^s,
\]

with $\kappa_s = 1 + 0.68/(1 + 0.52 \ln Q^2/Q_0^2)$,

with the initial scale of the CTEQ6m PDFs $Q_0^2 = 4 \, \text{GeV}^2$. $\tilde{H}$ is constructed using the Blümlein - Böttcher (BB) polarized PDF parametrization to fix the forward limit. Meson electroproduction data from HERA and HERMES have been considered to fix parameters for this GPD in the GK model.
Figure: The real part of the spacelike Compton Form Factor $\mathcal{H}(\xi)$ multiplied by $\xi$, as a function of $\xi$ in the double distribution model based on Kroll-Goloskokov (upper left) and MSTW08 (upper right) parametrizations, for $\mu_F^2 = Q^2 = 4 \text{ GeV}^2$ and $t = -0.1 \text{ GeV}^2$, at the Born order (dotted line), including the NLO quark corrections (dashed line) and including both quark and gluon NLO corrections (solid line).
Figure: The imaginary part of the spacelike Compton Form Factor $\mathcal{H}(\xi)$ multiplied by $\xi$, as a function of $\xi$ in the double distribution model based on Kroll-Goloskokov (upper left) and MSTW08 (upper right) parametrizations, for $\mu_F^2 = Q^2 = 4 \text{ GeV}^2$ and $t = -0.1 \text{ GeV}^2$, at the Born order (dotted line), including the NLO quark corrections (dashed line) and including both quark and gluon NLO corrections (solid line).
Figure: The real part of the \textit{timelike} Compton Form Factor $\mathcal{H}$ multiplied by $\eta$, as a function of $\eta$ in the double distribution model based on Kroll-Goloskokov (upper left) and MSTW08 (upper right) parametrizations, for $\mu_F^2 = Q^2 = 4 \text{ GeV}^2$ and $t = -0.1 \text{ GeV}^2$. Below the ratios of the NLO correction to LO result of the corresponding models.
Figure: The imaginary part of the *timelike* Compton Form Factor $\mathcal{H}$ multiplied by $\eta$, as a function of $\eta$ in the double distribution model based on Kroll-Goloshkov (upper left) and MSTW08 (upper right) parametrizations, for $\mu_F^2 = Q^2 = 4$ GeV$^2$ and $t = -0.1$ GeV$^2$. Below the ratios of the NLO correction to LO result of the corresponding models.
TCS and Bethe-Heitler contribution to exclusive lepton pair photoproduction.

Figure: The Feynman diagrams for the Bethe-Heitler amplitude.

Figure: Handbag diagrams for the Compton process in the scaling limit.
Figure: Kinematical variables and coordinate axes in the $\gamma p$ and $\ell^+\ell^-$ c.m. frames.
Interference

B–H dominant for not very high energies:

\[\text{Figure: LO (dotted) and NLO (solid) TCS and Bethe-Heitler (dash-dotted) contributions to the cross section as a function of} \ t \ \text{for} \ Q^2 = \mu^2 = 4 \text{GeV}^2 \ \text{integrated over} \ \theta \in (\pi/4; 3\pi/4) \ \text{and over} \ \phi \in (0; 2\pi) \ \text{for} \ E_\gamma = 10 \text{GeV}(\eta \approx 0.11).\]

The interference part of the cross-section for \(\gamma p \to \ell^+ \ell^- p\) with unpolarized protons and photons is given by:

\[\frac{d\sigma_{INT}}{dQ'^2 \ dt \ d\cos \theta \ d\varphi} \sim \cos \varphi \cdot \text{Re} \ \mathcal{H}(\eta, t)\]

Linear in GPD’s, odd under exchange of the \(l^+\) and \(l^-\) momenta \(\Rightarrow\) angular distribution of lepton pairs is a good tool to study interference term.
Rafayel Paremuzyan PhD thesis

Figure: $e^+e^-$ invariant mass distribution vs quasi-real photon energy. For TCS analysis $M(e^+e^-) > 1.1 \text{ GeV}$ and $s_{\gamma p} > 4.6 \text{ GeV}^2$ regions are chosen. Left graph represents e1-6 data set, right one is from e1f data set.
Theory vs experiment

R.Paremuzyan and V.Guzey:

\[ R = \frac{\int d\phi \cos \phi \int d\theta \, d\sigma}{\int d\phi \int d\theta \, d\sigma} \]

\[ Q^2 = 1.3 \, GeV^2 \quad E_\gamma = 3.536 \, GeV \]

**Figure:** Theoretical prediction of the ratio \( R \) for various GPDs models. Data points after combining both e1-6 and e1f data sets.
The photon beam *circular polarization* asymmetry:

\[ A = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-} \sim \sin \varphi \cdot \text{Im} \mathcal{H}(\eta, t) \]

**Figure:** Photon beam circular polarization asymmetry as a function of $\phi$, for $t = -0.1$ GeV$^2$, $Q^2 = \mu^2 = 4$ GeV$^2$, integrated over $\theta \in (\pi/4, 3\pi/4)$ and for $E_\gamma = 10$ GeV ($\eta \approx 0.11$).
Jefferson Lab PAC 39 Proposal

Timelike Compton Scattering and $J/\psi$ photoproduction on the proton in $e^+e^-$ pair production with CLAS12 at 11 GeV


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Approved experiment at Hall B, and LOI for Hall A.
In the case of a linearly polarized photon we now have a distinguished transverse direction given by the polarization vector, which we choose to point in the $x$-direction:

$$\epsilon(q)^\mu = \delta^{1\mu}$$

Momenta of other particles in $\gamma - p$ c.m. frame are given by:

$$q^\mu = (q^0, 0, 0, q^0)$$
$$p^\mu = (p^0, 0, 0, -q^0)$$
$$q'^\mu = (q'^0, \Delta_T \cos \Phi_h, \Delta_T \sin \Phi_h, q'^3)$$
$$p'^\mu = (p'^0, -\Delta_T \cos \Phi_h, -\Delta_T \sin \Phi_h, -q'^3)$$

where $\Phi_h$ is the angle between polarization vector and hadronic plane:

$$\sin \Phi_h = \epsilon(q) \cdot \vec{n} = \epsilon(q) \cdot \frac{\vec{p}' \times \vec{p}}{|\vec{p}' \times \vec{p}|}$$

where $\vec{n}$ is the vector normal to the hadronic plane.
Let us now turn to the contribution of the interference between the TCS and BH mechanisms to the differential $\gamma p \rightarrow l^+ l^- p$ cross section. For incoming photons with polarization vector as in Eq. (3) it, up to our accuracy, reads:

$$\frac{d\sigma^{(INT)}}{dQ^2 dt d\Omega_{l+l^-} d\Phi_h} = \frac{1}{2^{11/2} \pi^5} \frac{1}{s^2} \cdot \frac{1}{2} \sum \left( M_{TCS}(\epsilon) M_{BH}^*(\epsilon) + c.c. \right)$$

$$\equiv \frac{d\sigma^{(INT)}_{unpol}}{dQ^2 dt d\Omega_{l+l^-} d\Phi_h} + \frac{d\sigma^{(INT)}_{linpol}}{dQ^2 dt d\Omega_{l+l^-} d\Phi_h},$$

where:

$$\frac{d\sigma^{(INT)}_{unpol}}{dQ^2 dt d\Omega_{l+l^-} d\Phi_h} \sim \left( \frac{1 + \cos^2 \theta}{\sin \theta} \cos \phi \right) \text{Re} \left[ \mathcal{H} F_1 - \frac{t}{4M^2} \mathcal{E} F_2 - \eta \tilde{\mathcal{H}}(F_1 + F_2) \right],$$

$$\frac{d\sigma^{(INT)}_{linpol}}{dQ^2 dt d\Omega_{l+l^-} d\Phi_h} \sim -\left( \sin \theta \cos(2\Phi_h + 3\phi) \right) \text{Re} \left[ \mathcal{H} F_1 - \frac{t}{4M^2} \mathcal{E} F_2 + \eta \tilde{\mathcal{H}}(F_1 + F_2) \right].$$
We define two observables sensitive to the unpolarized and linearly polarized part of interference cross section. First one is similar (up to the terms formally of the order $t/Q^2$ or $M^2/Q^2$) to the R ratio defined in BDP in the case of an unpolarized photon beam:

$$
\tilde{R} = \frac{\int_0^{2\pi} d\Phi_h \int_0^{2\pi} d\phi \cos(\phi) \int_{\pi/4}^{3\pi/4} \sin \theta d\theta \frac{d\sigma}{dt dQ^2 d\Omega d\Phi_h} \int_0^{2\pi} d\Phi_h \int_0^{2\pi} d\phi \int_{\pi/4}^{3\pi/4} \sin \theta d\theta \frac{d\sigma}{dt dQ^2 d\Omega d\Phi_h}}{\int_0^{2\pi} d\Phi_h \int_0^{2\pi} d\phi \int_{\pi/4}^{3\pi/4} \sin \theta d\theta \frac{d\sigma}{dt dQ^2 d\Omega d\Phi_h}}.
$$

The second observable projects out the $d\sigma_{(INT)}^{linpol}$ part of the interference cross section:

$$
\tilde{R}_3 = \frac{2 \int_0^{2\pi} d\Phi_h \cos(2\Phi_h) \int_0^{2\pi} d\phi \cos(3\phi) \int_{\pi/4}^{3\pi/4} \sin \theta d\theta \frac{d\sigma}{dt dQ^2 d\Omega d\Phi_h} \int_0^{2\pi} d\Phi_h \int_0^{2\pi} d\phi \int_{\pi/4}^{3\pi/4} \sin \theta d\theta \frac{d\sigma}{dt dQ^2 d\Omega d\Phi_h}}{\int_0^{2\pi} d\Phi_h \int_0^{2\pi} d\phi \int_{\pi/4}^{3\pi/4} \sin \theta d\theta \frac{d\sigma}{dt dQ^2 d\Omega d\Phi_h}}.
$$

Making use of $\tilde{R}$ and $\tilde{R}_3$ we can define the following observable which is sensitive only to the interference term and which provides us with information about $\tilde{H}$:

$$
C = \frac{\tilde{R}}{\tilde{R}_3} = \frac{2 - 3\pi}{2 + \pi} \frac{\text{Re} \left[ \mathcal{H} F_1 - \frac{t}{4M^2} \mathcal{E} F_2 - \eta \tilde{H}(F_1 + F_2) \right]}{\text{Re} \left[ \mathcal{H} F_1 - \frac{t}{4M^2} \mathcal{E} F_2 + \eta \tilde{H}(F_1 + F_2) \right]}.
$$
How sensitive is $C$ on the values of $\tilde{H}$? We take $\tilde{H}_g = \{-1, 0, 1, 2, 3\} \cdot \tilde{H}_g^{GK}$.

Figure: $C$ as a function of $\eta$, for $Q^2 = 4 \text{ GeV}^2$ and $t = t_0$.

Figure: $C$ as a function of $|t|$, for $Q^2 = 4 \text{ GeV}^2$ and $\eta = 0.1$. 
Experimental possibilities

Hall D - flux of linearly polarized photons, are rates big enough?
Hall B - low-$Q^2$ tagger,

In the unpolarized electron scattering process, the virtual photon polarization is:

$$
\epsilon = \left[ 1 + 2 \frac{Q^2 + \nu^2}{Q^2} \tan^2(\theta_e'/2) \right]^{-1}
$$

where $\nu$ is the photon energy and $\theta_e'$ the electron scattering angle. The longitudinal polarization is given by $\epsilon_L = \frac{Q^2}{\nu^2} \epsilon$, and the polarization density matrix:

$$
\rho = \begin{pmatrix}
\frac{1}{2}(1 + \epsilon) & 0 & \sim \epsilon_L^{1/2} \\
0 & \frac{1}{2}(1 - \epsilon) & 0 \\
\sim \epsilon_L^{1/2} & 0 & \epsilon_L
\end{pmatrix}
$$

end the matrix describes real transverse photons.
Effective photon approximation

\[ \frac{d\sigma^{ep}}{dQ_{\gamma}^2 d\nu} \sim F(Q_{\gamma}^2, \nu) M^*_\lambda \rho^{\lambda\lambda'} M_{\lambda'} , \]

\[ \rho^{\lambda\lambda'} \approx \begin{pmatrix} \frac{1}{2} (1 + \epsilon) & 0 & 0 \\ 0 & \frac{1}{2} (1 - \epsilon) & 0 \\ 0 & 0 & 0 \end{pmatrix} , \]

so:

\[ \frac{d\sigma^{ep}}{dQ_{\gamma}^2 d\nu} = F(Q_{\gamma}^2, \nu) \left[ \frac{1}{2} (1 + \epsilon) \sigma^{xx}_{\gamma p} + \frac{1}{2} (1 - \epsilon) \sigma^{yy}_{\gamma p} \right] \]

\[ = F(Q_{\gamma}^2, \nu) \left[ (1 - \epsilon) \sigma^{unp}_{\gamma p} + \epsilon \sigma^{linpol}_{\gamma p} \right] \]

\[ C^{ep} = \frac{\tilde{R}^{ep}}{\tilde{R}_3^{ep}} = \frac{1 - \epsilon}{\epsilon} C \]
Approved CLAS12 experiment - "Meson Spectroscopy with low $Q^2$ electron scattering in CLAS12".

Figure 8: $Q^2$ and linear polarization of inelastic events within the geometrical and momentum acceptance of the FT.
Approved experiment - E12-12-001: "Timelike Compton Scattering and J/psi photoproduction on the proton in e+e- pair production with CLAS12 at 11 GeV."

Approved experiment - E12-12-005: "Meson spectroscopy with low $Q^2$ electron scattering in CLAS12"

Both in the same run group. Idea: extend the first one, by use of the low-$Q^2$ tagger from the second one.
Summary

- Differences of spacelike and timelike Compton Scattering important - good test of universality of GPDs,
- TCS already measured at CLAS 6 GeV, but much richer and more interesting kinematical region available after upgrade to 12 GeV.
- Accepted proposal for CLAS 12 GeV,
- LOI for Hall A,
- Linear polarization in TCS may give some information on $\tilde{H}$.
- Possible with a low-$Q^2$ tagger at CLAS? Or with a photon flux at Hall D?