# Phenomenology of unpolarized TMDs from SIDIS data
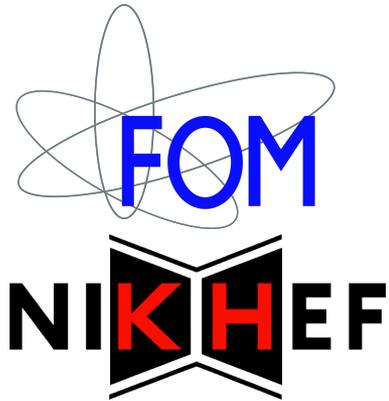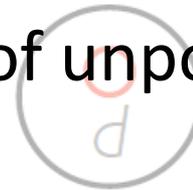
**Andrea Signori**

# Our recent work

## Investigations into the flavor dependence of partonic transverse momentum

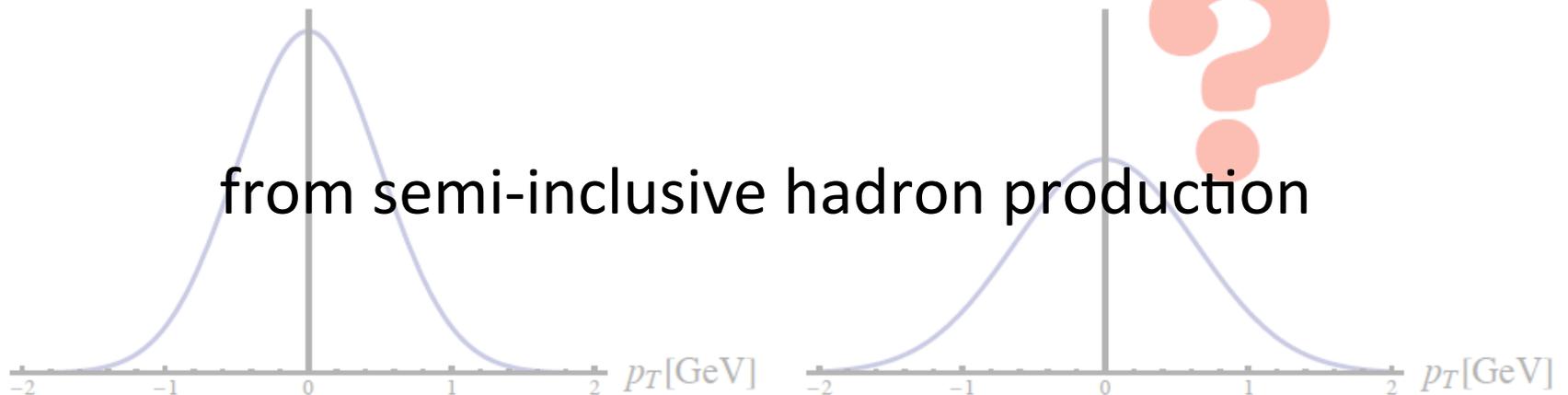Andrea Signori,[a,b] Alessandro Bacchetta,[c,d] Marco Radici[c] and Gunar Schnell[e,f]

# A new goal

Access the flavor dependence of unpolarized

transverse-momentum-dependent distributions (TMDs)

from semi-inclusive hadron production

$p_T$ [GeV] $\quad$ $p_T$ [GeV]

partonic transverse momenta

up quarks

down quarks

sea quarks

$p_{T,i}$

$x_i P$

$D_1^a(z, P_\perp^2, Q^2)$

collinear partonic momenta

$P$

photon

$p_{T,j}$

proton momentum

$p_{T,m}$  $p_{T,k}$  $p_{T,l}$  $x_j P$

$x_m P$

$x_k P$

$x_l P$
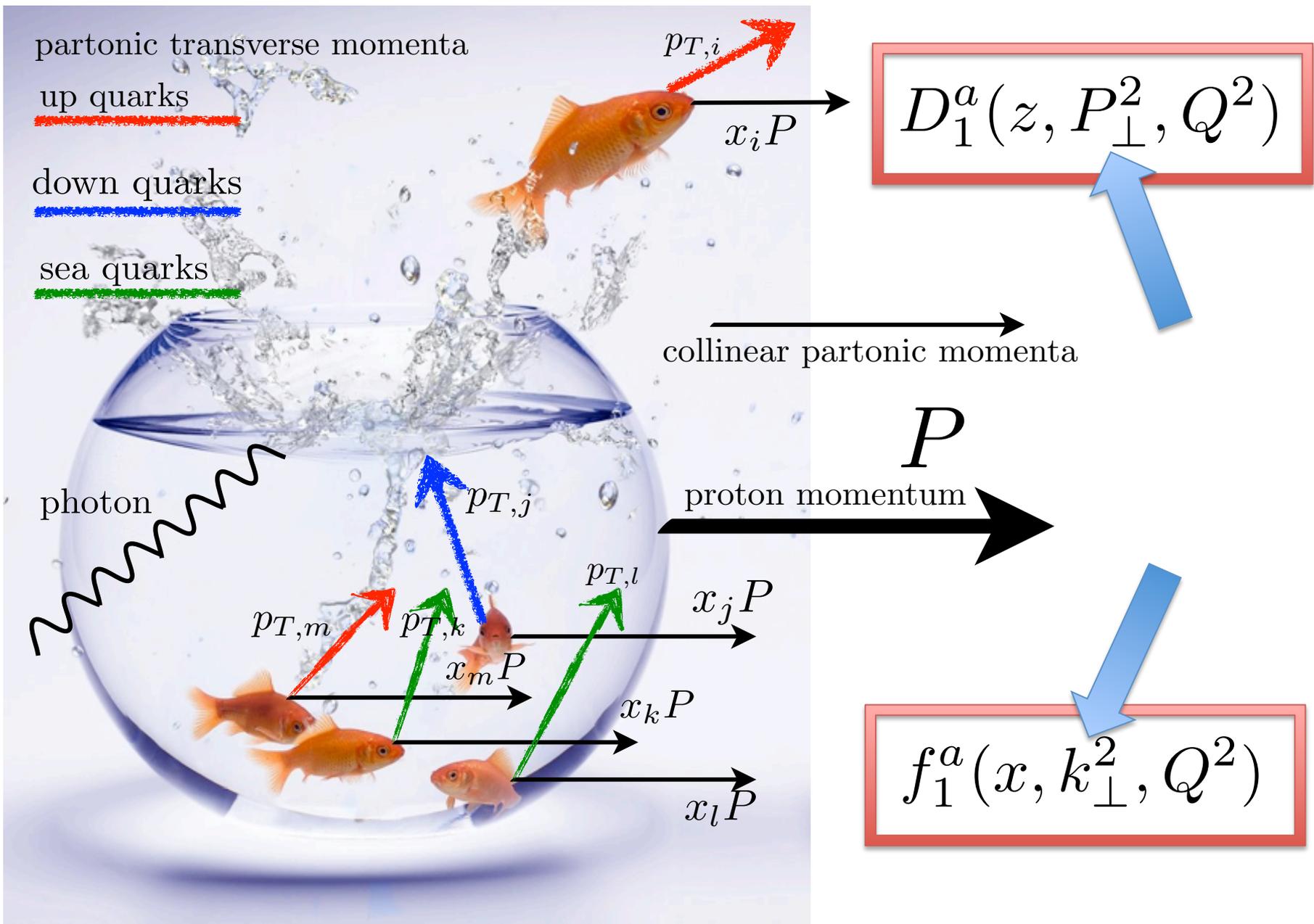
$f_1^a(x, k_\perp^2, Q^2)$

# TMDs in SIDIS

**Unpolarized**

## SIDIS

lepton          lepton

hadron

proton
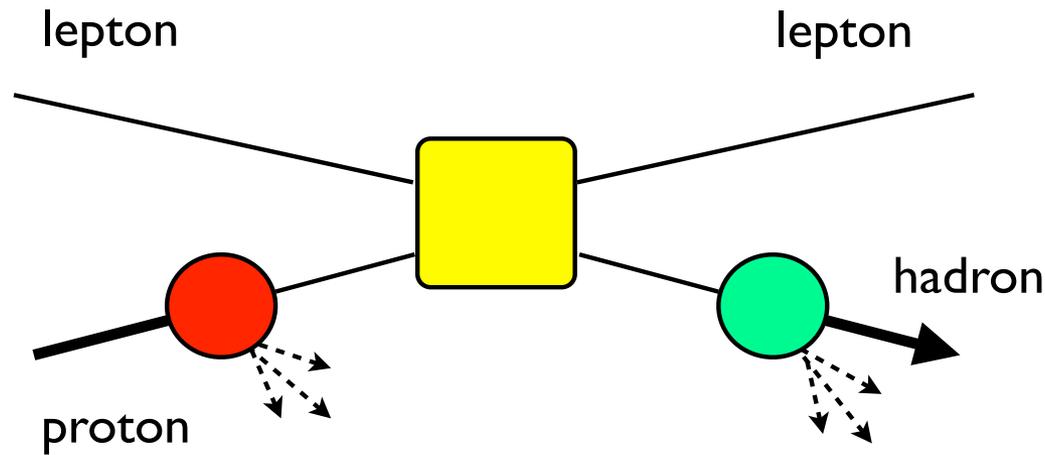
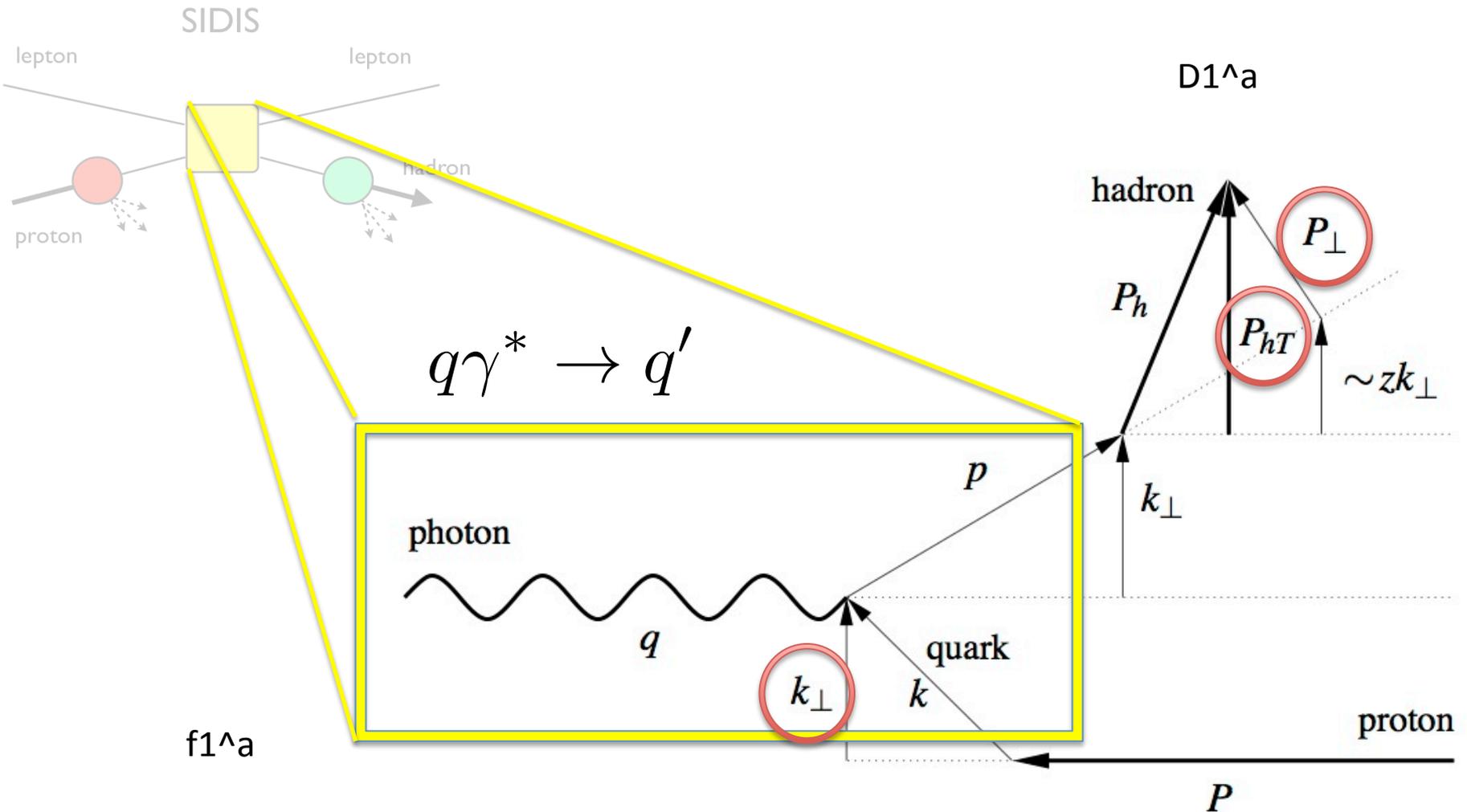*Parton model picture*

LO QED

*Zero-order QCD*

Leading twist
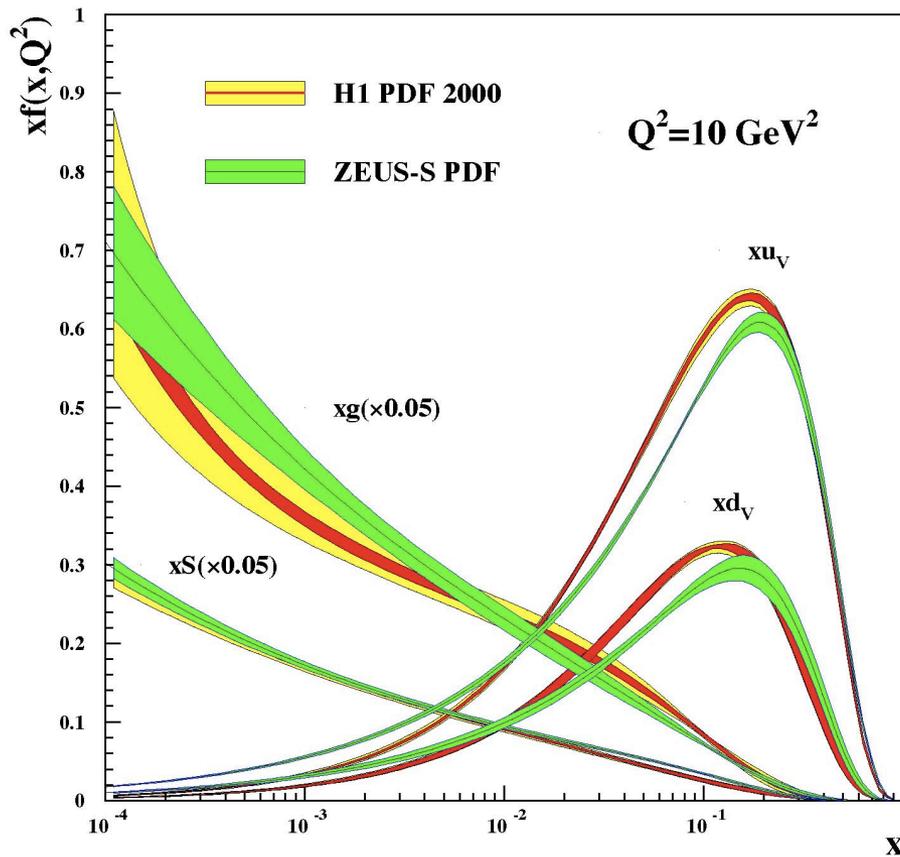
$$\sigma(P_{hT}^2) \sim \sum_a f_1^a(x, k_\perp^2) \otimes D_1^{a \to h}(z, P_\perp^2)$$

# Which transverse momenta ?



SIDIS

$q\gamma^* \to q'$

D1^a

f1^a

# Motivations

# Flavor in transverse space



Since the flavor dependence
In the collinear case is strong …

… WHY NOT
LOOKING FOR IT IN

$K_\perp$ DEPENDENCE

OF TMDs ?

# Flavor in transverse space

✓ **Lattice** QCD calculations
Musch *et al.*,  PRD **83** (11) 094507
[ … ]


✓ **Model** calculations
Chiral quark soliton model [ Scweitzer *et al.,* JHEP 1301 (913) 163 ]
Diquark spectator model [ Bacchetta *et al.,* PRD **78** (08) 074010 ]
Statistical approach [ Bourrely *et al.,* PRD **83** (11) 074008 ]
NJL-jet model [ Matevosyan *et al*.,  PRD **85** (12) 014021 ]
[ … ]


✓ **Previous** fits
JLab Hall C [ Asaturyan *et al*., (E00-108),  PRC **85** (12) 015202 ]

# Flavor in transverse space

With flavor dependence we can account *theoretically* for different cross sections for different target/final state hadron combinations.
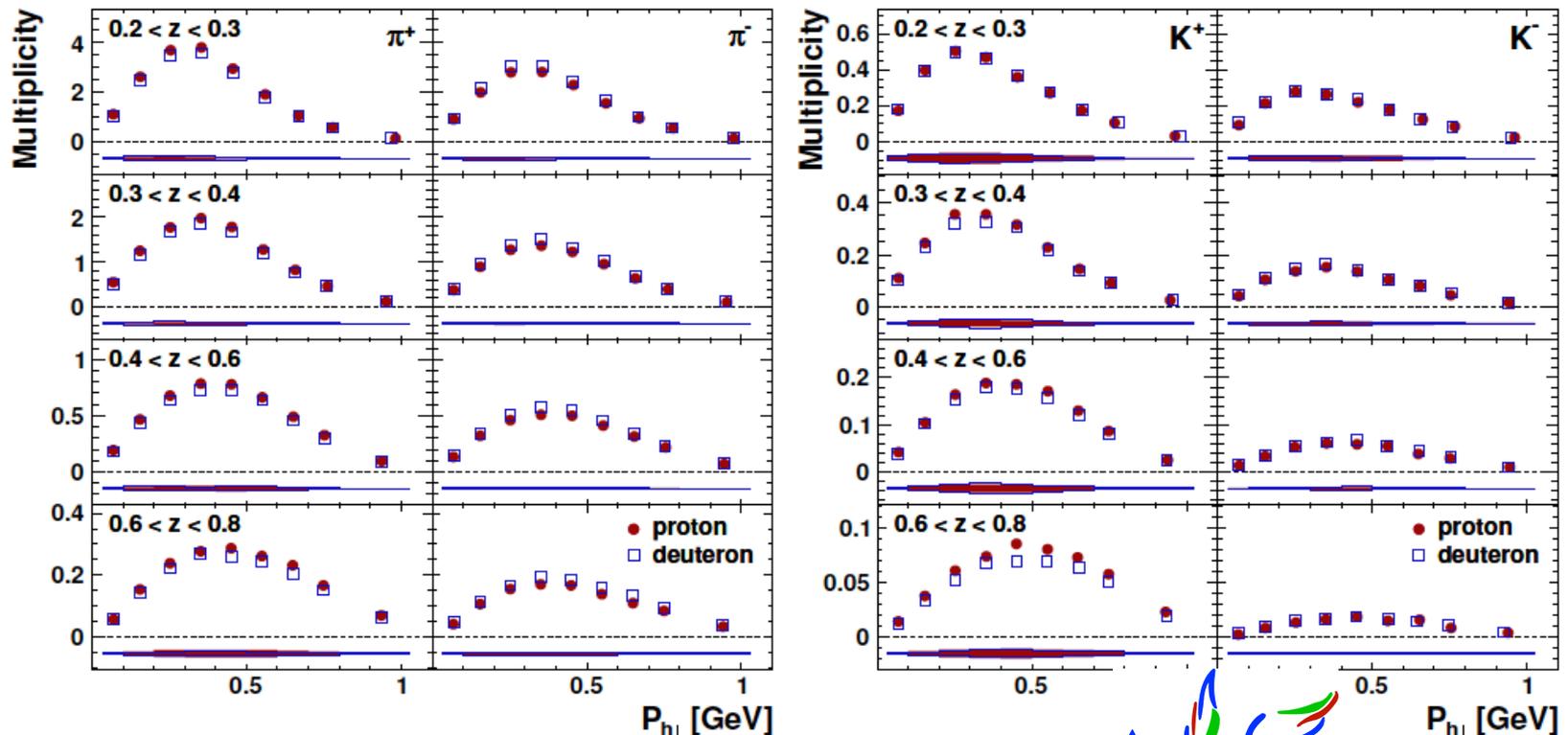
# Flavor dependent

# Gaussian TMDs

# Flavor dependent Gaussians

**_Different Gaussian parametrizations of TMD parts_**

$$f_1^a(x, k_\perp^2) = f_1^a(x) \boxed{\frac{1}{\pi \langle k_{\perp,a}^2 \rangle} \exp\left\{-\frac{k_\perp^2}{\langle k_{\perp,a}^2 \rangle}\right\}}$$

$$\langle k_{\perp,\mathrm{u_v}}^2 \rangle \neq \langle k_{\perp,\mathrm{d_v}}^2 \rangle \neq \langle k_{\perp,\mathrm{sea}}^2 \rangle$$
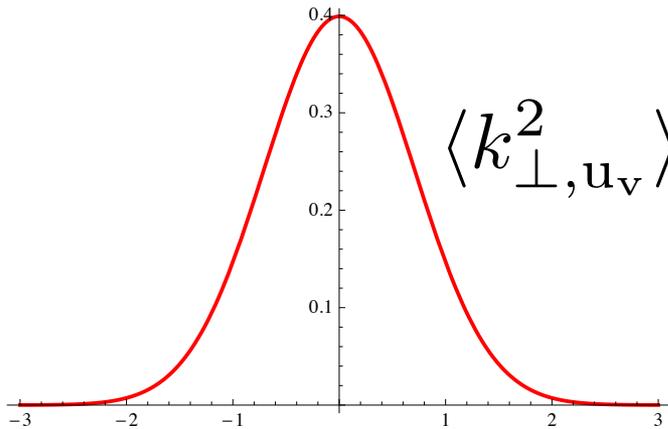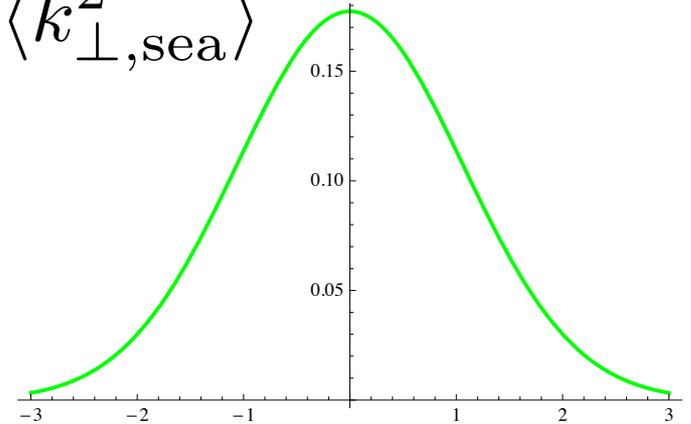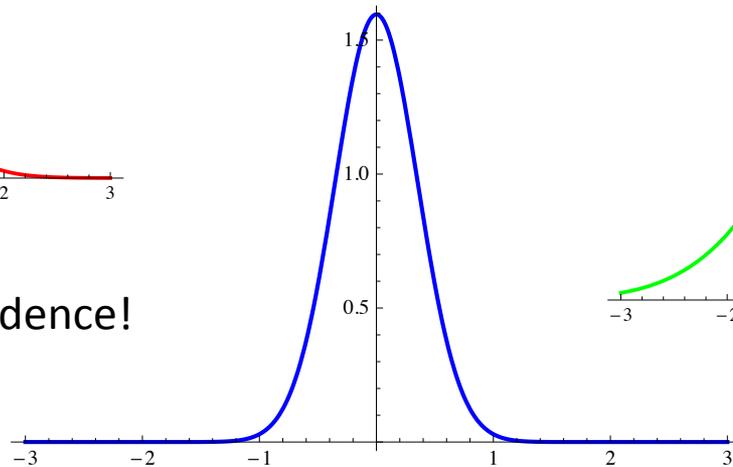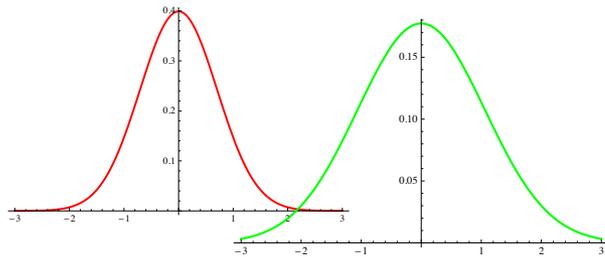
Simplified flavor dependence!

# Flavor dependent Gaussians

**_Different Gaussian parametrization of TMD parts_**

$$D_1^{a \to h}(z, P_\perp) = D_1^{a \to h}(z) \frac{1}{\pi \langle P_{\perp,a\to h}^2 \rangle} \exp \left\{ - \frac{P_\perp^2}{\langle P_{\perp,a\to h}^2 \rangle} \right\}$$

4 different combinations out of

$$\mathrm{u,\ d,\ s} \quad \Longleftrightarrow \quad \pi^\pm,\ K^\pm$$

$$\langle \boldsymbol{P}_{\perp,u\to\pi^+}^2 \rangle = \langle \boldsymbol{P}_{\perp,\bar{d}\to\pi^+}^2 \rangle = \langle \boldsymbol{P}_{\perp,\bar{u}\to\pi^-}^2 \rangle = \langle \boldsymbol{P}_{\perp,d\to\pi^-}^2 \rangle \equiv \langle \boldsymbol{P}_{\perp,\mathrm{fav}}^2 \rangle,$$

$$\langle \boldsymbol{P}_{\perp,u\to K^+}^2 \rangle = \langle \boldsymbol{P}_{\perp,\bar{u}\to K^-}^2 \rangle \equiv \langle \boldsymbol{P}_{\perp,uK}^2 \rangle,$$

$$\langle \boldsymbol{P}_{\perp,\bar{s}\to K^+}^2 \rangle = \langle \boldsymbol{P}_{\perp,s\to K^-}^2 \rangle \equiv \langle \boldsymbol{P}_{\perp,sK}^2 \rangle,$$

$$\langle \boldsymbol{P}_{\perp,\mathrm{all\ others}}^2 \rangle \equiv \langle \boldsymbol{P}_{\perp,\mathrm{unf}}^2 \rangle.$$
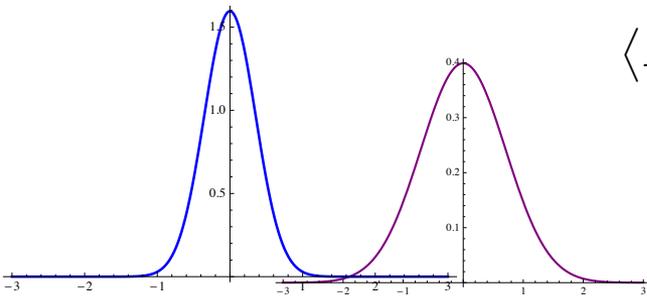
# Kinematic dependence

$$\langle \boldsymbol{k}_{\perp,\,q}^2 \rangle(x) = \langle \widehat{\boldsymbol{k}_{\perp,\,q}^2} \rangle \, \frac{(1-x)^\alpha \, x^\sigma}{(1-\hat{x})^\alpha \, \hat{x}^\sigma}$$

Flavor independent
Kinematic dependence

$$\langle \widehat{\boldsymbol{k}_{\perp,\,q}^2} \rangle = \langle \boldsymbol{k}_{\perp,\,q}^2 \rangle(\hat{x} = 0.1)$$

$$\langle \boldsymbol{P}_{\perp,\,q\to h}^2 \rangle(z) = \langle \widehat{\boldsymbol{P}_{\perp,\,q\to h}^2} \rangle \, \frac{(z^\beta + \delta)\,(1-z)^\gamma}{(\hat{z}^\beta + \delta)\,(1-\hat{z})^\gamma}$$

$$\langle \widehat{\boldsymbol{P}_{\perp,\,q\to h}^2} \rangle = \langle \boldsymbol{P}_{\perp,\,q\to h}^2 \rangle(\hat{z} = 0.5)$$

Flavor independent
Kinematic dependence

# Flavor analysis

| Parameters for TMD PDFs | | | | | |
|---|---|---|---|---|---|
| | $\langle \hat{\boldsymbol{k}}^2_{\perp, d_v} \rangle$ [GeV$^2$] | $\langle \hat{\boldsymbol{k}}^2_{\perp, u_v} \rangle$ [GeV$^2$] | $\langle \hat{\boldsymbol{k}}^2_{\perp, \text{sea}} \rangle$ [GeV$^2$] | $\alpha$ (random) | $\sigma$ (random) |

5 parameters

interval [0,2]          interval [-0.3,0.1]

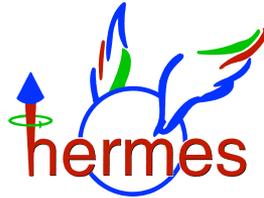| Parameters for TMD FFs | | | | | | |
|---|---|---|---|---|---|---|
| | $\langle \hat{\boldsymbol{P}}^2_{\perp, \text{fav}} \rangle$ [GeV$^2$] | $\langle \hat{\boldsymbol{P}}^2_{\perp, \text{unf}} \rangle$ [GeV$^2$] | $\langle \hat{\boldsymbol{P}}^2_{\perp, sK} \rangle$ [GeV$^2$] (random) | $\langle \hat{\boldsymbol{P}}^2_{\perp, uK} \rangle$ [GeV$^2$] | $\beta$ | $\delta$ | $\gamma$ |

7 parameters

interval [0.125,0.250]

# Analysis

# of HERMES data

# Hermes

$$e^{\pm} + \boxed{P/D} \to e^{\pm} + \{\pi^{\pm}/K^{\pm}\} + X$$

6 bins in x,
8 bins in z,
7 bins in $P_{hT}$,
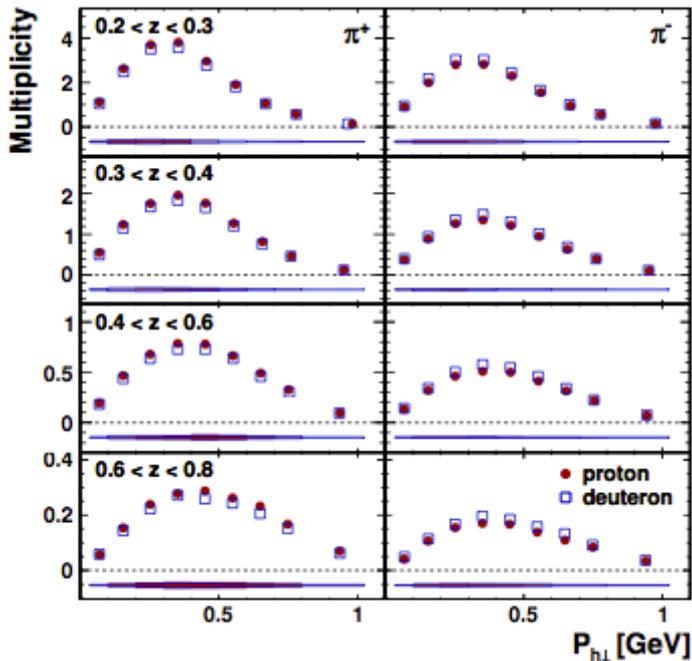2 targets, 4 final-state hadrons

hermes

Our selection

- Remove the first bin x-$Q^2$ ($Q^2 > 1.4$ GeV$^2$)

- $0.1 < z < 0.8$

- $P_{hT}^2 < Q^2/3$

2688 points

1538 analyzed points

limited $Q^2$ range $\Rightarrow$ safely neglect evolution <u>everywhere</u>

# Fitting procedure
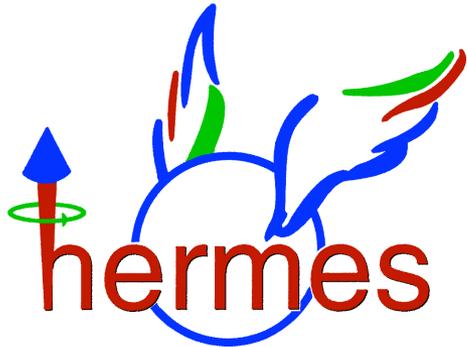


$\times$ **200**

Monte Carlo fit

***Distributions*** *of best-values*

# Fitting procedure



$m(x, z, P_{hT}, Q^2)$, proton target

$\pi^-$

$\langle x \rangle \sim 0.15$

$\langle Q^2 \rangle \sim 2.9 \text{ GeV}^2$

▲ $0.27 < z < 0.30$
■ $0.38 < z < 0.48$
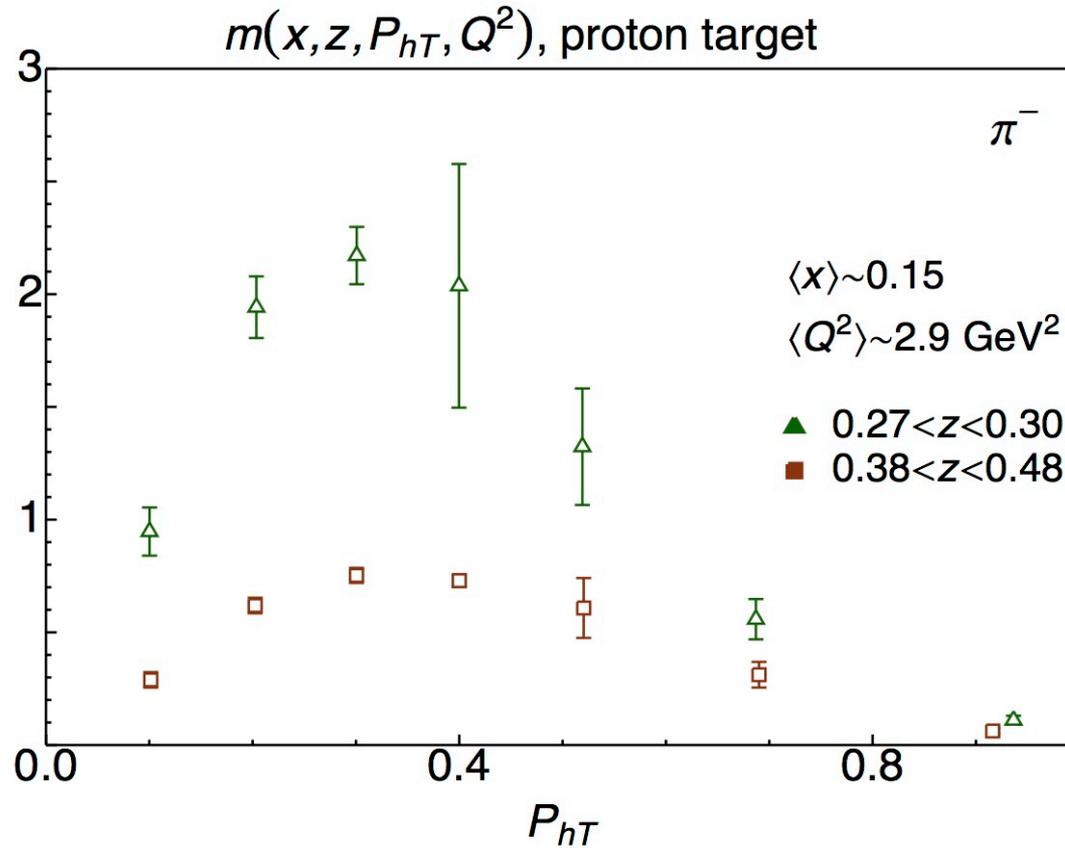
Sample of original data

# Fitting procedure



$m(x, z, P_{hT}, Q^2)$, proton target

$\pi^-$

$\langle x \rangle \sim 0.15$

$\langle Q^2 \rangle \sim 2.9$ GeV$^2$

▲ $0.27 < z < 0.30$
■ $0.38 < z < 0.48$

$P_{hT}$
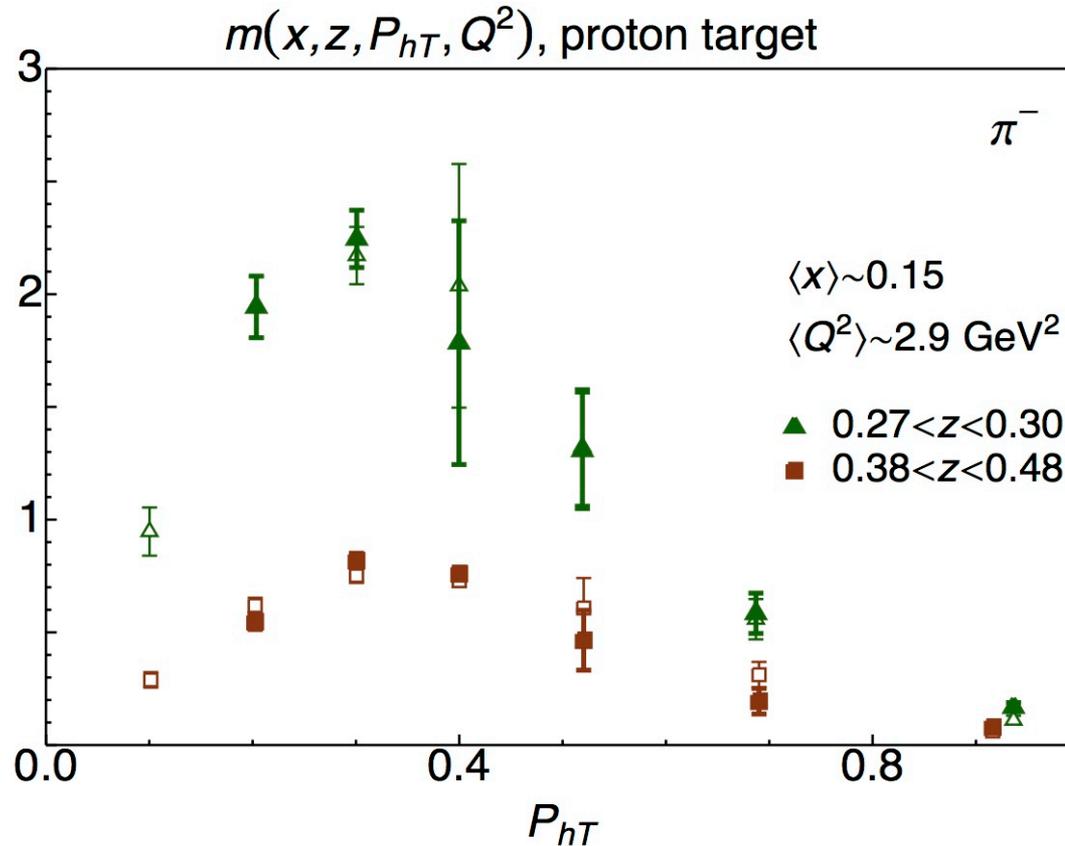
Replica of the original data with Gaussian noise

# Fitting procedure



$m(x, z, P_{hT}, Q^2)$, proton target

$\pi^-$

$\langle x \rangle \sim 0.15$

$\langle Q^2 \rangle \sim 2.9$ GeV$^2$

▲ $0.27 < z < 0.30$
■ $0.38 < z < 0.48$

$P_{hT}$
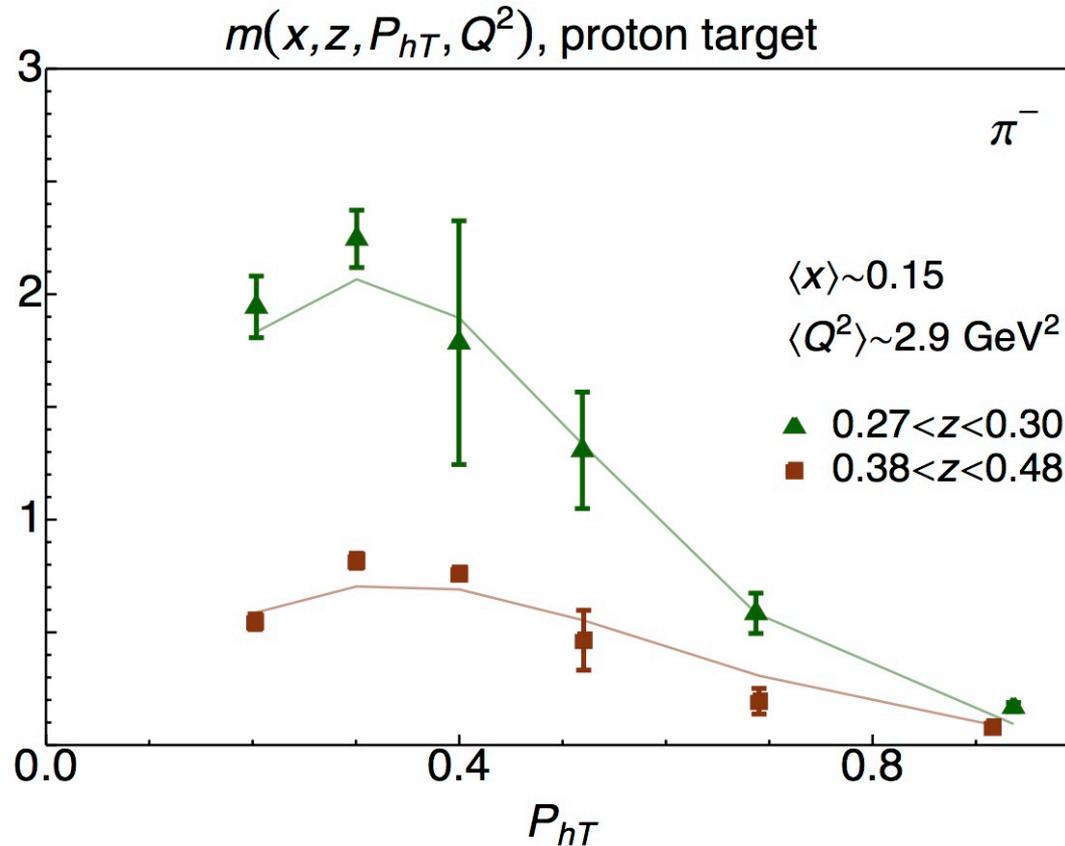
We fit the replicated data…

# Fitting procedure



$m(x, z, P_{hT}, Q^2)$, proton target

$\pi^-$

$\langle x \rangle \sim 0.15$

$\langle Q^2 \rangle \sim 2.9$ GeV$^2$

▲ $0.27 < z < 0.30$
■ $0.38 < z < 0.48$

$P_{hT}$
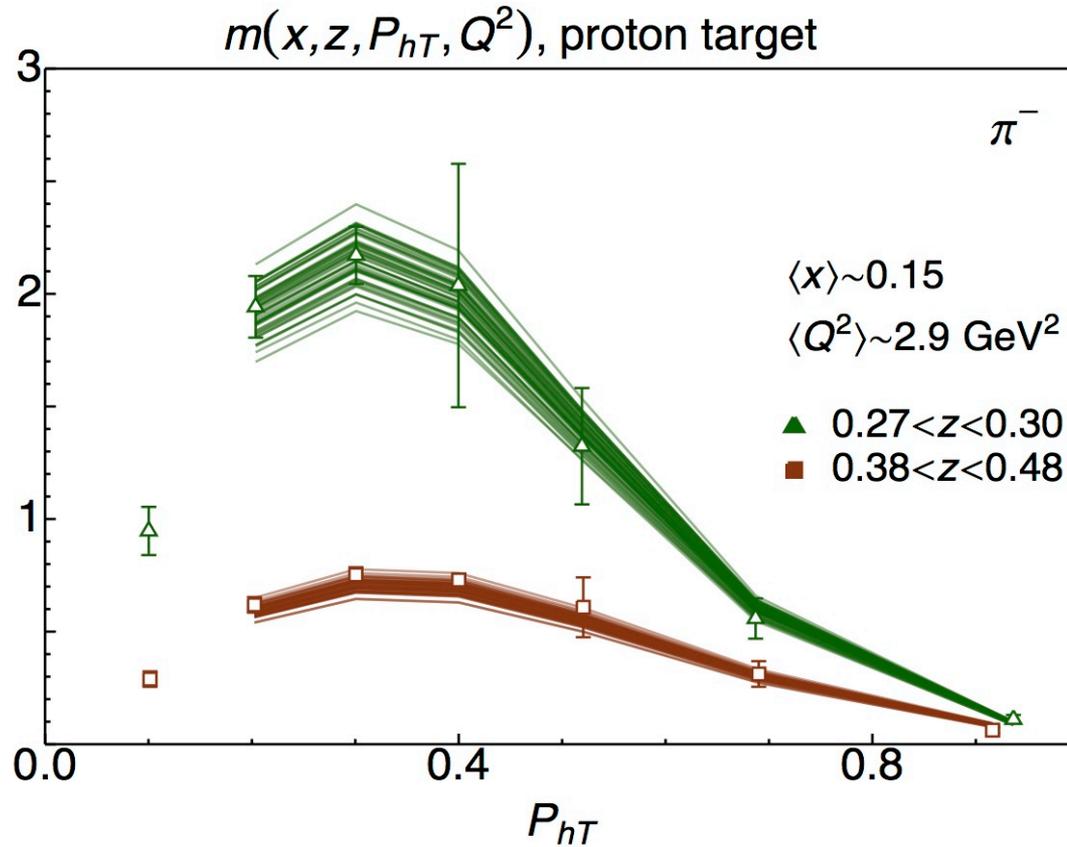
… repeating the fit over the 200 replicas

# Fitting procedure
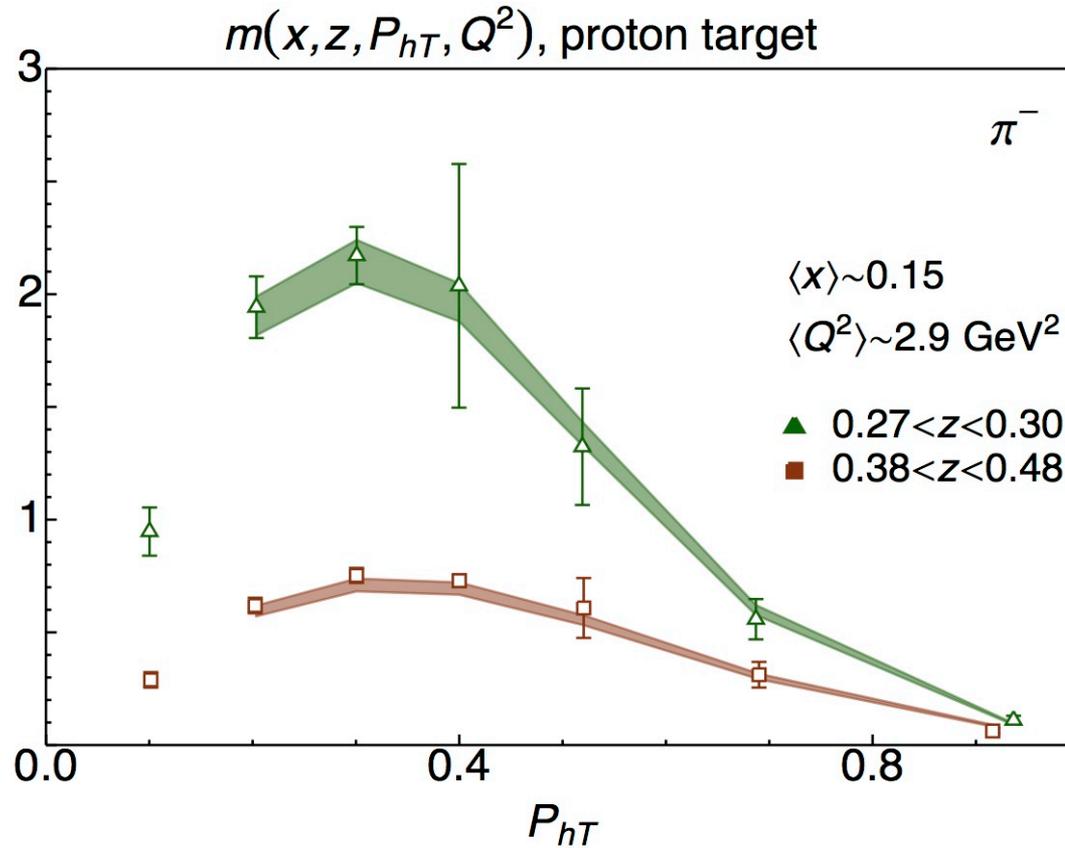


Plot of the 68% CL bands

# Fitting procedure

proton target      global  $\chi^2$ / d.o.f.  = 1.63 ± 0.12

no flavor dep.           1.72 ± 0.11

$\pi^-$
1.80 ± 0.27
1.83 ± 0.25

$\pi^+$
2.64 ± 0.21
2.89 ± 0.23

$K^-$
0.78 ± 0.15
0.87 ± 0.16

$K^+$
0.46 ± 0.07
0.43 ± 0.07



$m(x, z, P_{hT}^2, Q^2)$, proton target

$\langle x \rangle \sim 0.15$
$\langle Q^2 \rangle \sim 2.9$ GeV$^2$

0.10 < z < 0.20
0.27 < z < 0.30
0.38 < z < 0.48
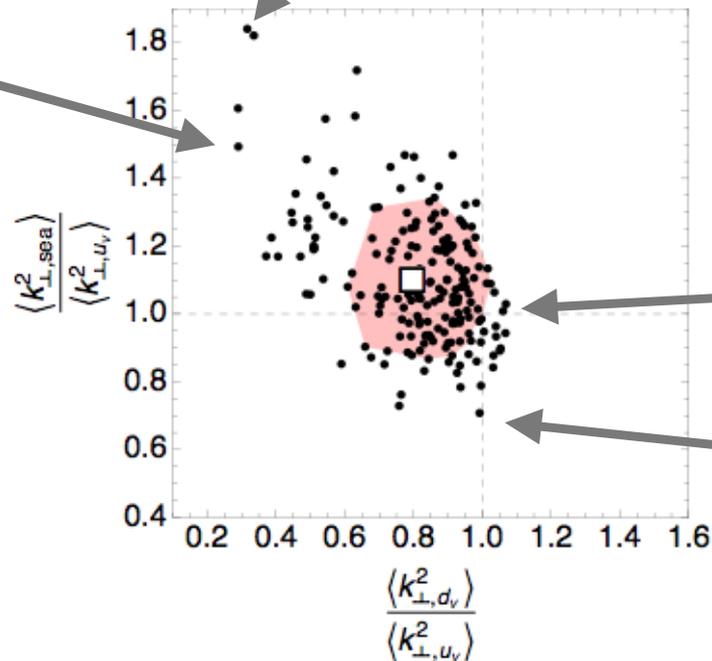0.60 < z < 0.80
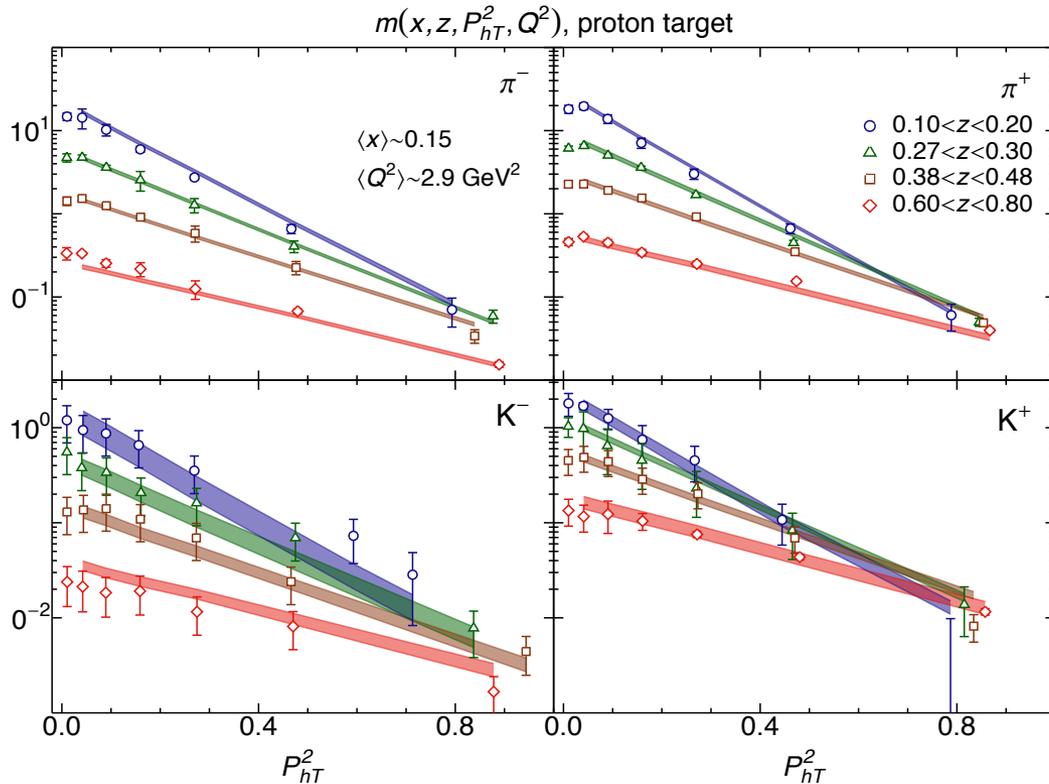
# TMD PDFs – full analysis

replica 149
$\chi^2/\text{dof} = 1.87$

replica 73
$\chi^2/\text{dof} = 1.70$

sea width

> (mostly)

$u_v$ width

replica 130
$\chi^2/\text{dof} = 1.77$

replica 186
$\chi^2/\text{dof} = 1.38$
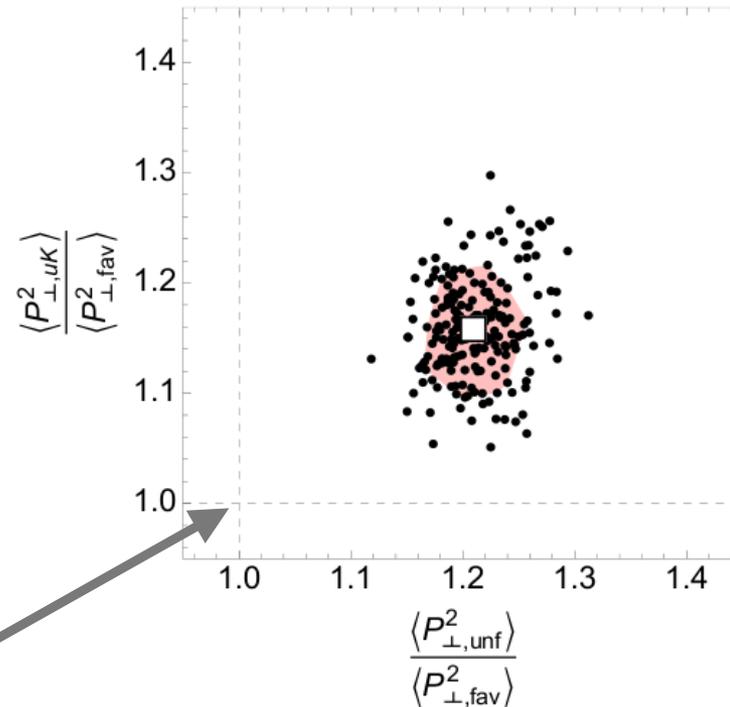
point of
no flavor dep.

$d_v$ width  < (mostly)  $u_v$ width

# TMD PDFs – full analysis

68% confidence intervals of best-fit parameters for TMD PDFs in the different scenarios

| Parameters for TMD PDFs | | | | |
|---|---|---|---|---|
| | Default | $Q^2 > 1.6 \text{ GeV}^2$ | Pions only | Flavor-indep. |
| $\langle \hat{\boldsymbol{k}}_{\perp,d_v}^2 \rangle$ [GeV$^2$] | $0.30 \pm 0.17$ | $0.33 \pm 0.19$ | $0.34 \pm 0.12$ | $0.30 \pm 0.10$ |
| $\langle \hat{\boldsymbol{k}}_{\perp,u_v}^2 \rangle$ [GeV$^2$] | $0.36 \pm 0.14$ | $0.37 \pm 0.17$ | $0.35 \pm 0.12$ | $0.30 \pm 0.10$ |
| $\langle \hat{\boldsymbol{k}}_{\perp,\text{sea}}^2 \rangle$ [GeV$^2$] | $0.41 \pm 0.16$ | $0.31 \pm 0.18$ | $0.29 \pm 0.13$ | $0.30 \pm 0.10$ |
| $\alpha$ (random) | $0.95 \pm 0.72$ | $0.93 \pm 0.70$ | $0.95 \pm 0.68$ | $1.03 \pm 0.64$ |
| $\sigma$ (random) | $-0.10 \pm 0.13$ | $-0.10 \pm 0.13$ | $-0.09 \pm 0.14$ | $-0.12 \pm 0.12$ |

# TMD FFs – full analysis

q→π  favored width
<
q→K  favored width



$$\frac{\langle P^2_{\perp,uK}\rangle}{\langle P^2_{\perp,\text{fav}}\rangle}$$

$$\frac{\langle P^2_{\perp,\text{unf}}\rangle}{\langle P^2_{\perp,\text{fav}}\rangle}$$

point of
no flavor dep.

q→π  favored width  <  unfavored

# TMD FFs – full analysis

68% confidence intervals of best-fit parameters for TMD FFs in the different scenarios

| Parameters for TMD FFs | | | | |
|---|---|---|---|---|
| | Default | $Q^2 > 1.6\ \mathrm{GeV}^2$ | Pions only | Flavor-indep. |
| $\left\langle \hat{\boldsymbol{P}}^2_{\perp,\mathrm{fav}} \right\rangle$ [GeV$^2$] | $0.15 \pm 0.04$ | $0.15 \pm 0.04$ | $0.16 \pm 0.03$ | $0.18 \pm 0.03$ |
| $\left\langle \hat{\boldsymbol{P}}^2_{\perp,\mathrm{unf}} \right\rangle$ [GeV$^2$] | $0.19 \pm 0.04$ | $0.19 \pm 0.05$ | $0.19 \pm 0.04$ | $0.18 \pm 0.03$ |
| $\left\langle \hat{\boldsymbol{P}}^2_{\perp,sK} \right\rangle$ [GeV$^2$] | $0.19 \pm 0.04$ | $0.19 \pm 0.04$ | - | $0.18 \pm 0.03$ |
| $\left\langle \hat{\boldsymbol{P}}^2_{\perp,uK} \right\rangle$ [GeV$^2$] | $0.18 \pm 0.05$ | $0.18 \pm 0.05$ | - | $0.18 \pm 0.03$ |
| $\beta$ | $1.43 \pm 0.43$ | $1.59 \pm 0.45$ | $1.55 \pm 0.27$ | $1.30 \pm 0.30$ |
| $\delta$ | $1.29 \pm 0.95$ | $1.41 \pm 1.06$ | $1.20 \pm 0.63$ | $0.76 \pm 0.40$ |
| $\gamma$ | $0.17 \pm 0.09$ | $0.16 \pm 0.10$ | $0.15 \pm 0.05$ | $0.22 \pm 0.06$ |

# TMD lib & plotter

**TMD lib** :
a library of parametrizations of TMDs and uPDFs
on the same footing of LHAPDF
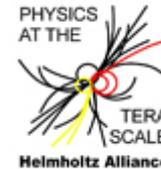
F. Hautmann, H. Jung
T. Rogers, P. Mulders, AS

## TMD Project

**Webpage maintained by:** Ted Rogers, Andrea Signori

This is the development page for the TMD project. The purpose of this project is to organize a repository theoretical and phenomenological studies of transverse-momentum-dependent parton distribution functions (PDFs) and fragmentation functions (TMD FFs). We provde access to parametrizations and fits of TMDs, with and without taking into account the perturbative QCD evolution.

Coming soon !

## High Energy Physics | TMD Plotter

| Home | TMD Plotter | Publications | HEP Links |

# Extensions

[ Amsterdam – Madrid – Pavia joint venture ]

Looking towards SIDIS + DY + l⁺l⁻ global fits ...

Collins (2011)
EIS (2012-2013)

$$\sigma \sim \int d^2\mathbf{b}\, e^{-i\mathbf{b}\cdot\mathbf{q}_T}\, \mathcal{H}\left[\frac{Q_f}{\mu}\right] \times$$

$$\left[C_{f/j} \otimes f_{j/N}\right](x, b_*, Q_0)\, \mathcal{F}^{NP}_{PDF} \times$$

Intrinsic (large b)
transverse momenta

Collinear OPE
for small $b_T$

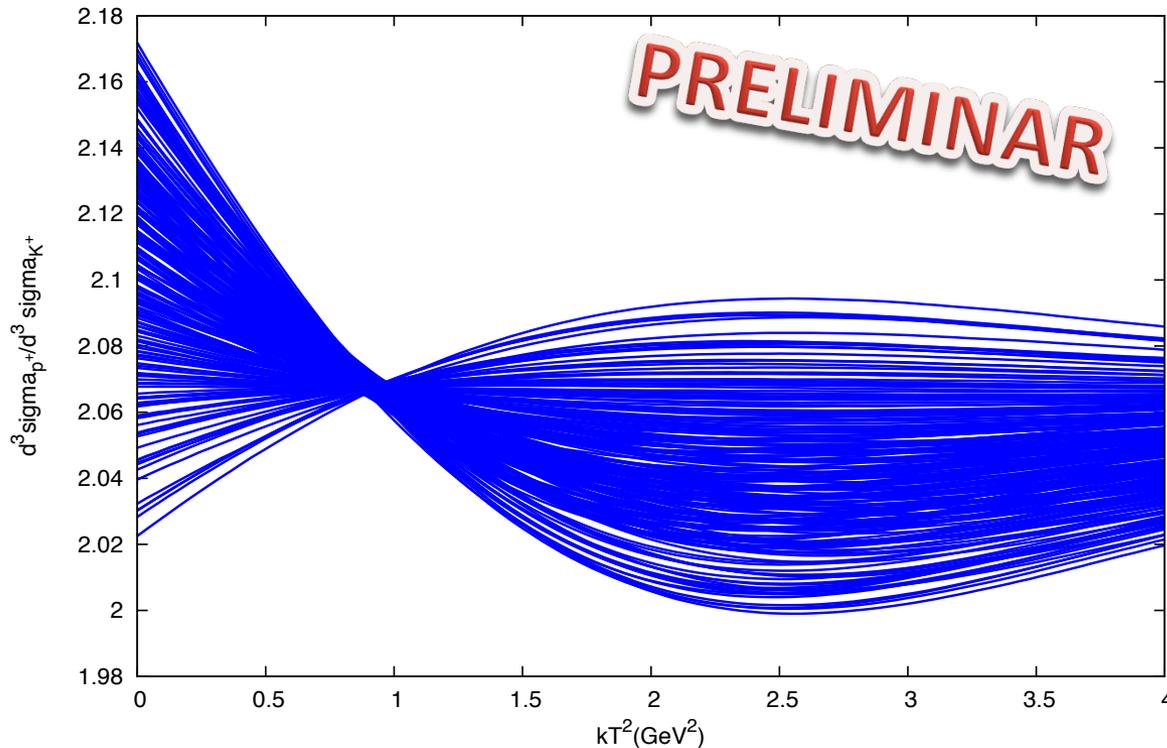$$\left[C_{D/i} \otimes D_{i/h}\right](z, b_*, Q_0)\, \mathcal{F}^{NP}_{FF} \times \quad \left(Q_0 \sim 1/b^*\right)$$

$$\exp\left\{\int_{Q_0}^{Q_f} \frac{d\mu}{\mu}\, \gamma_{PDF}\left[\ln\frac{Q_f^2}{\mu^2}, \alpha_S(\mu)\right]\right\}\left[\frac{Q_f}{Q_0}\right]^{-D(b^*,Q_0)+NP} \times$$

Perturbative
transverse momenta
and evolution
(resummed logs)

Soft evolution

$$\exp\left\{\int_{Q_0}^{Q_f} \frac{d\mu}{\mu}\, \gamma_{FF}\left[\ln\frac{Q_f^2}{\mu^2}, \alpha_S(\mu)\right]\right\}\left[\frac{Q_f}{Q_0}\right]^{-D(b^*,Q_0)+NP}$$

# e⁺e⁻ multiplicities

$$e^+ e^- \longrightarrow h \text{ jet } X$$

Pin down the most physical flavor dependent sets looking at the spectrum in transverse momentum of e⁺e⁻ collisions

Ratio of cross-section p+/K+ (y=0.2,z=0.6 - NLL)



Bands built out of the 200 replicas for TMD FF $D_1$

Waiting for the experimental values of these ratios..!

# Impact on HEP

Spin asymmetries – extractions of polarized TMDs

$$A_{\vec{e}\,\vec{N}}^{f(\phi_h,\phi_S)} \propto \frac{F_{\vec{e}\,\vec{N}}^{f(\phi_h,\phi_S)}}{F_{UU}} \propto \frac{\sum_q e_q^2 \ \mathrm{TMD\_PDF}^q \ \otimes_w \ \mathrm{TMD\_FF}^q}{\boxed{\sum_q e_q^2 \ f_1^q \otimes D_1^q}}$$

Monte Carlo generators dealing with
intrinsic transverse momentum

**Herwig++**       **gmc_trans
TMDgen**       **ResBos**

# Conclusions

**_There is a lot of room for flavor dependence_** _:_

- clear indication in TMD FFs that
  "q→π favored" width  <  "unfavored"  &  "q→K    favored"

- TMD PDFs: hints that  $d_v$ width < $u_v$ width < sea width

- if no K in final state:  sea width < $d_v$ ~ $u_v$ width

- flavor-independent fit performs worse but not ruled out

- anticorrelation: many intrinsic $\{k_\perp, P_\perp\}$ give same $P_{hT}$

# Backup slides

# Flavor in transverse space

valence picture of proton, #u / #d = 2

Less up at larger transverse momentum..!
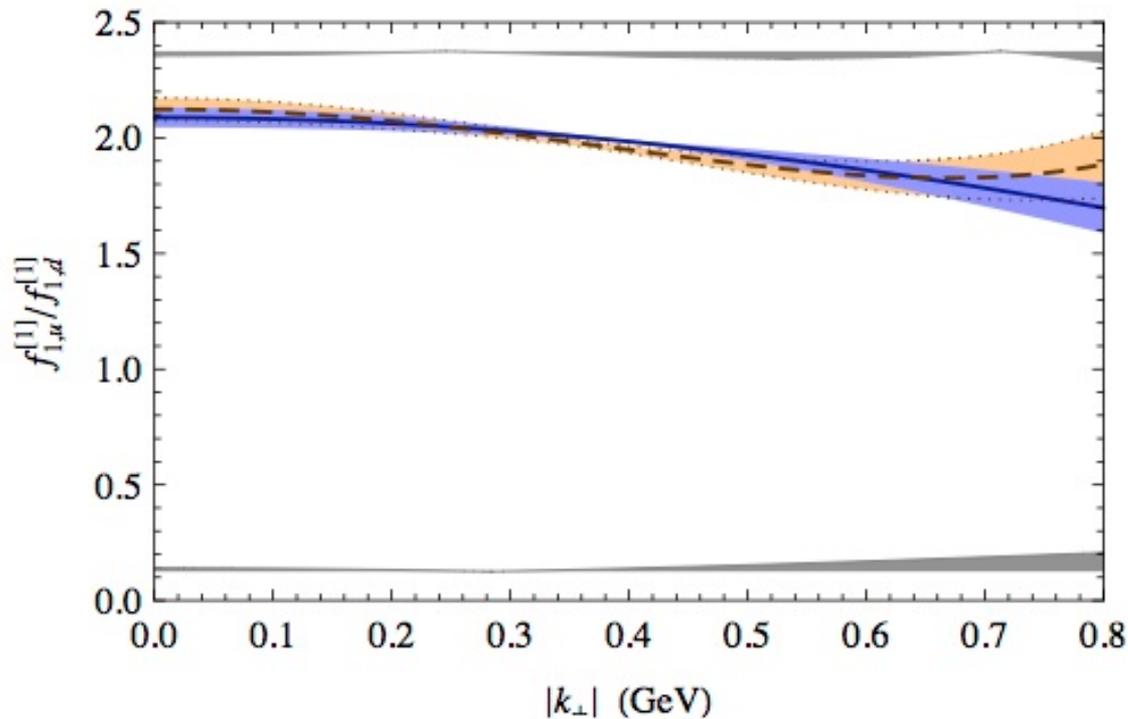


Lattice QCD

Musch *et al.*, P.R. D**83** (11) 094507

# Flavor in transverse space
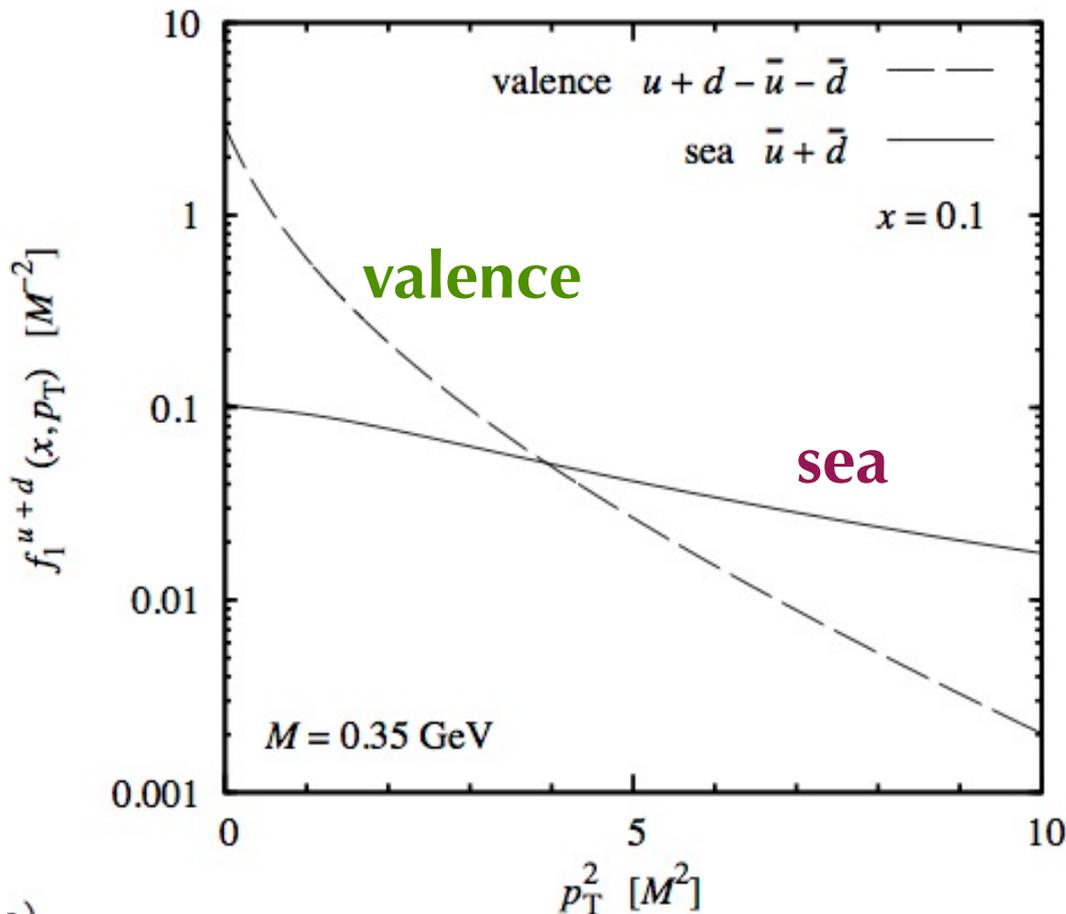


Chiral quark soliton model
(Schweitzer, Strikman, Weiss)
JHEP 1301 (913) 163

And other models…

Diquark spectator
(Bacchetta, Courtoy, Radici –
PRD 78 (08) 074010)

Statistical approach
(Bourrely, Buccella, Soffer
- PRD 83 (11) 074008

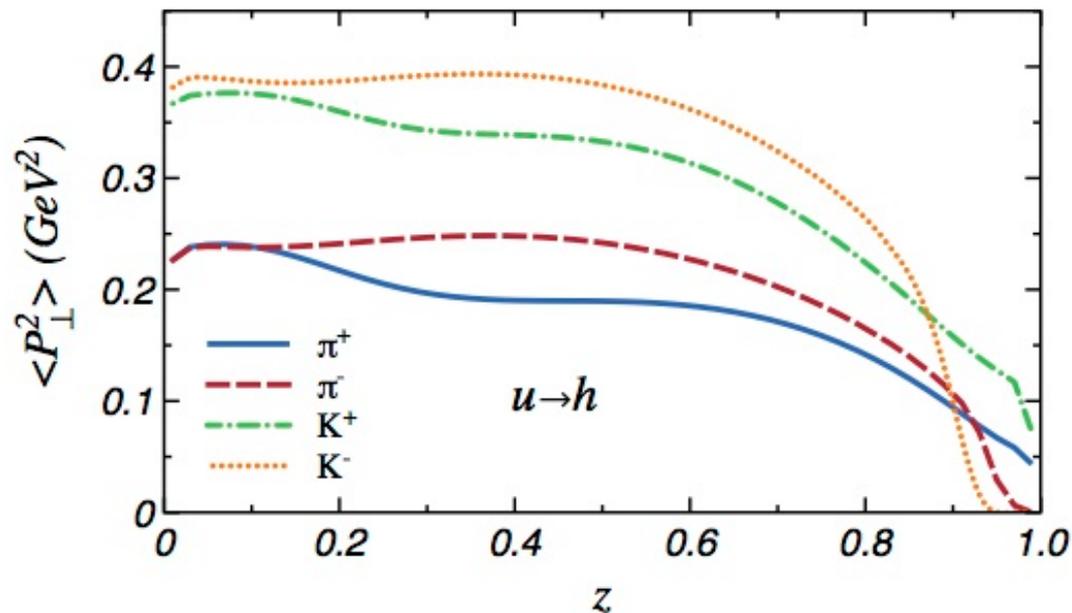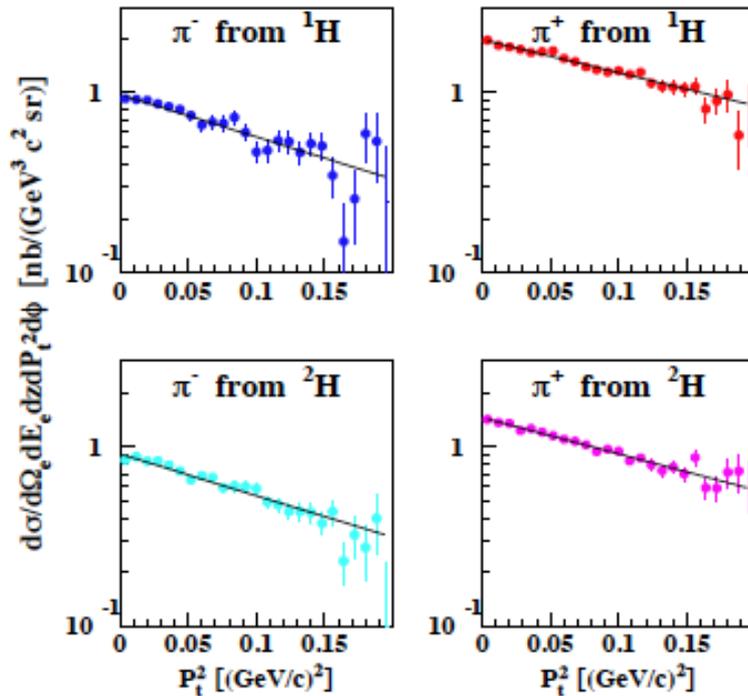# Flavor in transverse space
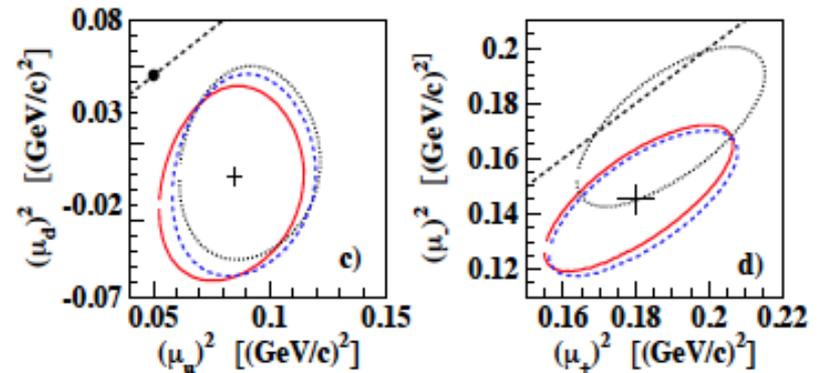
TMD FFs - NJL-jet model



FIG. 14. The averaged transverse momentum of $\pi$ and $K$ mesons emitted by a $u$ quark.

Matevosyan *et al.*, P. R. D**85** (12) 014021

# Flavor in transverse space



no kaons, no sea,
no *X-Z* dependence

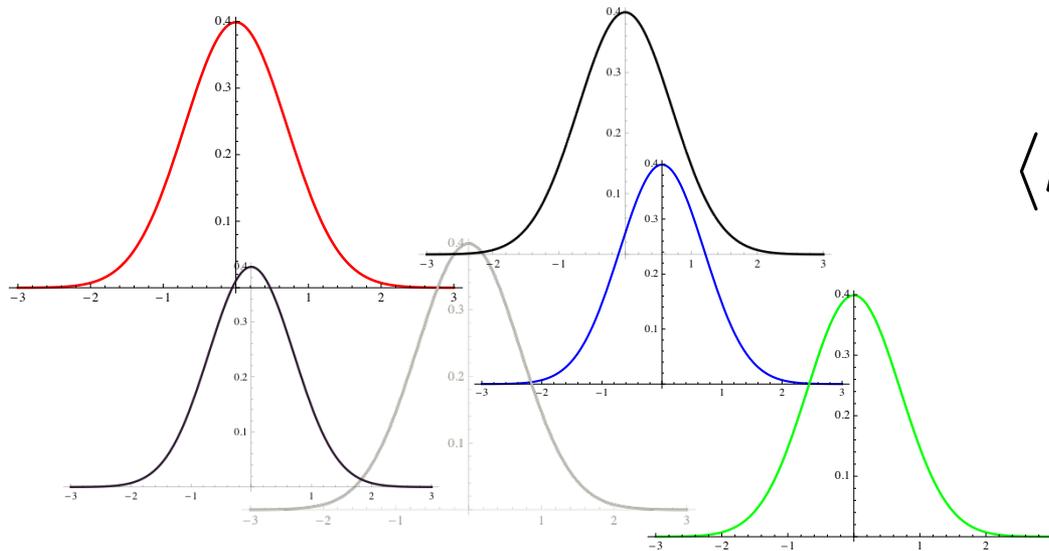up is wider than down

favored wider than unfavored

**Jefferson Lab**

Asaturyan *et al.* (E00-108),  P. R. **C85** (12) 015202

# Flavor independent Gaussianity

**Gaussian parametrization of TMD parts**

$$f_1^a(x, k_\perp^2) = f_1^a(x) \boxed{\frac{1}{\pi \langle k_\perp^2 \rangle} \exp\left\{ -\frac{k_\perp^2}{\langle k_\perp^2 \rangle} \right\}}$$



$$\langle k_{\perp,\mathrm{u}}^2 \rangle = \langle k_{\perp,\mathrm{d}}^2 \rangle = \dots$$
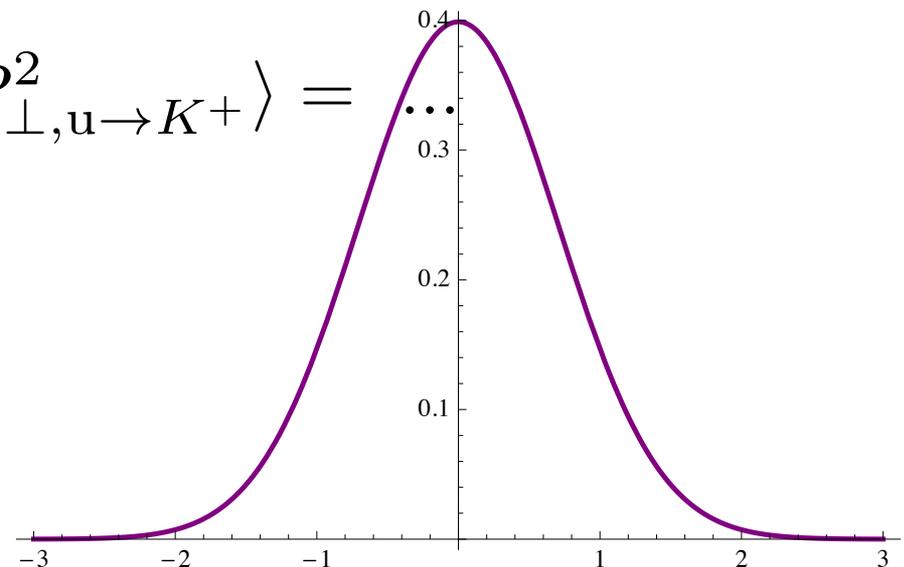
The same variance for all the flavors!

# Flavor independent Gaussianity

## *Gaussian parametrization of TMD parts*

$$D_1^{a \to h}(z, P_\perp^2) = D_1^{a \to h}(z) \boxed{\frac{1}{\pi \langle P_\perp^2 \rangle} \exp\left\{-\frac{P_\perp^2}{\langle P_\perp^2 \rangle}\right\}}$$

$$\langle P_{\perp, u \to \pi^+}^2 \rangle = \langle P_{\perp, u \to \pi^-}^2 \rangle = \langle P_{\perp, u \to K^+}^2 \rangle = \ldots$$

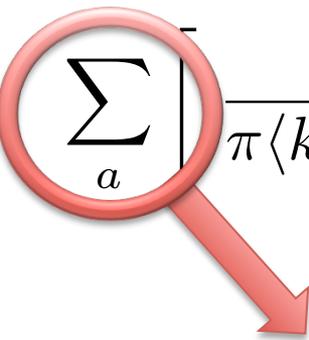The same variance
for all the fragmentation
processes

# Flavor analysis

$$\sigma(P_{hT}^2) \sim \sum_a f_1^a(x, k_\perp^2) \otimes D_1^{a \to h}(z, P_\perp^2)$$

***Flavor dependent case***

$$\sum_a \left[ \frac{1}{\pi \langle k_{\perp,a}^2 \rangle} \exp\left\{ -\frac{k_\perp^2}{\langle k_{\perp,a}^2 \rangle} \right\} \otimes \frac{1}{\pi \langle P_{\perp,a \to h}^2 \rangle} \exp\left\{ -\frac{P_\perp^2}{\langle P_{\perp,a \to h}^2 \rangle} \right\} \right]$$

**Simplified!**

Sum of Gaussian functions!

# Flavor analysis

$$\sigma(P_{hT}^2) \sim \sum_a \; f_1^a(x, k_\perp^2) \otimes D_1^{a \to h}(z, P_\perp^2)$$

**Flavor dependent case**

$$\sum_a \left[ \; \frac{1}{\pi \langle k_{\perp,a}^2 \rangle} \; \exp\left\{ -\frac{k_\perp^2}{\langle k_{\perp,a}^2 \rangle} \right\} \; \otimes \; \frac{1}{\pi \langle P_{\perp,a \to h}^2 \rangle} \; \exp\left\{ -\frac{P_\perp^2}{\langle P_{\perp,a \to h}^2 \rangle} \right\} \right]$$

in the convolution, for each flavor we get a Gaussian with width

$$\langle \boldsymbol{P}_{hT,q}^2 \rangle = z^2 \langle \boldsymbol{k}_{\perp,q}^2 \rangle + \langle \boldsymbol{P}_{\perp,q \to h}^2 \rangle$$

*Momenta are anticorrelated!*

# Results

'Tension' in the collinear case

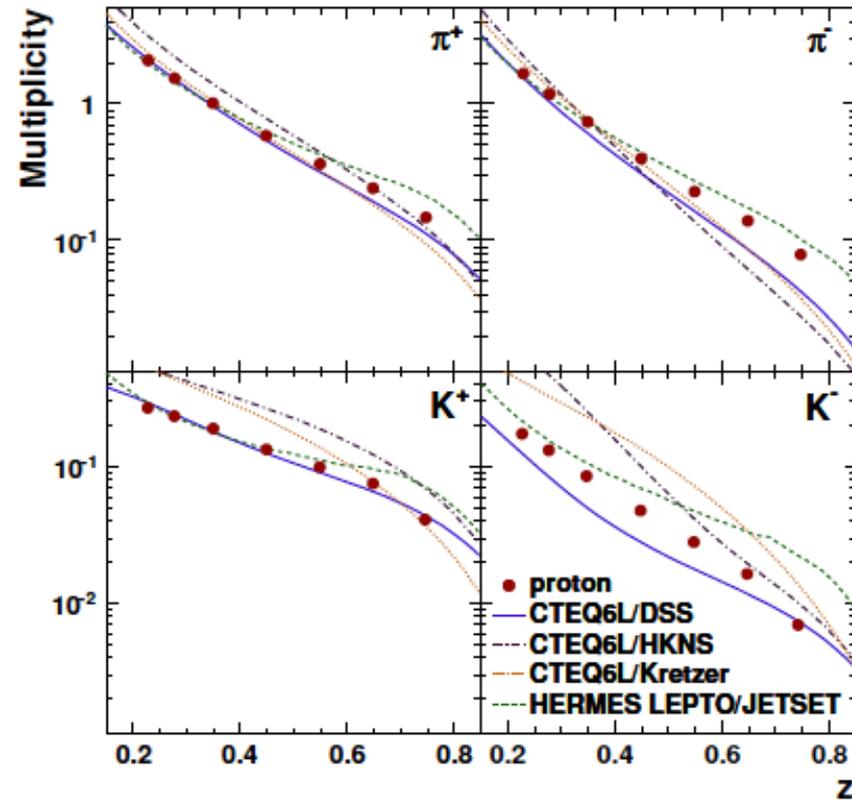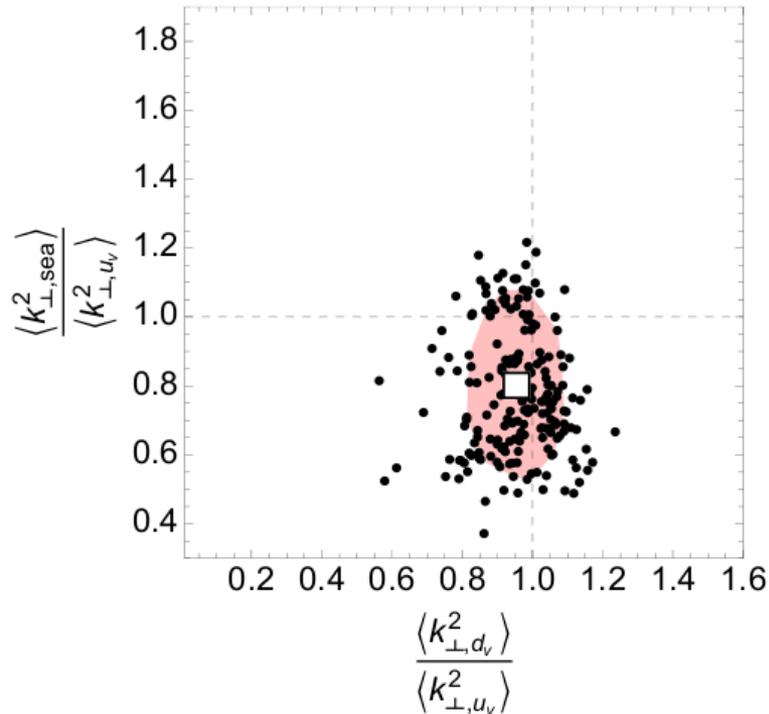| | $Q^2 > 1.4\,\text{GeV}^2$ |
|---|---|
| global | 2.86 |
| $p \to K^-$ | 2.25 |
| $p \to \pi^-$ | 3.39 |
| $p \to \pi^+$ | 1.87 |
| $p \to K^+$ | 0.89 |
| $D \to K^-$ | 4.26 |
| $D \to \pi^-$ | 5.05 |
| $D \to \pi^+$ | 3.33 |
| $D \to K^+$ | 1.80 |



**Table 2.** Values of $\chi^2/\text{d.o.f.}$ obtained from the comparison of the HERMES multiplicities $m_N^h(x, z, Q^2)$ with the theoretical prediction using the MSTW08LO collinear PDFs [8] and the DSS LO collinear FFs [48]. In all cases, the range $0.1 \le z \le 0.8$ was included.

# TMD PDFs – without Kaons

sea width
**<** (mostly)
$u_v$ width

Strange quarks
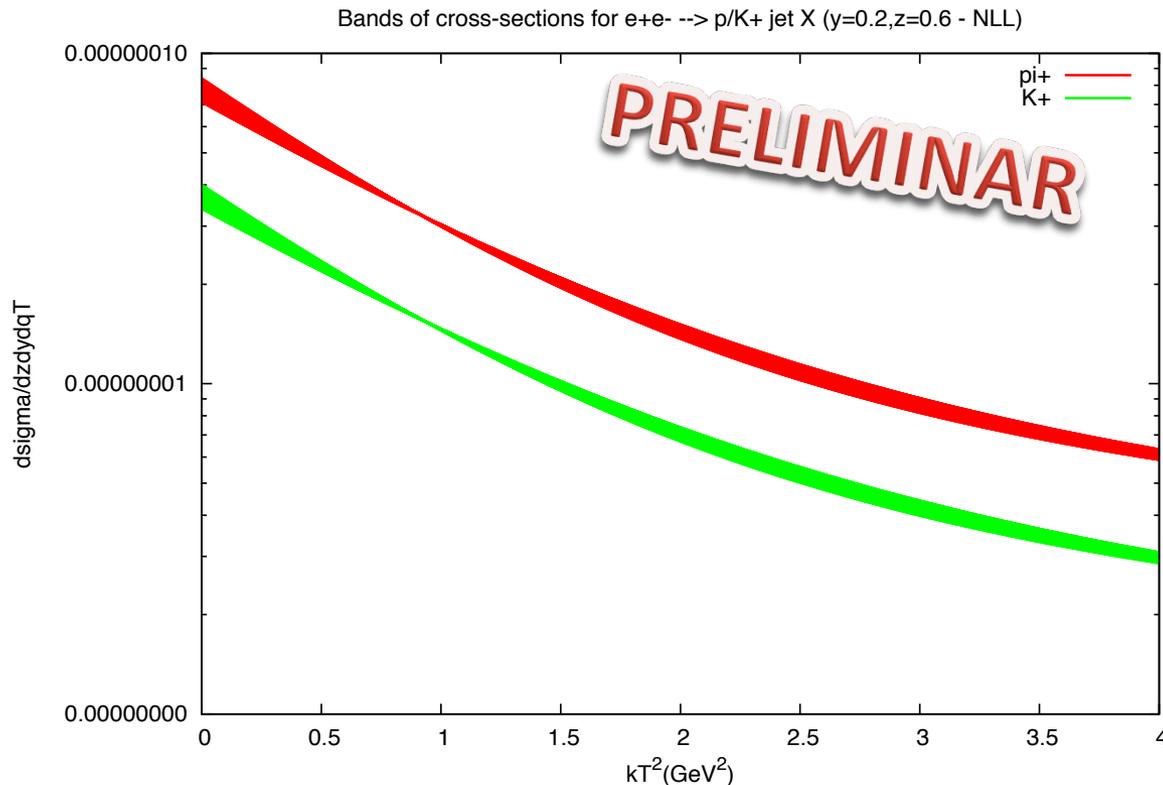are relevant !



point of
no flavor dep.

$d_v$ width  **~** (mostly)  $u_v$ width

# e⁺e⁻ multiplicities

$$e^+e^- \longrightarrow h \text{ jet } X$$

Pin down the most physical flavor dependent sets looking at the spectrum in transverse momentum of e⁺e⁻ collisions

Bands of cross-sections for e+e- --> p/K+ jet X (y=0.2,z=0.6 - NLL)



Bands built out of the 200 replicas for TMD FF $D_1$

# e⁺e⁻ multiplicities

$$e^+ e^- \longrightarrow h \text{ jet } X$$

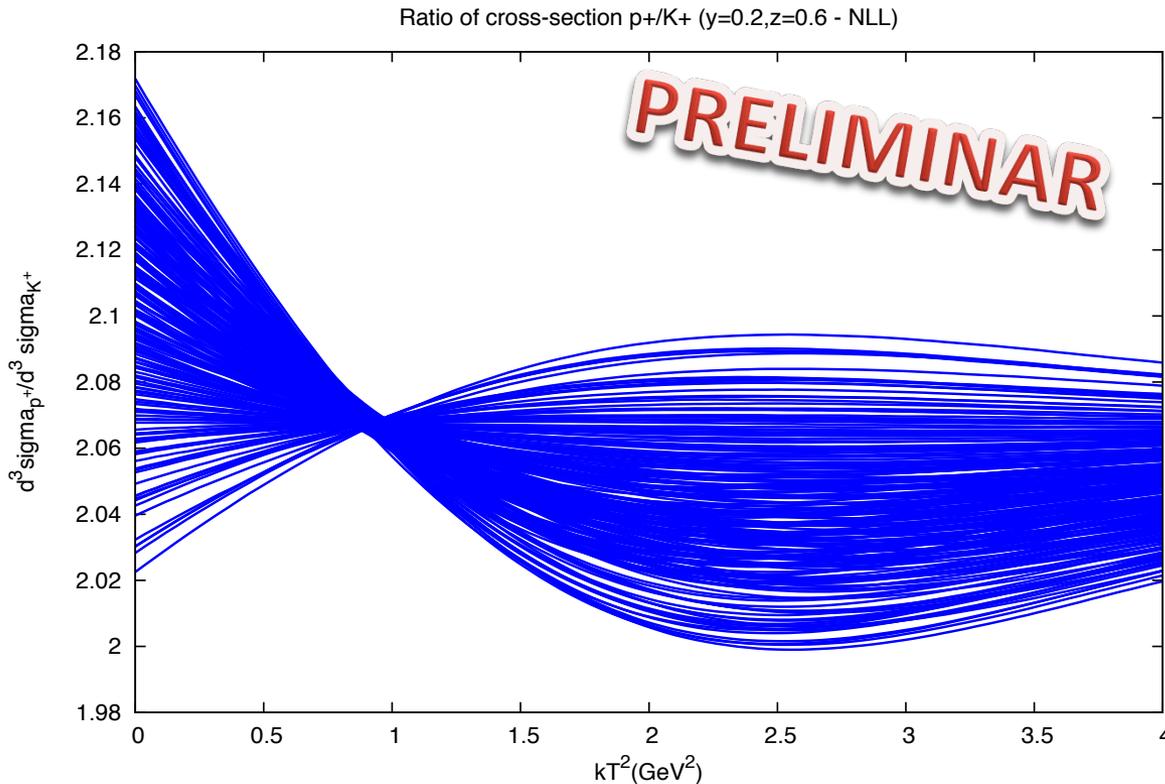Pin down the most physical flavor dependent sets looking at the spectrum in transverse momentum of e⁺e⁻ collisions

### Ratio of cross-section p+/K+ (y=0.2,z=0.6 - NLL)



Bands built out of the 200 replicas for TMD FF $D_1$

Waiting for the experimental values of these ratios ..!