b-CGC versus IP-Sat and combined HERA data

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XXII. International Workshop on Deep-Inelastic Scattering and Related Subjects (DIS 2014, Warsaw)
Motivation, introduction to saturation (b-CGC and IP-Sat model)
The importance of impact-parameter dependence of dipole amplitude.
Confronting the b-CGC and IP-Sat models with the recent H1+ZEUS combined data for $\sigma_r$, and also $F_2, F_2^{c\bar{c}}, F_L$, exclusive diffractive & DVCS data.
Conclusion

This talk is mainly based on:

See also:
Both DGLAP and BFKL are linear evolution equations: exponential growth of the gluon distributions at small-$x$

- **Linear $\Rightarrow$ unstable growth of the gluon distribution!**
Growth of the gluon distribution at small-\(x\), where do gluons go?

Unitarity or Froissart bound: \(\sigma_{\text{tot}} < c \ln^2(s)\): Gluon saturation at small-\(x\)
Charged hadron multiplicity does not change more than a factor $2 \div 4$ even gluon density increased more than an order of magnitude from RHIC to the LHC.
High energy/density: recombination processes $\implies$ saturation:
The number of partons created at a given step depends non-linearly on the number of partons present previously.

Nonlinear $\implies$ stable fixed point at high energy!
Unitarity, black disk limit and saturation

Connection between unitarization and saturation:

\[ \gamma^* \rightarrow \gamma^* \]

\[ \text{proton} \]

\[ N(x, r, b) \approx \alpha_s r^2 \frac{xG(x, 1/r^2)}{\pi R^2} \equiv \alpha_s n(x, Q^2 \sim 1/r^2) \]

Strong scattering \( N \sim 1 \iff \) High gluon density \( n \sim 1/\alpha_s \implies \) gluon saturation

To preserve unitarity \( \iff \) Multiple scattering is important: \( (\alpha_s n)^n \sim 1 \)
**Dilute regime:** Bjorken limit in QCD

\[ s \to \infty; \ Q^2 \to \infty; \ x \approx Q^2/s = \text{fixed} \]

**Dense regime:** Regge limit in QCD

- **IP-Sat:** probing the saturation from the DGLAP region.
- **b-CGC:** probing the saturation from the BFKL region.

\[ s \to \infty; \ x \to 0; \ Q^2 = \text{fixed} \]
Unified description of inclusive & exclusive processes in color-dipole factorization

**Exclusive diffractive process:** \( \psi_{q\bar{q}} \otimes \phi_{q\bar{q}} \otimes N^{q\bar{q} - p} \)

\[
A^{\gamma^* p \to \gamma_p (x, Q, \Delta)} = 2i \int d^2r \int_0^1 dz (\Psi^* \Psi)_{T, L} \int d^2b \ e^{-i[b-(1-z)r] \cdot \Delta} N(x, r, b)
\]

\[
\frac{d\sigma_{T, L}^{\gamma^* p \to Ep}}{dt} = \frac{1}{16\pi} \left| A^{\gamma^* p \to Ep}_{T, L} \right|^2, \quad t = -\Delta^2
\]

- With corrections from the real part of the amplitude and skewedness effect \( x \neq x' \)
- \((b \to 1/|t|)\): \( t \)-distributions access impact-parameter distribution of interactions

**Inclusive deep-inelastic scattering (DIS):** \( \psi_{q\bar{q}} \otimes \psi_{q\bar{q}} \otimes N^{q\bar{q} - p} \)

\[
\sigma_{L, T}^{\gamma^* p}(Q^2, x) = \text{Im} A_{T, L}^{\gamma^* p \to \gamma^* p(x, Q, \Delta = 0)}
\]

\[
= 2 \int d^2r \int_0^1 dz |\Psi_{L, T}(r, z; Q^2)|^2 \int d^2b N(x, r, b)
\]

- DIS is less sensitive to the \( b \)-dependence compared to exclusive diffractive process.
Impact-Parameter dependent Saturation (IP-Sat) model

- Kowalski, Teaney [hep-ph/0304189]
- Kowalski, Motyka, Watt [hep-ph/0606272]
- Rezaeian, Siddikov, Van de Klundert, Venugopalan [arXiv:1212.2974]

- Eikonalized DGLAP-evolved gluon density with Gaussian $b$-dependence (Glauber-Mueller amplitude):

\[
N(x, r, b) = 1 - \exp \left( -\frac{\pi^2 r^2}{2N_c} \alpha_s(\mu^2) xg(x, \mu^2) T_G(b) \right)
\]

\[
T_G(b) = \frac{1}{2\pi B_G} \exp \left( -\frac{b^2}{2B_G} \right)
\]

- Initial gluon distribution with a scale running with dipole size:

\[
xg(x, \mu_0^2) = A_g x^{-\lambda_g} (1 - x)^{5.6} \quad \mu^2 = C/r^2 + \mu_0^2
\]
Impact-parameter dependent Color Glass Condensate (b-CGC) model

- Watt, Kowalski [arXiv:0712.2670]
- Rezaeian, Schmidt [arXiv:1307.0825]


\[ N(x, r, b) = \begin{cases} 
N_0 \left( \frac{r Q_s}{2} \right)^{2\gamma_{\text{eff}}} & r Q_s \leq 2, \\
1 - \exp \left( -A \ln^2 (B r Q_s) \right) & r Q_s > 2
\end{cases} \]

Effective anomalous dimension and the saturation scale are defined as

\[ \gamma_{\text{eff}} = \gamma_s + \frac{1}{\kappa \lambda Y} \ln \left( \frac{2}{r Q_s} \right), \]

\[ Q_s \rightarrow Q_s(x, b) = \left( \frac{X_0}{x} \right)^{\frac{1}{2}} \exp \left\{ -\frac{b^2}{4\gamma_s B_{\text{CGC}}} \right\} \text{GeV} \]
IP-Sat and b-CGC models updated with the combined HERA data

- 4(5) free parameters in IP-Sat (b-CGC) models:
  - $B_G$ ($B_{CGC}$) is fixed from t-slope of exclusive $J/\Psi$ production;
  - Other parameters are fixed by DIS combined data for $\sigma_r$ for $x \leq 0.01$ and $Q^2 \in [0.75, 650]\text{GeV}^2$.

- Data for $F_2, F_L$ and $F_2^{c\bar{c}}$, exclusive diffractive (for $\rho$ and $\phi$) and DVCS are NOT included into the fit, but are predictions of the models.

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**old v. new fit obtained from combined H1+ZEUS data**

- Old IP-Sat and b-CGC parameters & combined $\sigma_r$ data: $\chi^2/d.o.f \approx 3$ → parameters of new fit are different with $\chi^2/d.o.f \approx 1$:
  - Old IP-Sat: $m_{u,d,s} = 50 - 100\text{ MeV}$, and $\lambda_g < 0$.
  - New IP-Sat: $m_{u,d,s} = 0$, and $\lambda_g > 0$ → makes more sense at small-$x$!
Slope of $t$-distribution of exclusive processes, a unified picture

\[ \frac{d\sigma}{dt} \approx e^{-B_D |t|} \quad \text{(large } Q^2) \quad \iff \quad Q_s^2(x, b) \approx Q_s^2(x) e^{-b^2/2B_D} \]

Supported by data and used in both b-CGC and IP-Sat models in dilute region.

- At a fixed $Q^2$, the typical dipole size is bigger for lighter vector meson $\Rightarrow$ validity of the above asymptotic expression is postponed to a higher $Q^2$.
- Universality of extracted impact-parameter distribution of the proton. 
  \textit{t-slope $B_D$ gives the width of saturation scale distribution in proton.}
The typical impact-parameter probed in the total $\gamma^* p$ cross-section is about \( b \approx 2 \div 3 \text{ GeV}^{-1} \) \( \Rightarrow \) Importance of b-dependence.

The typical dipole-size in the interaction depends on \( Q^2 \) (strongly), and \( x \) for a fixed \( b \): A larger \( Q^2 \) \( \rightarrow \) smaller \( r \).
Saturation scale of Proton extracted from combined HERA data: IP-Sat v. b-CGC

- $Q_s(x, b) < 1$ GeV at HERA kinematics.
- The power-law behavior of $Q_s^2 \approx x^{-\lambda}$ changes from $\lambda \approx 0.3$ (central) to $\lambda \approx 0.1$ (peripheral).
- Order of magnitude discrepancies in saturation scale extracted from different models $\Rightarrow$ sizable uncertainties in predictions of various observables.
- Current small-$x$ data do not put enough constrains on saturation models.
$F_{c\bar{c}}$, $F_2$ data were not included in the fit.

The difference among models can be considered as our current theoretical uncertainties $\Rightarrow$ significant uncertainties at small-$x$ $\Rightarrow$ Future exps with $x_B < 10^{-5}$ (LHeC, EIC) can constrain saturation models.
**IP-Sat v. b-CGC description of HERA data**

**FL and F2 data were not included in the fit.**
Two models give rather different $t$-distribution at large $t$.

- Large $t$ corresponds to small $b$: saturation scale are different in these two models at very small $x$ for central collisions.

See my afternoon talk in ”Future experiments session”

- $t$-dependence is described by $\sim \exp(-B_D t)$ at low-$t$, $B_D$ depends on $Q^2$, $W$ and $M^2$. 

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The slope of the t-distribution of exclusive diffractive processes, IP-Sat v. b-CGC

In the b-CGC model, there are non-trivial correlations between $b$ and $x \rightarrow W$ dependence of $B_D$. 
The $W$-dependence of the cross-section follows a power-law behavior
\[ \sigma \sim W^\delta \rightarrow \text{Indication of geometric scaling in diffractive data.} \]
Total $J/\psi$ cross-section as a function of $W_{\gamma p}$

Armesto and Rezaeian, arXiv:1402.4831, See also my talk in WG7, Tue

![Graph showing cross-section as a function of $W_{\gamma p}$]

- The LHCb 2014 data are in favour of the CGC/Saturation predictions.
- The $t$-distribution of diffractive photoproduction of vector mesons at large $|t|$ can discriminate among models.
Conclusion:

- Both the b-CGC and the IP-Sat models give generally good description of all HERA data at $x \leq 0.01$ including combined HERA data:
  - Universality (consistence) of saturation picture even though the details of two models are different:
    - $Q_s(x, b) < 1$ GeV at HERA kinematics.
    - The typical impact-parameter probed in the total $\gamma^* p$ cross-section is about $b \approx 2 \div 3$ GeV$^{-1}$.
  - Two models are different at very small $x$ and large $|t|$ ($x < 10^{-5}$ and $|t| > 1$).
  - The recent LHCb data (2014) in p+p collisions for exclusive diffractive photoproduction of $J/\Psi$ is favour of CGC/saturation predictions.