Next-to-eikonal corrections in the CGC

Tolga Altinoluk

Universidade de Santiago de Compostela

DIS - 2014 Warsaw, Poland

April 30, 2014

[T. A. , N. Armesto, G. Beuf, M. Martínez, C. Salgado, arXiv:1404.2219]

Eikonal approximation for high-energy (dense-dilute) scattering

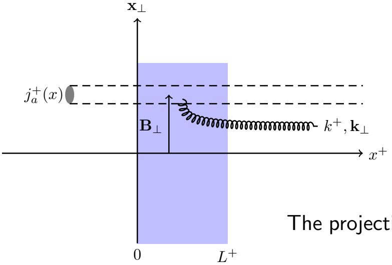
- dense target $\equiv$ classical background field $A^{\mu}_{a}(x)$
- dilute projectile $\equiv$ color charge $J^{\mu}_{a}(x)$

Coupling between the projectile and the target $\rightarrow \int d^4x J^{a}_{\mu}(x) A^{\mu}_{a}(x)$

In the high energy limit (within eikonal approximation):

- boosting the projectile:
  - $J^{+}_{a} \gg J^{i}_{a} \gg J^{-}_{a}$
  - $J^{\mu}_{a}(x) = \delta^{\mu +} J^{+}(x) \propto \delta(x^{-}) \Rightarrow$ projectile localized at $x^{-} = 0$
- boosting the target:
  - $A^{-}_{a} \gg A^{i}_{a} \gg A^{+}_{a}$
  - $A^{\mu}(x) \equiv \delta^{\mu -} A^{-}_{a}(x^{+}, \mathbf{x}) \propto \delta(x^{+}) \Rightarrow$ target localized at $x^{+} = 0$ & independent of x^{-}

Finite width target : relaxing the eikonal approximation



Consider a finite width target, but still assuming that

- $J_a^\mu(x) \propto \delta^{\mu+}\delta(x^-)$
- $A^\mu(x) \equiv \delta^{\mu-} A_a^-(x^+, x_\perp)$

The projectile $\rightarrow$ $j_a^\mu(x) = \delta^{\mu+}\delta(x^-)\,\mathcal{U}^{ab}(x^+, 0, \mathbf{x})\rho^b(\mathbf{x} - \mathbf{B})$

$$\downarrow$$

$$\boxed{\mathcal{P}_+ \exp\left\{ig \int_0^{x^+} dz^+ \; T \cdot \mathcal{A}^-\left(z^+, \mathbf{x}\right)\right\}^{ab}}$$

The single inclusive gluon spectrum for pA:

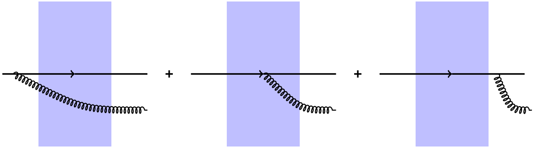
$$(2\pi)^3 \,(2k^+)\, \frac{dN}{dk^+ \, d^2\mathbf{k}}(\mathbf{B}) = \sum_{\lambda\,\text{phys.}} \left\langle \left\langle |\mathcal{M}_\lambda^a(\underline{k}, \mathbf{B})|^2 \right\rangle_p \right\rangle_A$$

gluon production amplitude

The single inclusive gluon cross section :

$$\frac{d\sigma}{dk^+ \, d^2\mathbf{k}} = \int d^2\mathbf{B} \; \frac{dN}{dk^+ \, d^2\mathbf{k}}(\mathbf{B})$$

$\mathcal{M}^a_\lambda(\underline{k},\mathbf{B}) \;=$



at LO in g, LSZ reduction formula $\Rightarrow \mathcal{M}^a_\lambda(\underline{k},\mathbf{B}) = \varepsilon^{\mu *}_\lambda \int d^4x\; e^{ik\cdot x}\; \Box_x A^a_\mu(x)$

power counting : dense target $\Rightarrow \mathcal{A}^-_a(x^+,\mathbf{x}) = \mathcal{O}(1/g)$

dilute projectile $\Rightarrow j^+_a(\underline{x}) = \mathcal{O}(g)$

perturbative expansion of the classical field : $A^\mu_a(x) = \mathcal{A}^\mu_a(x) + a^\mu_a(x) + \mathcal{O}(g^3)$

$$a^\mu_a(x) \Big\downarrow \; -i\int d^4y\; G^{\mu -}_R(x,y)_{ab}\; j^+_b(y)$$

In the LC gauge : $A^+_a = 0$ & $\varepsilon^{+*}_\lambda = 0 \Rightarrow \varepsilon^{\mu *}_\lambda\, \mathcal{A}^a_\mu(x) = -\varepsilon^{i*}_\lambda a^i_a$

$$\boxed{\mathcal{M}^a_\lambda(\underline{k},\mathbf{B}) = \varepsilon^{i*}_\lambda\,(2k^+) \lim_{x^+\to+\infty} \int d^2\mathbf{x} \int dx^-\; e^{ik\cdot x} \int d^4y\; G^{i-}_R(x,y)_{ab}\; j^+_b(y)}$$

$G^{\mu\nu}_R(x,y)_{ab}$ is the background retarded gluon propagator

Properties of the background retarded gluon propagator

$G_R^{\mu\nu}(x,y)_{ab}$ is the retarded solution of the linearized Yang-Mills equations around the classical field $\mathcal{A}_a^-(x^+,\mathbf{x})$:

$$\left[g_{\mu\nu}\left(\delta_{ab}\Box_x - 2ig\left(\mathcal{A}^-(x^+,\mathbf{x})\cdot T\right)_{ab}\partial_{x^-}\right) - \delta_{ab}\partial_{x^\mu}\partial_{x^\nu}\right] G_R^{\nu\rho}(x,y)_{bc} = i\, g_\mu{}^\rho\, \delta_{ac}\, \delta^{(4)}(x-y)$$

$\mathcal{A}_a^-(x^+,\mathbf{x})$ is independent of the x^-, so it is convenient to introduce the 1-d Fourier transform of $G_R^{\mu\nu}(x,y)_{ab}$

$$G_R^{\mu\nu}(x,y)_{ab} = \int \frac{\mathrm{d}k^+}{2\pi}\, e^{-ik^+(x^- - y^-)}\, \frac{1}{2(k^+ + i\epsilon)}\, \mathcal{G}_{k^+}^{\mu\nu}(\underline{x};\underline{y})_{ab}$$

The $(i-)$ component of the $\mathcal{G}_{k^+}^{\mu\nu}(\underline{x};\underline{y})_{ab}$ can be written as

$$\boxed{\mathcal{G}_{k^+}^{i-}(\underline{x};\underline{y})^{ab} = \frac{i}{k^+ + i\epsilon}\, \partial_{\mathbf{y}^i}\, \mathcal{G}_{k^+}^{ab}(\underline{x};\underline{y})}$$

$\mathcal{G}_{k^+}^{ab}(\underline{x};\underline{y})$ is the background scalar propagator.

The background scalar propagator satisfies the scalar Green's equation :

$$\left[\delta^{ab}\left(i\partial_{x^+}+\frac{\partial_{\mathbf{x}}^2}{2(k^+\!+i\epsilon)}\right)+g\Big(\mathcal{A}^-(\underline{x})\cdot T\Big)^{ab}\right]\mathcal{G}^{bc}_{k^+}(\underline{x};\underline{y})=i\,\delta^{ac}\,\delta^{(3)}(\underline{x}-\underline{y})$$

Schrödinger equation in $2+1$ dimensions in a space-time dependent matrix potential $-g\Big(\mathcal{A}^-(\underline{x})\cdot T\Big)^{ab}$.

The solution can be written formally as a path integral

$$\mathcal{G}^{ab}_{k^+}(\underline{x};\underline{y})=\theta(x^+\!-y^+)\int_{\mathbf{z}(y^+)=\mathbf{y}}^{\mathbf{z}(x^+)=\mathbf{x}}\mathcal{D}\mathbf{z}(z^+)\;\exp\left[\frac{ik^+}{2}\int_{y^+}^{x^+}dz^+\;\dot{\mathbf{z}}^2(z^+)\right]\times\;\mathcal{U}^{ab}\Big(x^+,y^+,\big[\mathbf{z}(z^+)\big]\Big)$$

with the Wilson line

$$\mathcal{U}^{ab}\Big(x^+,y^+,\big[\mathbf{z}(z^+)\big]\Big)\;=\;\mathcal{P}_+\exp\left\{ig\int_{y^+}^{x^+}dz^+\;T\cdot\mathcal{A}^-\Big(z^+,\mathbf{z}(z^+)\Big)\right\}^{ab}$$

following the Brownian trajectory $\mathbf{z}(z^+)$.

Discretization of the background propagator

Discretized form of the background propagator :

$$\mathcal{G}^{ab}_{k^+}(\underline{x};\underline{y}) = \lim_{N\to+\infty} \theta(x^+-y^+) \int \left(\prod_{n=1}^{N-1} \mathrm{d}^2\mathbf{z}_n\right) \left(\frac{-i(k^++i\epsilon)N}{2\pi(x^+-y^+)}\right)^N$$
$$\times \exp\left[\frac{i(k^++i\epsilon)N}{2(x^+-y^+)} \sum_{n=0}^{N-1} (\mathbf{z}_{n+1}-\mathbf{z}_n)^2\right] \mathcal{U}^{ab}(x^+,y^+,\{\mathbf{z}_n\})$$

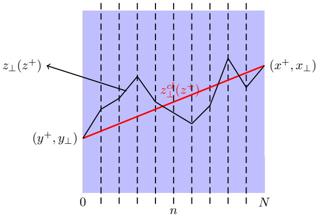
with the discertized Wilson line

$$\mathcal{U}^{ab}(x^+,y^+,\{\mathbf{z}_n\}) \;=\; \mathcal{P}_+\left\{\prod_{n=0}^{N-1} \exp\left[ig\frac{(x^+-y^+)}{N}\Big(\mathcal{A}^-(z_n^+,\mathbf{z}_n)\cdot T\Big)\right]\right\}^{ab}$$

with $z_n^+ = y^+ + \frac{n}{N}(x^+-y^+)$

Expansion of the background propagator

Perturbative expansion around free classical path:



eikonal limit : $\frac{k^+}{(x^+-y^+)} \gg Q_\perp^2$ in the problem

$\mapsto$ large k^+ limit (classical free path!)

$\Rightarrow$ perturbative expansion around the free classical path:

$\mathbf{z}_n = \mathbf{z}_n^{\mathrm{cl}} + \mathbf{u}_n$ with $\mathbf{z}_n^{\mathrm{cl}} = \mathbf{y} + \frac{n}{N}(\mathbf{x} - \mathbf{y})$

Then

$$\mathcal{G}^{ab}_{k^+}(\underline{x}; \underline{y}) = 2\pi i \mathcal{G}_{0,k^+}(\underline{x}; \underline{y}) \frac{(x^+ - y^+)}{k^+} \lim_{N \to +\infty} \int \left(\prod_{n=1}^{N-1} \mathrm{d}^2 \mathbf{u}_n \right)$$

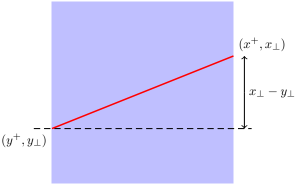
$$\times \mathcal{P}_+ \prod_{n=0}^{N-1} \left\{ \mathcal{G}_{0,k^+}\left(z^+_{n+1}, \mathbf{u}_{n+1}; z^+_n, \mathbf{u}_n\right) \exp\left[\frac{(x^+ - y^+)}{N} igT \cdot \mathcal{A}^-\left(z_n^+, \mathbf{z}_n^{\mathrm{cl}} + \mathbf{u}_n\right) \right] \right\}$$

- Taylor expand the gauge link around $\mathbf{u}_n = 0$ at each discretization step.
- Take the product of the Taylor expansion of all links and collect terms according to power of $\mathbf{u}_n$.

Expansion around the initial transverse position :

The first expansion is performed for fixed initial and final positions.

In the large k^+ limit , the result has to be re-expanded since $\mathbf{z}^{\rm cl}(z^+) - \mathbf{y}$ is small at each step.



After all :

$$\int d^2\mathbf{x}\; e^{-i\mathbf{k}\cdot\mathbf{x}}\; \mathcal{G}^{ab}_{k^+}(\underline{x};\underline{y}) = \theta(x^+ - y^+)\; e^{-i\mathbf{k}\cdot\mathbf{y}}\; e^{-ik^-(x^+-y^+)} \Big\{ \underbrace{\mathcal{U}(x^+, y^+, \mathbf{y})}_{\text{strict eikonal term}}$$

$$+ \frac{(x^+ - y^+)}{k^+} \mathbf{k}^i\, \mathcal{U}^i_{(1)}(x^+, y^+, \mathbf{y}) + i\frac{(x^+ - y^+)}{2k^+}\, \mathcal{U}_{(2)}(x^+, y^+, \mathbf{y}) \Big\}^{ab}$$

first corrections to strict eikonal limit

The explicit form of the corrections :

$$\mathcal{U}^{i,ab}_{(1)}(x^+,y^+,\mathbf{y}) = \int_{y^+}^{x^+} dz^+ \left(\frac{z^+ - y^+}{x^+ - y^+}\right) \Big\{\mathcal{U}(x^+,z^+,\mathbf{y}) \times \big[igT\cdot \partial_{\mathbf{y}^i} A^-(z^+,\mathbf{y})\big]\mathcal{U}(z^+,y^+,\mathbf{y})\Big\}^{ab}$$

and

$$\begin{aligned}\mathcal{U}^{ab}_{(2)}(x^+,y^+,\mathbf{y}) =& \int_{y^+}^{x^+} dz^+ \left(\frac{z^+ - y^+}{x^+ - y^+}\right) \Big\{\mathcal{U}(x^+,z^+,\mathbf{y})\big[igT\cdot \partial^2_{\mathbf{y}} A^-(z^+,\mathbf{y})\big]\mathcal{U}(z^+,y^+,\mathbf{y})\Big\}^{ab} \\ &+2\int_{y^+}^{x^+} dz^+ \int_{y^+}^{z^+} dw^+ \left(\frac{w^+ - y^+}{x^+ - y^+}\right) \Big\{\mathcal{U}(x^+,z^+,\mathbf{y})\big[igT\cdot \partial_{\mathbf{y}^i} A^-(z^+,\mathbf{y})\big] \\ &\times \mathcal{U}(z^+,w^+,\mathbf{y})\big[igT\cdot \partial_{\mathbf{y}^i} A^-(w^+,\mathbf{y})\big]\,\mathcal{U}(w^+,y^+,\mathbf{y})\Big\}^{ab}\end{aligned}$$

Reduced amplitude and the cross section

Defining Fourier transform of the color charge density

$$\rho^a(\mathbf{y}-\mathbf{B}) = \int \frac{d^2\mathbf{q}}{(2\pi)^2}\; e^{i\mathbf{q}\cdot(\mathbf{y}-\mathbf{B})}\; \tilde{\rho}^a(\mathbf{q})$$

and gluon-nucleus reduced amplitude

$$\mathcal{M}^a_\lambda(\underline{k},\mathbf{B}) \;=\; \int \frac{d^2\mathbf{q}}{(2\pi)^2}\; e^{-i\mathbf{q}\cdot\mathbf{B}}\; \overline{\mathcal{M}}^{ab}_\lambda(\underline{k},\mathbf{q})\;\; \tilde{\rho}^b(\mathbf{q})$$

The cross section is written as

$$k^+\,\frac{d\sigma}{dk^+\,d^2\mathbf{k}} = \int \frac{d^2\mathbf{q}}{(2\pi)^2}\,\varphi_p(\mathbf{q})\,\frac{\mathbf{q}^2}{4}\,\frac{1}{N_c^2-1}\sum_{\lambda\,\mathrm{phys.}}\Big\langle \overline{\mathcal{M}}^{ab}_\lambda(\underline{k},\mathbf{q})^\dagger\;\overline{\mathcal{M}}^{ab}_\lambda(\underline{k},\mathbf{q})\Big\rangle_A .$$

Expanded reduced amplitude reads

$$\overline{\mathcal{M}}^{ab}_\lambda(\underline{k},\mathbf{q}) = i\,\varepsilon^{i*}_\lambda\;\int d^2\mathbf{y}\; e^{-i(\mathbf{k}-\mathbf{q})\cdot\mathbf{y}}\,\Bigg\{2\left[\frac{\mathbf{k}^i}{\mathbf{k}^2}-\frac{\mathbf{q}^i}{\mathbf{q}^2}\right]\mathcal{U}(L^+,0;\mathbf{y})$$
$$+\frac{L^+}{k^+}\left[\delta^{ij}-2\frac{\mathbf{q}^i}{\mathbf{q}^2}\mathbf{k}^j\right]\mathcal{U}^j_{(1)}(L^+,0;\mathbf{y}) - i\,\frac{L^+}{k^+}\frac{\mathbf{q}^i}{\mathbf{q}^2}\mathcal{U}_{(2)}(L^+,0;\mathbf{y}) + O\left(\left(\frac{L^+}{k^+}\,\partial^2_\perp\right)^2\right)\Bigg\}^{ab}$$

By defining the new operators as

$$S_A(\mathbf{r},\mathbf{b}) = \frac{1}{N_c^2-1}\left\langle \mathrm{tr}\left[\mathcal{U}^\dagger\left(L^+,0;\mathbf{b}-\tfrac{\mathbf{r}}{2}\right)\mathcal{U}\left(L^+,0;\mathbf{b}+\tfrac{\mathbf{r}}{2}\right)\right]\right\rangle_A$$

$$\mathcal{O}^j_{(1)}(\mathbf{r},\mathbf{b}) = \frac{1}{N_c^2-1}\left\langle \mathrm{tr}\left[\mathcal{U}^\dagger\left(L^+,0;\mathbf{b}-\tfrac{\mathbf{r}}{2}\right)\mathcal{U}^j_{(1)}\left(L^+,0;\mathbf{b}+\tfrac{\mathbf{r}}{2}\right)\right]\right\rangle_A$$

$$\mathcal{O}_{(2)}(\mathbf{r},\mathbf{b}) = \frac{1}{N_c^2-1}\left\langle \mathrm{tr}\left[\mathcal{U}^\dagger\left(L^+,0;\mathbf{b}-\tfrac{\mathbf{r}}{2}\right)\mathcal{U}_{(2)}\left(L^+,0;\mathbf{b}+\tfrac{\mathbf{r}}{2}\right)\right]\right\rangle_A$$

Square of the reduced amplitude then reads

$$\frac{1}{N_c^2-1}\sum_{\lambda\,\text{phys.}}\left\langle \overline{\mathcal{M}}^{ab}_\lambda(\underline{k},\mathbf{q})^\dagger\,\overline{\mathcal{M}}^{ab}_\lambda(\underline{k},\mathbf{q})\right\rangle_A = \frac{1}{\mathbf{k}^2\,\mathbf{q}^2}\int_{\mathbf{b},\mathbf{r}} e^{-i(\mathbf{k}-\mathbf{q})\cdot\mathbf{r}}\Big\{4\,(\mathbf{k}-\mathbf{q})^2 S_A(\mathbf{r},\mathbf{b})$$

$$+\frac{L^+}{k^+}\Big[f(\mathbf{k}^i,\mathbf{q}^i)\big[\underbrace{\mathcal{O}^j_{(1)}(\mathbf{r},\mathbf{b})+\mathcal{O}^j_{(1)}(-\mathbf{r},\mathbf{b})}\big]+g(\mathbf{k},\mathbf{q})\big[\underbrace{\mathcal{O}_{(2)}(\mathbf{r},\mathbf{b})-\mathcal{O}_{(2)}(-\mathbf{r},\mathbf{b})}\big]\Big]\Big\}$$

Vanish upon integration over **b** due to rotational symmetry

$$\boxed{k^+\frac{d\sigma}{dk^+\,d^2\mathbf{k}} = \frac{1}{\mathbf{k}^2}\int\frac{d^2\mathbf{q}}{(2\pi)^2}\,\varphi_p(\mathbf{q})\,(\mathbf{k}-\mathbf{q})^2\int_{\mathbf{b},\mathbf{r}} e^{-i(\mathbf{k}-\mathbf{q})\cdot\mathbf{r}}\;S_A(\mathbf{r},\mathbf{b}) + \;O\left(\left(\frac{L^+}{k^+}\,\partial_\perp^2\right)^2\right)}$$

Well known $k_\perp$ factorization formula!!

Single transverse spin asymmetry : polarized target

Consider the process $p + A^{\uparrow} \to g + X$. The SSA is defined as

$$A_N = \frac{k^+ \frac{d\sigma^{\uparrow}}{dk^+ \, d^2\mathbf{k}} - k^+ \frac{d\sigma^{\downarrow}}{dk^+ \, d^2\mathbf{k}}}{k^+ \frac{d\sigma^{\uparrow}}{dk^+ \, d^2\mathbf{k}} + k^+ \frac{d\sigma^{\downarrow}}{dk^+ \, d^2\mathbf{k}}} = \frac{k^+ \frac{d\sigma^{\uparrow}}{dk^+ \, d^2\mathbf{k}} - k^+ \frac{d\sigma^{\downarrow}}{dk^+ \, d^2\mathbf{k}}}{2k^+ \frac{d\sigma}{dk^+ \, d^2\mathbf{k}}}$$

Transversely polarized target $\Rightarrow$ **s** dependent S_A, $\mathcal{O}^j_{(1)}$ and $\mathcal{O}_{(2)}$

Due to transverse rotational symmetry around the center of the target:

$S_A(-\mathbf{r}, -\mathbf{b}, -\mathbf{s}) = S_A(\mathbf{r}, \mathbf{b}, \mathbf{s})$
$\mathcal{O}^j_{(1)}(-\mathbf{r}, -\mathbf{b}, -\mathbf{s}) = -\mathcal{O}^j_{(1)}(\mathbf{r}, \mathbf{b}, \mathbf{s})$
$\mathcal{O}_{(2)}(-\mathbf{r}, -\mathbf{b}, -\mathbf{s}) = \mathcal{O}_{(2)}(\mathbf{r}, \mathbf{b}, \mathbf{s})$ Then:

$$k^+ \left(\frac{d\sigma^{\uparrow}}{dk^+ \, d^2\mathbf{k}} - \frac{d\sigma^{\downarrow}}{dk^+ \, d^2\mathbf{k}} \right) = \frac{L^+}{k^+} \int_{\mathbf{qrb}} \varphi_p(\mathbf{q}) \Big\{ f(\mathbf{k}^j, \mathbf{q}^j, \mathbf{r}) \mathcal{O}^j_{(1)}(\mathbf{r}, \mathbf{b}, \mathbf{s}) + g(\mathbf{k}, \mathbf{q}, \mathbf{r}) \mathcal{O}_{(2)}(\mathbf{r}, \mathbf{b}, \mathbf{s}) \Big\}$$

The strict eikonal term vanishes!!

Next-to-eikonal terms in the expansion become leading terms!!

Anology between twist-3 contributions to hard processes and next-to-eikonal contributions to high-energy processes??

Polarized gluon production in unpolarized pA

Consider a polarized gluon produced in unpolarized pA which then which then fragments in to polarized hadron preserving this asymmetry.

This asymmetry is given by difference between $\lambda = \pm 1$ helicity contributions of the produced gluon:

$$k^+ \frac{d\sigma^+}{dk^+\, d^2\mathbf{k}} - k^+ \frac{d\sigma^-}{dk^+\, d^2\mathbf{k}} = \int \frac{d^2\mathbf{q}}{(2\pi)^2}\, \varphi_p(\mathbf{q}) \frac{\mathbf{q}^2}{4} \frac{1}{N_c^2-1} \sum_{\lambda\,\text{phys.}} \lambda \, \Big\langle \overline{\mathcal{M}}^{ab}_{\lambda}(\underline{k}, \mathbf{q})^{\dagger}\, \overline{\mathcal{M}}^{ab}_{\lambda}(\underline{k}, \mathbf{q}) \Big\rangle_A$$

By using the identity

$$\sum_{\lambda\,\text{phys.}} \lambda\, \varepsilon^i_\lambda\, \varepsilon^{*j}_\lambda = -i\, \epsilon^{ij}$$

one gets

$$k^+ \frac{d\sigma^+}{dk^+\, d^2\mathbf{k}} - k^+ \frac{d\sigma^-}{dk^+\, d^2\mathbf{k}} = \frac{L^+}{k^+} \int \frac{d^2\mathbf{q}}{(2\pi)^2}\, \varphi_p(\mathbf{q})\, \mathbf{q}^2 \int_{\mathbf{b},\mathbf{r}} e^{-i(\mathbf{k}-\mathbf{q})\cdot\mathbf{r}}$$
$$\times \Bigg\{ -i \Bigg[\bigg(\frac{\mathbf{k}^i}{\mathbf{k}^2} - \frac{\mathbf{q}^i}{\mathbf{q}^2} \bigg)\, \epsilon^{ij} - 2\, \frac{(\epsilon^{il}\, \mathbf{k}^i\, \mathbf{q}^l)}{\mathbf{k}^2\, \mathbf{q}^2}\, \mathbf{k}^j \Bigg] \mathcal{O}^j_{(1)}(\mathbf{r}, \mathbf{b}) - \frac{(\epsilon^{ij}\, \mathbf{k}^i\, \mathbf{q}^j)}{\mathbf{k}^2\, \mathbf{q}^2}\, \mathcal{O}_{(2)}(\mathbf{r}, \mathbf{b}) \Bigg\}$$

Strict eikonal terms vanish leaving the next-to-eikonal terms as leading contribution!!!

Conclusions and Outlook

- We develop a systematic eikonal expansion of the retarded gluon propagator in a background field.
- We apply this method to single inclusive gluon cross section in order to study the corrections to the CGC beyond eikonal limit. The strict eikonal term provides the usual $k_\perp$-factorization formula, whereas the first next-to-eikonal corrections vanish for this particular observable.
- For SSA and polarized gluon production the strict eikonal terms vanish leaving the next-to-eikonal corrections as the leading contribution.
- The first non-vanishing contribution to strict eikonal limit of single inclusive gluon cross section arises at order $(L^+ Q_s^2/k^+)^2$ which can be obtained by continuing the expansion to higher orders. [In preparation]
- The same expansion can be applied to energy loss calculations in jet quenching physics. [In preparation]