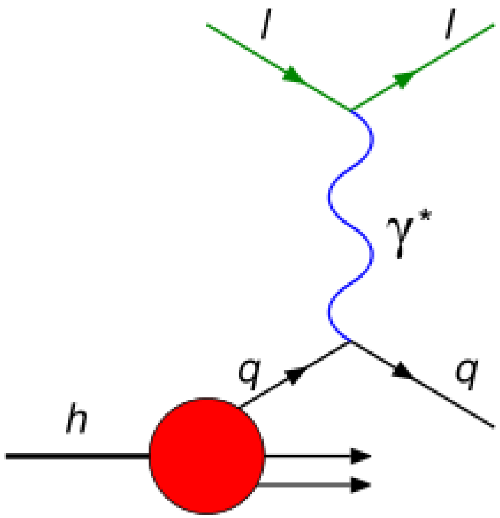
Transverse-spin gluon distribution function

Kazuhiro Tanaka (Juntendo U)

DIS



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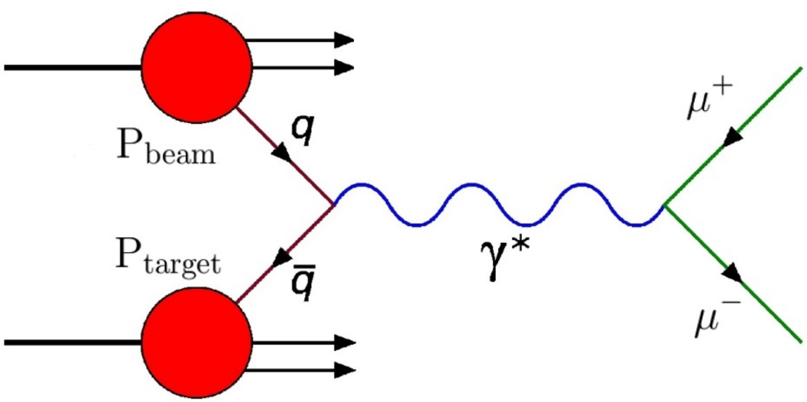

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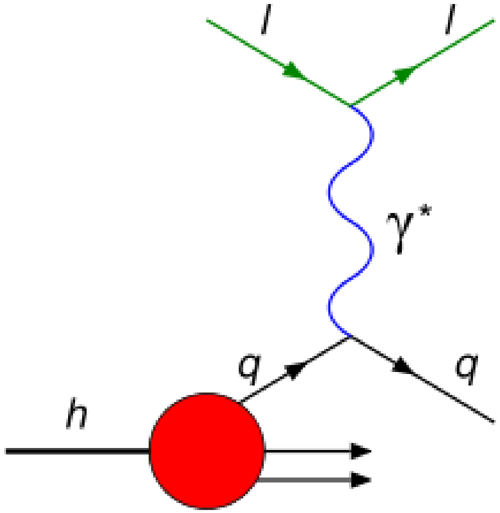

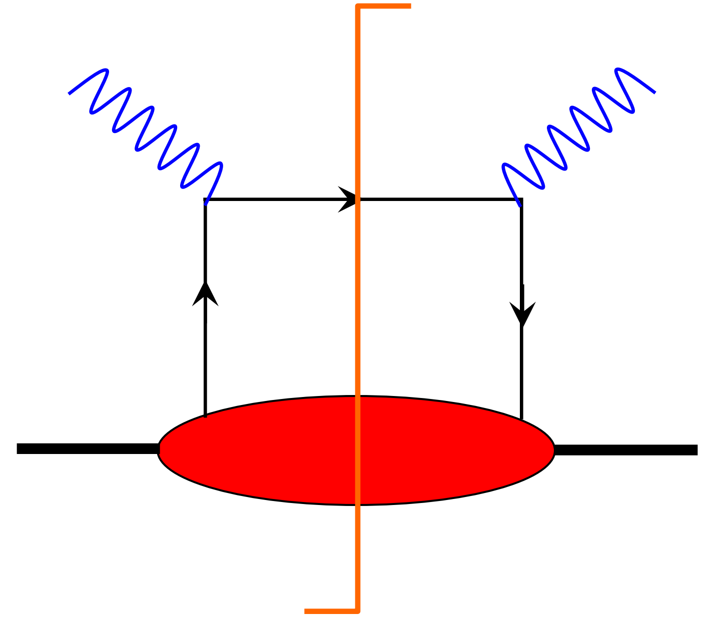

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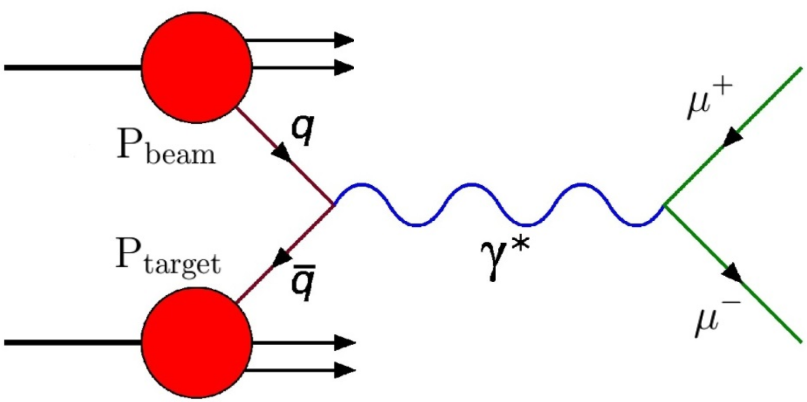
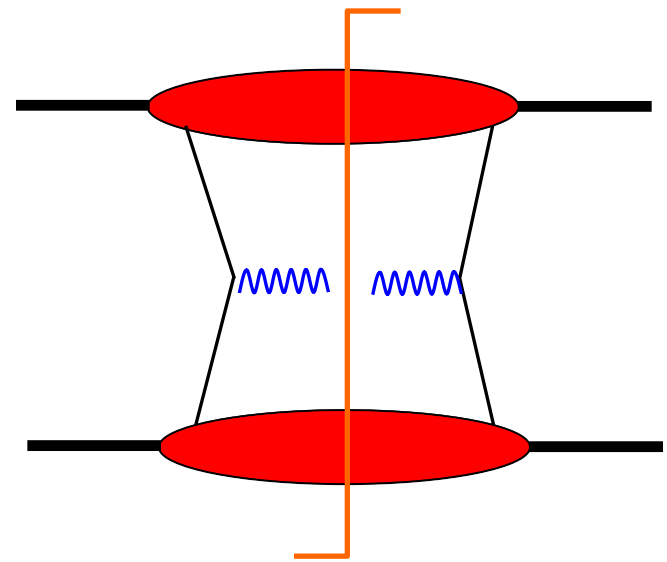
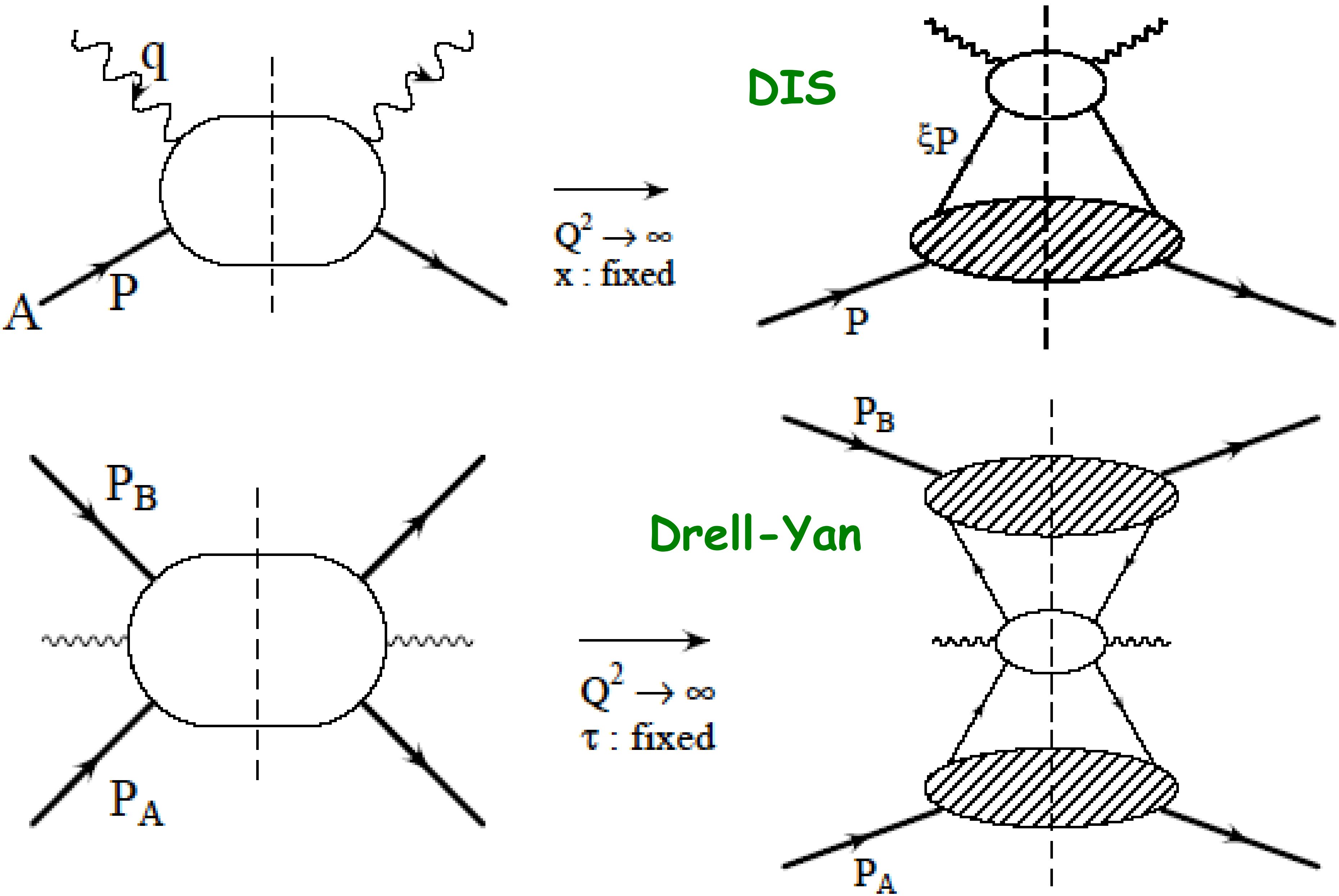
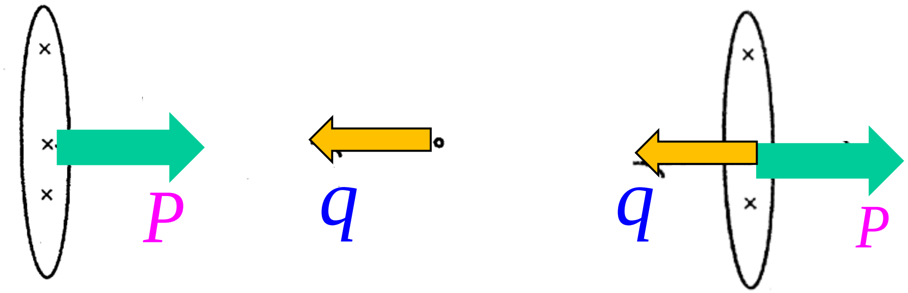




Hard processes in QCD

"hard $\otimes$ soft"

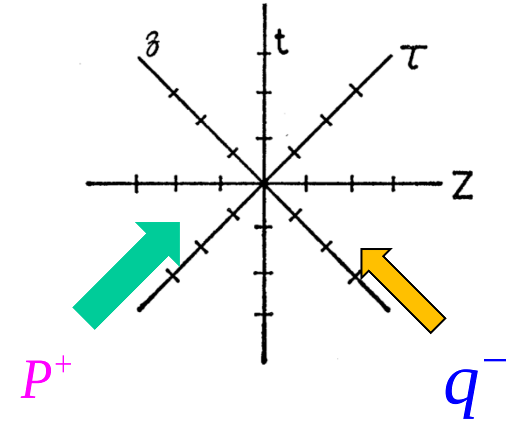


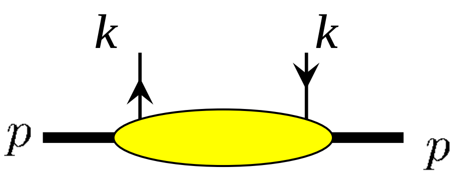


$$z^{\pm} = \frac{z^0 \pm z^3}{\sqrt{2}}$$

$$P^{\pm} = \frac{P^0 \pm P^3}{\sqrt{2}}$$

$$\left(P^- \approx 0\right)$$





$$\int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{ik^+ z^- - i\mathbf{k}_\perp \cdot \mathbf{z}_\perp} \left\langle P \middle| \psi^\dagger(0) \;\; \psi(z^+ = 0, z^-, \mathbf{z}_\perp) \middle| P \right\rangle$$

TMD

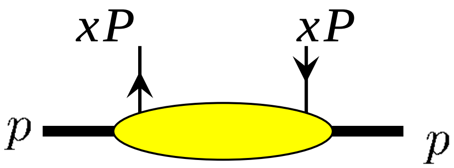
$$U(0; z^-, \mathbf{z}_\perp) = P \exp\left(ig \int_{(z^-, \mathbf{z}_\perp)}^{0} d\xi_\mu A^\mu(\xi) \right)$$

$$f(x) \sim \int \frac{dz^-}{4\pi} e^{i(xP^+) z^-} \left\langle P \middle| \psi^\dagger(0) \;\; \psi(z^+ = 0, z^-, \mathbf{z}_\perp = \mathbf{0}_\perp) \middle| P \right\rangle$$

PDF

$$U(0; z^-, \mathbf{z}_\perp = 0) = P \exp\left(ig \int_{z^-}^{0} d\xi^- A^+(\xi^-) \right)$$

Collins, Soper ('82)



$$f(x) \sim \int \frac{dz^-}{4\pi} e^{i(xP^+)z^-} \langle P | \psi^\dagger(0) \quad \psi(z^-) | P \rangle$$ **PDF**

$$n^\mu = \left(0, n^-, \mathbf{0}_\perp\right) \qquad P \cdot n = P^+ n^- = 1 \qquad z^- = \lambda n^-$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\, S | \bar{\psi}(0) \gamma^\sigma \psi(\lambda n) | P\, S \rangle = q(x) P^\sigma$$

$$S^2 = -M_N^2$$

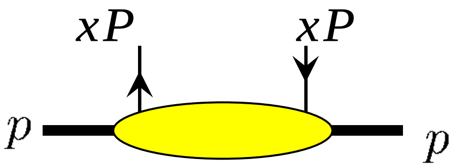
$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\, S | \bar{\psi}(0) \gamma^\sigma \gamma_5 \psi(\lambda n) | P\, S \rangle = \Delta q(x) \left(S \cdot n\right) P^\sigma + g_T(x) S_\perp^\sigma$$

$$S^\sigma = \left(S \cdot n\right) P^\sigma + S_\perp^\sigma$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\, S | \bar{\psi}(0) \sigma^{\mu\nu} i\gamma_5 \psi(\lambda n) | P\, S \rangle = \delta q(x) \frac{S_\perp^\mu P^\nu - S_\perp^\nu P^\mu}{M_N} + h_L(x) \left(P^\mu n^\nu - P^\nu n^\mu\right) M_N \left(S \cdot n\right)$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\, S | \bar{\psi}(0) \psi(\lambda n) | P\, S \rangle = M_N e(x)$$

Jaffe, Ji ('92)



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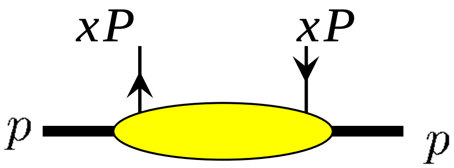

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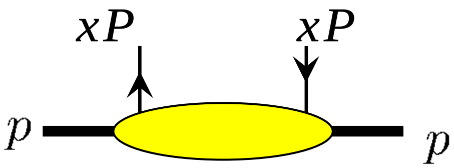
$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\ S | \bar{\psi}_j(0)\psi_i(\lambda n) | P\ S\rangle = \frac{1}{4}\left[q(x)\not{P} + \Delta q(x)\left(S\cdot n\right)\gamma_5 \not{P} + g_T(x)\gamma_5 \not{S}_\perp\right]_{ij}$$

$$S^\sigma = \left(S\cdot n\right) P^\sigma + S_\perp^\sigma$$



$$f(x) \sim \int \frac{dz^-}{4\pi} e^{i(xP^+)z^-} \langle P | \psi^\dagger(0) \quad \psi(z^-) | P \rangle$$ **PDF**

$$F^{+\nu}(0) F^{+\sigma}(z^-) \qquad F^{+\sigma} = \partial^+ A^\sigma$$

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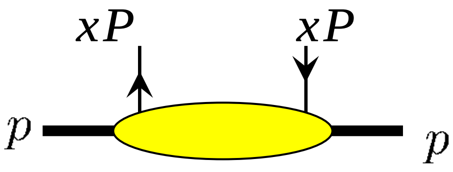

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$$S^\sigma = \left(S \cdot n\right) P^\sigma + S_\perp^\sigma$$

$$-\frac{(n^-)^2}{x} \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\, S \,|\, F^{+\nu}(0) F^{+\sigma}(\lambda n) \,|\, P\, S \rangle$$

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$$= \frac{1}{4} \left[G(x) g_\perp^{\nu\sigma} + \Delta G(x) i\epsilon^{\nu\sigma P n} \left(S \cdot n\right) + 2 G_{3E}(x) i\epsilon^{\nu\sigma\alpha n} S_{\perp\alpha} \right]$$

$$g_\perp^{\nu\sigma} = g^{\nu\sigma} - P^\nu n^\sigma - n^\nu P^\sigma \qquad \epsilon^{\nu\sigma P n} \equiv \epsilon^{\nu\sigma\alpha\beta} P_\alpha n_\beta$$

$$-\frac{(n^-)^2}{x}\int\frac{d\lambda}{4\pi}e^{i\lambda x}\langle P\ S\,|\,F^{+\nu}(0)F^{+\sigma}(\lambda n)\,|\,P\ S\rangle$$

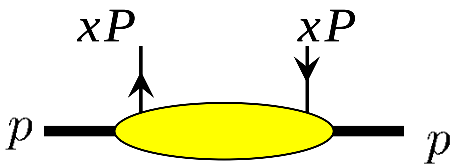
$$=\frac{1}{4}\left[G(x)g_\perp^{\nu\sigma}+\Delta G(x)i\epsilon^{\nu\sigma Pn}\left(S\cdot n\right)+2G_{3E}(x)i\epsilon^{\nu\sigma\alpha n}S_{\perp\alpha}\right]$$

$$-\frac{1}{x}\int\frac{d\lambda}{4\pi}e^{i\lambda x}\langle P\ S\,|\,F^{\mu\nu}(0)F^{\xi\sigma}(\lambda n)\,|\,P\ S\rangle$$

$$-\frac{(n^-)^2}{x}\int\frac{d\lambda}{4\pi}e^{i\lambda x}\langle P\ S\,|\,F^{+\nu}(0)F^{+\sigma}(\lambda n)\,|\,P\ S\rangle$$
$$=\frac{1}{4}\Big[G(x)g_\perp^{\nu\sigma}+\Delta G(x)i\epsilon^{\nu\sigma Pn}(S\cdot n)+2G_{3E}(x)i\epsilon^{\nu\sigma\alpha n}S_{\perp\alpha}\Big]$$

$$-\frac{1}{x}\int\frac{d\lambda}{4\pi}e^{i\lambda x}\langle P\ S\,|\,F^{\mu\nu}(0)F^{\xi\sigma}(\lambda n)\,|\,P\ S\rangle$$
$$=\frac{1}{4}\Big[G(x)\Big(P^\mu P^\nu g_\perp^{\nu\sigma}-P^\nu P^\xi g_\perp^{\mu\sigma}-P^\mu P^\sigma g_\perp^{\nu\xi}+P^\nu P^\sigma g_\perp^{\mu\xi}\Big)$$
$$+\Delta G(x)i(S\cdot n)\Big(P^\mu\epsilon^{\nu\sigma P\xi}-P^\nu\epsilon^{\mu\sigma P\xi}\Big)$$
$$+2G_{3E}(x)iS_{\perp\alpha}\Big(P^\mu\epsilon^{\nu\sigma\alpha\xi}-P^\nu\epsilon^{\mu\sigma\alpha\xi}\Big)$$
$$+2G_{3H}(x)i\Big(S_\perp^\mu\epsilon^{\nu\sigma P\xi}-S_\perp^\nu\epsilon^{\mu\sigma P\xi}\Big)\Big]$$

PT-inv. & hermiticity



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$$= \frac{1}{4}\left[G(x) g_\perp^{\nu\sigma} + \Delta G(x) i\epsilon^{\nu\sigma P n} \left(S \cdot n\right) + 2 G_{3E}(x) i\epsilon^{\nu\sigma\alpha n} S_{\perp\alpha} \right]$$

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$$M^{\mu\nu} \qquad M^{03} \qquad M^{12} = J^3$$

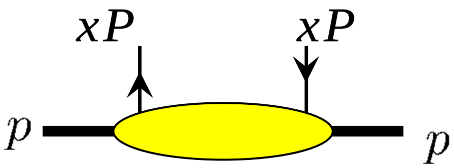
$$\left[M^{\mu\nu}, \psi(x)\right] = -\left(i\left(x^\mu \partial^\nu - x^\nu \partial^\mu\right) + \frac{1}{2}\sigma^{\mu\nu} \right)\psi(x)$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\, S \,|\, \psi^\dagger(0) \mathbf{1} \psi(\lambda n) \,|\, P\, S \rangle = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\, S \,|\, \overline{\psi}(0) \frac{1}{\sqrt{2}} \gamma^+ \psi(\lambda n) \,|\, P\, S \rangle \sim q(x)$$

$$\hat{s}^i = \frac{1}{2} \varepsilon^{ijk} \frac{\sigma^{jk}}{2} \qquad \left[\hat{s}^1, \hat{s}^2\right] = i\hat{s}^3$$

$$2\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\, S \,|\, \psi^\dagger(0) \hat{s}^3 \psi(\lambda n) \,|\, P\, S \rangle = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\, S \,|\, \overline{\psi}(0) \frac{1}{\sqrt{2}} \gamma^+ \gamma_5 \psi(\lambda n) \,|\, P\, S \rangle \sim \Delta q(x)$$

$$2\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\, S \,|\, \psi^\dagger(0) \hat{s}^\perp \psi(\lambda n) \,|\, P\, S \rangle = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\, S \,|\, \overline{\psi}(0) \gamma^\perp \gamma_5 \psi(\lambda n) \,|\, P\, S \rangle \sim g_T(x)$$



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PDF

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$M^{\mu\nu}$ $\qquad M^{03}$ $\qquad M^{12} = J^3$

$$\left[M^{\mu\nu}, \psi(x)\right] = -\left(i\left(x^\mu \partial^\nu - x^\nu \partial^\mu\right) + \frac{1}{2}\sigma^{\mu\nu}\right)\psi(x)$$

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$$2\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\, S \,|\, \psi^\dagger(0) \hat{s}^\perp \psi(\lambda n) \,|\, P\, S \rangle = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\, S \,|\, \bar{\psi}(0) \gamma^\perp \gamma_5 \psi(\lambda n) \,|\, P\, S \rangle \sim g_T(x)$$

$$\psi = \psi_{\text{good}} + \psi_{\text{bad}}$$

$$\bar{\psi}_{\text{good}}(0)\gamma^\perp\gamma_5\psi_{\text{bad}}(\lambda n) + \bar{\psi}_{\text{bad}}(0)\gamma^\perp\gamma_5\psi_{\text{good}}(\lambda n)$$

$$\psi_{\text{bad}}(\lambda n) = -i\int \frac{d\lambda'}{2\pi} \int \frac{d\xi}{2\xi} e^{-i\xi(\lambda-\lambda')} \not{n} \not{D}_\perp \psi_{\text{good}}(\lambda' n) \qquad D_\perp^\sigma = \partial_\perp^\sigma - igA_\perp^\sigma$$

$$\left[M^{\mu\nu}, F^{\alpha\beta}(x)\right] = -\left(i\left(x^{\mu}\partial^{\nu} - x^{\nu}\partial^{\mu}\right) + \Sigma^{\mu\nu}\right) F^{\alpha\beta}(x)$$

$$\Sigma^{\mu\nu} F^{\alpha\beta} = i\left(g^{\mu\alpha} F^{\nu\beta} - g^{\nu\alpha} F^{\mu\beta} - (\alpha \leftrightarrow \beta)\right)$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\ S \,|\, F^{\alpha\beta}(0)^{\dagger} \mathbf{1} F^{\alpha\beta}(\lambda n) \,|\, P\ S\rangle = -\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P\ S \,|\, F^{+\beta}(0) F^{+}{}_{\beta}(\lambda n) \,|\, P\ S\rangle \sim xG(x)$$

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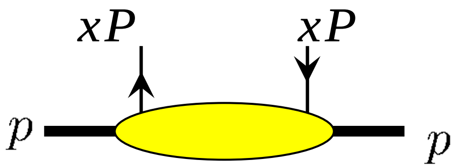
$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\ S \,|\, F^{\alpha\beta}(0)^{\dagger} \hat{s}^{\perp} F^{\alpha\beta}(\lambda n) \,|\, P\ S\rangle = \frac{-i}{\sqrt{2}} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P\ S \,|\, \widetilde{F}^{+\perp}(0) F^{+-}(\lambda n) + F^{+\perp}(0) F^{12}(\lambda n) + \cdots \,|\, P\ S\rangle$$



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$$S^2 = -M_N^2$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\, S | \bar{\psi}_j(0)\psi_i(\lambda n) | P\, S \rangle = \frac{1}{4}\left[q(x)\not{P} + \Delta q(x)\left(S\cdot n\right)\gamma_5 \not{P} + g_T(x)\gamma_5 \not{S}_\perp \right]_{ij}$$

$$S^\sigma = \left(S \cdot n\right) P^\sigma + S_\perp^\sigma$$

$$-\frac{(n^-)^2}{x} \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\, S | F^{+\nu}(0) F^{+\sigma}(\lambda n) | P\, S \rangle$$

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$$= \frac{1}{4}\left[G(x) g_\perp^{\nu\sigma} + \Delta G(x) i\epsilon^{\nu\sigma P n}\left(S\cdot n\right) + 2 G_{3E}(x) i\epsilon^{\nu\sigma\alpha n} S_{\perp\alpha} \right]$$

$$g_\perp^{\nu\sigma} = g^{\nu\sigma} - P^\nu n^\sigma - n^\nu P^\sigma \qquad \epsilon^{\nu\sigma P n} \equiv \epsilon^{\nu\sigma\alpha\beta} P_\alpha n_\beta$$

$$\left[M^{\mu\nu}, F^{\alpha\beta}(x)\right] = -\left(i\left(x^\mu\partial^\nu - x^\nu\partial^\mu\right) + \Sigma^{\mu\nu}\right)F^{\alpha\beta}(x)$$

$$\Sigma^{\mu\nu}F^{\alpha\beta} = i\left(g^{\mu\alpha}F^{\nu\beta} - g^{\nu\alpha}F^{\mu\beta} - (\alpha \leftrightarrow \beta)\right)$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x}\langle P\ S \,|\, F^{\alpha\beta}(0)^\dagger \mathbf{1} F^{\alpha\beta}(\lambda n) \,|\, P\ S\rangle = -\int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle P\ S \,|\, F^{+\beta}(0) F^{+}{}_{\beta}(\lambda n) \,|\, P\ S\rangle \sim xG(x)$$

$$\hat{s}^i = \frac{1}{2}\varepsilon^{ijk}\Sigma^{jk} \qquad \left[\hat{s}^1, \hat{s}^2\right] = i\hat{s}^3$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x}\langle P\ S \,|\, F^{\alpha\beta}(0)^\dagger \hat{s}^3 F^{\alpha\beta}(\lambda n) \,|\, P\ S\rangle = \int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle P\ S \,|\, iF^{+\beta}(0) \widetilde{F}^{+}{}_{\beta}(\lambda n) \,|\, P\ S\rangle \sim x\Delta G(x)$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x}\langle P\ S \,|\, F^{\alpha\beta}(0)^\dagger \hat{s}^\perp F^{\alpha\beta}(\lambda n) \,|\, P\ S\rangle = \frac{-i}{\sqrt{2}}\int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle P\ S \,|\, \widetilde{F}^{+\perp}(0) F^{+-}(\lambda n) + F^{+\perp}(0) F^{12}(\lambda n) + \cdots \,|\, P\ S\rangle$$

$$-\frac{(n^-)^2}{x}\int\frac{d\lambda}{4\pi}e^{i\lambda x}\langle P\ S\,|\,F^{+\nu}(0)F^{+\sigma}(\lambda n)\,|\,P\ S\rangle$$

$$=\frac{1}{4}\Big[\,G(x)g_\perp^{\nu\sigma}+\Delta G(x)i\epsilon^{\nu\sigma Pn}\,(S\cdot n)+2G_{3E}(x)i\epsilon^{\nu\sigma\alpha n}S_{\perp\alpha}\,\Big]$$

$$-\frac{1}{x}\int\frac{d\lambda}{4\pi}e^{i\lambda x}\langle P\ S\,|\,F^{\mu\nu}(0)F^{\xi\sigma}(\lambda n)\,|\,P\ S\rangle$$

$$=\frac{1}{4}\Big[\,G(x)\Big(P^\mu P^\nu g_\perp^{\nu\sigma}-P^\nu P^\xi g_\perp^{\mu\sigma}-P^\mu P^\sigma g_\perp^{\nu\xi}+P^\nu P^\sigma g_\perp^{\mu\xi}\Big)$$

$$+\Delta G(x)i\,(S\cdot n)\Big(P^\mu\epsilon^{\nu\sigma P\xi}-P^\nu\epsilon^{\mu\sigma P\xi}\Big)$$

$$+2G_{3E}(x)iS_{\perp\alpha}\Big(P^\mu\epsilon^{\nu\sigma\alpha\xi}-P^\nu\epsilon^{\mu\sigma\alpha\xi}\Big)$$

$$+2\,G_{3H}(x)i\Big(S_\perp^\mu\epsilon^{\nu\sigma P\xi}-S_\perp^\nu\epsilon^{\mu\sigma P\xi}\Big)\Big]$$

PT-inv. & hermiticity

$$\left[M^{\mu\nu}, F^{\alpha\beta}(x) \right] = -\left(i\left(x^{\mu}\partial^{\nu} - x^{\nu}\partial^{\mu} \right) + \Sigma^{\mu\nu} \right) F^{\alpha\beta}(x)$$

$$\Sigma^{\mu\nu} F^{\alpha\beta} = i\left(g^{\mu\alpha} F^{\nu\beta} - g^{\nu\alpha} F^{\mu\beta} - (\alpha \leftrightarrow \beta) \right)$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\, S \,|\, F^{\alpha\beta}(0)^{\dagger} \mathbf{1} F^{\alpha\beta}(\lambda n) \,|\, P\, S \rangle = -\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P\, S \,|\, F^{+\beta}(0) F^{+}{}_{\beta}(\lambda n) \,|\, P\, S \rangle \sim xG(x)$$

$$\hat{s}^{i} = \frac{1}{2} \varepsilon^{ijk} \Sigma^{jk} \qquad \left[\hat{s}^{1}, \hat{s}^{2} \right] = i\hat{s}^{3}$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\, S \,|\, F^{\alpha\beta}(0)^{\dagger} \hat{s}^{3} F^{\alpha\beta}(\lambda n) \,|\, P\, S \rangle = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P\, S \,|\, iF^{+\beta}(0) \widetilde{F}^{+}{}_{\beta}(\lambda n) \,|\, P\, S \rangle \sim x\Delta G(x)$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\, S \,|\, F^{\alpha\beta}(0)^{\dagger} \hat{s}^{\perp} F^{\alpha\beta}(\lambda n) \,|\, P\, S \rangle = \frac{-i}{\sqrt{2}} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P\, S \,|\, \widetilde{F}^{+\perp}(0) F^{+-}(\lambda n) + F^{+\perp}(0) F^{12}(\lambda n) + \cdots \,|\, P\, S \rangle$$

$$\sim x\left(G_{3E}(x) + G_{3H}(x) \right) \equiv xG_{T}(x)$$

$$\left[M^{\mu\nu}, F^{\alpha\beta}(x)\right] = -\left(i\left(x^{\mu}\partial^{\nu} - x^{\nu}\partial^{\mu}\right) + \Sigma^{\mu\nu}\right)F^{\alpha\beta}(x)$$

$$\Sigma^{\mu\nu}F^{\alpha\beta} = i\left(g^{\mu\alpha}F^{\nu\beta} - g^{\nu\alpha}F^{\mu\beta} - (\alpha\leftrightarrow\beta)\right)$$

$$\int\frac{d\lambda}{4\pi}e^{i\lambda x}\langle P\,S|F^{\alpha\beta}(0)^{\dagger}\,\mathbf{1}\,F^{\alpha\beta}(\lambda n)|P\,S\rangle = -\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle P\,S|F^{+\beta}(0)F^{+}{}_{\beta}(\lambda n)|P\,S\rangle \sim xG(x)$$

$$\hat{s}^{i} = \frac{1}{2}\varepsilon^{ijk}\Sigma^{jk} \qquad \left[\hat{s}^{1},\hat{s}^{2}\right] = i\hat{s}^{3}$$

$$\int\frac{d\lambda}{4\pi}e^{i\lambda x}\langle P\,S|F^{\alpha\beta}(0)^{\dagger}\,\hat{s}^{3}F^{\alpha\beta}(\lambda n)|P\,S\rangle = \int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle P\,S|iF^{+\beta}(0)\widetilde{F}^{+}{}_{\beta}(\lambda n)|P\,S\rangle \sim x\Delta G(x)$$

$$\int\frac{d\lambda}{4\pi}e^{i\lambda x}\langle P\,S|F^{\alpha\beta}(0)^{\dagger}\,\hat{s}^{\perp}F^{\alpha\beta}(\lambda n)|P\,S\rangle = \frac{-i}{\sqrt{2}}\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle P\,S|\widetilde{F}^{+\perp}(0)F^{+-}(\lambda n) + F^{+\perp}(0)F^{12}(\lambda n) + \cdots|P\,S\rangle$$

$$\sim x\left(G_{3E}(x) + G_{3H}(x)\right) \equiv xG_{T}(x)$$

$$F^{12} = \partial^{1}A^{2} - \partial^{2}A^{1} - ig[A^{1}, A^{2}]$$

$$F^{+-} = \partial^{+}A^{-} = \frac{1}{\partial_{-}}\left(D_{\perp j}F^{j+} + g\bar{\psi}t^{a}\gamma^{+}\psi t^{a}\right)$$

$$D_{\nu}F^{\mu\nu} = g\bar{\psi}t^{a}\gamma^{\mu}\psi t^{a} \qquad D_{\perp}^{\sigma} = \partial_{\perp}^{\sigma} - igA_{\perp}^{\sigma}$$

$$M^{\mu\nu} \qquad M^{03} \qquad M^{12} = J^3$$

$$\left[M^{\mu\nu}, \psi(x)\right] = -\left(i\left(x^\mu \partial^\nu - x^\nu \partial^\mu\right) + \frac{1}{2}\sigma^{\mu\nu} \right)\psi(x)$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\,S \,|\, \psi^\dagger(0) \mathbf{1} \psi(\lambda n) \,|\, P\,S\rangle = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\,S \,|\, \bar{\psi}(0) \frac{1}{\sqrt{2}} \gamma^+ \psi(\lambda n) \,|\, P\,S\rangle \sim q(x)$$

$$\hat{s}^i = \frac{1}{2}\varepsilon^{ijk}\frac{\sigma^{jk}}{2} \qquad \left[\hat{s}^1, \hat{s}^2\right] = i\hat{s}^3$$

$$2\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\,S \,|\, \psi^\dagger(0) \hat{s}^3 \psi(\lambda n) \,|\, P\,S\rangle = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\,S \,|\, \bar{\psi}(0) \frac{1}{\sqrt{2}} \gamma^+ \gamma_5 \psi(\lambda n) \,|\, P\,S\rangle \sim \Delta q(x)$$

$$2\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\,S \,|\, \psi^\dagger(0) \hat{s}^\perp \psi(\lambda n) \,|\, P\,S\rangle = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\,S \,|\, \bar{\psi}(0) \gamma^\perp \gamma_5 \psi(\lambda n) \,|\, P\,S\rangle \sim g_T(x)$$

$$\psi = \psi_{\text{good}} + \psi_{\text{bad}}$$

$$\bar{\psi}_{\text{good}}(0)\gamma^\perp\gamma_5\psi_{\text{bad}}(\lambda n) + \bar{\psi}_{\text{bad}}(0)\gamma^\perp\gamma_5\psi_{\text{good}}(\lambda n)$$

$$\psi_{\text{bad}}(\lambda n) = -i\int\frac{d\lambda'}{2\pi}\int\frac{d\xi}{2\xi} e^{-i\xi(\lambda-\lambda')} \not{n} \not{D}_\perp \psi_{\text{good}}(\lambda' n) \qquad D_\perp^\sigma = \partial_\perp^\sigma - igA_\perp^\sigma$$

$$M^{\mu\nu} \qquad M^{03} \qquad M^{12} = J^3$$

$$\psi = \psi_\uparrow + \psi_\downarrow$$

$$\left[M^{\mu\nu}, \psi(x)\right] = -\left(i\left(x^\mu \partial^\nu - x^\nu \partial^\mu\right) + \frac{1}{2}\sigma^{\mu\nu} \right)\psi(x)$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\ S \,|\, \psi^\dagger(0) \mathbf{1} \psi(\lambda n) \,|\, P\ S \rangle = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\ S \,|\, \bar{\psi}(0) \frac{1}{\sqrt{2}} \gamma^+ \psi(\lambda n) \,|\, P\ S \rangle \sim q(x)$$

$$\psi_\uparrow^\dagger(0)\psi_\uparrow(\lambda n) + \psi_\downarrow^\dagger(0)\psi_\downarrow(\lambda n)$$

$$\hat{s}^i = \frac{1}{2}\varepsilon^{ijk} \frac{\sigma^{jk}}{2} \qquad \left[\hat{s}^1, \hat{s}^2\right] = i\hat{s}^3$$

$$2\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\ S \,|\, \psi^\dagger(0) \hat{s}^3 \psi(\lambda n) \,|\, P\ S \rangle = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\ S \,|\, \bar{\psi}(0) \frac{1}{\sqrt{2}} \gamma^+ \gamma_5 \psi(\lambda n) \,|\, P\ S \rangle \sim \Delta q(x)$$

$$\psi_\uparrow^\dagger(0)\psi_\uparrow(\lambda n) - \psi_\downarrow^\dagger(0)\psi_\downarrow(\lambda n)$$

$$2\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\ S \,|\, \psi^\dagger(0) \hat{s}^\perp \psi(\lambda n) \,|\, P\ S \rangle = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\ S \,|\, \bar{\psi}(0) \gamma^\perp \gamma_5 \psi(\lambda n) \,|\, P\ S \rangle \sim g_T(x)$$

$$\psi_\uparrow^\dagger(0)\psi_\downarrow(\lambda n) + \psi_\downarrow^\dagger(0)\psi_\uparrow(\lambda n)$$

$$\psi = \psi_{\text{good}} + \psi_{\text{bad}}$$

$$\bar{\psi}_{\text{good}}(0)\gamma^\perp\gamma_5\psi_{\text{bad}}(\lambda n) + \bar{\psi}_{\text{bad}}(0)\gamma^\perp\gamma_5\psi_{\text{good}}(\lambda n)$$

$$\psi_{\text{bad}}(\lambda n) = -i\int \frac{d\lambda'}{2\pi} \int \frac{d\xi}{2\xi} e^{-i\xi(\lambda-\lambda')} \not{n} \not{D}_\perp \psi_{\text{good}}(\lambda' n) \qquad D_\perp^\sigma = \partial_\perp^\sigma - igA_\perp^\sigma$$

$$\left[M^{\mu\nu}, F^{\alpha\beta}(x)\right] = -\left(i\left(x^\mu\partial^\nu - x^\nu\partial^\mu\right) + \Sigma^{\mu\nu}\right)F^{\alpha\beta}(x)$$

$$\Sigma^{\mu\nu}F^{\alpha\beta} = i\left(g^{\mu\alpha}F^{\nu\beta} - g^{\nu\alpha}F^{\mu\beta} - (\alpha\leftrightarrow\beta)\right)$$

$$\int\frac{d\lambda}{4\pi}e^{i\lambda x}\langle P\ S\,|\,F^{\alpha\beta}(0)^\dagger \mathbf{1} F^{\alpha\beta}(\lambda n)\,|\,P\ S\rangle = -\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle P\ S\,|\,F^{+\beta}(0)F^{+}{}_{\beta}(\lambda n)\,|\,P\ S\rangle \sim xG(x)$$

$$\hat{s}^i = \frac{1}{2}\varepsilon^{ijk}\Sigma^{jk} \qquad \left[\hat{s}^1, \hat{s}^2\right] = i\hat{s}^3$$

$$\int\frac{d\lambda}{4\pi}e^{i\lambda x}\langle P\ S\,|\,F^{\alpha\beta}(0)^\dagger \hat{s}^3 F^{\alpha\beta}(\lambda n)\,|\,P\ S\rangle = \int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle P\ S\,|\,iF^{+\beta}(0)\widetilde{F}^{+}{}_{\beta}(\lambda n)\,|\,P\ S\rangle \sim x\Delta G(x)$$

$$\int\frac{d\lambda}{4\pi}e^{i\lambda x}\langle P\ S\,|\,F^{\alpha\beta}(0)^\dagger \hat{s}^\perp F^{\alpha\beta}(\lambda n)\,|\,P\ S\rangle = \frac{-i}{\sqrt{2}}\int\frac{d\lambda}{2\pi}e^{i\lambda x}\langle P\ S\,|\,\widetilde{F}^{+\perp}(0)F^{+-}(\lambda n) + F^{+\perp}(0)F^{12}(\lambda n) + \cdots\,|\,P\ S\rangle$$

$$\sim x\left(G_{3E}(x) + G_{3H}(x)\right) \equiv xG_T(x)$$

$$F^{12} = \partial^1 A^2 - \partial^2 A^1 - ig[A^1, A^2]$$

$$F^{+-} = \partial^+ A^- = \frac{1}{\partial_-}\left(D_{\perp j}F^{j+} + g\bar{\psi}t^a\gamma^+\psi t^a\right)$$

$$D_\nu F^{\mu\nu} = g\bar{\psi}t^a\gamma^\mu\psi t^a \qquad D_\perp^\sigma = \partial_\perp^\sigma - igA_\perp^\sigma$$

$$\left[M^{\mu\nu}, F^{\alpha\beta}(x)\right] = -\left(i\left(x^\mu\partial^\nu - x^\nu\partial^\mu\right) + \Sigma^{\mu\nu}\right)F^{\alpha\beta}(x)$$

$$\epsilon^\mu_{R/L} = (0, \mp 1, -i, 0)/\sqrt{2}$$

$$\Sigma^{\mu\nu}F^{\alpha\beta} = i\left(g^{\mu\alpha}F^{\nu\beta} - g^{\nu\alpha}F^{\mu\beta} - (\alpha \leftrightarrow \beta)\right)$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\ S \,|\, F^{\alpha\beta}(0)^\dagger \mathbf{1} F^{\alpha\beta}(\lambda n) \,|\, P\ S\rangle = -\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P\ S \,|\, F^{+\beta}(0) F^{+}{}_{\beta}(\lambda n) \,|\, P\ S\rangle \sim xG(x)$$

$$F^{+R}(0)^\dagger F^{+R}(\lambda n) + F^{+L}(0)^\dagger F^{+L}(\lambda n)$$

$$\hat{s}^i = \frac{1}{2}\varepsilon^{ijk}\Sigma^{jk}$$

$$\left[\hat{s}^1, \hat{s}^2\right] = i\hat{s}^3$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\ S \,|\, F^{\alpha\beta}(0)^\dagger \hat{s}^3 F^{\alpha\beta}(\lambda n) \,|\, P\ S\rangle = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P\ S \,|\, iF^{+\beta}(0) \widetilde{F}^{+}{}_{\beta}(\lambda n) \,|\, P\ S\rangle \sim x\Delta G(x)$$

$$F^{+R}(0)^\dagger F^{+R}(\lambda n) - F^{+L}(0)^\dagger F^{+L}(\lambda n)$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\ S \,|\, F^{\alpha\beta}(0)^\dagger \hat{s}^\perp F^{\alpha\beta}(\lambda n) \,|\, P\ S\rangle = \frac{-i}{\sqrt{2}} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P\ S \,|\, \widetilde{F}^{+\perp}(0) F^{+-}(\lambda n) + F^{+\perp}(0) F^{12}(\lambda n) + \cdots \,|\, P\ S\rangle$$

$$\sim x\left(G_{3E}(x) + G_{3H}(x)\right) \equiv xG_T(x)$$

$$\widetilde{F}^{+\perp}(0) F^{+-}(\lambda n) = \frac{F^{+R}(0)^\dagger + F^{+L}(0)^\dagger}{\sqrt{2}i} F^{30}(\lambda n) = \frac{F^{+R}(0)^\dagger + F^{+L}(0)^\dagger}{\sqrt{2}i} E^3(\lambda n)$$

$$F^{+\perp}(0) F^{12}(\lambda n) = \frac{F^{+R}(0)^\dagger - F^{+L}(0)^\dagger}{\sqrt{2}} H^3(\lambda n)$$

$$z^\pm = \frac{z^0 \pm z^3}{\sqrt{2}}$$

$$J_{\|} = \frac{1}{2} = L + \frac{1}{2}\Delta\Sigma + \Delta G$$

$$\Delta\Sigma = \int dx \Delta q(x) = \frac{1}{2P^{+}}\langle P\ S_{\|} \,|\, \bar{\psi}_f(0)\gamma^{+}\gamma_5\psi_f(0) \,|\, P\ S_{\|}\rangle = \frac{\sqrt{2}}{P^{+}}\langle P\ S_{\|} \,|\, \psi^{\dagger}(0)\hat{s}^3\psi(0) \,|\, P\ S_{\|}\rangle$$

$$\Delta G = \int dx \Delta G(x) = \frac{1}{P^{+2}}\int dx \frac{1}{x}\int \frac{d\lambda}{4\pi} e^{i\lambda x}\langle P\ S_{\|} \,|\, F^{\alpha\beta}(0)^{\dagger}\hat{s}^3 F^{\alpha\beta}(\lambda n) \,|\, P\ S_{\|}\rangle$$

$$M^{\mu\nu} \qquad M^{03} \qquad M^{12} = J^3$$

$$\left[M^{\mu\nu}, \psi(x)\right] = -\left(i\left(x^\mu \partial^\nu - x^\nu \partial^\mu\right) + \frac{1}{2}\sigma^{\mu\nu} \right)\psi(x)$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\, S \,|\, \psi^\dagger(0) \mathbf{1} \psi(\lambda n) \,|\, P\, S\rangle = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\, S \,|\, \bar{\psi}(0) \frac{1}{\sqrt{2}} \gamma^+ \psi(\lambda n) \,|\, P\, S\rangle \sim q(x)$$

$$\hat{s}^i = \frac{1}{2}\varepsilon^{ijk} \frac{\sigma^{jk}}{2} \qquad \left[\hat{s}^1, \hat{s}^2\right] = i\hat{s}^3$$

$$2\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\, S \,|\, \psi^\dagger(0) \hat{s}^3 \psi(\lambda n) \,|\, P\, S\rangle = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\, S \,|\, \bar{\psi}(0) \frac{1}{\sqrt{2}} \gamma^+ \gamma_5 \psi(\lambda n) \,|\, P\, S\rangle \sim \Delta q(x)$$

$$2\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\, S \,|\, \psi^\dagger(0) \hat{s}^\perp \psi(\lambda n) \,|\, P\, S\rangle = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\, S \,|\, \bar{\psi}(0) \gamma^\perp \gamma_5 \psi(\lambda n) \,|\, P\, S\rangle \sim g_T(x)$$

$$\psi = \psi_{\text{good}} + \psi_{\text{bad}}$$

$$\bar{\psi}_{\text{good}}(0) \gamma^\perp \gamma_5 \psi_{\text{bad}}(\lambda n) + \bar{\psi}_{\text{bad}}(0) \gamma^\perp \gamma_5 \psi_{\text{good}}(\lambda n)$$

$$\psi_{\text{bad}}(\lambda n) = -i \int \frac{d\lambda'}{2\pi} \int \frac{d\xi}{2\xi} e^{-i\xi(\lambda - \lambda')} \not{n} \not{D}_\perp \psi_{\text{good}}(\lambda' n) \qquad D_\perp^\sigma = \partial_\perp^\sigma - igA_\perp^\sigma$$

$$J_{\|} = \frac{1}{2} = L + \frac{1}{2}\Delta\Sigma + \Delta G$$

$$\Delta\Sigma = \int dx \Delta q(x) = \frac{1}{2P^{+}} \langle P\ S_{\|} \,|\, \bar{\psi}_f(0)\gamma^{+}\gamma_5 \psi_f(0) \,|\, P\ S_{\|} \rangle = \frac{\sqrt{2}}{P^{+}} \langle P\ S_{\|} \,|\, \psi^{\dagger}(0)\hat{s}^{3}\psi(0) \,|\, P\ S_{\|} \rangle$$

$$\Delta G = \int dx \Delta G(x) = \frac{1}{P^{+2}} \int dx \frac{1}{x} \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\ S_{\|} \,|\, F^{\alpha\beta}(0)^{\dagger}\hat{s}^{3} F^{\alpha\beta}(\lambda n) \,|\, P\ S_{\|} \rangle$$

$$\left[M^{\mu\nu}, F^{\alpha\beta}(x)\right] = -\left(i\left(x^{\mu}\partial^{\nu} - x^{\nu}\partial^{\mu}\right) + \Sigma^{\mu\nu}\right)F^{\alpha\beta}(x)$$

$$\Sigma^{\mu\nu}F^{\alpha\beta} = i\left(g^{\mu\alpha}F^{\nu\beta} - g^{\nu\alpha}F^{\mu\beta} - (\alpha \leftrightarrow \beta)\right)$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x}\langle P\ S \,|\, F^{\alpha\beta}(0)^{\dagger}\mathbf{1}F^{\alpha\beta}(\lambda n) \,|\, P\ S\rangle = -\int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle P\ S \,|\, F^{+\beta}(0)F^{+}{}_{\beta}(\lambda n) \,|\, P\ S\rangle \sim xG(x)$$

$$\hat{s}^{i} = \frac{1}{2}\varepsilon^{ijk}\Sigma^{jk} \qquad \left[\hat{s}^{1}, \hat{s}^{2}\right] = i\hat{s}^{3}$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x}\langle P\ S \,|\, F^{\alpha\beta}(0)^{\dagger}\hat{s}^{3}F^{\alpha\beta}(\lambda n) \,|\, P\ S\rangle = \int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle P\ S \,|\, iF^{+\beta}(0)\widetilde{F}^{+}{}_{\beta}(\lambda n) \,|\, P\ S\rangle \sim x\Delta G(x)$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x}\langle P\ S \,|\, F^{\alpha\beta}(0)^{\dagger}\hat{s}^{\perp}F^{\alpha\beta}(\lambda n) \,|\, P\ S\rangle = \frac{-i}{\sqrt{2}}\int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle P\ S \,|\, \widetilde{F}^{+\perp}(0)F^{+-}(\lambda n) + F^{+\perp}(0)F^{12}(\lambda n) + \cdots \,|\, P\ S\rangle$$

$$\sim x\left(G_{3E}(x) + G_{3H}(x)\right) \equiv xG_{T}(x)$$

$$F^{12} = \partial^{1}A^{2} - \partial^{2}A^{1} - ig[A^{1}, A^{2}]$$

$$F^{+-} = \partial^{+}A^{-} = \frac{1}{\partial_{-}}\left(D_{\perp j}F^{j+} + g\overline{\psi}t^{a}\gamma^{+}\psi t^{a}\right)$$

$$D_{\nu}F^{\mu\nu} = g\overline{\psi}t^{a}\gamma^{\mu}\psi t^{a} \qquad D_{\perp}^{\sigma} = \partial_{\perp}^{\sigma} - igA_{\perp}^{\sigma}$$

$$J_{\parallel} = \frac{1}{2} = L + \frac{1}{2}\Delta\Sigma + \Delta G$$

$$\Delta\Sigma = \int dx \Delta q(x) = \frac{1}{2P^+}\langle P\ S_{\parallel} \,|\, \bar{\psi}_f(0)\gamma^+\gamma_5\psi_f(0) \,|\, P\ S_{\parallel}\rangle = \frac{\sqrt{2}}{P^+}\langle P\ S_{\parallel} \,|\, \psi^\dagger(0)\hat{s}^3\psi(0) \,|\, P\ S_{\parallel}\rangle$$

$$\Delta G = \int dx \Delta G(x) = \frac{1}{P^{+2}}\int dx \frac{1}{x}\int \frac{d\lambda}{4\pi} e^{i\lambda x}\langle P\ S_{\parallel} \,|\, F^{\alpha\beta}(0)^\dagger \hat{s}^3 F^{\alpha\beta}(\lambda n) \,|\, P\ S_{\parallel}\rangle$$

$$J_{\|} = \frac{1}{2} = L + \frac{1}{2}\Delta\Sigma + \Delta G$$

$$\Delta\Sigma = \int dx \Delta q(x) = \frac{1}{2P^{+}} \langle P\ S_{\|} \,|\, \bar{\psi}_f(0)\gamma^{+}\gamma_5\psi_f(0) \,|\, P\ S_{\|}\rangle = \frac{\sqrt{2}}{P^{+}} \langle P\ S_{\|} \,|\, \psi^{\dagger}(0)\hat{s}^{3}\psi(0) \,|\, P\ S_{\|}\rangle$$

$$\Delta G = \int dx \Delta G(x) = \frac{1}{P^{+2}} \int dx \frac{1}{x} \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\ S_{\|} \,|\, F^{\alpha\beta}(0)^{\dagger}\hat{s}^{3}F^{\alpha\beta}(\lambda n) \,|\, P\ S_{\|}\rangle$$

$$J_T = \frac{1}{2} = L_T + \frac{1}{2}\Delta\Sigma_T + \Delta G_T$$

$$\Delta\Sigma_T = \int dx g_T(x) = \frac{1}{2M} \langle P\ S_{\perp} \,|\, \bar{\psi}_f(0)\gamma^{\perp}\gamma_5\psi_f(0) \,|\, P\ S_{\perp}\rangle = \frac{1}{M} \langle P\ S_{\perp} \,|\, \psi^{\dagger}(0)\hat{s}^{\perp}\psi(0) \,|\, P\ S_{\perp}\rangle$$

$$\Delta G_T = \int dx G_T(x) = \frac{1}{\sqrt{2}P^{+}M} \int dx \frac{1}{x} \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\ S_{\perp} \,|\, F^{\alpha\beta}(0)^{\dagger}\hat{s}^{\perp}F^{\alpha\beta}(\lambda n) \,|\, P\ S_{\perp}\rangle$$

$$M^{\mu\nu} \qquad M^{03} \qquad M^{12} = J^3$$

$$\left[M^{\mu\nu}, \psi(x)\right] = -\left(i\left(x^\mu \partial^\nu - x^\nu \partial^\mu\right) + \frac{1}{2}\sigma^{\mu\nu} \right)\psi(x)$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\,S \,|\, \psi^\dagger(0) \mathbf{1} \psi(\lambda n) \,|\, P\,S \rangle = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\,S \,|\, \bar{\psi}(0) \frac{1}{\sqrt{2}} \gamma^+ \psi(\lambda n) \,|\, P\,S \rangle \sim q(x)$$

$$\hat{s}^i = \frac{1}{2} \varepsilon^{ijk} \frac{\sigma^{jk}}{2} \qquad \left[\hat{s}^1, \hat{s}^2\right] = i\hat{s}^3$$

$$2\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\,S \,|\, \psi^\dagger(0) \hat{s}^3 \psi(\lambda n) \,|\, P\,S \rangle = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\,S \,|\, \bar{\psi}(0) \frac{1}{\sqrt{2}} \gamma^+ \gamma_5 \psi(\lambda n) \,|\, P\,S \rangle \sim \Delta q(x)$$

$$2\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\,S \,|\, \psi^\dagger(0) \hat{s}^\perp \psi(\lambda n) \,|\, P\,S \rangle = \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\,S \,|\, \bar{\psi}(0) \gamma^\perp \gamma_5 \psi(\lambda n) \,|\, P\,S \rangle \sim g_T(x)$$

$$\psi = \psi_{\text{good}} + \psi_{\text{bad}}$$

$$\bar{\psi}_{\text{good}}(0) \gamma^\perp \gamma_5 \psi_{\text{bad}}(\lambda n) + \bar{\psi}_{\text{bad}}(0) \gamma^\perp \gamma_5 \psi_{\text{good}}(\lambda n)$$

$$\psi_{\text{bad}}(\lambda n) = -i \int \frac{d\lambda'}{2\pi} \int \frac{d\xi}{2\xi} e^{-i\xi(\lambda - \lambda')} \not{n} \not{D}_\perp \psi_{\text{good}}(\lambda' n) \qquad D_\perp^\sigma = \partial_\perp^\sigma - igA_\perp^\sigma$$

$$J_{\parallel} = \frac{1}{2} = L + \frac{1}{2}\Delta\Sigma + \Delta G$$

$$\Delta\Sigma = \int dx \Delta q(x) = \frac{1}{2P^{+}} \langle P\ S_{\parallel} | \overline{\psi}_f(0)\gamma^{+}\gamma_5 \psi_f(0) | P\ S_{\parallel}\rangle = \frac{\sqrt{2}}{P^{+}} \langle P\ S_{\parallel} | \psi^{\dagger}(0)\hat{s}^{3}\psi(0) | P\ S_{\parallel}\rangle$$

$$\Delta G = \int dx \Delta G(x) = \frac{1}{P^{+2}} \int dx \frac{1}{x} \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\ S_{\parallel} | F^{\alpha\beta}(0)^{\dagger} \hat{s}^{3} F^{\alpha\beta}(\lambda n) | P\ S_{\parallel}\rangle$$

$$J_{T} = \frac{1}{2} = L_{T} + \frac{1}{2}\Delta\Sigma_{T} + \Delta G_{T}$$

$$\Delta\Sigma_{T} = \int dx g_{T}(x) = \frac{1}{2M} \langle P\ S_{\perp} | \overline{\psi}_f(0)\gamma^{\perp}\gamma_5 \psi_f(0) | P\ S_{\perp}\rangle = \frac{1}{M} \langle P\ S_{\perp} | \psi^{\dagger}(0)\hat{s}^{\perp}\psi(0) | P\ S_{\perp}\rangle$$

$$\Delta G_{T} = \int dx G_{T}(x) = \frac{1}{\sqrt{2}P^{+}M} \int dx \frac{1}{x} \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\ S_{\perp} | F^{\alpha\beta}(0)^{\dagger} \hat{s}^{\perp} F^{\alpha\beta}(\lambda n) | P\ S_{\perp}\rangle$$

$$\left[M^{\mu\nu}, F^{\alpha\beta}(x)\right] = -\left(i\left(x^{\mu}\partial^{\nu} - x^{\nu}\partial^{\mu}\right) + \Sigma^{\mu\nu}\right)F^{\alpha\beta}(x)$$

$$\Sigma^{\mu\nu}F^{\alpha\beta} = i\left(g^{\mu\alpha}F^{\nu\beta} - g^{\nu\alpha}F^{\mu\beta} - (\alpha \leftrightarrow \beta)\right)$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x}\langle P\,S\,|\,F^{\alpha\beta}(0)^{\dagger}\mathbf{1}F^{\alpha\beta}(\lambda n)\,|\,P\,S\rangle = -\int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle P\,S\,|\,F^{+\beta}(0)F^{+}{}_{\beta}(\lambda n)\,|\,P\,S\rangle \sim xG(x)$$

$$\hat{s}^{i} = \frac{1}{2}\varepsilon^{ijk}\Sigma^{jk} \qquad \left[\hat{s}^{1}, \hat{s}^{2}\right] = i\hat{s}^{3}$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x}\langle P\,S\,|\,F^{\alpha\beta}(0)^{\dagger}\hat{s}^{3}F^{\alpha\beta}(\lambda n)\,|\,P\,S\rangle = \int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle P\,S\,|\,iF^{+\beta}(0)\widetilde{F}^{+}{}_{\beta}(\lambda n)\,|\,P\,S\rangle \sim x\Delta G(x)$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x}\langle P\,S\,|\,F^{\alpha\beta}(0)^{\dagger}\hat{s}^{\perp}F^{\alpha\beta}(\lambda n)\,|\,P\,S\rangle = \frac{-i}{\sqrt{2}}\int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle P\,S\,|\,\widetilde{F}^{+\perp}(0)F^{+-}(\lambda n) + F^{+\perp}(0)F^{12}(\lambda n) + \cdots\,|\,P\,S\rangle$$

$$\sim x\left(G_{3E}(x) + G_{3H}(x)\right) \equiv xG_{T}(x)$$

$$F^{+-} = \partial^{+}A^{-} = \frac{1}{\partial_{-}}\left(D_{\perp j}F^{j+} + g\bar{\psi}t^{a}\gamma^{+}\psi t^{a}\right)$$

$$F^{12} = \partial^{1}A^{2} - \partial^{2}A^{1} - ig[A^{1}, A^{2}]$$

$$D_{\nu}F^{\mu\nu} = g\bar{\psi}t^{a}\gamma^{\mu}\psi t^{a} \qquad D_{\perp}^{\sigma} = \partial_{\perp}^{\sigma} - igA_{\perp}^{\sigma}$$

$$J_{\parallel} = \frac{1}{2} = L + \frac{1}{2}\Delta\Sigma + \Delta G$$

$$\Delta\Sigma = \int dx \Delta q(x) = \frac{1}{2P^{+}} \langle P\ S_{\parallel} | \overline{\psi}_f(0)\gamma^{+}\gamma_5\psi_f(0) | P\ S_{\parallel}\rangle = \frac{\sqrt{2}}{P^{+}} \langle P\ S_{\parallel} | \psi^{\dagger}(0)\hat{s}^{3}\psi(0) | P\ S_{\parallel}\rangle$$

$$\Delta G = \int dx \Delta G(x) = \frac{1}{P^{+2}} \int dx \frac{1}{x} \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\ S_{\parallel} | F^{\alpha\beta}(0)^{\dagger}\hat{s}^{3}F^{\alpha\beta}(\lambda n) | P\ S_{\parallel}\rangle$$

$$J_T = \frac{1}{2} = L_T + \frac{1}{2}\Delta\Sigma_T + \Delta G_T$$

$$\Delta\Sigma_T = \int dx g_T(x) = \frac{1}{2M} \langle P\ S_{\perp} | \overline{\psi}_f(0)\gamma^{\perp}\gamma_5\psi_f(0) | P\ S_{\perp}\rangle = \frac{1}{M} \langle P\ S_{\perp} | \psi^{\dagger}(0)\hat{s}^{\perp}\psi(0) | P\ S_{\perp}\rangle$$

$$\Delta G_T = \int dx G_T(x) = \frac{1}{\sqrt{2}P^{+}M} \int dx \frac{1}{x} \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\ S_{\perp} | F^{\alpha\beta}(0)^{\dagger}\hat{s}^{\perp}F^{\alpha\beta}(\lambda n) | P\ S_{\perp}\rangle$$

$$J_{\parallel} = \frac{1}{2} = L + \frac{1}{2}\Delta\Sigma + \Delta G$$

$$\Delta\Sigma = \int dx \Delta q(x) = \frac{1}{2P^{+}} \langle P\, S_{\parallel} \,|\, \bar{\psi}_f(0)\gamma^{+}\gamma_5 \psi_f(0) \,|\, P\, S_{\parallel} \rangle = \frac{\sqrt{2}}{P^{+}} \langle P\, S_{\parallel} \,|\, \psi^{\dagger}(0)\hat{s}^{3}\psi(0) \,|\, P\, S_{\parallel} \rangle$$

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$$\Delta G_{T} = \int dx G_{T}(x) = \frac{1}{\sqrt{2}P^{+}M} \int dx \frac{1}{x} \int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\, S_{\perp} \,|\, F^{\alpha\beta}(0)^{\dagger}\hat{s}^{\perp} F^{\alpha\beta}(\lambda n) \,|\, P\, S_{\perp} \rangle$$

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$$\Delta\Sigma_T = \int dx g_T(x) = \frac{1}{2M}\langle P\ S_{\perp} | \bar{\psi}_f(0)\gamma^{\perp}\gamma_5\psi_f(0) | P\ S_{\perp}\rangle = \frac{1}{M}\langle P\ S_{\perp} | \psi^{\dagger}(0)\hat{s}^{\perp}\psi(0) | P\ S_{\perp}\rangle \quad = \Delta\Sigma$$

$$\Delta G_T = \int dx G_T(x) = \frac{1}{\sqrt{2}P^{+}M}\int dx \frac{1}{x}\int \frac{d\lambda}{4\pi} e^{i\lambda x}\langle P\ S_{\perp} | F^{\alpha\beta}(0)^{\dagger}\hat{s}^{\perp}F^{\alpha\beta}(\lambda n) | P\ S_{\perp}\rangle$$

$$\Delta G_T = \Delta G \quad ?$$

$\Delta G_T = \Delta G$?

$$G_T(x) = G_{3E}(x) + G_{3H}(x)$$

$$\left[M^{\mu\nu}, F^{\alpha\beta}(x)\right] = -\left(i\left(x^\mu \partial^\nu - x^\nu \partial^\mu\right) + \Sigma^{\mu\nu}\right) F^{\alpha\beta}(x)$$

$$\epsilon^\mu_{R/L} = (0, \mp 1, -i, 0)/\sqrt{2} \qquad \Sigma^{\mu\nu} F^{\alpha\beta} = i\left(g^{\mu\alpha} F^{\nu\beta} - g^{\nu\alpha} F^{\mu\beta} - (\alpha \leftrightarrow \beta)\right)$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\ S \,|\, F^{\alpha\beta}(0)^\dagger \mathbf{1} F^{\alpha\beta}(\lambda n) \,|\, P\ S\rangle = -\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P\ S \,|\, F^{+\beta}(0) F^{+}{}_{\beta}(\lambda n) \,|\, P\ S\rangle \sim xG(x)$$

$$F^{+R}(0)^\dagger F^{+R}(\lambda n) + F^{+L}(0)^\dagger F^{+L}(\lambda n)$$

$$\hat{s}^i = \frac{1}{2} \varepsilon^{ijk} \Sigma^{jk} \qquad \left[\hat{s}^1, \hat{s}^2\right] = i\hat{s}^3$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\ S \,|\, F^{\alpha\beta}(0)^\dagger \hat{s}^3 F^{\alpha\beta}(\lambda n) \,|\, P\ S\rangle = \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P\ S \,|\, iF^{+\beta}(0) \widetilde{F}^{+}{}_{\beta}(\lambda n) \,|\, P\ S\rangle \sim x\Delta G(x)$$

$$F^{+R}(0)^\dagger F^{+R}(\lambda n) - F^{+L}(0)^\dagger F^{+L}(\lambda n)$$

$$\int \frac{d\lambda}{4\pi} e^{i\lambda x} \langle P\ S \,|\, F^{\alpha\beta}(0)^\dagger \hat{s}^\perp F^{\alpha\beta}(\lambda n) \,|\, P\ S\rangle = \frac{-i}{\sqrt{2}} \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P\ S \,|\, \widetilde{F}^{+\perp}(0) F^{+-}(\lambda n) + F^{+\perp}(0) F^{12}(\lambda n) + \cdots \,|\, P\ S\rangle$$

$$\sim x\left(G_{3E}(x) + G_{3H}(x)\right) \equiv xG_T(x)$$

$$\widetilde{F}^{+\perp}(0) F^{+-}(\lambda n) = \frac{F^{+R}(0)^\dagger + F^{+L}(0)^\dagger}{\sqrt{2}i} F^{30}(\lambda n) = \frac{F^{+R}(0)^\dagger + F^{+L}(0)^\dagger}{\sqrt{2}i} E^3(\lambda n)$$

$$F^{+\perp}(0) F^{12}(\lambda n) = \frac{F^{+R}(0)^\dagger - F^{+L}(0)^\dagger}{\sqrt{2}} H^3(\lambda n)$$

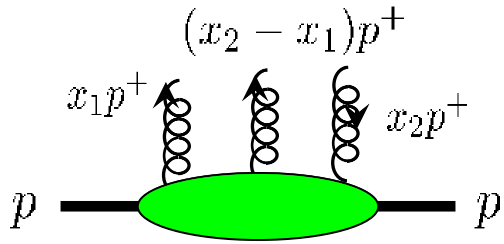
$$z^\pm = \frac{z^0 \pm z^3}{\sqrt{2}}$$

$\Delta G_T = \Delta G$ **?**

$$G_T(x) = G_{3E}(x) + G_{3H}(x)$$

$$G_{3E}(x) = \int_x^{\epsilon(x)} dx' \frac{\Delta G(x')}{x'} + [\text{genuine tw.3}]$$

Hatta, KT, Yoshida (2013)



$$\langle P\, S_\perp | F^{+\perp} F^{+\perp} F^{+\perp} | P\, S_\perp \rangle$$

$$\langle P\, S_\perp | \overline{\psi} F^{+\perp} \psi | P\, S_\perp \rangle$$

$$G_{3H}(x) = [\text{genuine tw.3}]$$

Kodaira, Nasuno, Tochimura, KT, Yasui (1998)

Braun, Korchemsky, Manashov (2001)

$$\int dx G_{3E}(x) = \Delta G$$

$$\int dx G_{3H}(x) = 0$$

$$\Delta G_T = \int dx G_T(x) = \Delta G \quad \textbf{!}$$

$$J_{\parallel} = \frac{1}{2} = L + \frac{1}{2}\Delta\Sigma + \Delta G$$

$$\Delta\Sigma = \int dx \Delta q(x) = \frac{1}{2P^+}\langle P\ S_{\parallel} | \bar{\psi}_f(0)\gamma^+\gamma_5\psi_f(0) | P\ S_{\parallel}\rangle = \frac{\sqrt{2}}{P^+}\langle P\ S_{\parallel} | \psi^\dagger(0)\hat{s}^3\psi(0) | P\ S_{\parallel}\rangle$$

$$\Delta G = \int dx \Delta G(x) = \frac{1}{P^{+2}}\int dx \frac{1}{x}\int \frac{d\lambda}{4\pi} e^{i\lambda x}\langle P\ S_{\parallel} | F^{\alpha\beta}(0)^\dagger \hat{s}^3 F^{\alpha\beta}(\lambda n) | P\ S_{\parallel}\rangle$$

$$J_T = \frac{1}{2} = L_T + \frac{1}{2}\Delta\Sigma_T + \Delta G_T$$

$$\Delta\Sigma_T = \int dx g_T(x) = \frac{1}{2M}\langle P\ S_{\perp} | \bar{\psi}_f(0)\gamma^{\perp}\gamma_5\psi_f(0) | P\ S_{\perp}\rangle = \frac{1}{M}\langle P\ S_{\perp} | \psi^\dagger(0)\hat{s}^{\perp}\psi(0) | P\ S_{\perp}\rangle \quad = \Delta\Sigma$$

$$\Delta G_T = \int dx G_T(x) = \frac{1}{\sqrt{2}P^+ M}\int dx \frac{1}{x}\int \frac{d\lambda}{4\pi} e^{i\lambda x}\langle P\ S_{\perp} | F^{\alpha\beta}(0)^\dagger \hat{s}^{\perp} F^{\alpha\beta}(\lambda n) | P\ S_{\perp}\rangle$$

$$\Delta G_T = \Delta G \quad !$$

$$\int dx G_{3E}(x) = \Delta G$$

$$\int dx G_{3H}(x) = 0$$

$$J_{\parallel} = \frac{1}{2} = L + \frac{1}{2}\Delta\Sigma + \Delta G$$

$$L = L_q + L_g$$

Chen, et al.; Wakamatsu; Hatta; Lorce, Pasquini; …

$$\Delta\Sigma = \int dx \Delta q(x) = \frac{1}{2P^+}\langle P\ S_{\parallel} | \bar{\psi}_f(0)\gamma^+\gamma_5\psi_f(0) | P\ S_{\parallel}\rangle = \frac{\sqrt{2}}{P^+}\langle P\ S_{\parallel} | \psi^\dagger(0)\hat{s}^3\psi(0) | P\ S_{\parallel}\rangle$$

$$\Delta G = \int dx \Delta G(x) = \frac{1}{P^{+2}}\int dx \frac{1}{x}\int \frac{d\lambda}{4\pi} e^{i\lambda x}\langle P\ S_{\parallel} | F^{\alpha\beta}(0)^\dagger \hat{s}^3 F^{\alpha\beta}(\lambda n) | P\ S_{\parallel}\rangle$$

$$J_T = \frac{1}{2} = L_T + \frac{1}{2}\Delta\Sigma_T + \Delta G_T$$

$$\left(L_T\right)_{q,g} \Leftarrow \frac{P^3}{2(P^0 + M)}\bar{C}_{q,g}$$

Hatta, KT, Yoshida (2013)

$$\Delta\Sigma_T = \int dx g_T(x) = \frac{1}{2M}\langle P\ S_{\perp} | \bar{\psi}_f(0)\gamma^{\perp}\gamma_5\psi_f(0) | P\ S_{\perp}\rangle = \frac{1}{M}\langle P\ S_{\perp} | \psi^\dagger(0)\hat{s}^{\perp}\psi(0) | P\ S_{\perp}\rangle = \Delta\Sigma$$

$$\Delta G_T = \int dx G_T(x) = \frac{1}{\sqrt{2}P^+ M}\int dx \frac{1}{x}\int \frac{d\lambda}{4\pi} e^{i\lambda x}\langle P\ S_{\perp} | F^{\alpha\beta}(0)^\dagger \hat{s}^{\perp} F^{\alpha\beta}(\lambda n) | P\ S_{\perp}\rangle$$

$$\Delta G_T = \Delta G \quad !$$

$$\int dx G_{3E}(x) = \Delta G$$

$$\int dx G_{3H}(x) = 0$$

Summary

transverse-spin gluon distribution

spin-operator representation for

bilocal operator definitions of PDFs

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P\ S | \Phi^\dagger(0) \begin{Bmatrix} \mathbf{1} \\ \hat{s}^i \end{Bmatrix} \Phi(\lambda n) | P\ S \rangle$$

$$\Phi = \psi, F^{\mu\nu}$$

$q(x)$	$\Delta q(x)$	$g_T(x)$
$G(x)$	$\Delta G(x)$	$G_T(x)$
density	helicity	flip
		"transverse spin"

$$G_T(x) = G_{3E}(x) + G_{3H}(x)$$

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P\ S | F^{\mu\nu}(0) F^{\xi\sigma}(\lambda n) | P\ S \rangle$$

$$F^{+\perp} F^{+-} \qquad F^{+\perp} F^{12}$$

$$\int dx G_T(x) = \int dx \Delta G(x) = \Delta G$$

$$\int dx x^{n-1} G_T(x)$$

Hard processes with $G_T(x)$?