Gluon saturation beyond (naive) leading logs

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Outline

- Dilute-dense scattering at high energy
  $\rightarrow$ Common introduction with the next 2 talks.
- Kinematical constraint for the BFKL equation
- Consistent Leading Logs from DIS at NLO
- Kinematical constraint for the BK equation

G.B., *to appear in* Phys. Rev. D, arXiv:1401.0313

Dilute-dense scattering at high-energy

High energy scattering:

projectile : momentum $q^\mu \simeq \delta^{\mu+} q^+$

target : momentum $P^\mu \simeq \delta^{\mu-} P^-$

$\Rightarrow$ Mandelstam s variable: $s \simeq 2P^- \, q^+$

Eikonal approximation: Take the high-energy limit $s \to +\infty$ and drop power-suppressed contributions.

Semi-classical approximation: At weak coupling g, dense target $\to$ random classical background field $\mathcal{A}_a^\mu(x) = O(1/g)$.

In the semi-classical approximation, the eikonal limit can be obtained by an infinite boost $P^- \to +\infty$ of the target field $\mathcal{A}_a^\mu(x)$. Hence:

- Only the $\mathcal{A}_a^-$ component is relevant
- Infinite Lorentz dilation: $\mathcal{A}_a^\mu(x)$ independent of x^-
- Infinite Lorentz contraction: $\mathcal{A}_a^\mu(x) \propto \delta(x^+)$ (shockwave)

Eikonal dilute-dense scattering

Recipe for *dilute-dense* processes at high-energy,
following Bjorken, Kogut and Soper (1971):

- Decompose the projectile on a Fock basis at the time $x^+ = 0$, with appropriate Light-Front wave-functions.
- Each parton n scatters independently on the target via a light-like Wilson line $U_{\mathcal{R}_n}(\mathbf{x}_n)$ through the target:

$$U_{\mathcal{R}_n}(\mathbf{x}_n) = \mathcal{P}_+ \exp\left[ig \int dx^+ \; T^a_{\mathcal{R}_n} \, A^-_a(x^+, \mathbf{x}_n)\right]$$

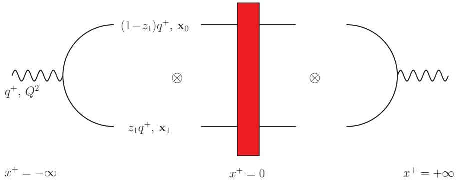
with $\mathcal{R}_n = A$, F or $\bar{F}$ for g, q or $\bar{q}$ partons.

- Include final-state evolution of the projectile remnants.

Comments:

1. Light-cone gauge $A^+_a = 0$ strongly recommended!
2. At this stage, no apparent dependence on s ...

Example: Dipole factorization for DIS at LO



$$\sigma^{\gamma}_{T,L} = \frac{4N_c\,\alpha_{em}}{(2\pi)^2} \sum_f e_f^2 \int \mathrm{d}^2\mathbf{x}_0\, \mathrm{d}^2\mathbf{x}_1 \int_0^1 \mathrm{d}z_1 \times \mathcal{I}^{q\bar{q},LO}_{T,L}(x_{01}, z_1, Q^2) \left[1 - \left\langle \mathcal{S}_{01} \right\rangle_\eta \right]$$

Bjorken, Kogut, Soper (1971); Nikolaev, Zakharov (1990)

Dipole operator: $\mathcal{S}_{01} = \frac{1}{N_c} \mathrm{Tr}\left(U_F(\mathbf{x}_0)\, U_F^\dagger(\mathbf{x}_1) \right)$

η: regulator of rapidity divergence of Wilson lines $U_F(\mathbf{x}_n)$.

Corrections beyond the eikonal approximation

At least three sources of corrections to the eikonal approximation:

1. Other components of the target background field $\mathcal{A}_a^{\mu}(x)$
2. Dynamics of the target: x^- dependence of $\mathcal{A}_a^{\mu}(x)$
3. Finite width L^+ of the target along x^+

- In the context of jet quenching and in-medium energy loss: full finite width effects are included, but not the other effects.
  $\rightarrow$ Further approximation (like harmonic potential for BDMPS-Z) required to deal with quantum diffusion of the projectile inside the target.

- For scattering in the high-energy limit, power-suppressed finite L^+ correction can be calculated systematically without further approximation.
  $\Rightarrow$ Power series in $L^+ Q_s^2/q^+ \ll 1$. (Note that $L^+/q^+ \propto 1/s$)
  Altinoluk, Armesto, G. B., Martinez and Salgado, arXiv:1404.2219.
  $\rightarrow$ See talk by Tolga Altinoluk

High-energy OPE in the eikonal limit

High-energy Operator Product Expansion for eikonal dense-dilute scattering observables:

$$\sigma = \mathcal{I}^{LO} \otimes \left\langle \hat{\mathcal{O}}_0 \right\rangle_\eta + \alpha_s \mathcal{I}^{NLO}(\eta) \otimes \left\langle \hat{\mathcal{O}}_1 \right\rangle_\eta + \alpha_s^2 \mathcal{I}^{NNLO}(\eta) \otimes \left\langle \hat{\mathcal{O}}_2 \right\rangle_\eta + O(\alpha_s^3)$$

where the operators $\hat{\mathcal{O}}_n$ are products of Wilson lines and $\mathcal{I}^{N^nLO}$ calculable coefficients.
Balitsky (1996)

Dependence on the regulator η has to cancel in the sum.
$\Rightarrow$ RG evolution for the operators: JIMWLK equation

$$\partial_\eta \left\langle \hat{\mathcal{O}}_n \right\rangle_\eta = - \left\langle H \, \hat{\mathcal{O}}_n \right\rangle_\eta$$

Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner, ... (1997-2002)

Natural factorization scale choice: $\eta \sim \log(1/s)$
(but scheme-dependent)

B-JIMWLK and BK evolutions

RG evolution for the dipole operator:

$$\partial_\eta \langle \mathbf{S}_{01} \rangle_\eta = -\langle H\, \mathbf{S}_{01} \rangle_\eta = \frac{2\alpha_s C_F}{\pi} \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi} \frac{x_{01}^2}{x_{02}^2\, x_{21}^2} \langle \mathbf{S}_{012} - \mathbf{S}_{01} \rangle_\eta$$
$$= \bar{\alpha} \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi} \frac{x_{01}^2}{x_{02}^2\, x_{21}^2} \langle \mathbf{S}_{02}\mathbf{S}_{21} - \mathbf{S}_{01} \rangle_\eta$$

with $\bar{\alpha} = N_c \alpha_s / \pi$, and the $q\bar{q}g$ tripole operator

$$\mathbf{S}_{012} \equiv \frac{1}{N_c C_F} \mathrm{Tr}\Big(U_F(\mathbf{x}_0) t^a U_F^\dagger(\mathbf{x}_1) t^b \Big)\, U_A^{ba}(\mathbf{x}_2) = \frac{N_c}{2C_F} \left[\mathbf{S}_{02}\, \mathbf{S}_{21} - \frac{1}{N_c^2} \mathbf{S}_{01} \right]$$

New operator $\langle \mathbf{S}_{012} \rangle_\eta$ or $\langle \mathbf{S}_{02}\mathbf{S}_{21} \rangle_\eta$ appears $\Rightarrow$ only the first equation in Balitsky's infinite hierarchy.

In practice: truncate the hierarchy with the approx $\langle \mathbf{S}_{02}\mathbf{S}_{21} \rangle_\eta \simeq \langle \mathbf{S}_{02} \rangle_\eta \langle \mathbf{S}_{21} \rangle_\eta$ to get the BK equation.
Balitsky (1996); Kovchegov (1999)

NLO revolution for gluon saturation: observables

So far, most calculations and studies have been performed at LO in $\bar{\alpha}$, with resummations of leading logs $(\bar{\alpha}\log(1/s))^n$ using to the BK/JIMWLK equations.

However, NLO calculations are now available for the two simplest observables:

- NLO corrections to DIS structure functions
  Balitsky, Chirilli (2011)
  G.B. (2012)
- NLO corrections to forward single inclusive particle production in pA (or pp)
  Chirilli, Xiao, Yuan (2012)

NLO revolution for gluon saturation: evolution equations

The NLO corrections to the evolution equations are also known, allowing in principle NLL resummations: $\bar{\alpha}(\bar{\alpha}\log(1/s))^n$

- BK equation at NLO:
  Balitsky, Chirilli (2008)

- NLO corrections to the JIMWLK equation and Balitsky's hierarchy
  Balitsky, Chirilli (2013); Kovner, Lublinsky, Mulian (2013)
  $\rightarrow$ See talk by Yair Mulian

Moreover: Proof that observables like DIS or like particle production obey the same NLL equation (despite crossing of Wilson lines from the complex conjugate amplitude to the amplitude)
Mueller, Munier (2012)

All-orders corrections to the high-energy evolution

Naive perturbative expansion in $\bar{\alpha}$ of the BK and JIMWLK (and BFKL) equations is badly behaved

$\Rightarrow$ All-orders resummations of several effects needed to make the expansion stable

Running coupling: Crucial impact to phenomenological studies, but rather well understood. Up-to-date prescriptions:

- For BK: Balitsky's prescription (2007)
- For JIMWLK: Lappi-Mäntysaari prescription (2013)

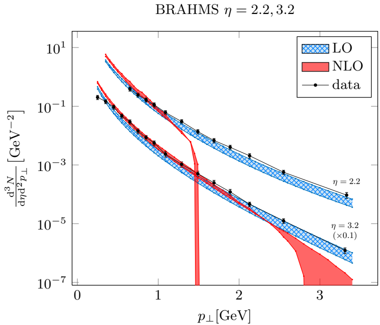
$\rightarrow$ Both select, in all possible limits, the minimal dipole size for the scale in $\bar{\alpha}$.

Kinematical contraint: Improvement of kinematics required to make BK, JIMWLK and BFKL consistent at finite energies.
$\rightarrow$ Main topic of the rest of this talk!

Moreover, it might be possible (needed?) to resum the full DGLAP evolution into the BK and JIMWLK equations in various limits.

NLO revolution for gluon saturation: phenomenology?

First practical study at NLO+LL accuracy:



Forward single inclusive hadron production in pA

Staśto, Xiao, Zaslavsky (2013)

Good at small $p_\perp$, but large negative NLO corrections at large $p_\perp$!

$\Rightarrow$ More work needed to fully understand and solve this problem... (More on this issue at the end of the talk.)

Kinematical constraint for BFKL

Approximations required in the derivation of the BFKL equation are valid only if successive gluon are strongly ordered in k^+ and in k^- simultaneously:

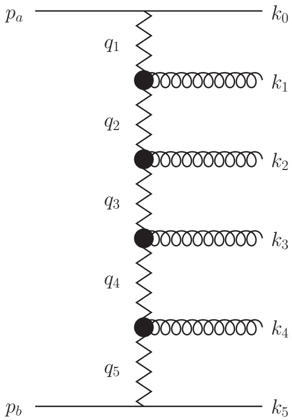
$$k_0^+ \gg k_1^+ \gg \cdots \gg k_n^+ \gg \dots$$

and

$$k_0^- \ll k_1^- \ll \cdots \ll k_n^- \ll \dots$$

In each factorization scheme, only the ordering along the chosen evolution variable is guarantied.

$\Rightarrow$ (Small) inconsistency on the standard version of the BFKL equation!



Kinematical constraint for BFKL

Example: factorization scheme with regulator $\eta \sim \log(k^+) \Rightarrow$ strong ordering in k^+.

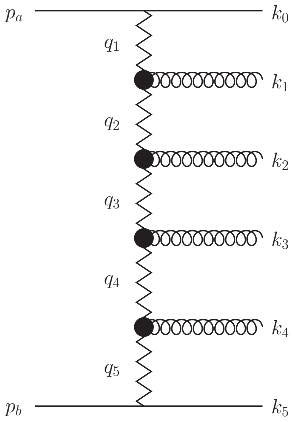
Then, ordering in k^- has to be imposed in the BFKL equation, by a restriction on the $\mathbf{k}_\perp$ integration.

$\rightarrow$ Kinematical constraint

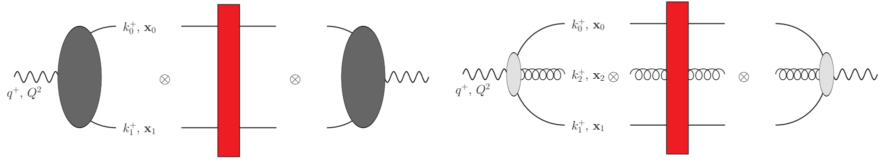
Ciafaloni (1988)
Catani, Fiorani, Marchesini (1990)
Andersson, Gustafson, Kharraziha, Samuelsson (1996)
Kwieciński, Martin, Sutton (1996)

Analog in Mellin space: Salam (1998)

First study in mixed-space:
Motyka, Staśto (2009)



DIS at high energy at NLO (fixed order)



$$\sigma^{\gamma}_{T,L}(Q^2, x_{Bj}) = 2\,\frac{2N_c\,\alpha_{em}}{(2\pi)^2}\sum_f e_f^2 \int \mathrm{d}^2\mathbf{x}_0 \int \mathrm{d}^2\mathbf{x}_1 \int_0^1 \mathrm{d}z_1$$
$$\times\Bigg\{\mathcal{I}^{q\bar{q},LO}_{T,L}(x_{01}, z_1, Q^2)\,\Big[1+\mathcal{O}(N_c\alpha_s)\Big]\,\Big[1-\langle\mathcal{S}_{01}\rangle_0\Big]$$
$$+\frac{2\alpha_s C_F}{\pi}\int_{z_{\min}}^{1-z_1}\frac{\mathrm{d}z_2}{z_2}\int\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\,\mathcal{I}^{q\bar{q}g}_{T,L}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,z_1,z_2,Q^2)\,\Big[1-\langle\mathcal{S}_{012}\rangle_0\Big]\Bigg\}$$

with $z_n = k_n^+/q^+$ and $z_{\min} = \frac{x_{Bj}}{Q^2}\,\frac{Q_0^2}{x_0}$.
G.B. (2012); see also Balitsky, Chirilli (2011).

Extraction of LL from NLO

The LL contained in the NLO term seems consistent with BK

$$\bar{\alpha} \int_{z_{\min}}^{z_f} \frac{\mathrm{d}z_2}{z_2} \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi} \mathcal{I}_{T,L}^{q\bar{q}g}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, z_1, z_2, Q^2) \left[1 - \langle \mathcal{S}_{012} \rangle_0 \right]$$

$$\sim \mathcal{I}_{T,L}^{q\bar{q},LO}(x_{01}, z_1) \quad \bar{\alpha} \log\left(\frac{z_f}{z_{\min}}\right) \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi} \frac{x_{01}^2}{x_{02}^2\, x_{21}^2} \left[1 - \langle \mathcal{S}_{012} \rangle_0 \right]$$

because $\mathcal{I}_{T,L}^{q\bar{q}g}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,z_1,z_2{=}0) = \frac{x_{01}^2}{x_{02}^2\, x_{21}^2} \quad \mathcal{I}_{T,L}^{q\bar{q},LO}(x_{01},z_1)$

However: for any small but finite $z_2 = k_2^+/q^+$

$$\mathcal{I}_{T,L}^{q\bar{q}g}(\mathbf{x}_0,\mathbf{x}_1,\mathbf{x}_2,z_1,z_2) \ll \frac{x_{01}^2}{x_{02}^2\, x_{21}^2} \quad \mathcal{I}_{T,L}^{q\bar{q},LO}(x_{01},z_1)$$

when $z_1(1-z_1)x_{01}^2 \ll z_2 x_{02}^2 \simeq z_2 x_{21}^2$.

$\Rightarrow$ Splitting of a dipole into much larger dipoles do not participate to LL's !

Physical interpretation: splitting into too large dipoles violate lifetime ordering of the fluctuations.

Corrected real gluon emission kernel

Real emission contribution to the usual LL:

$$\bar{\alpha}\,\frac{\mathrm{d}k_2^+}{k_2^+}\,\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\,\frac{x_{01}^2}{x_{02}^2 x_{21}^2}\,\left\langle \mathcal{S}_{02}\,\mathcal{S}_{21} - \frac{1}{N_c^2}\mathcal{S}_{01}\right\rangle_{Y_2^+=\log(k_2^+/k_{\min}^+)}$$

Ordering in k^+ guarantied by the choice of factorization scheme/evolution in k^+, at k_f^+.

Modification: forbid gluon emission at large distance, multiplying the real contribution by $\theta\left(k_f^+\,x_{01}^2 - k_2^+\,\min(x_{02}^2, x_{21}^2)\right)$

$\rightarrow$ Mixed-space analog of the k^- ordering (kinematical constraint).

Same general idea as in the previous study in mixed space:
Motyka, Staśto (2009)

However: several issues there, in particular the treatment of virtual corrections.

Calculating virtual corrections from unitarity

Assume the kinematical constraint to preserve the probabilistic interpretation of the parton cascade.

Evolution of $\langle S_{01} \rangle$ over a finite range $Y_f^+ = \log(k_f^+/k_{\min}^+)$:

$$\begin{aligned}\langle S_{01} \rangle_{Y_f^+} &= \langle S_{01} \rangle_0 \times D_{01}(Y_f^+) + \bar{\alpha} \int_0^{Y_f^+} \mathrm{d}Y_2^+ \, D_{01}(Y_f^+ - Y_2^+) \\ &\quad \times \int \frac{\mathrm{d}^2\mathbf{x_2}}{2\pi} \, \frac{x_{01}^2}{x_{02}^2 x_{21}^2} \, \theta\left(Y_f^+ - Y_2^+ - \log\left(\frac{\min(x_{02}^2, x_{21}^2)}{x_{01}^2} \right) \right) \\ &\quad \times \left\langle \mathcal{S}_{02}\, \mathcal{S}_{21} - \frac{1}{N_c^2} \mathcal{S}_{01} \right\rangle_{Y_2^+}\end{aligned}$$

with the probability $D_{01}(Y^+)$ of no splitting for the dipole 01 in the range Y^+.

Calculating virtual corrections from unitarity

In the vacuum (absence of target), $S_{01} = S_{02} = S_{21} = 1$.
$\to$ equation determining $D_{01}(Y^+)$.

Solution:

$$D_{01}(Y^+) = \exp\left[-\bar{\alpha}\,\frac{2C_F}{N_c}\int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\,\frac{x_{01}^2}{x_{02}^2 x_{21}^2}\,\left(Y^+ - \Delta_{012}\right)\,\theta\left(Y^+ - \Delta_{012}\right)\right]$$

where the shift Δ_{012} should behave as

$$\begin{aligned}\Delta_{012} &= 0 \qquad \text{for} \quad x_{02}^2 \lesssim x_{01}^2 \quad \text{and} \quad x_{21}^2 \lesssim x_{01}^2 \\ \Delta_{012} &\sim \log\left(\frac{x_{02}^2}{x_{01}^2}\right) \sim \log\left(\frac{x_{21}^2}{x_{01}^2}\right) \qquad \text{for} \quad x_{01}^2 \ll x_{02}^2 \sim x_{21}^2\end{aligned}$$

Possible choices:

$$\begin{aligned}\Delta_{012} &= \max\left\{0,\, \log\left(\frac{\min(x_{02}^2, x_{21}^2)}{x_{01}^2}\right)\right\} \\ \text{or} \quad \Delta_{012} &= \max\left\{0,\, \log\left(\frac{|\mathbf{x}_{02}\cdot\mathbf{x}_{21}|}{x_{01}^2}\right)\right\}\end{aligned}$$

Kinematically constrained BK equation (kcBK)

Rewriting the new evolution equation as a differential equation and discarding irrelevant terms explicitly of order NLL:

$$\partial_{Y^+} \left\langle \mathcal{S}_{01} \right\rangle_{Y^+} = \bar{\alpha} \int \frac{\mathrm{d}^2\mathbf{x}_2}{2\pi} \frac{x_{01}^2}{x_{02}^2\, x_{21}^2}\, \theta(Y^+ - \Delta_{012}) \times \left\{ \left\langle \mathcal{S}_{02}\mathcal{S}_{21} - \frac{1}{N_c^2}\mathcal{S}_{01} \right\rangle_{Y^+ - \Delta_{012}} - \left(1 - \frac{1}{N_c^2}\right) \left\langle \mathcal{S}_{01} \right\rangle_{Y^+} \right\}$$

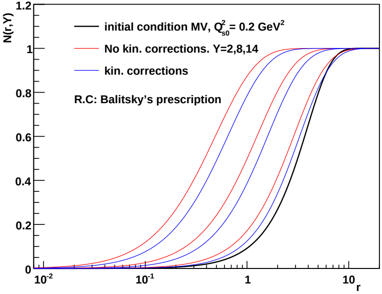
G.B., arXiv:1401.0313

Slows down the BK evolution:

- Restriction of phase space by the theta function
- Shift of the Y^+ argument of the dipole amplitude in the real term but not in the virtual term.

Large effect especially at small Y^+.

Numerics for kcBK

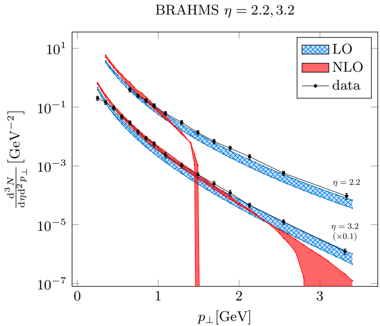


Main effects of the kinematical constraint (running coupling case):

- slows down the beginning of the Y^+ evolution, especially for softer initial Q_s
- at large Y^+: $\sim$ constant rescaling of the saturation scale

Work in progress; Albacete, Armesto, G.B., Milhano

Back on the issues at NLO+LL for pA



Staśto, Xiao, Zaslavsky (2013)

Possible origin of large negative NLO corrections: Use of the standard BK instead of kcBK to subtract LL's from NLO terms.

Alternative explanation: subtraction of LL's from NLO terms not done in a consistent factorization scheme.
Kang, Vitev, Xing (2014)

Conclusion

The kinematical constraint is required to make high-energy evolution equations consistent at finite energies.

It allows to

- resum the LLs actually present in observables at NLO and beyond $\rightarrow$ from naive LL to consistent LL accuracy!
- reproduce the correct DLL limit of DGLAP
- enforce strong x^+-time ordering of the fluctuations along the evolution

Kinematically consistent BK equation (kcBK) now available.

- Impact on phenomenological studies?
- What about JIMWLK?