KAON COLLINS MEASUREMENTS @ BELLE

XXII. International Workshop on Deep-Inelastic Scattering and Related Subjects
Warsaw, 28 April - 2 May 2014

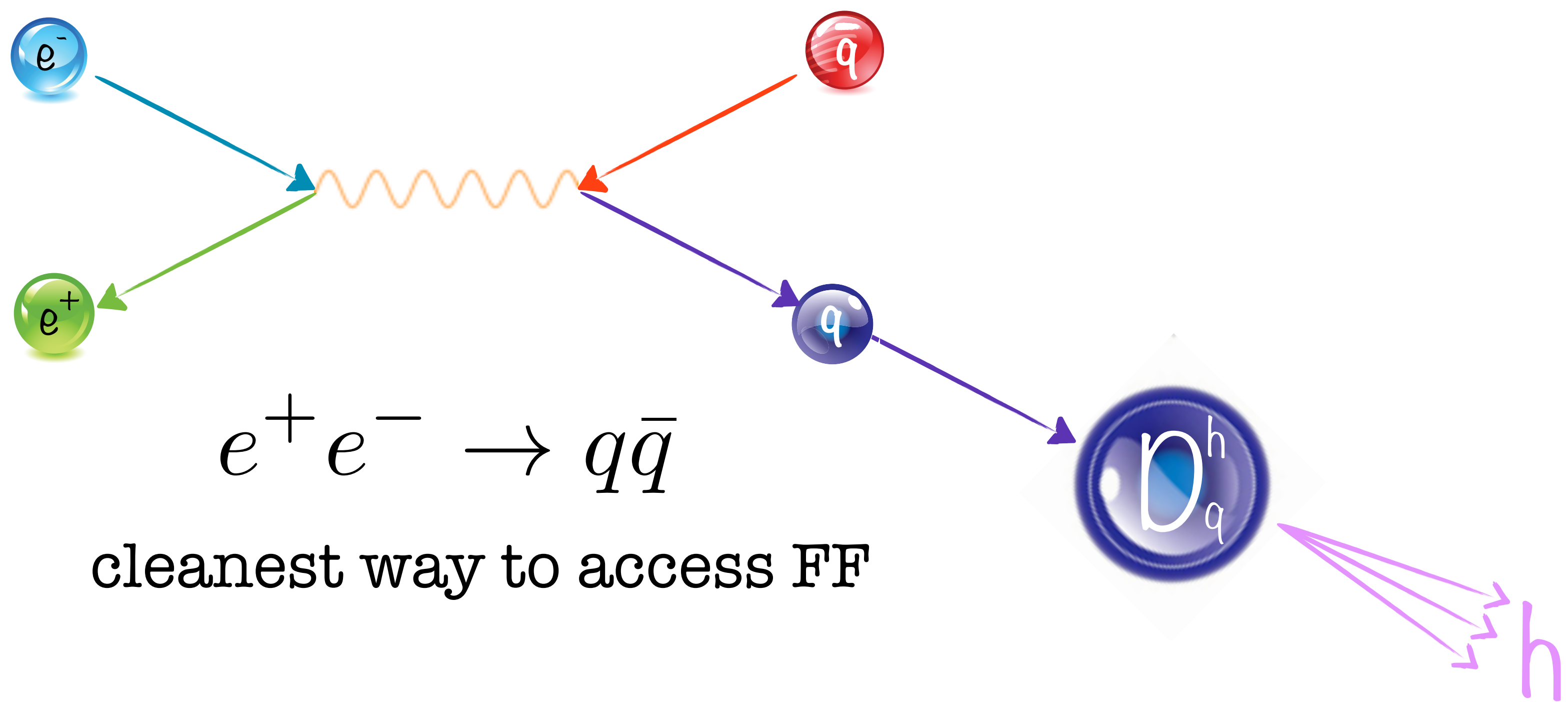
Francesca Giordano, for the BELLE collaboration

DIS2014

ILLINOIS
UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN

BELLE

Fragmentation functions



$$e^+e^- \rightarrow q\bar{q}$$

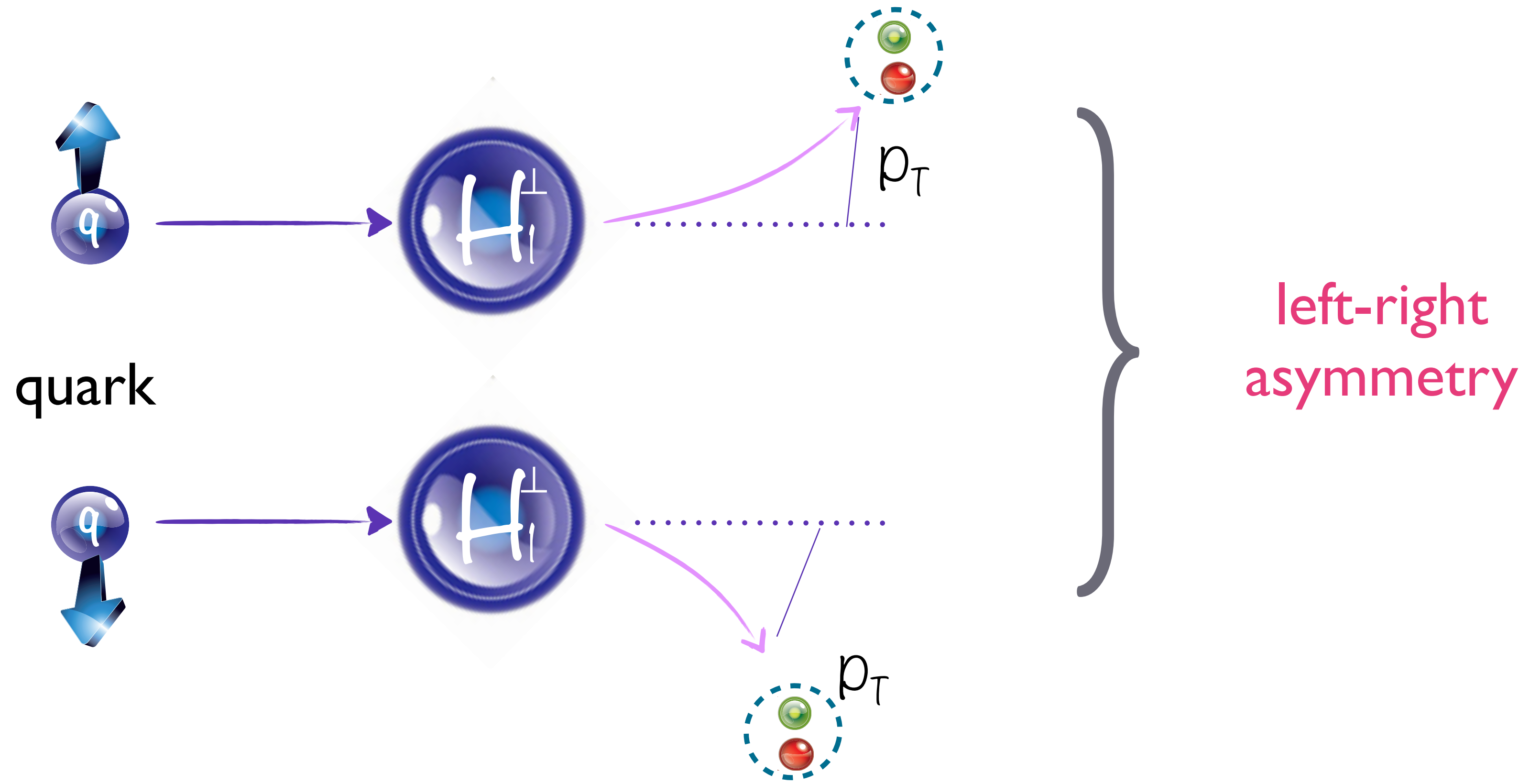
cleanest way to access FF

- Universal: can be used to study the nucleon structure when combined with SIDIS and hadronic reactions data
  (FF contribute to hadron production cross sections, azimuthal spin asymmetries...)

$$A^h_{LL} = \frac{\sigma^{\rightrightarrows} - \sigma^{\rightleftarrows}}{\sigma^{\rightleftarrows} + \sigma^{\rightleftarrows}}$$

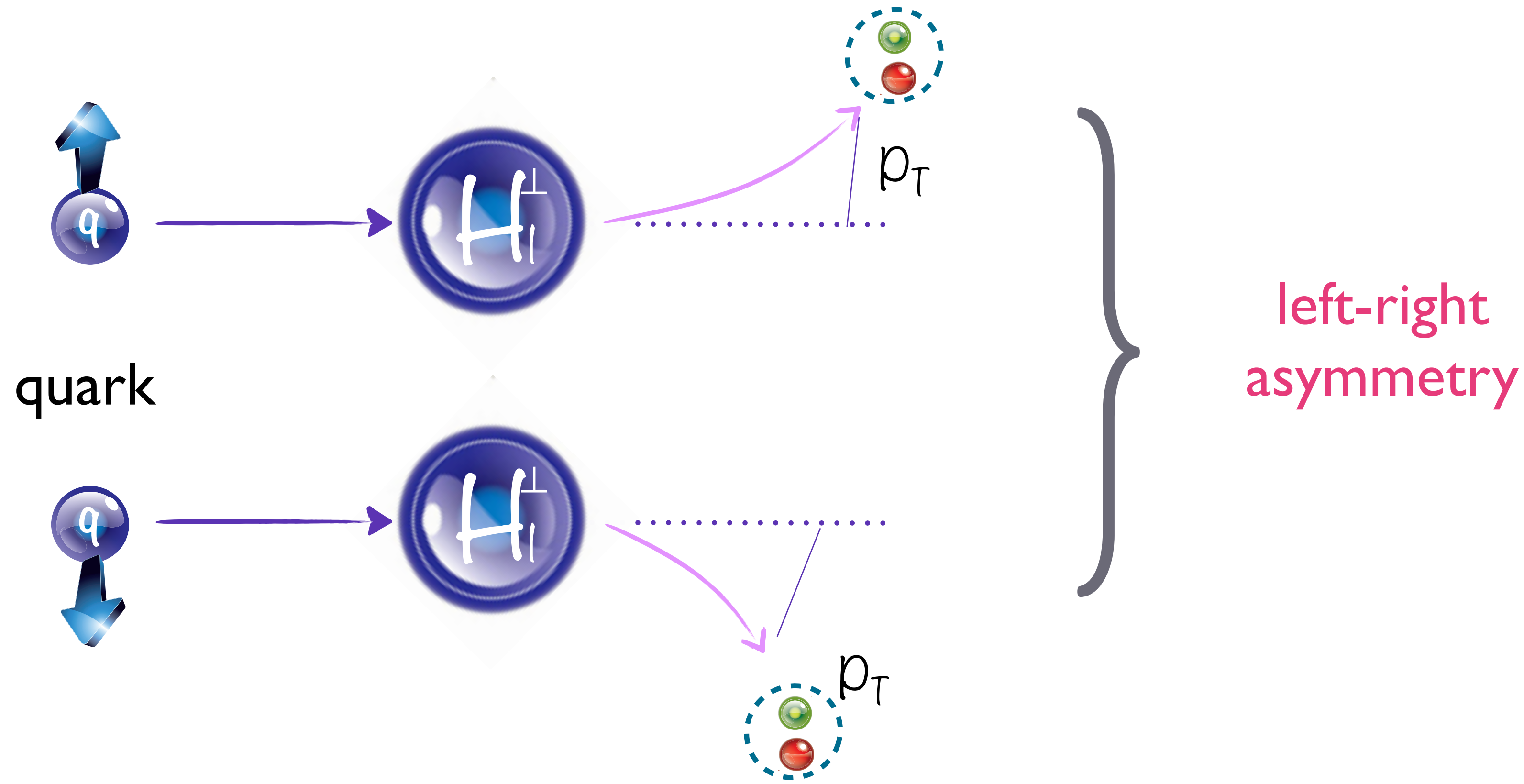
$$A^h_{UT} = \frac{\sigma^{\uparrow\upuparrows} - \sigma^{\uparrow\downdownarrows}}{\sigma^{\uparrow\upuparrows} + \sigma^{\uparrow\downdownarrows}}$$

Collins Fragmentation



Collins mechanism: correlation between the parton transverse spin and the direction of final hadron

Collins Fragmentation

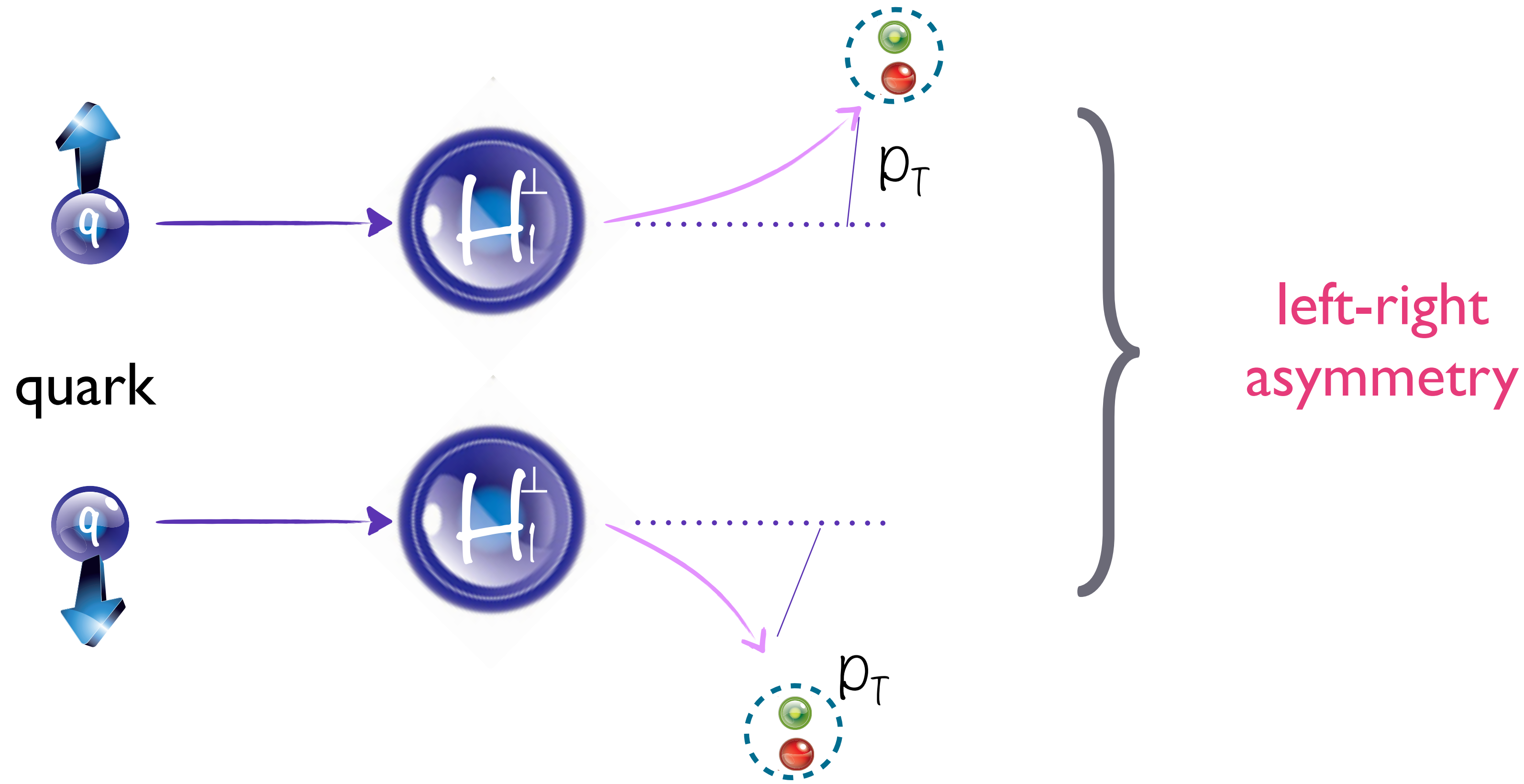


Collins mechanism: correlation between the parton transverse spin and the direction of final hadron

strictly related to the outgoing hadron tranverse momentum

TMD!

Collins Fragmentation



Collins mechanism: correlation between the parton transverse spin and the direction of final hadron

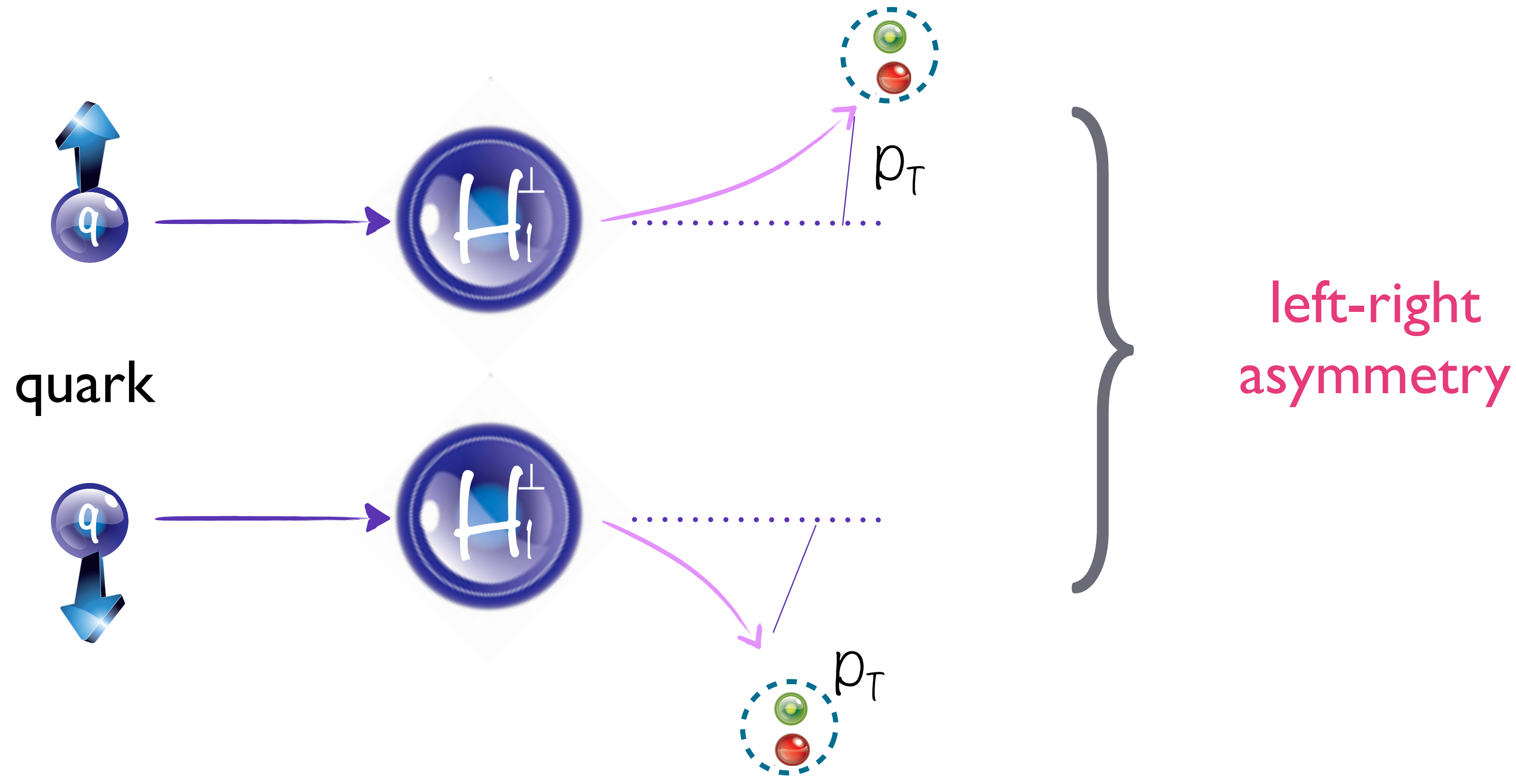
strictly related to the outgoing hadron tranverse momentum

TMD!

Chiral odd!

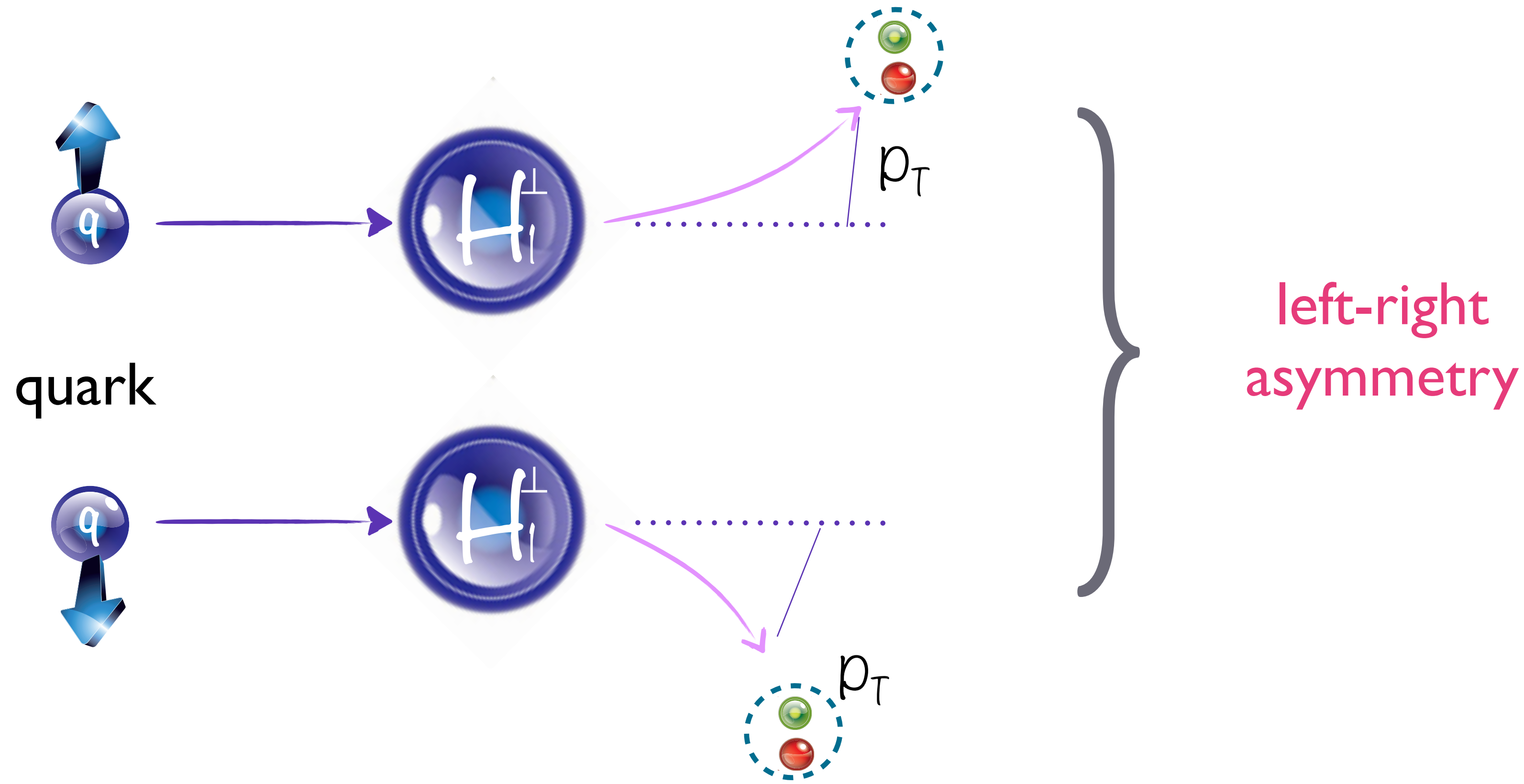
$$\underbrace{\underset{\text{chiral odd}}{X} \otimes \underset{\text{chiral odd}}{H_1^\perp}}_{\text{chiral even}}$$

Collins fragmentation function



In e+e- reaction, there is no fixed transverse axis to define azimuthal angles to, and even if there were one the net quark polarization would be 0

Collins fragmentation function



In e+e- reaction, there is no fixed transverse axis to define azimuthal angles to, and even if there were one the net quark polarization would be 0

But if we look at the whole event, even though the q and $\bar{q}$ spin directions are unknown, they must be parallel

$$e^+e^- \to q\,\bar{q} \;\to h_1\, h_2\, X$$

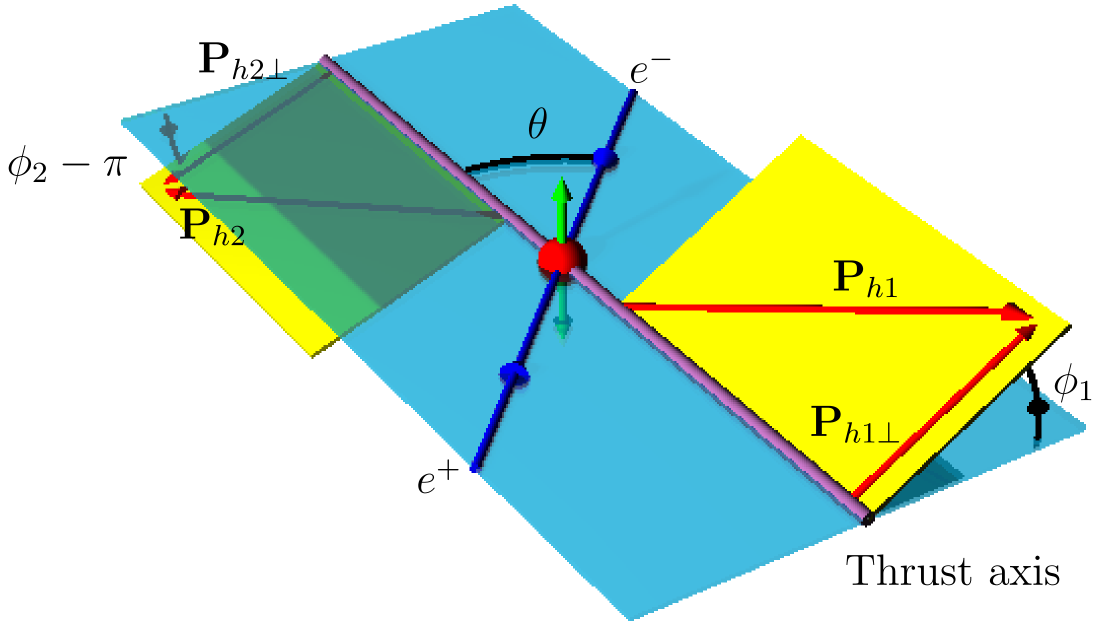
$$h = \pi,\ K$$

Reference frames

$$e^+e^- \to q\,\bar{q} \to h_1\,h_2\,X \qquad h = \pi,\ K$$

$\phi_1+\phi_2$ method:

hadron azimuthal angles with respect to the $q\bar{q}$ axis proxy

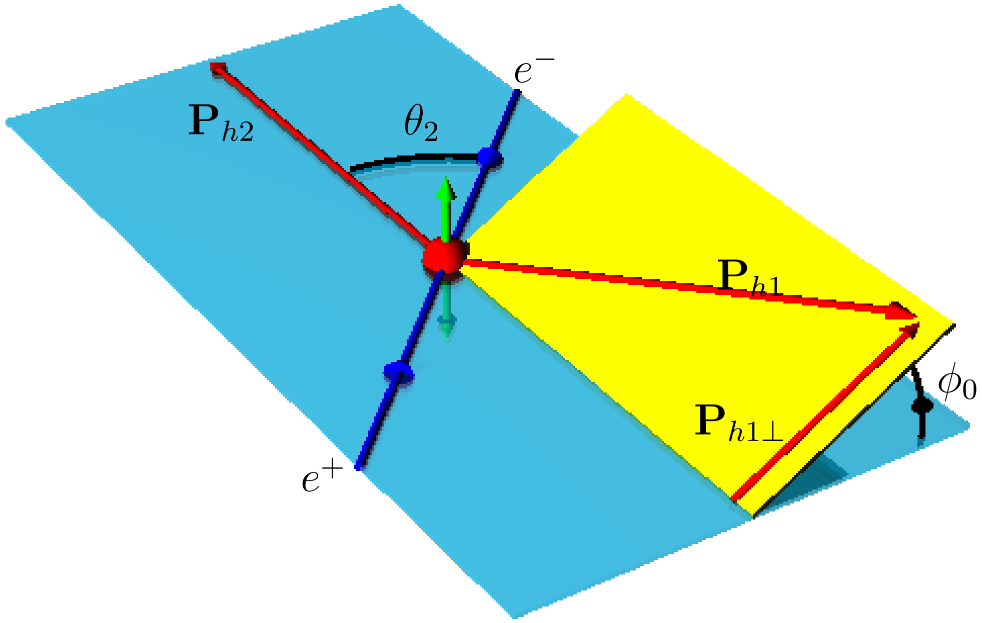


reference plane (in blue) given by the e+e- direction and the $q\bar{q}$ axis

Thrust axis= proxy for the $q\bar{q}$ axis

ϕ_0 method:

hadron 1 azimuthal angle with respect to hadron 2

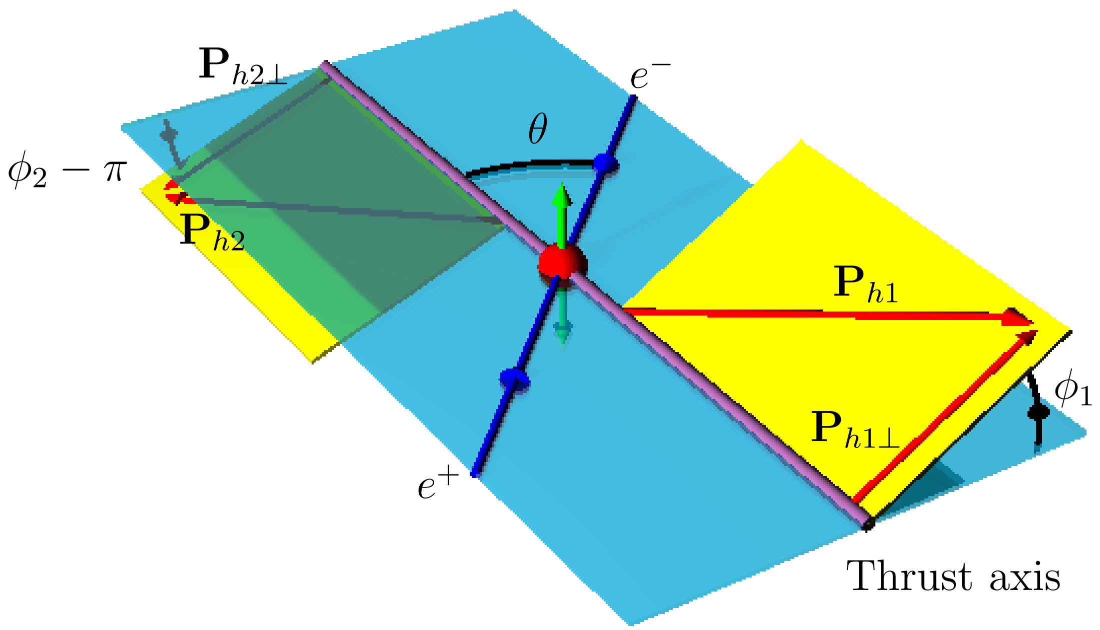


reference plane (in blue) given by the e+e- direction and one of the hadron

Reference frames

$\phi_1+\phi_2$ method:

hadron azimuthal angles with respect to the $q\bar{q}$ axis proxy

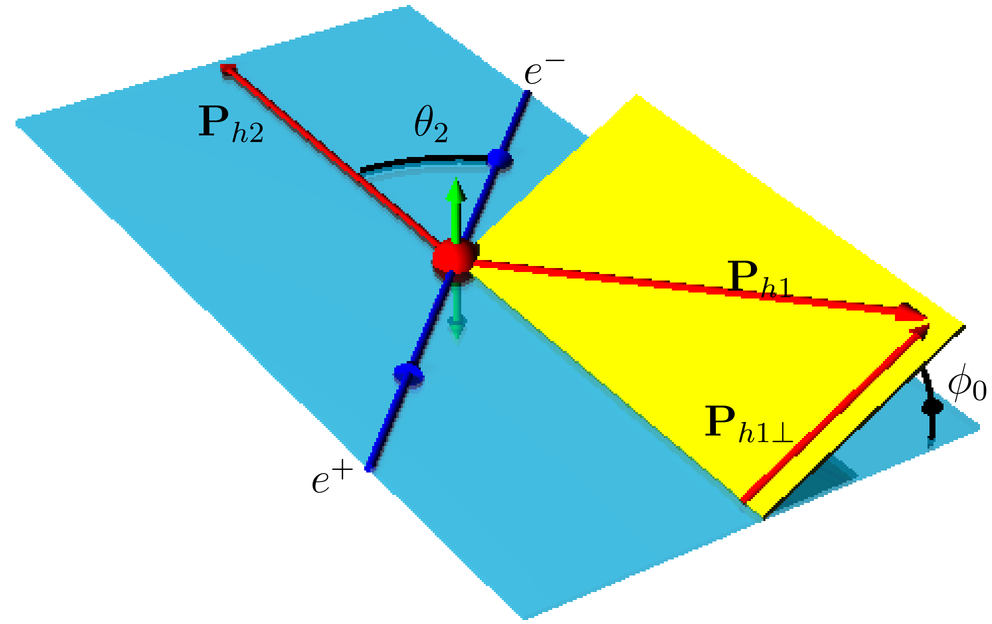


$$\sigma \sim \mathcal{M}_{12}\Big(1 + \frac{\sin^2\theta_T}{1+\cos^2\theta_T}\, cos(\phi_1+\phi_2)\frac{H_1^{\perp[1]}(z_1)\bar{H}_1^{\perp[1]}(z_2)}{D_1^{[0]}(z_1)\bar{D}_1^{[0]}(z_2)}\Big)$$

$$F^{[n]}(z_i) \equiv \int d|k_T|^2\left[\frac{|k_T|}{M_i}\right]^{[n]} F(z_i,|k_T|^2)$$

ϕ_0 method:

hadron 1 azimuthal angle with respect to hadron 2



$$\sigma \sim \mathcal{M}_0\Big(1 + \frac{\sin^2\theta_2}{1+\cos^2\theta_2}\, cos(2\phi_0)\mathcal{F}\Big[\frac{H_1^{\perp}(z_1)\bar{H}_1^{\perp}(z_2)}{D_1^{\perp}(z_1)\bar{D}_1^{\perp}(z_2)}\Big]\Big)$$

$$\mathcal{F}[X] = \sum_{q\bar{q}}\int [2\hat{\mathbf{h}}\cdot\mathbf{k_{T1}}\,\hat{\mathbf{h}}\cdot\mathbf{k_{T2}} - \mathbf{k_{T1}}\cdot\mathbf{k_{T2}}]$$
$$d^2\mathbf{k_{T1}}\,d^2\mathbf{k_{T2}}\,\delta^2(\mathbf{k_{T1}}+\mathbf{k_{T2}}-\mathbf{q_T})X$$

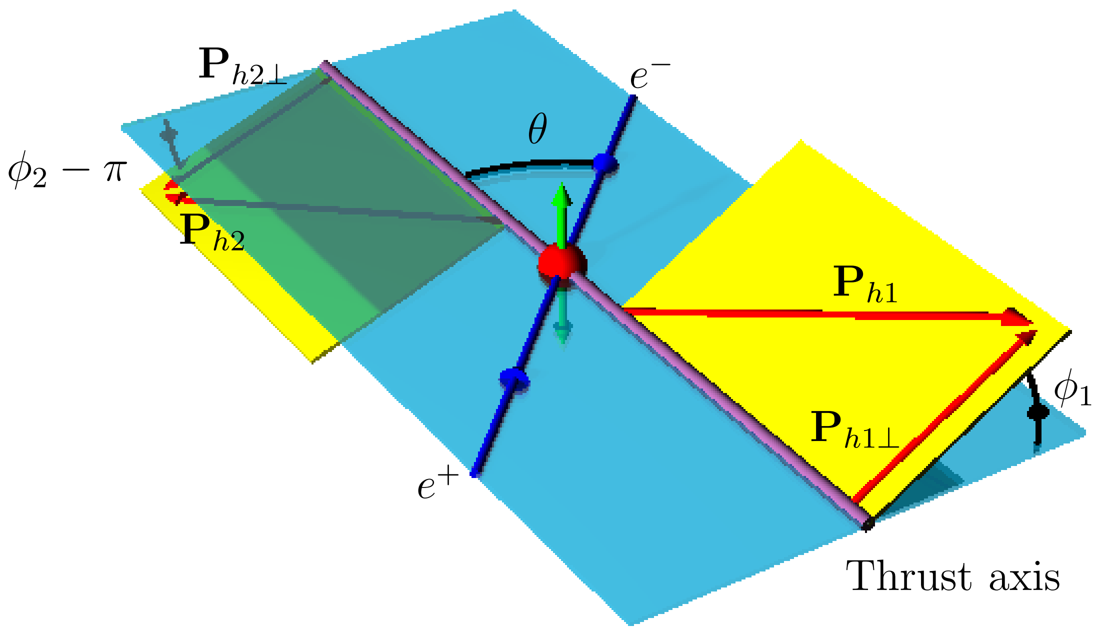
$$k_{Ti} = z_i\, p_{Ti}$$

D. Boer
Nucl.Phys.B806:23,2009

Reference frames

$\phi_1+\phi_2$ method:

hadron azimuthal angles with respect to the $q\bar{q}$ axis proxy



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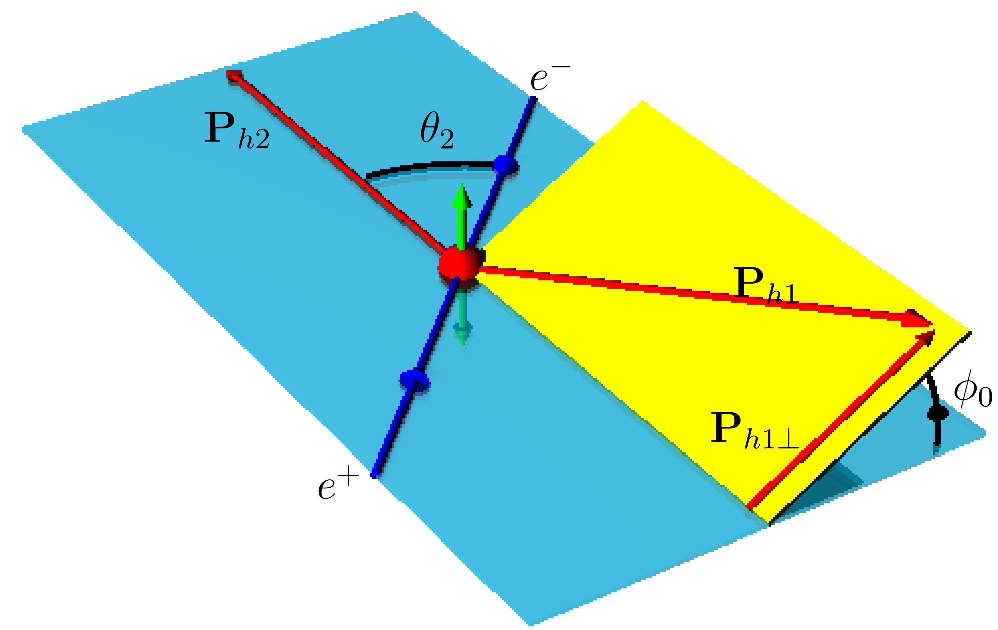

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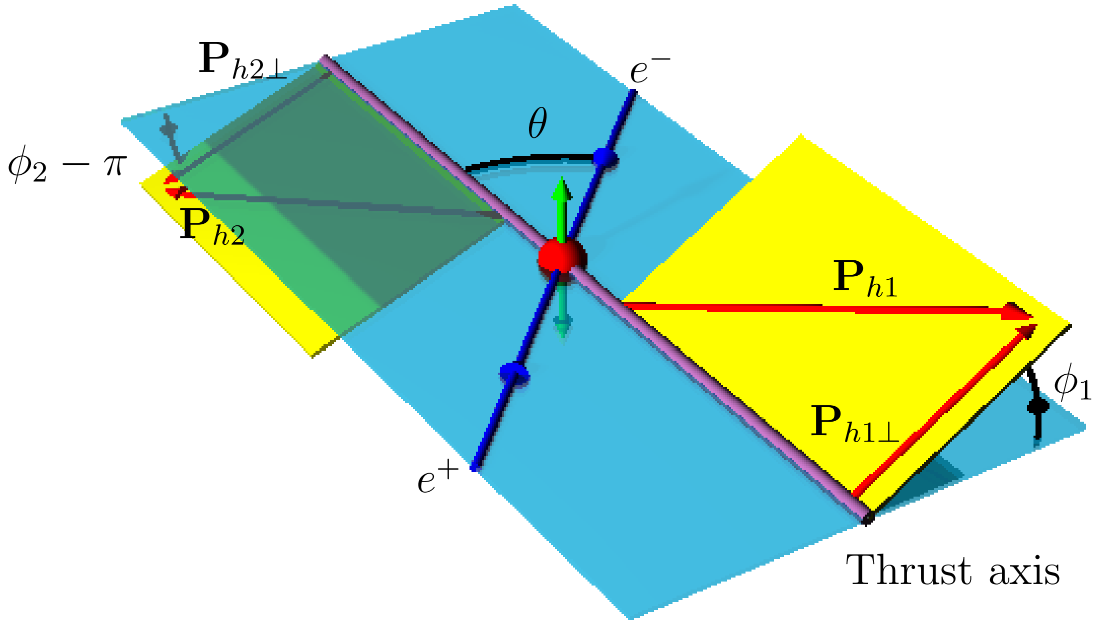

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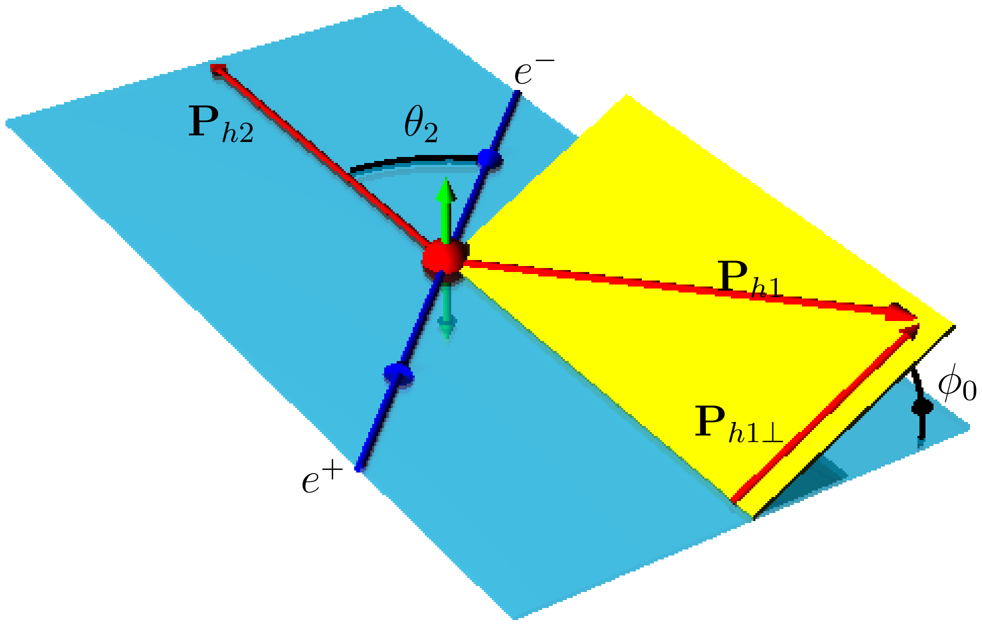

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D. Boer
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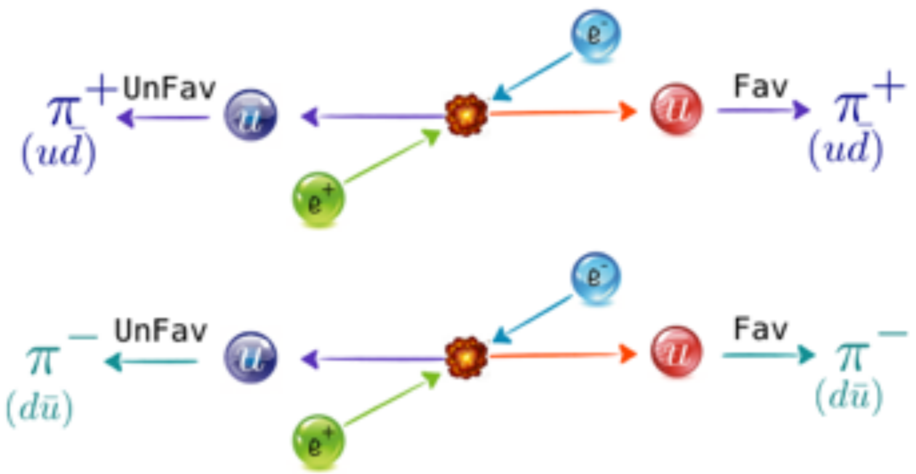
Product of 2 Collins FFs

Like-sign couples



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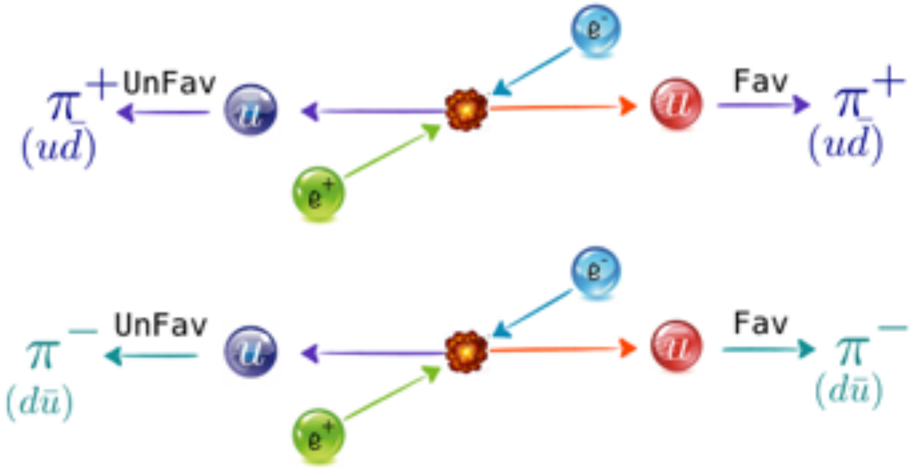
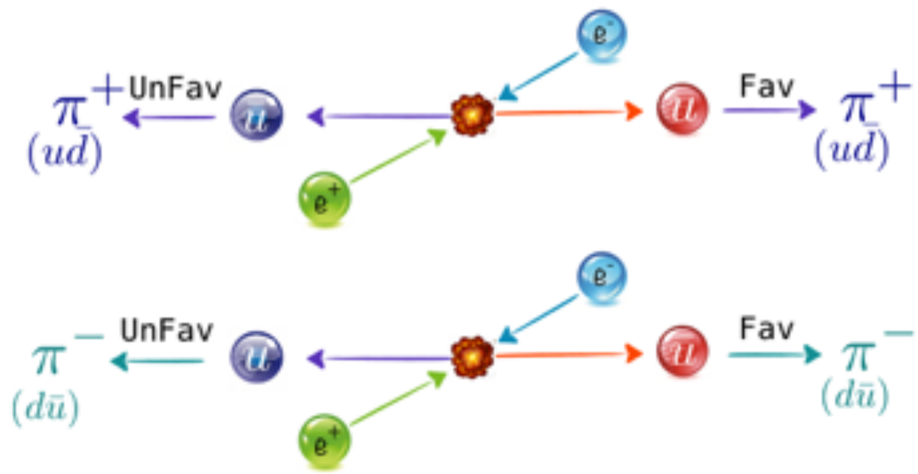


$$[\text{Fav} \cdot \text{UnFav}] + [\text{UnFav} \cdot \text{UnFav}]$$

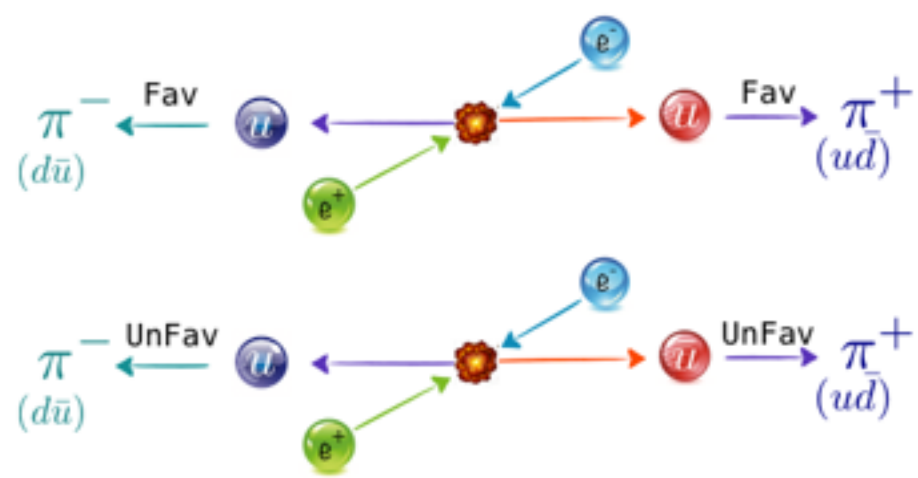
Product of 2 Collins FFs

Like-sign couples



[Fav · UnFav] + [UnFav · UnFav]

Unlike-sign couples



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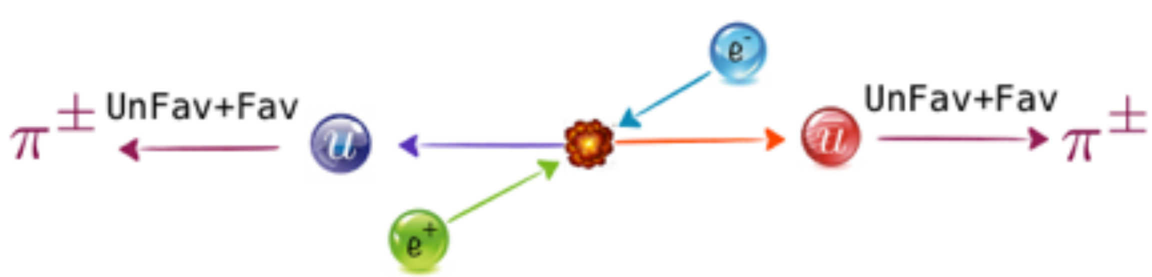
All charges couples



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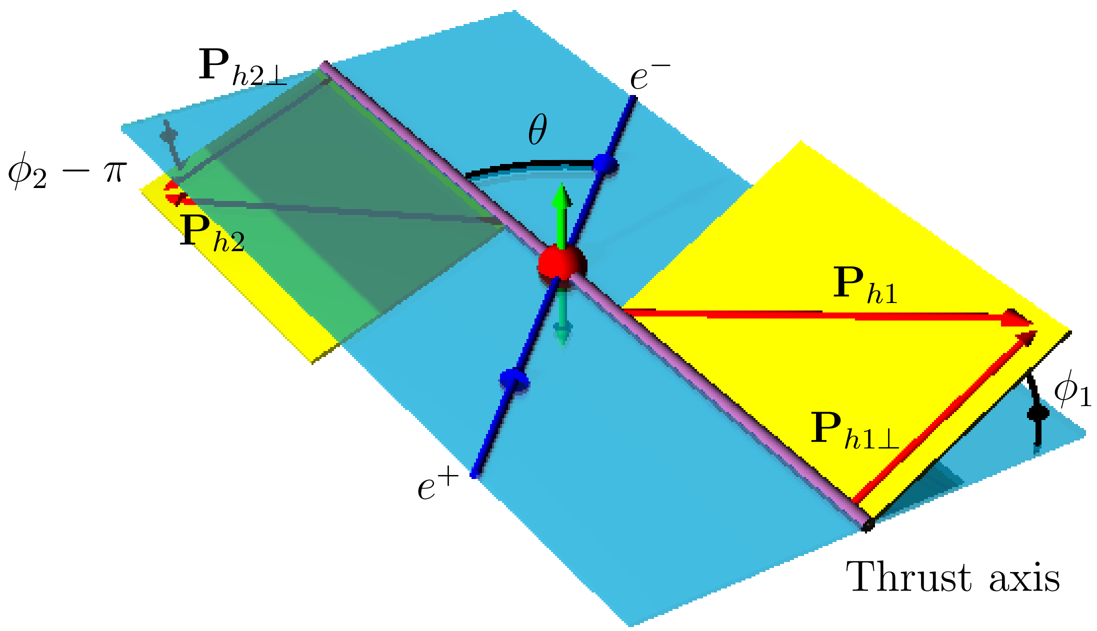
Reference frames

$$e^+e^- \to q\,\bar{q} \to h_1\,h_2\,X$$

$$h = \pi,\ K$$

ϕ_1+ϕ_2 method:

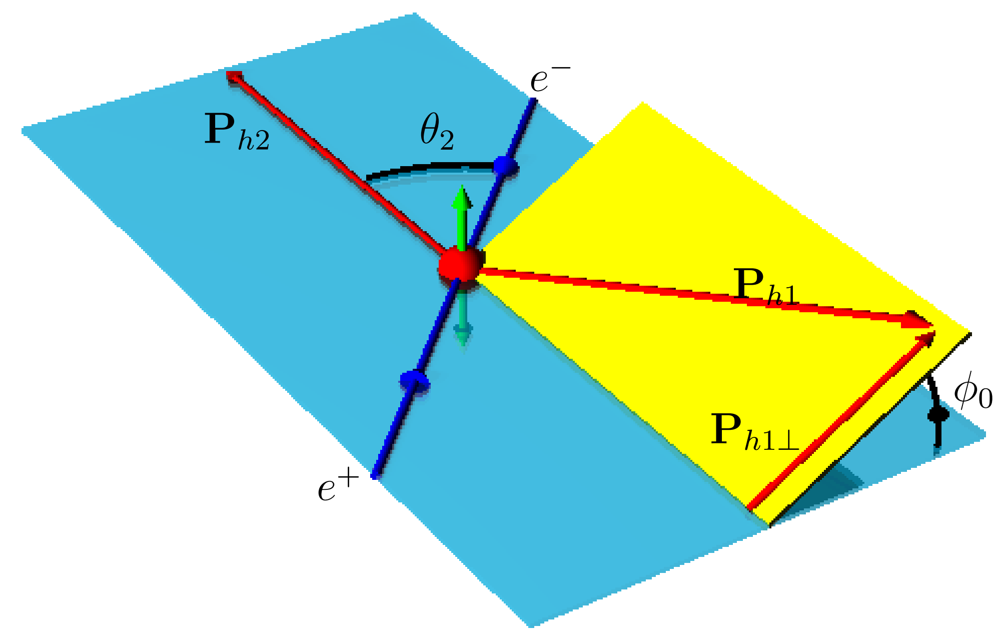
hadron azimuthal angles with respect to the $q\bar{q}$ axis proxy



$$\mathcal{R}_{12} = \frac{N_{12}(\phi_1 + \phi_2)}{\langle N_{12}\rangle}$$

ϕ_0 method:

hadron 1 azimuthal angle with respect to hadron 2



$$\mathcal{R}_0 = \frac{N_0(\phi_0)}{\langle N_0\rangle}$$

Double-ratios

But! Acceptance and radiation effects also contribute to azimuthal asymmetries!

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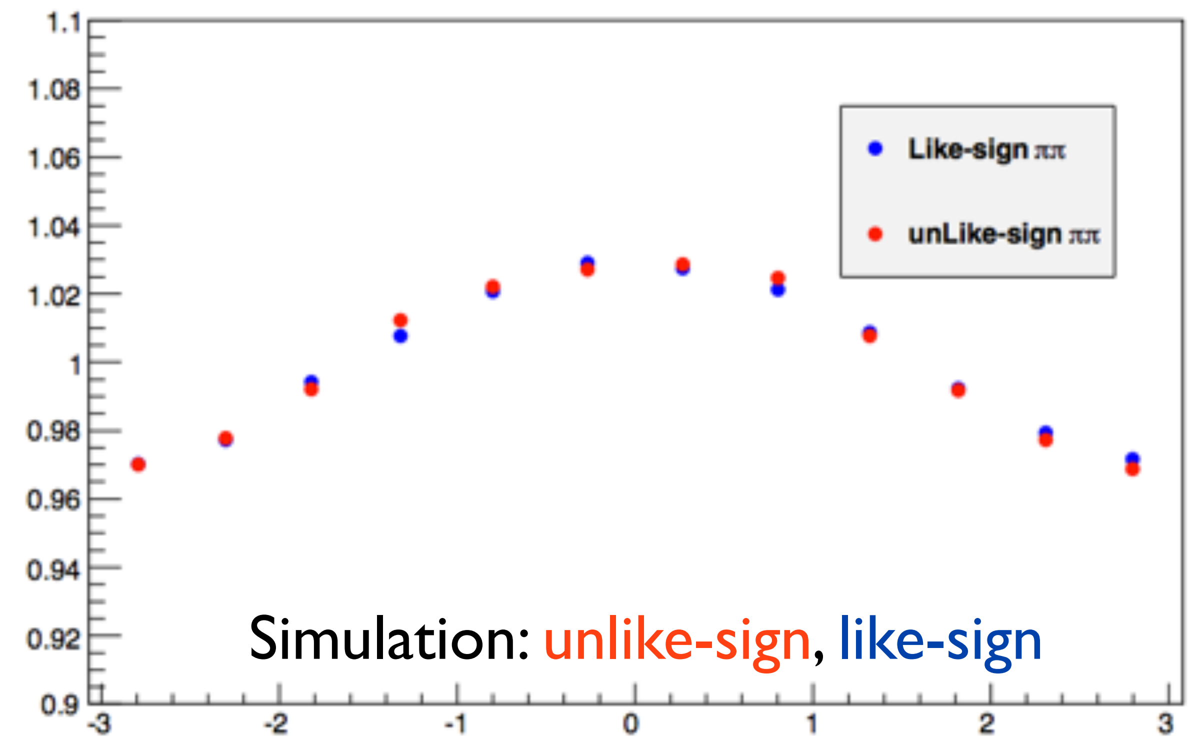
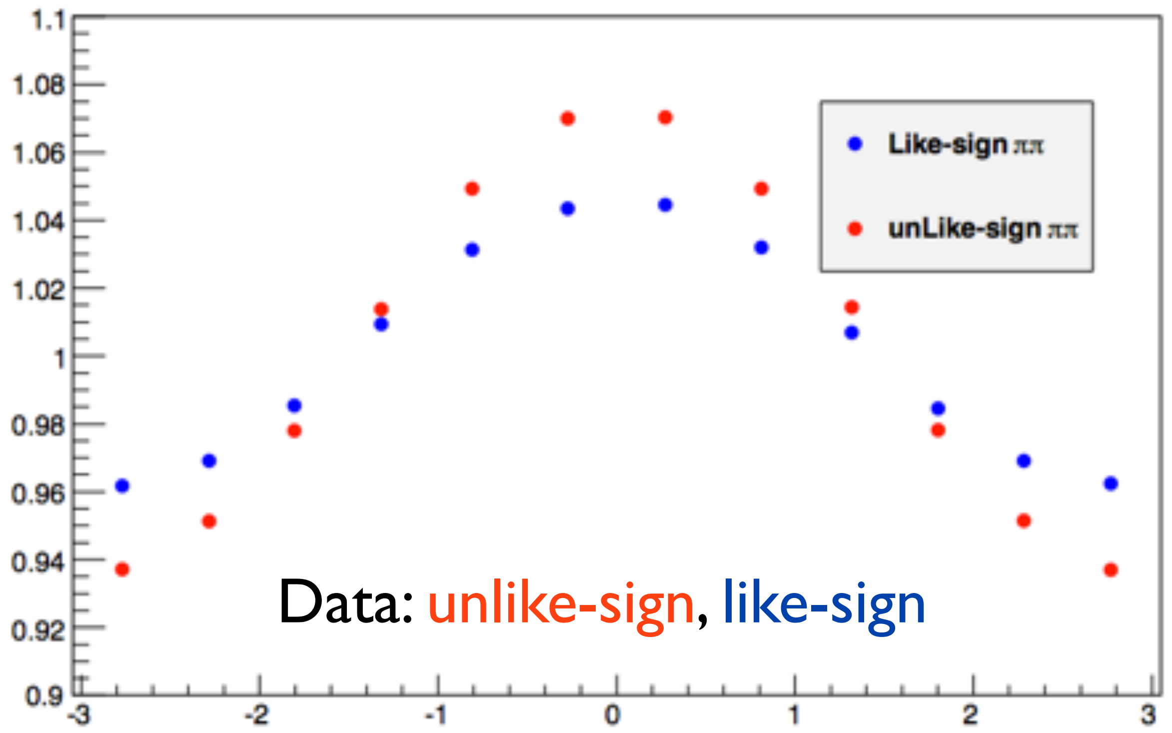
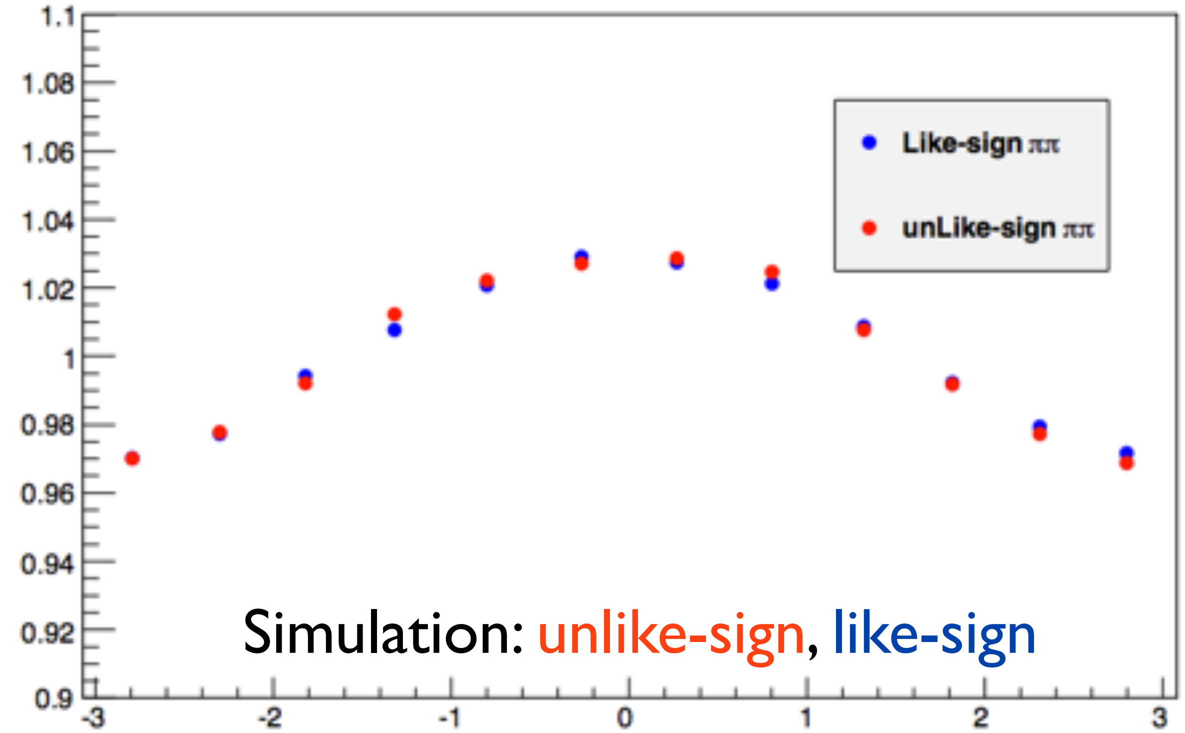
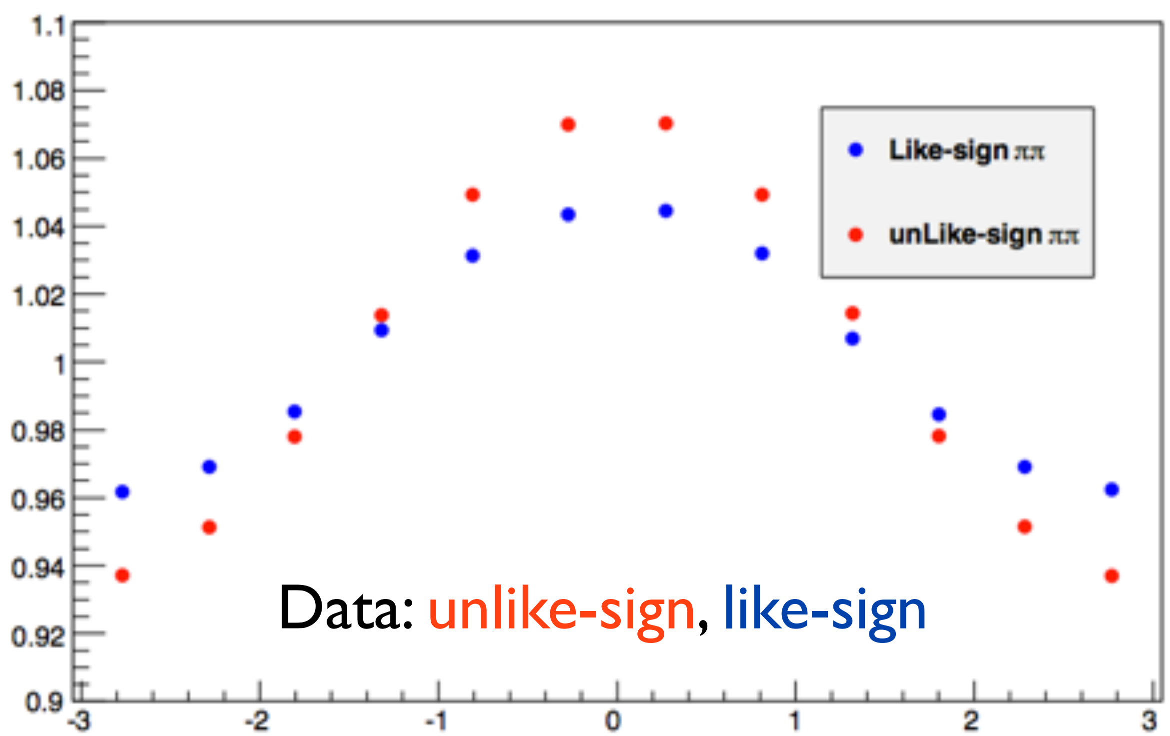


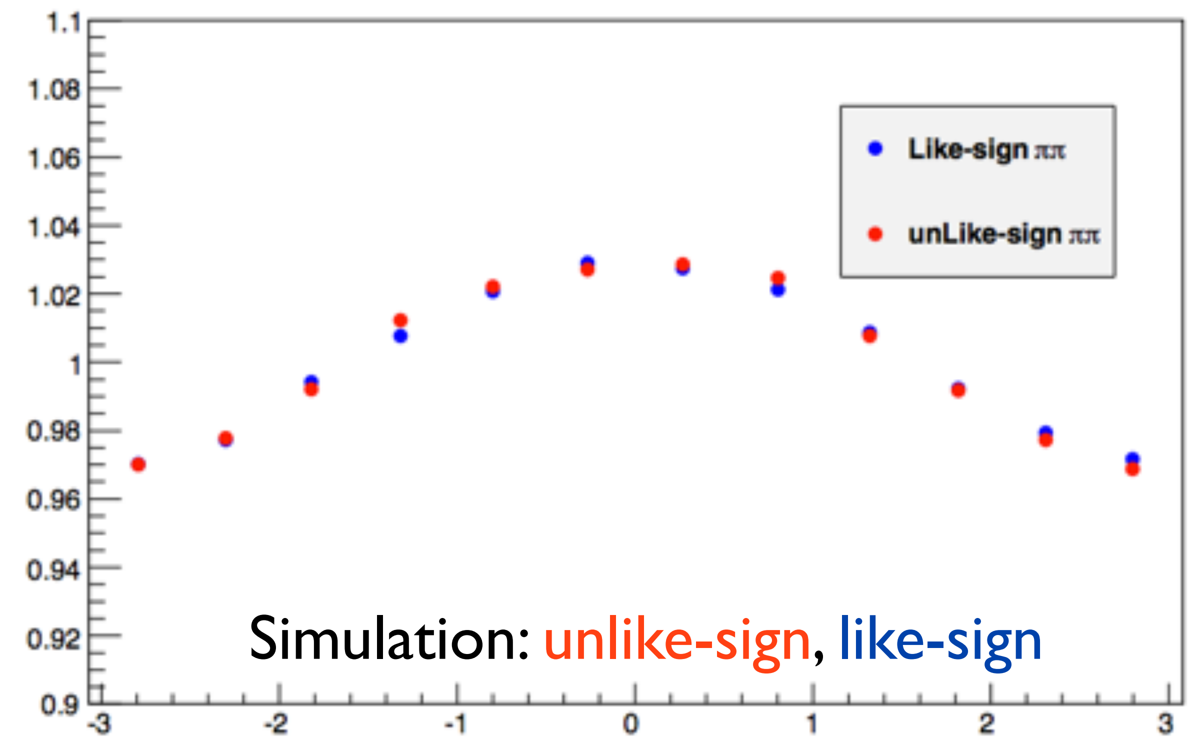
Double-ratios





Double-ratios





To reduce such non-Collins effects:
divide the sample of hadron couples in unlike-sign and like-sign (or All-charges),
and extract the asymmetries of the super ratios between these 2 samples:

Unlike-sign couples / Like-sign couples

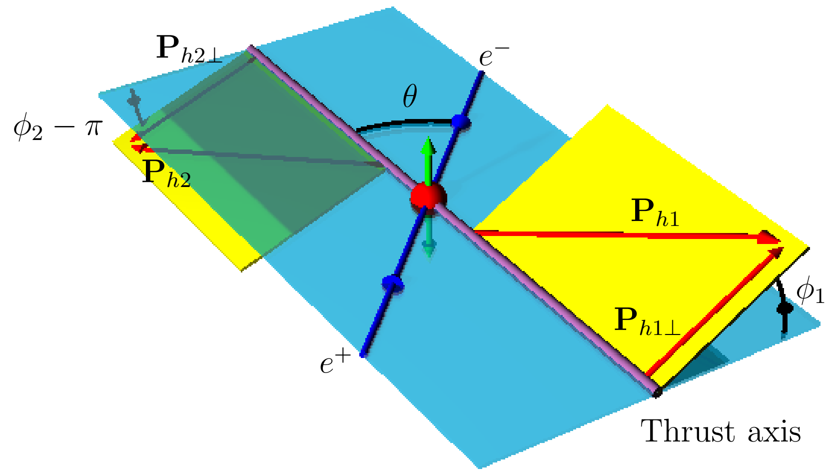
$$\mathcal{D}_{ul}^{h_1 h_2} = \mathcal{R}^U / \mathcal{R}^L$$

Unlike-sign couples / All charges

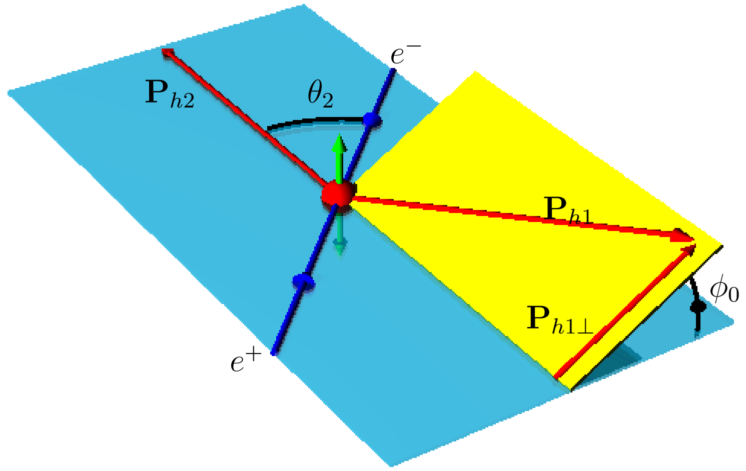
$$\mathcal{D}_{uc}^{h_1 h_2} = \mathcal{R}^U / \mathcal{R}^C$$

Double-ratios

ϕ_1+ϕ_2 method



ϕ_0 method



$$\sigma \sim \mathcal{M}_{12}\Big(1 + \frac{\sin^2\theta_T}{1+\cos^2\theta_T} cos(\phi_1+\phi_2) \frac{H_1^{\perp[1]}(z_1)\bar{H}_1^{\perp[1]}(z_2)}{D_1^{[0]}(z_1)\bar{D}_1^{[0]}(z_2)}\Big)$$

$$\sigma \sim \mathcal{M}_0\Big(1 + \frac{\sin^2\theta_2}{1+\cos^2\theta_2} cos(2\phi_0) \mathcal{F}\Big[\frac{H_1^{\perp}(z_1)\bar{H}_1^{\perp}(z_2)}{D_1^{\perp}(z_1)\bar{D}_1^{\perp}(z_2)}\Big]\Big)$$

$$\mathcal{D}_{ul}^{h_1h_2} = \mathcal{R}^U/\mathcal{R}^L$$

Fitted by

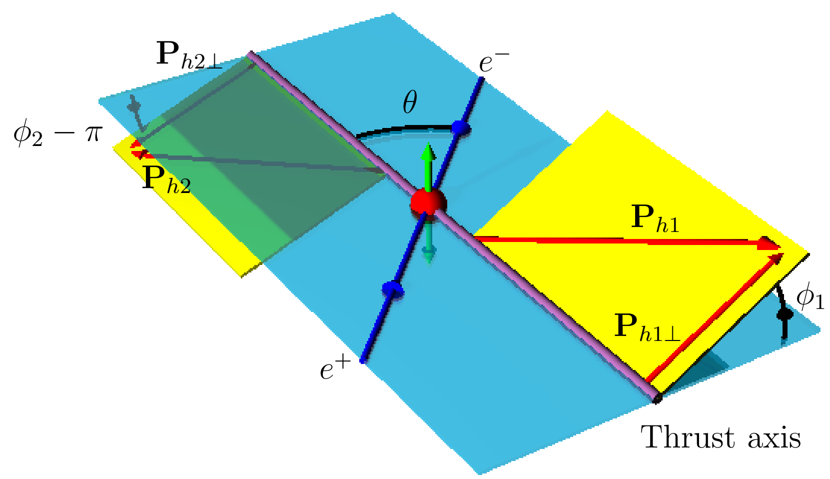
$$\mathcal{D}_{uc}^{h_1h_2} = \mathcal{R}^U/\mathcal{R}^C$$

$$\mathcal{B}_{12}(1 + \mathcal{A}_{12}\cos(\phi_1+\phi_2))$$

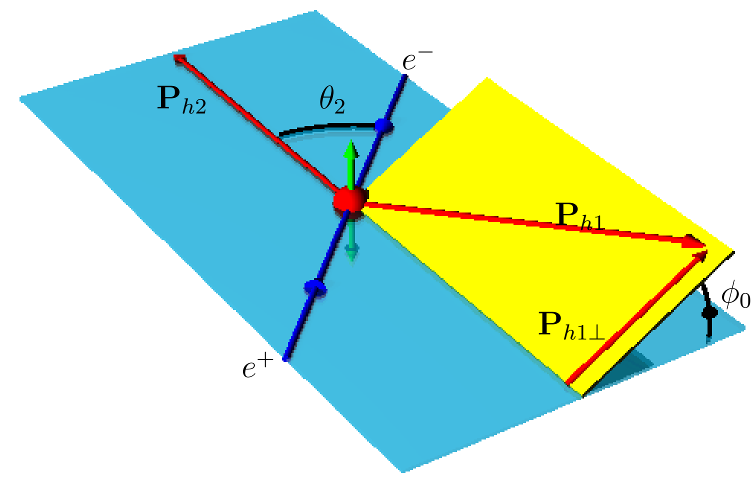
$$\mathcal{B}_0(1 + \mathcal{A}_0\cos(2\phi_0))$$

Double-ratios

$\phi_1+\phi_2$ method



ϕ_0 method



$$\sigma \sim \mathcal{M}_{12}\Big(1+\frac{\sin^2\theta_T}{1+\cos^2\theta_T}\,cos(\phi_1+\phi_2)\,\frac{H_1^{\perp[1]}(z_1)\bar{H}_1^{\perp[1]}(z_2)}{D_1^{[0]}(z_1)\bar{D}_1^{[0]}(z_2)}\Big)$$

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$$A_{12}=\frac{\sin^2\theta}{1+\cos^2\theta}\,\frac{H_1^{\perp[1]}(z_1)\bar{H}_1^{\perp[1]}(z_2)}{D_1^{[0]}(z_1)\bar{D}_1^{[0]}(z_2)}$$

$$A_0=\frac{\sin^2\theta}{1+\cos^2\theta}\,\mathcal{F}\Big[\frac{H_1^{\perp}(z_1)\bar{H}_1^{\perp}(z_2)}{D_1^{\perp}(z_1)\bar{D}_1^{\perp}(z_2)}\Big]$$

Fitted by

$$\mathcal{B}_{12}(1+\mathcal{A}_{12}\cos(\phi_1+\phi_2))$$

$$\mathcal{B}_0(1+\mathcal{A}_0\cos(2\phi_0))$$

Kinematic variables

$$z \equiv \frac{E_h}{E_p}$$ hadron energy fraction with respect to parton

$\boxed{z_1, z_2}$

p_T component of hadron momentum transverse to reference direction

1. $\phi_1+\phi_2$ method: the thrust axis $\boxed{p_{T1}, p_{T2}}$
2. ϕ_0 method: hadron 2 $\boxed{p_{T0}}$

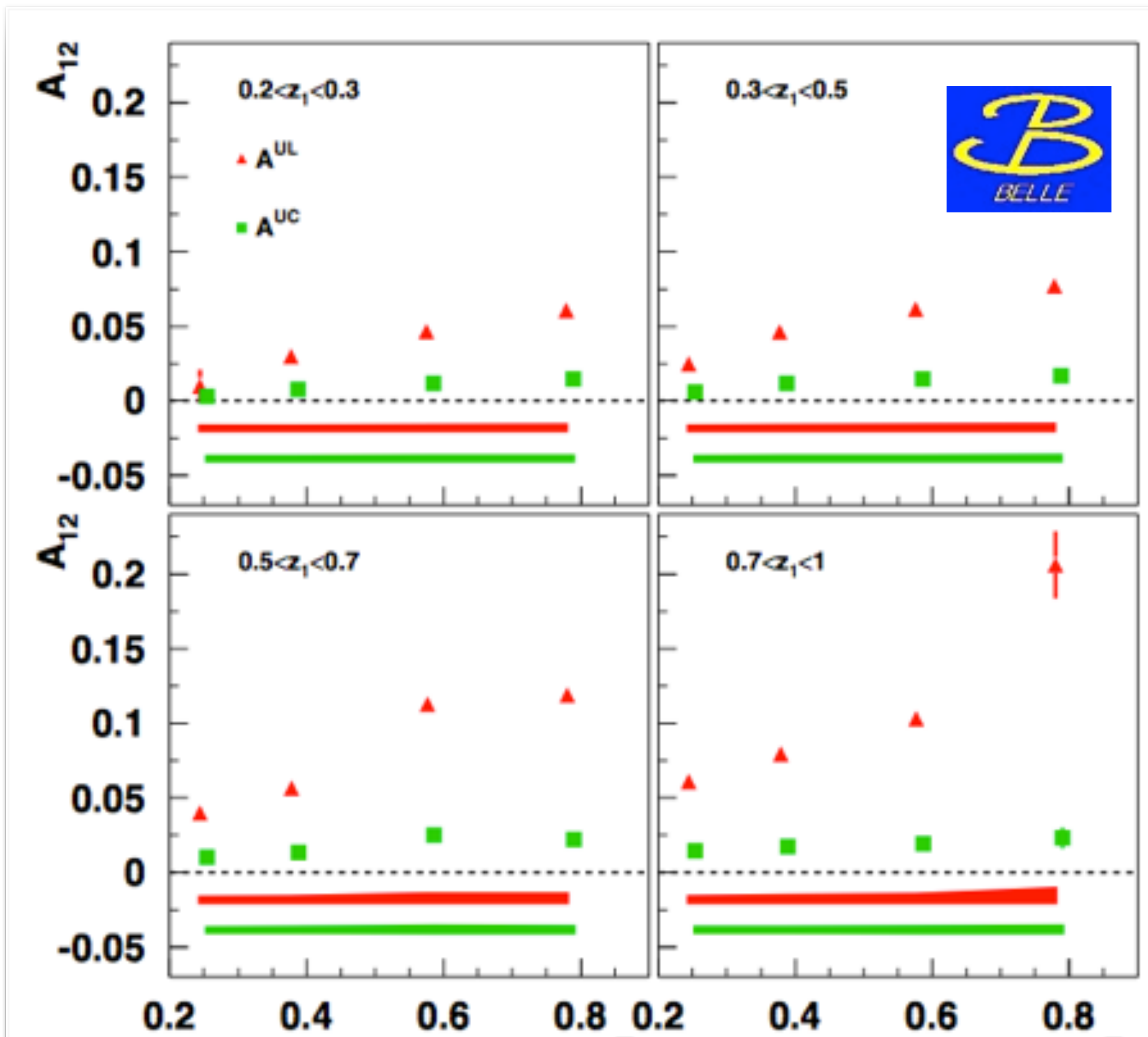
q_T component of virtual photon momentum transverse to the h_1h_2 axis in the frame where h_1 and h_2 are back-to-back

z	0.2	0.25	0.3	0.42	1				
p_{T12}	0	0.13	0.3	0.5	3				
p_{T0}	0	0.13	0.25	0.4	0.5	0.6	0.75	1	3
q_T	0	0.5	1	1.25	1.5	1.75	2	2.25	2.5
$\sin^2\theta/(1+\cos^2\theta)$	0.4	0.45	0.5	0.6	0.7	0.8	0.9	0.97	1

Belle publications
PRL 96,232002, (2006)
PRD 78, 032011 (2008)

Published results: $\pi\pi$

$\phi_1+\phi_2$ method



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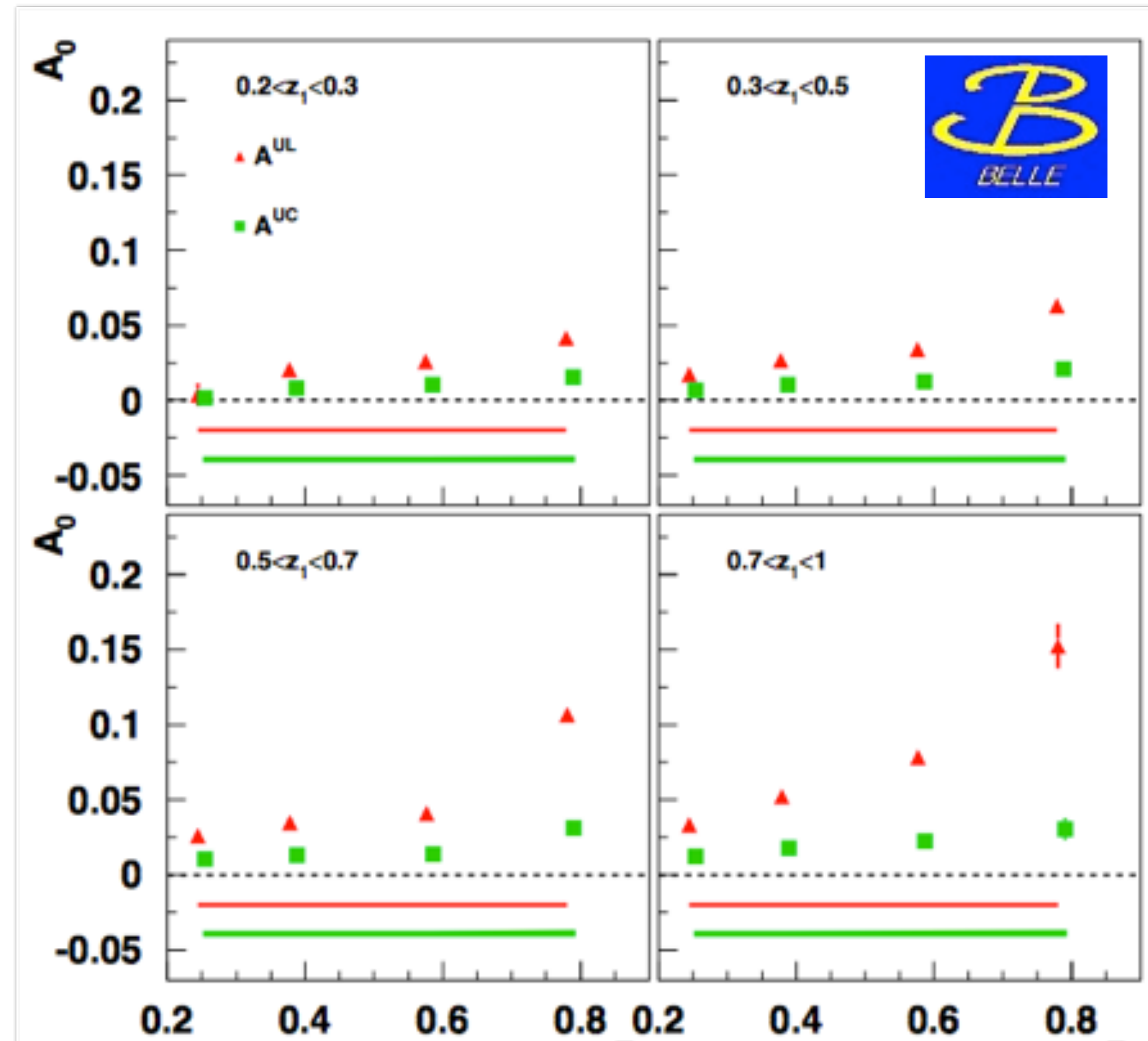
ϕ_0 method



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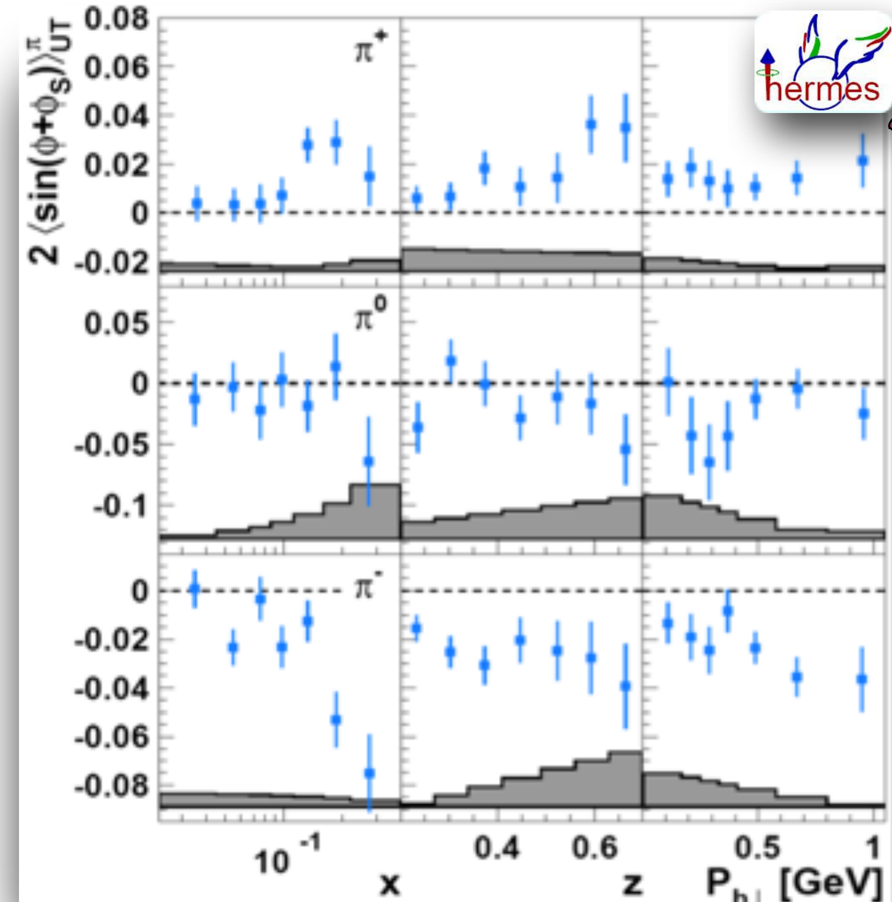
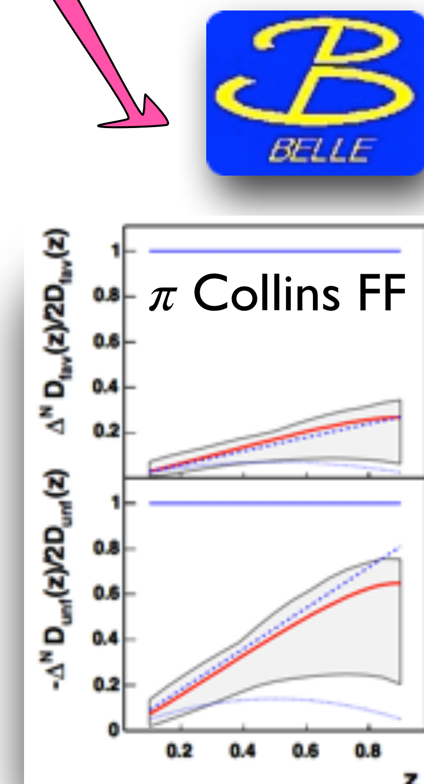
Collins amplitudes in SIDIS

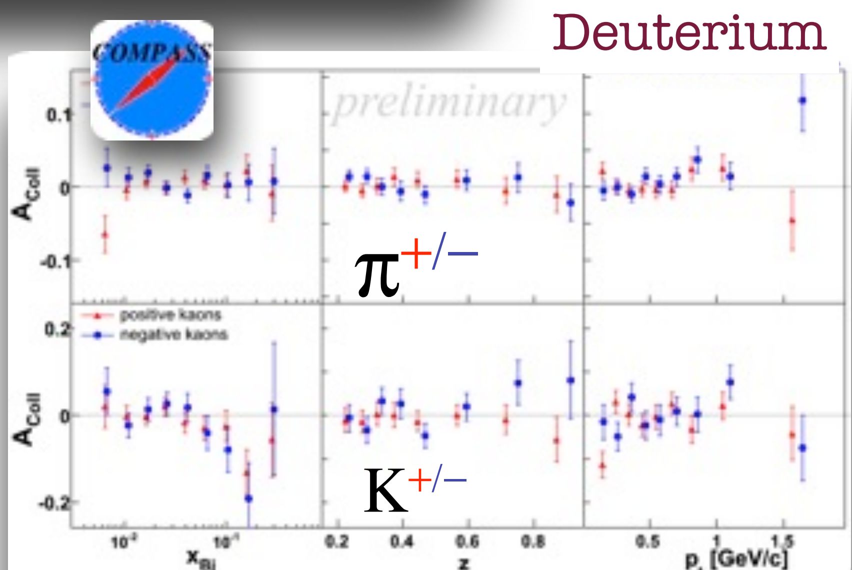
$$A_{UT} \propto h_1 \otimes H_1^{\perp}$$

[Airapetian *et al.*, Phys. Lett. B 693 (2010) 11-16]





Deuterium



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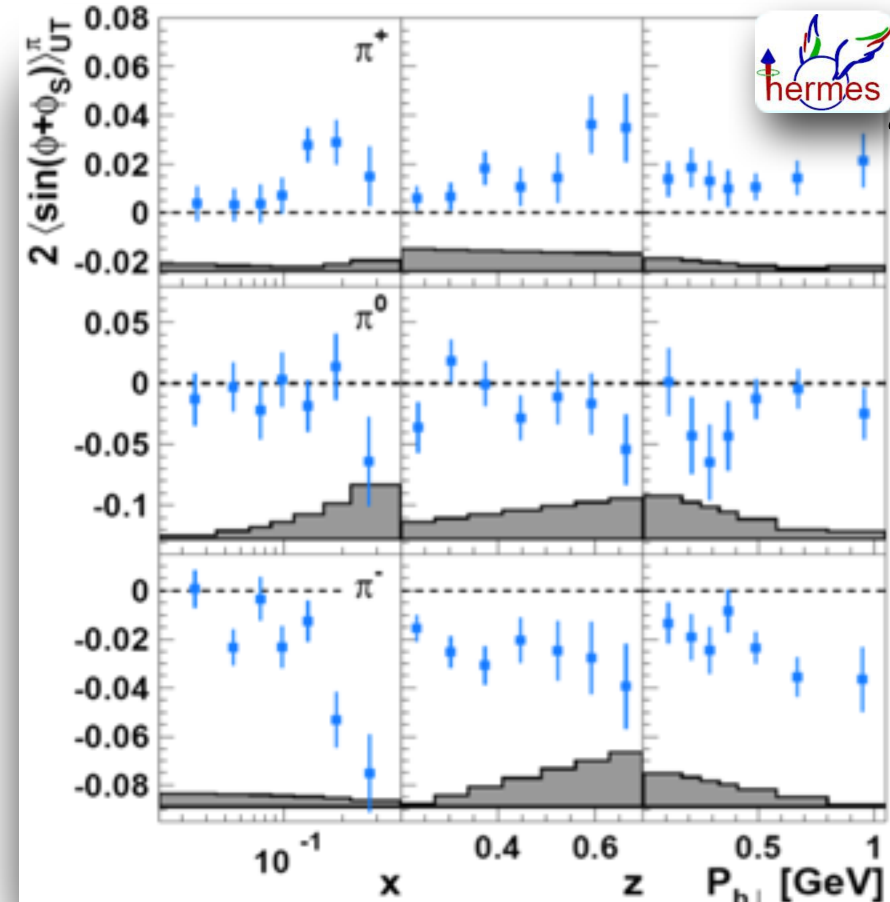
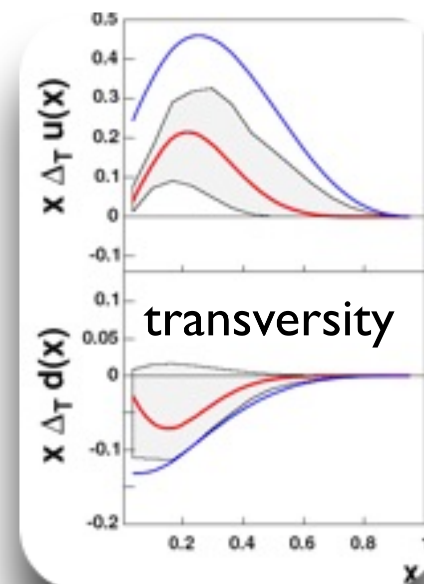
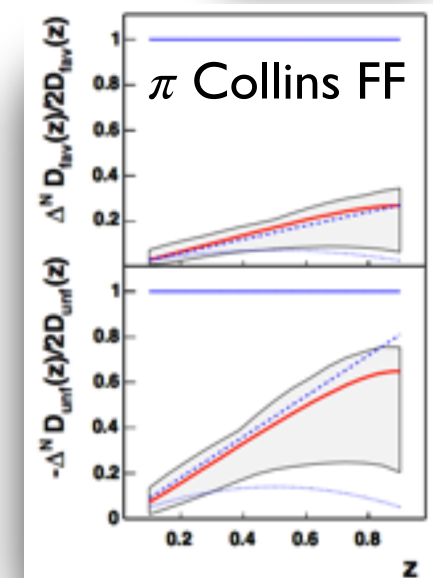
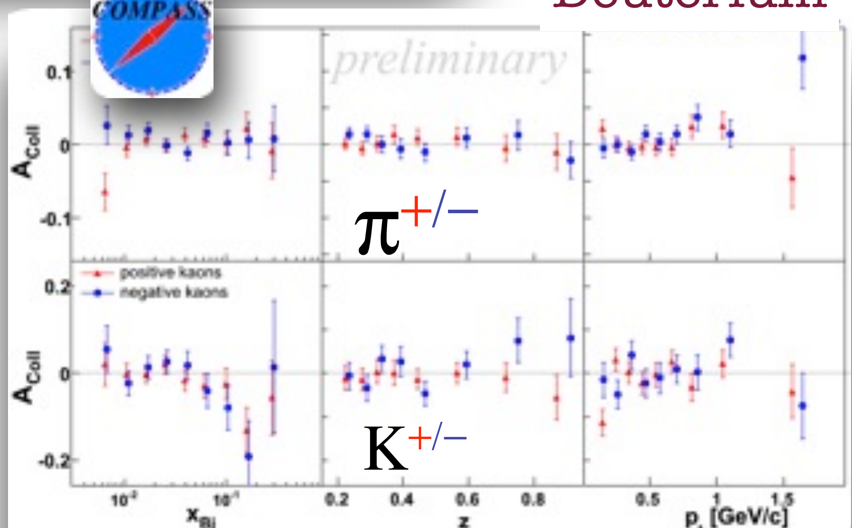


Anselmino et al.
Phys.Rev. D75 (2007)





Deuterium



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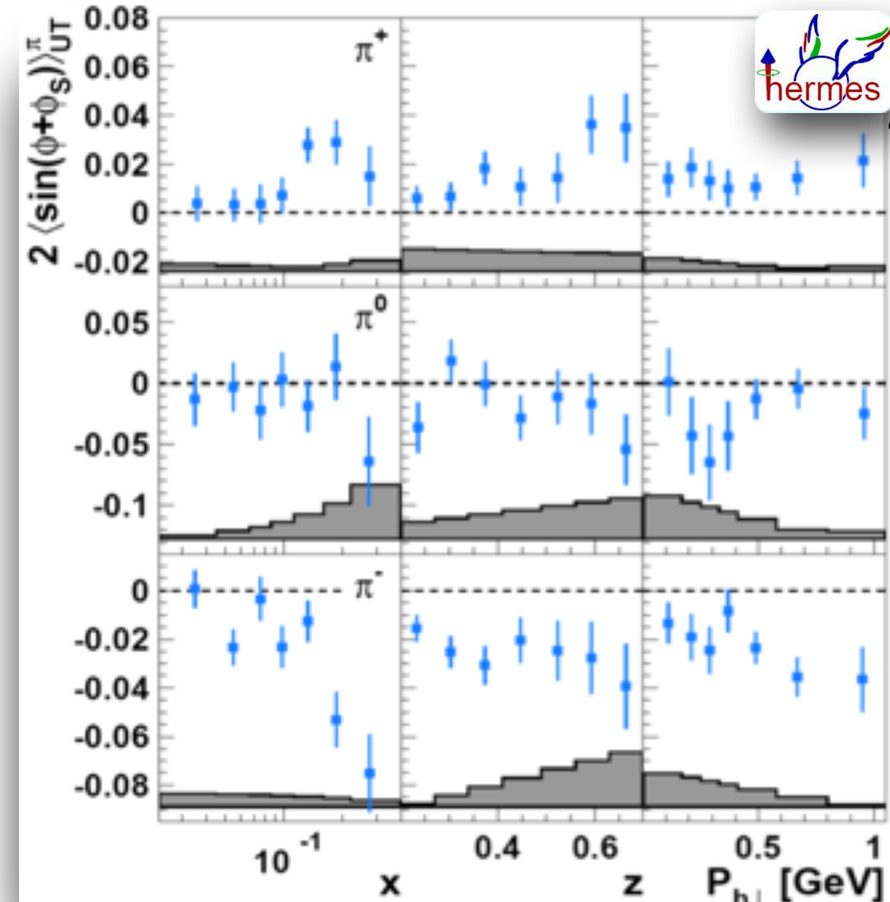
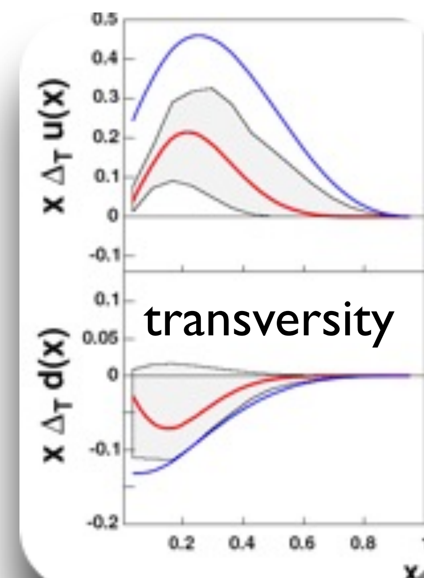
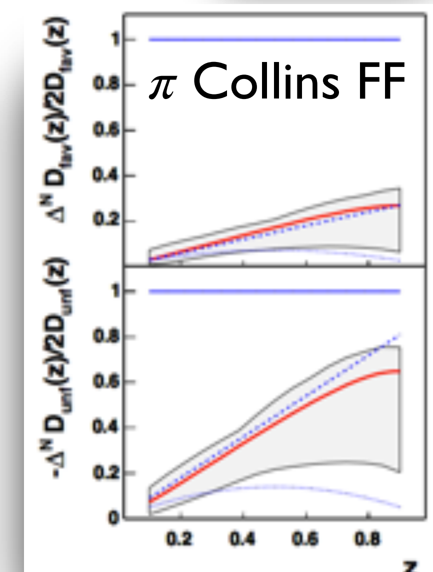
[Airapetian *et al.*, Phys. Lett. B 693 (2010) 11-16]

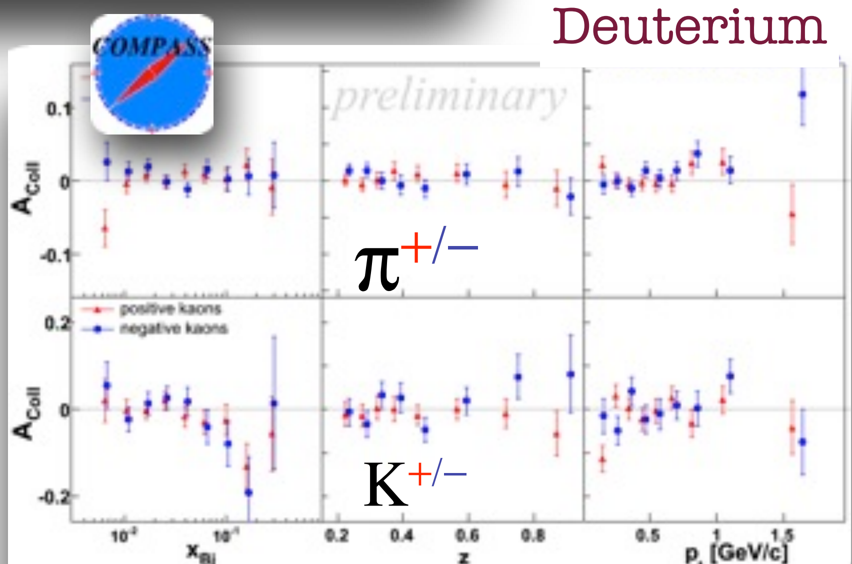


Anselmino et al.
Phys.Rev. D75 (2007)

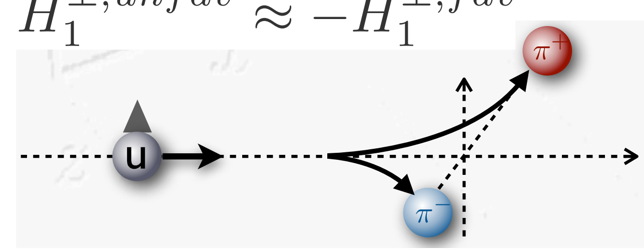




Deuterium

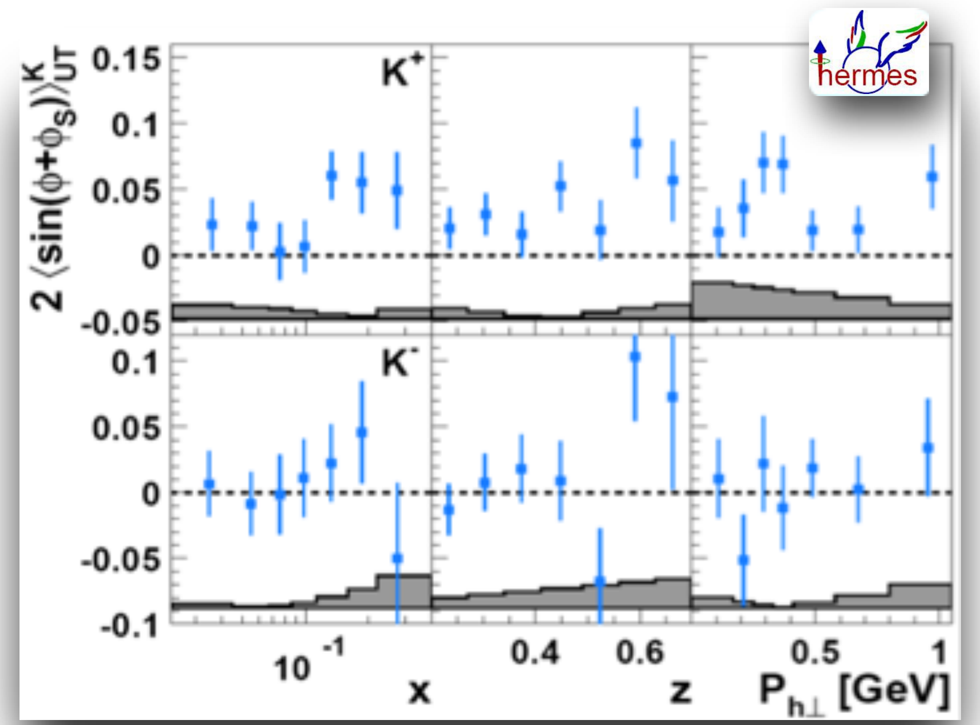


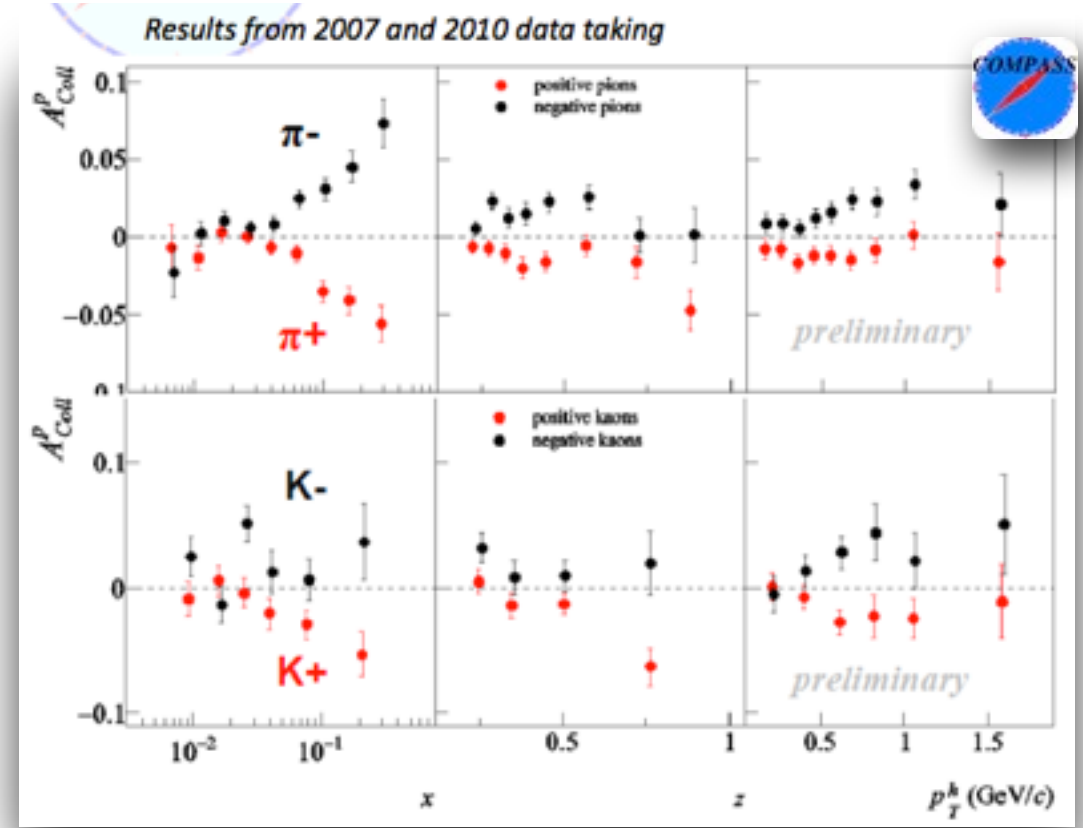
$$H_1^{\perp,unfav} \approx -H_1^{\perp,fav}$$



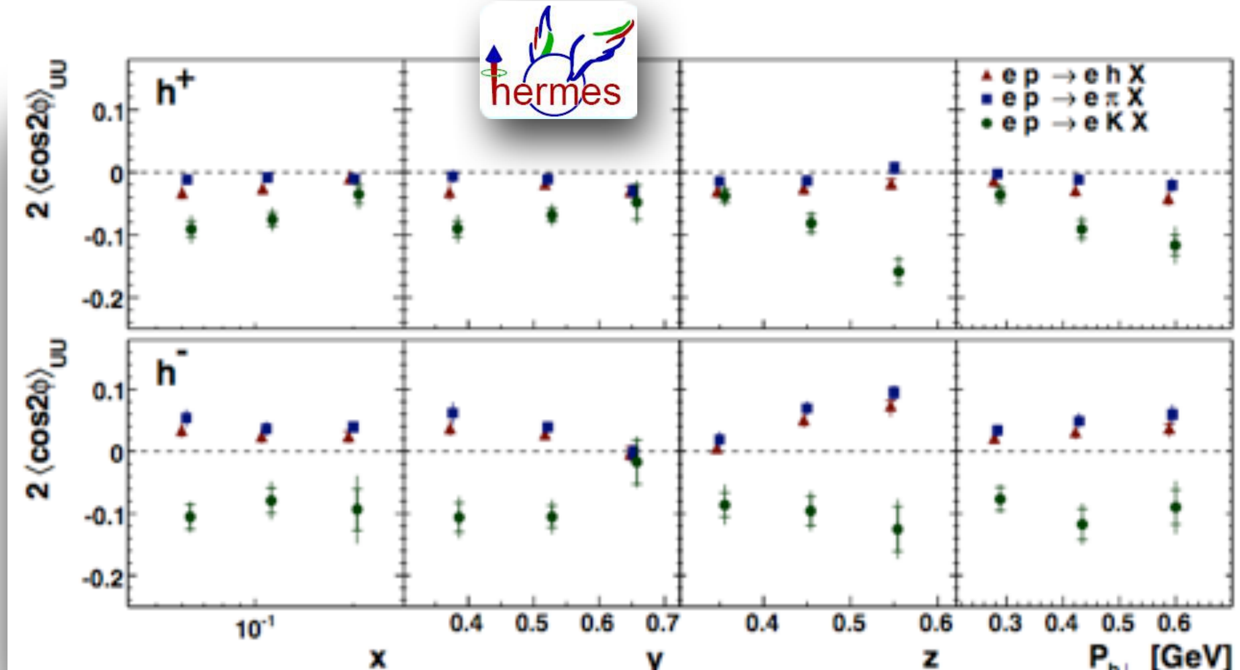
Collins amplitudes in SIDIS

$$A_{UT} \propto h_1 \otimes H_1^{\perp}$$





$$A_{UU} \propto h_1^{\perp} \otimes H_1^{\perp}$$



Collins amplitudes in SIDIS

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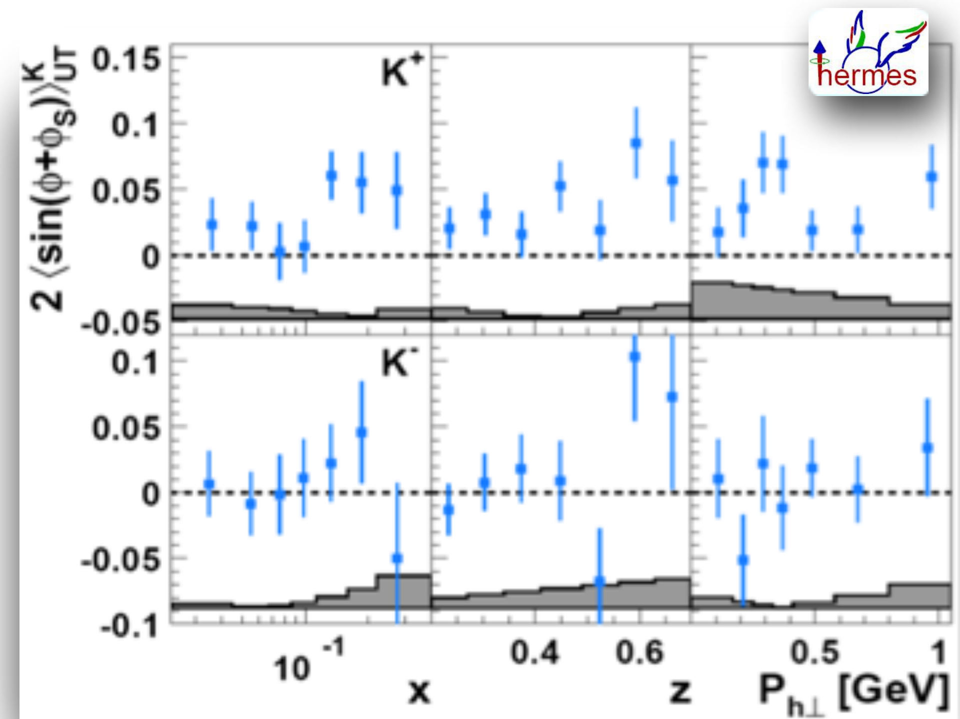
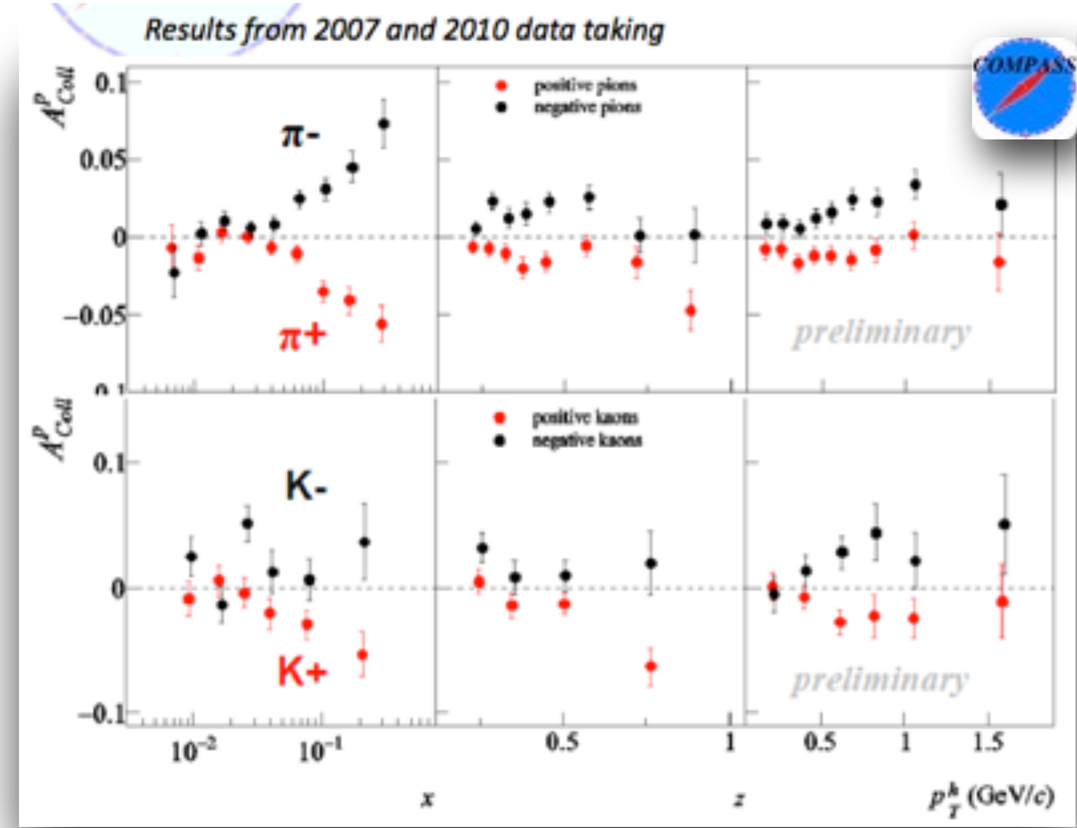
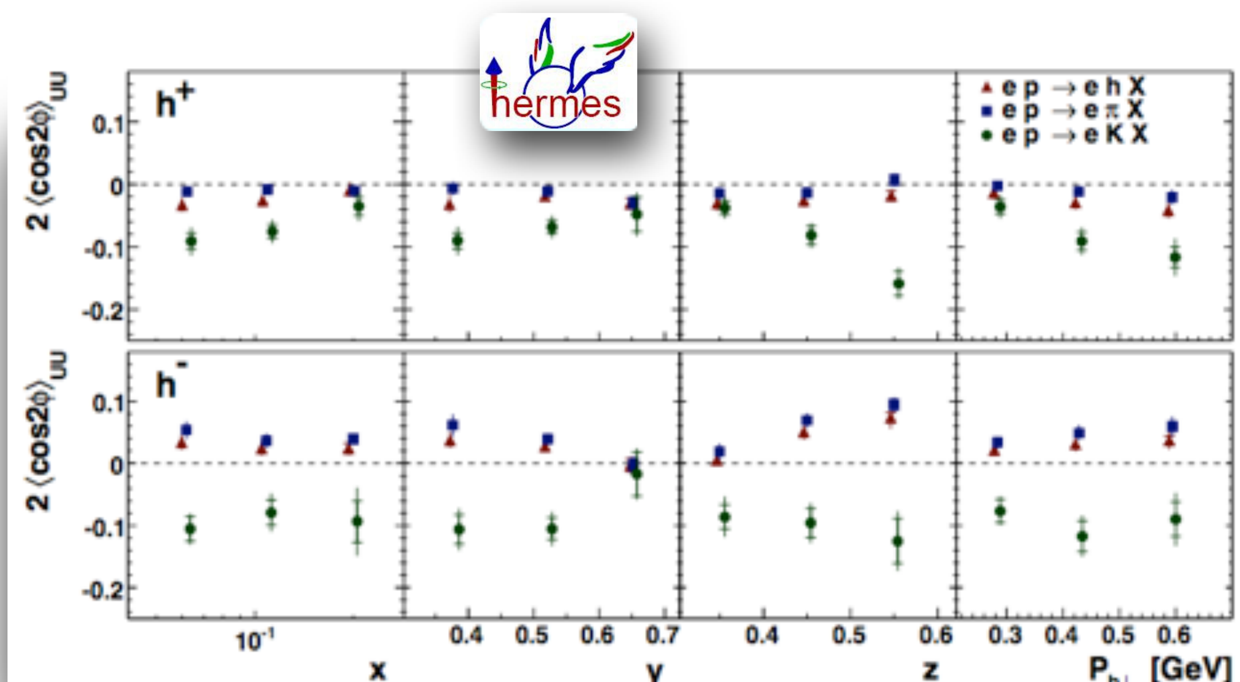


K⁺ amplitudes larger than π^+?



$$A_{UU} \propto h_1^\perp \otimes H_1^\perp$$



What's new?

	z	q_T	$\sin^2\theta/(1+\cos^2\theta)$	p_T
$\mathcal{D}_{ul}^{\pi\pi} = \mathcal{R}^{U\pi\pi}/\mathcal{R}^{L\pi\pi}$	√	√	√	New!
$\mathcal{D}_{ul}^{\pi k} = \mathcal{R}^{U\pi k}/\mathcal{R}^{L\pi k}$	New!	New!	New!	New!
$\mathcal{D}_{ul}^{kk} = \mathcal{R}^{Ukk}/\mathcal{R}^{Lkk}$	New!	New!	New!	New!

	z	q_T	$\sin^2\theta/(1+\cos^2\theta)$	p_T
$\mathcal{D}_{uc}^{\pi\pi} = \mathcal{R}^{U\pi\pi}/\mathcal{R}^{C\pi\pi}$	√	√	√	New!
$\mathcal{D}_{uc}^{\pi k} = \mathcal{R}^{U\pi k}/\mathcal{R}^{C\pi k}$	New!	New!	New!	New!
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What's new?

	z	q_T	$\sin^2\theta/(1+\cos^2\theta)$	p_T
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$\mathcal{D}_{ul}^{\pi k} = \mathcal{R}^{U\pi k}/\mathcal{R}^{L\pi k}$	New!	New!	New!	New!
$\mathcal{D}_{ul}^{kk} = \mathcal{R}^{Ukk}/\mathcal{R}^{Lkk}$	New!	New!	New!	New!

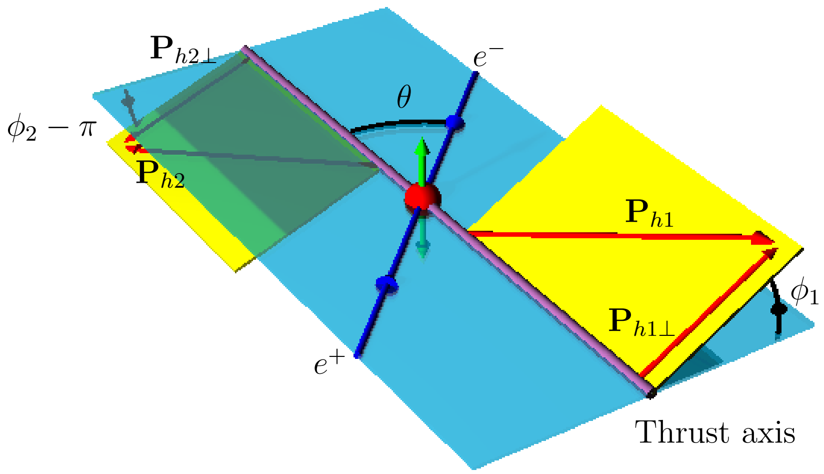
	z	q_T	$\sin^2\theta/(1+\cos^2\theta)$	p_T
$\mathcal{D}_{uc}^{\pi\pi} = \mathcal{R}^{U\pi\pi}/\mathcal{R}^{C\pi\pi}$	√	√	√	New!
$\mathcal{D}_{uc}^{\pi k} = \mathcal{R}^{U\pi k}/\mathcal{R}^{C\pi k}$	New!	New!	New!	New!
$\mathcal{D}_{uc}^{kk} = \mathcal{R}^{Ukk}/\mathcal{R}^{Ckk}$	New!	New!	New!	New!

Word of caution: this analysis is mainly aimed at kaons, so kinematic cuts and binning are optimized for kaons, and the same values used for pion too.

→ $\pi\pi$ results cannot be compared directly to published results

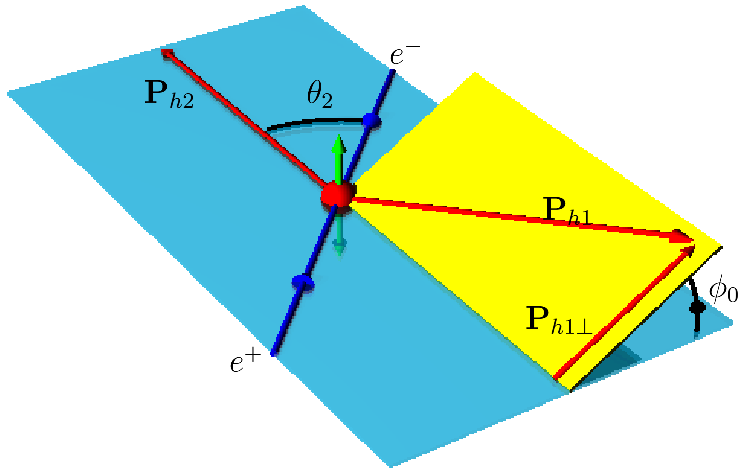
What's new?

$\phi_1+\phi_2$ method



$$\sigma \sim \mathcal{M}_{12}\Big(1+\frac{\sin^2\theta_T}{1+\cos^2\theta_T}\,cos(\phi_1+\phi_2)\frac{H_1^{\perp[1]}(z_1)\bar{H}_1^{\perp[1]}(z_2)}{D_1^{[0]}(z_1)\bar{D}_1^{[0]}(z_2)}\Big)$$

ϕ_0 method



$$\sigma \sim \mathcal{M}_0\Big(1+\frac{\sin^2\theta_2}{1+\cos^2\theta_2}\,cos(2\phi_0)\mathcal{F}\Big[\frac{H_1^{\perp}(z_1)\bar{H}_1^{\perp}(z_2)}{D_1^{\perp}(z_1)\bar{D}_1^{\perp}(z_2)}\Big]\Big)$$

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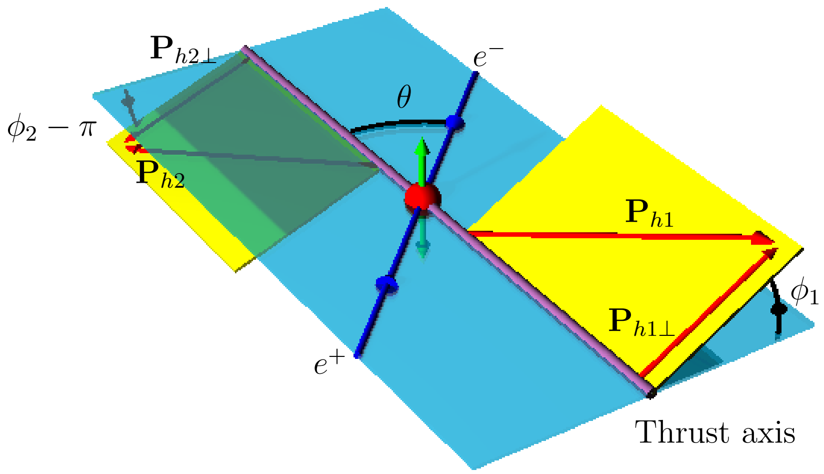

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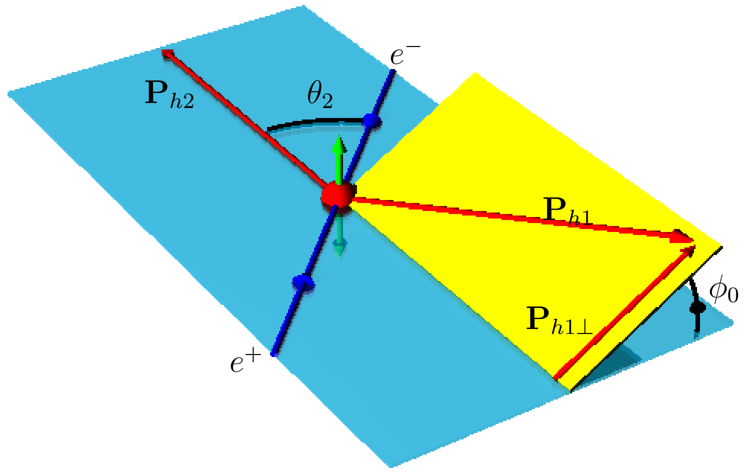

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Both interesting: different integration of FFs in p_{Ti}, might provide information on the Collins p_T dependence

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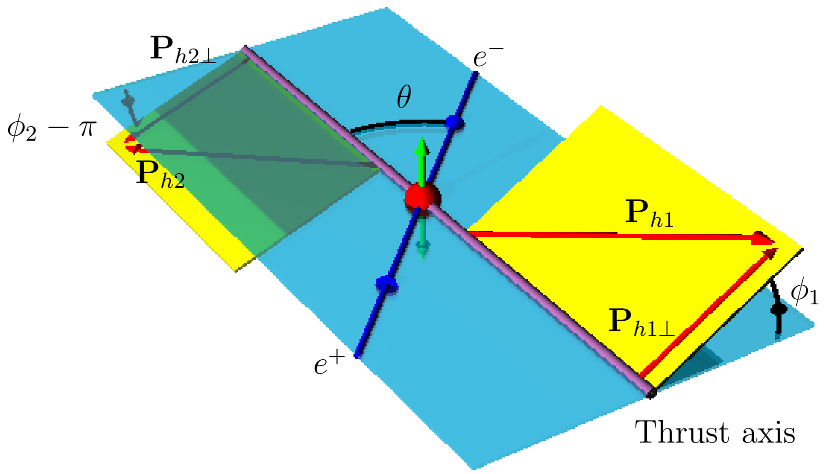

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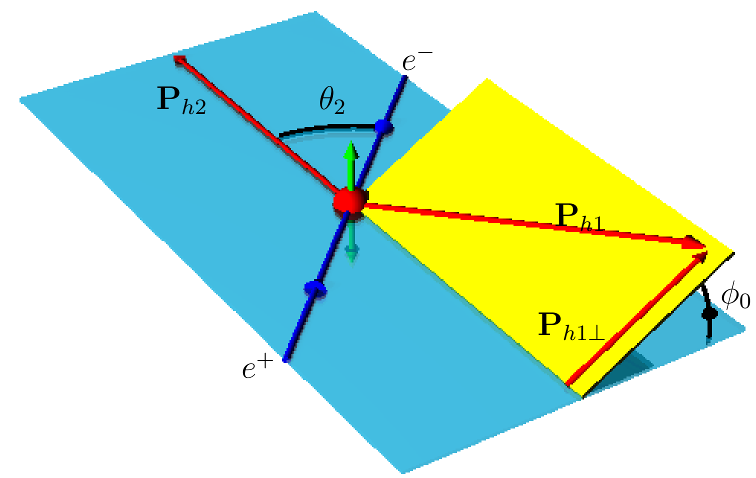

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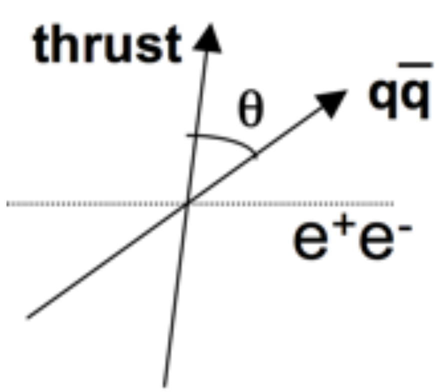


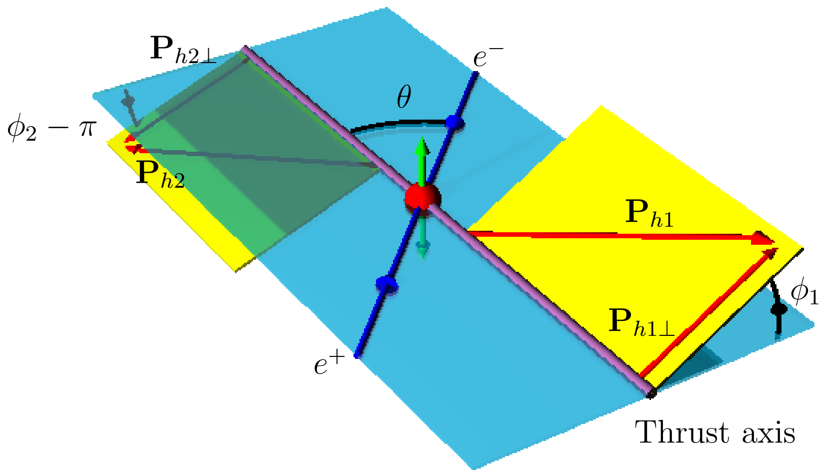
Both interesting: different integration of FFs in p_{Ti}, might provide information on the Collins p_T dependence

Advantage: more intuitive
Technically more complicated: require the determination of a $q\bar{q}$ proxy (Thrust axis)

Advantage: more convoluted
Technically simpler

What's new?

$\phi_1+\phi_2$ method



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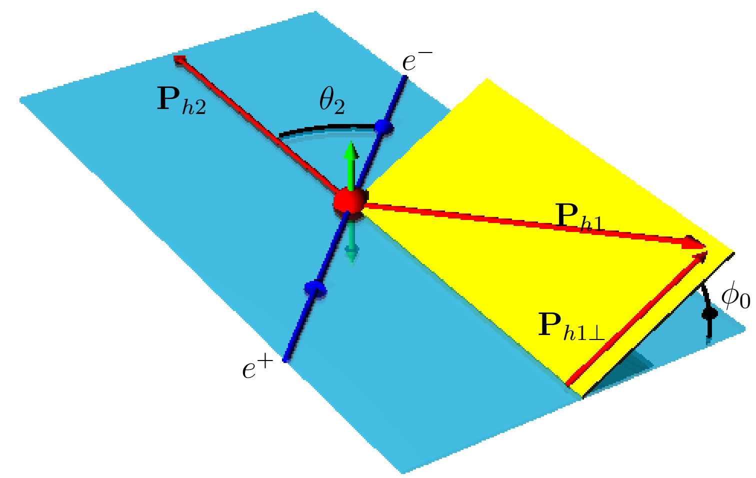
ϕ_0 method



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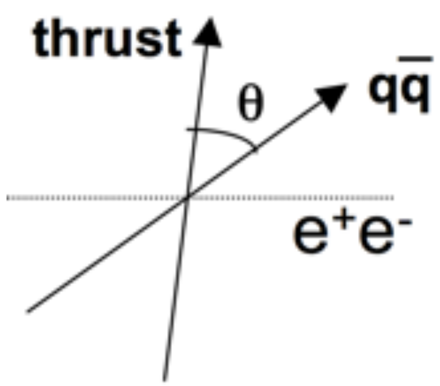

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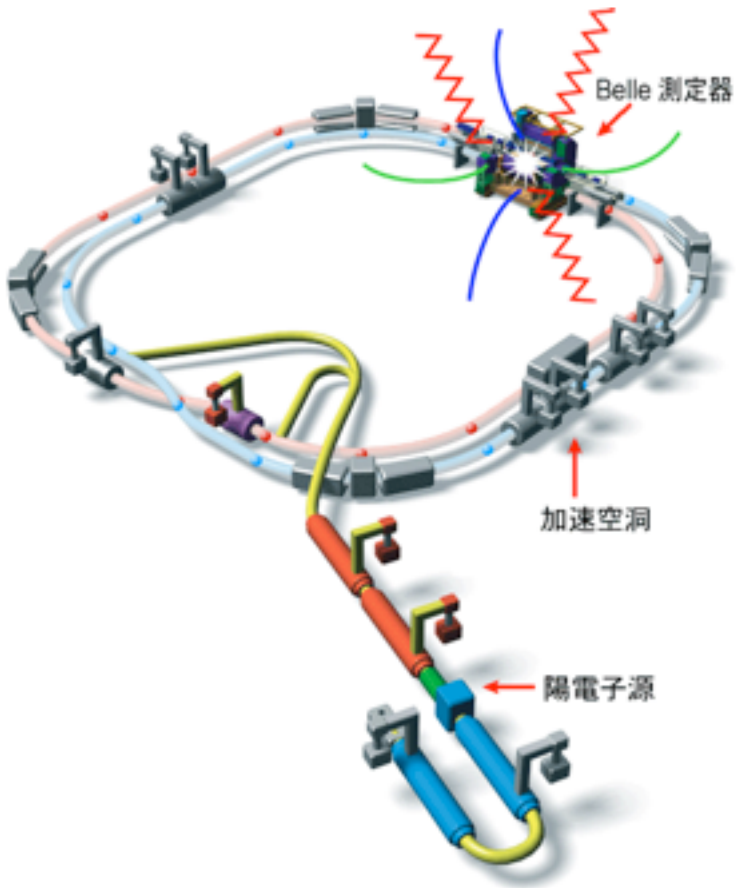
BELLE @ KEKB

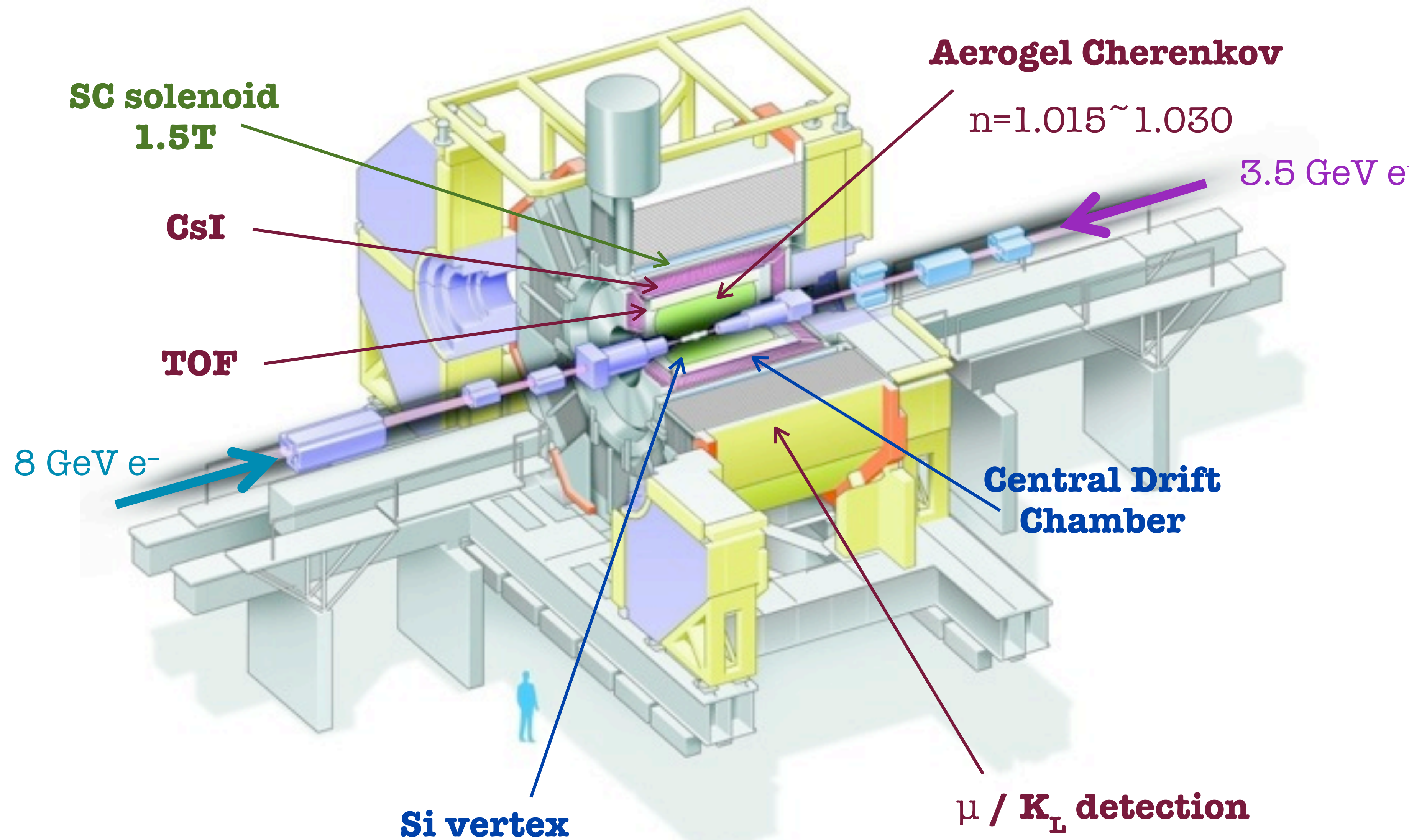

Harbin
哈尔滨
黑龙江
Sapporo
札幌
Aomori
青森
Morioka
盛岡
Akita
秋田
Sea of Japan
Sendai
仙台
Niigata
新潟
日本
(Japan)
Nagano
長野
Fukushima
福島
(South Korea)
Daejeon
대전
Daegu
대구
山东
Tottori
鳥取
Nagoya
名古屋
東京
Chiba
千葉
Hiroshima
広島
Gwangju
광주
Busan
부산
Osaka
大阪
Shizuoka
静岡
Fukuoka
福岡
Saga
佐賀
Kochi
高知
Nanjing
南京
江苏
Shanghai
上海
Kumamoto
熊本
Miyazaki
宮崎
安徽
Kagoshima
鹿児島
Hangzhou
杭州
East
China Sea
浙江
KEKB
BELLE

BELLE @ KEKB

Asymmetric
e^+ (3.5 GeV) e^- (8 GeV) collider

BELLE @ KEKB





Asymmetric e+ e- collider

On resonance: $\sqrt{s}$ = 10.58 GeV (e+ e- $\rightarrow \Upsilon(4S) \rightarrow B\bar{B}$)

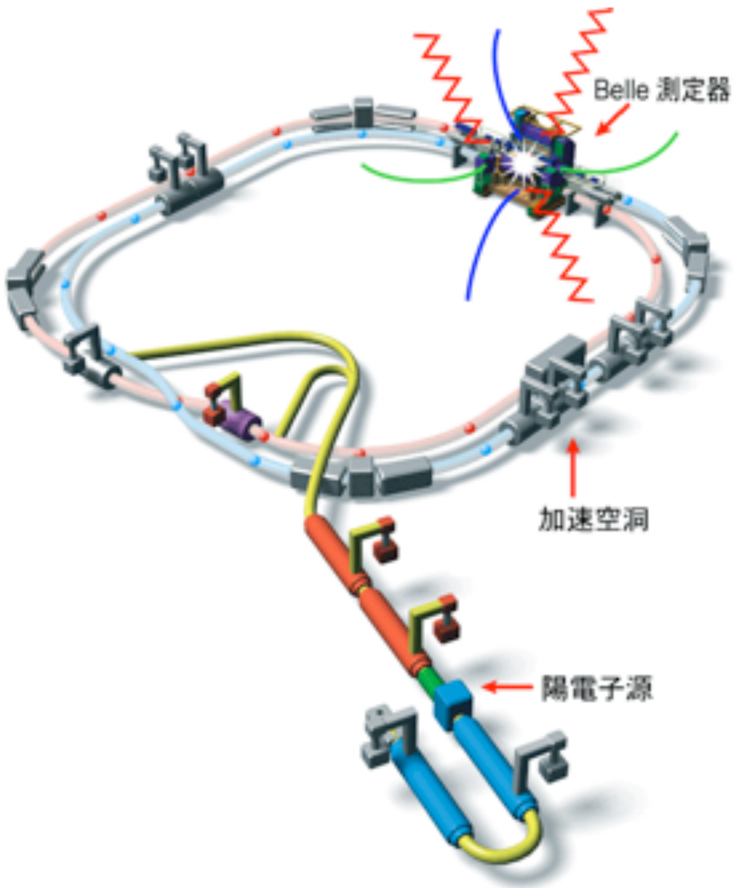
Off resonance $\sqrt{s}$ = 10.52 GeV (e+ e- $\rightarrow q\bar{q}$ (q=u,d,s,c))

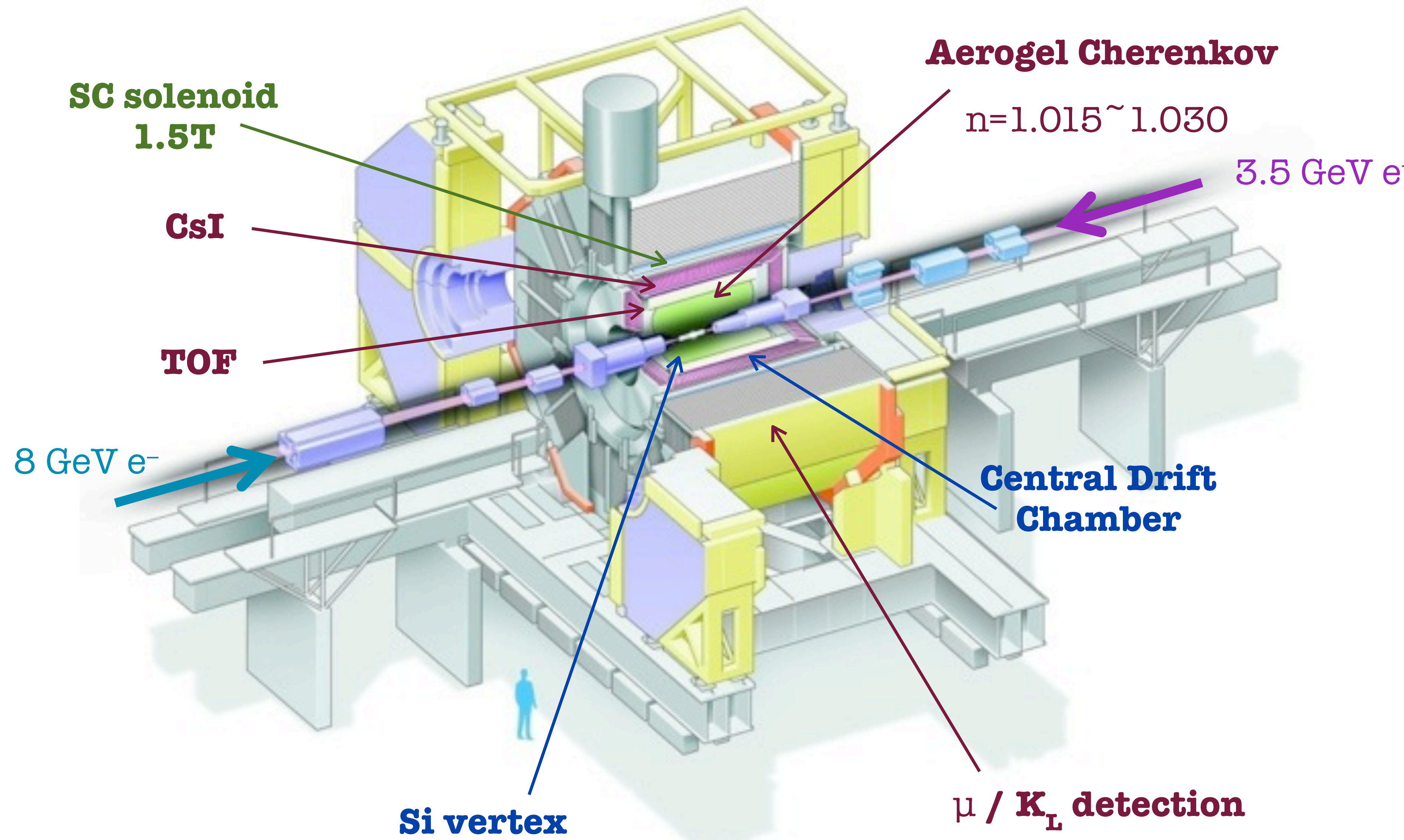
Good tracking Θ [17^0;150^0]

Good PID: $\varepsilon(\pi) \gtrsim 90\%$

$\varepsilon(K) \gtrsim 85\%$

BELLE @ KEKB





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This analysis uses ~790 fb^{-1}

Good tracking Θ [17^0;150^0]

Good PID: $\varepsilon(\pi) \gtrsim 90\%$

$\varepsilon(K) \gtrsim 85\%$

Particle ID correction

$$N^{j,raw} = P_{ij} N^i$$

$i = \pi, K$

$j = e, \mu, \pi, K, p$

Perfect PID ⇨ $j = i$

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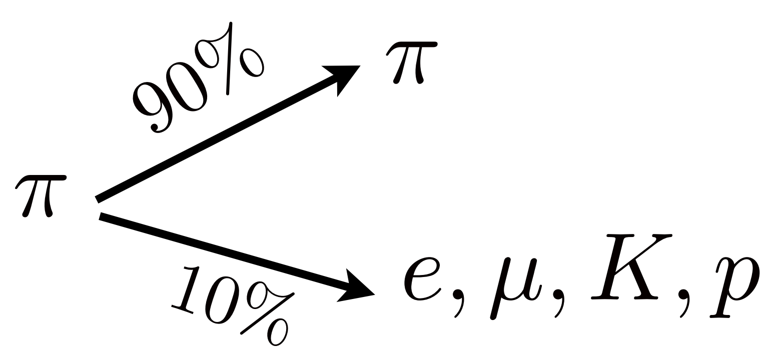


$$P_{ij} = \begin{pmatrix} P_{e\to e} & P_{e\to\mu} & P_{e\to\pi} & P_{e\to K} & P_{e\to p} \\ P_{\mu\to e} & P_{\mu\to\mu} & P_{\mu\to\pi} & P_{\mu\to K} & P_{\mu\to p} \\ P_{\pi\to e} & P_{\pi\to\mu} & P_{\pi\to\pi} & P_{\pi\to K} & P_{\pi\to p} \\ P_{K\to e} & P_{K\to\mu} & P_{K\to\pi} & P_{K\to K} & P_{K\to p} \\ P_{p\to e} & P_{p\to mu} & P_{p\to\pi} & P_{p\to K} & P_{p\to p} \end{pmatrix}$$

Particle ID correction

$$i = \pi, K$$

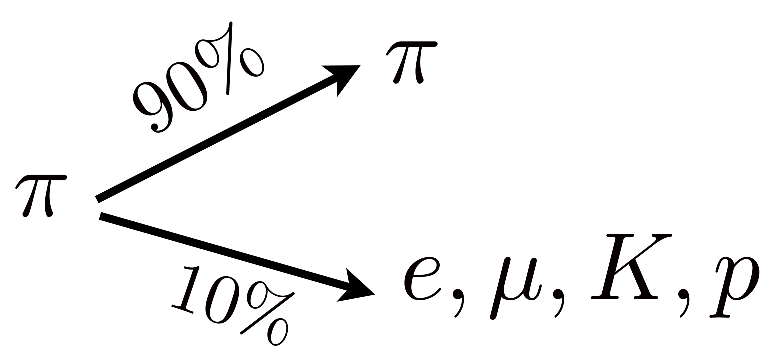
$$j = e, \mu, \pi, K, p$$

$$N^{j,raw} = P_{ij} N^i$$

$$N^i = P_{ij}^{-1} N^{j,raw}$$

Perfect PID ⇨ $j = i$

ε(π) ≳ 90% ε(K) ≳ 85%



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How to determine the P_{ij}?

How to determine the P_{ij}?

From data!

How to determine the P_{ij}?

From data!

$$D^* \longrightarrow D^0$$
$$\searrow \pi^+_{slow}$$

How to determine the P_{ij}?

$$D^* \rightarrow D^0 \, \pi^+_{slow}, \qquad D^0 \rightarrow K^- \, \pi^+_{fast}$$

From data!

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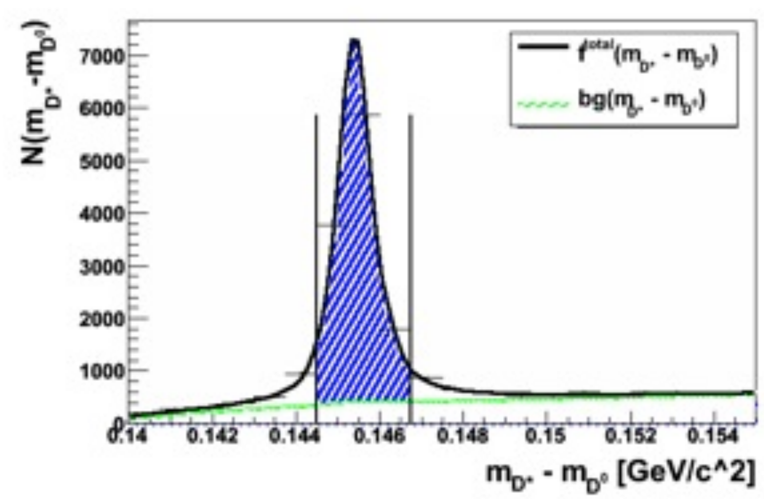
$$m_D^* - m_D^0$$



Negative hadron = K^-
(no PID likelihood used)

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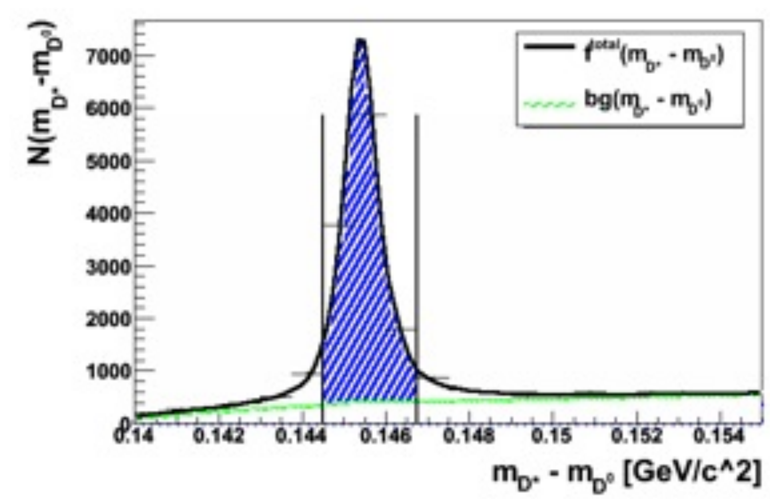
$$m_D^* - m_D^0$$



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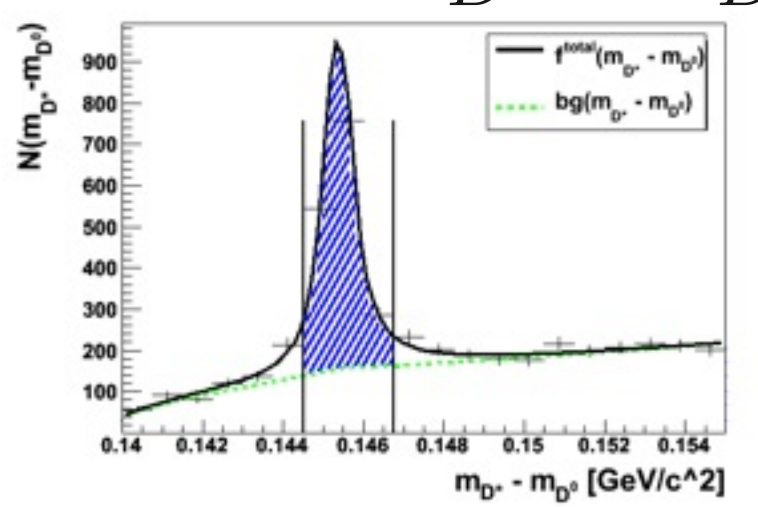
How to determine the P_{ij}?

$$D^* \longrightarrow D^0 \longrightarrow K^-$$
$$D^* \longrightarrow \pi^+_{slow} \qquad D^0 \longrightarrow \pi^+_{fast}$$

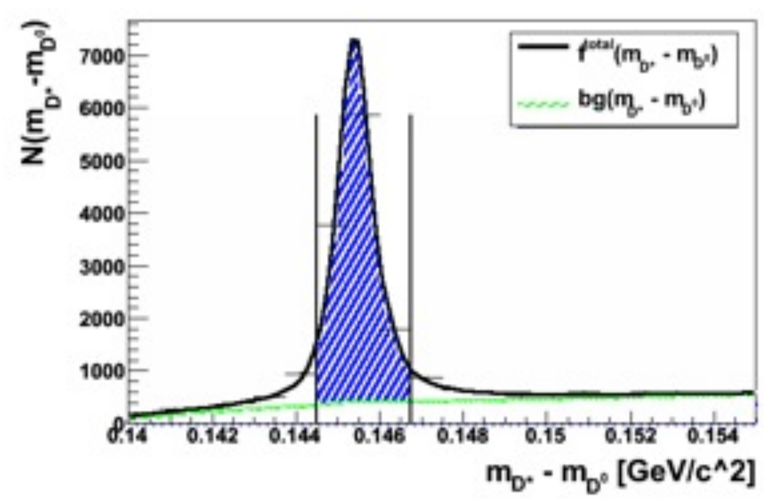
From data!

$$m_D^* - m_D^0$$



Negative hadron identified as π^-

$$m_D^* - m_D^0$$



Negative hadron = K^-
(no PID likelihood used)

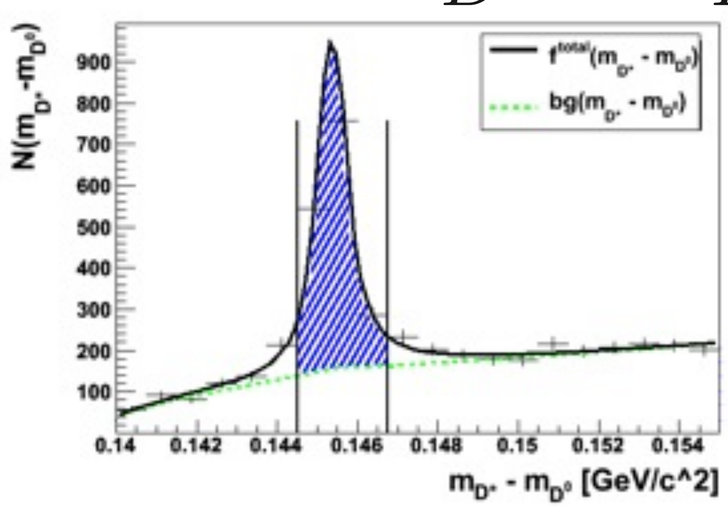
How to determine the P_{ij}?

From data!

$$D^* \rightarrow D^0 \, \pi^+_{slow}$$

$$D^0 \rightarrow K^- \, \pi^+_{fast}$$

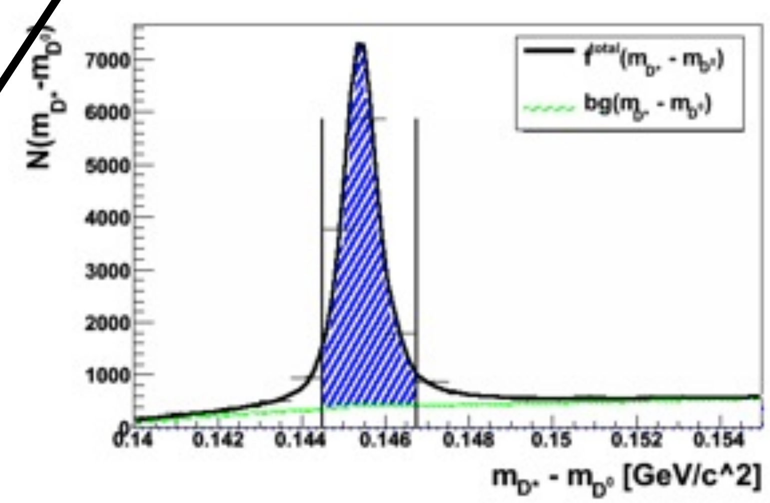
$m_D^* - m_D^0$



Negative hadron identified as π^-

$$P_{K^- \rightarrow \pi^-}$$

$m_D^* - m_D^0$



Negative hadron = K^-
(no PID likelihood used)

How to determine the P_{ij}?

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From data!

How to determine the P_{ij}?

$$D^* \longrightarrow D^0, \quad D^* \longrightarrow \pi^+_{slow}$$

From data!

How to determine the P_{ij}?

From data!

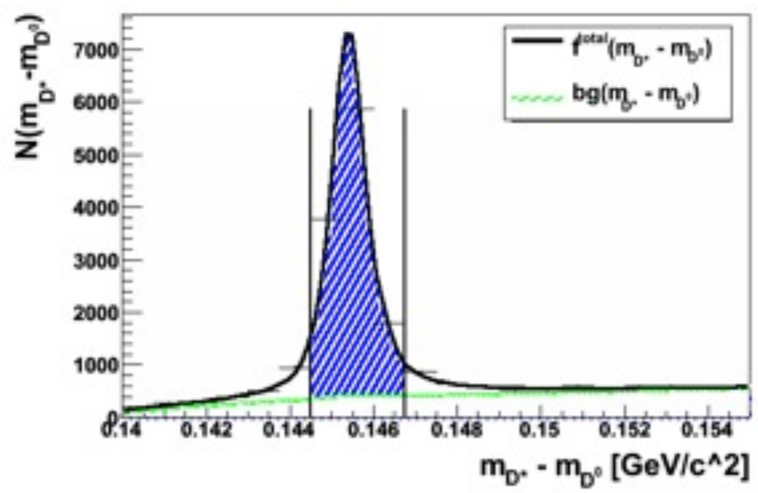
$$D^* \longrightarrow D^0 \longrightarrow K^-$$
$$D^* \longrightarrow \pi^+_{slow}, \quad D^0 \longrightarrow \pi^+_{fast}$$

How to determine the P_{ij}?

From data!

$$D^* \to D^0 \, \pi^+_{slow}, \qquad D^0 \to K^- \, \pi^+_{fast}$$

$$m_D^* - m_D^0$$



Negative hadron = K^-
(no PID likelihood used)

How to determine the P_{ij}?

From data!

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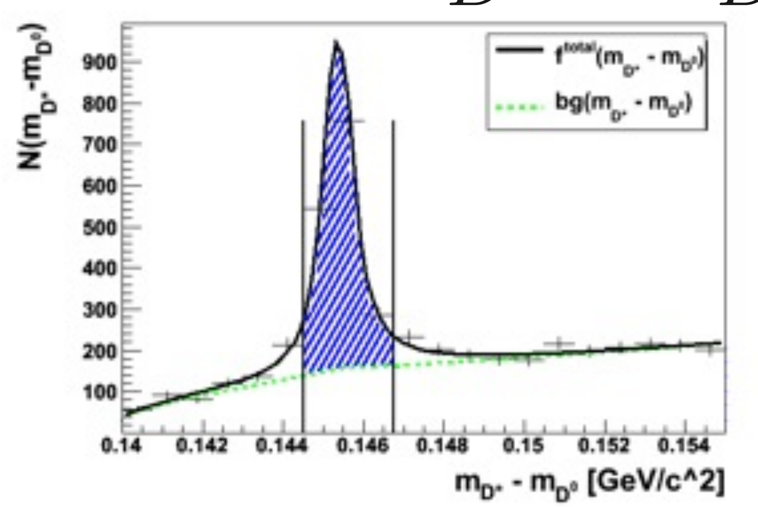
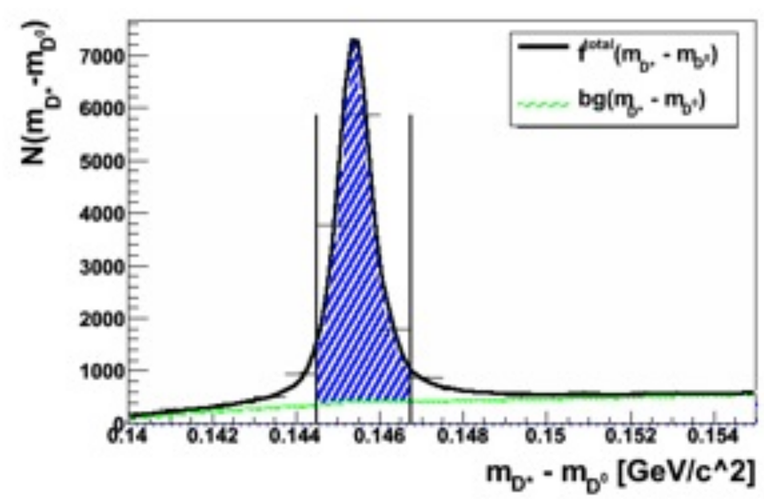
$$D^* \longrightarrow \pi^+_{slow} \qquad D^0 \longrightarrow \pi^+_{fast}$$

$$m_D^* - m_D^0$$



Negative hadron identified as π^-

$$m_D^* - m_D^0$$



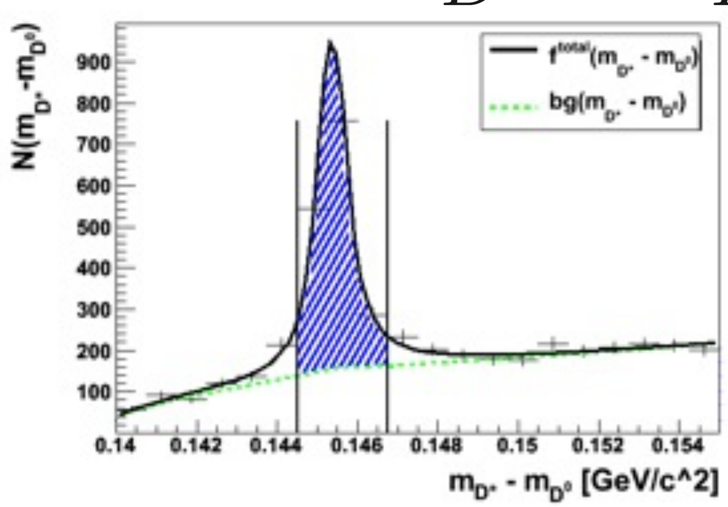
Negative hadron = K^- (no PID likelihood used)

How to determine the P_{ij}?

From data!

$$D^* \longrightarrow D^0 \longrightarrow K^-$$
$$D^* \longrightarrow \pi^+_{slow}, \quad D^0 \longrightarrow \pi^+_{fast}$$

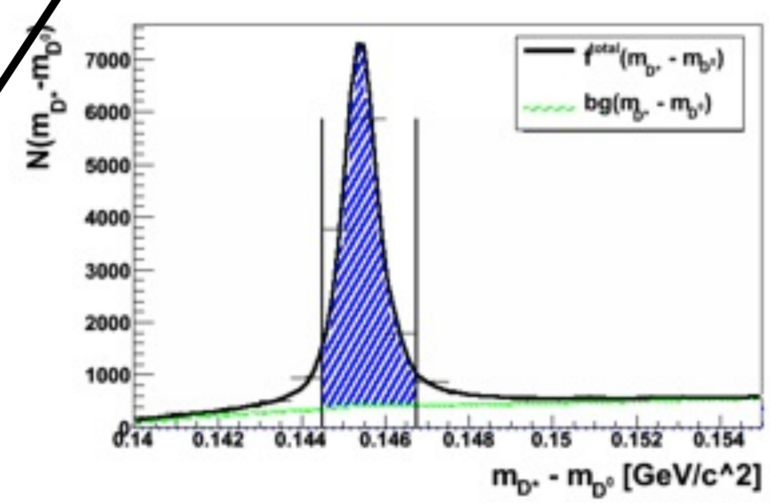
$m_D^* - m_D^0$



Negative hadron identified as π^-

$$P_{K^- \to \pi^-}$$

$m_D^* - m_D^0$



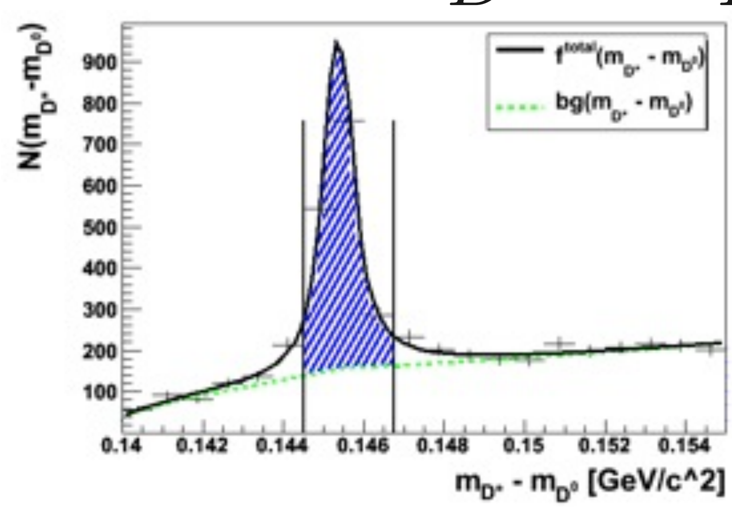
Negative hadron = K^-
(no PID likelihood used)

How to determine the P_{ij}?

From data!

$$D^* \to D^0 \, \pi^+_{slow}, \quad D^0 \to K^- \, \pi^+_{fast}$$

$m_D^* - m_D^0$

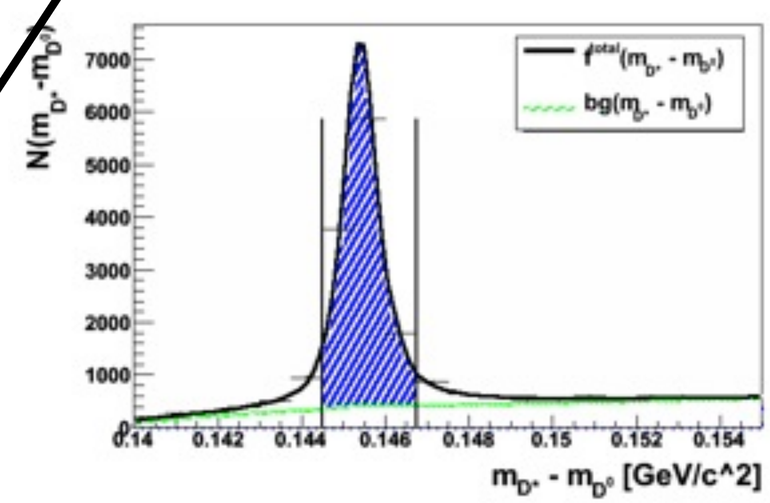


Negative hadron identified as π^-

K^-

$$P_{K^- \to \pi^-}$$

$m_D^* - m_D^0$



$$P_{K^- \to K^-}$$

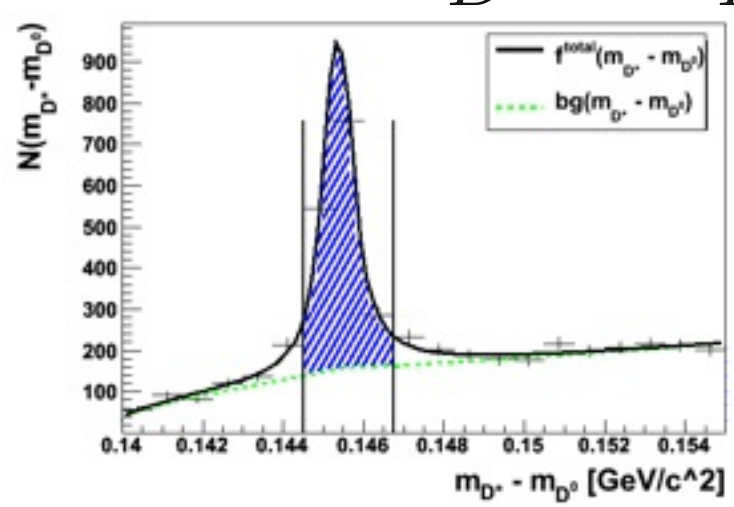
Negative hadron = K^-
(no PID likelihood used)

How to determine the P_{ij}?

From data!

$$D^* \longrightarrow D^0 + \pi^+_{slow}$$
$$D^0 \longrightarrow K^- + \pi^+_{fast}$$

$m_D^* - m_D^0$



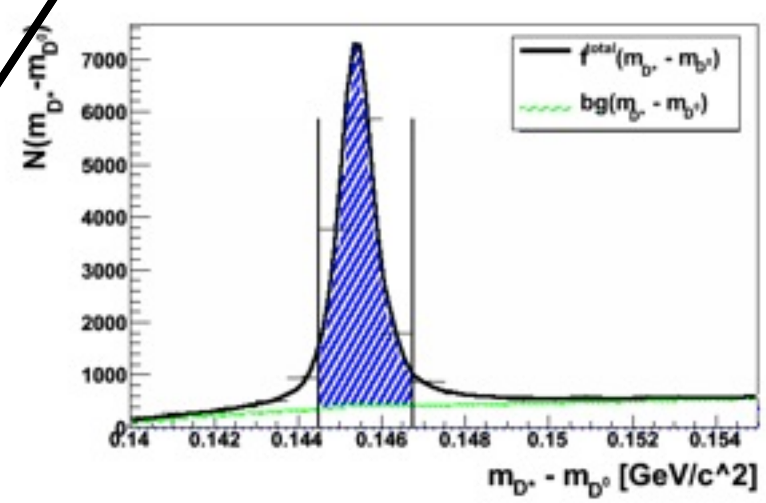
Negative hadron identified as π^-

K^-

$\bar{p}$

$$P_{K^- \rightarrow \pi^-}$$

$m_D^* - m_D^0$



Negative hadron = K^- (no PID likelihood used)

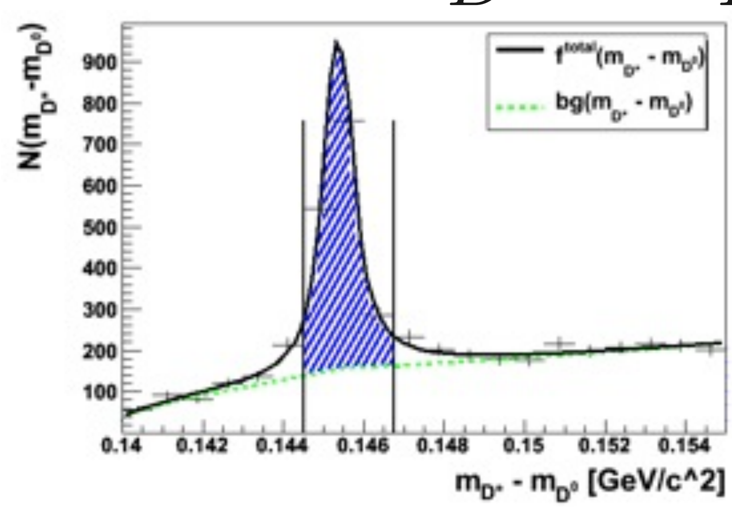
$$P_{K^- \rightarrow K^-}$$
$$P_{K^- \rightarrow \bar{p}}$$

How to determine the P_{ij}?

$$D^* \rightarrow D^0 \rightarrow K^- \quad \pi^+_{fast}$$
$$D^* \rightarrow \pi^+_{slow}$$

From data!

$m_D^* - m_D^0$

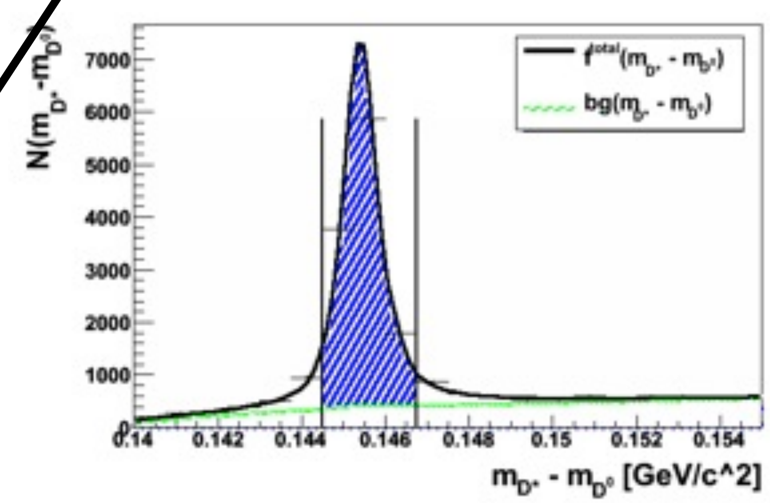


Negative hadron identified as π^-

K^- $\bar{p}$ μ^-

$P_{K^- \rightarrow \pi^-}$

$m_D^* - m_D^0$



Negative hadron = K^- (no PID likelihood used)

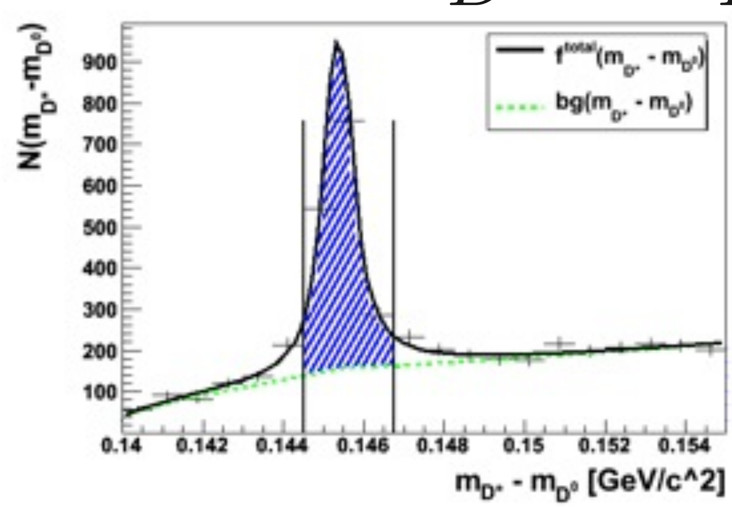
$P_{K^- \rightarrow K^-}$
$P_{K^- \rightarrow \bar{p}}$
$P_{K^- \rightarrow \mu^-}$

How to determine the P_{ij}?

$$D^* \rightarrow D^0 \rightarrow K^-$$
$$\pi^+_{slow} \qquad \pi^+_{fast}$$

From data!

$$m_D^* - m_D^0$$

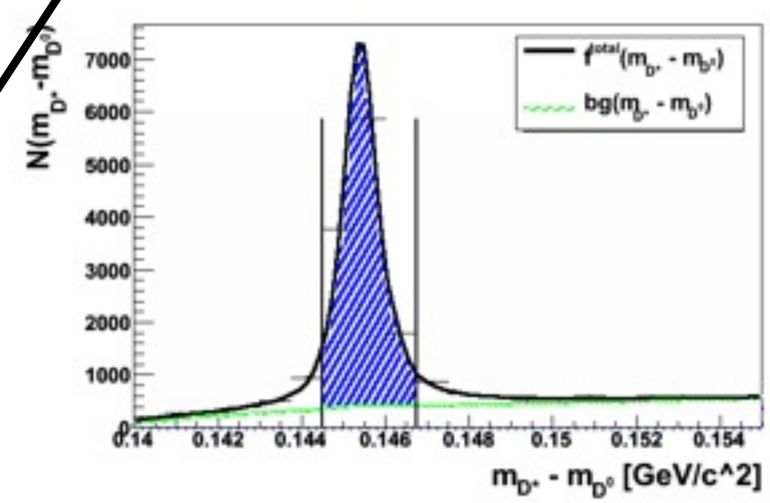


Negative hadron identified as π^-

K^- $\bar{p}$ μ^- e^-

$$P_{K^- \rightarrow \pi^-}$$

$$m_D^* - m_D^0$$



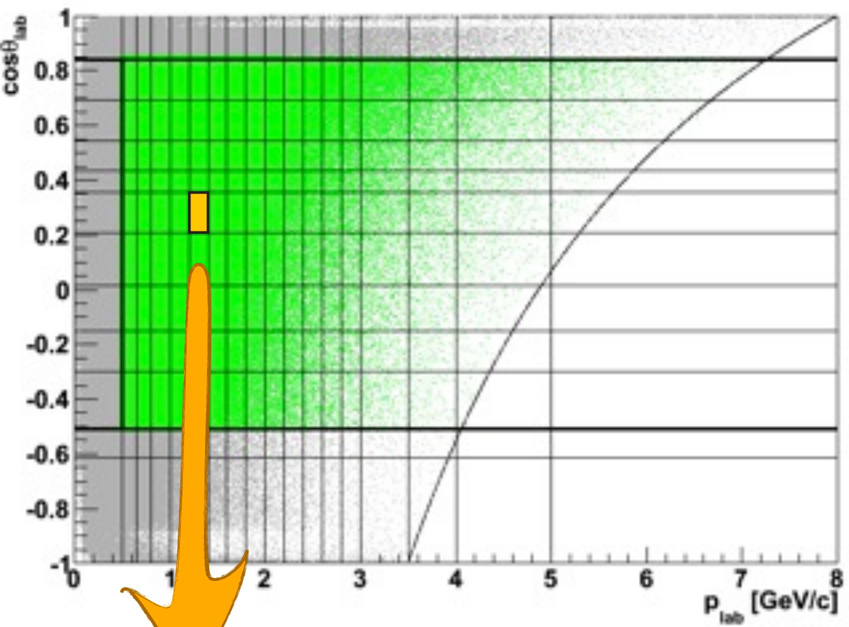
Negative hadron = K^-
(no PID likelihood used)

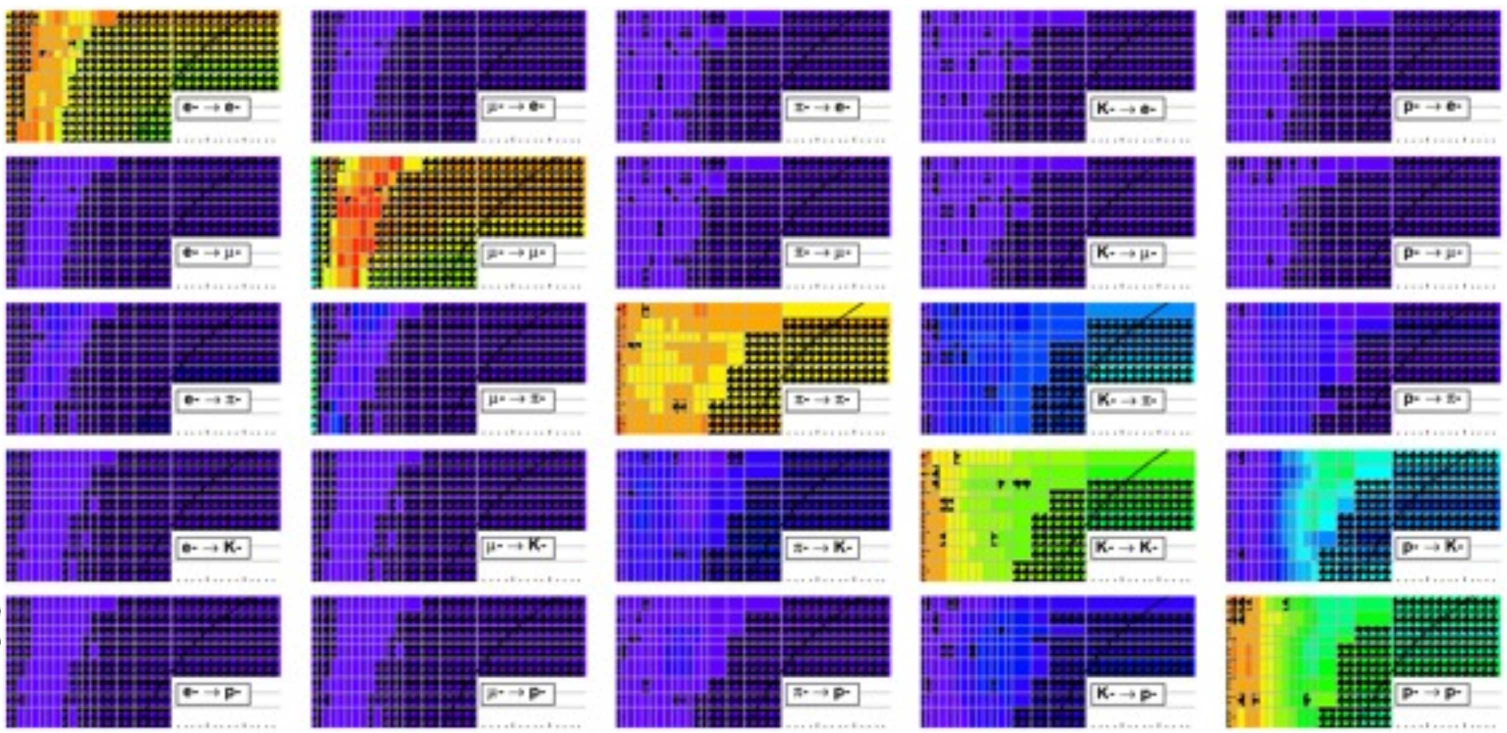
$$P_{K^- \rightarrow K^-}$$
$$P_{K^- \rightarrow \bar{p}}$$
$$P_{K^- \rightarrow \mu^-}$$
$$P_{K^- \rightarrow e^-}$$

2D correction

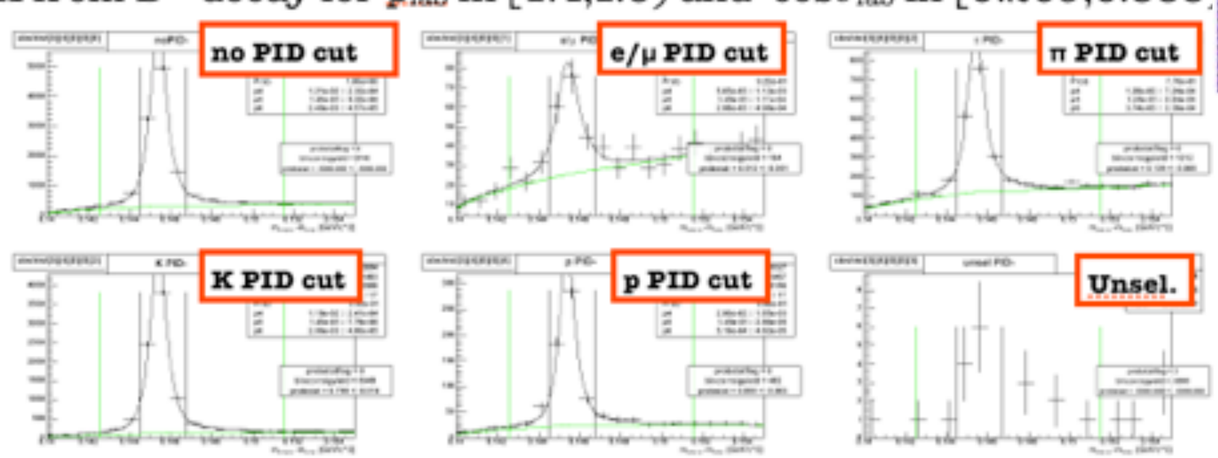
Detector performance depends on momentum and scattering angle!

$$P_{ij} \Rightarrow P_{ij}(p,\theta)$$

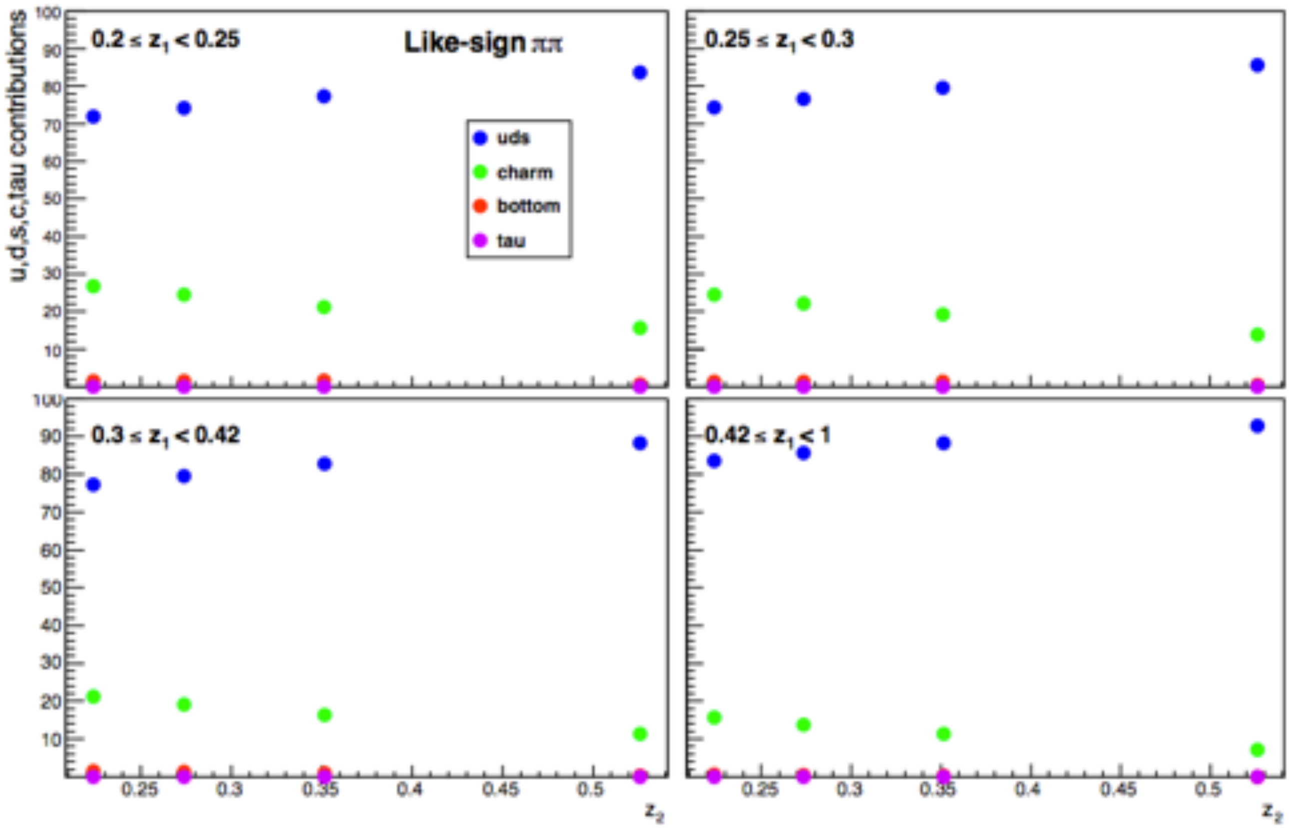




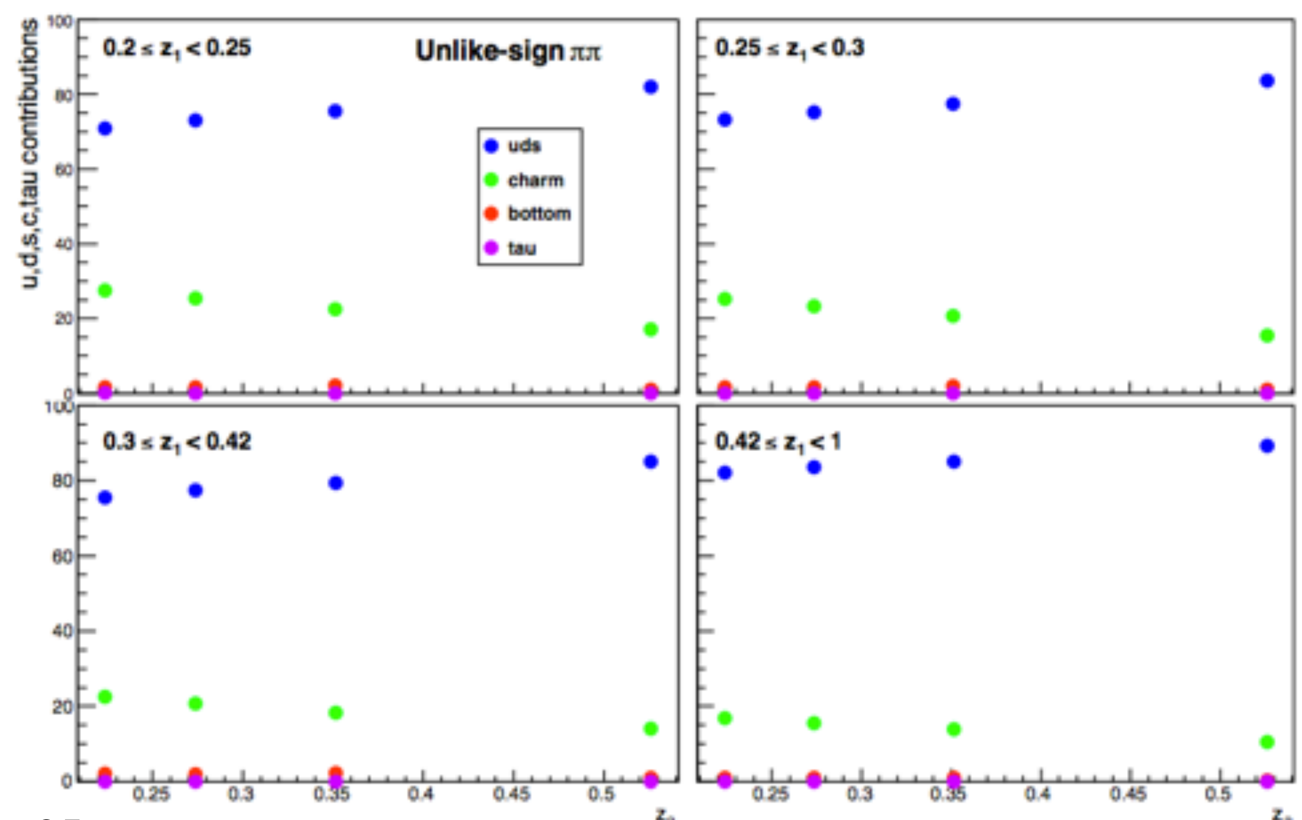
K from D* decay for $p_{\rm lab}$ in [1.4,1.6) and $\cos\theta_{\rm lab}$ in [0.209,0.355]



uds-charm-bottom-tau contributions

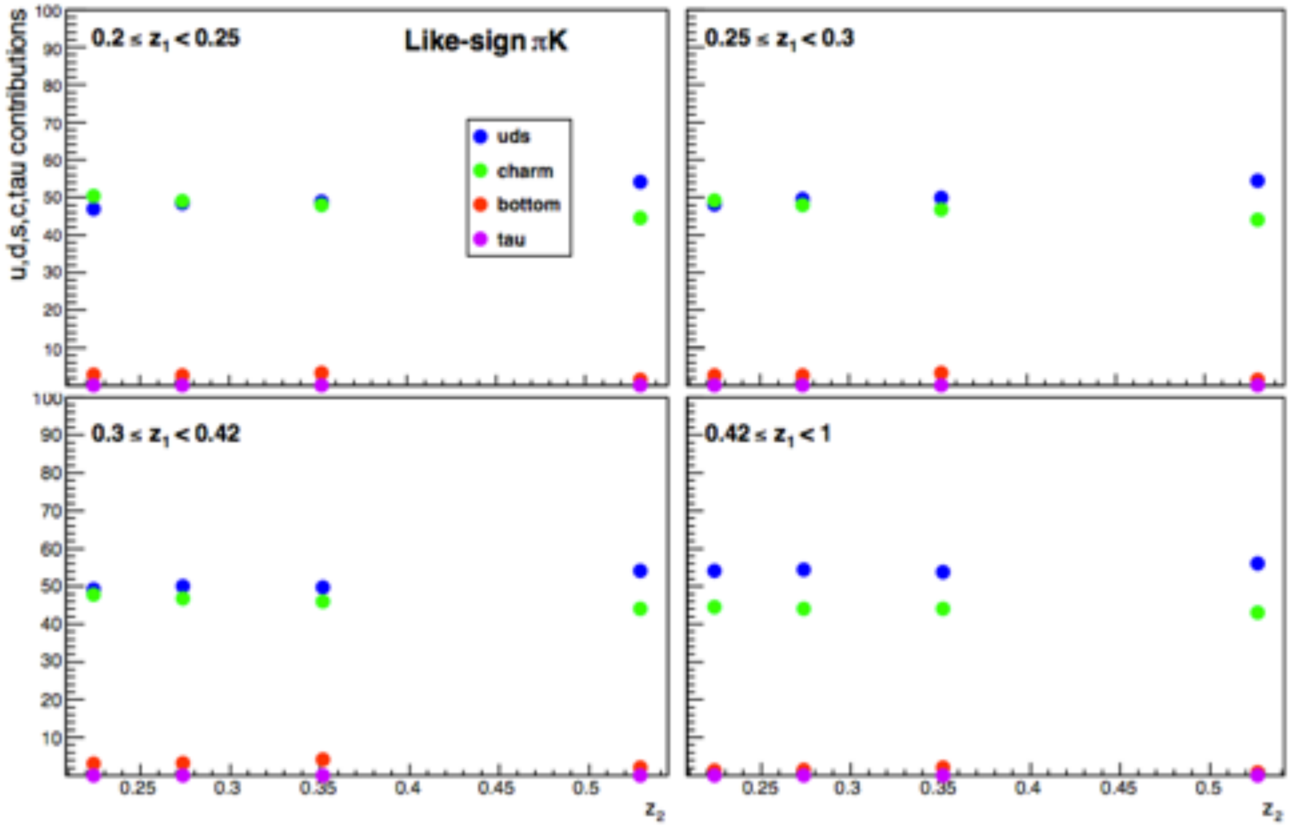


$\pi\pi$ couples

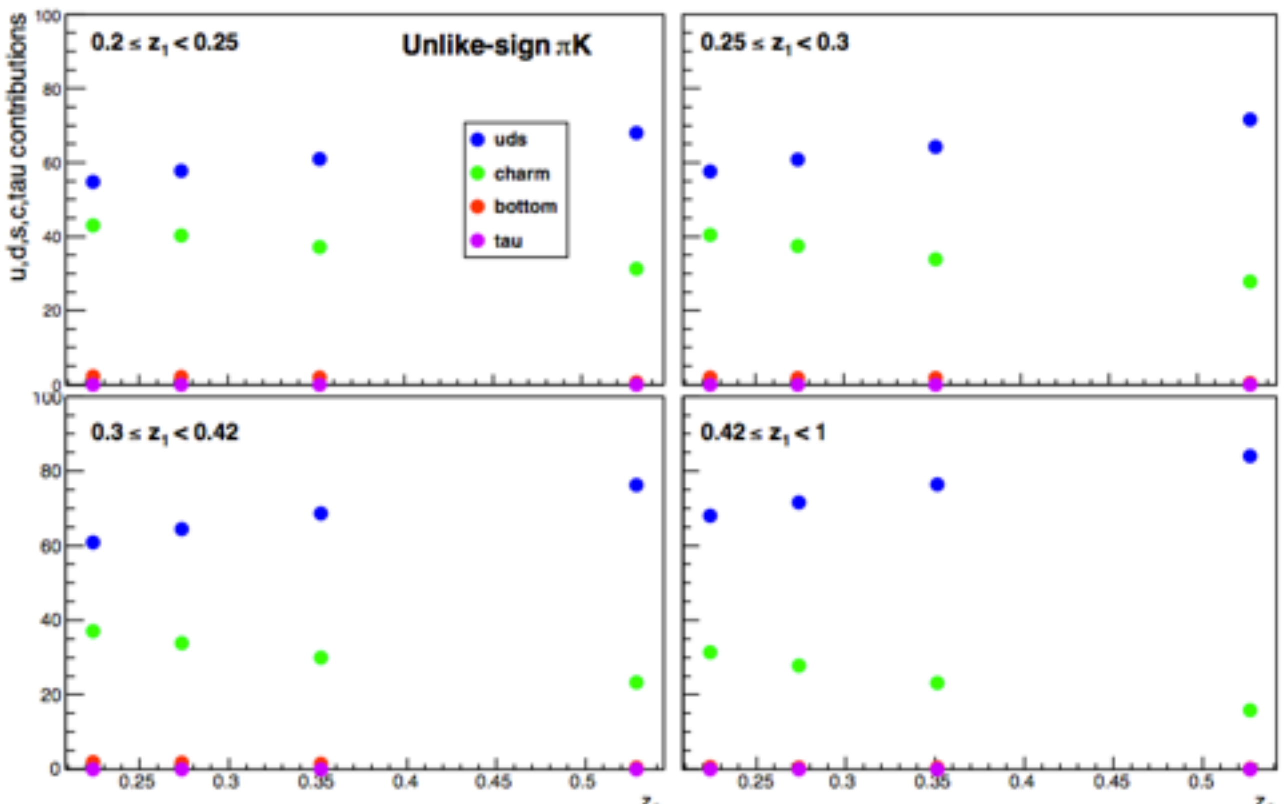


Published $\pi\pi$ studied a charm enhanced data and found charm contribute only as dilution
=> charm contribution corrected out

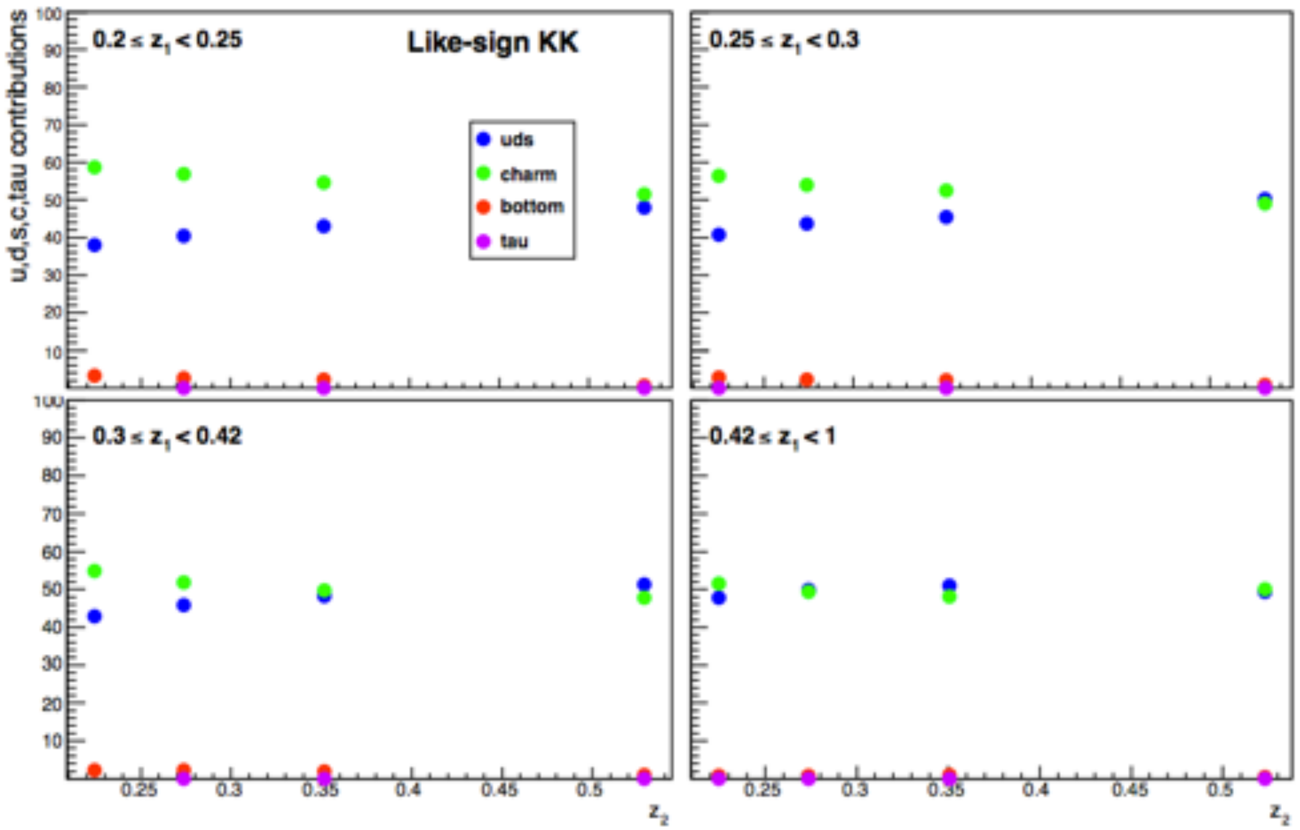
uds-charm-bottom-tau contributions



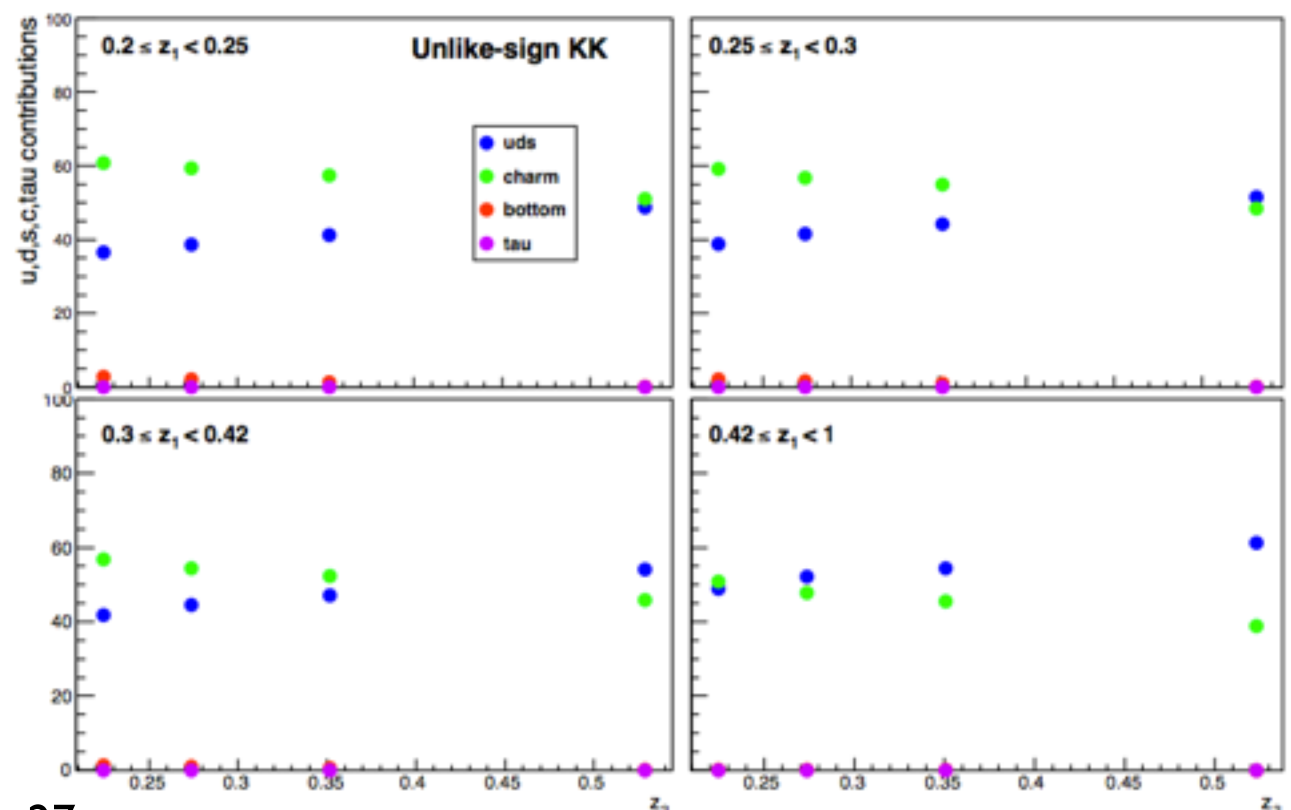
πK couples



uds-charm-bottom-tau contributions



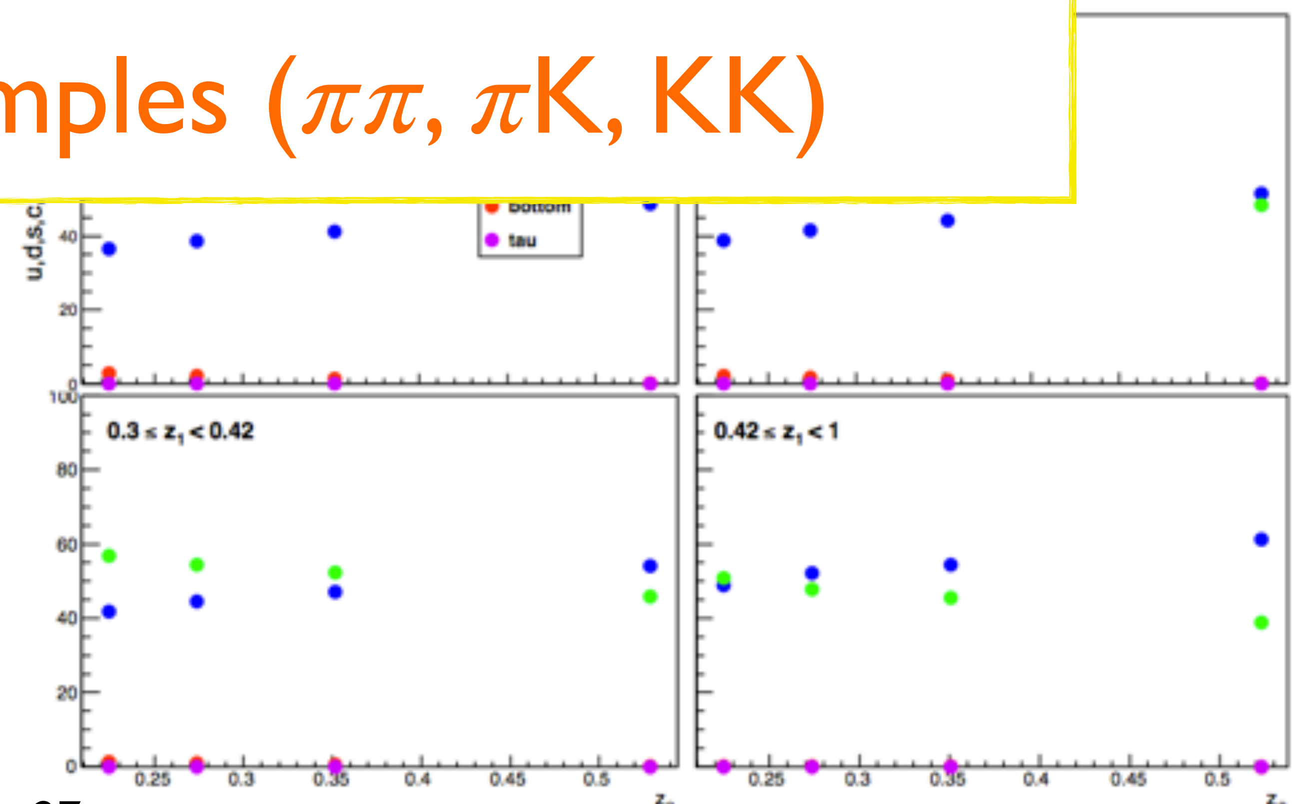
KK couples



uds-charm-bottom-tau contributions

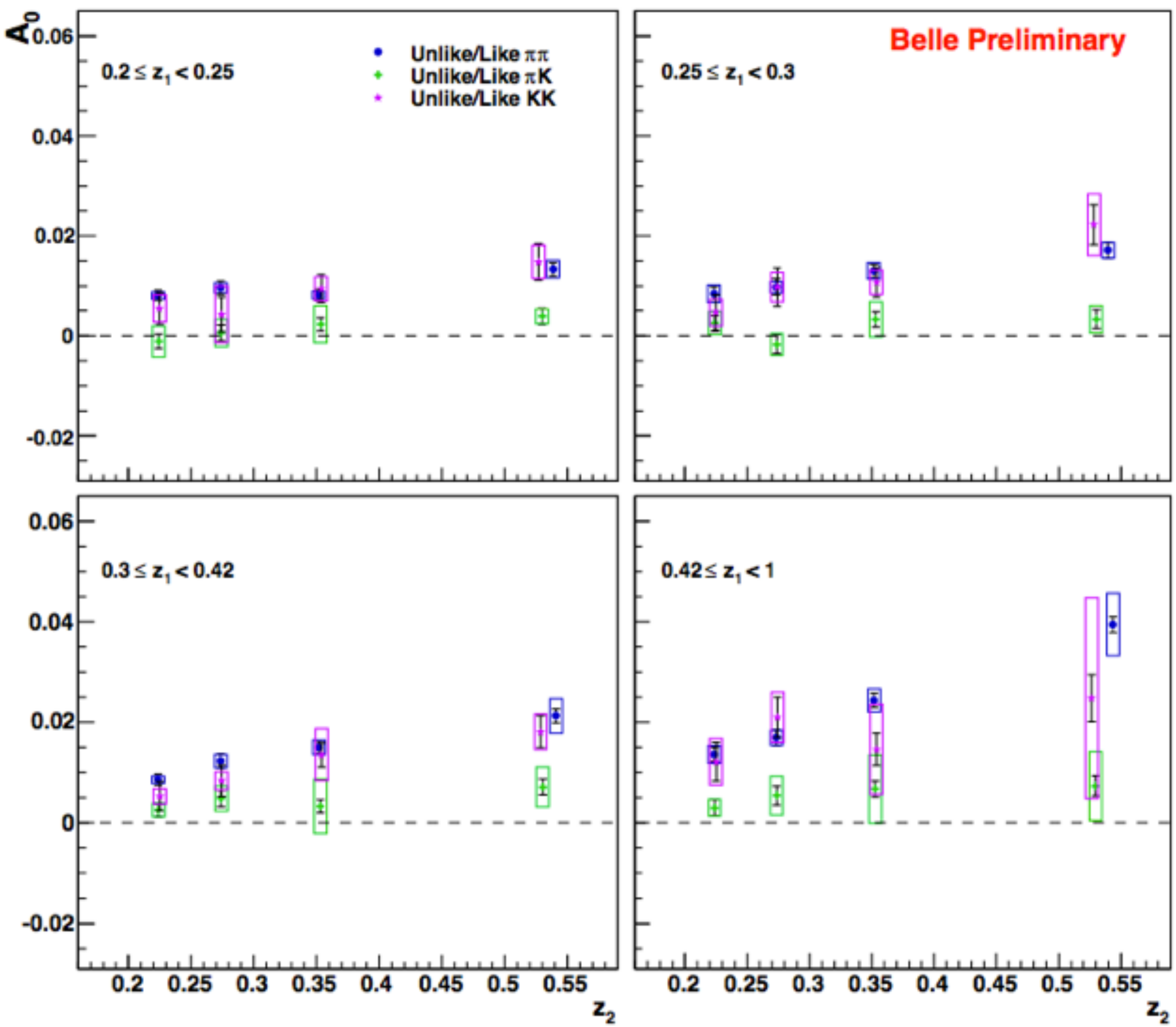
KK couples

For the moment charm contribution
is not being corrected out
in any of the samples ($\pi\pi$, πK, KK)



Collins asymmetries

ϕ_0 asymmetries

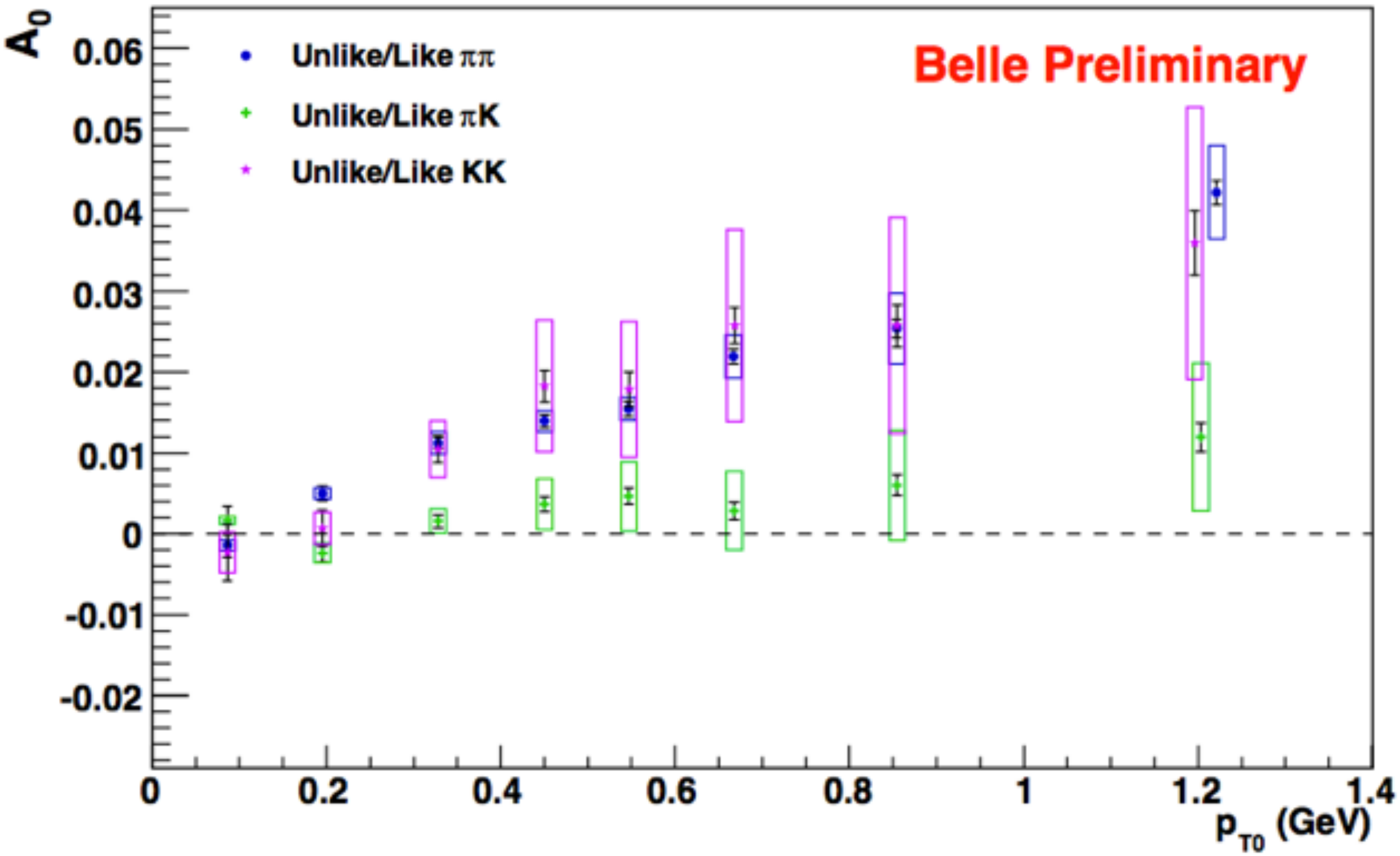


$\pi\pi$ => non-zero asymmetries, increase with z_1, z_2

πK => asymmetries compatible with zero

KK => non-zero asymmetries, increase with z_1,z_2 similar size of pion-pion

ϕ_0 asymmetries

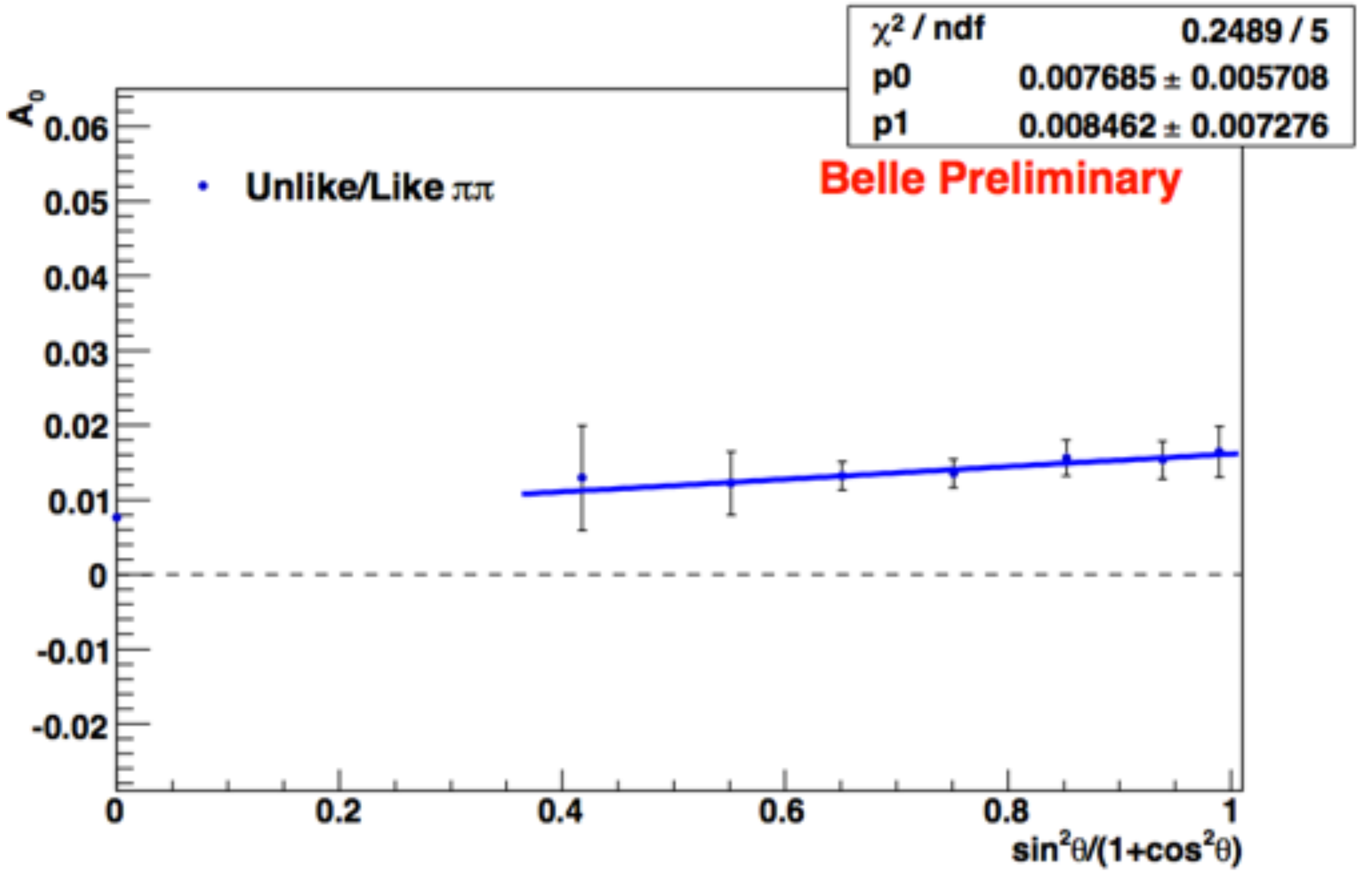


$\pi\pi$ => non-zero asymmetries, increase with z_1, z_2

πK => asymmetries compatible with zero

KK => non-zero asymmetries, increase with z_1,z_2 similar size of pion-pion

$\pi\pi$ versus $\sin^2\theta/(1+\cos^2\theta)$



$$A_0 = \frac{\sin^2\theta}{1+\cos^2\theta}\mathcal{F}\left[\frac{H_1^{\perp}(z_1)\bar{H}_1^{\perp}(z_2)}{D_1^{\perp}(z_1)\bar{D}_1^{\perp}(z_2)}\right]$$

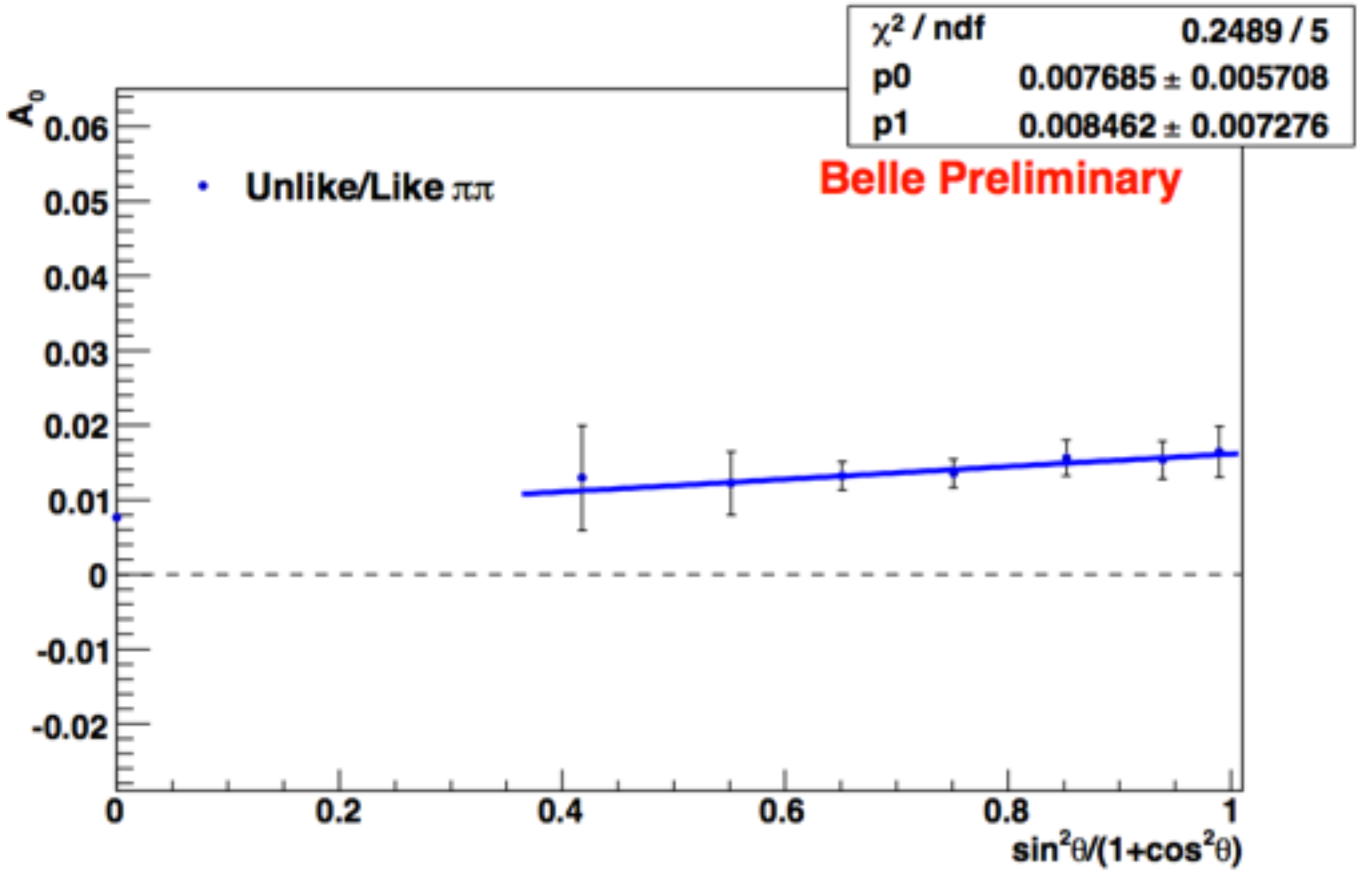
linear in $\sin^2\theta/(1+\cos^2\theta)$,

go to 0 for $\sin^2\theta/(1+\cos^2\theta) \to 0$

fit form: $p_0 + p_1 \sin^2\theta/(1+\cos^2\theta)$

p0 forced to 0

$\pi\pi$ versus $\sin^2\theta/(1+\cos^2\theta)$

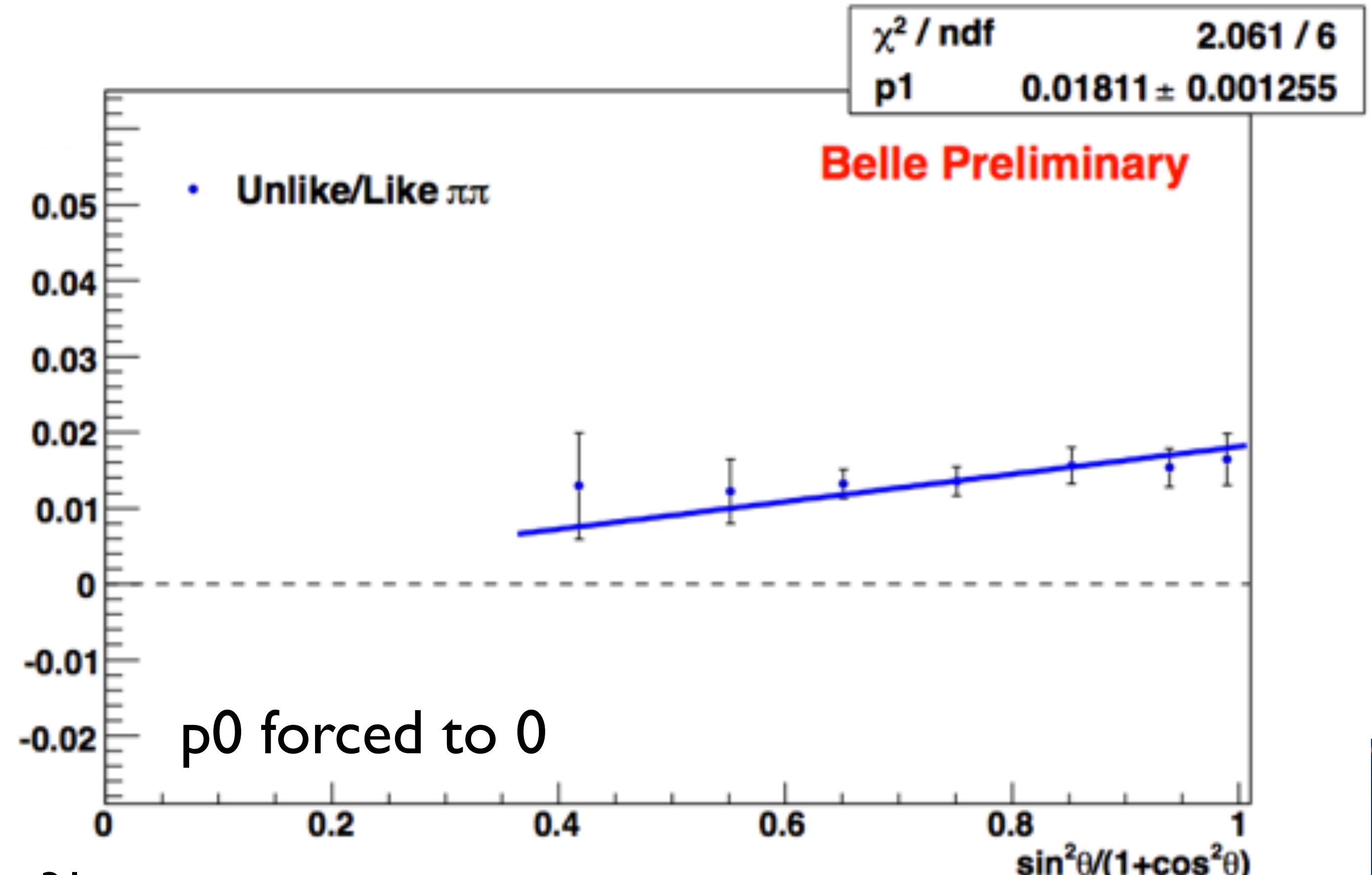


$$A_0 = \frac{\sin^2\theta}{1+\cos^2\theta}\,\mathcal{F}\left[\frac{H_1^\perp(z_1)\bar{H}_1^\perp(z_2)}{D_1^\perp(z_1)\bar{D}_1^\perp(z_2)}\right]$$

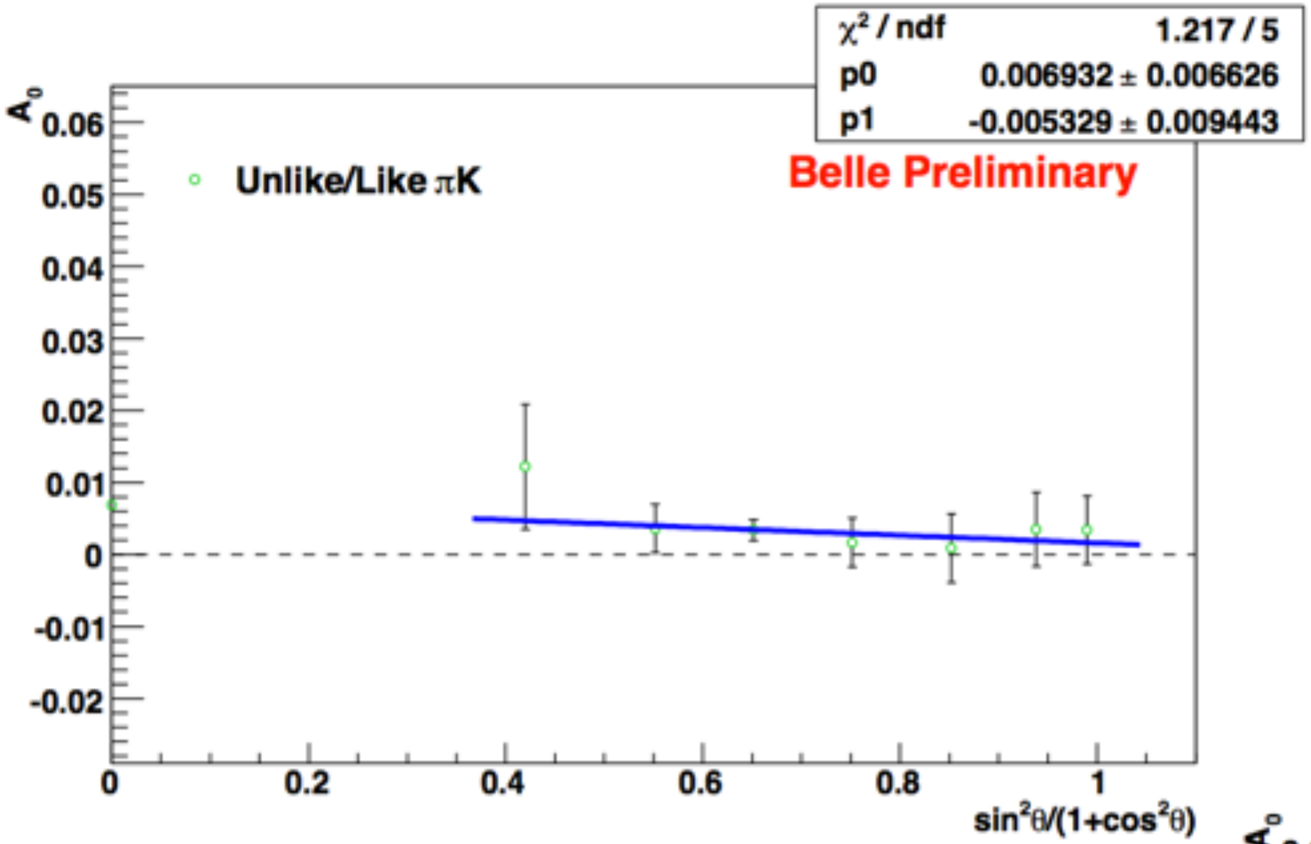
linear in $\sin^2\theta/(1+\cos^2\theta)$,

go to 0 for $\sin^2\theta/(1+\cos^2\theta) \rightarrow 0$

fit form: $p_0 + p_1 \sin^2\theta/(1+\cos^2\theta)$



πK versus $\sin^2\theta/(1+\cos^2\theta)$

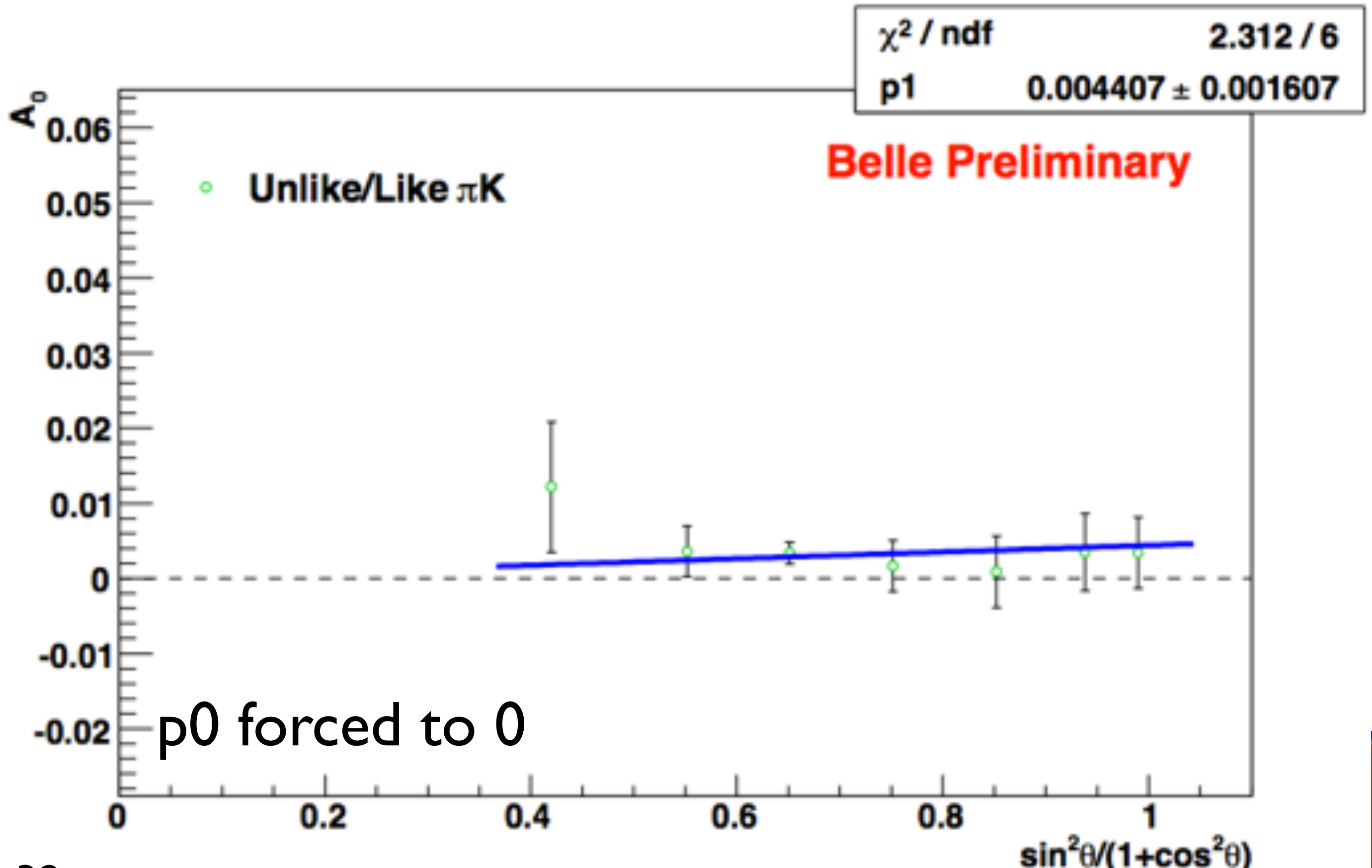


$$A_0 = \frac{\sin^2\theta}{1+\cos^2\theta}\mathcal{F}\left[\frac{H_1^\perp(z_1)\bar{H}_1^\perp(z_2)}{D_1^\perp(z_1)\bar{D}_1^\perp(z_2)}\right]$$

linear in $\sin^2\theta/(1+\cos^2\theta)$,

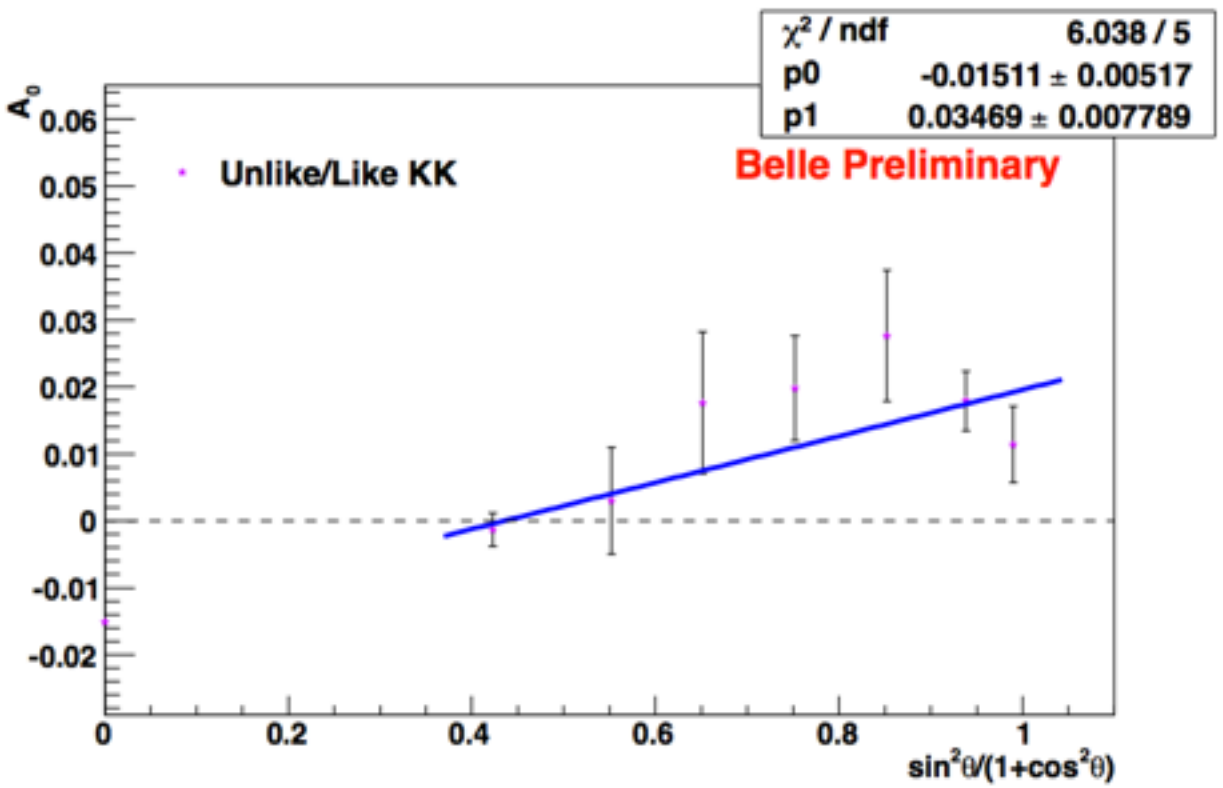
go to 0 for $\sin^2\theta/(1+\cos^2\theta) \rightarrow 0$

fit form: $p_0 + p_1 \sin^2\theta/(1+\cos^2\theta)$



p0 forced to 0

KK versus $\sin^2\theta/(1+\cos^2\theta)$

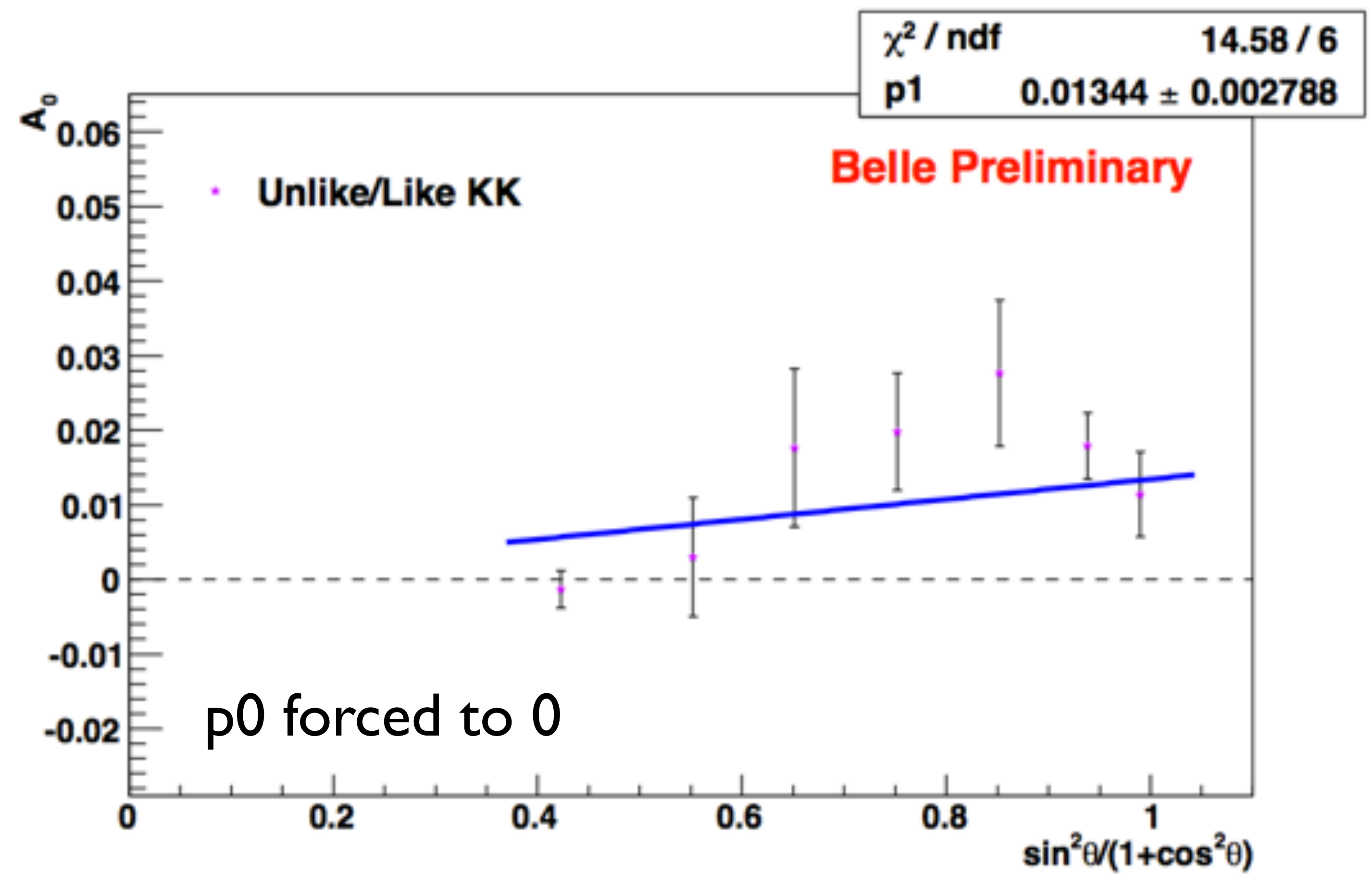


$$A_0 = \frac{\sin^2\theta}{1+\cos^2\theta}\,\mathcal{F}\left[\frac{H_1^{\perp}(z_1)\bar{H}_1^{\perp}(z_2)}{D_1^{\perp}(z_1)\bar{D}_1^{\perp}(z_2)}\right]$$

linear in $\sin^2\theta/(1+\cos^2\theta)$,

go to 0 for $\sin^2\theta/(1+\cos^2\theta) \to 0$

fit form: $p_0 + p_1 \sin^2\theta/(1+\cos^2\theta)$



Fragmentation contributions

$u,\ d \to \pi\ (u\bar{d},\ \bar{u}d)$

$$D^{fav} = D_u^{\pi^+} = D_d^{\pi^-} = D_{\bar{u}}^{\pi^-} = D_{\bar{d}}^{\pi^+}$$
$$D^{dis} = D_u^{\pi^-} = D_d^{\pi^+} = D_{\bar{u}}^{\pi^+} = D_{\bar{d}}^{\pi^-}$$

$s \to \pi\ (u\bar{d},\ \bar{u}d)$

$$D^{dis}_{s\to\pi} = D_s^{\pi^+} = D_s^{\pi^-} = D_{\bar{s}}^{\pi^+} = D_{\bar{s}}^{\pi^-}$$

$u,\ d \to K\ (u\bar{s},\ \bar{u}s)$

$$D^{fav}_{u\to K} = D_u^{K^+} = D_{\bar{u}}^{K^-}$$
$$D^{dis}_{u,d\to K} = D_u^{K^-} = D_{\bar{u}}^{K^+} = D_d^{K^+} = D_{\bar{d}}^{K^-} = D_d^{K^-} = D_{\bar{d}}^{K^+}$$

$s \to K\ (u\bar{s},\ \bar{u}s)$

$$D^{fav}_{s\to K} = D_s^{K^-} = D_{\bar{s}}^{K^+}$$
$$D^{dis}_{s\to K} = D_s^{K^+} = D_{\bar{s}}^{K^-}$$

In the end we are left with 7 possible fragmentation functions:

$$D^{fav},\ D^{dis},\ D^{dis}_{s\to\pi},\ D^{fav}_{u\to K},\ D^{dis}_{u,d\to K},\ D^{fav}_{s\to K},\ D^{dis}_{s\to K}$$

Assuming charm contribute only as a dilution

Fragmentation contributions

For pion-pion couples:

$$D^{\frac{U_{\pi\pi}}{L_{\pi\pi}}} \propto 1 \quad + \cos 2\phi_0 \frac{\sin^2\theta}{1+cos^2\theta} \left(\frac{5H_1^{fav} H_2^{fav} + 5H_1^{dis} H_2^{dis} + 2H_{1\,s\to\pi}^{dis} H_{2\,s\to\pi}^{dis}}{5D_1^{fav} D_2^{fav} + 5D_1^{dis} D_2^{dis} + 2D_{1\,s\to\pi}^{dis} D_{2\,s\to\pi}^{dis}} - \frac{5H_1^{fav} H_2^{dis} + 5H_1^{dis} H_2^{fav} + 2H_{1\,s\to\pi}^{dis} H_{2\,s\to\pi}^{dis}}{5D_1^{fav} D_2^{dis} + 5D_1^{dis} D_2^{fav} + 2D_{1\,s\to\pi}^{dis} D_{2\,s\to\pi}^{dis}} \right)$$

For pion-Kaon couples:

$$D^{\frac{U_{\pi K}}{L_{\pi K}}} \propto 1 + \cos 2\phi_0 \frac{\sin^2\theta}{1+cos^2\theta} \times$$

$$\left(\frac{4H_1^{fav} H_{2\,K}^{fav} + H_{1K}^{dis}(5H_2^{dis} + H_2^{fav}) + H_{2K}^{dis}(5H_1^{dis} + H_1^{fav}) + 4\,H_{1K}^{fav} H_2^{fav} + H_{1s\to\pi}^{dis}(H_{2s\to K}^{dis} + H_{2s\to K}^{fav}) + H_{2s\to\pi}^{dis}(H_{1s\to K}^{fav} + H_{1s\to K}^{dis})}{4D_1^{fav} D_{2\,K}^{fav} + D_{1K}^{dis}(5D_2^{dis} + D_2^{fav}) + D_{2K}^{dis}(5D_1^{dis} + D_1^{fav}) + 4\,D_{1K}^{fav} D_2^{fav} + D_{1s\to\pi}^{dis}(D_{2s\to K}^{dis} + D_{2s\to K}^{fav}) + D_{2s\to\pi}^{dis}(D_{1s\to K}^{fav} + D_{1s\to K}^{dis})} \right.$$

$$\left. \frac{H_{2\,K}^{dis}(5H_1^{fav} + H_1^{dis}) + 4\,H_{1\,K}^{fav} H_2^{dis} + 4H_1^{dis} H_{2K}^{fav} + H_{1K}^{dis}(5H_2^{fav} + H_2^{dis}) + H_{1s\to\pi}^{dis}(H_{2s\to K}^{fav} + H_{2s\to K}^{dis}) + (H_{1s\to K}^{dis} + H_{1s\to K}^{fav})H_{2s\to\pi}^{dis}}{D_{2\,K}^{dis}(5D_1^{fav} + D_1^{dis}) + 4\,D_{1\,K}^{fav} D_2^{dis} + 4D_1^{dis} D_{2K}^{fav} + D_{1K}^{dis}(5D_2^{fav} + D_2^{dis}) + D_{1s\to\pi}^{dis}(D_{2s\to K}^{fav} + D_{2s\to K}^{dis}) + (D_{1s\to K}^{dis} + D_{1s\to K}^{fav})D_{2s\to\pi}^{dis}} \right)$$

For Kaon-Kaon couples:

$$D^{\frac{U_{KK}}{L_{KK}}} \propto 1 \quad + \cos 2\phi_0 \frac{\sin^2\theta}{1+cos^2\theta} \left(\frac{4H_{1K}^{fav} H_{2\,K}^{fav} + 6H_{1\,K}^{dis} H_{2K}^{dis} + H_{1s\to K}^{dis} H_{2s\to K}^{dis} + H_{1s\to K}^{fav} H_{2s\to K}^{fav}}{4D_{1K}^{fav} D_{2\,K}^{fav} + 6D_{1\,K}^{dis} D_{2K}^{dis} + D_{1s\to K}^{dis} D_{2s\to K}^{dis} + D_{1s\to K}^{fav} D_{2s\to K}^{fav}} - \frac{4H_{1K}^{fav} H_{2\,K}^{dis} + 4H_{1\,K}^{dis} H_{2K}^{fav} + 2H_{1K}^{dis} H_{2K}^{dis} + H_{1s\to K}^{dis} H_{2s\to K}^{fav} + H_{1s\to K}^{fav} H_{2s\to K}^{dis}}{4D_{1K}^{fav} D_{2\,K}^{dis} + 4D_{1\,K}^{dis} D_{2K}^{fav} + 2D_{1K}^{dis} D_{2K}^{dis} + D_{1s\to K}^{dis} D_{2s\to K}^{fav} + D_{1s\to K}^{fav} D_{2s\to K}^{dis}} \right)$$

Fragmentation contributions

For pion-pion couples:

$$D^{\frac{U_{\pi\pi}}{L_{\pi\pi}}} \propto 1 \quad + \cos 2\phi_0 \frac{\sin^2\theta}{1+cos^2\theta} \left(\frac{5H_1^{fav} H_2^{fav} + 5H_1^{dis} H_2^{dis} + 2H^{dis}_{1\, s\to\pi} H^{dis}_{2\, s\to\pi}}{5D_1^{fav} D_2^{fav} + 5D_1^{dis} D_2^{dis} + 2D^{dis}_{1\, s\to\pi} D^{dis}_{2\, s\to\pi}} - \frac{5H_1^{fav} H_2^{dis} + 5H_1^{dis} H_2^{fav} + 2H^{dis}_{1\, s\to\pi} H^{dis}_{2\, s\to\pi}}{5D_1^{fav} D_2^{dis} + 5D_1^{dis} D_2^{fav} + 2D^{dis}_{1\, s\to\pi} D^{dis}_{2\, s\to\pi}} \right)$$

For pion-Kaon couples:

$$D^{\frac{U_{\pi K}}{L_{\pi K}}} \propto 1 + \cos 2\phi_0 \frac{\sin^2\theta}{1+cos^2\theta} \times$$

$$\left(\frac{4H_1^{fav} H^{fav}_{2\,K} + H^{dis}_{1K}(5H_2^{dis} + H_2^{fav}) + H^{dis}_{2K}(5H_1^{dis} + H_1^{fav}) + 4\,H^{fav}_{1K} H_2^{fav} + H^{dis}_{1s\to\pi}(H^{dis}_{2s\to K} + H^{fav}_{2s\to K}) + H^{dis}_{2s\to\pi}(H^{fav}_{1s\to K} + H^{dis}_{1s\to K})}{4D_1^{fav} D^{fav}_{2\,K} + D^{dis}_{1K}(5D_2^{dis} + D_2^{fav}) + D^{dis}_{2K}(5D_1^{dis} + D_1^{fav}) + 4\,D^{fav}_{1K} D_2^{fav} + D^{dis}_{1s\to\pi}(D^{dis}_{2s\to K} + D^{fav}_{2s\to K}) + D^{dis}_{2s\to\pi}(D^{fav}_{1s\to K} + D^{dis}_{1s\to K})} \right.$$

$$\left. \frac{H^{dis}_{2\,K}(5H_1^{fav} + H_1^{dis}) + 4\,H^{fav}_{1\,K} H_2^{dis} + 4H_1^{dis} H^{fav}_{2K} + H^{dis}_{1K}(5H_2^{fav} + H_2^{dis}) + H^{dis}_{1s\to\pi}(H^{fav}_{2s\to K} + H^{dis}_{2s\to K}) + (H^{dis}_{1s\to K} + H^{fav}_{1s\to K})H^{dis}_{2s\to\pi}}{D^{dis}_{2\,K}(5D_1^{fav} + D_1^{dis}) + 4\,D^{fav}_{1\,K} D_2^{dis} + 4D_1^{dis} D^{fav}_{2K} + D^{dis}_{1K}(5D_2^{fav} + D_2^{dis}) + D^{dis}_{1s\to\pi}(D^{fav}_{2s\to K} + D^{dis}_{2s\to K}) + (D^{dis}_{1s\to K} + D^{fav}_{1s\to K})D^{dis}_{2s\to\pi}} \right)$$

For Kaon-Kaon couples:

$$D^{\frac{U_{KK}}{L_{KK}}} \propto 1 \quad + \cos 2\phi_0 \frac{\sin^2\theta}{1+cos^2\theta} \left(\frac{4H^{fav}_{1K} H^{fav}_{2\,K} + 6H^{dis}_{1\,K} H^{dis}_{2K} + H^{dis}_{1s\to K} H^{dis}_{2s\to K} + H^{fav}_{1s\to K} H^{fav}_{2s\to K}}{4D^{fav}_{1K} D^{fav}_{2\,K} + 6D^{dis}_{1\,K} D^{dis}_{2K} + D^{dis}_{1s\to K} D^{dis}_{2s\to K} + D^{fav}_{1s\to K} D^{fav}_{2s\to K}} - \frac{4H^{fav}_{1K} H^{dis}_{2\,K} + 4H^{dis}_{1\,K} H^{fav}_{2K} + 2H^{dis}_{1K} H^{dis}_{2K} + H^{dis}_{1s\to K} H^{fav}_{2s\to K} + H^{fav}_{1s\to K} H^{dis}_{2s\to K}}{4D^{fav}_{1K} D^{dis}_{2\,K} + 4D^{dis}_{1\,K} D^{fav}_{2K} + 2D^{dis}_{1K} D^{dis}_{2K} + D^{dis}_{1s\to K} D^{fav}_{2s\to K} + D^{fav}_{1s\to K} D^{dis}_{2s\to K}} \right)$$

Not so easy! A full phenomenological study needed!

Summary & outlook

- ϕ_0 asymmetries
  - present similar features for $\pi\pi$ and KK couples
  - very small/compatible with zero for πK couples
  - for $\pi\pi$ and πK the $\sin^2\Theta/(1+\cos^2\Theta)$ dependence of asymmetries are not inconsistent with a linear dependence going to zero
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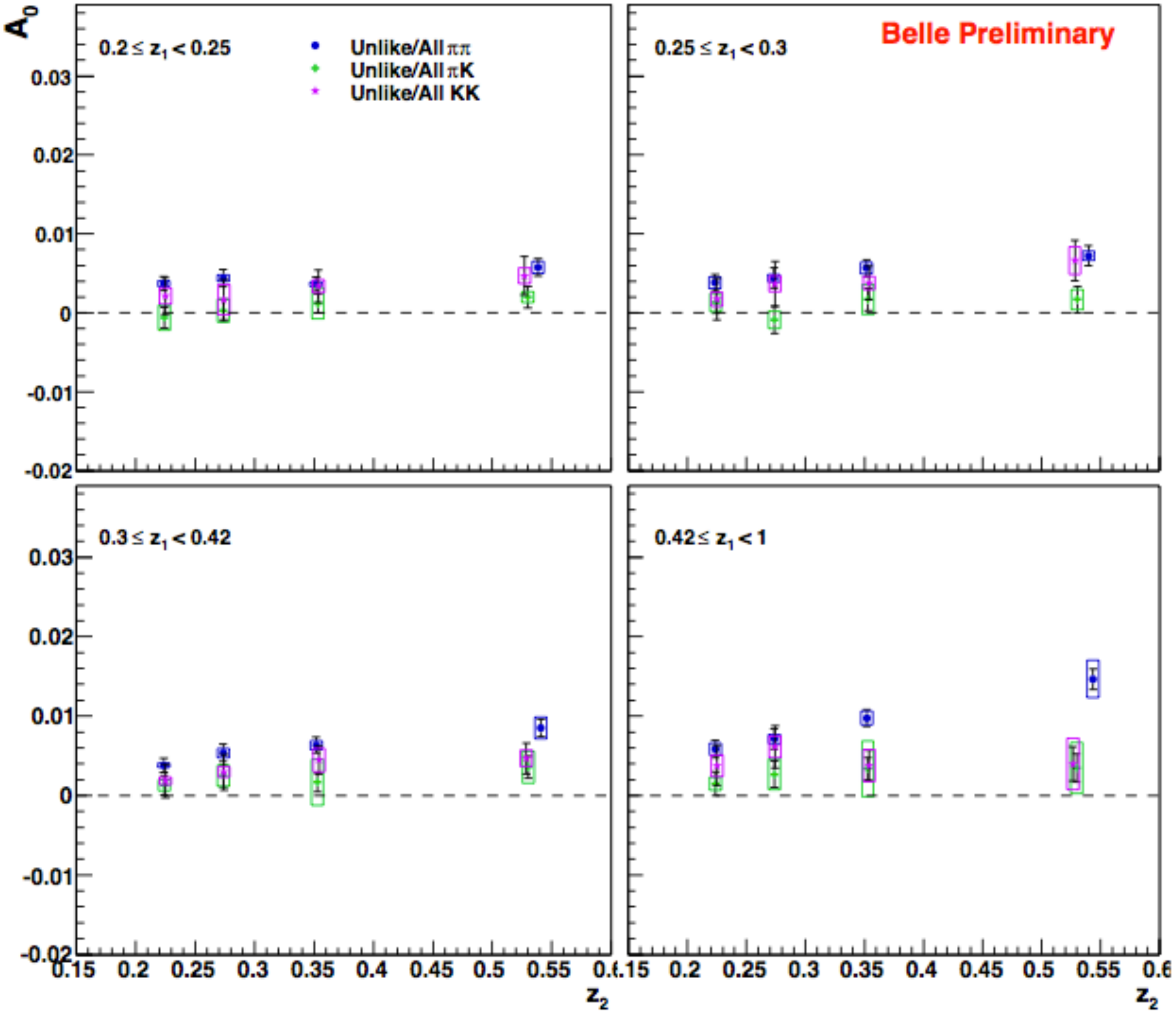
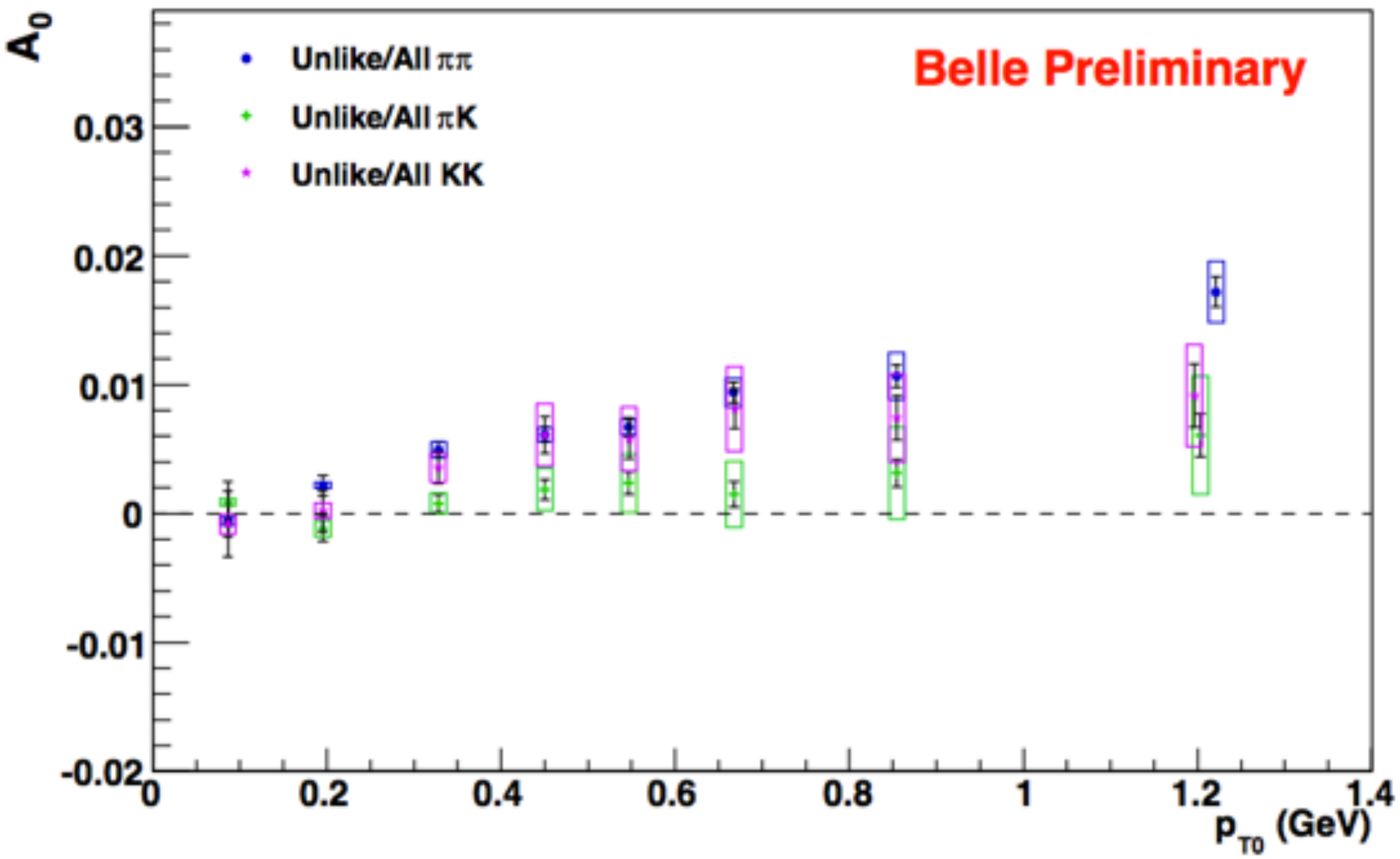
Stay tuned!

Backups

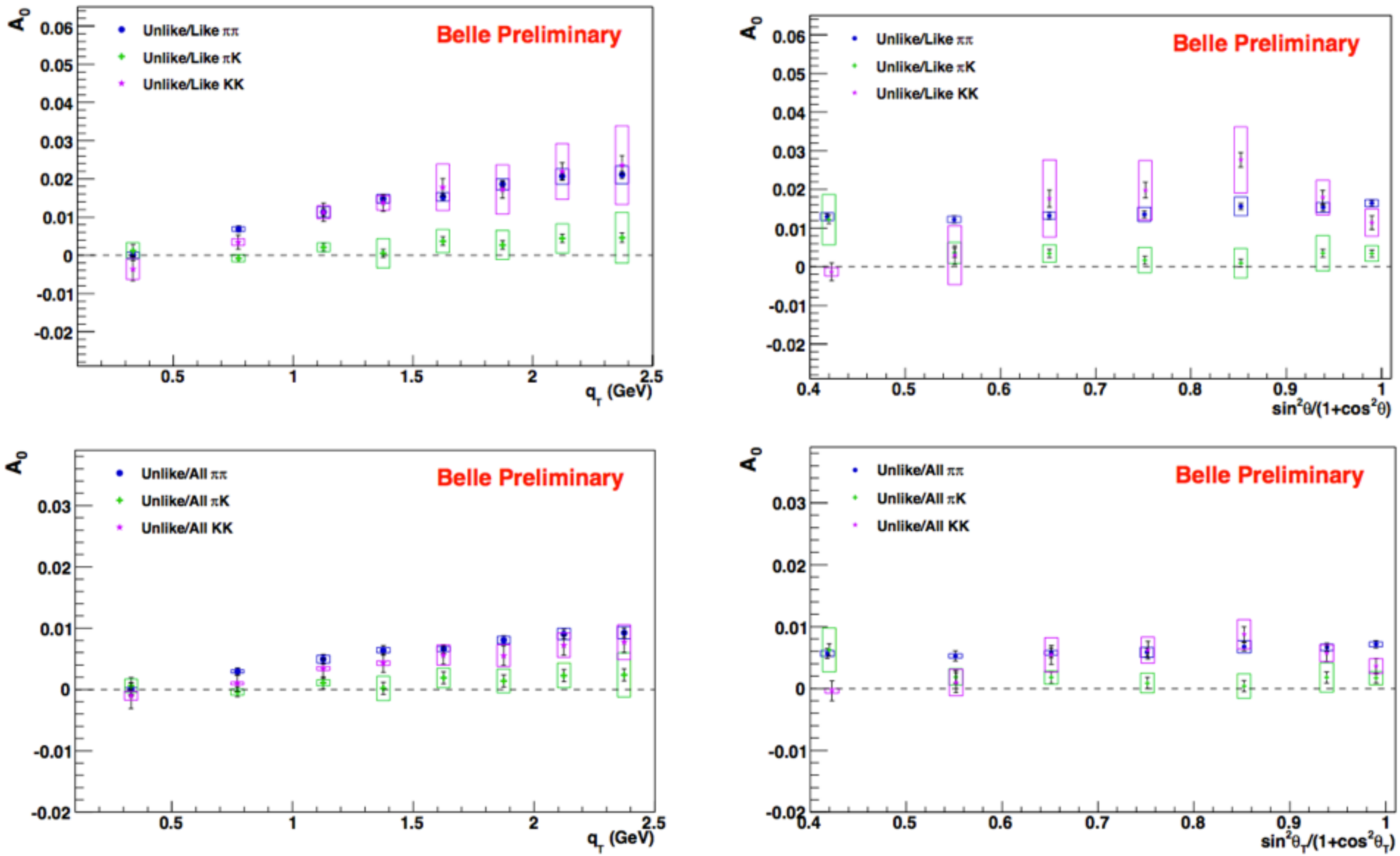
ϕ_0 asymmetries



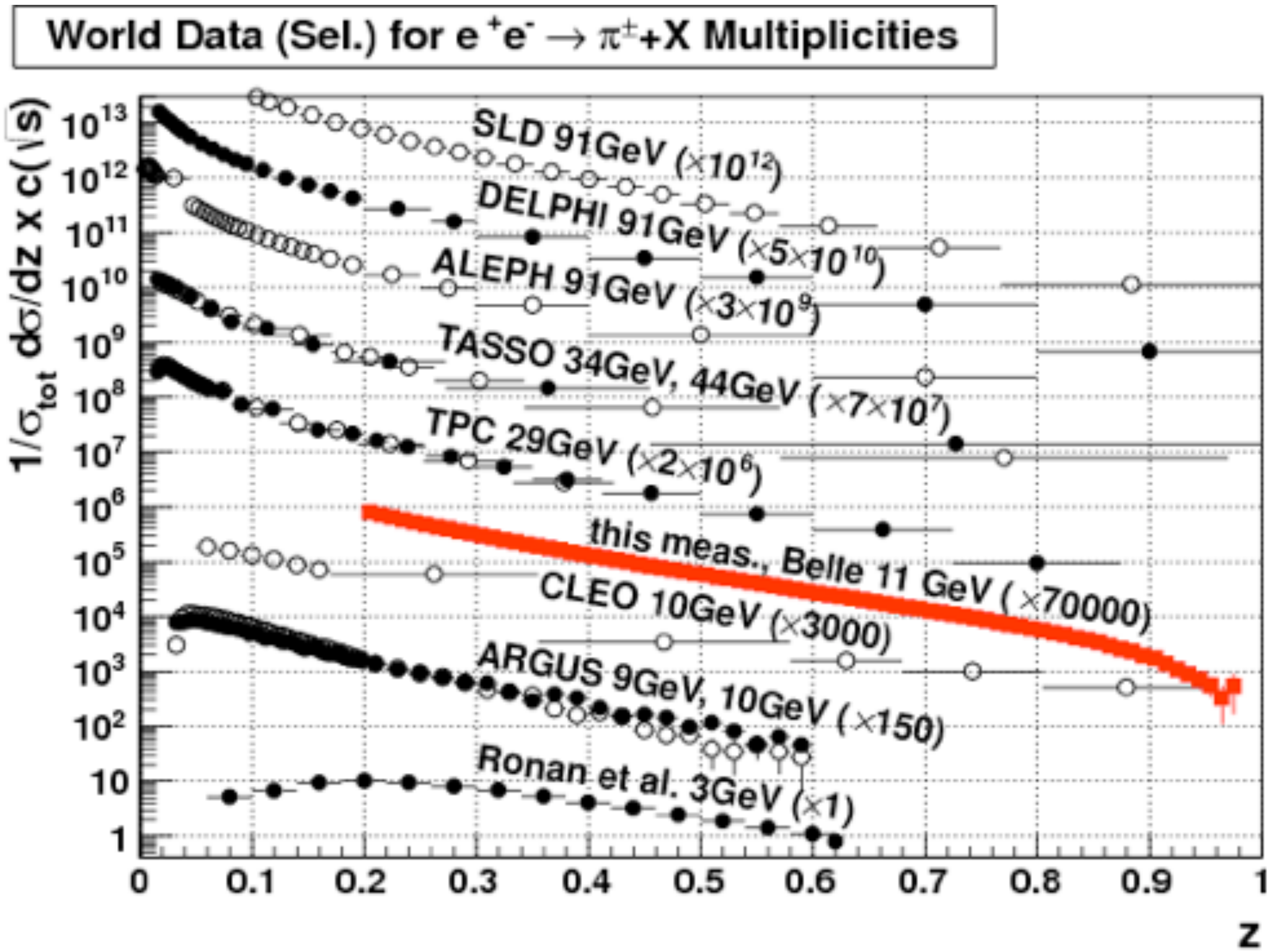
ϕ_0 asymmetries

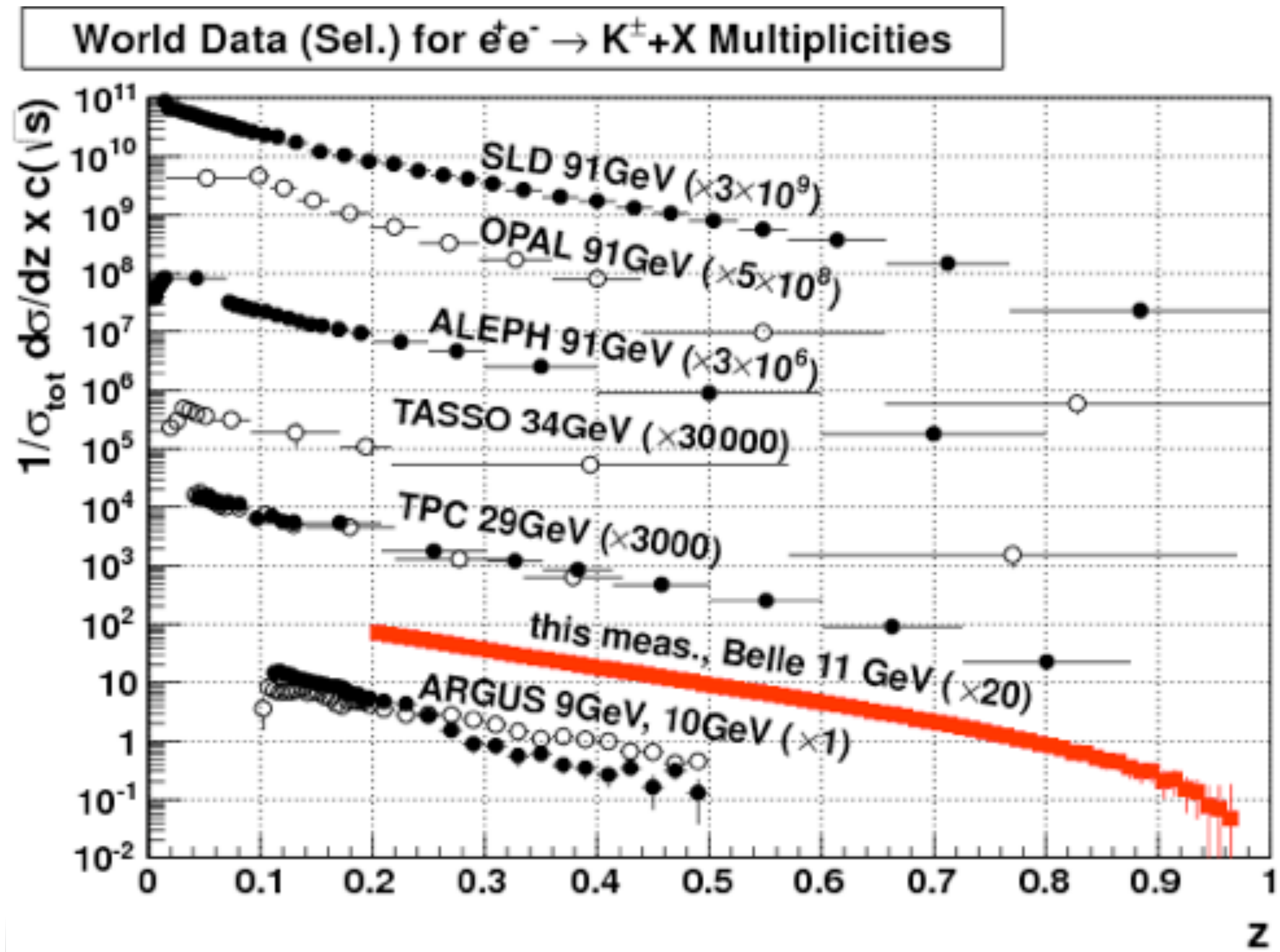


More ϕ_0 asymmetries



e+e- world data





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