Running of the charm quark mass

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- Heavy quark treatment in pQCD
- HERA charm data combination
- Measurement of the charm quark mass running
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The HERA $ep$ collisions experiments

HERA ring

- HERA accelerator was unique lepton-proton collider
- $e^\pm$ and $p$ were brought to collision with $\sqrt{s} = 318\,\text{GeV}$

H1 and ZEUS experiments collected $0.5\ fb^{-1}$ per experiment

H1 and ZEUS detectors

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Deep Inelastic Scattering

Deep Inelastic Scattering diagram.

\[ Q^2 = - (k - k')^2 \] – boson virtuality,

\[ x = \frac{Q^2}{2P \cdot (k - k')} \] – Bjorken x

\[ y = \frac{P \cdot (k - k')}{P \cdot k} \] – inelasticity

\( Q^2 > 1 \text{GeV}^2 : \text{DIS} \)
HERA measurements of DIS cross sections cover wide range of $Q^2$. Can be used to determine sets of quark and gluon momentum distributions in the proton.
Measurements of parton density functions at HERA

Previous HERAI inclusive $F_2$ measurements for neutral current and charged current reactions have been combined and used in global QCD fit of parton density functions (known as PDF).

PDFs are used in factorization theorem and essential for theoretical predictions.
Charm production in $ep$ scattering

Charm contribution in DIS $\rightarrow 10\%-30\%$.

- At HERA charm mainly produced by boson-gluon fusion (sensitive to the gluon density in the proton)
- Double-differential cross-sections of heavy quarks production can be described in terms of the reduced cross sections $\sigma_{red.}(Q^2, x)$:

$$
\frac{d\sigma^{q\bar{q}}(e^{\pm}p)}{dx dQ^2} = \frac{2\pi \alpha^2}{xQ^4} \left[1 + (1 - y)^2\right] \sigma_{red.}^{q\bar{q}}(Q^2, x)
$$
Heavy Quark Scheme in QCD Analysis defines treatment of heavy flavours in perturbative expansion. Factorisation:

\[ F_2^V(x, Q^2) = \sum_{i=q,\bar{q},g} \int_x^1 dz \times C_2^{i,V}(\frac{x}{z}, Q^2, \mu_F, \mu_R, \alpha_S) \times f_i(z, \mu_F, \mu_R) \]

where \( i \) - number of active flavours in the proton

- **Fixed Flavour Number Scheme (FFNS)**: heavy quarks are massive, produced in boson-gluon fusion as just they a final-state particle and not as a parton
**Quark mass definitions**

**Pole quark mass**
- Based on (unphysical) concept of quark being a free parton
- Pole mass is ambiguous up to corrections of $O(\Lambda_{QCD})$

**Running quark mass ($\overline{MS}$)**
- $\overline{MS}$ (minimal subtraction scheme) mass definition $m(\mu_R)$ realizes running mass (scale dependence)
- Renormalization group equation (mass anomalous dimension $\gamma$)

$$
\left( \mu^2_R \frac{\delta}{\delta \mu^2_R} + \beta(\alpha_s) \frac{\delta}{\delta \alpha_s} \right) m(\mu_R) = \gamma(\alpha_s) m(\mu_R)
$$
Running quark mass

Can HERA measure charm quark mass at different scales?
9 different reduced charm cross sections measurements were combined in recent H1/ZEUS paper (Eur. Phys. J. C73 (2013) 2311).

Combination showed good consistency of data with $\frac{\chi^2}{n_{dof}} = 62/103$. 
Fully accounts for correlations. Significant reduction of uncertainties.
Good description of combined data ABM predictions (FFNS using $\overline{MS}$ mass definition.

- We can use it to measure charm quark mass.
Charm mass was measured performing mass scan in global QCD fit of NC+CC+charm in FFNS with $\overline{MS}$ charm mass definition.

$$m_{c^{MS}}(m_c) = 1.26 \pm 0.05_{\text{exp}} \pm 0.03_{\text{mod}} \pm 0.02_{\text{param}} \pm 0.02_{\alpha_s} \text{ GeV}$$

PDG value: $m_{c^{MS}}(m_c) = 1.275 \pm 0.025 \text{ GeV}$
Charm quark mass running demonstration needs a measurement of $m_c$ at different scales. 
→ use charm data from different $Q^2$ regions.
Technically OpenQCDRad (code that is used for QCD predictions) returns $m_c(m_c)$ value. However physically we measure $m_c$ at the scale of used data, so translation back from $m_c(m_c)$ to actually measured $m_c(\mu)$ required.
Measurement of the charm quark mass running

Combined data are grouped into 6 subsets choosing bins close in $Q^2$
Scale was chosen as:
\[ \mu = \sqrt{Q^2 + 4m_c^2}, \]
Measurement of the charm quark mass running

Procedure is similar to that of [Eur.Phys.J.C73 (2013) 2311] with following modifications:

1) PDFs for different $m_c(m_c)$ are extracted using the inclusive HERA-I data only
2) Calculate charm cross sections for each of the subsets based on these PDFs for different $m_c(m_c)$
3) Perform a $\chi^2$ scan for each of the subsets
Uncertainties of $m_c$ measurement

Fit uncertainty
- Was estimated by taking $\Delta \chi^2 = 1$ (dominant uncertainty)

Parametrisation
- Adding extra parameter in the PDF parametrisation

Model uncertainty
- Variation of the strangness suppression factor
- Lower cut on $Q^2$ for inclusive data
- The evolution starting scale
- The $b$-quark mass

Theory
- Variation of $\alpha_s$

- Variation of the factorisation and renormalization scales of heavy quarks by factor 2
  (were found to be one of the dominant systematical uncertainty)
Measurement of the charm mass at different scales

Red point - $m_c(m_c)$ PDG value (1.275), its uncert. represented by band, Black points - this measurement $m_c(m_c)(Q^2)$, full uncertainties with scales variation (outer error bars) and without (inner error bars)
Measurement of the charm quark mass running

From $m_c(m_c)$ it was translated back to $m_c(\mu)$ by 1-loop formula:

$$m_c(\mu) = m_c(m_c) \left( \frac{\alpha_s(\mu)}{\pi} \right)^{\frac{1}{\beta_0}} \left( \frac{\alpha_s(m_c)}{\pi} \right)^{\frac{1}{\beta_0}}$$

Where $\beta_0$ for $N_f=3$ is $\frac{9}{4}$

$$\mu = \sqrt{Q^2 + 4m_c^2},$$

This formula is the same that is used in the QCD fit (OpenQCDRad).


$Q^2$ was chosen to be log average between $Q^2$ of used bins
Red square - $m_c(m_c)$ PDG value (1.275), uncert. represented by band, Black points - this measurement $m_c(\mu)$, $\mu = \sqrt{Q^2 + 4m_c^2}$
First measurement of the charm quark mass running has been presented based on HERA data.

Measured charm masses are in good agreement with expected QCD evolution based on the PDG value.

It shows internal consistency of using running mass treatment in QCD.
Charm mass measurement

- $\chi^2$ mass scan had been performed by fitting charm data in FFNS ABM($\overline{MS}$) scheme (OPENQCDRAD program) using HeraFitter package with following setup:
  - FFNS ABM (running mass)
  - Evolution starting scale set to $Q_0=1.4 \text{ GeV}^2$
  - PDF parametrisation with 13 parameters
  - H12011 $\chi^2$ function definition
  - $\alpha_s(M_z)=0.105$
  - Data below $Q^2 = 3.5 \text{ GeV}^2$ removed
  - $m_b(m_b)$ was set to 4.75
  - Renormalization and factorization scale was set to $\sqrt{Q^2 + 4m_q^2}$