Perturbative contributions to rare B-meson decays

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1. Introduction
2. $B_{s(d)} \longrightarrow \ell^+\ell^-$
3. $\bar{B} \longrightarrow X_s\gamma$
4. $b \longrightarrow s\,\ell^+\ell^-$
5. Summary

B-meson or Kaon decays occur at low energies, at scales $\mu \ll M_W$.

We pass from the full theory of electroweak interactions to an effective theory by removing the high-energy degrees of freedom, i.e. integrating out the W-boson and all the other particles with $m \sim M_W$.

$$\mathcal{L}_{\text{(full EW}\times\text{QCD)}} \longrightarrow \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QED}\times\text{QCD}}\left(\begin{smallmatrix}\text{quarks} \neq t \\ \text{\& leptons}\end{smallmatrix}\right) + N \sum_n C_n(\mu)\, Q_n$$

Q_n – **local interaction terms (operators),** C_n – **coupling constants (Wilson coefficients)**

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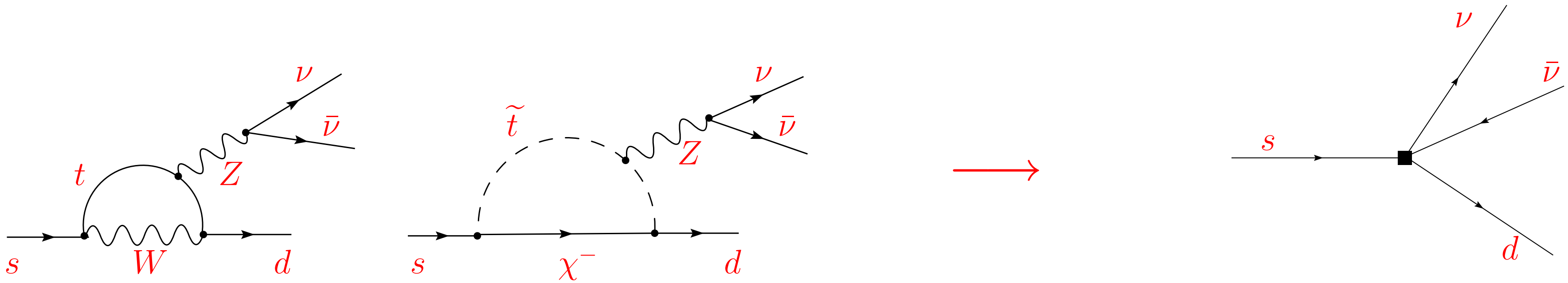
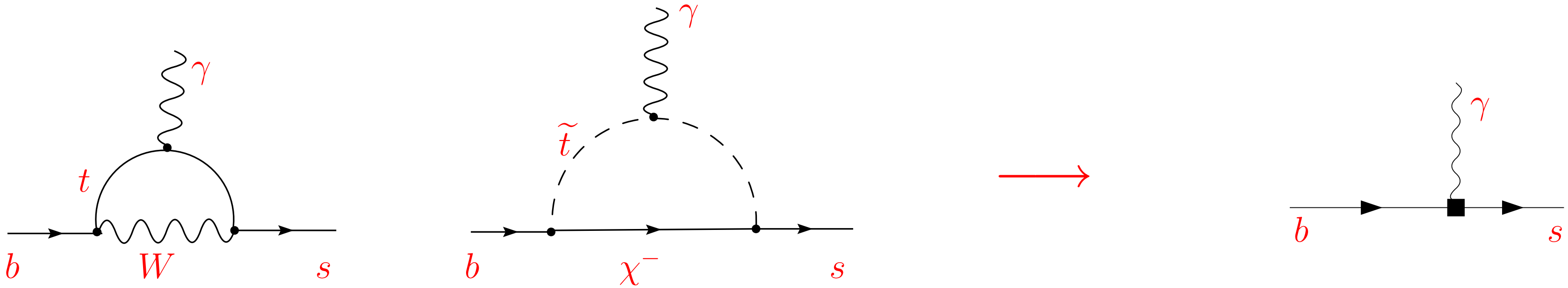
Information on the electroweak-scale physics is encoded in the values of $\boldsymbol{C_i(\mu)}$ **, e.g.,**





This is a modern version of the Fermi theory for weak interactions. It is "nonrenormalizable" in the traditional sense but actually renormalizable. It is also predictive because all the C_i are calculable, and only a finite number of them is necessary at each given order in the (external momenta)/M_W expansion.

Advantages: **Resummation of** $\left(\alpha_s \ln \frac{M_W^2}{\mu^2}\right)^n$ **using renormalization group, easier account for symmetries.**

$B_s \to \mu^+\mu^-$ — the flavour physics highlight of the LHC

- It is a strongly suppressed, loop-generated process in the SM. Its average time-integrated branching ratio (with final-state photon bremsstrahlung included) reads:

$$\overline{\mathcal{B}}_{\rm SM} = (3.65 \pm 0.23) \times 10^{-9}$$

[C. Bobeth, M. Gorbahn, T. Hermann, MM, E. Stamou and M. Steinhauser, Phys. Rev. Lett. 112 (2014) 101801]

- It is very sensitive to new physics even in models with Minimal Flavour Violation (MFV). Enhancements by orders of magnitude are possible even when constraints from all the other measurements are taken into account.

- It has a clear experimental signature: **PEAK** in the dimuon invariant mass.

- Recently measured branching ratios

$$\overline{\mathcal{B}}_{\rm exp} = \begin{cases} \left(2.9\,^{+1.1}_{-1.0}\right) \times 10^{-9}, & \text{LHCb [Phys. Rev. Lett. 111 (2013) 101805]} \\ \left(3.0\,^{+1.0}_{-0.9}\right) \times 10^{-9}, & \text{CMS [Phys. Rev. Lett. 111 (2013) 101804]} \end{cases}$$

Combined: $\overline{\mathcal{B}}_{\rm exp} = (\,2.9 \pm 0.7\,) \times 10^{-9}$ [CMS-PAS-BPH-13-007, LHCb-CONF-2013-012]

Operators (dim 6) that matter for $B_s \to \mu^+\mu^-$ read

$$Q_A = \left(\bar{b}\gamma^\alpha\gamma_5 s\right)\left(\bar{\mu}\gamma_\alpha\gamma_5\mu\right)$$ – the only relevant one in the SM

$$Q_{S(P)} = \left(\bar{b}\gamma_5 s\right)\left(\bar{\mu}(\gamma_5)\mu\right) = \frac{i(\bar{b}\gamma^\alpha\gamma_5 s)\partial_\alpha(\bar{\mu}(\gamma_5)\mu)}{m_b+m_s} + \boxed{E} + \boxed{T}$$

E: vanishing by EOM

T: total derivative

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E: vanishing by EOM

T: total derivative

Necessary non-perturbative input: $\langle 0|\bar{b}\gamma^\alpha\gamma_5 s|B_s(p)\rangle = ip^\alpha f_{B_s}$

Recent lattice determinations of the B_s-meson decay constant:

$f_{B_s} =$

225.0(4.0) MeV,	HPQCD (r),	arXiv:1110.4510
224.0(5.0) MeV,	HPQCD (nr),	arXiv:1302.2644
234.0(6.0) MeV,	ROME,	arXiv:1212.0301
242.0(9.5) MeV,	FNAL/MILC,	arXiv:1112.3051
232.0(10) MeV,	ETM,	arXiv:1107.1441
219.0(12) MeV,	ALPHA,	arXiv:1210.6524
235.4(12) MeV,	RBC/UKQCD,	arXiv:1404.4670
224.0(14) MeV,	ALPHA,	arXiv:1404.3590

Flavour Lattice Averaging Group (FLAG), arXiv:1310.8555 gives

$$f_{B_s} = 227.7(4.5)\ \text{MeV}.$$

Average time-integrated branching ratio:

$$\overline{\mathcal{B}}(B_s \to \mu^+\mu^-) = \frac{|N|^2 M_{B_s}^3 f_{B_s}^2}{8\pi\Gamma_H^s}\beta\left(\left|rC_A - uC_P\right|^2 F_P + \left|u\beta C_S\right|^2 F_S\right) + \mathcal{O}(\alpha_{em}),$$

where $N = \frac{V_{tb}^* V_{ts} G_F^2 M_W^2}{\pi^2}$, $r = \frac{2m_\mu}{M_{B_s}}$, $\beta = \sqrt{1-r^2}$, $u = \frac{M_{B_s}}{m_b+m_s}$,

$$F_P = 1 - \frac{\Delta\Gamma^s}{\Gamma_L^s}\sin^2\left[\tfrac{1}{2}\phi_s^{\rm NP} + \arg(rC_A - uC_P)\right] \xrightarrow{\text{SM CP}} 1 ,$$

$$F_S = 1 - \frac{\Delta\Gamma^s}{\Gamma_L^s}\cos^2\left[\tfrac{1}{2}\phi_s^{\rm NP} + \arg C_S\right] \xrightarrow{\text{SM CP}} \frac{\Gamma_H^s}{\Gamma_L^s}$$

derived following [K. de Bruyn *et al.*, Phys. Rev. Lett. 109 (2012) 041801]

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derived following [K. de Bruyn *et al.*, Phys. Rev. Lett. 109 (2012) 041801]

Numerical data: $M_{B_s} = 5.36677(24)$ GeV, $\Delta M_{B_s} = 11.64(5)$ meV (milli-eV), $\tau_{\rm decay}/T_{\rm oscillation} \simeq 28$

In the limit of no CP-violation, mass eigenstates are CP eigenstates:

Heavier, CP-odd: $B_s^H = \frac{1}{\sqrt{2}}(B_s + \bar{B}_s)$, annihilated by $\bar{b}\gamma_5 s + \bar{s}\gamma_5 b$, lifetime $\tau_H = 1.615(21)$ ps

Lighter, CP-even: $B_s^L = \frac{1}{\sqrt{2}}(B_s - \bar{B}_s)$, annihilated by $\bar{b}\gamma_5 s - \bar{s}\gamma_5 b$, lifetime $\tau_L = 1.516(11)$ ps

Our interactions in this limit are all CP-even:

$$Q_A + Q_A^\dagger = \left[(\bar{b}\gamma^\alpha\gamma_5 s) + (\bar{s}\gamma^\alpha\gamma_5 b)\right](\bar{\mu}\gamma_\alpha\gamma_5\mu)$$

$$Q_P + Q_P^\dagger = \left[(\bar{b}\gamma_5 s) + (\bar{s}\gamma_5 b)\right](\bar{\mu}\gamma_5\mu)$$

} annihilate B_s^H, produce CP-odd dimuons

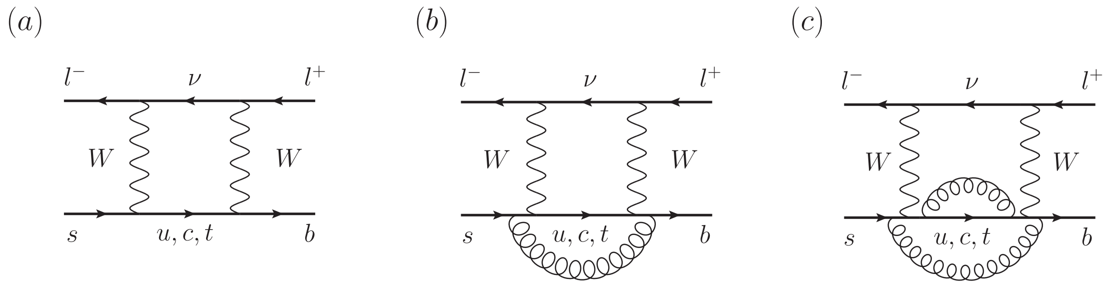
$$Q_S + Q_S^\dagger = \left[(\bar{b}\gamma_5 s) - (\bar{s}\gamma_5 b)\right](\bar{\mu}\mu)$$

} annihilates B_s^L, produces CP-even dimuons

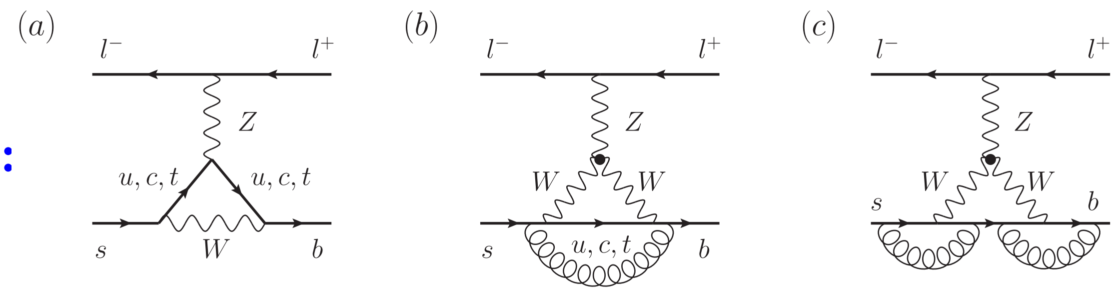
Evaluation of the NNLO QCD matching corrections in the SM

[T. Hermann, MM, M. Steinhauser, JHEP 1312 (2013) 097]

W-boxes:
(1LPI)



Z-penguins:
(1LPI)

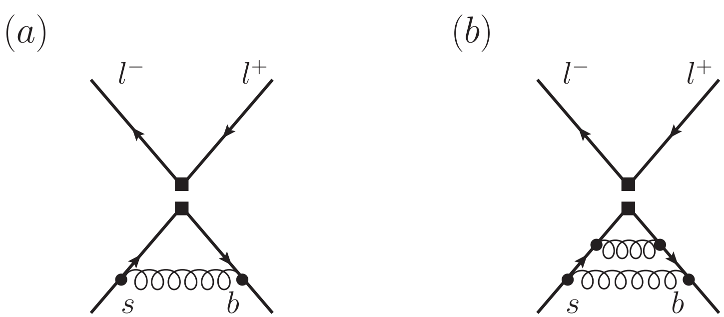


All the external momenta and light masses have been set to zero $\Rightarrow$ No loop diagrams on the effective theory side.

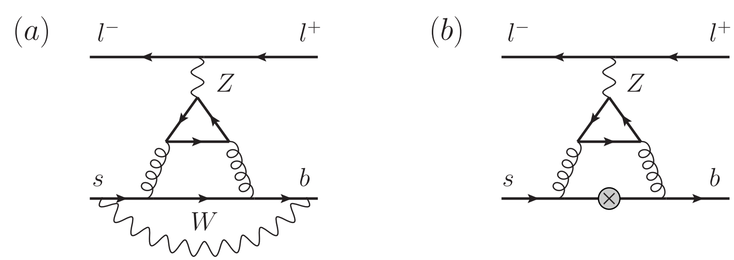
Subtleties: (i) counterterms with finite parts $\sim \bar{b}_L \not{D} s_L$

(ii) evanescent operators: $E_B = (\bar{b}\gamma_\nu\gamma_\rho\gamma_\sigma\gamma_5 s)(\bar{\mu}\gamma^\sigma\gamma^\rho\gamma^\nu\gamma_5\mu) - 4(\bar{b}\gamma_\alpha\gamma_5 s)(\bar{\mu}\gamma^\alpha\gamma_5\mu)$

$E_T = \mathrm{Tr}\,(\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^\alpha\gamma_5)(\bar{b}\gamma_\nu\gamma_\rho\gamma_\sigma s)(\bar{\mu}\gamma_\alpha\gamma_5\mu) + 24(\bar{b}\gamma_\alpha\gamma_5 s)(\bar{\mu}\gamma^\alpha\gamma_5\mu)$



Renormalization of E_B

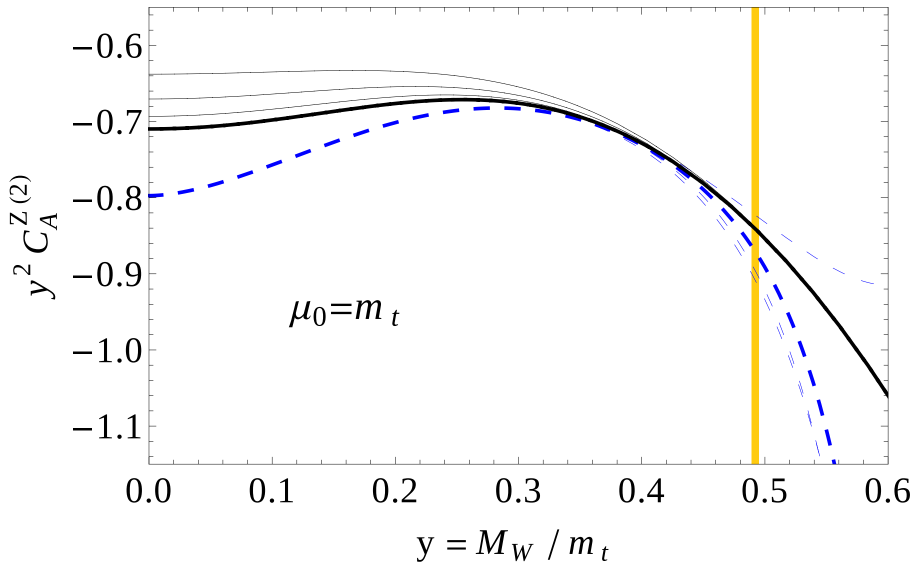


Diagrams generating E_T

Perturbative series for the Wilson coefficient at $\mu = \mu_0 \sim m_t, M_W$:

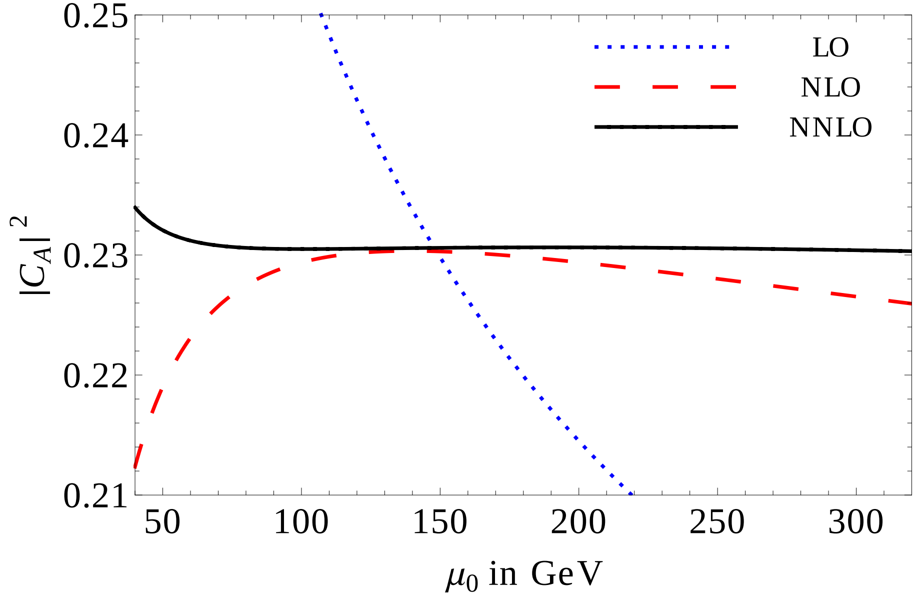
$$C_A(\mu_0) = C_A^{(0)}(\mu_0) + \frac{\alpha_s}{4\pi} C_A^{(1)}(\mu_0) + \left(\frac{\alpha_s}{4\pi}\right)^2 C_A^{(2)}(\mu_0) + \frac{\alpha_{em}}{4\pi} \Delta_{\mathrm{EW}} C_A(\mu_0) + \ldots$$

The top quark mass is $\overline{\mathrm{MS}}$-renormalized at μ_0 with respect to QCD, and on shell with respect to the EW interactions. Both α_s and α_{em} are $\overline{\mathrm{MS}}$-renormalized at μ_0 in the effective theory.



$$C_A^{(n)} = C_A^{W,(n)} + C_A^{Z,(n)}$$

To deal with single-scale tadpole integrals, we expand around $y = 1$ (solid lines) and around $y = 0$ (dashed lines), where $y = M_W/m_t$. The expansions reach $(1-y^2)^{16}$ and y^{12}, respectively. The blue band indicates the physical region.



Matching scale dependence of $|C_A|^2$ gets significantly reduced. The plot corresponds to $\Delta_{\mathrm{EW}} C_A(\mu_0) = 0$. However, with our conventions for m_t and the global normalization, μ_0-dependence is due to QCD only.

NNLO fit (with $\Delta_{\mathrm{EW}} C_A(\mu_0) = 0$):

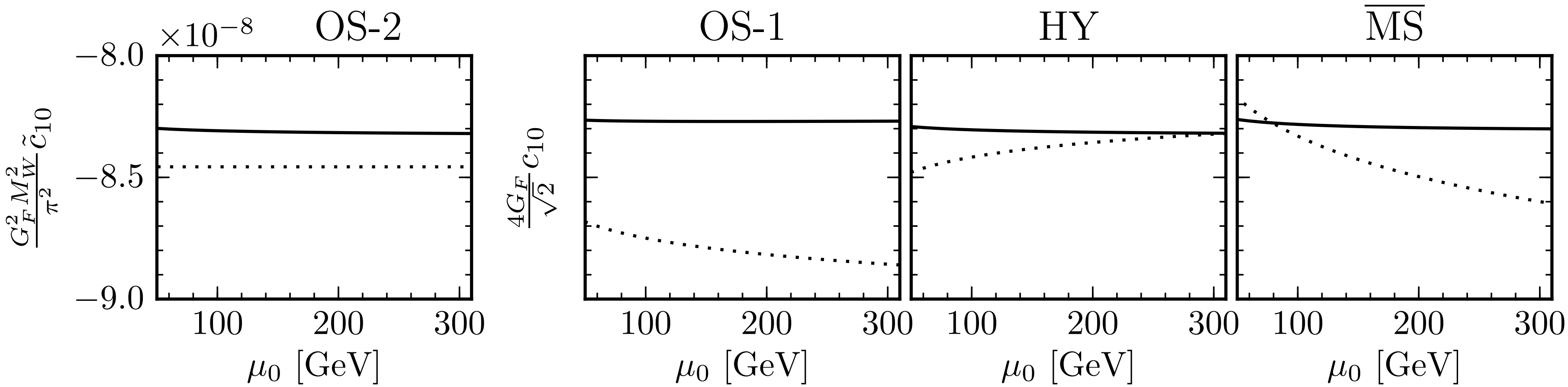
$$C_A = 0.4802 \left(\frac{M_t}{173.1}\right)^{1.52} \left(\frac{\alpha_s(M_Z)}{0.1184}\right)^{-0.09} + \mathcal{O}(\alpha_{em})$$

Evaluation of the NLO EW matching corrections in the SM

[C. Bobeth, M. Gorbahn, E. Stamou, Phys. Rev. D 89 (2014) 034023]

Method: similar to the NNLO QCD case. Two-loop integrals with three mass scales are present.

Dependence of the final result on μ_0 in various renormalization schemes (dotted – LO, solid – NLO):



In all the four plots: no QCD corrections to C_A included, $m_t(m_t)$ w.r.t. QCD used.

OS-2 scheme: Global normalization factor in $\mathcal{L}_{\rm eff}$ set to $N = V_{tb}^* V_{ts}\, G_F^2 M_W^2/\pi^2$

Masses at the LO renormalized on-shell w.r.t. EW interactions (including M_W in N)

Plotted quantity: $-2C_A\, G_F^2 M_W^2/\pi^2$ in GeV^{-2}

NLO EW matching correction to the BR: -3.7%

other schemes: Global normalization factor in $\mathcal{L}_{\rm eff}$ set to $4V_{tb}^* V_{ts}\, G_F/\sqrt{2}$

At the LO, $\alpha_{em}(\mu_0)$ used

$\overline{\rm MS}$: Masses and $\sin^2\theta_W$ renormalized at μ_0

OS-1: Masses as in OS-2, $\sin^2\theta_W$ on-shell

HY (hybrid): Masses as in OS-2, $\sin^2\theta_W$ as in $\overline{\rm MS}$.

SM predictions for all the branching ratios $\overline{\mathcal{B}}_{q\ell} \equiv \overline{\mathcal{B}}(B_q \to \ell^+\ell^-)$

[C. Bobeth, M. Gorbahn, T. Hermann, MM, E. Stamou, M. Steinhauser, PRL 112 (2014) 101801]

$$
\begin{aligned}
\overline{\mathcal{B}}_{se} \times 10^{14} &= (8.54 \pm 0.13)\, R_{t\alpha}\, R_s = 8.54 \pm 0.55, \\
\overline{\mathcal{B}}_{s\mu} \times 10^{9} &= (3.65 \pm 0.06)\, R_{t\alpha}\, R_s = 3.65 \pm 0.23, \qquad (\text{LHCb \& CMS}: \ 2.9 \pm 0.7) \\
\overline{\mathcal{B}}_{s\tau} \times 10^{7} &= (7.73 \pm 0.12)\, R_{t\alpha}\, R_s = 7.73 \pm 0.49, \\
\overline{\mathcal{B}}_{de} \times 10^{15} &= (2.48 \pm 0.04)\, R_{t\alpha}\, R_d = 2.48 \pm 0.21, \\
\overline{\mathcal{B}}_{d\mu} \times 10^{10} &= (1.06 \pm 0.02)\, R_{t\alpha}\, R_d = 1.06 \pm 0.09, \qquad (\text{LHCb \& CMS}: \ 3.6^{+1.6}_{-1.4}) \\
\overline{\mathcal{B}}_{d\tau} \times 10^{8} &= (2.22 \pm 0.04)\, R_{t\alpha}\, R_d = 2.22 \pm 0.19,
\end{aligned}
$$

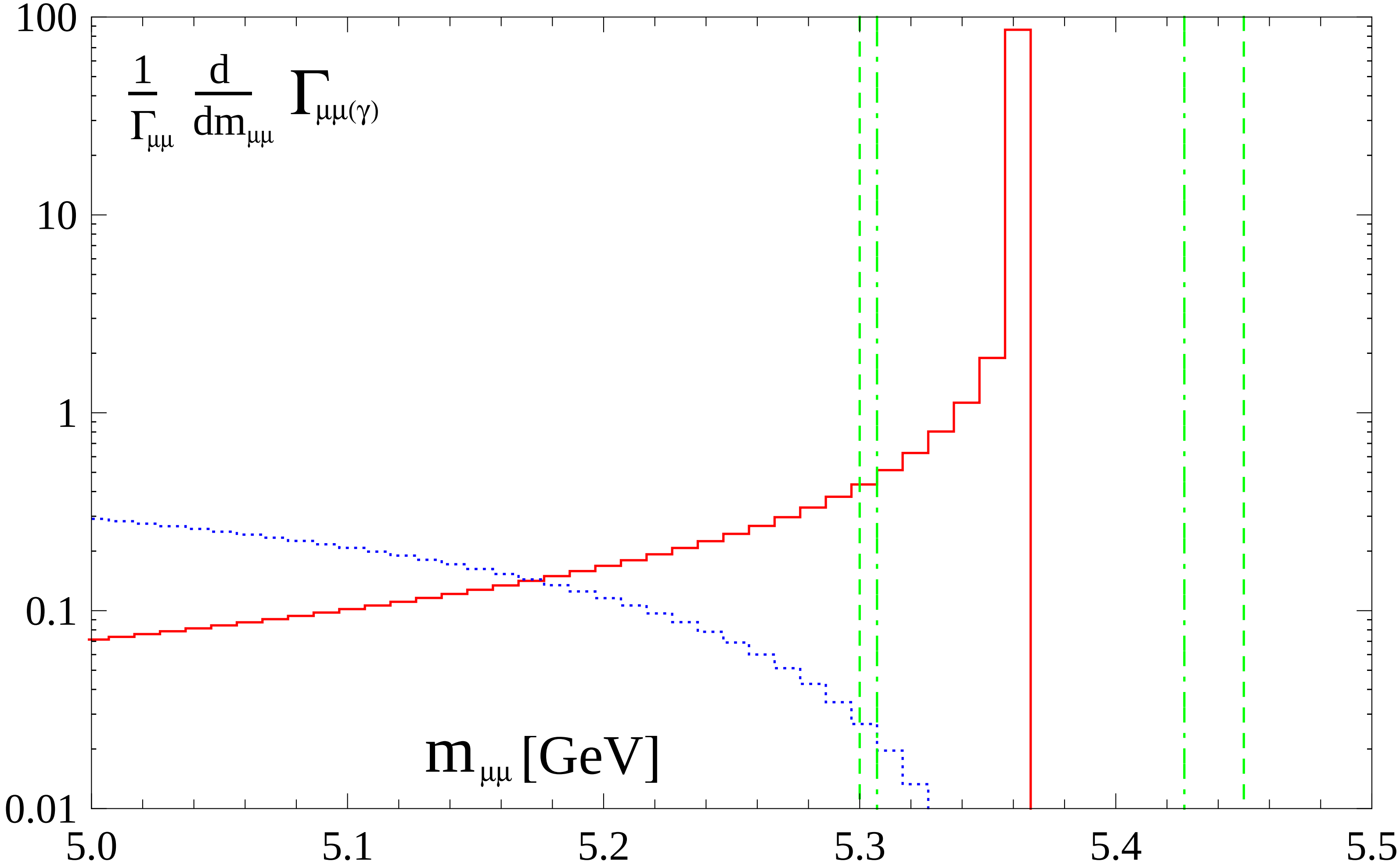
where

$$
\begin{aligned}
R_{t\alpha} &= \left(\frac{M_t}{173.1\ \text{GeV}}\right)^{3.06} \left(\frac{\alpha_s(M_Z)}{0.1184}\right)^{-0.18}, \\
R_s &= \left(\frac{f_{B_s}[\text{MeV}]}{227.7}\right)^2 \left(\frac{|V_{cb}|}{0.0424}\right)^2 \left(\frac{|V_{tb}^\star V_{ts}/V_{cb}|}{0.980}\right)^2 \frac{\tau_H^s\ [\text{ps}]}{1.615}, \\
R_d &= \left(\frac{f_{B_d}[\text{MeV}]}{190.5}\right)^2 \left(\frac{|V_{tb}^\star V_{td}|}{0.0088}\right)^2 \frac{\tau_d^{\text{av}}\ [\text{ps}]}{1.519}.
\end{aligned}
$$

Sources of uncertainties	f_{B_q}	CKM	τ_H^q	M_t	α_s	other parametric	non-parametric	$\sum$	
$\overline{\mathcal{B}}_{s\ell}$	4.0%	4.3%	1.3%	1.6%	0.1%	< 0.1%	1.5%	6.4%	⟶ 4.7% (?)
$\overline{\mathcal{B}}_{d\ell}$	4.5%	6.9%	0.5%	1.6%	0.1%	< 0.1%	1.5%	8.5%	

In the case of $\overline{\mathcal{B}}_{s\ell}$, the main uncertainty (4.2%) originates from $|V_{cb}| = 0.0424(9)$ that comes from a recent fit to the inclusive semileptonic data [P. Gambino and C. Schwanda, arXiv:1307.4551].

Radiative tail in the dimuon invariant mass spectrum



Ggreen vertical lines – experimental windows (→ MC)

Red line – no photon and/or radiation only from the muons. It vanishes when $m_\mu \longrightarrow 0$.

Blue line – remainder due to radiation from the quarks. IR-safe because B_s is neutral.

Phase-space suppressed but survives in the $m_\mu \longrightarrow 0$ limit.

Interference between the two contributions is negligible – suppressed both by phase-space and $m_\mu^2/M_{B_s}^2$.

Inclusive weak radiative B-meson decay

SM estimate [hep-ph/0609232]:

$$\mathcal{B}(\bar{B} \to X_s\gamma)^{\rm SM}_{E_\gamma > 1.6\ \rm GeV} = (3.15 \pm 0.23) \times 10^{-4}$$

Contributions to the total TH uncertainty (summed in quadrature):

5% non-perturbative, 3% from the interpolation in m_c

3% higher order $\mathcal{O}(\alpha_s^3)$, 3% parametric

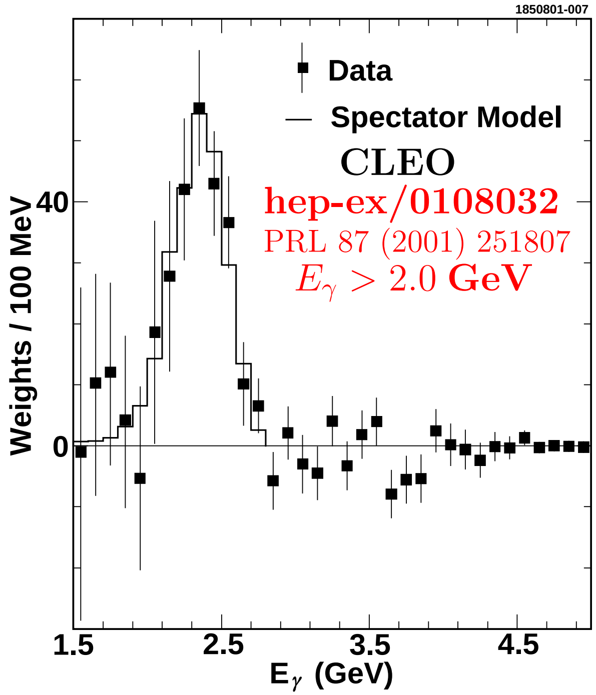
Experimental world average (HFAG, 2.08.2012):

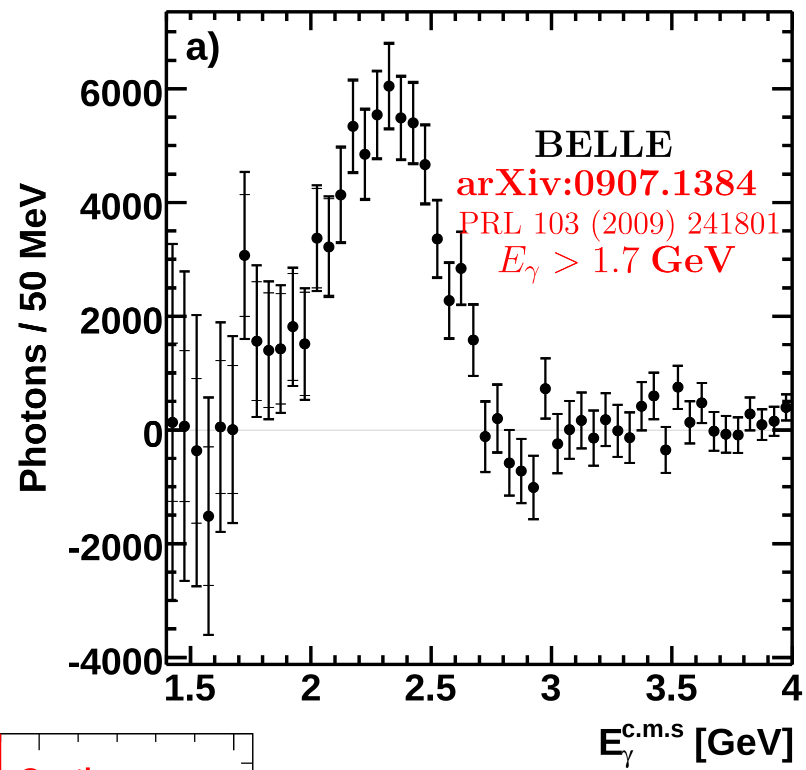
$$\mathcal{B}(\bar{B} \to X_s\gamma)^{\rm EXP}_{E_\gamma > 1.6\ \rm GeV} = (3.43 \pm 0.21 \pm 0.07) \times 10^{-4}$$

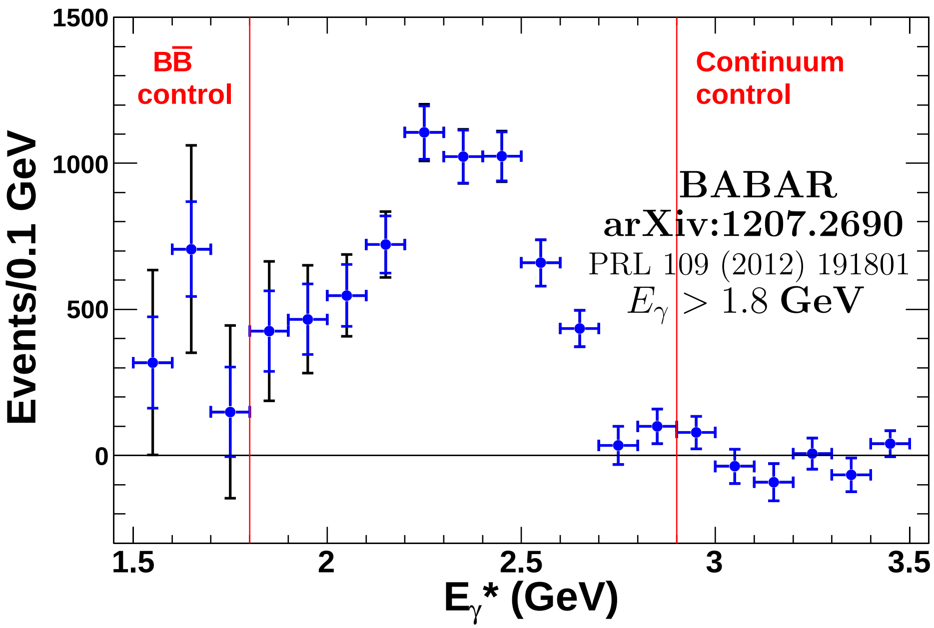
Experiment agrees with the SM at better than $\sim 1\sigma$ level.

Uncertainties: TH $\sim 7\%$, EXP $\sim 6.5\%$.

The "raw" photon energy spectra in the inclusive measurements







The peaks are centred around

$$\tfrac{1}{2} m_b \simeq 2.35 \text{ GeV}$$

which corresponds to a two-body $b \to s\gamma$ decay.

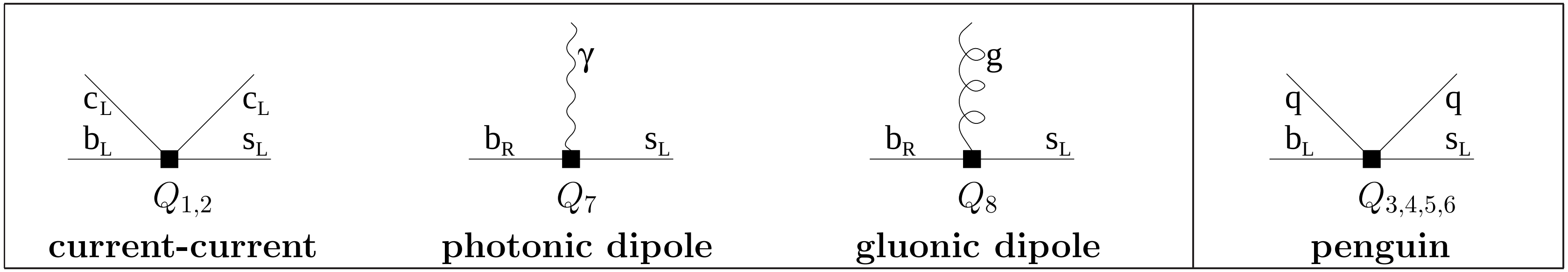
Broadening is due to (mainly):

- perturbative gluon bremsstrahlung,
- motion of the b quark inside the $\bar{B}$ meson,
- motion of the $\bar{B}$ meson in the $\Upsilon(4S)$ frame.

Decoupling of W, Z, t, H^0 $\Rightarrow$ effective weak interaction Lagrangian:

$$L_{\text{weak}} \sim \sum C_i(\mu_b)\, Q_i$$

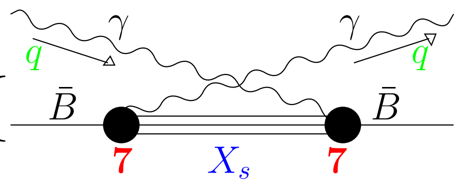
8 operators matter in the SM when the higher-order EW and/or CKM-suppressed effects are neglected:



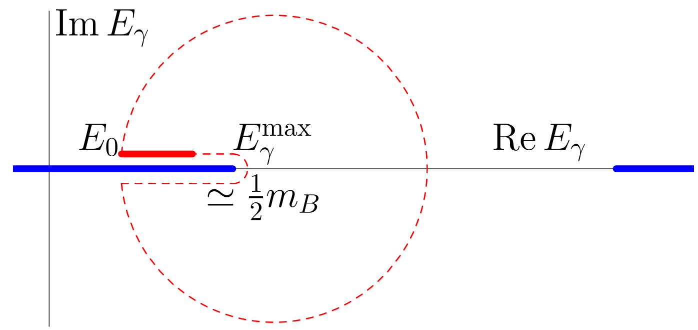
$$\Gamma(\bar{B} \to X_s \gamma)_{E_\gamma > E_0} = |C_7|^2\, \Gamma_{77}(E_0) + (\text{other})$$

Optical theorem:

$$\frac{d\Gamma_{77}}{dE_\gamma} \sim \text{Im}\{ \ldots \} \equiv \text{Im}\, A$$



Integrating the amplitude A over E_γ:



OPE on the ring $\Rightarrow$ Non-perturbative corrections to $\Gamma_{77}(E_0)$ form a series in $\frac{\Lambda_{\text{QCD}}}{m_b}$ and α_s that begins with

$$\frac{\mu_\pi^2}{m_b^2},\ \frac{\mu_G^2}{m_b^2},\ \frac{\rho_D^3}{m_b^3},\ \frac{\rho_{LS}^3}{m_b^3}, \ldots;\ \frac{\alpha_s \mu_\pi^2}{(m_b - 2E_0)^2},\ \frac{\alpha_s \mu_G^2}{m_b(m_b - 2E_0)}; \ldots,$$

where $\mu_\pi, \mu_G, \rho_D, \rho_{LS} = \mathcal{O}(\Lambda_{\text{QCD}})$ are extracted from the semileptonic $\bar{B} \to X_c e \bar{\nu}$ spectra and the B–$B^\star$ mass difference.

NNLO QCD corrections to $\bar{B} \to X_s \gamma$

The relevant perturbative quantity:

$$\frac{\Gamma[b \to X_s \gamma]_{E_\gamma > E_0}}{|V_{cb}/V_{ub}|^2 \, \Gamma[b \to X_u e \bar{\nu}]} = \left| \frac{V_{ts}^* V_{tb}}{V_{cb}} \right|^2 \frac{6 \alpha_{\rm em}}{\pi} \underbrace{\sum_{i,j} C_i \, C_j \, K_{ij}}_{P(E_0)}$$

Expansions of the Wilson coefficients and K_{ij}:

$$C_i(\mu_b) = C_i^{(0)}(\mu_b) + \frac{\alpha_s(\mu_b)}{4\pi} C_i^{(1)}(\mu_b) + \left(\frac{\alpha_s(\mu_b)}{4\pi} \right)^2 C_i^{(2)}(\mu_b) + \ldots$$

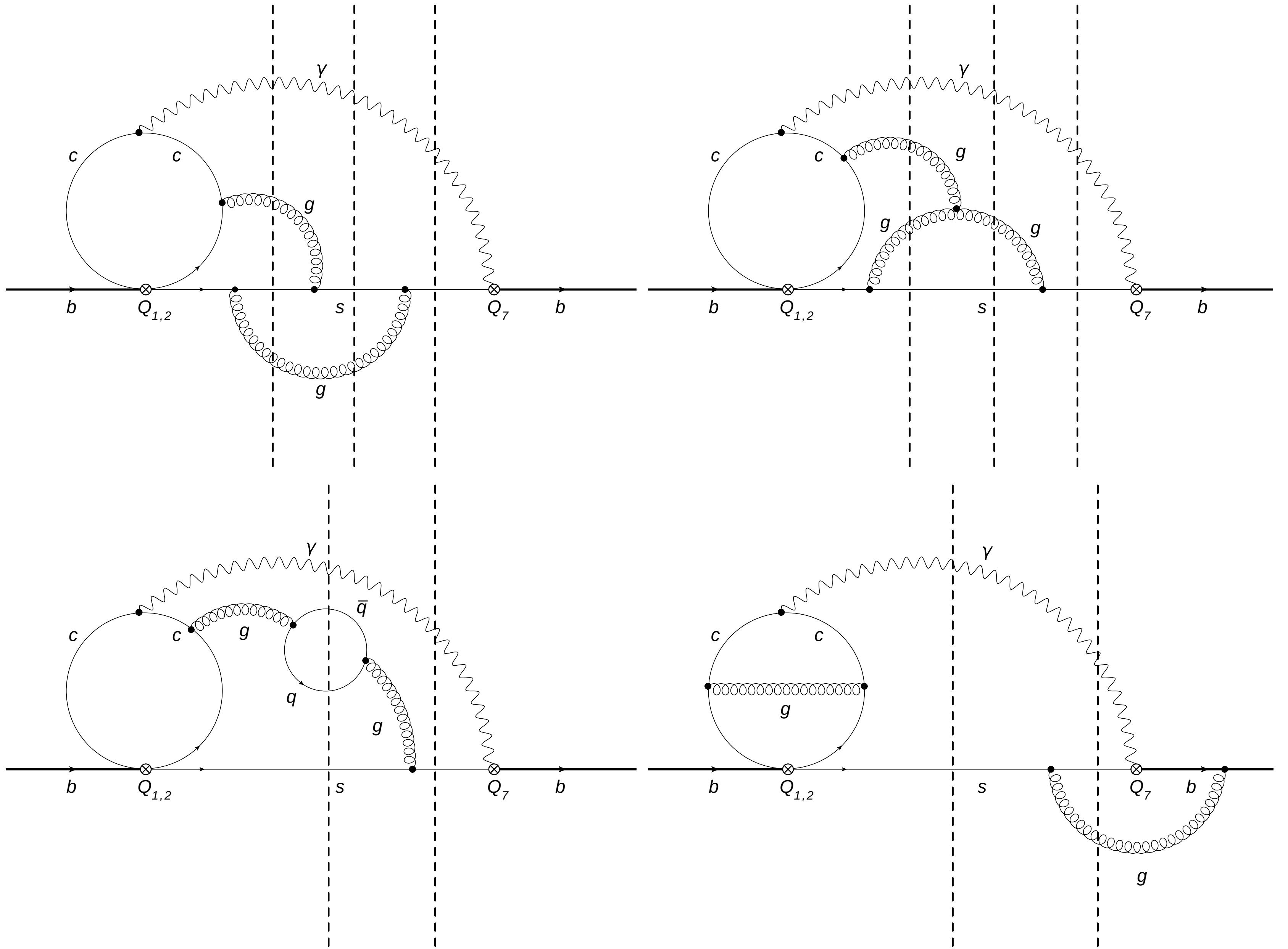
$$K_{ij} = K_{ij}^{(0)} + \frac{\alpha_s(\mu_b)}{4\pi} K_{ij}^{(1)} + \left(\frac{\alpha_s(\mu_b)}{4\pi} \right)^2 K_{ij}^{(2)} + \ldots \qquad \mu_b \sim \frac{m_b}{2}$$

Most important at the NNLO: $K_{77}^{(2)}$, $K_{27}^{(2)}$ **and** $K_{17}^{(2)}$.

They depend on $\frac{\mu_b}{m_b}$, $\frac{E_0}{m_b}$ **and** $r = \frac{m_c}{m_b}$.

Evaluation of $K_{27}^{(2)}$ and $K_{17}^{(2)}$ for $m_c = 0$:

[M. Czakon, P. Fiedler, T. Huber, MM, T. Schutzmeier, M. Steinhauser, to be published]



Results for the NNLO corrections:

$$K_{27}^{(2)}(r,E_0) = A_2 + F_2(r,E_0) + 3f_q(r,E_0) + f_b(r) + f_c(r) + \frac{8}{3}\phi_{27}^{(1)}(r,E_0)\ln r$$
$$+ \left[(4L_c - x_m)\, r\frac{d}{dr} + x_m E_0 \frac{d}{dE_0}\right] f_{NLO}(r,E_0) + \frac{416}{81}x_m$$
$$+ \left(\frac{10}{3}K_{27}^{(1)} - \frac{2}{3}K_{47}^{(1)} - \frac{208}{81}K_{77}^{(1)} - \frac{35}{27}K_{78}^{(1)} - \frac{254}{81}\right)L_b - \frac{5948}{729}L_b^2,$$

$$K_{17}^{(2)}(r,E_0) = -\frac{1}{6}K_{27}^{(2)}(r,E_0) + A_1 + F_1(r,E_0) + \left(\frac{94}{81} - \frac{3}{2}K_{27}^{(1)} - \frac{3}{4}K_{78}^{(1)}\right)L_b - \frac{34}{27}L_b^2,$$

where $F_i(0,0) \equiv 0$, $A_1 \simeq 22.605$, $A_2 \simeq -$ ~~37.314~~ -81.179 (?) from the present calculation.

Correction due to $\mathcal{O}(\epsilon)$ term in one of the master integrals. Currently being cross-checked.

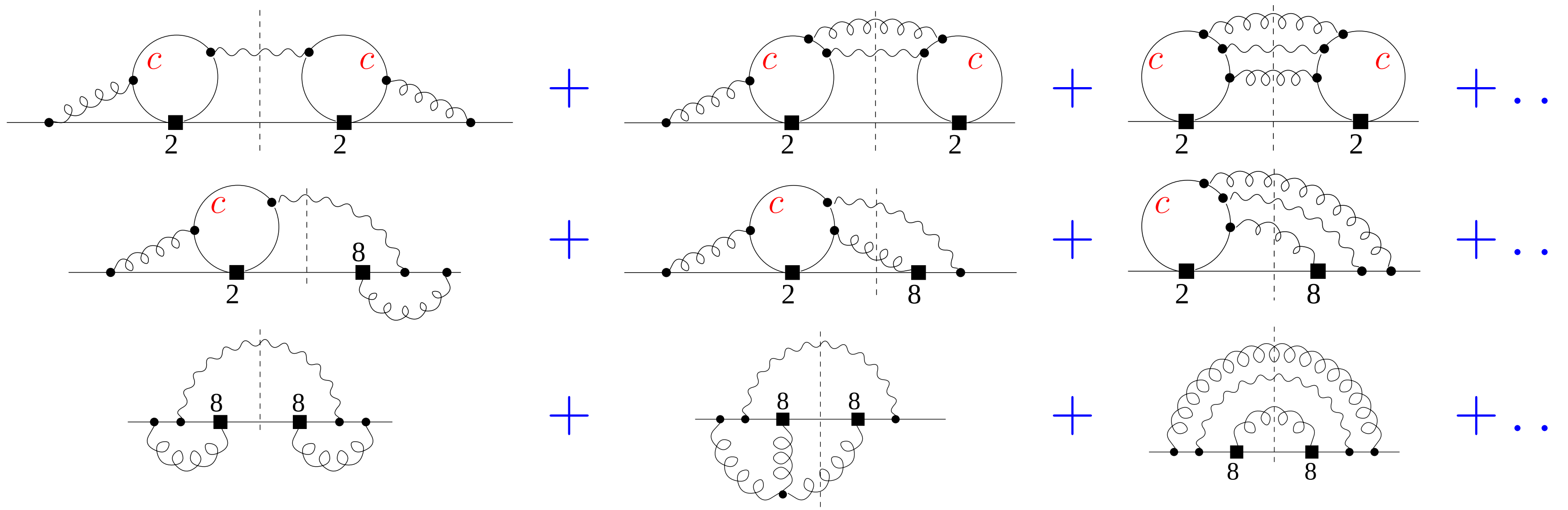
Next, we interpolate in m_c by assuming that $F_i(r,0)$ are linear combinations of $f_q(r,0)$, $f_{NLO}(r,0)$, $r\frac{d}{dr}f_{NLO}(r,0)$ and a constant term. The known large-r behaviour of F_i [hep-ph/0609241] and the condition $F_i(0,0) \equiv 0$ fix these linear combinations in a unique manner.

Interferences not involving the photonic dipole operator are treated as follows:

K_{22}**:** (and analogous K_{11} & K_{12})

K_{28}**:** (and analogous K_{18})

K_{88}**:**



Two-particle cuts are known (just $|\mathrm{NLO}|^2$).

Three- and four-particle cuts are known in the BLM approximation only. The NLO+(NNLO BLM) corrections are not big $(+3.8\%)$.

Incorporating other perturbative contributions evaluated after the previous phenomenological analysis in hep-ph/0609232:

1. Four-loop mixing (current-current) $\to$ (gluonic dipole)
   M. Czakon, U. Haisch, MM, JHEP 0703 (2007) 008 [hep-ph/0612329]

2. Diagrams with massive quark loops on the gluon lines
   R. Boughezal, M. Czakon and T. Schutzmeier, JHEP 0709 (2007) 072 [arXiv:0707.3090]
   H. M. Asatrian, T. Ewerth, H. Gabrielyan and C. Greub, Phys. Lett. B 647 (2007) 173 [hep-ph/0611123]
   T. Ewerth, Phys. Lett. B 669 (2008) 167 [arXiv:0805.3911]

3. Complete interference (photonic dipole)–(gluonic dipole)
   H. M. Asatrian, T. Ewerth, A. Ferroglia, C. Greub and G. Ossola,
   Phys. Rev. D 82 (2010) 074006 [arXiv:1005.5587]

4. New BLM corrections to contributions from 3-body and 4-body final states for interferences not involving the photonic dipole:
   A. Ferroglia and U. Haisch, Phys. Rev. D 82 (2010) 094012 [arXiv:1009.2144]
   MM and M. Poradziński, Phys. Rev. D 83 (2011) 014024 [arXiv:1009.5685]

5. LO contributions from $b \to s\gamma q\bar{q}$, $(q = u, d, s)$ from the four quark operators ("penguin" ones or CKM-suppressed ones).
   M. Kamiński, MM and M. Poradziński, Phys. Rev. D 86 (2012) 094004 [arXiv:1209.0965]

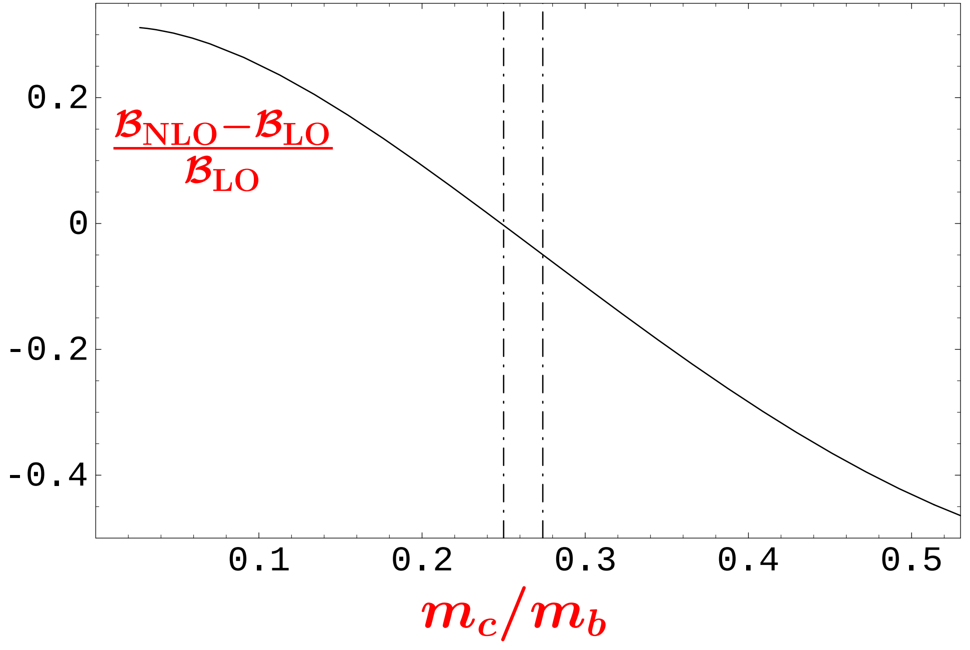
Taking into account new non-perturbative analyses:

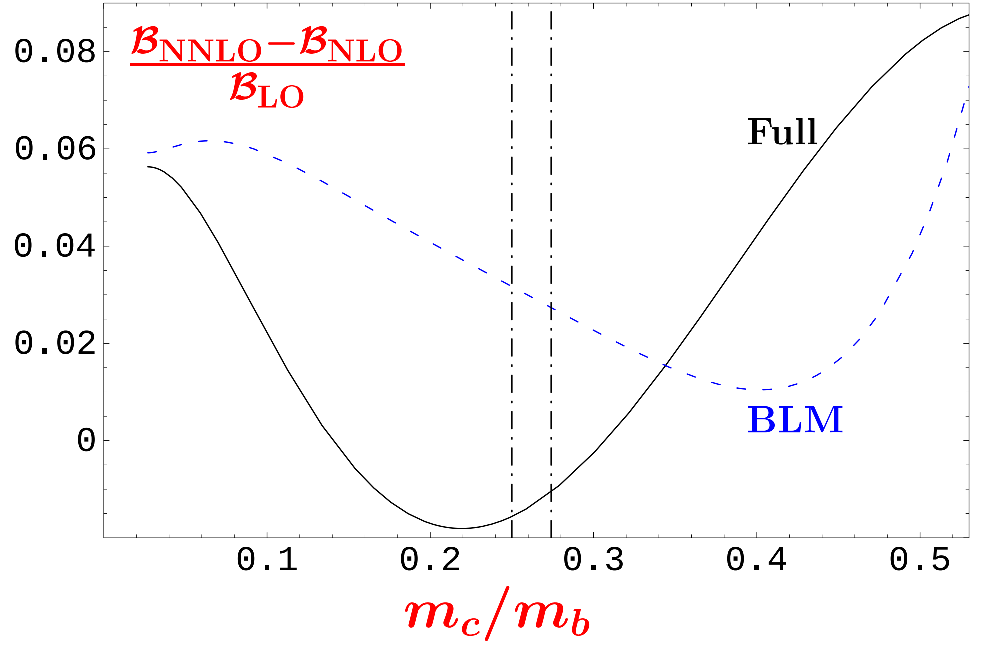
M. Benzke, S. J. Lee, M. Neubert and G. Paz, JHEP 1008 (2010) 099 [arXiv:1003.5012]
T. Ewerth, P. Gambino and S. Nandi, Nucl. Phys. B 830 (2010) 278 [arXiv:0911.2175]

Updating the parameters (Parametric uncertainties go down to 2.4%)

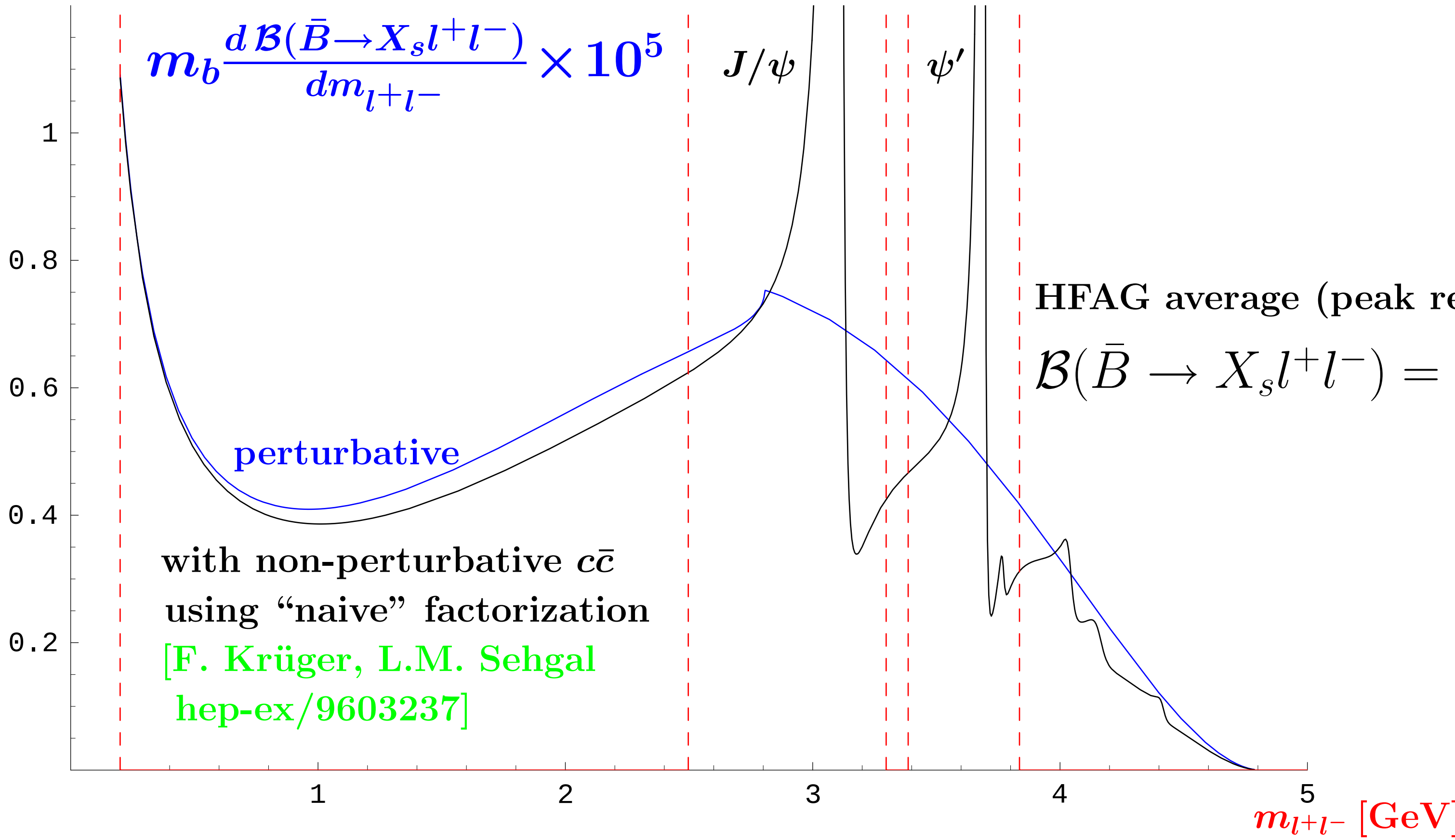
P. Gambino, C. Schwanda, Phys. Rev. D 89 (2014) 014022

Relative NLO and NNLO QCD corrections to $\mathcal{B}(\bar{B} \to X_s \gamma)$ and their dependence on m_c/m_b





Dilepton mass spectrum in $\bar{B} \to X_s l^+ l^-$ $(l = e \text{ or } \mu)$



HFAG average (peak regions removed):

$$\mathcal{B}(\bar{B} \to X_s l^+ l^-) = (3.66 \pm 0.77) \times 10^{-6}$$

$$\left(\hat{s} = \frac{m_{l^+l^-}^2}{m_b^2}\right)$$

$$\frac{d\Gamma(\bar{B} \to X_s l^+ l^-)}{d\hat{\mathbf{s}}} = \frac{G_F^2 m_{b,\text{pole}}^5 |V_{ts}^* V_{tb}|^2}{48\pi^3} \left(\frac{\alpha_{\text{em}}}{4\pi}\right)^2 (1-\hat{\mathbf{s}})^2 \times$$

$$\left\{(1+2\hat{\mathbf{s}})\left(|\mathbf{C_9^{\text{eff}}}(\hat{\mathbf{s}})|^2 + |\mathbf{C_{10}^{\text{eff}}}(\hat{\mathbf{s}})|^2\right) + \left(4 + \frac{8}{\hat{\mathbf{s}}}\right)|\mathbf{C_7^{\text{eff}}}(\hat{\mathbf{s}})|^2 + 12\,\text{Re}\left(\mathbf{C_7^{\text{eff}}}(\hat{\mathbf{s}})\mathbf{C_9^{\text{eff}*}}(\hat{\mathbf{s}})\right)\right\} + \mathbf{R}.$$

$$C_i^{\text{eff}}(\hat{\mathbf{s}}) = C_i(\mu_{\mathrm{b}}) + \textbf{(loop corrections)}(\hat{\mathbf{s}}).$$

R **stands for small bremsstrahlung contributions and for the non-perturbative corrections.**

$$Q_7 \sim m_b \left(\bar{s}\sigma^{\alpha\beta} P_R b\right) F_{\alpha\beta}$$

$$Q_9 \sim (\bar{s}\gamma^\alpha P_L b)\left(\bar{\ell}\gamma_\alpha \ell\right)$$

$$Q_{10} \sim (\bar{s}\gamma^\alpha P_L b)\left(\bar{\ell}\gamma_\alpha \gamma_5 \ell\right)$$

Known terms in the perturbative expansions of $C_{7,10}$ at $\mu_b \sim m_b/2$:

$$C_i(\mu_b) = C_i^{(0)}(\mu_b) + \frac{\alpha_s}{4\pi} C_i^{(1)}(\mu_b) + \left(\frac{\alpha_s}{4\pi}\right)^2 C_i^{(2)}(\mu_b) + \frac{\alpha_{em}}{4\pi} \Delta_{\rm EW} C_i(\mu_b)$$

However, for $C_9(\mu_b)$, we only have:

$$C_9(\mu_b) = \frac{4\pi}{\alpha_s} C_9^{(-1)}(\mu_b) + C_9^{(0)}(\mu_b) + \frac{\alpha_s}{4\pi} C_9^{(1)}(\mu_b) + \mathcal{O}\left(\frac{\alpha_{em}}{\alpha_s^2}, \frac{\alpha_{em}}{\alpha_s}\right)$$

Recall:

$$\frac{4\pi}{\alpha_s} C_i^{(-1)}(\mu_b) \xrightarrow{\alpha_s \to 0} \frac{4}{9} \ln \frac{\mu_0^2}{\mu_b^2} \simeq \frac{4}{9} \ln \frac{M_W^2}{m_b^2} \quad \text{(electroweak logarithm)}$$

$$\frac{4\pi}{\alpha_s} C_i^{(-1)}(\mu_b) \simeq C_i^{(0)}(\mu_b) \quad \text{(numerically)}$$

Is there enough motivation to calculate $C_9^{(2)}$ and $\Delta_{\rm EW} C_9(\mu_b)$?

Summary

- **Combining the recently calculated NNLO QCD and NLO EW corrections to $\overline{\mathcal{B}}(B_s \to \mu^+\mu^-)$, we find a significant reduction of the non-parametric theoretical uncertainties ($\sim 8\% \to\sim 1.5\%$).**

- **Dominant NNLO corrections to $\mathcal{B}(\bar{B} \to X_s\gamma)$ will soon be known not only in the large m_c limit, but also at $m_c = 0$. If the current result survives, no reduction of uncertainties with respect to the 2006 estimate is expected, except for the parametric one.**

- **New measurements of $\bar{B} \to X_s\ell^+\ell^-$ are expected from Belle II and LHCb. Upgrading our perturbative knowledge of C_9 to the same level as $C_{7,10}$ might become necessary.**