Transverse momentum dependent (TMD) parton distribution functions (PDF) in Laguerre polynomial basis

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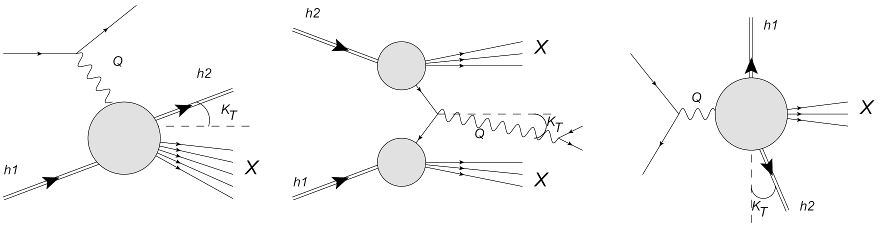


Transverse momentum dependent (TMD) parton distribution functions (PDFs) appears in the hard processes which involves two hadrons.

In the regime $\boxed{Q^2 \gg k_T^2 \sim b_T^{-2} \gg \Lambda^2}$ there are TMD factorization theorems [Collins,Soper,82; Collins,Soper,Sterman, 85]

$$W^{\mu\nu}(Q,k_T) \simeq \; H^{\mu\nu}\left(\frac{Q^2}{\mu^2}, x_1, x_2\right)$$
$$\otimes \int \frac{d^2 b_T}{(2\pi)^2} e^{-i(b_T k_T)} \underbrace{F^{uns.}(x_1, b_T; \mu, \delta^+) D^{uns.}(x_2, b_T; \mu, \delta^-)}_{\text{unsubtracted TMD PDFs}} \underbrace{S(b_T, \delta^+, \delta^-)}_{\text{soft factor}}$$

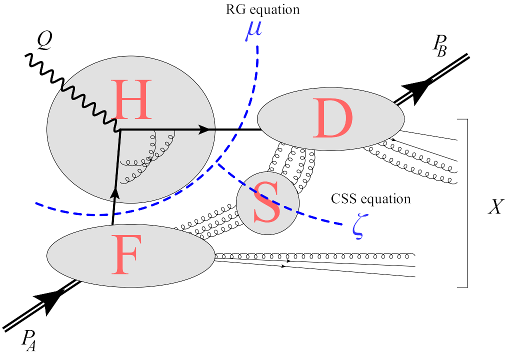
- $\delta^{\pm}$ are regularization parameters for rapidity divergences

- Factorization of hard part id is controlled by parameter μ (RG equation):

$$\mu^2 \frac{d}{d\mu^2} \ln F(x, b_T; \mu, \zeta) = \gamma_F \left(\alpha; \frac{\zeta}{\mu}\right) \qquad \left[= \frac{\alpha_s}{\pi} C_F \left(\frac{3}{2} - \ln\left(\frac{\zeta}{\mu^2}\right)\right) + \mathcal{O}(\alpha_s^2)\right]$$

- Factorization of Glauber region is controlled by parameter ζ (CSS equation [Collins,Soper,Sterman,85]):

$$\zeta \frac{d}{d\zeta} \ln F(x, b_T; \mu, \zeta) = K(b_T; \mu) \qquad \left[= -\frac{\alpha_s}{\pi} C_F \ln\left(\frac{\mu^2 b_T^2}{4e^{-2\gamma_E}}\right) + \mathcal{O}(\alpha_s^2)\right]$$



- At $b_T \to 0$ ($k_T \to \infty$) is singular and does not match integrated PDFs

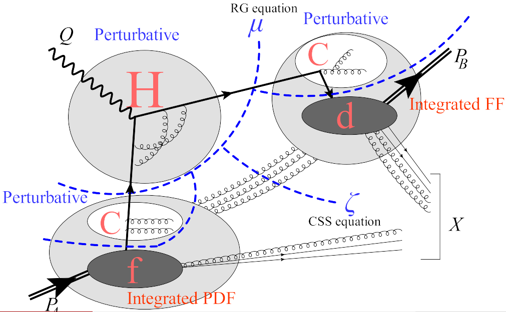
$$\lim_{b_T \to 0} F(x, b_T; \mu, \zeta) \not\to f(x, \mu)$$

- The matching of integrated and TMD PDF requires OPE at small-b_T [Collins,Soper,82]

(assuming $b_T \sim Q^{-1}$)

$$F_{f/P}(x, b_T, \mu, \zeta) = \sum_j \int_x^1 \frac{dz}{z} \underbrace{C_{f/j}\left(\frac{x}{z}, b_T; \zeta, \mu\right)}_{\text{coef. function}} \underbrace{f_{j/P}(z, \mu)}_{PDF} + \mathcal{O}\left((\Lambda b_T)^a\right).$$

This expression is a result of "collinear" factorization between scales $\Lambda^2 \ll k_T^2(b_T^{-2})$.



For the practical application one needs $b_T \gg Q^{-1}$

$$F_{f/H}(x,b_T;\mu,\zeta) = \underbrace{\overbrace{\sum_j \left(C_{f/j} * f_{j/H}\right)[x,b_T;\mu_b]}^{\text{OPE at } b_T \to 0}\overbrace{R(b_T;\mu,\zeta;\mu_b)}^{\text{evolution}}}_{\text{perturbative input}}\overbrace{\exp\left(g_1(x,b_T) + g_2(b_T)\ln\left(\frac{\zeta}{\zeta_0}\right)\right)}^{\text{nonperturbative factor}}$$

Nonperturbative factor

- Minimal (Gaussian) anzatz $g_1 \;\propto\; g_2 \;\propto\; -a_1 b^2$
- Non-minimal anzatz $g_2 \;\propto -a_1 b^2 + a_2 b^4 + \ldots$ or $g_2 \;\propto\; b^2 \ln\left(1 + c_1 b^2\right)$
  e.g. [Aidala,et al 2014]

Minimal anzatz should be modified at larger b_T.
Can we obtain any perturbative modification to it?

The nonperturbative factor behaves as

~ 1 at $b_T \to 0$, for matching with coll.fac.	$\sim e^{-b_T^2}$ intermediate region,	$\sim ?$ at $b_T \to \infty$, truly non-pert. regime

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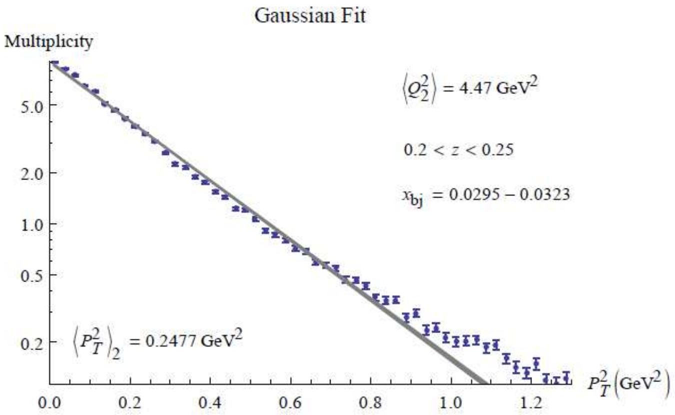
[Aidala, et al, 14]



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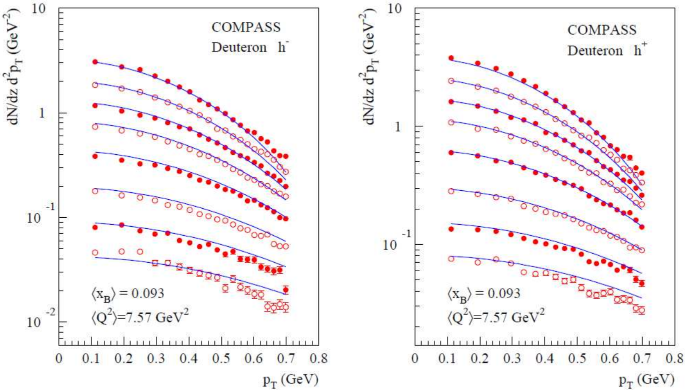
[Echavarria, et al, 14]



Phenomenologically motivated OPE

- In principle, perturbative QCD should work in some non-zero range of $0 < b_T^2 \ll \Lambda_{QCD}^{-2}$. In this range one can perform OPE with reasonable convergence of perturbative corrections to coefficient functions

$$\underbrace{O(x,b_T)}_{\substack{\text{TMD PDF operator}\\ \sim \bar{q}(b_T)..q(0)}} = \sum_{n=0}^{\infty} \underbrace{G_n^{(T)}(x,b_T)}_{\substack{\sim b_T^n \\ \text{contains } \ln(b_T)}} \otimes \underbrace{O_n^{(T)}(x)}_{\substack{\text{PDF operator}\\ \sim \bar{q}(0)..\partial_T^n..q(0)}}$$

- The matrix element of O_0 is PDF. The matrix elements of O_n are unknown.
- At intermediate b_T there is no reason to expect any suppression of higher terms.

Standard approach

$$O(x,b_T) = \underbrace{G_0(x,b_T) \otimes O_0(x)}_{\text{leading at } b_T \to 0} \times \underbrace{(\ 1 + \ ....)}_{\text{nonperturbative factor}}$$

- The Taylor-like OPE is not saturated by the first terms.
- Possible solution is to use another basis.

Demands to the operator basis at intermediate b_T

- Transverse locality $\rightarrow$ polynomial in ∂_T
- Orthogonality $\rightarrow$ orthogonal polynomial in ∂_T
- Infinite support in b_T $\rightarrow$ Hermite or Laguerre polynomials
- Symmetry constrains (??)

- Both polynomials has Gaussian generation function, $\Rightarrow$ it guaranties the leading Gaussian behavior of OPE
- Both basis includes unity, $\Rightarrow$ the leading term is integrated PDF.
- Hermite are suitable for the polarized TMDs while Laguerre are suitable for unpolarized TMDs.

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Phenomenologically motivated OPE

Operator definition of TMD PDF

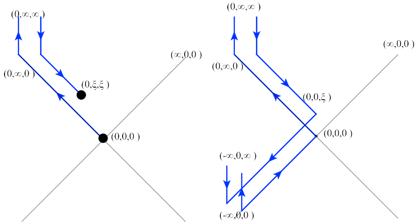
we use [Echevarria,Idilbi,Scimemi,12-13] (EIS) definition

$$\text{TMD PDF} \qquad F(x,b_T;\mu,\zeta) = \overbrace{\underbrace{Z(\mu)}_{\text{UV renorm.}} \quad \underbrace{(S(b_T,\zeta\delta,\delta))^{-\frac{1}{2}}}_{\substack{\text{cancel}\\ \text{rapidity}\\ \text{divergences}}} F^{uns.}(x,b_T;\mu,\delta)}^{\text{free of divergences}}$$

- δ-regularization is defined as $(k^\pm \mp i0)^{-1} \to (k^\pm \mp i\delta)^{-1}$

$$F^{uns.}(x,b_T,\delta) = \int \frac{d\xi^-}{(2\pi)} e^{-ix\xi^- p_+} \langle p|\bar{q}(\xi) W\left(\xi,0;\mathcal{C}_\delta\right) q(0)|p\rangle\Big|_{\xi^+=0,\xi_\perp=b_T}$$

$$S(b_T,\delta^+,\delta^-) = \langle 0|\mathrm{tr}\left[W\left(b_T,0;\mathcal{C}_{\delta^+}\right) W\left(0,b_T;\mathcal{C}_{\delta^-}\right)\right]|0\rangle$$



Laguarre based OPE at leading order (free theory)

$$O(x,b_T)=\underbrace{\sum_{n=0,2,..}^{\infty}\overbrace{\left(\frac{b_T^2}{4B_T^2}\right)^{\frac{n}{2}}e^{-\frac{b_T^2}{4B_T^2}}}^{\text{prototype of nonper. factor}}\left(1+\mathcal{O}(\alpha_s^2)\right)\mathbb{O}_n(x,B_T)}_{\text{independent on } B_T}$$

$$\mathbb{O}_n(x,B_T)\propto\int\frac{d\xi^-}{2\pi}e^{-ixp_+\xi^-}\bar{q}\left(\frac{\xi^-}{2}\right)\left[\frac{\xi^-}{2},\infty^-\right]\gamma^+L_n\left(\overleftrightarrow{\partial}_T^2B_T^2\right)\left[\infty^-,-\frac{\xi^-}{2}\right]q\left(-\frac{\xi^-}{2}\right)$$

- At $B_T\to\infty$ turns to the standard OPE
- Every individual term is dependent on B_T
- The leading order results to the Gaussian factor, we may assume that contribution of $n>0$ terms is small at small and intermediate b_T.

It can be seen as an extraction (by hands) of desired distribution from the higher order term,
one can extract any factor(smooth) in such a way.

The important point is quantum corrections

No twist-expansion, — no hierarchy!

The operators in Laguerre basis strongly mix in loops. The mix includes operators of kind

$$\underbrace{\bar q\gamma^+(\partial_T^2)^n q}_{\text{operators from Laguerre basis}} \qquad \underbrace{\bar q\partial^2(\partial_T^2)^{n-1} q}_{\text{higher order operators}} \qquad \underbrace{\bar q q(\partial_T^2)^{n-2}\bar q q}_{\text{operators with more then two fields}} \qquad etc.$$

- The admixture of four-field operators appears only at two-loop level, (6-field at three-loop, etc.)
- There is no admixture of $\partial^2(\partial_T^2)^{n-1}$ operator in $(\partial_T^2)^n$ (but not the opposite)
- The correction to the Laguerre operators mix in triangular way, i.e L_n influence on L_k only if $n < k$ (due to orthogonality!)

At one loop the actual calculation is reduced to

$$C_n^{\text{1-loop}}(x, b_T, \mu) = \left(\lim_{b_T\to 0}\lim_{\epsilon\to 0} - \lim_{\epsilon\to 0}\lim_{b_T\to 0}\right)\mathcal{G}^{\text{1-loop}}(x, b_T, \mu)\Big|_{L_n},$$

$\mathcal{G}$ is a 2-point matrix element of O.

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We have calculated next order correction to coefficient function of operator $\mathbb{O}_n$
for q/q and q/g
evolution kernels, expressions etc. see [1402.3182]

The only practically interesting case is L_0

Modified expression for TMD PDF

$$F_{q/H}(x,b_T;\mu,\zeta) = \sum_j \int_x^1 \frac{dz}{z} \overbrace{\mathbb{C}_{q/j}\left(\frac{x}{z},b_T;\mu,\zeta\right)}^{\text{modified coef.functions}} \overbrace{f_{j/H}(x,\mu)}^{\text{int. PDF}} + \overbrace{\mathcal{O}_1}^{\text{small?}},$$

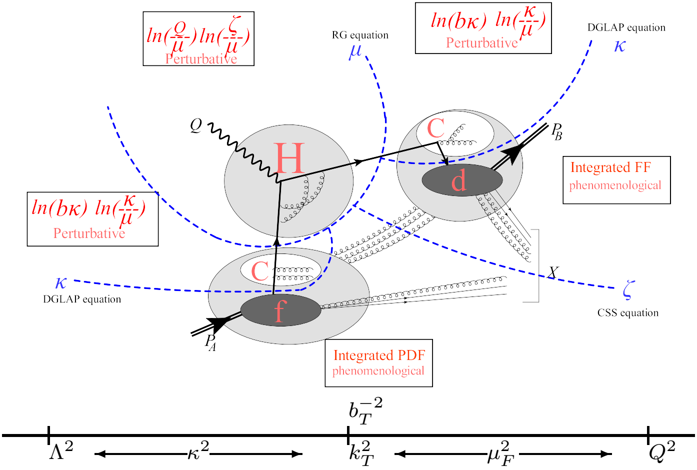
$$\begin{aligned}
\mathbb{C}_{q/q}(x,b_T,\mu,\zeta) &= e^{-\frac{b_T^2}{4B_T^2}}\delta(1-x) + \\
&2a_s C_F e^{-\frac{x^2 b_T^2}{4B_T^2}}\Bigg[-L_T P_{qq}(x) + \delta(\bar{x})\left(\frac{3}{2}L_T - \frac{1}{2}L_T^2 - \frac{\pi^2}{12} + L_T \ln\left(\frac{\mu^2}{\zeta}\right)\right) \\
&+ \bar{x}x^2 \frac{b_T^2}{B_T^2} L_T \left(\frac{x^2}{8}\frac{b_T^2}{B_T^2} - 1\right) - \frac{x^4\bar{x}}{4}\left(\frac{b_T^2}{B_T^2}\right)^2 + \frac{3x^2\bar{x}}{2}\frac{b_T^2}{B_T^2}\Bigg] + \mathcal{O}(a_s^2), \\
\mathbb{C}_{q/g}(x,b_T,\mu,\zeta) &= 2a_s e^{-\frac{x^2 b_T^2}{4B_T^2}}(-P_{qg}(x)L_T + 2x\bar{x}) + \mathcal{O}(a_s^2).
\end{aligned}$$

$$L_T = \ln\left(\frac{\mu^2 b_T^2}{4e^{-2\gamma_E}}\right)$$

Renormalization group kernel and CSS kernel are the same as in the standard approach.

The result can not be presented as a factor, it is a generalized function!

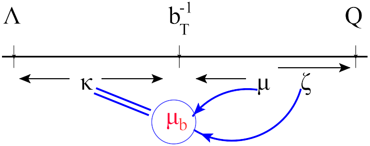
Tale(s) on three scales



$$d\sigma \propto \underbrace{H\left(\frac{Q}{\mu}\right) \otimes \underbrace{\overbrace{\left(C(b\kappa_1, \frac{\zeta}{\mu}, \frac{\mu}{\kappa_1}) \otimes f(\kappa_1)\right)}^{\kappa\text{-independent}} \otimes \overbrace{\left(C(b\kappa_2, \frac{\zeta}{\mu}, \frac{\mu}{\kappa_2}) \otimes f(\kappa_2)\right)}^{\kappa\text{-independent}}}_{\zeta\text{-independent}}}_{\mu\text{-independent}}$$

One cannot choose scales such that all logarithms are small.
Traditional approach e.g [Aybat,Rogers,11] is to evolve $(\mu,\zeta) \to (\mu_b, \mu_b)$ and set $\kappa = \mu_b$.

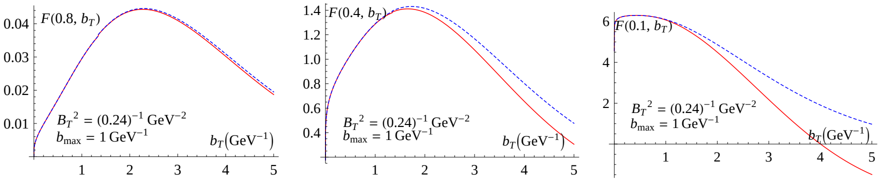
$$\mu_b : \quad \begin{cases} (\mu_b b_T) \to \text{const} & b_T \to 0 \\ \alpha(\mu_b) < 1 & b_T \to \infty \end{cases} \qquad \mu_b^2 = 4e^{-2\gamma_E}\left(\frac{1}{b_T^2} + \frac{1}{b_{\text{max}}^2}\right)$$



$$F_{q/H}(x, b_T; \mu, \zeta) = \sum_j \int_x^1 \frac{dz}{z} \mathbb{C}_{q/j}\left(\frac{x}{z}, b_T; \mu_b, \mu_b^2\right) f_{j/H}(x, \mu_b) \times$$

$$\exp\left[-2a_s(\mu_b)C_F \ln\left(\frac{\zeta}{\mu_b^2}\right) L_T^b + 2C_F \int_{\mu_b}^{\mu} \frac{d\mu'^2}{\mu'^2} a_s(\mu') \left(\frac{3}{2} - \ln\left(\frac{\zeta}{\mu'^2}\right)\right)\right]$$

$$L_T^b = \ln\left(b_T^2 \mu_b^2 / 4e^{-2\gamma_E}\right).$$



- Effect of quantum correction is higher at smaller x
- Higher order corrections result to b_T^n terms, which flatten out the initial Gaussian distribution.

Conclusion

Phenomenological OPE

- The introduction of "nonperturbative" information can be done in "perturbative" way via phenomenological OPE.
- Chousing appropriate basis one can obtain the nonperturbative factor of a desired form at leading order
- The perturbative corrections would slightly violate this form, resulting to the better matching between perturbative and nonperturbative regions

TMD PDF in Laguerre basis

- The general Gaussian form of TMD PDF implies the Laguerre operator basis
- The LO and NLO matching coefficients are calculated
- The corrections flattens the tail of Gaussian distribution
- The correction is less significant at $x \sim 1$ and more significant at small x.
- For the check of the method the accurate phenomenological study is needed.