

Phenomenological aspects of evolution in transverse momentum dependent fragmentation and PDF's

Ignazio Scimemi (UCM)

based on:

arXive: 1402.0869, with M.G. Echevarría (NIKHEF), A. Idilbi (Penn U.),

arXive: 1405., with U. D' Alesio (Cagliari), M.G. Echevarría (NIKHEF), S. Melis (Turin)

and also PLB726(2013)795,
EPJC 73(2013)2636,
JHEP1207(2012)002

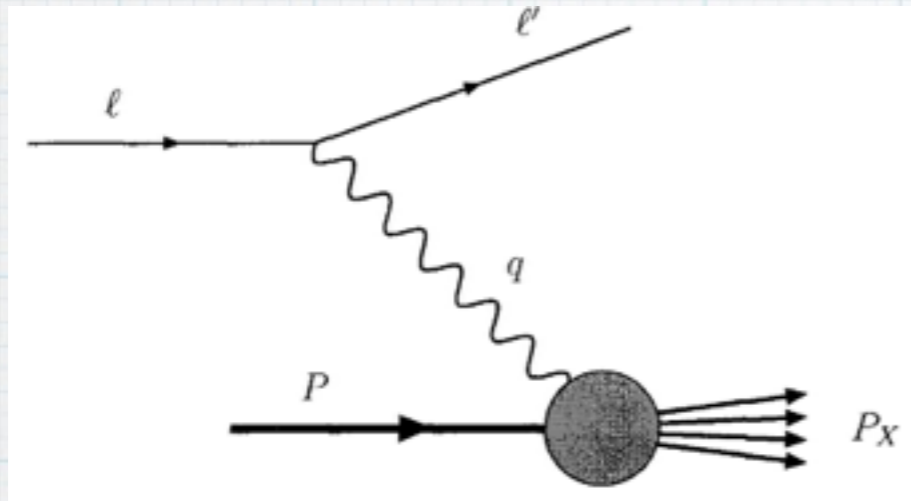
DIS2014, Warsaw

Topics and outline

- * Spin dependent observables and transverse momentum dependent observables need factorization theorems with TMD's
- * TMD's are the fundamental **non-perturbative** objects to be used in factorization theorems in (un-)polarized Drell-Yan, SIDIS, e^+e^- to 2 jets (multi-jets?)
- * Properties of TMD's:
 - 1) The evolution of all TMD's is universal (alike PDF and FF it is process independent)
 - 2) The evolution of all TMD's is spin independent and it is the same for TMDPDF and TMDFF
- * Extraction of unpolarized TMDPDF from Drell-Yan and Z-boson production using completely resummed TMD's at NNLL

Preliminary

Factorization theorem



SIDIS as a study case:
both PDF and FF

$$q^2 \gg q_T^2$$

Fact. scale

$$l(k) + N(P) \rightarrow l'(k') + h(P_h) + X(P_X)$$

$$W^{\mu\nu} = H(Q^2/\mu^2) \frac{2}{N_c} \sum_q e_q \int d^2 k_{n\perp} d^2 k_{\bar{n}\perp} \delta^{(2)}(\mathbf{q}_\perp + \mathbf{k}_{n\perp} - \mathbf{k}_{\bar{n}\perp})$$

$$\times \text{Tr} [F(x, \mathbf{k}_{n\perp}, S; Q^2/\alpha, \mu^2) \gamma^\mu D(z, \hat{P}_{h\perp}, S_h; Q^2/\alpha, \mu^2) \gamma^\nu]$$

Hard coeff

TMDPDF

TMDFF

Soft splitting number

$$\mathbf{k}_{\bar{n}\perp} = -\hat{P}_{h\perp}/z$$

$$\zeta_F = Q^2/\alpha$$

$$\zeta_D = \alpha Q^2$$

A lot of TMD's

Mulders-Tangerman '96,
 Boer Mulders ' 98
 Mulders, 2001 (gluons)
 Boer, Mulders, Collins
 Mulders, Buffing, Mukherjee 2013

TMDPDF

Nucleon
Polarization

Quark Polarization

	U	L	T
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1, h_{1T}^\perp

Similar structures for
 Gluons as initial states: in
 Higgs production both f_1^g and $h_1^{\perp g}$

$$\tilde{F}_{f/N}^{[\gamma^+]}(x, \mathbf{b}_\perp, S; \zeta_F, \mu^2) = \tilde{f}_1 - \frac{\epsilon_\perp^{ij} \mathbf{b}_\perp^i \mathbf{S}_\perp^j}{ib_T M_N} \tilde{f}_{1T}^{\perp(1)},$$

$$\tilde{F}_{f/N}^{[\gamma^+ \gamma_5]}(x, \mathbf{b}_\perp, S; \zeta_F, \mu^2) = \lambda \tilde{g}_{1L} + \frac{(\mathbf{b}_\perp \cdot \mathbf{S}_\perp)}{ib_T M_N} \tilde{g}_{1T}^{(1)},$$

$$\tilde{F}_{f/N}^{[i\sigma^{i+} \gamma_5]}(x, \mathbf{b}_\perp, S; \zeta_F, \mu^2) = \mathbf{S}_\perp^i \tilde{h}_1 + \frac{\lambda \mathbf{b}_\perp^i}{ib_T M_N} \tilde{h}_{1L}^{\perp(1)}$$

$$- \frac{(\mathbf{b}_\perp^i \mathbf{b}_\perp^j + \frac{1}{2} b_T^2 g_\perp^{ij}) \mathbf{S}_\perp^j}{(i)^2 b_T^2 M_N^2} \tilde{h}_{1T}^{\perp(2)} - \frac{\epsilon_\perp^{ij} \mathbf{b}_\perp^j}{ib_T M_N} \tilde{h}_1^{\perp(1)}$$

T-odd distributions

$$f_{1T,DIS}^\perp = -f_{1T,DY}^\perp$$

TMDFF

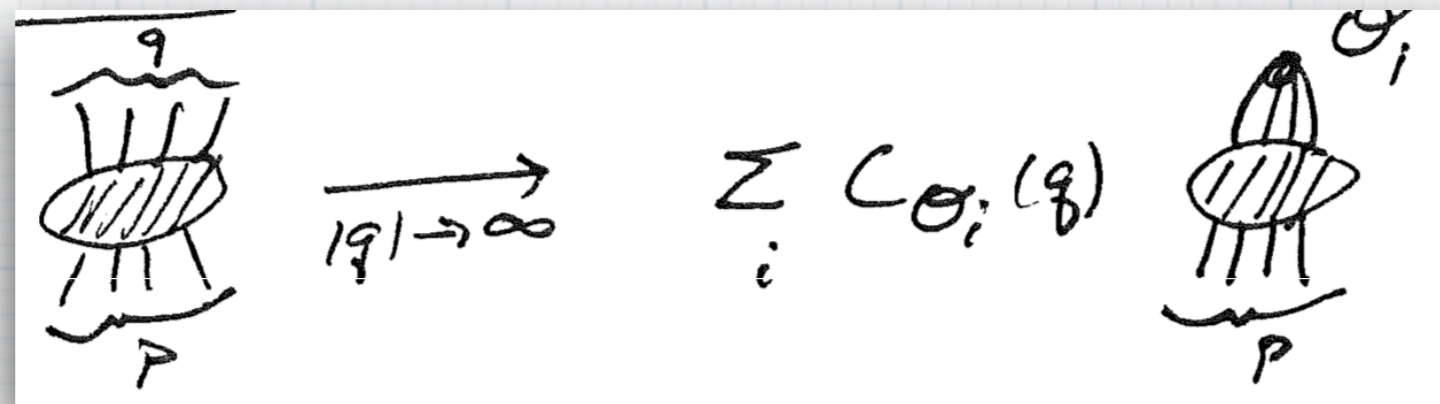
$$\tilde{D}_{h/f}^{[\gamma^-]}(z, \mathbf{b}_\perp, S_h; \zeta_D, \mu^2) = \tilde{D}_1 - \frac{\epsilon_\perp^{ij} \mathbf{b}_\perp^i \mathbf{S}_{h\perp}^j}{(-ib_T) M_h} \tilde{D}_{1T}^{\perp(1)},$$

$$\tilde{D}_{h/f}^{[\gamma^- \gamma_5]}(z, \mathbf{b}_\perp, S_h; \zeta_D, \mu^2) = \lambda \tilde{G}_{1L} + \frac{(\mathbf{b}_\perp \cdot \mathbf{S}_{h\perp})}{(-ib_T) M_h} \tilde{G}_{1T}^{(1)},$$

$$\tilde{D}_{h/f}^{[i\sigma^{i-} \gamma_5]}(z, \mathbf{b}_\perp, S_h; \zeta_D, \mu^2) = \mathbf{S}_{h\perp}^i \tilde{H}_1 + \frac{\lambda \mathbf{b}_\perp^i}{(-ib_T) M_h} \tilde{H}_{1L}^{\perp(1)}$$

$$- \frac{(\mathbf{b}_\perp^i \mathbf{b}_\perp^j + \frac{1}{2} b_T^2 g_\perp^{ij}) \mathbf{S}_{h\perp}^j}{(-ib_T)^2 M_h^2} \tilde{H}_{1T}^{\perp(2)} - \frac{\epsilon_\perp^{ij} \mathbf{b}_\perp^j}{(-ib_T) M_h} \tilde{H}_1^{\perp(1)}$$

Origin of factorization and EFT



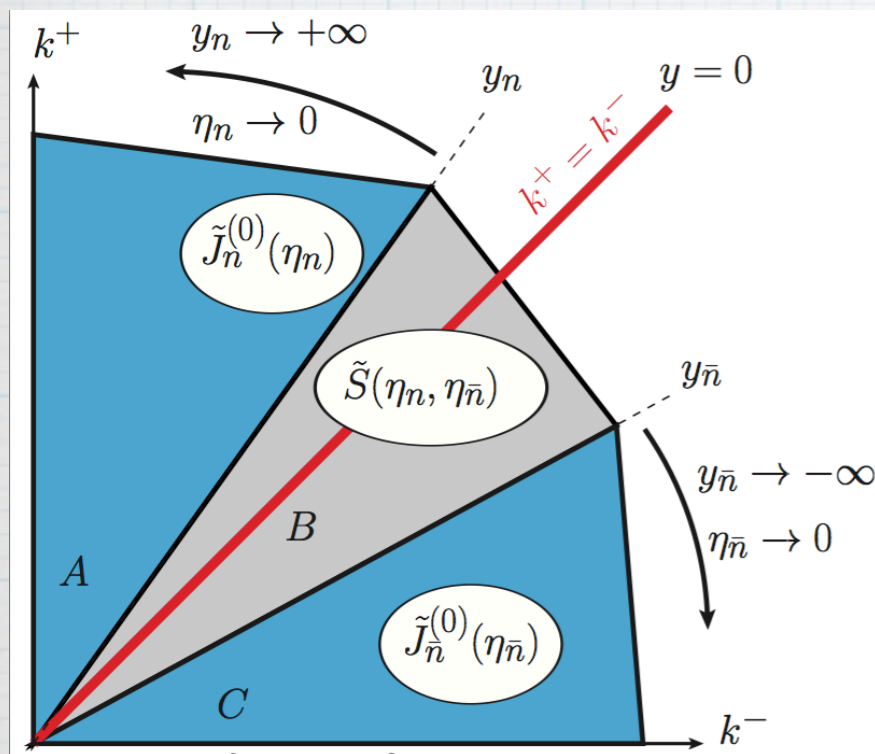
Drawing from J. Preskill lectures

In Green function, when the momenta "q" of some fields are much larger than the others we can "factor out" the hard momenta onto Wilson coefficient and effective matrix elements and use RGE on coefficients to resum large logs (Wilson, Zimmerman, Callan, Symanzyk '70-'72)

Wilson coefficients and hard factors depend only on the high scale "q" and the factorization scale " μ ": no other (IR-sensitive) scale.

Same philosophy in the construction of EFT: identification of modes, effective field theory Lagrangians, factorized cross sections, Wilson coefficients, ... 5

Factorization: SCET-II as an intermediate step



Using power counting we have collinear, anti-collinear, and soft sectors

$$H(Q^2) \tilde{J}_n^{(0)}(\eta_n) \tilde{S}(\eta_n, \eta_{\bar{n}}) \tilde{J}_{\bar{n}}^{(0)}(\eta_{\bar{n}})$$

None of these sectors is well defined: rapidity divergences

(+, -; perp)

$$k_n \sim Q(1, \lambda^2, \lambda) \rightarrow y \gg 0$$

$$k_{\bar{n}} \sim Q(\lambda^2, 1, \lambda) \rightarrow y \ll 0$$

$$k_s \sim Q(\lambda, \lambda, \lambda) \rightarrow y \approx 0$$

$$\lambda \sim \frac{q_T}{Q}$$

multipole expansion fixes arguments

$$J_n^{(0)}(0^+, y^-, y_\perp) = \frac{1}{2} \sum_{\sigma_1} \langle N_1(P, \sigma_1) | \bar{\chi}_n(0^+, y^-, y_\perp) \frac{\not{n}}{2} \chi_n(0) | N_1(P, \sigma_1) \rangle$$

$$J_{\bar{n}}^{(0)}(y^+, 0^-, y_\perp) = \frac{1}{2} \sum_{\sigma_1} \langle N_2(\bar{P}, \sigma_2) | \bar{\chi}_{\bar{n}}(0) \frac{\not{\bar{n}}}{2} \chi_{\bar{n}}(y^+, 0^-, y_\perp) | N_2(\bar{P}, \sigma_2) \rangle$$

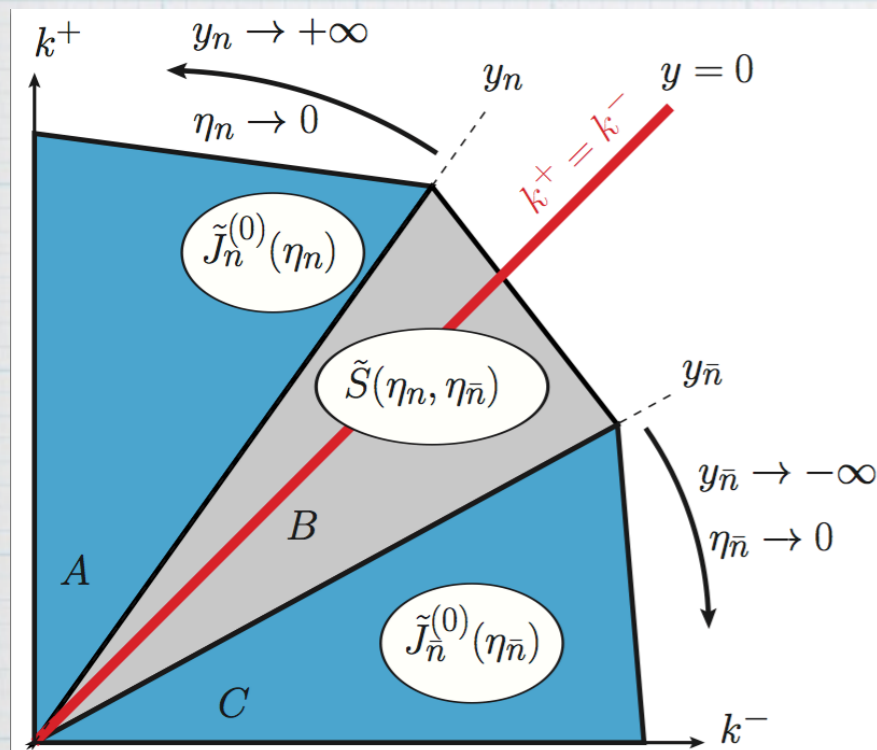
$$S(0^+, 0^-, y_\perp) = \langle 0 | \text{Tr} \bar{T} [S_n^{T\dagger} S_n^T](0^+, 0^-, y_\perp) T [S_{\bar{n}}^{T\dagger} S_{\bar{n}}^T](0) | 0 \rangle, \quad \chi = W^{T\dagger} \xi$$

Boosts mix soft and collinear modes (same invariant mass)

Rapidity divergences

- * Modes can be distinguished only by their rapidity, so need a rapidity regulator (Manohar, Stewart, 2006)
- * All properties of TMD are regulator independent
- * We performed our calculation on-the-light cone and using delta-regulator (Chiu, Fuhrer, Hoang Manohar, 2009). Checks with other regulators agree (Collins 2011, Chiu, Jain, Neill, Rothstein 2012, ..)

Rapidity divergences at one loop:



$$\tilde{J}_{n1}^{(0)}(\Delta^-) = \frac{\alpha_s C_F}{2\pi} \left\{ \delta(1-x) \left[\frac{2}{\epsilon_{UV}^2} - \frac{2}{\epsilon_{UV}} \ln \frac{\Delta^-}{\mu^2} + \frac{3}{2\epsilon_{UV}} - \frac{1}{4} - \frac{2\pi^2}{12} - L_T^2 \right] + \frac{3}{2} L_T - 2L_T \ln \frac{\Delta^-}{\mu^2} - (1-x) \ln(1-x) - \mathcal{P}_{q/q} \ln \frac{\Delta^-}{\mu^2} - L_T \mathcal{P}_{q/q} \right\}$$

Pure collinear matrix elements are "per se" ill defined

Definition of TMD's

Positive and negative rapidity quanta can be collected into 2 different TMDs because of the splitting of the soft function

$$\tilde{S}(b_T; \frac{Q^2 \mu^2}{\Delta^+ \Delta^-}, \mu^2) = \tilde{S}_-(b_T; \zeta_F, \mu^2; \Delta^-) \tilde{S}_+(b_T; \zeta_D, \mu^2; \Delta^+),$$

$$\tilde{S}_-(b_T; \zeta_F, \mu^2; \Delta^-) = \sqrt{\tilde{S}\left(\frac{\Delta^-}{p^+}, \alpha \frac{\Delta^-}{\bar{p}^-}\right)},$$

$$\tilde{S}_+(b_T; \zeta_D, \mu^2; \Delta^+) = \sqrt{\tilde{S}\left(\frac{1}{\alpha} \frac{\Delta^+}{p^+}, \frac{\Delta^+}{\bar{p}^-}\right)}$$

$$\zeta_F = Q^2 / \alpha$$

$$\zeta_D = \alpha Q^2$$

TMDPDF

$$\ln F_{ij}(x, \mathbf{b}_\perp, S; \zeta_F, \mu^2; \Delta^-) = \ln \tilde{\Phi}_{ij}^{(0)}(x, \mathbf{b}, S; \mu^2; \Delta^-) + \ln \tilde{S}_-(b_T; \zeta_F, \mu^2; \Delta^-)$$

TMDFF

$$\ln D_{ij}(x, \mathbf{b}_\perp, S_h; \zeta_D, \mu^2; \Delta^+) = \ln \tilde{\Delta}_{ij}^{(0)}(x, \mathbf{b}, S_h; \mu^2; \Delta^+) + \ln \tilde{S}_+(b_T; \zeta_D, \mu^2; \Delta^+)$$

Soft function: structure and properties

In the high q_T limit: $Q \gg q_T \gg \Lambda_{QCD}$ the hadronic tensor is

$$\tilde{M} = H(Q^2/\mu^2) \tilde{C}(x_n, z_{\bar{n}}, L_\perp, Q^2/\mu^2) f_n(x_n, \Delta^-/\mu^2) d_{\bar{n}}(z_{\bar{n}}, \Delta^+/\mu^2)$$

and PDF, f_n , and FF, d_n , have single log dependence on UV/IR cutoff (Korchemsky, Radyushkin 1987)

$$\ln f_n = \mathcal{R}_{f1}(x_n, \alpha_s) + \mathcal{R}_{f2}(x_n, \alpha_s) \ln \frac{\Delta^-}{\mu^2}$$

$$\ln d_{\bar{n}} = \mathcal{R}_{f1}(z_{\bar{n}}, \alpha_s) + \mathcal{R}_{f2}(z_{\bar{n}}, \alpha_s) \ln \frac{\Delta^+}{\mu^2}$$

Soft function: structure and properties

All this implies that each **pure** collinear and soft sectors are of the form

$$\ln \tilde{J}_n^{(0)} = \mathcal{R}_{n1}(x_n, \alpha_s, L_\perp) + \mathcal{R}_{n2}(x_n, \alpha_s, L_\perp) \ln \frac{\Delta^-}{\mu^2},$$

$$\ln \tilde{J}_{\bar{n}}^{(0)} = \mathcal{R}_{\bar{n}1}(x_{\bar{n}}, \alpha_s, L_\perp) + \mathcal{R}_{\bar{n}2}(x_{\bar{n}}, \alpha_s, L_\perp) \ln \frac{\Delta^+}{\mu^2},$$



$$\ln \tilde{S} = \mathcal{R}_{s1}(\alpha_s, L_\perp) + 2D(\alpha_s, L_\perp) \ln \frac{\Delta^- \Delta^+}{Q^2 \mu^2}.$$

- * Each sector is linear in logs of the rapidity scale
- * Each collinear sector depends just on 1 IR/rapidity regulator
- * Each collinear sector depends solely on the corresponding collinear momentum. It is not possible that a Q dependence arise in a pure collinear sector, Q dependence arises only in the soft sector.
- * The soft function is linear in the logs of rapidity regulator to cancel the corresponding logs in collinear sectors (In QCD there are no rapidity divergences)
- * The soft function is Hermitian, so it is the same for DIS, DY and ee to 2 jets

Splitting of the soft function

$$\ln \tilde{S} = \mathcal{R}_s(b_T, \alpha_s) + 2D(b_T, \alpha_s) \ln \left(\frac{\Delta^+ \Delta^-}{Q^2 \mu^2} \right)$$

$$\ln \tilde{S}_- = \frac{1}{2} \mathcal{R}_s(b_T, \alpha_s) + D(b_T, \alpha_s) \ln \left(\frac{(\Delta^-)^2}{\zeta_F \mu^2} \right),$$

$$\ln \tilde{S}_+ = \frac{1}{2} \mathcal{R}_s(b_T, \alpha_s) + D(b_T, \alpha_s) \ln \left(\frac{(\Delta^+)^2}{\zeta_D \mu^2} \right)$$

Using single log dependence

$$\zeta_F = Q^2 / \alpha$$

$$\zeta_D = \alpha Q^2$$

Q-dependence of TMD's

$$\frac{d}{d \ln \zeta_F} \ln \tilde{F}_{f/N}^{[\Gamma]}(x, \mathbf{b}_\perp, S; \zeta_F, \mu^2) = -D(b_T; \mu^2),$$

$$\frac{d}{d \ln \zeta_D} \ln \tilde{D}_{h/f}^{[\Gamma]}(z, \mathbf{b}_\perp, S_h; \zeta_D, \mu^2) = -D(b_T; \mu^2).$$

The Q-dependence of the TMD is dictated by the soft function:
spin independent

The D -function can be defined for arbitrary b_T as $\frac{d \ln S}{d \ln Q^2} \equiv -2D$

Evolution kernel for TMD's

$$\tilde{F}_{f/N}^{[\Gamma]}(x, \mathbf{b}_\perp, S; \zeta_{F,f}, \mu_f^2) = \tilde{F}_{f/N}^{[\Gamma]}(x, \mathbf{b}_\perp, S; \zeta_{F,i}, \mu_i^2) \tilde{R}(b_T; \zeta_{F,i}, \mu_i^2, \zeta_{F,f}, \mu_f^2),$$

$$\tilde{D}_{h/f}^{[\Gamma]}(z, \mathbf{b}_\perp, S_h; \zeta_{D,f}, \mu_f^2) = \tilde{D}_{h/f}^{[\Gamma]}(z, \mathbf{b}_\perp, S_h; \zeta_{D,i}, \mu_i^2) \tilde{R}(b_T; \zeta_{D,i}, \mu_i^2, \zeta_{D,f}, \mu_f^2),$$

$$\tilde{R}(b; \zeta_i, \mu_i^2, \zeta_f, \mu_f^2) = \exp \left\{ \int_{\mu_i}^{\mu_f} \frac{d\bar{\mu}}{\bar{\mu}} \gamma \left(\alpha_s(\bar{\mu}), \ln \frac{\zeta_f}{\bar{\mu}^2} \right) \right\} \left(\frac{\zeta_f}{\zeta_i} \right)^{-D(b_T; \mu_i)}$$

$$\gamma_H = -\gamma_F \left(\alpha_s(\mu), \ln \frac{\zeta_F}{\mu^2} \right) - \gamma_D \left(\alpha_s(\mu), \ln \frac{\zeta_D}{\mu^2} \right)$$

$$\gamma_{F,D} \left(\alpha_s(\mu), \ln \frac{\zeta_{F,D}}{\mu^2} \right) = -\Gamma_{\text{cusp}}(\alpha_s(\mu)) \ln \frac{\zeta_{F,D}}{\mu^2} - \gamma^V(\alpha_s(\mu))$$

$$\frac{dD}{d \ln \mu} = \Gamma_{\text{cusp}}$$

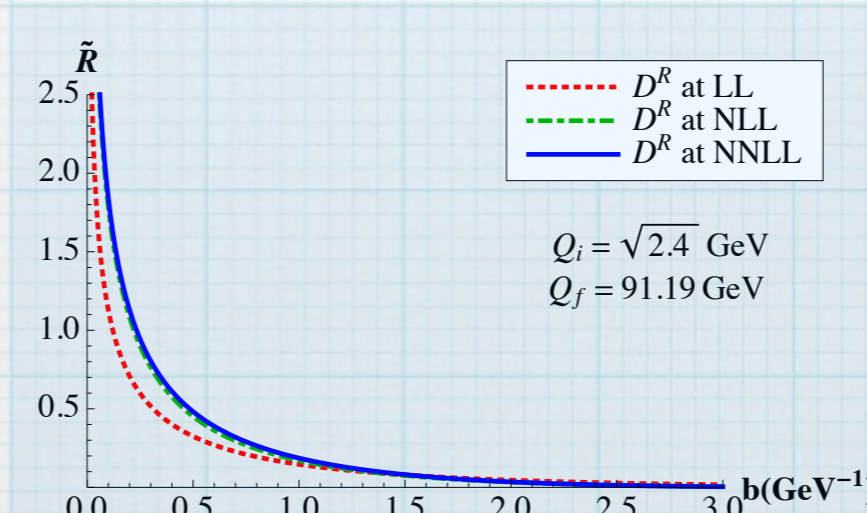
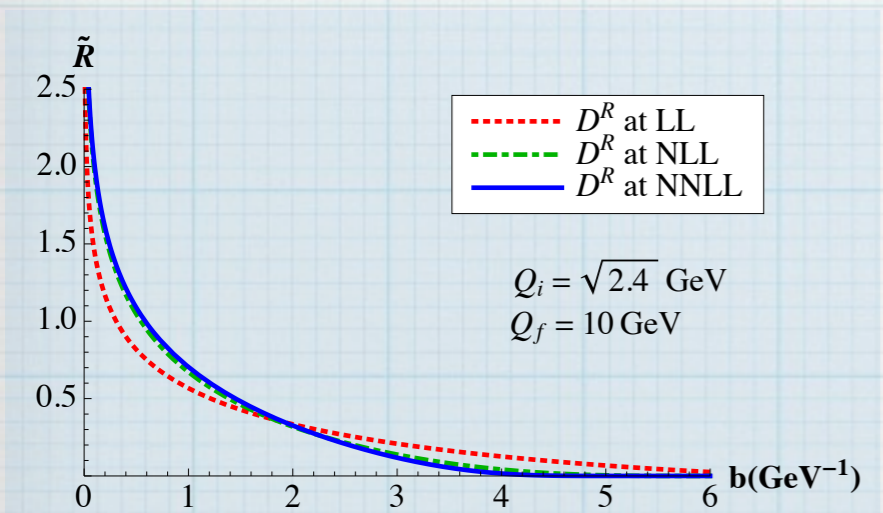
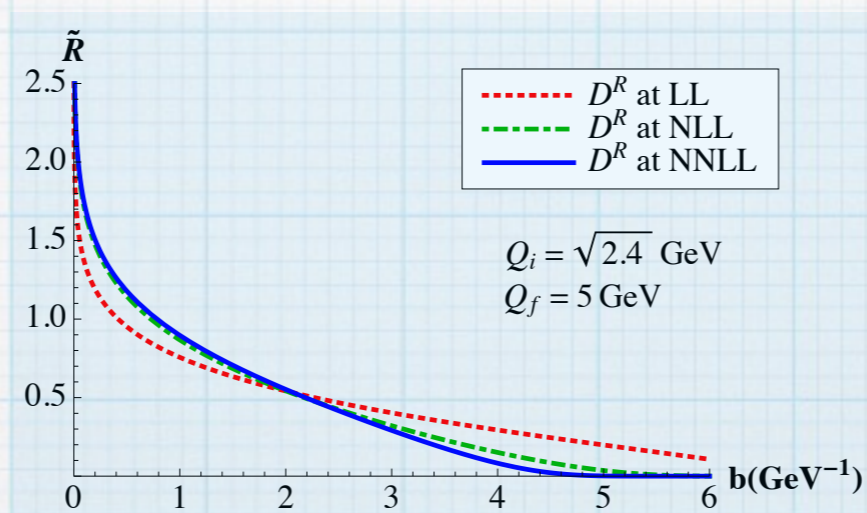
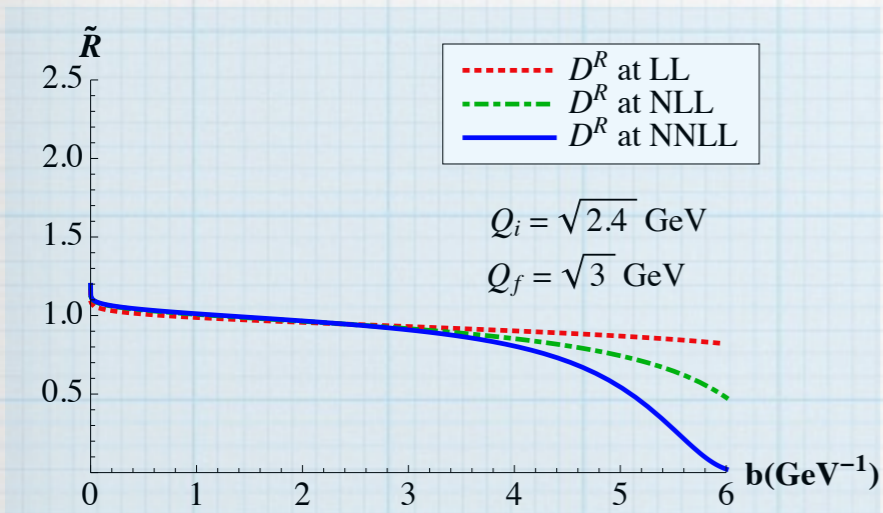
The perturbative expansion of the \mathcal{D} is valid in limited (but large, using **resummation**) portion of Impact Parameter Space

$$D(b; Q_i) = D(b; \mu_b) + \int_{\mu_b}^{Q_i} \frac{d\bar{\mu}}{\bar{\mu}} \Gamma_{\text{cusp}}; \quad \mu_b = 2e^{-\gamma_E}/b$$

$$D(b; Q_i) = -\frac{\Gamma_0}{2\beta_0} \ln \frac{\alpha_s(Q_i)}{\alpha_s(\mu_b)} \longrightarrow D(b; Q_i) = -\frac{\Gamma_0}{2\beta_0} \ln(1 - X) \quad \text{Landau pole}$$

$$\alpha_s(\mu_b) = \alpha_s(Q)/(1 - X)$$

Plots for evolution kernel



* Very good convergence up to $b=4-5/\text{GeV}$ in all cases

* The region sensitive to the Landau pole is strongly suppressed $b > 5/\text{GeV}$

* For $Q_f = M_z$ we are sensitive only to $b < 1.5/\text{GeV}$ region

* For $Q_f = 3-5 \text{ GeV}$ we are sensitive only to $b < 4/\text{GeV}$ region

* For $Q_f < 2 \text{ GeV}$ we can be sensitive to the Landau pole region

Studying processes at different energies one explores different regions in IPS

Unpolarized TMDPF: construction and fits

* Basic test, preliminary to all spin dependent analysis, many ingredients as in standard QCD.

* More or less standard recipe for TMD construction (CSS, ...):

o take the asymptotic limit of the TMDPDF

$$\tilde{F}_{q/N}(x, \vec{b}, Q_i, \mu) = \left(\frac{Q_i^2 b^2}{4e^{2\gamma_E}} \right)^{-D_R(b, \mu)} \sum_j \tilde{C}_{q \leftarrow j}(x, \vec{b}_\perp, \mu) \otimes f_{j/N}(x; \mu) \otimes M_q(x, \vec{b}, Q_i)$$

OPE to PDF, valid for $q_T \gg \Lambda_{QCD}$

PDF

Process independent
Non-perturbative correction

Common to all analysis:
Florence (Catani et al.), Zurich (Gehrmann. et al)

o Exponentiation of part of the coefficient and **complete resummation of the logs in the exponent**

(Kodaira, Trentadue 1982, Becher, Neubert Wilhelm 2011)

$$\tilde{C}_{q \leftarrow j}(x, \vec{b}_\perp, \mu) \equiv \exp(h_\Gamma - h_\gamma) \hat{C}_{q \leftarrow j}(x, \vec{b}_\perp, \mu)$$

$$\frac{dh_\Gamma}{d \ln \mu} = \Gamma_{cusp} L_\perp$$

$$\frac{dh_\gamma}{d \ln \mu} = \gamma^V$$

$$h_\Gamma^R(b, \mu) = \int_{\alpha_s(1/\hat{b})}^{\alpha_s(\mu)} d\alpha' \frac{\Gamma_{cusp}^F(\alpha')}{\beta(\alpha')} \int_{\alpha_s(1/\hat{b})}^{\alpha'} \frac{d\alpha}{\beta(\alpha)}$$

and finally write $a(1/b)$ in terms of $a(\mu)$ and fix $\mu=Q_i$.
Logs are minimized with the choice $Q_i=Q_0+q_T$

Experimental Data

	E288 200	E288 300	E288 400	R209
points	35	35	49	6
\sqrt{s}	19.4 GeV	23.8 GeV	27.4 GeV	62 GeV
E_{beam}	200 GeV	300 GeV	400 GeV	-
Beam/Target	p Cu	p Cu	p Cu	p p
M range used	4-9 GeV	4-9 GeV	5-9 and 10.5-14 GeV	5-8 and 11-25 GeV
Other kin. var	$y=0.4$	$y=0.21$	$y=0.03$	
Observable	$Ed^3\sigma/d^3p$	$Ed^3\sigma/d^3p$	$Ed^3\sigma/d^3p$	$d\sigma/dq_T^2$

Expected to be insensitive to Landau pole region



	CDF Run I	D0 Run I	CDF Run II	D0 Run II
points	32	16	41	9
\sqrt{s}	1.8 TeV	1.8 TeV	1.96 TeV	1.96 TeV
σ	248 ± 11 pb	221 ± 11.2 pb	256 ± 15.2 pb	255.8 ± 16.7 pb

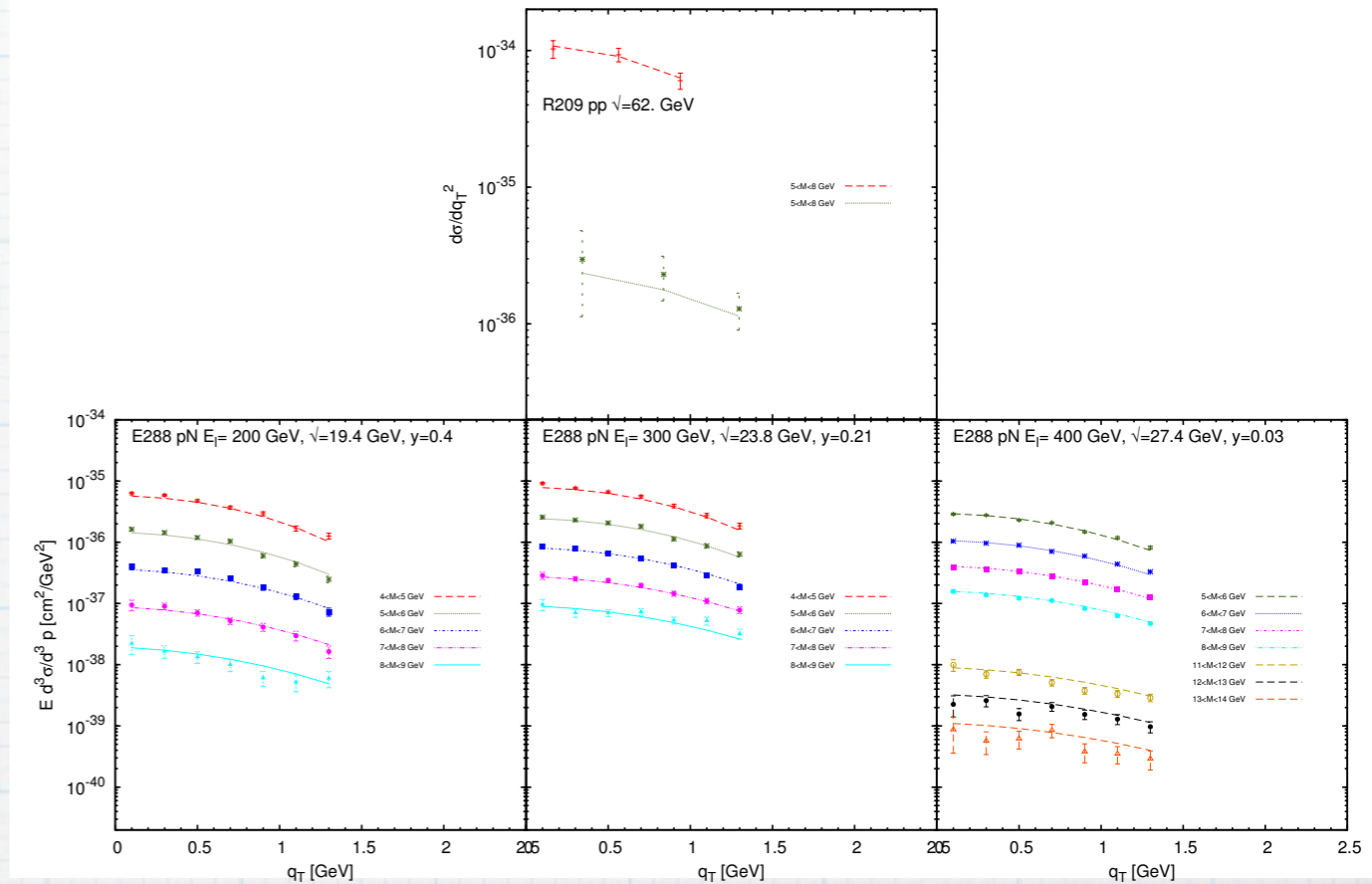
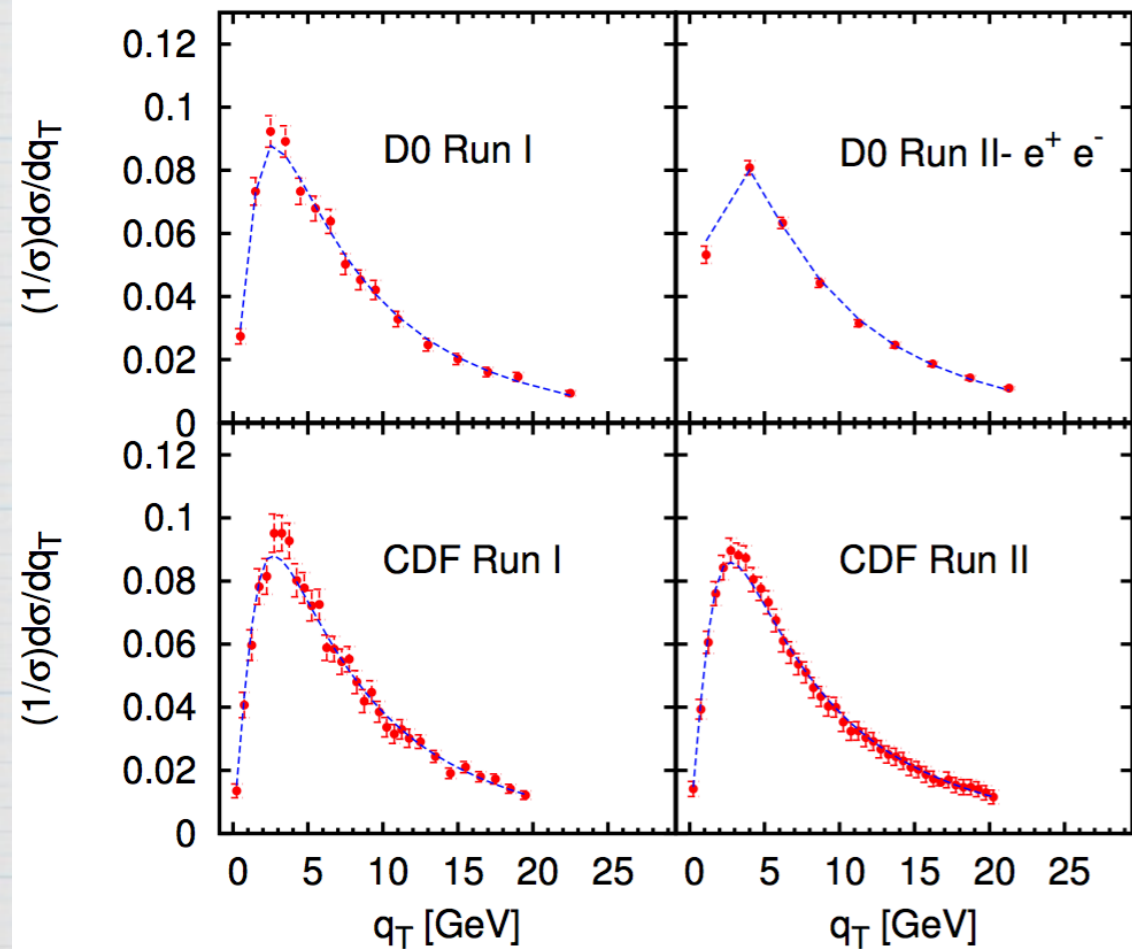
Z, run I: Becher, Neubert, Wilhelm 2011
Catani et al. 2009

Theoretical settings

- * Two matching scales used; fixed $Q_i=2$ GeV, and not fixed $Q_i=2$ GeV+ q_T : similar results at NNLL
- * Checked both NLL and NNLL
- * Several sets of PDF checked (MSTW, CTEQ)
- * Checked several form of non-perturbative models: gaussian, exponential, Q-dependence, ...

$$M_q(x, \vec{b}, Q_i) = \exp[-\lambda_1 b](1 + b^2 \lambda_2 + \dots)$$

Preliminary results at NNLL



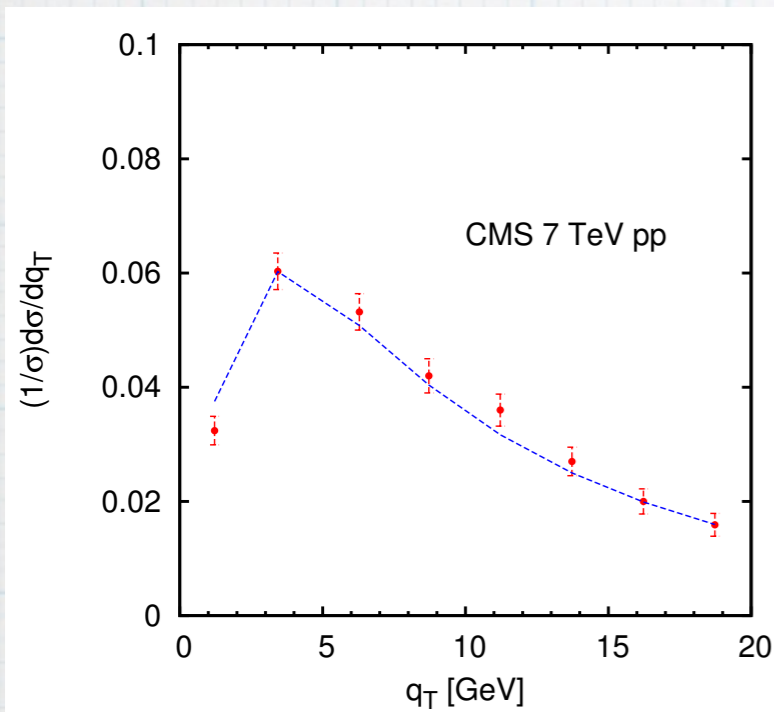
$\lambda_1 = 0.35 \pm \dots \text{ GeV}$ Error fixing
 $\lambda_2 = 0.17 \pm \dots \text{ GeV}^2$ in progress

Z-boson data are (fairly) sensitive to functional non-perturbative form (gaussian vs exponential) and (poorly) sensitive just to λ_1 . In order to fix it we need the global fit

$$\frac{\chi^2}{\text{points}} \Big|_Z \simeq 0.6$$

$$\frac{\chi^2}{\text{points}} \Big|_{\text{global}} \simeq 1.1$$

Preliminary results at NNLL



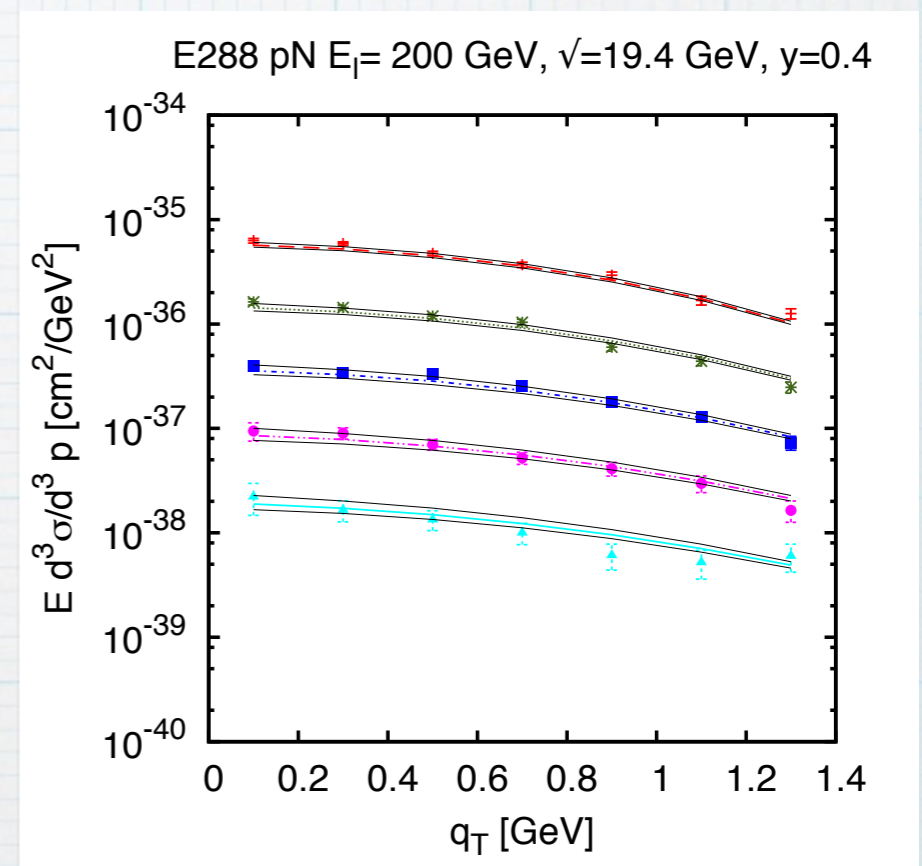
Prediction for Z production with CMS data.
Bin width not shown

$$\frac{\chi^2}{\text{points}} \simeq 0.9$$

Scale dependence

$$m_c \sim 1.3 \text{ GeV} < Q_0 < 2.7 \text{ GeV}$$

1st bin energy



- * Data are not sensitive to: the Landau pole region in IPS, non-perturbative Q -dependence. The study of flavor dependence needs to include also W -production data....

Conclusions

- * The correct measurement of non-perturbative effects in transverse momentum dependent observables requires the definition of TMDs
- * Pure collinear and soft matrix elements are not well defined when the dependence on transverse momentum is included. The proper transverse momentum distributions must be defined as a combination of collinear and soft parts (rapidity divergence problem).
- * The soft matrix elements are responsible for Q -dependence of TMD's. TMD's are universal (the same for DIS, DY, $ee \rightarrow 2j$). **The evolution of TMDPDF and TMDFF is the same and spin independent.**
- * **First fits for unpolarized TMDPDF in DY. Data with $4 < Q/\text{GeV} < 10$ can fix non-perturbative parameters, which have some impact on vector boson production and DY processes in LHC. More data required. SIDIS and $ee \rightarrow 2j$ analysis to be done.**
- * **TMD non-perturbative QCD effects should be included in high precision LHC observables.**
- * **Analysis of spin dependent observables including evolution is starting now. Data from Belle, Compass, JLab, LHC..**