Variable flavor number scheme for final state jets

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based on work with
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Outline

1. Classical DIS: $x \sim \mathcal{O}(1)$
2. DIS in the endpoint region $x \to 1$
3. Event shapes in e^+e^--collisions
4. Summary

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Factorization for $x \sim \mathcal{O}(1)$

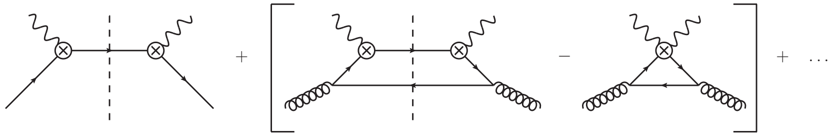
Factorization theorem for massless quarks:

$$F_{1,2} \sim \sum_{i=q,\bar{q}} \sum_{j=q,\bar{q},g} H_{i,j}(\mu_H) \otimes \Phi_{j/P}(\mu_\Phi)$$

Collins, Soper, Sterman (1988)

Bauer, Fleming, Pirjol, Rothstein, Stewart (2002)

$\rightarrow H_{i,j}(\mu_H \sim Q)$: matching between full QCD and low energy description



$\rightarrow \Phi_{j/P}(\mu_\Phi \sim \Lambda_{\mathrm{QCD}})$: pdf = matrix element of low energy effective operator

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goals for factorization with massive quarks ($m \gg \Lambda_{\rm QCD}$):

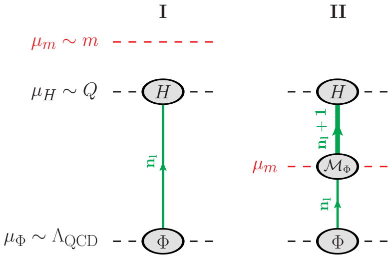
$\rightarrow$ resummation of all logarithms $\ln\left(\frac{Q}{\Lambda_{\rm QCD}}\right)$, $\ln\left(\frac{m}{\Lambda_{\rm QCD}}\right)$, $\ln\left(\frac{Q}{m}\right)$

$\rightarrow$ correct limits for $H_{i,j}$ (decoupling for $m \rightarrow \infty$ + massless limit for $m \rightarrow 0$)

$\rightarrow$ continuous description for arbitrary masses

$\Rightarrow$ use CWZ renormalization for pdfs and α_s: ACOT scheme

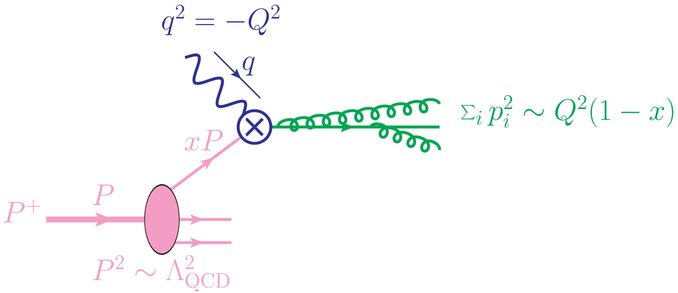
Collins, Wilczek, Zee (1978), Aivazis, Collins, Olness, Tung (1994)



Outline

Scales for $x \to 1$

- $x \to 1$: experimentally barely accessible (small pdfs!)
  but: nontrivial factorization setup → interesting as a showcase for concepts
- use factorization theorem for $x \sim \mathcal{O}(1)$?
  → unresummed logarithms in $H_{i,j}$: $\ln(1-x) = \ln\left(\frac{Q^2(1-x)}{Q^2}\right)$ ↯
  ↔ additional scale: final state jet invariant mass $\sum_i p_i^2 = s \sim Q^2(1-x)$
- here: $1 - x \gg \Lambda^2_{\mathrm{QCD}}/Q^2 \to s \gg \Lambda^2_{\mathrm{QCD}}$



Massless factorization theorem for $x \to 1$

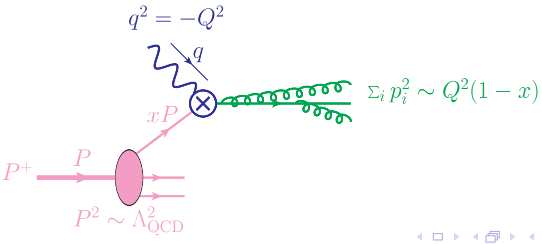
Factorization theorem for massless quarks:

$$F_{1,2} \sim \sum_{i=q,\bar{q}} H_{\text{DIS}}(\mu_H) J_{\text{DIS}}(\mu_J) \otimes \Phi_{i/P}(\mu_\Phi)\,[1 + \mathcal{O}(1-x)]$$

`Sterman 1987, Manohar (2003), Becher, Neubert, Pecjak (2006), ...`

Ingredients:

- at $\mu_H \sim Q$: hard function $H_{\text{DIS}}(\mu_H) = |C(\mu_H)|^2$
  $\to$ $C(\mu_H)$: current matching between full QCD and low-energy description (local!)
- at $\mu_J \sim Q\sqrt{1-x}$: final state jet function $J_{\text{DIS}}(\mu_J)$
  $\to$ jet rate in terms of its invariant mass (nonlocal!)
- at $\mu_\Phi \sim \Lambda_{\text{QCD}}$: pdf $\Phi_{q/P}(\mu_\Phi)$

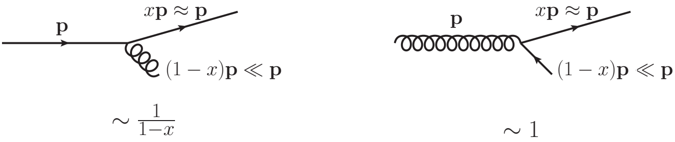


Massive quark effects

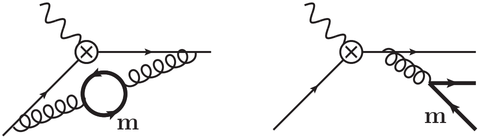
Factorization theorem for massless quarks:

$$F_{1,2} \sim \sum_{i=q,\bar{q}} H_{\rm DIS}(\mu_H) J_{\rm DIS}(\mu_J) \otimes \Phi_{i/P}(\mu_\Phi)$$

- Note: only flavor-diagonal (non-singlet) contributions in matching and evolution



- for massive quarks: massive threshold corrections also flavor-diagonal
  $\Rightarrow$ no generation of massive quarks as initial state of the hard interaction
  $\Rightarrow$ only “secondary“ massive corrections to light quark inititated processes



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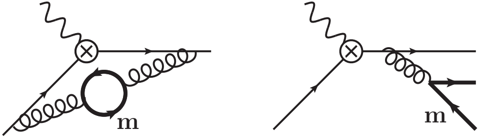


- aim: factorization setup with secondary massive quarks incorporating
  $\rightarrow$ summation of large logarithms
  $\rightarrow$ correct limits for H_{DIS}, J_{DIS} (decoupling for $m \to \infty$ + massless limit for $m \to 0$)
  $\rightarrow$ continuous behavior in between with correct LO terms in the power counting

$\Rightarrow$ achieved by proper renormalization conditions $\leftrightarrow$ CWZ

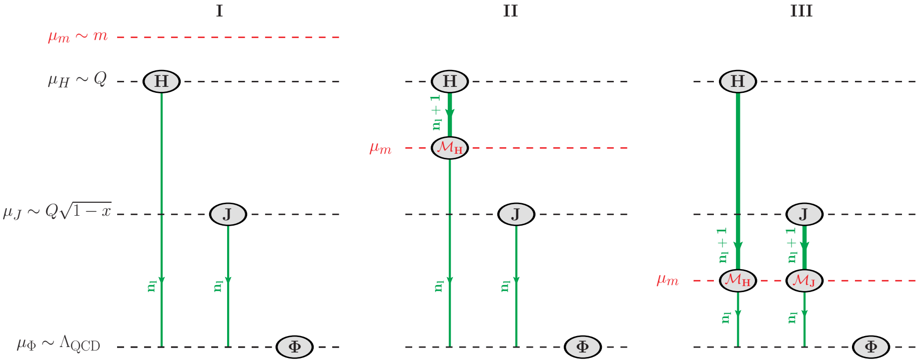
Gritschacher, Hoang, Jemos, P.P. (2013),
Gritschacher, Hoang, Jemos, Mateu, P.P. (2014)

Mass factorization: Overview

scaling hierarchies for a heavy quark ($m \gg \Lambda_{\rm QCD}$) in the endpoint region ($1 - x \ll 1$):

$$\text{I. } m > Q\,, \quad \text{II. } Q > m > Q\sqrt{1-x}\,, \quad \text{III. } Q\sqrt{1-x} > m > \Lambda_{\rm QCD}\,,$$

here: top-down evolution → final renormalization scale $\mu = \mu_\Phi$



Factorization theorems

- I. $m > Q$: use OS renormalization for current, jet function, pdf and α_s

$$\boxed{\frac{d\sigma}{d\tau} \sim H_l^{(n_l)}(\mu_H) U_H^{(n_l)}(\mu_H, \mu_\Phi)\, J^{(n_l)}(\mu_J) \otimes U_J^{(n_l)}(\mu_J, \mu_\Phi) \otimes \Phi^{(n_l)}(\mu_\Phi)}$$

only full QCD contributions to hard current matching $\to H_l^{(n_l)}(\mu_H)$
$\Rightarrow$ decoupling for $m \gg Q$, but mass-singularities for $m \to 0$

Factorization theorems

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$$\frac{d\sigma}{d\tau} \sim H_I^{(n_l)}(\mu_H) U_H^{(n_l)}(\mu_H, \mu_\Phi)\, J^{(n_l)}(\mu_J) \otimes U_J^{(n_l)}(\mu_J, \mu_\Phi) \otimes \Phi^{(n_l)}(\mu_\Phi)$$

- II. $Q > m > Q\sqrt{1-x}$: use $\overline{\mathrm{MS}}$ renormalization for current and α_s above μ_m

$$\frac{d\sigma}{d\tau} \sim H_{II}^{(n_l+1)}(\mu_H) U_H^{(n_l+1)}(\mu_H, \mu_m)\, \mathcal{M}_H(\mu_m) U_H^{(n_l)}(\mu_m, \mu_\Phi) \times J^{(n_l)}(\mu_J) \otimes U_J^{(n_l)}(\mu_J, \mu_\Phi) \otimes \Phi^{(n_l)}(\mu_\Phi)$$

$\to$ finite subtractions to $H_I^{(n_l)}(\mu_H)$ due to different scheme (non-vanishing low energy current diagrams!)

$\Rightarrow$ $H_{II}^{(n_l+1)}(\mu_H)$ has correct massless limit for $m \ll Q$

below μ_m: OS renormalization

$\to$ massive threshold contribution $\mathcal{M}_H(\mu_m) \leftrightarrow$ scheme change

Factorization theorems

- I. $m > Q$: use OS renormalization for current, jet fct, pdf and α_s

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- III. $Q\sqrt{1-x} > m$: use $\overline{\text{MS}}$ renormalization for current, jet fct and α_s above μ_m

$$\frac{d\sigma}{d\tau} \sim H_{II}^{(n_l+1)}(\mu_H) U_H^{(n_l+1)}(\mu_H, \mu_m)\, \mathcal{M}_H(\mu_m) U_H^{(n_l)}(\mu_m, \mu_\Phi) \times J^{(n_l+1)}(\mu_J) \otimes U_J^{(n_l+1)}(\mu_J, \mu_m) \otimes \mathcal{M}_J(\mu_m) \otimes U_J^{(n_l)}(\mu_m, \mu_\Phi) \otimes \Phi^{(n_l)}(\mu_\Phi)$$

$\rightarrow$ modification of the jet function due to massive quark contributions
$\Rightarrow$ correct massless limit for $m \ll Q\sqrt{1-x}$

$$J^{(n_l+1)}(s, m, \mu_J) = J_0^{(n_l+1)}(s, \mu_J) + \delta J_m^{\text{dist}}(s, m, \mu_J) + \theta(s - 4m^2)\delta J_m^{\text{real}}(s, m)$$

below μ_m: OS renormalization
$\rightarrow$ massive threshold contribution $\mathcal{M}_J(\mu_m) \leftrightarrow$ scheme change

Massive threshold corrections

Example: threshold correction in jet sector

bare jet function:

$$J^{\text{bare}} = Z_J^{OS} \otimes J^{\text{OS}} = Z_J^{\overline{\text{MS}}} \otimes J^{\overline{\text{MS}}}$$

in OS renormalization:

$$J^{\text{OS}}(s, m, \mu) = J^{(n_l)}(s, \mu) + \theta(s - 4m^2)\delta J_m^{\text{real}}(s, m) \xrightarrow{m \gg s} J^{(n_l)}(s, \mu)$$

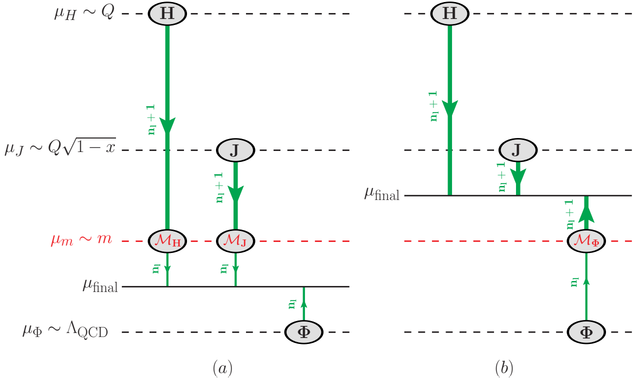
in $\overline{\text{MS}}$ renormalization:

$$J^{\overline{\text{MS}}}(s, m, \mu) = J^{(n_l+1)}(s, \mu) + \delta J_m^{\text{dist}}(s, m, \mu) + \theta(s - 4m^2)\delta J_m^{\text{real}}(s, m) \xrightarrow{m \ll s} J^{(n_l+1)}(s, \mu)$$

$$\boxed{\Rightarrow \mathcal{M}_J(s, m, \mu) = J^{\text{OS}}(s, m, \mu) \otimes (J^{\overline{\text{MS}}}(s, m, \mu))^{-1}}$$

$\rightarrow$ matching condition directly related to jet function
$\rightarrow$ continuity by construction

Consistency conditions for $Q\sqrt{1-x} > m > \Lambda_{\rm QCD}$



physical cross section independent of $\mu_{\rm final}$ $\to$ (a) and (b) equivalent
$\to$ relation between evolution factors

$$U_H^{(n_f)} \times U_J^{(n_f)} = \left(U_\Phi^{(n_f)}\right)^{-1} \quad \text{for } n_f = n_l, n_l+1$$

$\to$ relation between massive threshold contributions

$$\boxed{\mathcal{M}_H \times \mathcal{M}_J = \mathcal{M}_\Phi}$$

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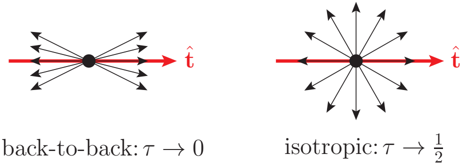
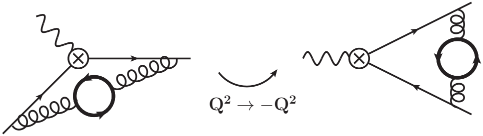
Event shapes

- event shapes: geometric description of final state kinematics
- thrust: $\tau \equiv 1 - \max_{\hat{\mathbf{t}}} \frac{\sum_i |\hat{\mathbf{t}}\cdot\vec{p}_i|}{\sum_i E_i} \in [0, \frac{1}{2}]$



- here endpoint region $\tau \to 0$ very important: large values for thrust distribution!
- discussion for event shapes: analogous to DIS (crossed process)



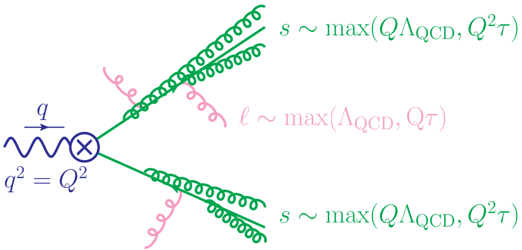
Factorization theorem for massless quarks

Massless factorization theorem for $\tau \ll 1$:

$$\frac{d\sigma}{d\tau} \sim H_\tau(\mu_H) J_\tau(\mu_J) \otimes S_\tau(\mu_S) \left[1 + \mathcal{O}(\tau)\right]$$

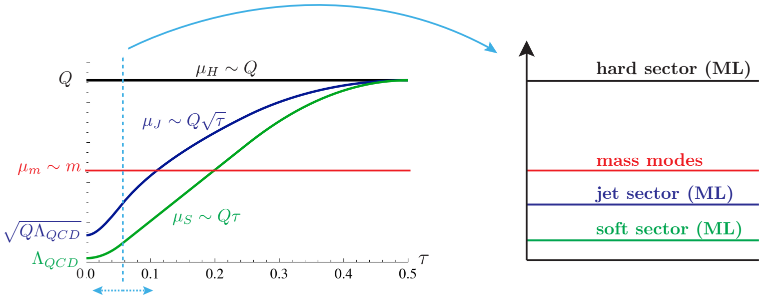
Berger, Kucs, Sterman (2003), Fleming, Hoang, Mantry, Stewart (2007), Bauer, Fleming, Lee, Sterman (2008),...

- compared to DIS: $H_\tau = H_{\rm DIS}(Q^2 \to -Q^2)$, $J_\tau \to 2J_{\rm DIS}$
- main difference concerns soft physics: $S_\tau \leftrightarrow \Phi_{i/P}$
  $\rightarrow$ in tail region ($\tau \gg \Lambda_{\rm QCD}/Q$): $\mu_S \sim Q\tau \gg \Lambda_{\rm QCD}$: $S_\tau = \hat{S} \otimes S^{\rm model}$



Scale hierarchies with massive quarks

- profile functions: Parametrization of renormalization scales in terms of thrust
  $\rightarrow$ continuous transition between peak, tail and far-tail region
- include massive quark effects $\rightarrow$ scales and hierarchies:



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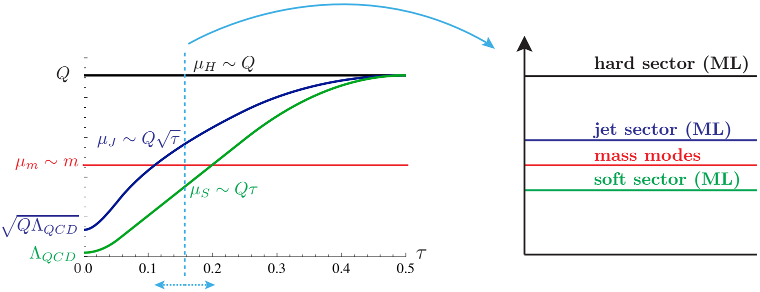

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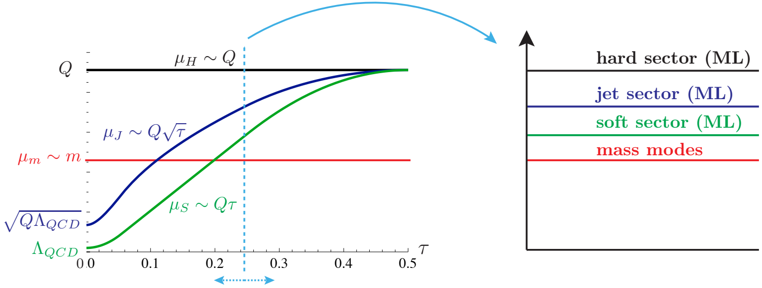
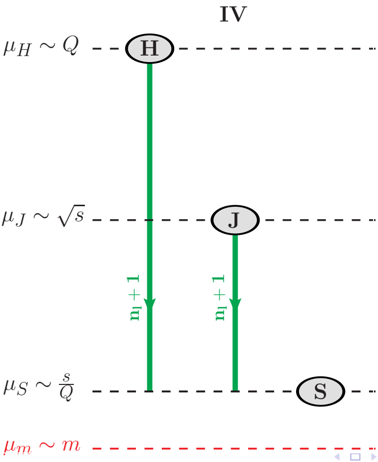


Setup for secondary massive quarks

- Setup for event shapes = Setup for DIS (same structure for factorization theorem)
- now: additional hierarchy possible $m < Q\tau \sim \mu_S$
  $\rightarrow \overline{\mathrm{MS}}$ renormalization for all structures
  $\Rightarrow$ evolution always including massive flavor, massive contributions to soft function



Outline

Summary & Outlook

- setup for secondary massive quark effects with various kinematical scales
- use of proper renormalization schemes crucial for resummation of all logarithms and correct limiting behavior
- universal structures due to consistency conditions
- calculation of ingredients for DIS (for $x \to 1$) and thrust (for $\tau \to 0$) at $\mathcal{O}(\alpha_s^2)$
- possible applications:
  1. analysis of low energy collider data (e.g. event shapes)
  2. application to other endpoint processes (e.g. hard photoproduction)
  3. mass effects on TMDPDFs/beam functions
  4. ...

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Thank you!

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$$F_{1,2} \sim \sum_{i=q,\bar{q}} H_{\rm DIS}(\mu_H) J_{\rm DIS}(\mu_J) \otimes \Phi_{i/P}(\mu_\Phi)\,[1+\mathcal{O}(1-x)]\ (*)$$

- where is this factorization theorem valid?
  $\to$ naive expectation: for $1-x \ll 1$ (say $x \gtrsim 0.9$?)
  $\to$ however: dominant terms from $\xi \approx x$ due to pdf suppression $\leftrightarrow$ $x/\xi \approx 1$

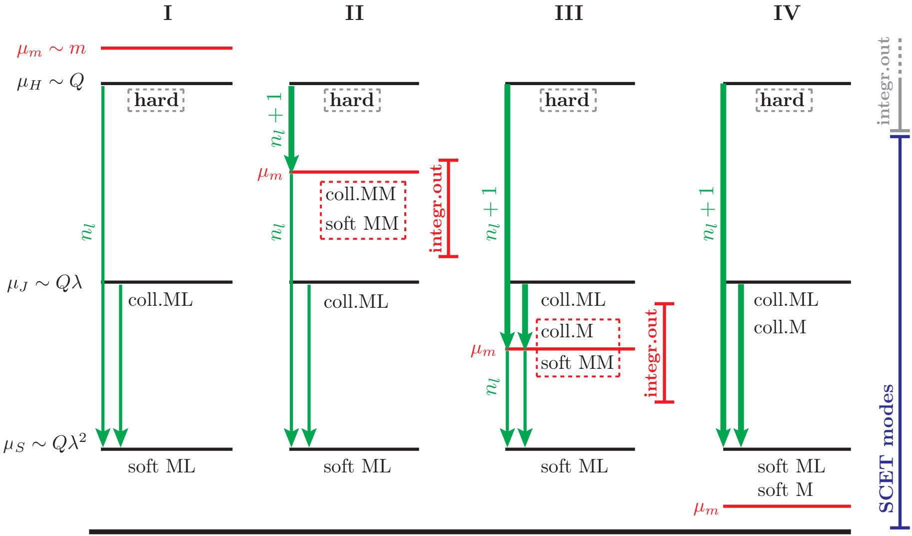
$$F_{1,2} \sim \sum_{i=q,\bar{q}} \sum_{j=q,\bar{q},g} \int_x^1 \frac{d\xi}{\xi} H_{i,j}\left(\frac{x}{\xi}\right) \underbrace{\Phi_{j/P}(\xi)}_{\sim(1-\xi)^n}$$

  $\to$ factorization theorem $(*)$ can be sensible even up to $x \sim 0.5$
- full description including all relevant terms in classical and endpoint region

$$F_{1,2} \sim \sum_{i=q,\bar{q}} H_{\rm DIS} J_{\rm DIS} \otimes \Phi_{i/P} + \sum_{i=q,\bar{q}} \sum_{j=q,\bar{q},g} \left(H_{i,j} - H_{i,j}^{\rm singular}\right) \otimes \Phi_{j/P}$$

  $\to$ singular terms in $H_{i,j}$= leading terms for $x \to 1$

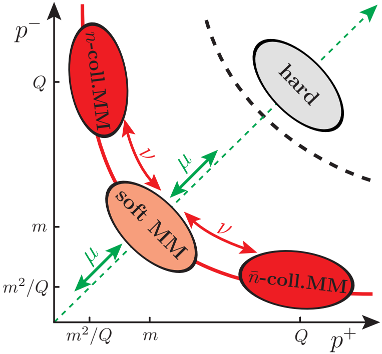
Mass mode setup: Summary



MM = mass-mode, ML = massless, M = massive

Rapidity logs in massive threshold coefficients

Massive threshold coefficients contain rapidity logarithms
e.g. in $\mathcal{M}_H$: large logarithm $\sim \ln\left(\frac{Q^2}{m^2}\right)$

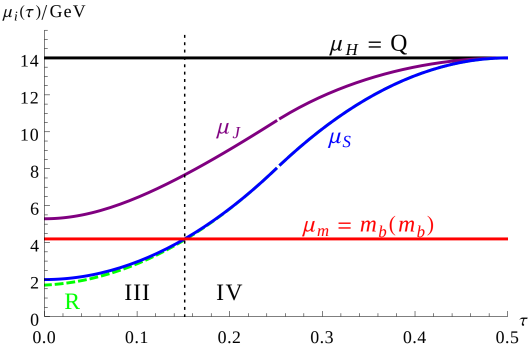


$\rightarrow$ rapidity RGE/collinear anomaly: resummation via exponentiation

`Chiu, Golf, Kelley, Manohar (2008), Becher, Neubert (2011)`
`Chiu, Jain, Neill, Rothstein (2012)`

Analysis of secondary massive bottom effects

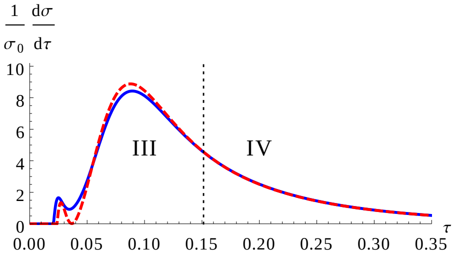
- analysis for $Q = 14, 22, 35$ GeV $\leftrightarrow$ bottom mass effects relevant
- ingredients for analysis at $\mathcal{O}(\alpha_s^2)$ in the dijet region $\tau \ll 1$ $\surd$
- numerical code incl. a nonperturbative model function $\surd$
- profile functions for $Q = 14$ GeV:



Secondary massive bottom effects for $Q = 14$ GeV

comparison between massless and massive thrust distribution
ML: $n_l = 5$, M: $n_l = 4$ & massive b ($m_b = 4.2$ GeV)

massive vs. massless



relative deviation massive vs. massless
$\mu_m = m$, $\mu_m = m/2$, $\mu_m = 2m$