# Detailed study of the $K_{e4}$ decay mode properties

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on behalf of the NA48/2 Collaboration

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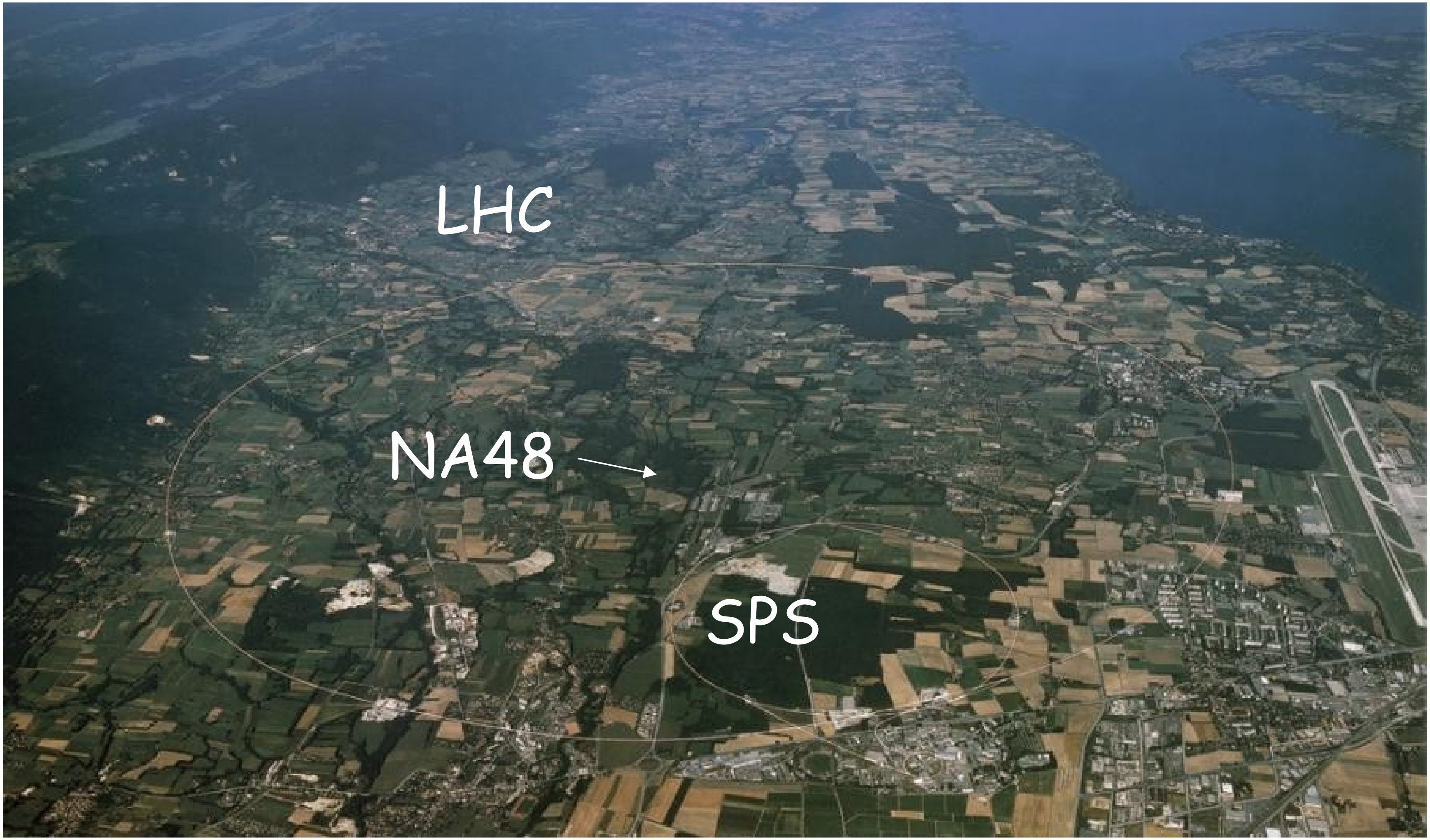
XII International Workshop on Deep-Inelastic Scattering 2014 and Related Subjects

# Outline

- NA48/2 beam line and detector
- Ke4 introduction
- **NA48/2:** $K_{e4}$ event selection, Form Factors and Bramching ratios:
    - $K^{\pm} \rightarrow \pi^{+}\pi^{-}e^{\pm}\nu$, called $K_{e4}(+-)$
    - $K^{\pm} \rightarrow \pi^{0}\pi^{0}e^{\pm}\nu$, called $K_{e4}(00)$
- Summary

# NA48/2 – a fixed target experiment at CERN SPS.

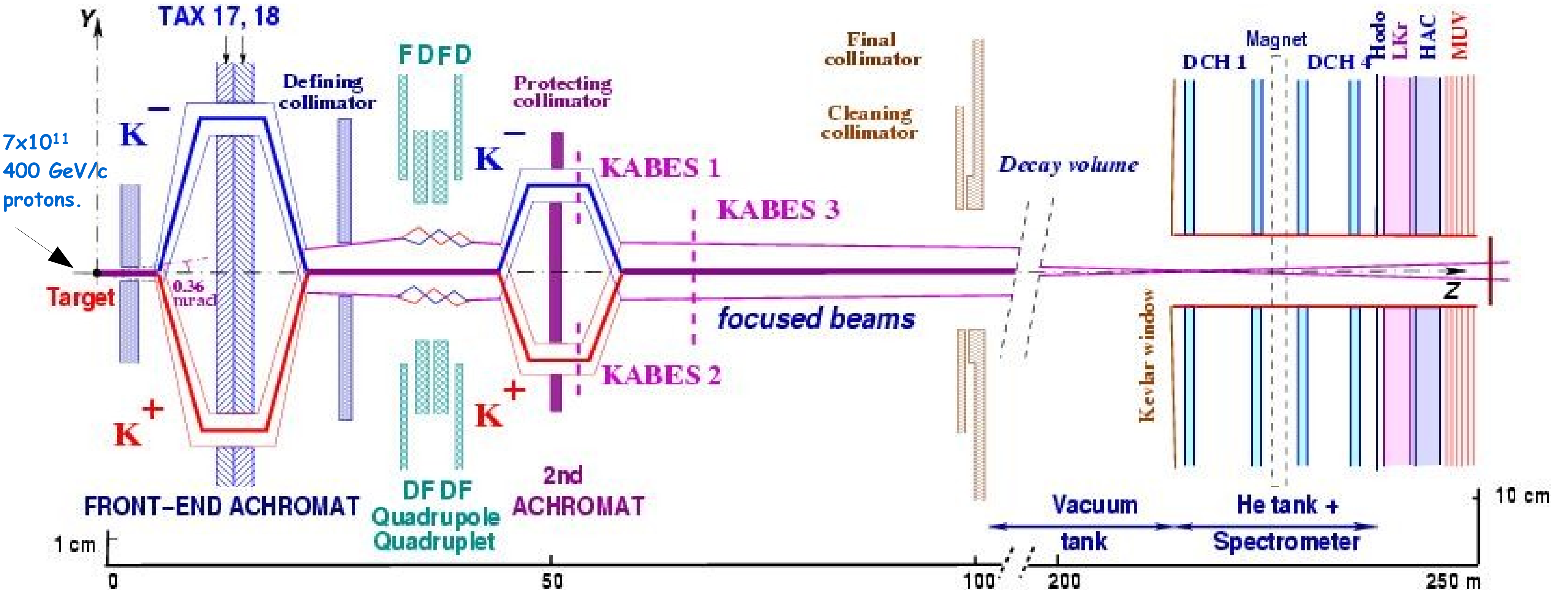


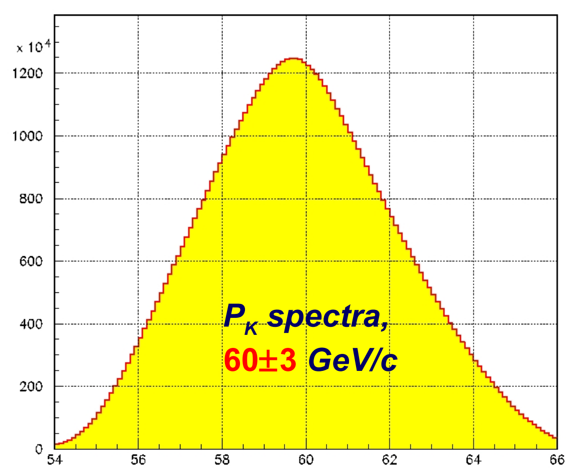

The main goal was to search for direct CP violation in $3\pi$ decays of charged kaons.

High statistics collection gives an excellent opportunity for rare decay measurements.

/ 2003 and 2004 ~6 months data taking/

# Simultaneous $K^+$ and $K^-$ beams




2-3M K/spill ($\pi$/K~10),$\pi$-decay products stay in the beam pipe.
Flux ratio K+/K- ≈1.8

Beams coincide within ~ 1mm all along 114m decay volume.

# The NA48/2 detector

## Liquid Krypton EM calorimeter (Lkr)

High granularity (13248 cells 2x2cm$^2$)

Quasi-homogeneous (7m$^3$ liquid Kr, 27$X_0$)

$\sigma(E)/E=(3.2\%/E^{1/2}) + (9\%/E) + 0.42\%$ [E in GeV]

$\sigma x=\sigma y \sim 1.5$ mm for E=10 GeV

E/p ratio used for e/π discrimination

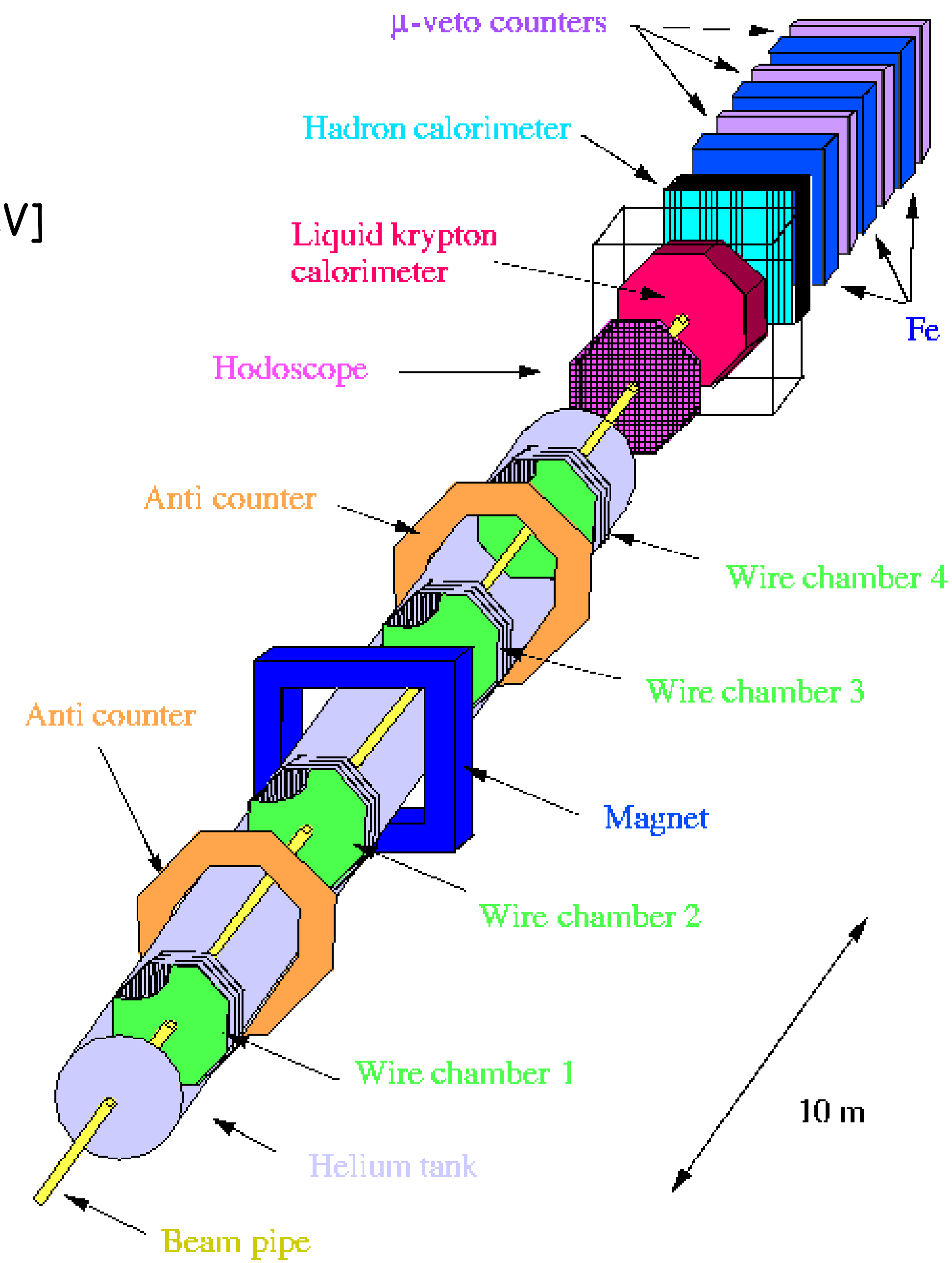
## Hodoscope

fast trigger; precise time measurement
$\sigma_T$=150ps

## Magnetic spectrometer

4 drift chambers and dipole magnet

$\sigma(p)/p= (1.02 + 0.044*p)\%$ [p in GeV/c]



# $K_{e4}$ Introduction – decay amplitude

The Ke4 amplitude is a product of weak lepton current and (V-A) hadron current:

$$\frac{G_F}{\sqrt{2}} V_{us}^* \bar{u_\nu} \gamma_\lambda (1-\gamma_5) v_e \langle \pi^+\pi^- | V^\lambda - A^\lambda | K^+ \rangle$$

**R** enters in the decay rate multiplied by lepton mass squared -> this term is negligible for $K_{e4}$

where

$$\langle \pi^+\pi^- | A^\lambda | K^+ \rangle = -\frac{i}{m_K} (F(\mathbf{p}_{\pi^+} + \mathbf{p}_{\pi^-})^\lambda + G(\mathbf{p}_{\pi^+} - \mathbf{p}_{\pi^-})^\lambda + R(\mathbf{p}_e + \mathbf{p}_\nu)^\lambda)$$

$$\langle \pi^+\pi^- | V^\lambda | K^+ \rangle = -\frac{H}{m_K^3} \epsilon^{\lambda\mu\rho\sigma} (\mathbf{p}_{\pi^+} + \mathbf{p}_{\pi^-} + \mathbf{p}_e + \mathbf{p}_\nu)_\mu) \times (\mathbf{p}_{\pi^+} + \mathbf{p}_{\pi^-})_\rho)(\mathbf{p}_{\pi^+} + \mathbf{p}_{\pi^-})_\sigma)$$

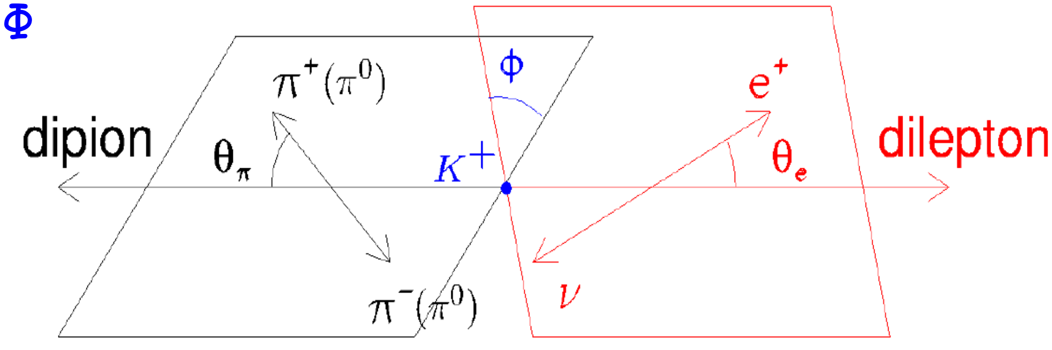
**p** is the 4-momentum of each particle, F, G, R are three axial-vector and H one vector complex Form Factors.

***F,G,R,H*** are Form Factors (FF) which depend on the decay Lorentz invariants, so their parameterisation (or some tabulation) is needed to describe data.

# $K_{e4}$ Introduction – formalism

$K_{e4}(+-)$ – $S_\pi(M^2_{\pi\pi})$, $S_e(M^2_{e\nu})$, $\cos\theta_\pi$, $\cos\theta_e$ and $\Phi$

$K_{e4}(00)$ – $S_\pi(M^2_{\pi\pi})$, $S_e(M^2_{e\nu})$ and $\cos\theta_e$




Cabibbo-Maksymowicz

*Phys. Rev. 137 (1965)*

Partial Wave expansion of the decay amplitude into s and p waves (*Pais-Treiman, Phys.Rev. 168, 1968*) +Watson theorem (T – invariance) for $\delta_l^I$

$\delta_s = \delta^0_0$ and $\delta_p = \delta^1_1$

F,G – 2 complex Axial Form Factors

$F = F_s e^{i\delta s} + F_p e^{i\delta p} \cos(\theta_\pi)$

$G = G_p e^{i\delta g}$

H – 1 complex Vector Form Factor

$H = H_p e^{i\delta h}$

Map the distribution of the Cabibbo-Maksymowicz variables in the five-dimensional space with 4 real Form factors and only one phase shift, assuming identical phases for p-wave Form factors $F_p$, $G_p$, $H_p$ .

$K_{e4}(+-)$ – the fit parameters (real) are : $F_s$ $F_p$ $G_p$ $H_p$ and $\delta = \delta_s - \delta_p$

$K_{e4}(00)$ – reduces to S wave only (one complex Form factor $F = F_s e^{i\delta s}$) , the fit parameter is only one $F_s$

# $K^{\pm} \to \pi^+\pi^- e^{\pm}\nu$ event selection

**Event reconstruction:**

* 3 tracks, reconstructed by the magnetic spectrometer,

* forming a vertex within the decay volume;

* Opposite sign $2\pi$ (''Right Sign'')

* 1 electron ($E_{LKr} / P_{DCH} \sim 1$)

* No MUV hit associated with tracks

**Main background sources: $K^{\pm} \to \pi^+\pi^-\pi^{\pm}$**

*case of $K^+$ :*

**a** $K^+ \to [\pi^+$ misident. as $e^+]\ \pi^+ \pi^-$

$K^+ \to [\pi^+ \to e^+\nu]\ \pi^+ \pi^-$

contributes twice more to ''Right Sign'' events than to ''Wrong Sign''

misident. lost

**b** $K^+ \to [\pi^0 \to e^+ e^- \gamma]\ \pi^0 \pi^+$ almost negligible

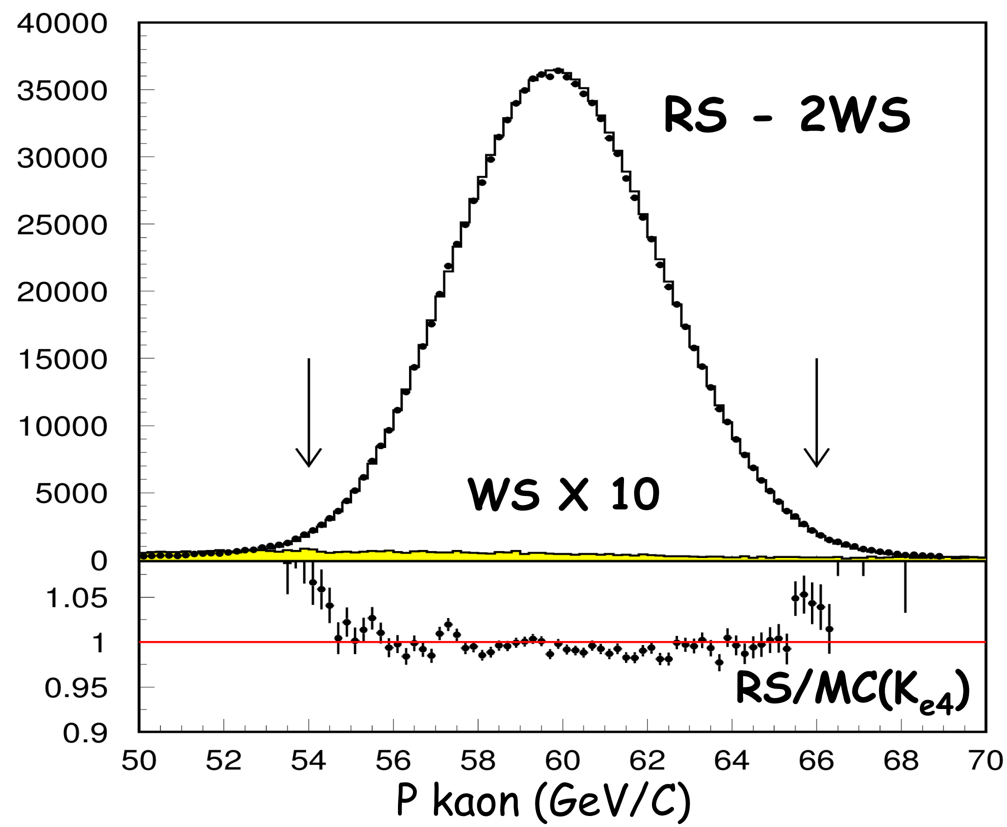
**"Right Sign" events:**

RS = $e^+\pi^+\pi^-$, 2 $\pi^+$ can decay

**''Wrong Sign'' events:**

WS = $e^-\pi^+\pi^+$, 1 $\pi^-$ can decay

Total background is below 1% , estimated from WS events (contribution **a** is dominant) and checked by MC.



# $K^{\pm} \rightarrow \pi^{+}\pi^{-}e^{\pm}\nu$ relative Form Factors: fit results

**Form factors (normalized to $f_s$)**

*[ Eur.Phys. C70 (2010) 635 ]*

NA48/2 total statistics

(2003+2004)

Series expansion with:

$q^2 = S_\pi/(4m_\pi^2) - 1$

$S_e/(4m_\pi^2)$

$$F_s = f_s(1+f'_s/f_s q^2+f''_s/f_s q^4+f'_e/f_s S_e/4m_\pi^2)$$

$$F_s = f_p/f_s$$

$$G_p=f_s(g_p/f_s + g'_p/f_s q^2)$$

$$H_p = h_p/f_s$$

| | value | stat. | syst. |
|---|---|---|---|
| $f_s/f_s$ | 0.152 | ±0.007 | ±0.005 |
| $f''_s/f_s$ | -0.073 | ±0.007 | ±0.006 |
| $f'_e/f_s$ | 0.068 | ±0.006 | ±0.007 |
| $f_p/f_s$ | -0.048 | ±0.003 | ±0.004 |
| $g_p/f_s$ | 0.868 | ±0.010 | ±0.010 |
| $g'_p/f_s$ | 0.089 | ±0.017 | ±0.013 |
| $h_p/f_s$ | -0.398 | ±0.015 | ±0.008 |

*correlations*

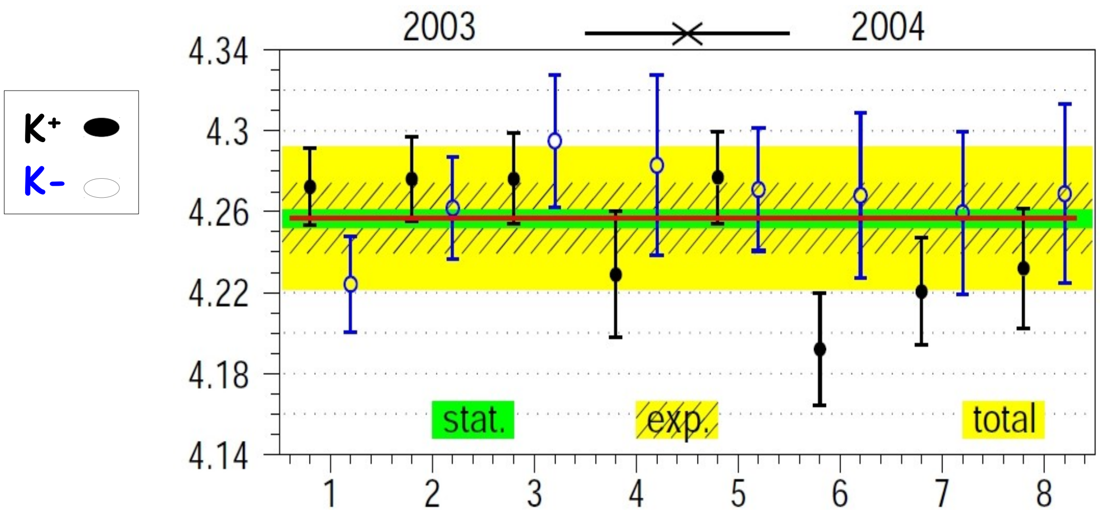
| | $f''_s/f_s$ | $f'_e/f_s$ | | $g_p/f_s$ |
|---|---|---|---|---|
| $f'_s/f_s$ | -0.954 | 0.080 | $g'_p/f_s$ | -0.914 |
| $f''_s/f_s$ | | 0.019 | | |

# $K^{\pm} \to \pi^+\pi^- e^{\pm}\nu$ / $K_{e4}(+-)$ / branching fraction

* Use $K^{\pm} \to \pi^+\pi^-\pi^{\pm}$ channel for normalization

* Number of signal ($1.1 \times 10^6$), number of normalization ($1.9 \times 10^9$) and number of background (0.95% of Ke4) events

* $Br(K^{\pm} \to \pi^+ \pi^- \pi^{\pm}) = (5.59 \pm 0.04)\%$




| Relative systematic uncertainty | % |
|---|---|
| Acceptance, beam geom. | 0.18 |
| Muon vetoing | 0.16 |
| Accidental activity | 0.21 |
| Particle ID | 0.09 |
| background | 0.07 |
| Radiative effects | 0.08 |
| Trigger efficiency | 0.11 |
| Simulation statistics | 0.05 |
| Total systematics | 0.37 |
| External error [Br(K3π)] | 0.72 |

*$K^-$ : first measurement*

$BR(K_{e4}(+)) = (4.255 \pm 0.008) \times 10^{-5}$ $BR(K_{e4}(-)) = (4.261 \pm 0.011) \times 10^{-5}$

**$BR(K_{e4}(+-)) = (4.257 \pm 0.004_{stat.} \pm 0.016_{syst.} \pm 0.031_{ext.}) \times 10^{-5} = (4.257 \pm 0.035) \times 10^{-5}$** **0.8% rel.err.**

**PDG 2012: $(4.09 \pm 0.1) \times 10^{-5}$ 2.4% rel.err.**

Absolute form factor value (for $|V_{us}| = 0.2252 \pm 0.0009$ from PDG 2012)

$F_s(q2=0, Se=0) = 5.705 \pm 0.003_{stat} \pm 0.017_{syst} \pm 0.031_{ext}$

Published in
*Phys.Lett. B715 (2012) 105*

# $K_{e4}$(+-) decay and ππ scattering lengths

The S-wave ππ scattering lengths a0 and a2 (I=0 and I=2) are precisely predicted by ChPT [NPB 603 (2001) 125, PRL 86 (2001) 5008]

Two statistically independent measurements by the NA48/2:

* From the cusp in $M_{\pi^0\pi^0}$ in $K^\pm \to \pi^\pm\pi^0\pi^0$ decay *[Eur.Phys.J. C64(2009)589]]*
* From the phase shift $\delta(M_{\pi\pi})=\delta_s-\delta_p$ in Ke4(+-) decay *[Eur. Phys.J. C70(2010)635]*

Different theoretical inputs:

Roy equations and isospin breaking correction vs. re-scattering in the final state and ChPT expansion

Large overlap in the $a^0_0$ and $a^2_0$ plane.

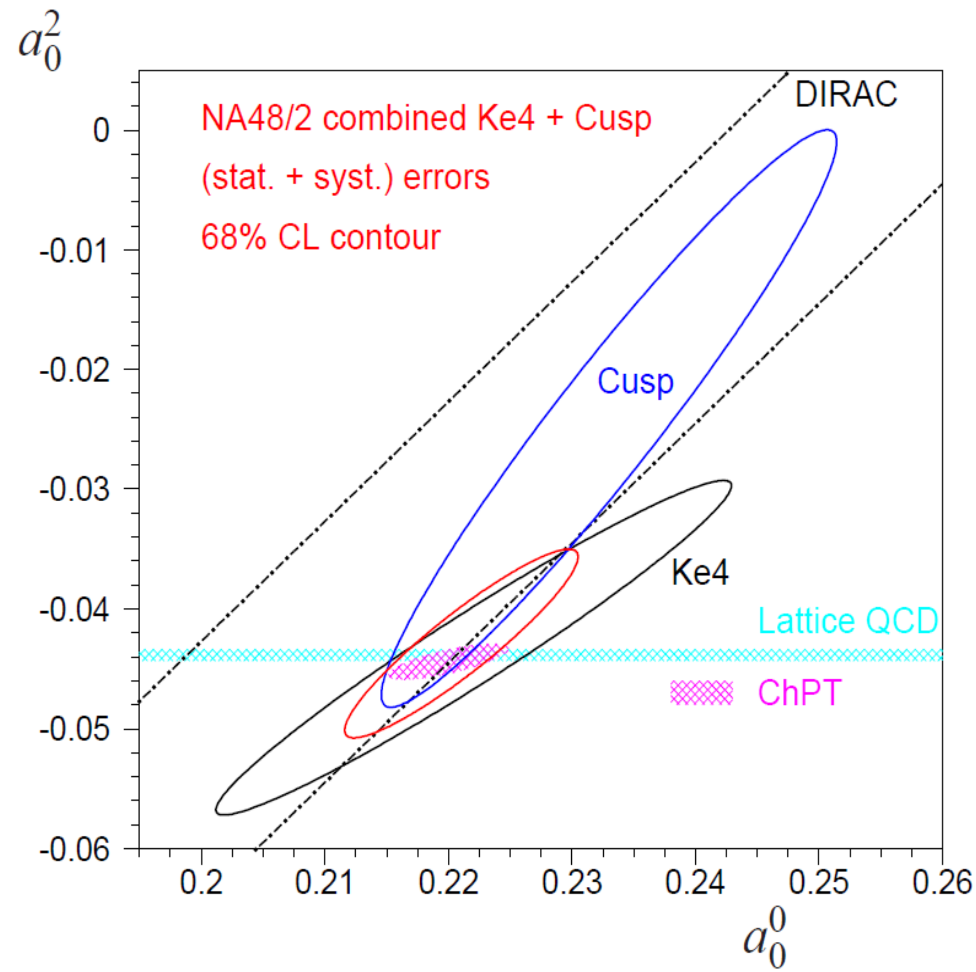
Impressive agreement with ChPT !

**combined ππ scattering lengths result**

$a^0_0 = 0.2210 \pm 0.0047_{stat.} \pm 0.0040_{syst.}$

$a^2_0 = -0.0429 \pm 0.0044_{stat.} \pm 0.0028_{syst.}$

$a^0_0 - a^2_0 = 0.2639 \pm 0.0020_{stat.} \pm 0.0015_{syst.}$



# $K^{\pm} \to \pi^0\pi^0 e^{\pm}\nu$ event selection

**Event reconstruction:**

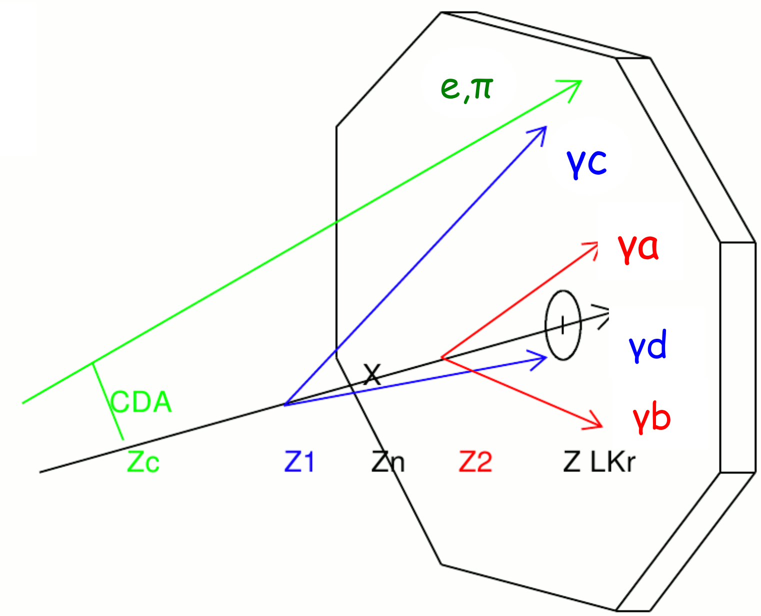
* find 2 Lkr γ-cluster pairs (ab) & (cd) in time (±2.5 ns) and energy > 3 GeV

* decay positions $Z_1$ and $Z_2$ assuming $\pi^0 \to \gamma\gamma$

  $Z_n=(Z_1+Z_2)/2$ within the decay volume

  $D_{zn}=|Z_1-Z_2|< 500$ cm

* Combined with charged track (Zc at CDA to the beam line) if $D_Z = |Z_c-Z_n| < 800$ cm




**Electron identification:**

* LKr cluster associated to track is in-time (±10 ns) with track and $2\pi^0$
* $E_{LKr}/P_{DCH} \sim 1$ [0.9-1.1]
* Extra rejection using a dedicated discriminant variable. It is a linear combination of variables related to shower properties and trained on real and fake electrons from data.

| Background rejection | |
|---|---|
| Fake-electron background ($K^{\pm} \to \pi^0\pi^0\pi^{\pm}$) | 0.65 % |
| Decay electron background ($K^{\pm} \to \pi^0\pi^0\pi^{\pm}$; $\pi^{\pm} \to e^{\pm}\nu$) | 0.12 % |
| Accidental track or photon | 0.23 % |

**Total BGR ~ 1%**

# $K^{\pm} \to \pi^0\pi^0 e^{\pm}\nu$ relative to $K^{\pm} \to \pi^0\pi^0\pi^{\pm}$

**Signal/normalization kinematic separation**

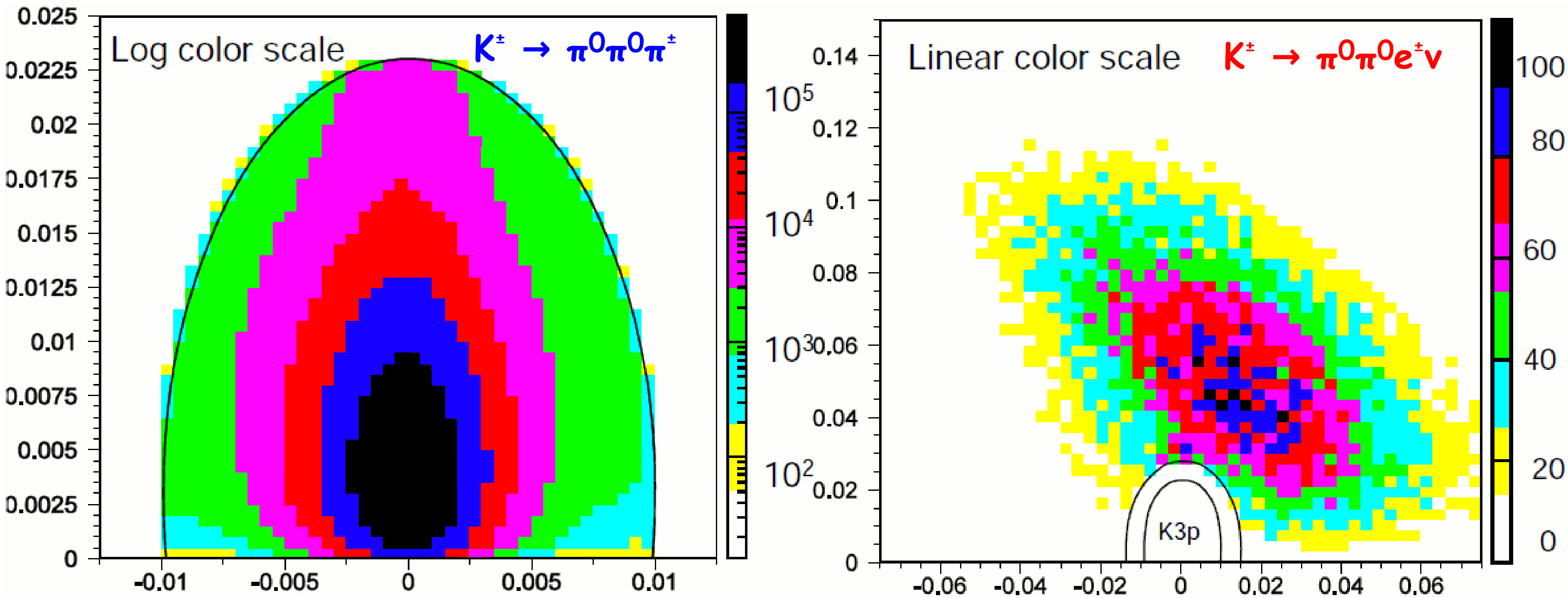
* Assign $m_{\pi}$ to the charged track, plot $P_t$ to the beam vs invariant mass

* Cut $K^{\pm} \to \pi^0\pi^0\pi^{\pm}$ events with a small $P_t$ and close to the kan PDG mass

* Cut $S_{e\nu} < 0.25\ (GeV/c^2)^2$ ,rejects 0.5% candidates (mis-reconstructed tracks in fake electrons and accidentals)

* No extra close cluster E > 3 GeV

Elliptic cut separates ~93 x $10^6$ $K^{\pm} \to \pi^0\pi^0\pi^{\pm}$ from ~65000 $K_{e4}$ candidates




$P_t$(GeV/c) vs $(M_{3p} - M_K)$ (GeV/c$^2$)

# $K_{e4}$ (00) Form Factor measurement – principle

* Because of two identical particles in the final state, the $\pi^0 \pi^0$ system cannot be in a l=1 state and only the S-wave term contributes to the partial wave expansion of the form factors ($F_s$ ).

* The differential rate depends only on 3 kinematic variables:

$$d^3\Gamma = \frac{G_F^2 |V_{us}|^2}{2(4\pi)^6 m_K^5} \rho(S_\pi, S_e)\ J_3(S_\pi, S_e, \cos\theta_e) \times\ dS_\pi\ dS_e\ d\cos\theta_e$$

$$J_3 = |XF_s|^2(1 - \cos 2\theta_e) = 2|XF_s|^2 \sin^2\theta_e$$

| |
|---|
| $\rho(S,Se)$ – phase space factor |
| $X=0.5*\Lambda^{1/2}(M^2_K,S_\pi,S_e)$ |
| $\Lambda(a,b,c)=a^2+b^2+c^2-2(ab+ac+bc)$ |

* Differential rate in the $(S_\pi,S_e)$ plane is proportional to $|F_s|^2$.

* No $F_s$ dependence with $\theta_e$ angle, $F_s$ must be studied only in the $(S_\pi,S_e)$ plane !

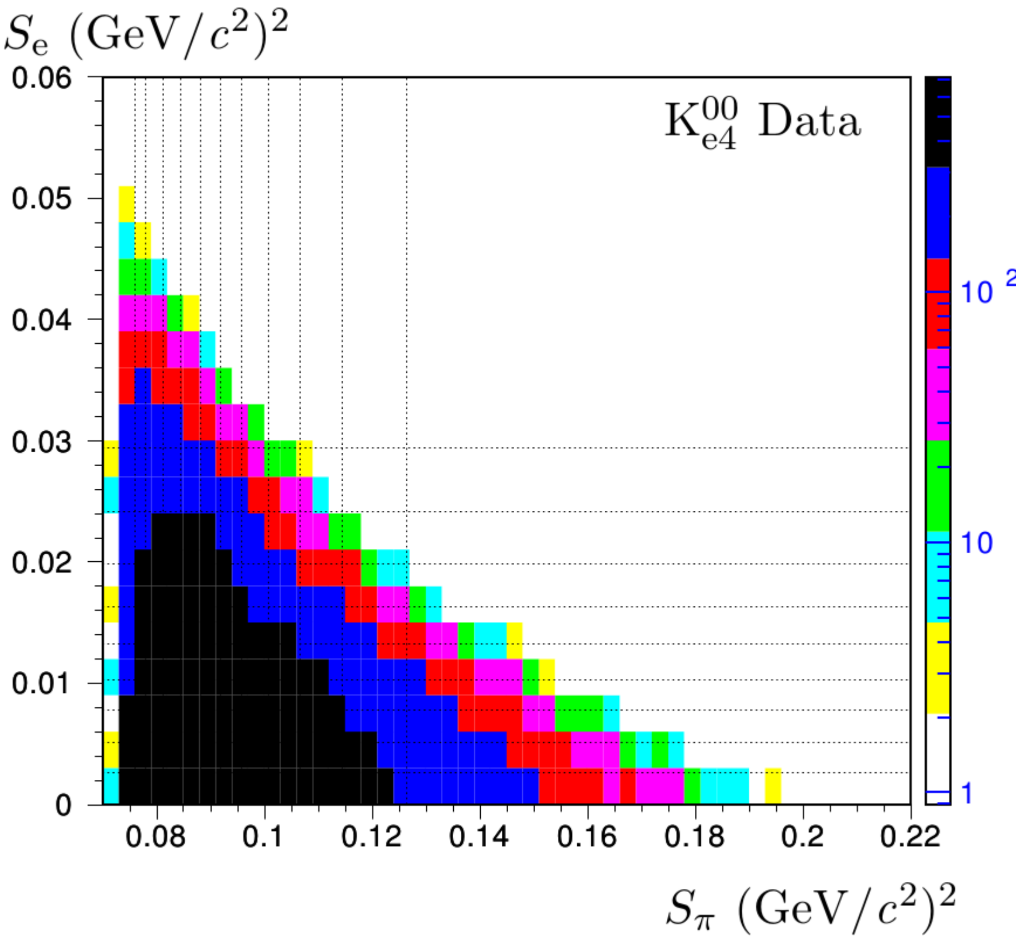
* Subtract background in the 2d-plane.

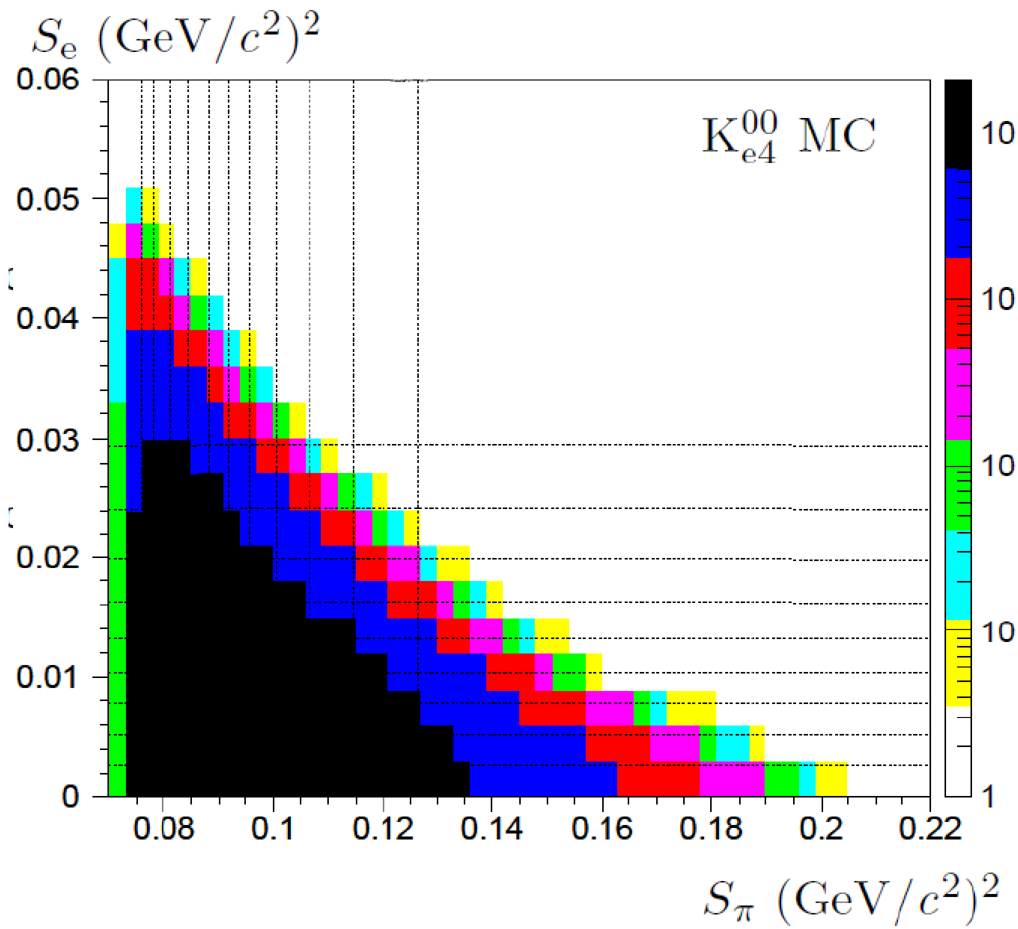
* Compare to the same distribution obtained from simulation including acceptance, resolution, trigger efficiency, radiative corrections and kinematic factors but using a constant form factor.

* Switch to dimensionless variables: $q2=(S_\pi/4m^2_{\pi+} - 1)$ and $S_e/4m^2_{\pi+}$

* Define a grid of 10 equal population bins in $S_\pi$ above the $2m_{\pi+}$ treshold and two equal population bins below (10 bins with 6000 events each, 2 bins with 3000 events each), 10 bins in $S_e$ (300 or 600 events in 2d-bins).

# Form Factor measurement: 2d plot ($S_\pi$, $S_e$)




~ 65 000 $K_{e4}$ candidates + background

~ $100\times10^6$ $K_{e4}$ simulated events with constant $F_s$

# Fit procedure

2d fit function:

$G = N ( 1 + a X + b X^2 + c Y )^2$    X > 0, above treshold

$G = N ( 1 + d (|X/(1+X)|)^{1/2} + c Y )^2$    X < 0, below treshold

Dimensionless variables:

$X=q^2=S_\pi/(4m^2_{\pi+}) -1$

$Y=S_e/(4m^2_{\pi+})$

**To minimize:**

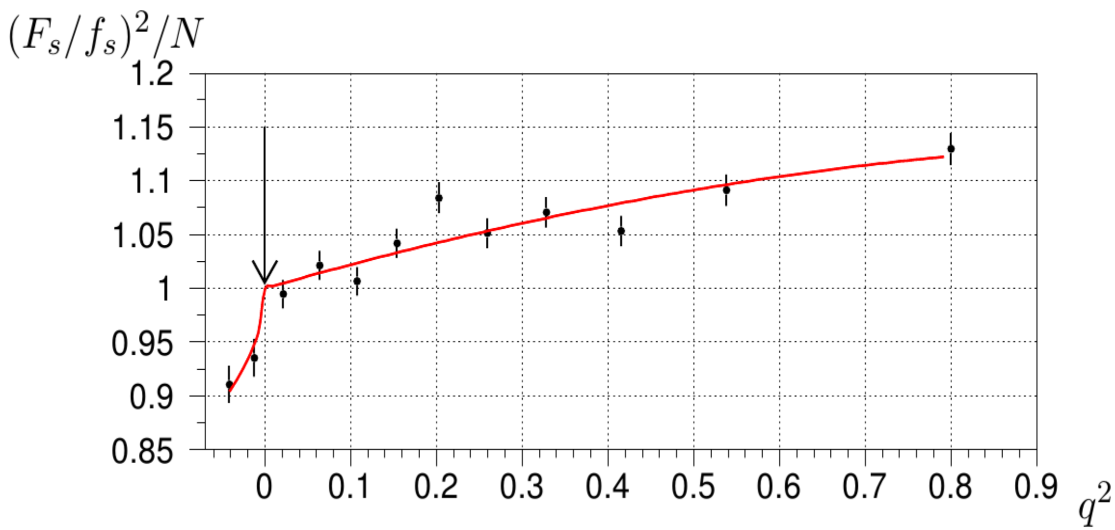
$$\chi^2 = \sum_{i=1}^{12}\sum_{j=1}^{10}((n_{ij}/m_{ij} - G(X_i, Y_j, \hat{p}))/\sigma_{ij})^2$$

fit parameters = a,b,c,d

$n_{ij}$ = Data - BGR

$m_{ij}$ = MC with $F_s$=1

$X_i$, $Y_j$ are the barycenters of the bin ij.



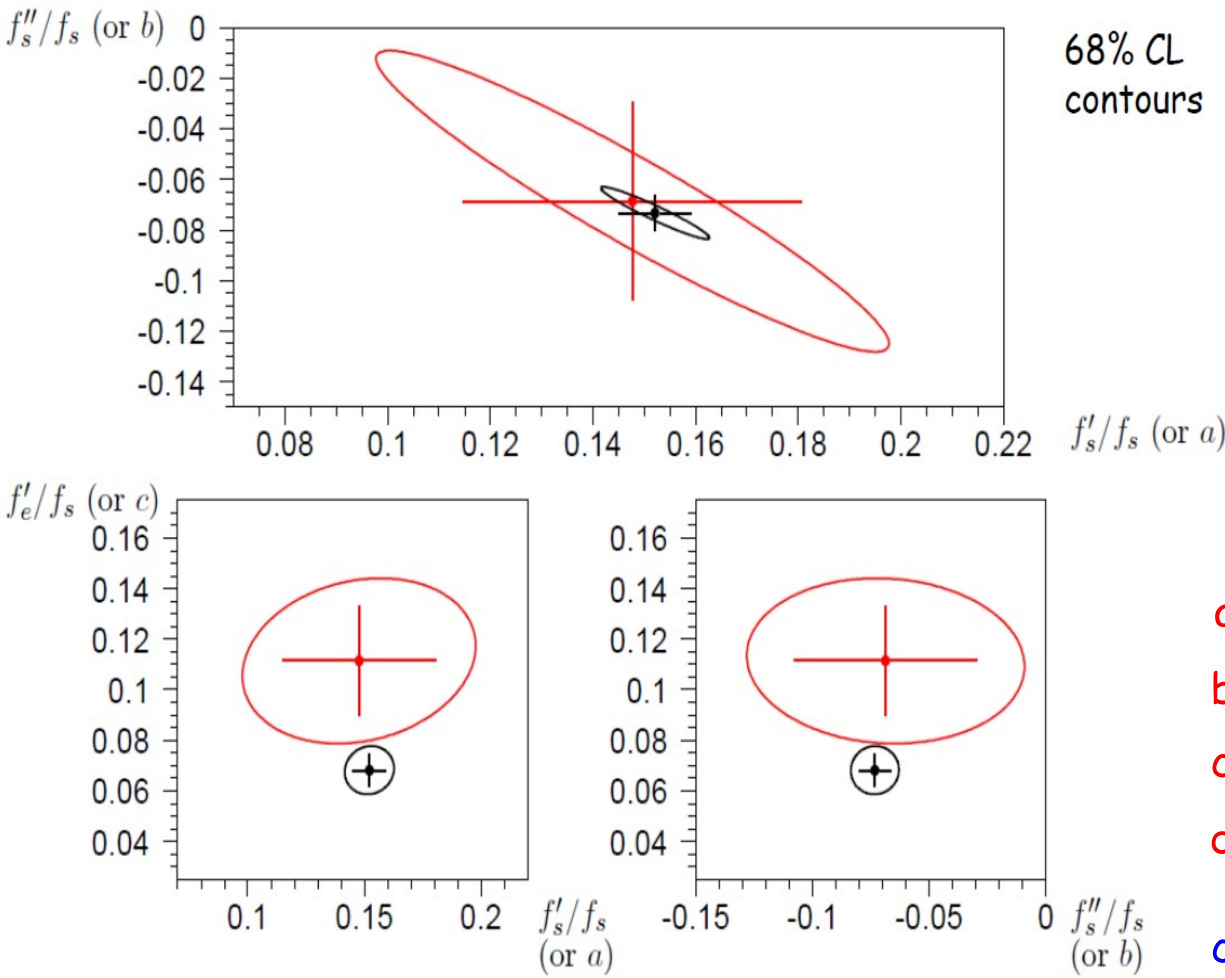

We observe the cusp-like behavior of Form Factor $S_\pi$ dependence with a threshold at $4m^2_{\pi+}$

# $F_s/f_s$ Form Factor comparison

## Ke4(+-) and Ke4(00)

### Final/ Preliminary

$$F_s = f_s(1+f'_s/f_s q^2+f''_s/f_s q^4+f'_e/f_s S_e/4m_\pi^2)$$




- Similar q2 and Se dependence
- Same correlations
- Consistent within statistical errors

a = 0.149 ± 0.033$_{stat}$ ± 0.014$_{syst}$

b = – 0.070 ± 0.039$_{stat}$ ± 0.013$_{syst}$

c = 0.113 ± 0.022$_{stat}$ ± 0.007$_{syst}$

d = – 0.256 ± 0.049$_{stat}$ ± 0.016$_{syst}$

chi$^2$/ndf =101.4/107: 63% probability

# $K^\pm \to \pi^0\pi^0 e^\pm \nu$ / $K_{e4}(00)$ / branching fraction

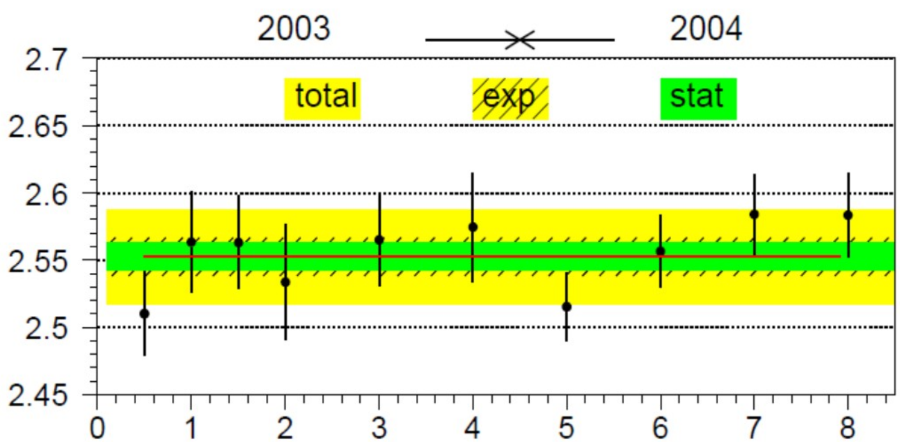
* Use $K^\pm \to \pi^0\pi^0\pi^\pm$ channel for normalization

* Number of signal (65210), number of normalization ($93.5 \times 10^6$) and number of background (650) events

* $Br(K^\pm \to \pi^0\pi^0\pi^\pm) = (1.761 \pm 0.022)\%$ - source of external error

*trigger efficiency: ε(Ke4)=96.06% and ε(K3π)=97.42%

| Systematic Uncertainty (% to Br value) | |
|---|---|
| Acceptance | 0.15 |
| Form Factor | 0.17 |
| Background | 0.25 |
| Trigger cut | 0.04 |
| Radiative effects | 0.20 |
| Simulation statistics | 0.09 |
| Trigger efficiency | 0.03 |
| Total | 0.40 |




PDG 2012 : $(2.2 \pm 0.4)\ 10^{-5}$ 18% rel.err.

***Preliminary:***

$BR(K_{e4}(+-)) = (2.552 \pm 0.010_{stat.} \pm 0.010_{syst.} \pm 0.032_{ext.}) \times 10^{-5} = (2.552 \pm 0.035) \times 10^{-5}$ 1.4% rel.err.

Absolute form factor value (no radiative corr. for $|V_{us}| = 0.2252 \pm 0.0009$ from PDG 2012)

$(1+\delta_{EM})\ F_s(q2=0, Se=0) = 6.079 \pm 0.012_{stat} \pm 0.027_{syst} \pm 0.046_{ext}$

# Summary

* 1.11 millons of reconstructed $K^{\pm} \rightarrow \pi^{+}\pi^{-}e^{\pm}\nu$ /$K_{e4}$(+-) / and
~65000 of $K^{\pm} \rightarrow \pi^{0}\pi^{0}e^{\pm}\nu$ /$K_{e4}$(00) / decays (2003+2004 data).

* Improved branching fractions:
Br Ke4(+-) = $(4.257 \pm 0.035)\times10^{-5}$ *[Phys.Lett. B715 (2012) 105]* (3 times better/PDG)
Br $K_{e4}$(00) = $(2.552 \pm 0.035) \times10^{-5}$ *[preliminary]* (13 times better/PDG)

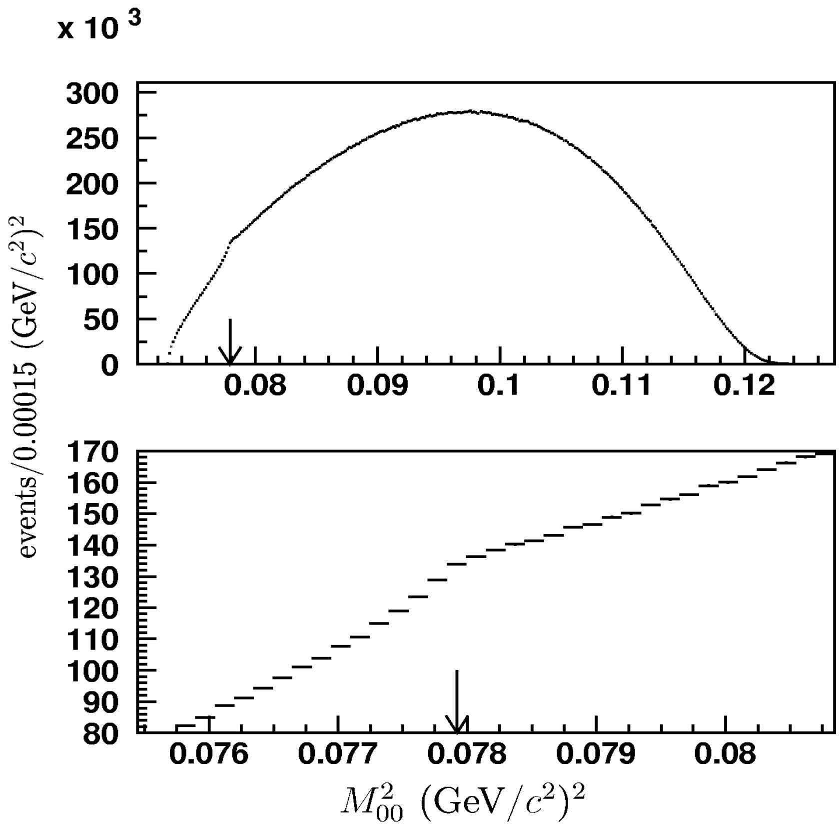
* $K_{e4}$(00) $F_s$ form factor is compatible with the $K_{e4}$(+-) one above $2m_{\pi^+}$ threshold. Deficit below can be due to ($\pi\pi$) final state charge exchange scattering.

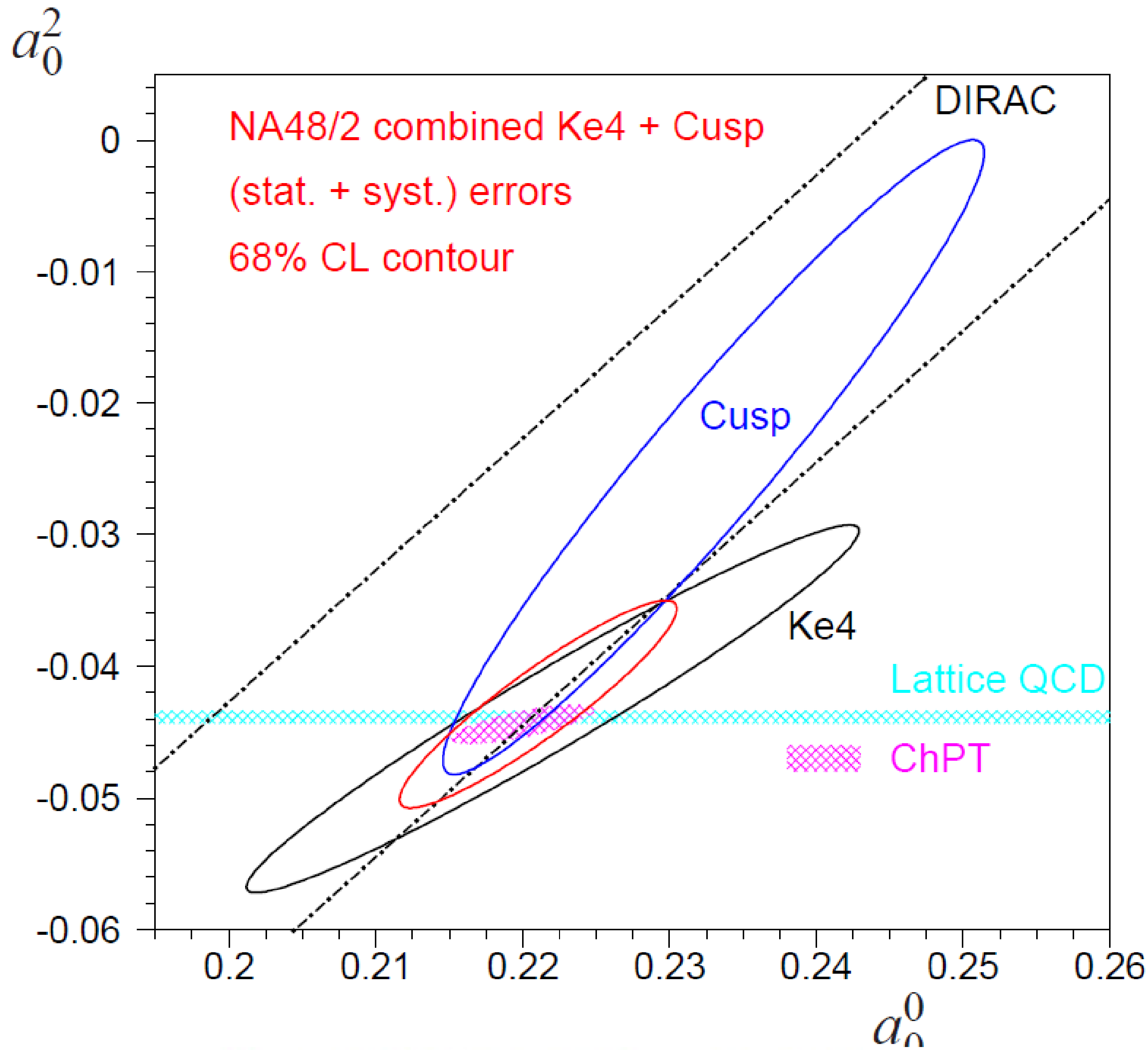
# Spares

# $\pi\pi$ scattering lengths measurement from phase shift $\delta(M_{\pi\pi}) = \delta_s - \delta_p$

**Cusp in $K^\pm \to \pi^\pm\pi^0\pi^0$**

Published in EPJ C64(2009)589



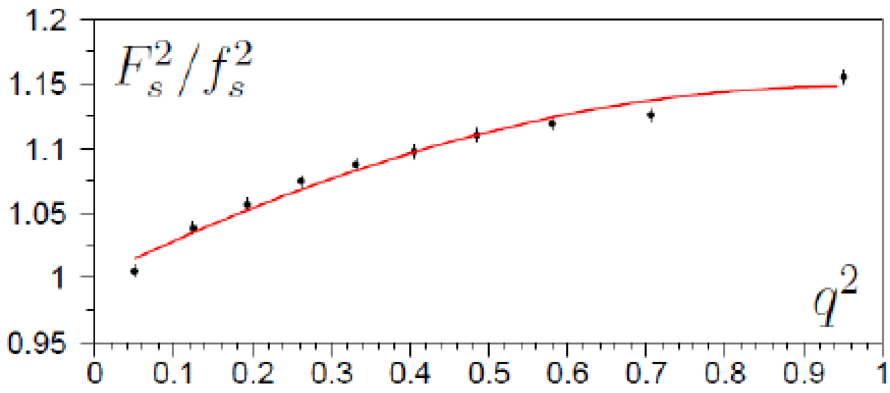

$$a_0^0 = 0.2210 \pm 0.0047_{\text{stat}} \pm 0.0040_{\text{syst}},$$

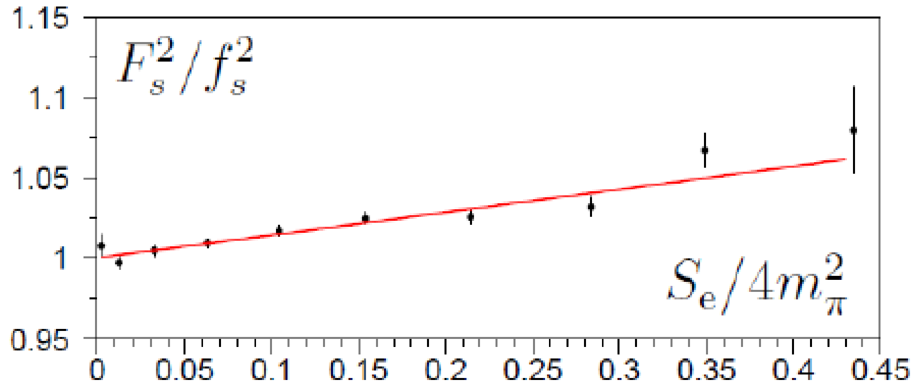
$$a_0^2 = -0.0429 \pm 0.0044_{\text{stat}} \pm 0.0028_{\text{syst}},$$

$$a_0^0 - a_0^2 = 0.2639 \pm 0.0020_{\text{stat}} \pm 0.0015_{\text{syst}}.$$

Published in
[Eur.Phys. C70 (2010) 635]

# Ke4(+-)




$$\Gamma(K_{e4})/\Gamma(K_{3\pi}) = \frac{N_s - N_b}{N_n} \cdot \frac{A_n\ \varepsilon_n}{A_s\ \varepsilon_s}$$

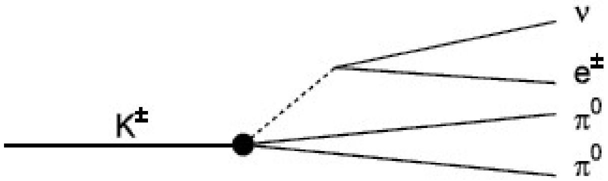
$$\mathrm{BR}(K_{e4}) = \frac{N_s - N_b}{N_n} \cdot \frac{A_n\ \varepsilon_n}{A_s\ \varepsilon_s} \cdot \mathrm{BR}(K_{3\pi})$$

$$\mathrm{BR}(K_{e4}) = \tau_{K^\pm} \cdot (|V_{us}| \cdot f_s)^2 \cdot \int d\Gamma_5\ /\ (|V_{us}| \cdot f_s)^2$$

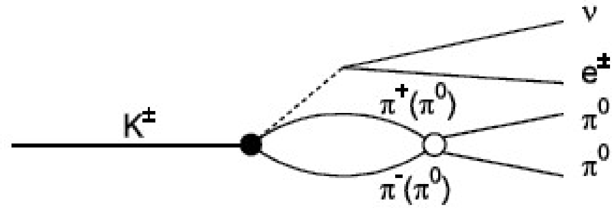
$$d\Gamma_5 = \frac{G_F^2 |V_{us}|^2}{2\hbar(4\pi)^6 m_K^5}\ \rho(S_\pi, S_e)\ J_5(S_\pi, S_e, \cos\theta_\pi, \cos\theta_e, \phi)\ dS_\pi\ dS_e\ d\cos\theta_\pi\ d\cos\theta_e\ d\phi$$

# $K_{e4}$(00) Form Factor interpretation by analogy

1-loop calculation for $3\pi$ decays: Cabibbo, PRL 93(2004)121801




Tree level M0




1-loop M1

Above threshold: $|M|^2 = |M0 + i\, M1|^2 = M0^2 + M1^2$

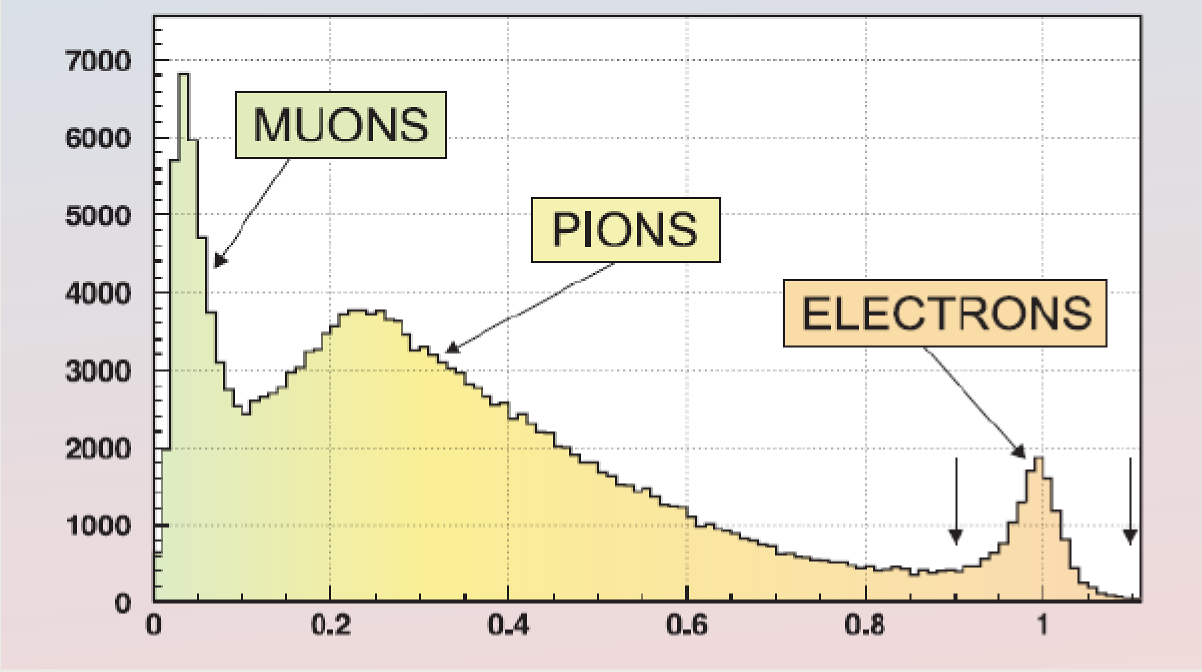
Below threshold : $|M|^2 = |M0 + M1|^2 = M0^2 + M1^2 + 2\, M0\, M1$

$q2 = S\pi/4m\pi{+}^2 - 1$ $\sigma\pi = \sqrt{(4m\pi{+}^2/S\pi - 1)} = \sqrt{(|q2|/(1+q2))}$

M0 = unperturbed amplitude: $Fs = fs\,(1 + a\, q2 + b\, q4 + c\, Se/4m\pi{+}^2)$

M1 = scattering amplitude: $-\,2/3\,(a0-a2)\, fs \sqrt{(|q2|/(1+q2))}$

## Particle Identification (both modes):




- **electron-ID:**

0.9 < E/p < 1.1 complemented by shower-shape properties

- **charged pion-ID:**

E/p < 0.8