Kinematical constraint revisited and nonlinear evolution at large values of coupling constant

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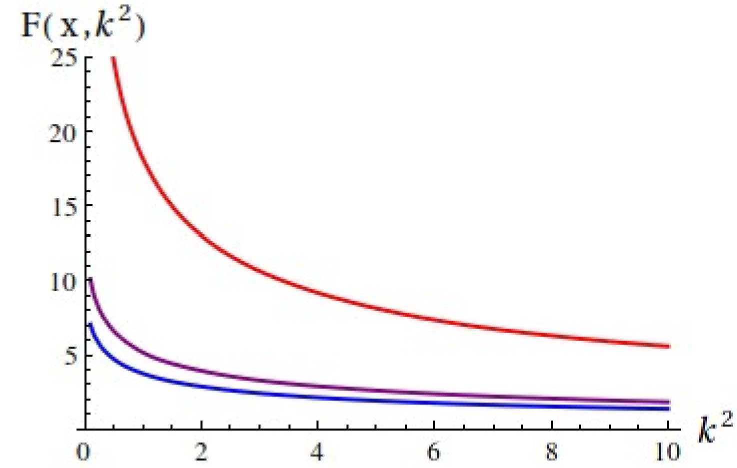
The BFKL equation and its solution

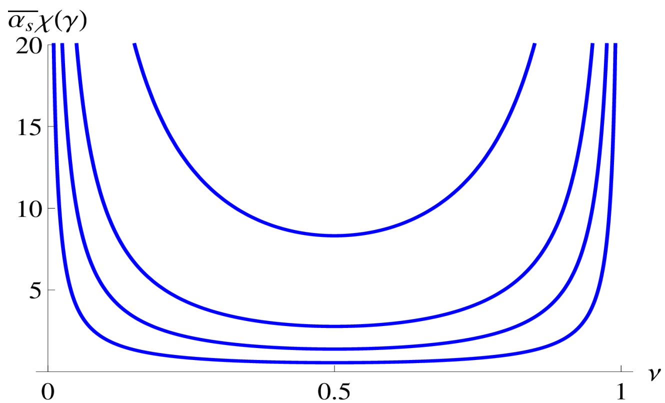


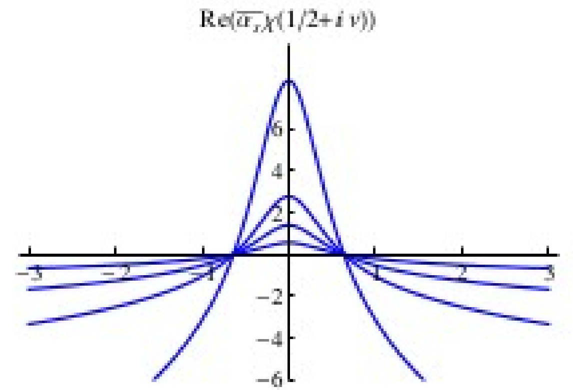
$$f(x,k^2) = f_0(x,k^2) + \bar{\alpha}_s k^2 \int_{x/x_0}^{1} \frac{dz}{z} \int_0^\infty \frac{dl^2}{l^2} \left[\frac{f(x/z,l^2) - f(x/z,k^2)}{|l^2-k^2|} + \frac{f(x/z,k^2)}{\sqrt{4l^4+k^4}} \right]$$

$$f(x,k^2) = \frac{1}{2\pi i} \int d\gamma (k^2)^\gamma f(x_0,\gamma) \left(\frac{x}{x_0}\right)^{-\bar{\alpha}_s \chi(\gamma)}.$$

$$\chi(\gamma) = 2\psi(1) - \psi(1-\gamma) - \psi(\gamma).$$

$$\mathcal{F}(x,k^2) = \mathcal{F}(x_0,1/2) \frac{1}{\sqrt{4\pi \ln(x_0/x) 1/2\lambda'}} e^{\lambda \ln(x_0/x) - 1/2 \ln(k^2/k_0^2)} e^{\frac{-\ln(k^2/k_0^2)^2}{4 1/2\lambda' \ln(x_0/x)}}$$





$$\partial_Y \mathcal{F}(Y,\rho) = \frac{1}{2}\lambda' \partial_\rho^2 \mathcal{F}(Y,\rho) + \frac{1}{2}\lambda' \partial_\rho \mathcal{F}(Y,\rho) + (\lambda + \lambda'/8)\mathcal{F}(Y,\rho).$$

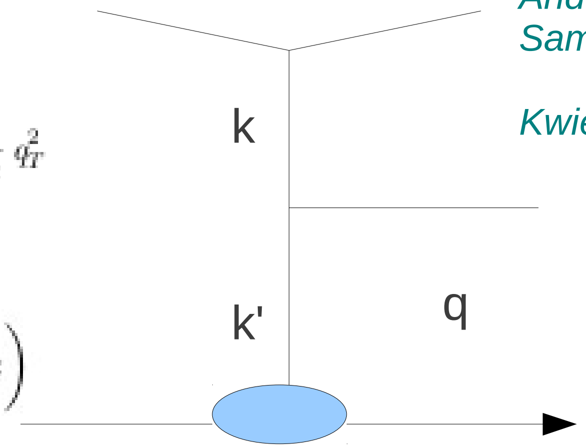
The kinematical constraint effects

Andersson,Gustafson
Samuelsson, Kharraziha '96

Kwiecinski, Martin, Sutton '97

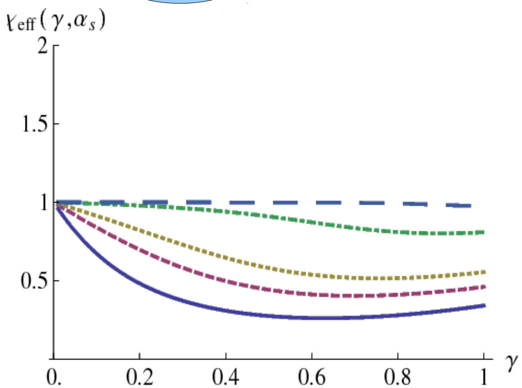
$$k^2 = k^+k^- - k_T^2$$

$$k_T^2 > |k^+k^-| \qquad k^+k^- \simeq -\frac{k^+}{q^+}q_T^2 = -\frac{k^+}{k'^+ - k^+}q_T^2 = -\frac{z}{1-z}q_T^2$$

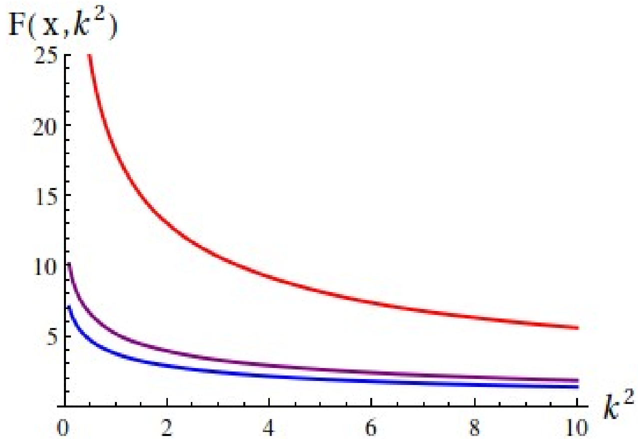


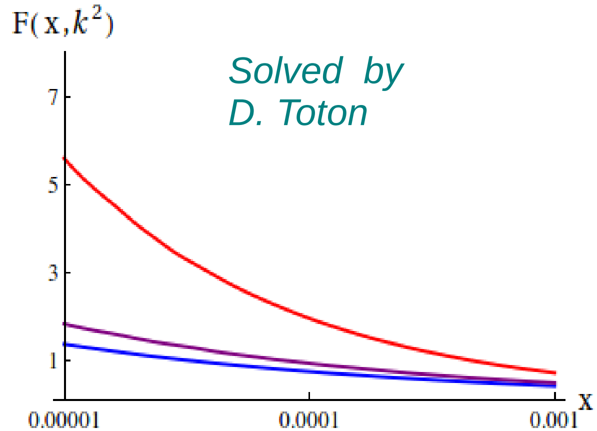
$$\theta\left(\frac{k_T^2}{q_T^2} - z\right)$$ antilollinearl imit $$l \gg k \qquad \theta\left(\frac{k_T^2}{l_T^2} - z\right)$$

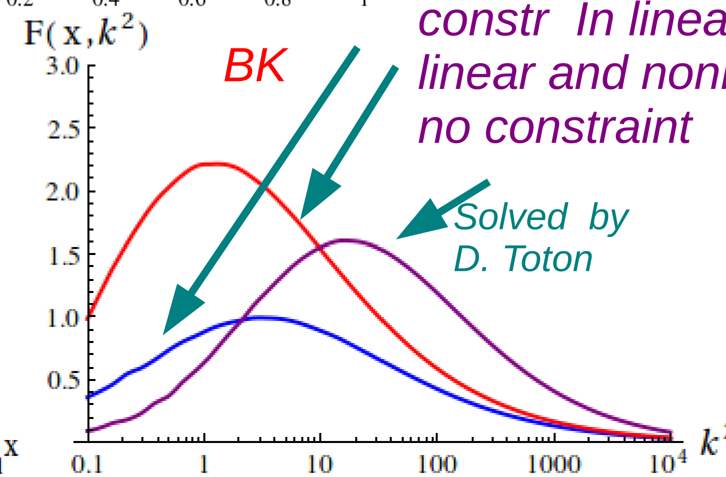
$$f(x,k^2) = f_0(x,k^2) + \bar{\alpha}_s \int_x^1 \frac{dz}{z}\int_0^\infty \frac{dl^2}{l^2}\left[\frac{\theta(k^2/l^2 - z)f(x/z,l^2) - f(x/z,k^2)}{|k^2 - l^2|} + \frac{f(x/z,k)}{\sqrt{(4l^4 + k^4)}}\right]$$



$$f(x,k^2) = \frac{1}{2\pi i}\int d\gamma (k^2)^\gamma \frac{1}{2\pi i}\int d\omega\, x^{-\omega} \frac{\omega \bar{f}_0(\omega,\gamma)}{\omega - \bar{\alpha}_s \chi_{k.c.}(\gamma,\omega,)}$$







Stasto model for resummed BFKL with kinematical constraint and DGLAP effects

Stasto, '07

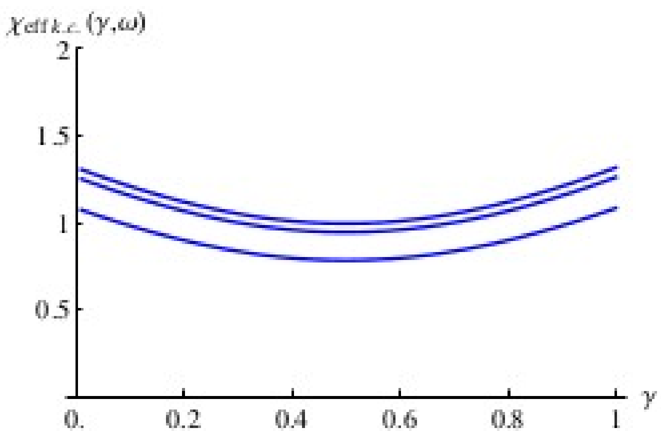
$$f(x,k^2) = f_0(x,k^2) + \bar{\alpha}_s k^2 \int_x^1 \frac{dz}{z} \int_0^\infty \frac{dl^2}{l^2} \left[\frac{f(x/z,l^2)\theta(l-kz)\theta(k/z-l) - f(x/z,k^2)}{|l^2-k^2|} + \frac{f(x/z,k^2)}{\sqrt{4l^4+k^4}} \right]$$

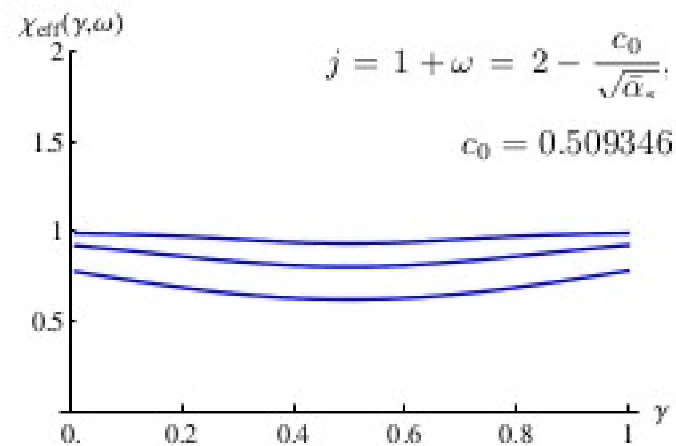
$$\chi_{k.c.}(\gamma,\omega) = 2\psi(1) - \psi(1-\gamma+\omega/2) - \psi(\gamma+\omega/2).$$

$$\chi_{eff}(\gamma,\omega) = \bar{\alpha}_s \chi_{k.c.}(\gamma,\omega)\,(1+A\omega)$$

Crucial behaviour vanishing eigenvalue when $\omega \to 1$

Contains DGLAP anaomalous Dimension at LO in $\ln Q^2$





The leading order in AdS/CFT *Brower, Polchinski, Strassler*

$$j = 1+\omega = 2 - \frac{c_0}{\sqrt{\bar{\alpha}_s}}, \qquad c_0 = 1/\pi$$

Higher orders:

Costa, Goncalves, Penedones' 12
Kotokov, Lipatov '13
Janik '14

Stasto model for BFKL with kinematical constraint and DGLAP effects

Kutak, Surowka, '13

$$f(x,k^2) = f_0(x,k^2) + \bar{\alpha}_s k^2 \int_x^1 \frac{dz}{z} \int_0^\infty \frac{dl^2}{l^2} \left[\frac{f(x/z,l^2)\theta(l-kz)\theta(k/z-l) - f(x/z,k^2)}{|l^2-k^2|} + \frac{f(x/z,k^2)}{\sqrt{4l^4+k^4}} \right]$$

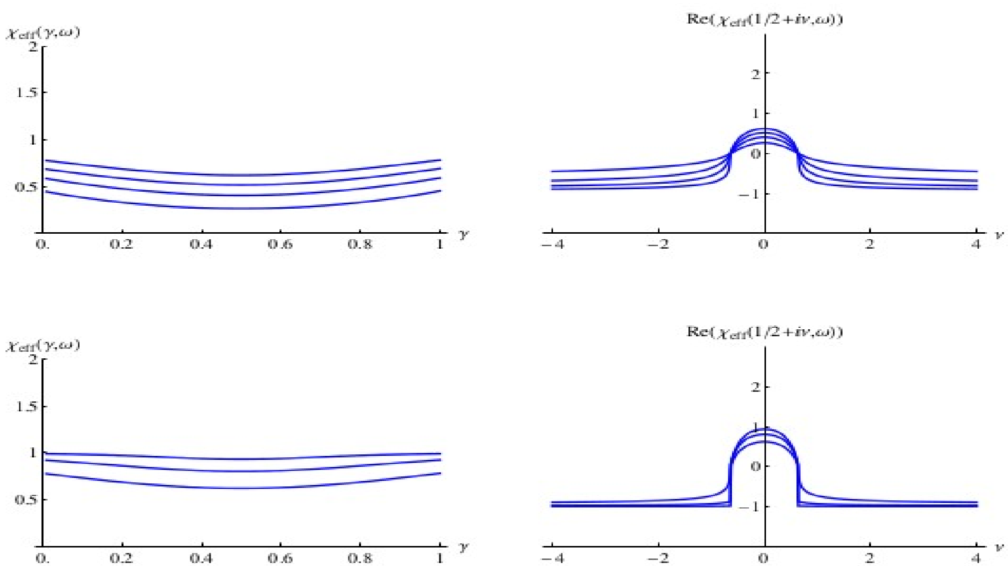
we choose the symmetric form of k.c.

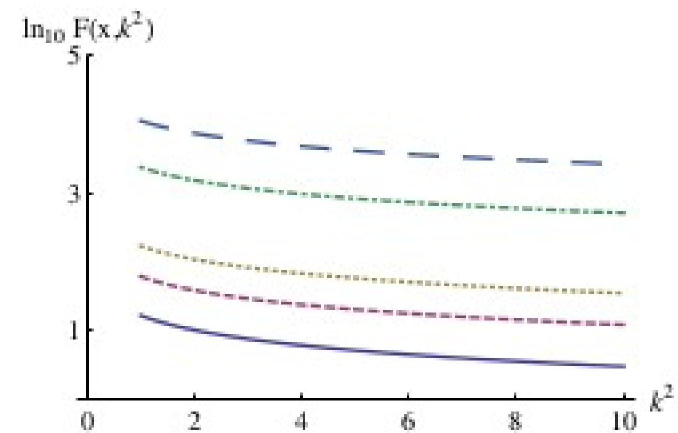
$$\chi_{k.c.}(\gamma,\omega) = 2\psi(1) - \psi(1-\gamma+\omega/2) - \psi(\gamma+\omega/2).$$

$$f(x,k^2) = \frac{1}{2\pi i}\int d\gamma (k^2)^\gamma \frac{1}{2\pi i}\int d\omega\, x^{-\omega} \frac{\omega \overline{f}_0(\omega,\gamma)}{\omega - \bar{\alpha}_s \chi_{k.c.}(\gamma,\omega,)}$$

$$\omega = \mathrm{Re}\left(\chi_{eff\, k.c.}(1/2+i\nu,\omega)\right) \qquad \chi_{eff}(\gamma,\omega) = \bar{\alpha}_s \chi_{k.c.}(\gamma,\omega)\,(1+A\omega)$$

$$f(x,k^2) = \frac{1}{2\pi i}\int d\gamma (k^2)^\gamma f(x_0,\gamma)\left(\frac{x}{x_0}\right)^{-\bar{\alpha}_s\chi(\gamma)}.$$

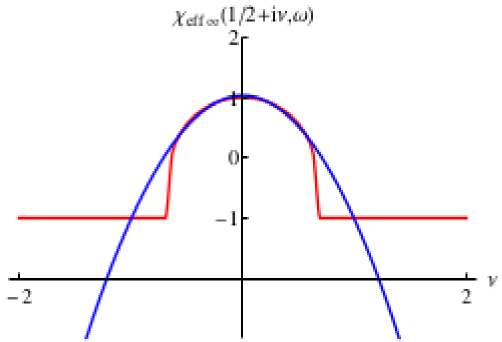




BFKL equation at strong coupling

Kutak, Surowka, '13

Fit function $\chi_{eff\,\infty}(\omega, 1/2 + i\nu) = \sum_{n=-M}^{N} A_n \nu^n$



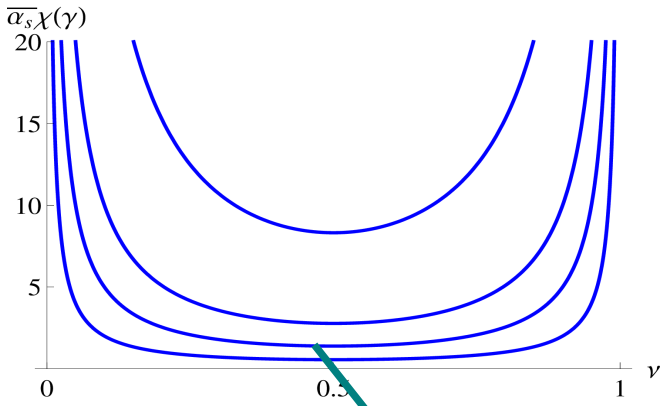
$$\chi_{eff\,\infty}(\omega, 1/2+i\nu) = P_{10}(\nu)\theta(\nu+0.683)\theta(0.683-\nu) - \theta(-\nu-0.683) - \theta(\nu-0.683)$$

Works very well: $\chi_{eff\,\infty}(\omega, 1/2 + i\nu) = 1.02795 - 2.04635\nu^2 \equiv \lambda_{st} - \frac{1}{2}\lambda'_{st}\nu^2$

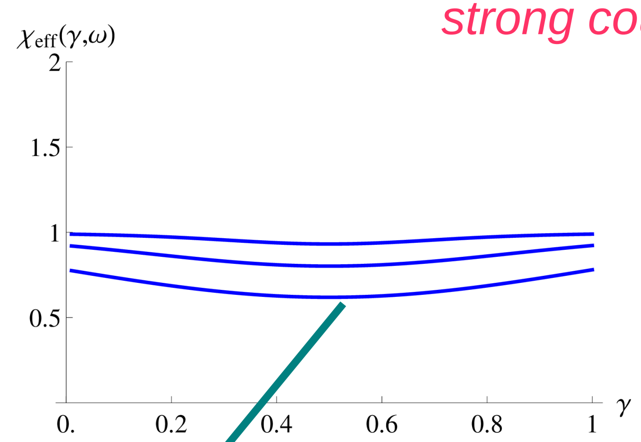
$$\partial_Y \Phi(Y,\rho) = \frac{1}{2}\lambda'_{st}\partial_\rho^2\Phi(Y,\rho) + \frac{1}{2}\lambda'_{st}\partial_\rho\Phi(Y,\rho) + (\lambda_{st} + \lambda'_{st}/8)\Phi(Y,\rho) \quad \lambda'_{st} = 4.08,\ \lambda_{st} = 1.02$$

Gluon density at the large coupling values

weak coupling



strong coupling



Stasto '07

$$\chi(\gamma) = 2\psi(1) - \psi(1-\gamma) - \psi(\gamma)$$

weak coupling

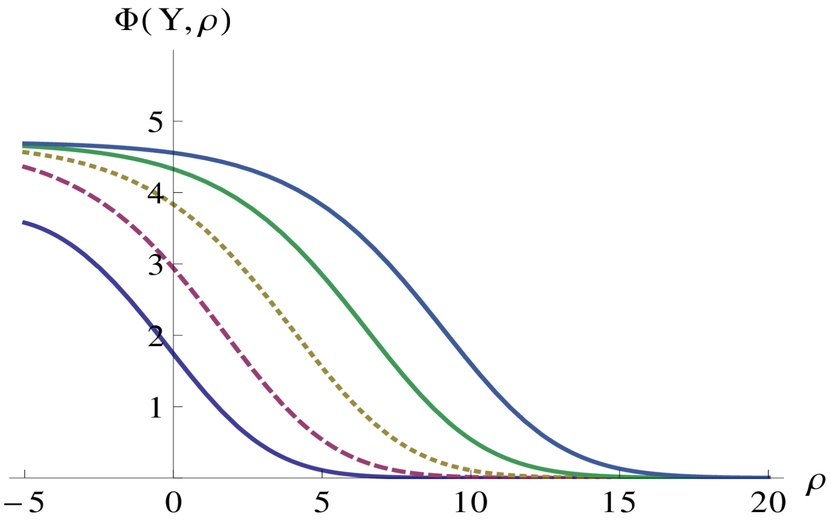
strong coupling

Kutak, Surowka '13

critical point dominates
at large coupling

WW density at the large coupling values

Kutak, Surowka, '13



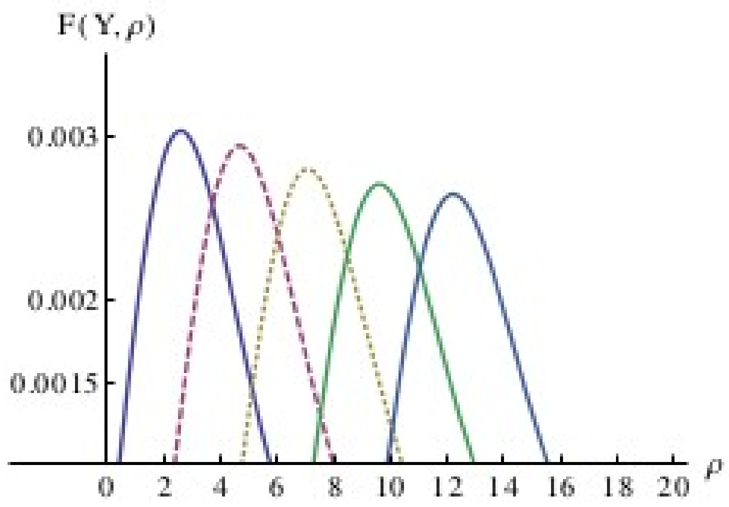
Nonlinear nonlinear equation valid at strong coupling limit

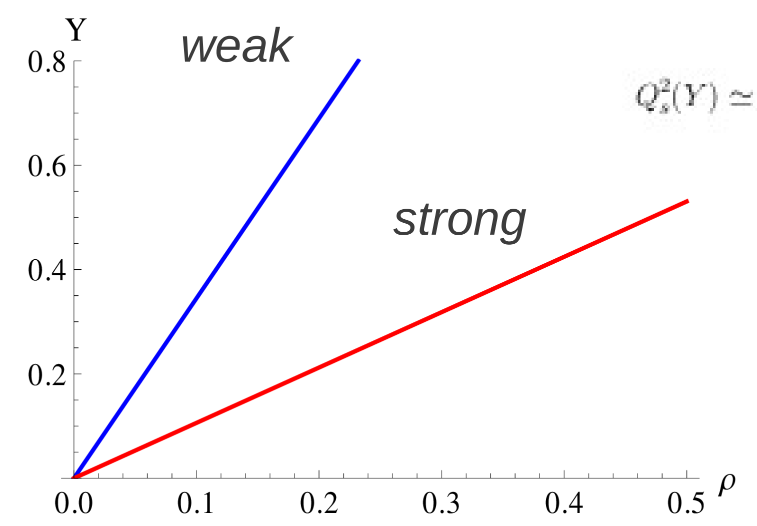
$$\partial_Y \Phi(Y,\rho) = \frac{1}{2}\lambda'_{st}\partial_\rho^2\Phi(Y,\rho) + \frac{1}{2}\lambda'_{st}\partial_\rho\Phi(Y,\rho) + (\lambda_{st} + \lambda'_{st}/8)\Phi(Y,\rho) - \frac{\bar{\alpha}_s}{\pi R^2}\Phi^2(Y,\rho)$$

Saturation scale at large values of couplng constant

$$\mathcal{F}_{BK}(Y,\rho) = \frac{N_c}{4\pi\alpha_s}\partial_\rho^2\Phi(Y,\rho)$$

$$\partial_\rho \mathcal{F}_{BK}(Y,\rho)|_{\rho=\ln Q_s^2(Y)} = 0$$





$Q_s^2(Y) \simeq e^{1.06\,Y}$

Similar behaviour as in
Mueller, Shoshi, Xiao '10

Outlook

- *Entropy at large coupling*
- *Full range in running coupling effect*
- *Just for curiosity check the cross section for inclusive production*
- *Perhaps formulate directly in momentum space*