Comprehensive Bayesian Analysis of Rare (Semi)leptonic and Radiative B Decays

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in collaboration with
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based on arxiv:1310.2478

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Deep Inelastic Scattering 2014 – WG 5: Heavy Flavours
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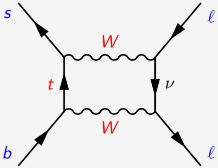
DFG FOR 1873
quark flavour physics and effective field theories

Flavor Changing Neutral Current (FCNC)

- expand amplitudes in $G_\mathrm{F} \sim 1/M_W^2$ (OPE)
- operators (matrix elem. below $\mu_b \simeq m_b$)

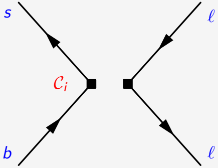
$$\mathcal{O}_i \equiv \left[\bar{s}\Gamma_i b\right]\left[\bar{\ell}\Gamma'_i\ell\right]$$



- Wilson coefficients (above $\mu_b \simeq m_b$)

$$\mathcal{C}_i \equiv \mathcal{C}_i(M_W, M_Z, m_t, \ldots)$$

- use $\mathcal{C}_i = \mathcal{C}_i(\mu_b = 4.2\mathrm{GeV})$



Effective Hamiltonian

$$\mathcal{H} = -\frac{4G_\mathrm{F}}{\sqrt{2}}\frac{\alpha_e}{4\pi}\left[V_{tb}V_{ts}^* \sum_i \mathcal{C}_i\mathcal{O}_i + O\left(V_{ub}V_{us}^*\right)\right] + \mathrm{h.c.}$$

Effective Field Theory for $b \to s\ell^+\ell^-$ FCNCs

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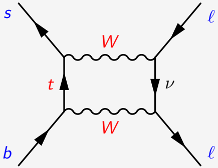

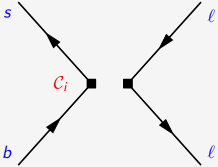

Operators

$$\mathcal{O}_{7(')} = \frac{m_b}{e}[\bar{s}\sigma^{\mu\nu}P_{R(L)}b]F_{\mu\nu}$$

$$\mathcal{O}_{9(')} = [\bar{s}\gamma^\mu P_{L(R)}b][\bar{\ell}\gamma_\mu\ell]$$

$$\mathcal{O}_{10(')} = [\bar{s}\gamma^\mu P_{L(R)}b][\bar{\ell}\gamma_\mu\gamma_5\ell]$$

Effective Field Theory for $b \to s\ell^+\ell^-$ FCNCs

Flavor Changing Neutral Current (FCNC)

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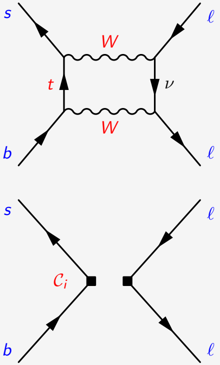

Decay Modes

$B \to K^*\ell^+\ell^-$	$B_s \to \mu^+\mu^-$	$B \to K\ell^+\ell^-$
$B \to K^*\gamma$	$B \to X_s\ell^+\ell^-$	$B \to X_s\gamma$

Model-Independent Framework

Definition of model-independent for the purpose of this work:

Basis of Operators $\mathcal{O}_i$

- include as many $\mathcal{O}_i$ beyond SM as needed/as few as possible
- balancing act, test statistically if choice of basis describes data well!

Wilson Coefficients $\mathcal{C}_i$

- treat $\mathcal{C}_i$ as uncorrelated, generalized couplings
- constrain their values from data
- model builders: confront new physics models with constraints

Fit Scenarios: SM(ν-only)

SM-like Coefficients

- fix $\mathcal{C}_{7,9,10}$ to SM values (NNLL, $\mathcal{C}_7^{\mathrm{SM}} \simeq -0.3$, $\mathcal{C}_9^{\mathrm{SM}} \simeq -\mathcal{C}_{10}^{\mathrm{SM}} \simeq 4.2$)

Chirality-flipped Coefficients

- fix $\mathcal{C}_{7'} = m_s/m_b\,\mathcal{C}_7^{\mathrm{SM}}$, fix $\mathcal{C}_{9',10'} = 0$

Nuisance Parameters

- fit nuisance parameters
- informative (Gaussian or LogGamma) priors
  - form factors:
    - light-cone sum rules [Khodjamirian/Mannel/Pivovarov/Wang '10]
    - lattice $B \to K$ [HPQCD], $B \to K^*$ [Horgan/Liu/Meinel/Wingate '13]
  - power corrections: power-counting assumptions
  - CKM: tree-level fit [UTfit]
  - quark masses [PDG]

Fit Scenarios: SM Basis

SM-like Coefficients

- fit $\mathcal{C}_{7,9,10}$

Chirality-flipped Coefficients

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  - quark masses [PDG]

Fit Scenarios: SM+SM′ Basis

SM-like Coefficients

- fit $\mathcal{C}_{7,9,10}$

Chirality-flipped Coefficients

- fit $\mathcal{C}_{7',9',10'}$

Nuisance Parameters

- fit nuisance parameters
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  - form factors:
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  - power corrections: power-counting assumptions
  - CKM: tree-level fit [UTfit]
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Sensitivity to Fit Parameters

Wilson Coefficients

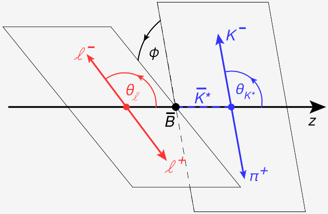
	$\mathcal{C}_{7(')}$	$\mathcal{C}_{9(')}$	$\mathcal{C}_{10(')}$	
$B_s \to \mu^+\mu^-$	–	–	✓	
$B \to X_s\gamma$	✓	–	–	
$B \to X_s\ell^+\ell^-$	✓	✓	✓	
$B \to K^*\gamma$	✓	–	–	
$B \to K^*\ell^+\ell^-$	✓	✓	✓	12 CP-avg. angular observables
$B \to K\ell^+\ell^-$	✓	✓	✓	3 CP-avg. angular observables

Hadronic Matrix Elements

- interplay between $B \to X_s\{\gamma, \ell^+\ell^-\}$ and $B \to K^*\{\gamma, \ell^+\ell^-\}$
- some $B \to K^*\ell^+\ell^-$ observables at large q^2 are ...
  - ... form-factor insensitive by construction
  - ... dominantly sensitive to form factor ratios

(Angular) Observables in $B \to K^* \ell^+ \ell^-$

- kinematics
  - dilepton mass squared q^2
  - three angles
- complicated diff. decay width
  - 12(+) angular observables J_n
  - express all observables through J_n
  - compose observ. from J_n with specific benefits, e.g. A_{FB}, F_L, $P'_{4,5,6}$, ...



Definitions

$$\Gamma \sim 3J_{1c} + 6J_{1s} - J_{2c} - 2J_{2s} \qquad A_{\mathrm{FB}} \sim \frac{J_{6s}}{\Gamma} \qquad F_L \sim \frac{3J_{1c} - J_{2c}}{\Gamma}$$

$$P'_4 \sim \frac{+J_4}{\sqrt{-J_{2s}J_{2c}}} \qquad P'_5 \sim \frac{+J_5}{2\sqrt{-J_{2s}J_{2c}}} \qquad P'_6 \sim \frac{-J_7}{2\sqrt{-J_{2s}J_{2c}}}$$

Analysis of 92 Individual Measurements

$B \to K^*\ell^+\ell^-$ $q^2 \in [1,6]\mathbf{GeV}^2$, $q^2 \geq M^2_{\psi'}$

- $\mathcal{B}$, A_{FB}, F_{L}, A_{T}^2
- $A_{\mathrm{T}}^{\mathrm{re}}$, P'_4, P'_5, P'_6
- ATLAS, BaBar, Belle, CDF, CMS, LHCb

$B \to K^*\gamma$

- $\mathcal{B}$, $S_{K^*\gamma}$, $C_{K^*\gamma}$
- BaBar, Belle, CLEO

$B \to K\ell^+\ell^-$ $q^2 \in [1,6]\mathbf{GeV}^2$, $q^2 \geq M^2_{\psi'}$

- $\mathcal{B}$
- BaBar, Belle, CDF, LHCb

$B \to X_s\gamma$ $E^{\gamma}_{\mathbf{min}} = 1.8\,\mathbf{GeV}$

- $\mathcal{B}$
- BaBar, Belle, CLEO

$B_s \to \mu^+\mu^-$

- $\int \mathrm{d}\tau\, \mathcal{B}(\tau)$
- CMS, LHCb

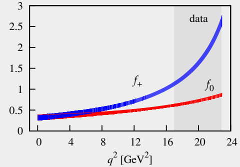
$B \to X_s\ell^+\ell^-$ $q^2 \in [1,6]\mathbf{GeV}^2$

- $\mathcal{B}$
- BaBar, Belle

Further Theory Constraints

$B \to K$ Form Factors from Lattice QCD (LQCD) [HPQCD 1306.2384]

- $B \to K$ form factors available from LQCD
  - data only at high q^2: 17 – 23 GeV^2
  - no data points given
- reproduce 3 data points from z-parametrization
  - $q^2 \in \{17, 20, 23\}$ GeV^2
  - use as constraint, incl. covariance matrix

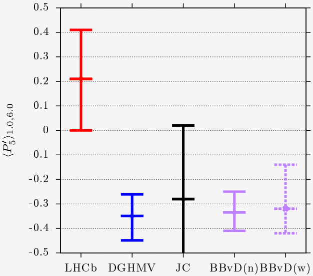


$B \to K^*$ Form Factor (FF) Relation at $q^2 = 0$

- FF $V, A_1 \propto \xi_\perp + \ldots$ [Charles et al. hep-ph/9901378]
  - no α_s corrections [Burdmann/Hiller hep-ph/0011266, Beneke/Feldmann hep-ph/0008255]
  - Large Energy Limit: $V(0) = (1.33 \pm 0.4) \times A_1(0)$
- LCSR constraint: $\xi_{||}(0) = 0.10^{+0.03}_{-0.02}$, to avoid $\xi_{||}(q^2) \propto A_0(q^2) < 0$.

- **LHCb** measurement [1308.1707]
  - deviation from SM prediction in form factor-free obs. $\langle P_5' \rangle_{[1,6]}$
  - LHCb uses one SM prediction (**DGHMV**)

    [Descotes-Genon/Hurth/Matias/Virto 1303.5794]



- however: further SM prediction exist, much larger uncertainty (**JC**)

  [Jäger/Camalich 1212.2263]

- our take on SM prediction

$$\langle P_5' \rangle_{[1,6]} = -0.34^{+0.09}_{-0.08} \text{ (BBvD, nominal priors)}$$
$$\langle P_5' \rangle_{[1,6]} = -0.32^{+0.18}_{-0.10} \text{ (BBvD, wide priors)}$$

see also backups for $P'_{4,6}$ and $[2, 4.3]$ bins

difference: treatment of unknown power corrections
(form factor corrections, $\bar{c}c$ resonances)

Results SM(ν-only)

Pull Values at Best-Fit Point

- largest pulls

-3.5σ	$\langle F_L \rangle_{[1,6]}$, BaBar 2012	$+2.6\sigma$	$\langle \mathcal{B} \rangle_{[16,19.21]}$, Belle 2009
-2.6σ	$\langle F_L \rangle_{[1,6]}$, ATLAS 2013	$+2.2\sigma$	$\langle A_{\text{FB}} \rangle_{[16,19]}$, ATLAS 2013
-2.4σ	$\langle P'_4 \rangle_{[14.18,16]}$, LHCb 2013	$+2.1\sigma$	$\langle P'_5 \rangle_{[1,6]}$, LHCb 2013

ATLAS, BaBar $\langle F_L \rangle_{[1,6]}$ problematic due to isospin averaging

p Values

- p value 0.10 without Lattice $B \to K^*$ FFs
- reduces to 0.04 with $B \to K^*$ Lattice input

Summary

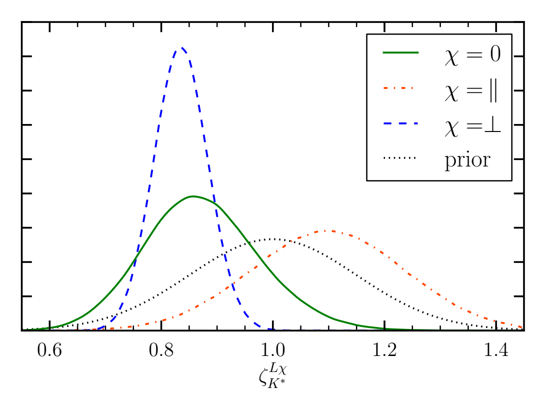
- good fit, no New Physics signal
- we find power corrections on top of QCDF results at large recoil

Parametrization of Power Corrections @ Large Recoil

- six parameters $\zeta_\chi^{L(R)}$ for the $[1, 6]$ bin

$$A_\chi^{L(R)}(q^2) \mapsto \zeta_\chi^{L(R)} A_\chi^{L(R)}(q^2)\,, \quad \chi = \perp, \parallel, 0$$

- on top of QCDF correction to transversity amplitudes

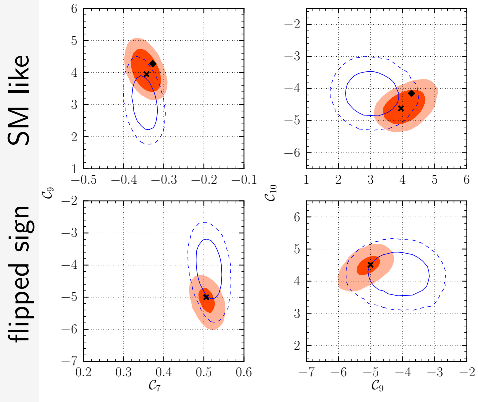


SM(ν-only)

- tension diluted by parameters $\zeta_\chi^{L(R)}$
- shift by $\simeq -20\%$ for $\zeta_{\perp,\parallel}^L$
- shift by $\simeq +10\%$ for ζ_0^L
- few percents for ζ_χ^R

improved understanding of power corrections desirable

Results (SM Basis)



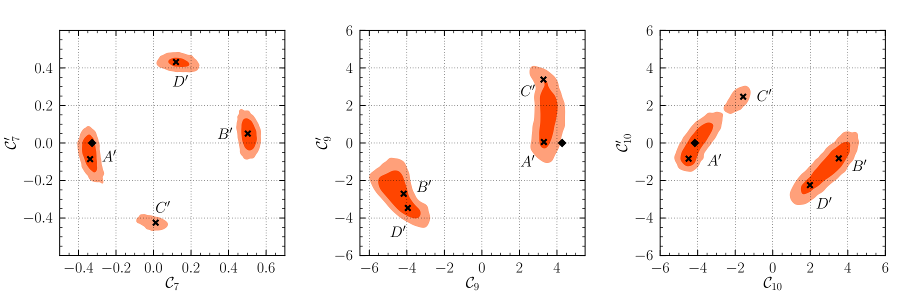
◆: Standard Model, ×: best-fit point

(light-) red: 68% prob (95% prob) for full dataset

post HEP'13

- SM-like uncertainty reduced by ~ 2 compared to 2012
- SM at the border of 1σ
- flipped-sign barely allowed at 1σ (26% of evidence)
- cannot confirm NP findings
  - in $(\mathcal{C}_7, \mathcal{C}_9)$
    [Descotes-Genon et al. 1307.5683]
- $\zeta_\chi^{L(R)}$ as in SM(ν-only)
- p value: 0.08 (@SM-like sol.)

Results (SM+SM′ Basis)



♦: Standard Model, ×: best-fit points, (light-) red: 68% prob (95% prob) for full dataset

post HEP’13

- four solutions A' through D'
  - $A' =$ SM like, 39% of ev.
  - $B' =$ flipped signs, 41% of ev.
  - C', D' suppressed: 5% and 15% of evidence
- for A' (SM-like sol.)
  - p value 0.09
  - $\mathcal{C}_9 - \mathcal{C}_9^{\rm SM} = -0.8^{+0.2}_{-0.5}$
  - 2σ deviation from SM
  - $\zeta_\chi^{L(R)}$ decrease wrt. SM(ν-only) and SM basis

(Statistical) Model Comparison

- model comparison using Bayes factor and model priors
- compare scenarios only at SM-like solution $A(')$
- adjust priors to contain only $A(')$
- results without $B \to K^*$ lattice form factors
  - SM(ν-only) wins over SM basis: odds of 93:1
  - SM(ν-only) wins over SM+SM$'$ basis: odds of 19:1
- results with $B \to K^*$ lattice form factors
  - SM(ν-only) wins over SM basis: odds of 97:1
  - SM+SM$'$ basis wins over SM(ν-only): odds of 5:1

Conclusion

- all three scenarios describe $b \to s(\gamma, \ell^+\ell^-)$ data well
- SM(ν-only) wins comparison with SM and SM+SM′
  - subleading power correction on top of QCDF: 10–20%
- several tensions in all scenarios
  - $\langle P_5' \rangle_{[1,6]}$ reduced pull in fit due to power corrections
  - $\langle F_L \rangle_{[1,6]}$ from BaBar, ATLAS (both preliminary) persist
  - $\langle P_4' \rangle_{[14.18,16]}$ LHCb persists
- new physics signal only for
  - SM+SM′ basis (2σ, model comparison highly FF-dependent)
- data also allows inference of form factor parameters
- looking forward to further LHC analyses (2012 datasets) and the prospects of Belle-II

Backup Slides

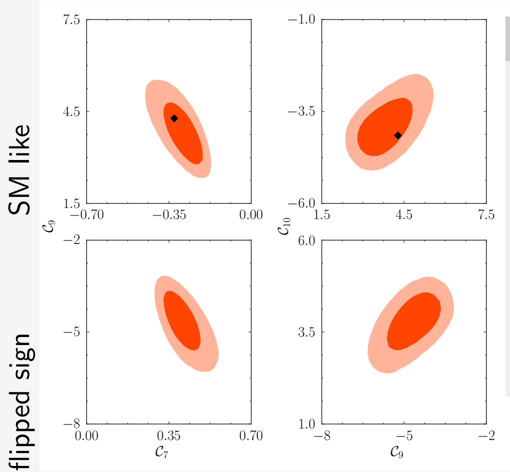
Standard Model Predictions for $P'_{4,5,6}$

- toy Monte Carlo using priors + theory constraints (FFs)
- calculate observable for 10^5 samples
- find minimal 68% intervals

	q^2 [GeV^2]	$\langle P'_4 \rangle$	$\langle P'_5 \rangle$	$10^2 \times \langle P'_6 \rangle$
BBvD	$[1, 6]$	$+0.46\ ^{+0.12}_{-0.11}$	$-0.335\ ^{+0.085}_{-0.075}$	$-6.4\ \pm 1.7$
	$[2, 4.3]$	$+0.48\ ^{+0.11}_{-0.10}$	$-0.315\ ^{+0.074}_{-0.090}$	$-7.2\ ^{+1.5}_{-2.2}$
LHCb†	$[1, 6]$	$+0.58\ ^{+0.33}_{-0.36}$	$+0.21\ ^{+0.20}_{-0.21}$	$+18\ \pm 21$
	$[2, 4.3]$	$+0.74\ ^{+0.11}_{-0.53}$	$+0.29\ ^{+0.40}_{-0.39}$	$+15\ ^{+38}_{-36}$

†: [LHCb 1308.1707], adjusted to theory convention

2012 Results (SM Basis)

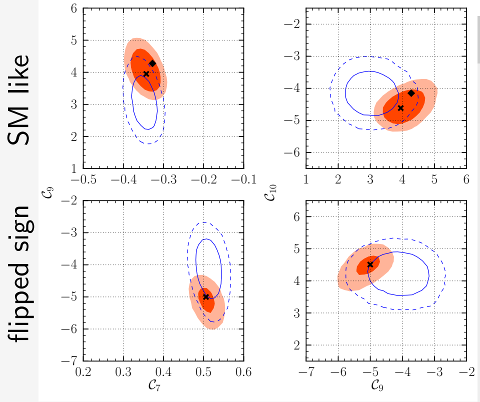


early 2012

- figure from [1205.1838]
- no $B \to X_s\{\gamma, \ell^+\ell^-\}$
- only LHCb bound on $B_s \to \mu^+\mu^-$
- $B \to K^{(*)}\ell^+\ell^-$: $\mathcal{B}, A_{\rm FB}, F_L, A_T^{(2)}, S_3$
- $B \to K^*\gamma$: $\mathcal{B}, S_{K^*\gamma}, C_{K^*\gamma}$

◆: Standard Model

Results (SM Basis, Selection)



◆: Standard Model,   ×: best-fit point

(light-) red: 68% pr (95% pr) for full dataset

blue solid (blue dashed): 68% pr (95% pr) for selection

post HEP'13 (selection)

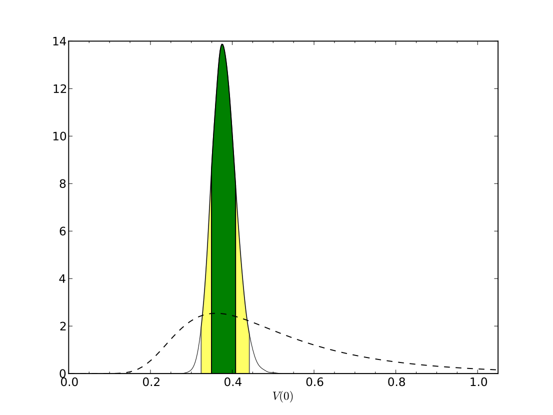
- with $B \to X_s\{\gamma, \ell^+\ell^-\}$
- $B_s \to \mu^+\mu^-$ from LHCb and CMS
- same data as [Descotes-Genon/Matias/Virto 1307.5683] exclusive decays limited:
  - only $B \to K^*\ell^+\ell^-$!
  - only LHCb data!
  - only $q^2 \in [1, 6]\text{GeV}^2$
- best-fit point similar to [Descotes-Genon et al. 1307.5683]
  - less tension, only $\lesssim 2\sigma$
  - $\mathcal{C}_9 - \mathcal{C}_9^{\rm SM} \simeq -1.3 \pm 0.5$

Further Theory Constraints

$B \to K^*$ Form Factors from Lattice QCD [Horgan/Liu/Meinel/Wingate 1310.3722]

- first unquenched $B \to K^*$ form factors
- based on "full QCD" 2+1 ensembles with improved staggered action [MILC]
- however: stable K^* due to unphysical K^* mass
- used optionally only, since there is a large systematic error attached

Results for $B \to K^*$ Form Factors

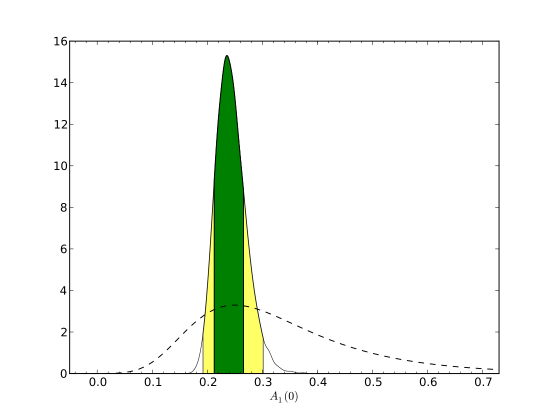


dashed: prior [Khodjamirian/Mannel/Pivovarov/Wang 1006.4945]

solid: posterior, green: 68% CL, yellow: 95% CL

- more precise than prior
- $B \to K^*$: $\xi_\perp$ from
  - $B \to X_s \gamma$
  - $B \to K^* \gamma$
  - $B \to K^* \ell^+ \ell^-$
  - theory input
- results @ 68% CL
  - $V(0) = 0.37^{+0.03}_{-0.02}$

Results for $B \to K^*$ Form Factors



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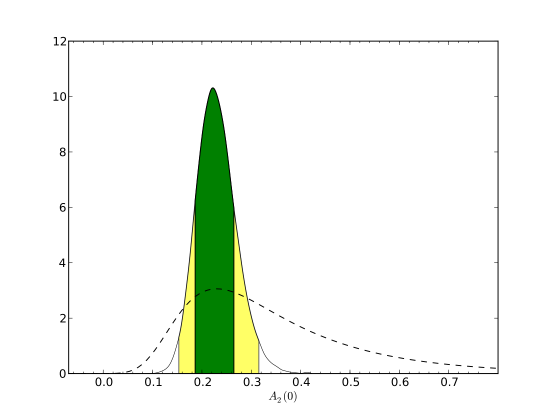
- results @ 68% CL
  - $V(0) = 0.37^{+0.03}_{-0.02}$
  - $A_1(0) = 0.24 \pm 0.03$

Results for $B \to K^*$ Form Factors



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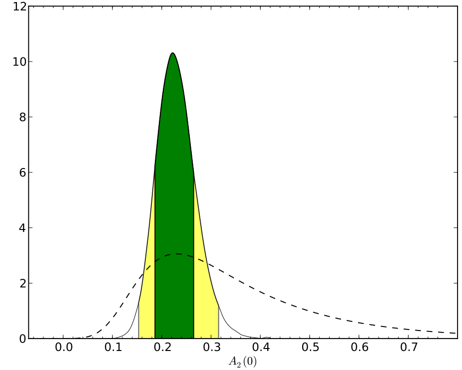
  - $B \to X_s\gamma$
  - $B \to K^*\gamma$
  - $B \to K^*\ell^+\ell^-$
  - theory input
- results @ 68% CL
  - $V(0) = 0.37^{+0.03}_{-0.02}$
  - $A_1(0) = 0.24 \pm 0.03$
  - $A_2(0) = 0.22 \pm 0.04$

Results for $B \to K^*$ Form Factors



dashed: prior [Khodjamirian/Mannel/Pivovarov/Wang 1006.4945]

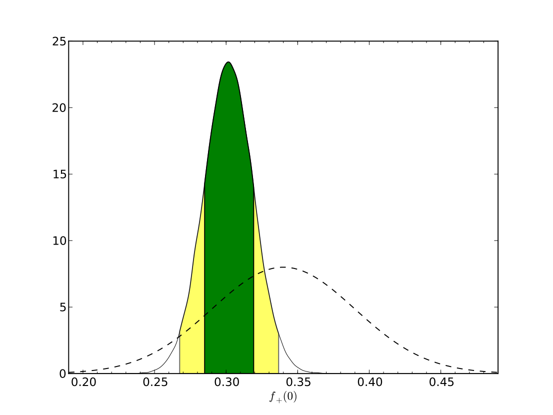
solid: posterior, green: 68% CL, yellow: 95% CL

- more precise than prior
- $B \to K^*$: $\xi_\perp$ from
  - $B \to X_s\gamma$
  - $B \to K^*\gamma$
  - $B \to K^*\ell^+\ell^-$
  - theory input
- results @ 68% CL
  - ratio of central values

$$V(0)/A_1(0) \simeq 1.5$$
$$A_2(0)/A_1(0) \simeq 0.9$$

agree w/ (SE2 full) [Hambrock/Hiller/Schacht/Zwicky 1308.4379]

Results for $B \to K$ Form Factors

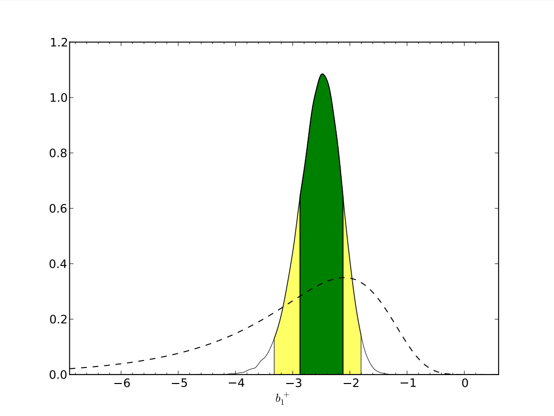


dashed: prior [Khodjamirian/Mannel/Pivovarov/Wang 1006.4945],
modified to accomodate [Ball/Zwicky hep-ph/0406232]

solid: posterior, green: 68% CL, yellow: 95% CL

- more precise than prior
- $B \to K$:
  - $B \to K\ell^+\ell^-$
  - Lattice
- results @ 68% CL
  - $f_+(0) = 0.30 \pm 0.02$

Results for $B \to K$ Form Factors



dashed: prior [Khodjamirian/Mannel/Pivovarov/Wang 1006.4945],

solid: posterior, green: 68% CL, yellow: 95% CL

- more precise than prior
- $B \to K$:
  - $B \to K\ell^+\ell^-$
  - Lattice
- results @ 68% CL
  - $f_+(0) = 0.30 \pm 0.02$
  - $b_1^+ = -2.5 \pm 0.4$

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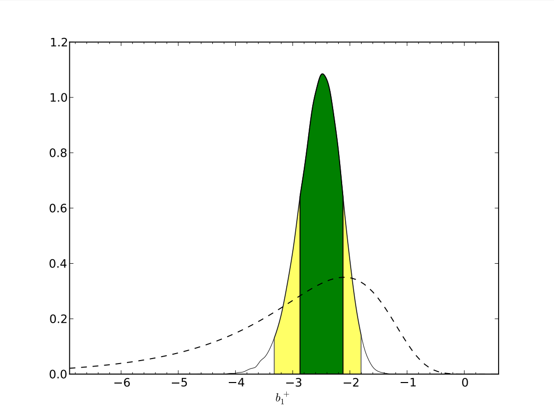

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- more precise than prior
- $B \to K$:
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  - Lattice
- results @ 68% CL
  - $f_+(0) = 0.30 \pm 0.02$
  - $b_1^+ = -2.5 \pm 0.4$
- small tension
  - LHCb lo q^2: -1.4σ
  - LHCb hi q^2: $+1.1\sigma$
  - Lattice: $+0.5\sigma$

Priors and Parametrizations (I)

Form Factors [Khodjamirian et al. 1006.4945]

- values @ $q^2 = 0$ and slope: two parameters per FF
- z-parametrization
- asymmetric priors, use LogGamma function

CKM [update of hep-ph/0012308]

- Wolfenstein parametrization
- UTfit pre-Moriond2013, tree-level data only

Quark Masses [PDG]

Priors and Parametrizations (I) - Subleading

parametrize unknown subleading contributions

$B \to K^* \ell^+ \ell^-$

- lo q^2: 6 parameters, one scaling factor per amplitude
- hi q^2: 3 parameters

$B \to K \ell^+ \ell^-$

- lo q^2: 1 parameter
- hi q^2: 1 parameter

for all: Gaussian with 1σ interval $\pm\Lambda_{\text{QCD}}/m_b \simeq \pm 0.15$

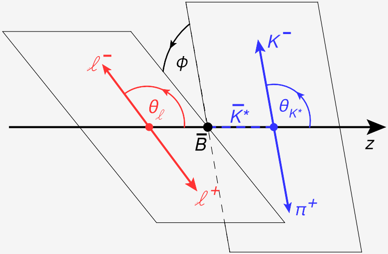
A Note on p Values

- test statistic: function of data and model (parameters) $\chi^2 = \chi^2(D, \vec{\theta})$
- only one data set observed $D_{obs} \Rightarrow \chi^2_{obs}$
- $p \equiv P(\chi^2 > \chi^2_{obs})$

But how to fix $\vec{\theta}$?

1. this work
   $\vec{\theta} = (\vec{\mathcal{C}}, \vec{\nu})$ at (local) mode of posterior, $\chi^2 \sim \frac{(x-\mu)^2}{\sigma^2_{exp}}$
2. Descotes-Genon et al. [1307.5683]
   $\vec{\theta} = \vec{\mathcal{C}}$ at (local) mode of likelihood, $\chi^2 \sim \frac{(x-\mu)^2}{\sigma^2_{exp} + \sigma^2_{theo}}$

Kinematics of $\bar{B} \to \bar{K}\pi\ell^+\ell^-$



Kinematic Variables

$$4m_\ell^2 \leq q^2 \leq (M_B - M_{K^*})^2$$

$$-1 \leq \cos\theta_\ell \leq 1$$

$$-1 \leq \cos\theta_{K^*} \leq 1$$

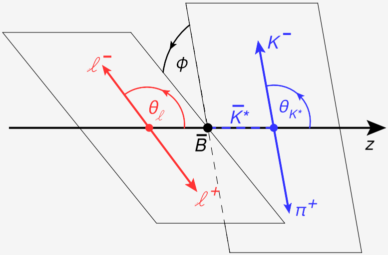
$$0 \leq \phi \leq 2\pi$$

$$\left[(M_K + M_\pi)^2 \leq k^2 \leq (M_B - \sqrt{q^2})^2\right]$$

On-shell and S-Wave

- one usually assumes on-shell decay of P-wave K^* ($\sim \sin\theta_{K^*}, \cos\theta_{K^*}$)
- for high precision: consider width of K^*, and $J = 0$ (S-wave) ($\not\sim \theta_{K^*}$) $K\pi$-final-state from K_0^* and *non-resonant background*

Kinematics of $\bar{B} \to \bar{K}\pi\ell^+\ell^-$



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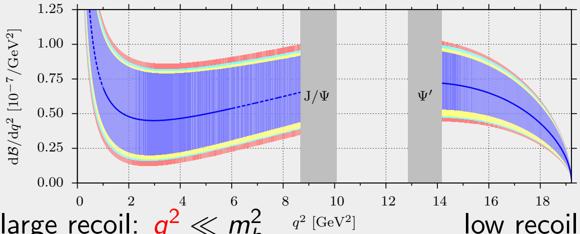
Large vs. Low Recoil (for illustration)



large recoil: $q^2 \ll m_b^2$ low recoil: $q^2 \simeq m_b^2$

Differential Decay Rate for pure P-wave state

$$
\begin{aligned}
\frac{d^4\Gamma}{dq^2 d\cos\theta_\ell d\cos\theta_{K^*} d\phi} \sim\ & J_{1s}\sin^2\theta_{K^*} + J_{1c}\cos^2\theta_{K^*} \\
& + (J_{2s}\sin^2\theta_{K^*} + J_{2c}\cos^2\theta_{K^*})\cos 2\theta_\ell \\
& + (J_3\cos 2\phi + J_9\sin 2\phi)\sin^2\theta_{K^*}\sin^2\theta_\ell \\
& + (J_4\sin 2\theta_{K^*})\sin 2\theta_\ell\cos\phi \\
& + (J_5\sin 2\theta_{K^*})\sin\theta_\ell\cos\phi \\
& + (J_{6s}\sin^2\theta_{K^*} + J_{6c}\cos^2\theta_{K^*})\cos\theta_\ell \\
& + (J_7\sin 2\theta_{K^*})\sin\theta_\ell\sin\phi \\
& + (J_8\sin 2\theta_{K^*})\sin 2\theta_\ell\sin\phi\,,
\end{aligned}
$$

$J_i \equiv J_i(q^2)$: 12 angular observables

Angular Distribution [Krüger/Matias '05, Altmannshofer et al. '08, Blake/Egede/Shires '12]

Differential Decay Rate for mixed P- and S-wave state

$$\begin{aligned}\frac{d^4\Gamma}{dq^2 d\cos\theta_\ell d\cos\theta_{K^*} d\phi} \sim\ & J_{1s}\sin^2\theta_{K^*} + J_{1c}\cos^2\theta_{K^*} + J_{1i}\cos\theta_{K^*}\\ &+ (J_{2s}\sin^2\theta_{K^*} + J_{2c}\cos^2\theta_{K^*} + J_{2i}\cos\theta_{K^*})\cos 2\theta_\ell\\ &+ (J_3\cos 2\phi + J_9\sin 2\phi)\sin^2\theta_{K^*}\sin^2\theta_\ell\\ &+ (J_4\sin 2\theta_{K^*} + J_{4i}\cos\theta_{K^*})\sin 2\theta_\ell\cos\phi\\ &+ (J_5\sin 2\theta_{K^*} + J_{5i}\cos\theta_{K^*})\sin\theta_\ell\cos\phi\\ &+ (J_{6s}\sin^2\theta_{K^*} + J_{6c}\cos^2\theta_{K^*})\cos\theta_\ell\\ &+ (J_7\sin 2\theta_{K^*} + J_{7i}\cos\theta_{K^*})\sin\theta_\ell\sin\phi\\ &+ (J_8\sin 2\theta_{K^*} + J_{8i}\cos\theta_{K^*})\sin 2\theta_\ell\sin\phi\,,\end{aligned}$$

$J_i \equiv J_i(q^2, k^2)$: 12 angular observables, no further needed [Bobeth/Hiller/DvD '12]

Conclusion: remove S-wave in exp. analysis

- angular analysis [Egede/Blake/Shires '12]
- side-band analysis (for $J_{1s,1c,2s,2c}$) [Bobeth/Hiller/DvD '12]