

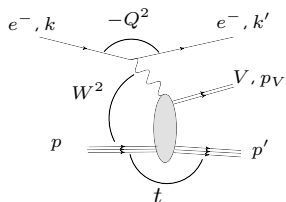
# Leptoproduction of vector meson from the small x to the valence region

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DIS2014, Warsaw





## Hard exclusive vector meson lepton production from small $x$ to the valence region

- Important exclusive process for at least two communities :
  - Small- $x$  community (  $k_T$ -factorization, dipole models )
  - GPD community ( Collinear factorization framework )
- Comparison between the two descriptions

## Probing the non-perturbative physics of the nucleon and the meson

- Information on the **nucleon Generalized Parton Distributions (GPDs)** :

- intrinsically non-forward amplitudes :  $|t|_{\min} \sim \frac{4m^2\xi^2}{1-\xi^2}$

⇒ Skewness effects

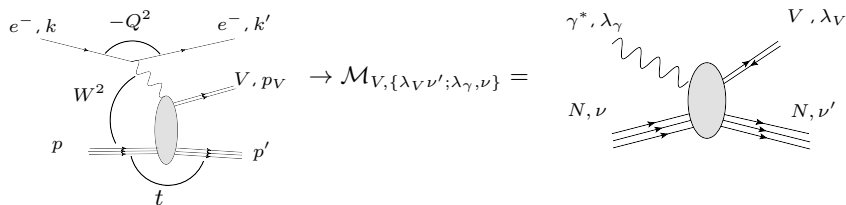
- DVMP ⇒ scan the flavor content of the nucleon  
(via quark diagram contributions)

$$\rho = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}), \quad \phi = s\bar{s}, \quad \omega = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}), \dots$$

- Small  $x$  : Information on the **dipole cross-section**  $\hat{\sigma}(x, \underline{r})$ :

- Saturation effects and transverse distribution of gluons
- Skewness factor (Shuvaev, Golec-Biernat, Martin, Ryskin, '99)

- Information on the **wavefunction** or the distribution amplitudes of the vector meson (VM)

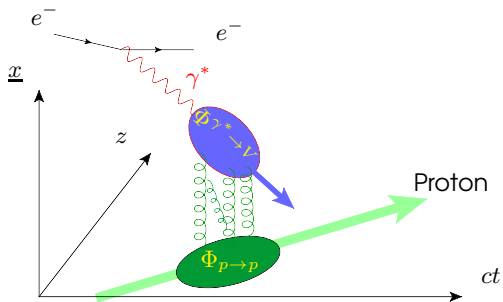


- Spin density matrix elements (SDME) linked to the helicity amplitudes :

(Schilling Wolf, '73) & (Dielh, '07)

- small  $x$  HERA (H1 and ZEUS)
- mid- $x$  region: COMPASS, HERMES, E665, NMC
- valence region: CLAS

## Part I : DVMP within $k_T$ -factorization and dipole models



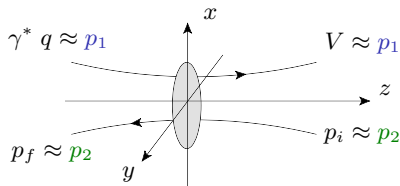
### Sudakov Basis

- Light-cone vectors  $p_1$  and  $p_2$  :

$$p_1^2 = p_2^2 = 0$$

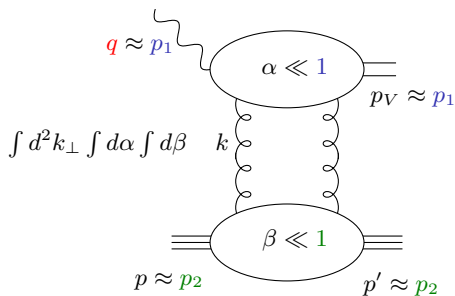
$$2p_1 \cdot p_2 = s$$

- We can choose:



- Euclidean notation :  $\underline{k}^2 = -k_\perp^2$

- $k = \alpha p_1 + \beta p_2 + k_\perp$



- Writing  $g_{\mu\nu} = \frac{p_{2\mu} p_{1\nu} + p_{1\mu} p_{2\nu}}{s/2} + g_{\perp\mu\nu}$

- Eikonal approximation :

$$p_1 \text{ --- } \text{---} p_1 + k \sim p_1$$

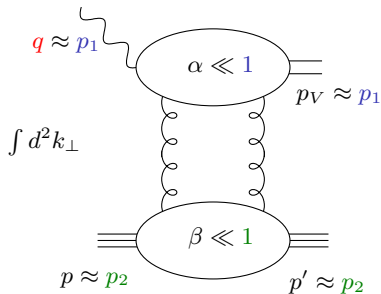
$$\gamma^\mu g_{\mu\nu} \rightarrow \frac{\not{p}_2}{p_1 \cdot p_2} p_{1\nu} + \dots$$

- Next-to-Eikonal approximation at small- $x$  in pA gluon production  
See talk (T. Altinoluk)

- Impact factor representation of the helicity amplitudes

$$\mathcal{M}_{\lambda_V \lambda_\gamma} = is \int \frac{d^2 \underline{k}}{(2\pi)^2 (\underline{k}^2)^2} \Phi^{\gamma^* (\lambda_\gamma) \rightarrow V (\lambda_V)}(\underline{k}) \Phi^{P \rightarrow P}(-\underline{k})$$

- $k = \alpha p_1 + \beta p_2 + k_\perp$



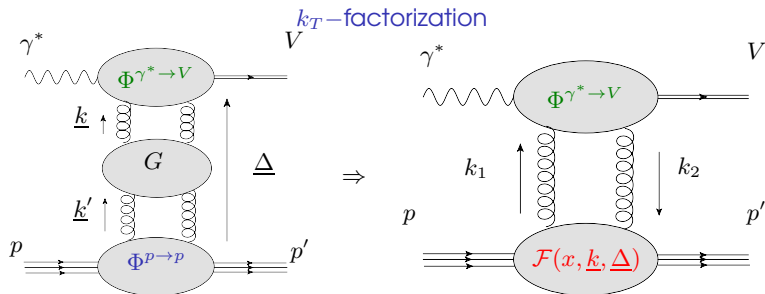
$$\Phi^{\gamma^* \rightarrow V}(\underline{k}^2) \propto \int d\beta \mathcal{M}^{\gamma^* g \rightarrow Vg}(0, \beta, \underline{k})$$

$$\Phi^{P \rightarrow P}(\underline{k}^2) \propto \int d\alpha \mathcal{M}^{pg \rightarrow pg}(\alpha, 0, \underline{k})$$

- Impact factor representation of the helicity amplitudes

$$\mathcal{M}_{\lambda_V \lambda_\gamma} = is \int \frac{d^2 \underline{k}}{(2\pi)^2 (\underline{k}^2)^2} \Phi^{\gamma^*(\lambda_\gamma) \rightarrow V(\lambda_V)}(\underline{k}) \Phi^{P \rightarrow P}(-\underline{k})$$

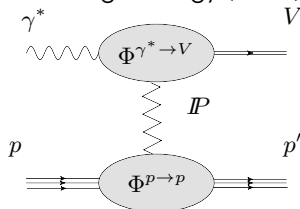




$$\begin{aligned}
 \mathcal{M}_{\lambda_V \lambda_\gamma}(s, t) &= \frac{is}{(2\pi)^2} \int \frac{d^2 \underline{k}}{k^2} \Phi^{\gamma^* \rightarrow V}(\underline{k}, \underline{\Delta} - \underline{k}) \int \frac{d^2 \underline{k}'}{k'^2} \Phi^{p \rightarrow p'}(-\underline{k}', -\underline{\Delta} + \underline{k}') \\
 &\quad \times \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^\omega G_\omega(\underline{k}, \underline{k}', \underline{\Delta}) \\
 &\equiv is \int \frac{d^2 \underline{k}}{k^2 (\underline{\Delta} - \underline{k})^2} \Phi^{\gamma^* \rightarrow V}(\underline{k}, \underline{\Delta} - \underline{k}) \mathcal{F}(x, \underline{k}, \underline{\Delta})
 \end{aligned}$$

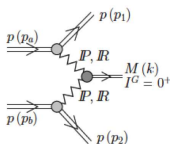
### Regge theory point of view

- At high energy (low  $x$ )



- Pomeron  $IP$  exchange in  $t$ -channel
- Pomeron contains the  $s$ -dependence
- Fits (Donnachie, Landshoff, '92) of experimental data  $\Rightarrow$  Pomeron intercept  $\alpha_P(0) = 1.08 \Leftrightarrow \sigma_{tot} \propto s^{0.08}$ !

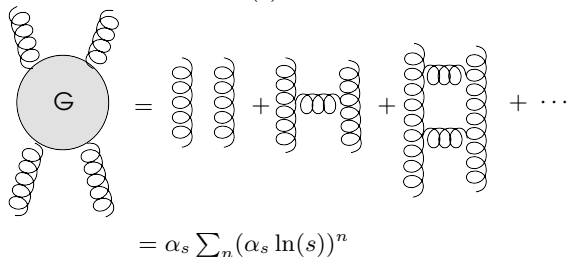
- Pomeron couplings to hadrons:



Double Pomeron exchange in central meson production  
See talk (P. Lebiedowicz)

### BFKL resummation

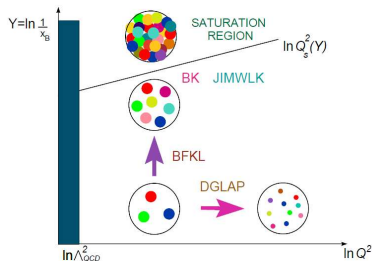
- Resummation in  $\alpha_s \ln(s)$ :


$$= \alpha_s \sum_n (\alpha_s \ln(s))^n$$

- BFKL equation [Balitsky, Fadin, Kuraev, Lipatov, '77, '78](#)
- LO BFKL solution = LLx Hard QCD Pomeron leads to  $\sigma_{tot} \sim s^{\alpha_P - 1}$  with  $\alpha_P > 1$  Pomeron intercept
- Probing BFKL dynamics in Mueller-Navelet jets  
See talks. [\(B. Ducloué\)](#), [\(R. Maciula\)](#), [\(G. Safronov\)](#)
- NLLx evolution  $\Rightarrow$  NLLx impact factors :  
Heavy quark NLLx impact factors : See talk [\(M. Deak\)](#)

## Saturation and pQCD evolution equations

- QCD evolution equations



- $Q^2$  evolution driven by DGLAP equation  
Dokshitzer-Gribov-Lipatov-Altarelli-Parisi, '72, '77
- small  $x$  evolution driven by BFKL equation in the diluted regime and BK-JIMWLK equation in the dense regime (Balitsky-Kovchegov '96, Jalilian-Marian-Iancu-McLerran-Weigert-Leonidov-Kovner, '97, '99, '01).

- Low- $x$  evolution equations:

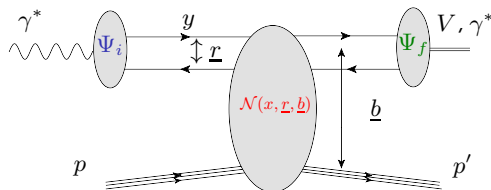
See talks : (K. Kutak) (gluon evolution at strong coupling), (G. Beuf) (Improv. Kinematics), (Y. Mulian) (NLO JIMWLK)

- Dipole model approach, a convenient scheme to introduce saturation effects

## Color dipole factorization scheme

- Impact parameter space representation of the amplitudes in the infinite momentum frame

Nikolaev, Zakharov, '91, Mueller, '90



- Initial  $\Psi_i$  and final  $\Psi_f$  states wave functions.
- Universal dipole/target scattering amplitude  $\mathcal{N}(x, \underline{r}, \underline{b})$ :
  - DIS structure functions, DIS diffractive, exclusive processes ..
  - See talk (H. Mäntysäari): initial condition for the dipole scattering amplitude from DIS

- Amplitude for DVMP (Kowalski, Motyka, Watt, '06):

$$\mathcal{M}_{\lambda_V \lambda_\gamma}(Q^2, x, t) = is \int dy \int d\underline{r} \int d^2\underline{b} \Psi_{\lambda_V}^*(y, \underline{r}) \Psi_{\lambda_\gamma}(y, \underline{r}) e^{-i(\underline{b} - \bar{y}\underline{r}) \cdot \underline{\Delta}} \mathcal{N}(x, \underline{r}, \underline{b})$$

Next talk: dipole models with  $\underline{b}$ -dependence (A. Rezaeian)

- In the limit  $\underline{\Delta} = 0$ , i.e.  $|t| = |t|_{min}$ ,

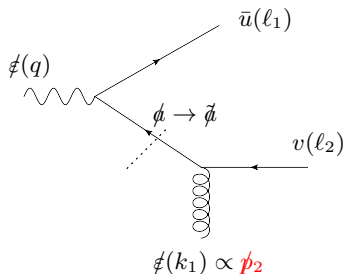
$$\mathcal{M}_{\lambda_V \lambda_\gamma}(Q^2, x) = is \int dy \int d\underline{r} \Psi_{\lambda_V}^*(y, \underline{r}) \Psi_{\lambda_\gamma}(y, \underline{r}) \hat{\sigma}(x, \underline{r})$$

- Dipole cross-section and unintegrated gluon density at Born level

$$\hat{\sigma}(x, \underline{r}) = \frac{N^2 - 1}{4} \frac{4\pi\alpha_s}{N} \int d^2\underline{k} \frac{1}{(\underline{k}^2)^2} \mathcal{F}(x, \underline{k}) \left(1 - e^{-i\underline{k} \cdot \underline{r}}\right) \left(1 - e^{i\underline{k} \cdot \underline{r}}\right)$$

- Skewness effects can be taken into account in dipole cross-section model (Shuvaev, Golec-Biernat, Martin, Ryskin, '99)

### Factorization of the wavefunctions and models



- Eikonal approximation leads to (Anikin, Wüsthof, '99)

$$\phi p_2 \rightarrow \tilde{\phi} p_2 \propto v(\tilde{a})\bar{v}(\tilde{a}) p_2, \text{ such that } \tilde{a}^2 = 0$$

- The wavefunction factorizes

$$\Psi_{\gamma^*}(y, \underline{\ell}) \propto \frac{\bar{u}(l_1)\phi(q)v(\tilde{a})}{a^2 + i\epsilon} \Big|_{\ell_1 = y p_1 + \ell_\perp + \frac{\ell^2}{2y s} p_2}$$

- Vector meson wavefunction: need to be modeled  
See talk (M. Djuric) on nucleon and meson holographic wavefunctions in ADS

## Models of dipole cross-section

- Small- $x$  evolution
  - Initial condition for the dipole cross-section at a given rapidity from DIS structure functions
  - Evolution with rc-BK equation (Balitsky, '07)

See talk (H. Mäntysaari) for details

- DGLAP evolution (Bartels, Golec-Biernat, Kowalski, '02)

$$\hat{\sigma}(x, \underline{r}) = \sigma_0 \left( 1 - \exp \left\{ - \frac{\pi^2 \underline{r}^2 \alpha_s(\mu^2(\underline{r}^2)) xg(x, \mu^2(\underline{r}^2))}{3\sigma_0} \right\} \right)$$

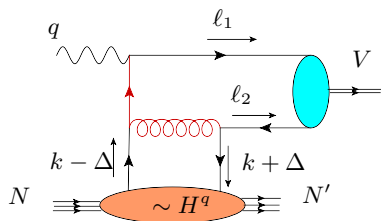
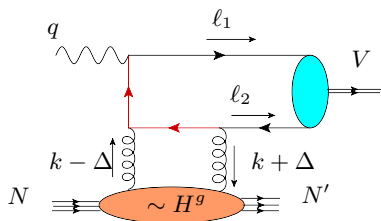
See talk (Kowalski) for details



## Part II : Modified perturbative approach and connection to dipole models

## Description of exclusive processes within Collinear factorization approach

- Description of DVMP, DVCS, TCS, ... in the Bjorken limit
- Collinear factorization proven for LT amplitude  $\mathcal{M}_{V,\{0+;0+\}}$   
(Collins, Frankfurt, Strikman, '97, Radyushkin, '97)
- Set of GPDs,  $H(x, \xi, t)$ ,  $E(x, \xi, t)$ ,  $\tilde{H}(x, \xi, t)$ ,  $\tilde{E}(x, \xi, t)$
- Quark and Gluon contributions:



## Gluon contribution in MPA

$$\mathcal{M}_V^g = \int dx \int dy \int \frac{d^2 \ell}{(2\pi)^2}$$

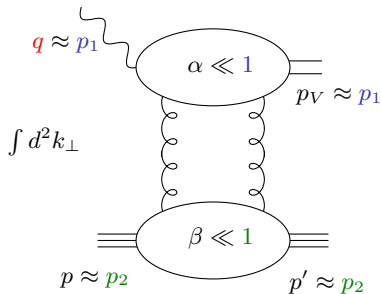
$\ell_q = yp_1 + \ell_\perp + \frac{\ell_\perp^2}{2s_y} p_2$

$\langle V(p_1) | \bar{\psi}_i(z_2) \psi_j(0) | 0 \rangle \rightarrow p'_{1,ij} \Psi_V(y, \ell_\perp)$

$\langle N(p') | A(z_1) A^*(0) | N(p) \rangle \rightarrow \sum_\lambda \not{\epsilon}_\perp^{(\lambda)} \not{\epsilon}_\perp^{(\lambda)*} \frac{H^g(x, \xi, t)}{x^2 - (\xi - i\epsilon)^2}$

- Neglect then  $\frac{\ell_\perp^2}{y\bar{y}Q^2}$  **terms in numerator** in the MPA spirit
- Fourier transform in transverse space  $\rightarrow$  impact parameter space
- Sudakov form factor (Sterman, Li '92)  
(Resums soft gluon emissions from the quark-antiquark dipole)

- $k = \alpha p_1 + \beta p_2 + k_\perp$



$$\Phi^{\gamma^* \rightarrow V}(\underline{k}^2) \propto \int d\beta \mathcal{M}^{\gamma^* g \rightarrow Vg}(0, \beta, \underline{k})$$

$$\Phi^{P \rightarrow P}(\underline{k}^2) \propto \int d\alpha \mathcal{M}^{pg \rightarrow pg}(\alpha, 0, \underline{k})$$

- Impact factor representation of the helicity amplitudes

$$\mathcal{M}_{\lambda_V \lambda_\gamma} = is \int \frac{d^2 \underline{k}}{(2\pi)^2 (\underline{k}^2)^2} \Phi^{\gamma^*(\lambda_\gamma) \rightarrow V(\lambda_V)}(\underline{k}) \Phi^{P \rightarrow P}(-\underline{k})$$

Results in the two different approaches in the limit  $t \sim 0$

- **MPA** result for the helicity amplitude  $\gamma_L^* N(p) \rightarrow V_L N(p')$   
(A.B. in preparation):

$$\begin{aligned} \text{Im } \mathcal{M}_{V,\{0+,0+\}}^g &= - \int_0^1 dy \int d^2 \underline{r} \sum_f C_V^f \left( \hat{\Psi}_V(y, -\underline{r}) \hat{\Psi}_{\gamma_L^*}^f(y, \underline{r}) \right) \\ &\times \left( \frac{\pi \sqrt{2\pi}}{N_c y \bar{y}} \alpha_s \frac{H^g(\xi, \xi, 0)}{2\xi} \right) \end{aligned}$$

- **$k_T$ -factorization** result for the helicity amplitude  $\gamma_L^* N(p) \rightarrow V_L N(p')$  :  
(A.B., Szymanowski, Wallon, '13)

$$\begin{aligned} \text{Im } \mathcal{M}_{V,\{0+,0+\}}^g &= - \int dy \int d^2 \underline{r} \sum_f C_V^f \left( \hat{\Psi}_V(y, -\underline{r}) \hat{\Psi}_{\gamma_L^*}^f(y, \underline{r}) \right) \\ &\times \left( \frac{s \hat{\sigma}(x, \underline{r})}{2\sqrt{2\pi}} \right) \end{aligned}$$

- At small- $x$  :  $\xi \approx x/2$

## Interpretation

- Forward dipole cross-section (Frankfurt, Radyushkin, Strikman, '97) :

$$\hat{\sigma}(x, \underline{r}) = \frac{\pi^2}{N_c} \underline{r}^2 \alpha_s x g(x) \quad (\text{color transparency})$$

- Comparing the results for DVMP:

$$\hat{\sigma}(x, \underline{r}) \leftrightarrow \frac{\pi^2}{N_c} \left( \frac{4}{y\bar{y}Q^2} \right) \alpha_s H^g(\xi, \xi, 0) = \frac{\pi^2}{N_c} \underline{r}_0^2 \alpha_s H^g(\xi, \xi, 0)$$

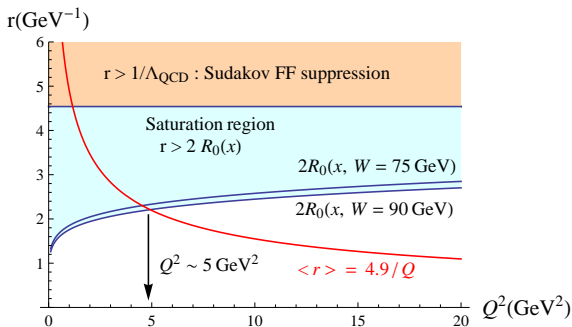
with  $\underline{r}_0^2 = \frac{4}{y\bar{y}Q^2}$

# Analogy between the results

- Evaluating the average  $\langle \underline{r}_0^2 \rangle$  over the amplitude  $\Rightarrow \sqrt{\langle \underline{r}_0^2 \rangle} \sim \frac{4.9}{Q}$
- When does the average size of dipole  $\langle r_0^2 \rangle$  enter the saturation region ?

$$\hat{\sigma}(x, \underline{r}) = \sigma_0 \left( 1 - e^{-\frac{r^2}{4R_0^2(x)}} \right) \quad , \text{ saturation for } |\underline{r}| > 2R_0(x)$$

(Golec-Biernat, Wüsthof, '98)



- $\langle r_0^2 \rangle \sim$  independent on the normalization of the wavefunctions.

## Contribution from gluons and quarks

- Within MPA : real and imaginary parts of gluon and quark contributions
- Replacement  
(extracting the flavor dependence of the overlap of wavefunction):

$$\sum_f e_f C_V^f \hat{\sigma}(x, \underline{r})$$
$$\longleftrightarrow \sum_f e_f C_V^f \frac{\pi^2}{N_c} \left( \frac{4}{y\bar{y}Q^2} \right) \alpha_s \left\{ \int_0^1 dx \frac{2\xi H^g(\xi, \xi, t) + 2x C_F \xi H_{\text{singlet}}^f(\xi, \xi, t)}{(x - \xi + i\epsilon)(x + \xi - i\epsilon)} \right\}$$

- Same  $\underline{r}_0^2 = \frac{4}{y\bar{y}Q^2}$  for quark and gluon contribution.



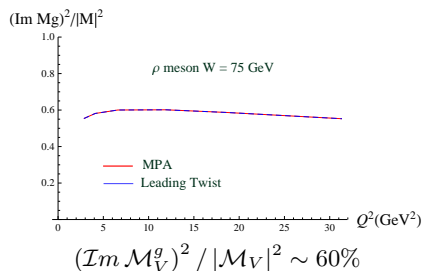
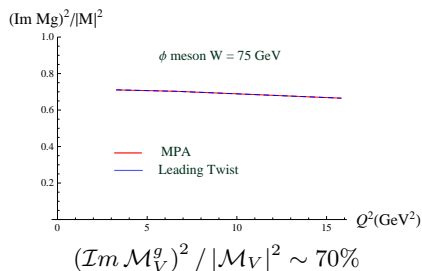
## Model dependences

- Models and parameters from (Kroll, Goloskokov, '08) for :
  - GPDs with evolution approximated by the DGLAP evolution
  - Wavefunctions model (Gaussian ansatz)

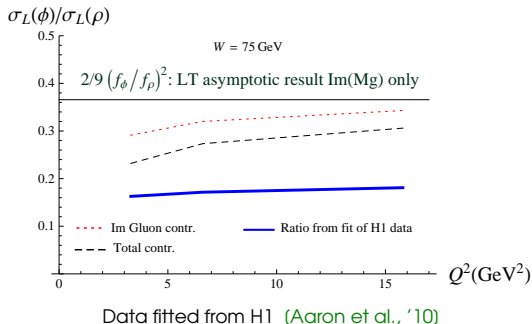
$$\hat{\Psi}_V(y, \underline{r}) \propto \text{Leading twist DA} \times \exp\left(-\frac{r^2}{4a_V^2} y\bar{y}\right)$$

- Kroll&Goloskokov GPD model based on double distribution ansatz (Musatov, Radyushkin, '00)

$$H(x, \xi) = \int_{-1}^1 d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(\beta + \xi\alpha - x) f(\beta, \alpha, t')$$



- Sea quark contribution (via interference term) not negligible in MPA approach with GK GPDs based on (CTEQ6M, '02) fits ( $10^{-4} < x < 0.5$  and  $4 < Q^2 < 40$  GeV<sup>2</sup>)

Ratio  $\sigma_L(\phi)/\sigma_L(\rho)$ 

- Red curve :  $\sim |\text{Im } \mathcal{M}_\phi^g|^2 / |\text{Im } \mathcal{M}_\rho^g|^2$
- Black curve :  $\sim |\mathcal{M}_\phi^g|^2 / |\mathcal{M}_\rho^g|^2$
- No additional factor from the normalization of the  $\phi$  and  $\rho$  wavefunctions
- Quark GPD contributions  $\Rightarrow$  decreases the ratio of  $\sigma_L(\phi)/\sigma_L(\rho)$

## From small $x$ region to valence region

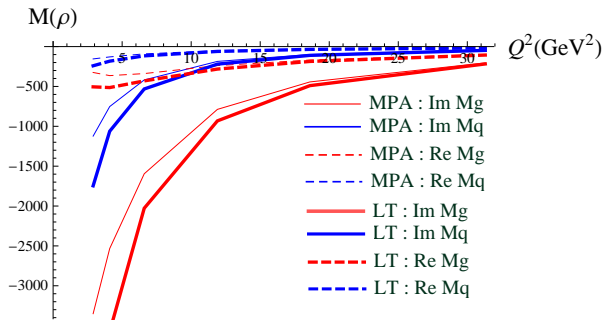
- Results for  $\mathcal{M}_{V,\{0,+,0+\}}$  in MPA exhibits the factorization of the overlap of the wavefunctions as in the dipole model picture allowing to link the MPA result to the dipole cross-section.
- Perspectives:
  - Other Helicity amplitudes :  $\mathcal{M}_{V,\{++,++\}}, \dots$
  - PARTONS (PARTonic Tomography Of Nucleon Software) project (collaborations between CPhT, IPN, lfu, LPT):
    - Software to extract of GPDs from observables with uncertainties on parameters
    - with systematic comparison between theoretical GPD related predictions and data
    - experimental results and theoretical predictions databases
  - Preliminary work on the DVMP observables
    - Descriptions of other helicity amplitudes with Natural (N) and Unatural parity exchange (U)  $\Rightarrow$  Sensitivity to other GPDs such as  $\mathcal{M}_{V,\{++,++\}}^U \sim \tilde{H}$ ,
 
$$\mathcal{M}_{V,\{\mu+, \mu+\}}^N \sim H(x, \xi, t') \quad \mathcal{M}_{V,\{++,++\}}^U \sim \tilde{H}(x, \xi, t')$$

$$\mathcal{M}_{V,\{\mu-, \mu+\}}^N \sim E(x, \xi, t') \quad \mathcal{M}_{V,\{+-,++\}}^U \sim \tilde{E}(x, \xi, t')$$
 related to SDME and spin asymmetries.
    - Implementation of NLO GPD evolutions

Thanks to my collaborators for encouraging discussions on this work

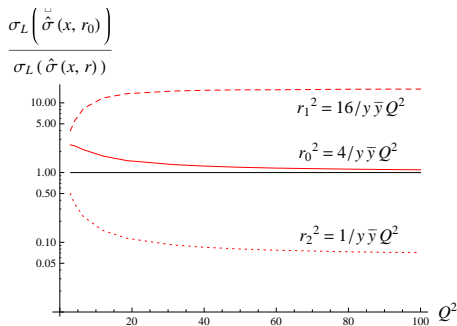
P. Kroll  
C. Lorcé  
C. Mezrag  
H. Moutarde  
S. Munier  
B. Pire  
F. Sabatié  
L. Szymanowski  
S. Wallon

# Contributions of other amplitudes

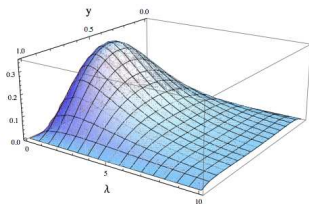


# Large $Q^2$ limit check

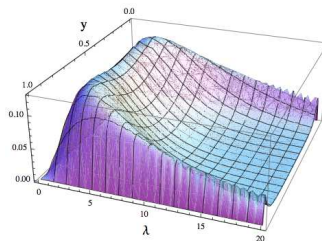
- $\sigma_L \propto \left| \int dy \int d^2 \underline{r} (\Psi_V^* \Psi_\gamma)(y, \underline{r}) \hat{\sigma}(x, r) \right|^2$
- Expect to find same numerical values at large  $Q^2$  using :
  - a dipole cross-section model (here GBW model, (Golec-Biernat, Wüsthof, '98)):  
 $\hat{\sigma}(x, r) = \sigma_0 (1 - e^{-r^2/(4R_0(x)^2)})$
  - a dipole cross-section model with  $\underline{r}^2 \rightarrow \underline{r}_0^2 = \frac{4}{y \bar{y} Q^2}$



- Helicity amplitude  $\mathcal{M}_{V,\{++,++\}}$  within MPA .. under study
- The overlap of wavefunctions appearing within  $k_T$ -factorization  $\Psi_V^*(y, \underline{r})\Psi_{\gamma^*}(y, \underline{r})$ :



$\lambda_\gamma = \lambda_V = 0$   
Leading twist DA

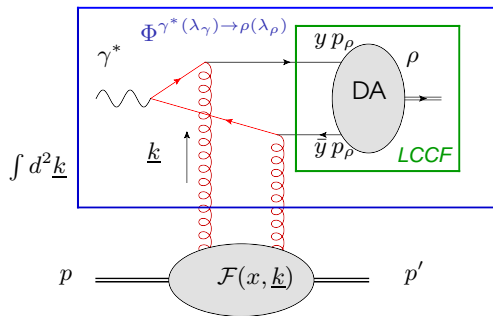


$\lambda_\gamma = \lambda_V = \pm$   
Twist 3 combination of DAs  
(in Wandzura-Wilczek approx.)  
(A.B., Szymanowski, Wallon, '13)

- Expected to be sensitive to the Sudakov form factor suppression



## Factorization of helicity amplitudes



- **Twist 2:**

- $\gamma_L^* \rightarrow \rho_L (\equiv T_{00})$
- $\gamma_T^* \rightarrow \rho_L (\equiv T_{01})$

Ginzburg, Panfil, Serbo, '85

- **Twist 3, in the limit  $t \sim 0$ :**

- $\gamma_T^* \rightarrow \rho_T (\equiv T_{11})$

Anikin, Ivanov, Pire, Szymanowski, Wallon, '10

