Understanding b-initiated processes at the LHC

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Work in collaboration with
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The bottom quark is the only quark such that

\[ \Lambda_{QCD} \ll m \ll M (m_W, m_t, m_H, m_Z) \]

b-quark phenomenology plays a crucial role at the LHC, from flavor physics to Higgs characterization and measurements as a window to New Physics.

A deeper understanding is crucial for to achieve rigorous predictions for this class of processes, which appear as both BSM signals and irreducible background at the LHC.
Outline

- Bottom production at the LHC
- 4F and 5F schemes
- Questions and Puzzles
- The investigation
- Results and generalization
- Conclusions and outlook
**b production at the LHC**

- Bottom quarks can enter in processes at the LHC

- Dominant strong production (gluon splitting) can take place in the initial or final state. E.g. bottom-initiated Higgs production

**t-channel kinematics**

*Initial state*

**s-channel kinematics**

*Final state*
**b production at the LHC**

- Bottom quarks can enter in processes at the LHC

- Dominant strong production (gluon splitting) can take place in the initial or final state. E.g. bottom-initiated Higgs production

\[ \alpha_S^2 \log^2 \frac{\hat{s}}{m_b^2} \]
These logs for $m_b << \hat{s}$, might be large, possibly spoiling perturbation theory

$$\alpha_S(\hat{s}) \log \frac{\hat{s}}{m_b^2} \approx 1$$

A way out is to define a 5 flavor QCD effective field theory where the effects of such logs are resummed using DGLAP equations into fragmentation functions and $b$-pdf’s, in the final and initial state respectively.

$$\sum_{n=1}^{\infty} \left( \alpha_S \log \frac{\mu_F^2}{m_b^2} \right)^n$$
From 4F to 5F scheme

Take for simplicity a process with one bottom in the initial state (Single top)
From 4F to 5F scheme

Start from 4F and integrate over t-channel propagator

\[
\frac{1}{t - m_b^2} \sim \frac{1}{p_T^2 + m_b^2}
\]

\[
t = (p_b - p_g)^2, \quad p_T^2 = p_T,\bar{b}
\]

Contribution to the cross-section

\[
\int_{0}^{p_T,\text{max}} \frac{d p_T^2}{p_T^2 + m_b^2} = \log \left( \frac{p_T,\text{max}}{m_b^2} \right)
\]

Coefficient of the logarithm in the collinear limit

Splitting function

\[ P_{g \rightarrow q\bar{q}} \times \]

matrix elements with splitting removed
From 4F to 5F scheme

* When logs are dominant

\[
\frac{d\sigma(qg \to q't\bar{b})}{d \log p_T^{2,\text{max}}} \sim \left(\frac{\alpha_s}{4\pi}\right) \left[ \int \frac{dx}{x} P_{g\to q\bar{q}g} \right] \times \hat{\sigma}(qb \to q't)
\]

* But the first part resembles the evolution equation for a quark:

\[
\frac{dq}{d \log \mu^2} \sim \left(\frac{\alpha_s}{4\pi}\right) \int \frac{dx}{x} [P_{g\to q\bar{q}g} + P_{q\to qgq}]
\]

* So when logarithms dominate, can replace this description by

\[
\sigma(qg \to q't\bar{b}) \sim \sigma(q\tilde{b} \to q't) \quad \tilde{b}(x) \sim \left(\frac{\alpha_s}{2\pi}\right) \log \left(\frac{\mu^2}{m_b^2}\right) \left[ \int \frac{dz}{z} P_{g\to q\bar{q}g} \right]
\]

- DGLAP evolution of b-PDF = resummation of full tower of leading logs
- Only the first logarithm goes to the PDF, following logs moved to higher orders
4F versus 5F scheme

At all orders 5F and 4F descriptions should agree; order by order they differ:

4F

- It does not resum possibly large logs, yet it does have them explicitly
- Computing higher orders is more difficult
- Mass effects are there at any order
- MC at LO and NLO no problem

5F

- It does resum initial state large logs into b PDFs leading to more stable predictions
- Computing higher orders is easier
- pT of bottom enter at higher orders
- Implementation in MC depend on the gluon splitting model in the parton showers
Questions #1

How the issue was historically raised

Factor of 10 difference??? Is this the effects of the logs? How can that be?
Questions #1

Two important ingredients helped in bringing predictions closer:

- Inclusion of higher order corrections
  
  - [Harlander and Kilgore, 2003]
  
  - [Dittmaier, Krämer, Spira '04]
  
  - [Dawson, Jackson, Reina, Wackeroth '04]
  
  - [Hirschi et al. 1103.0621]

- Scale choices: better agreement when smaller than naive choices $M_H$. 

M. Kraemer, 2010
However:

✴ Why the agreement is so good around $M_H = 100$ GeV and the uncertainty band comparable?

✴ Why the agreements gets worse at large $M_H$? The logs should have more space to develop at smaller $M_H / \sqrt{S}$ ...

✴ How is the smaller scale choice $M_H / 4$ justified?

✴ Is this behavior only proper of $b\bar{b} \rightarrow$ Higgs or it is general?
 Differences at natural scales $M_t$ become smaller at lower scales, $\mu \sim M_t/4$. Why?

* At LHC both scale dependences are rather mild. 4F is as good as 5F. Where is the need for resummation?

* Differences are smaller at the LHC than Tevatron. Why? The logs should have more space to develop at the LHC...


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**Questions #2**

To answer last question take t-channel single top production

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J. Campbell et al, JHEP 0910 (2009) 042
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What happens for a heavier top?

F. Maltoni, G. Ridolfi, M. Ubiali, JHEP 1207 (2012) 022
Questions #3

What about other more exclusive observables?

- This observable is NLO only in the 4F calculation.
- A 4F calculation is much more EXP handy and useful in actual analyses.
- Slightly softer in 4F (2 → 3), particularly at the Tevatron

J. Campbell et al, PRL 102 (2009) 182003
Single top, t-channel hadro-production: Acceptance for spectator b-quark (second-hardest jet)

At the LHC 5F and 4F results are much close to each others for all pT, while at Tevatron the difference is much larger: why?
All these apparent puzzles can be merged into a simple and consistent picture by taking into account two main results:

F. Maltoni, G. Ridolfi, M. Ubiali, JHEP 1207 (2012) 022

\[ \tilde{b}^{O(\alpha_s^2)}(x, \mu) \]

Similar result obtained by R. Thorne [1402.3536] and F. Olness et al [0812.3371]

The resummation effects of the initial state logs into the b-PDFs are important only at large x!

Larger effect at large-x:

\[ x \approx \frac{M^2}{S_{\text{had}}} \]
Single top production in 5F scheme @ LHC14

- $m_T = 172$ GeV
- $m_T = 400$ GeV
- $m_T = 800$ GeV

The resummation effects of the initial state logs into the b-PDFs are important only at large $x$!

The heavier the particle, the larger is the impact of resummation

\[
\langle x \rangle = 10^{-2}
\]

\[
\langle x \rangle = 5 \cdot 10^{-2}
\]

\[
\langle x \rangle = 10^{-1}
\]
The possibly large ratios $\frac{M^2}{m_b^2}$ are always accompanied by universal phase space factors

$$\log \left( \frac{M^2}{m_b^2} h(z) \right), \quad z = \frac{M^2}{\hat{s}}$$

$$= \log \left( \frac{\tilde{\mu}^2}{m_b^2} \right), \quad \tilde{\mu}^2 = M^2 < h(z) >$$

For the LHC processes that we have considered, the larger the mass the stronger the suppression. The opposite happens for DIS processes: $\tilde{\mu} < M_t$
How is determine \( h(z) \) determined? Take the simplest case

\[
\sigma^{5F}(\tau) = \left( \frac{\pi \sqrt{2}}{3} G_F \tau \right) \int_{\tau}^{1} \frac{dz}{z} \mathcal{L}_{ug} \left( \frac{\tau}{z}, \mu_F^2 \right) \frac{\alpha_S}{2\pi} P_{qg}(z) \log \frac{\mu_F^2}{m_b^2} + \ldots
\]

\[
\sigma^{4F}(\tau) = \left( \frac{\pi \sqrt{2}}{3} G_F \tau \right) \int_{\tau}^{1} \frac{dz}{z} \mathcal{L}_{ug} \left( \frac{\tau}{z} \right) \frac{\alpha_S}{2\pi} P_{qg}(z) L_{DY} + O(m_b^0)
\]

\[
z = \frac{M_W^2}{s}
\]

Take collinear limit

\[
\hat{\sigma}^{4F}(z) = \int_{t_-}^{t_+} dt \frac{d\hat{\sigma}}{dt}(s, t, \alpha_S).
\]

\[
\hat{\sigma}^{4F}(z) = \frac{\alpha_s}{2\pi} \left( \pi \frac{\sqrt{2}}{3} G_F \right) z^2 + \frac{(1 - z)^2}{2} \log \left[ \frac{M_W^2}{m_b^2} \right] + O(m_b^0),
\]

\[
L_{DY} \equiv \log \left[ \frac{M_W^2}{m_b^2} \right]
\]
The distribution of \((1-z)^2/z\) and \(t\) tend to suppress \(L_{DY}\) and suppression is stronger the larger is the mass of the produced particle

\[
\log \frac{\tilde{\mu}_F^2}{m_b^2} = \frac{\int_1^{1/z} \frac{dz}{z} \mathcal{L}_{ug} \left( \frac{\tau}{z} \right) P_{qq}(z) \log \left[ \frac{M_W^2}{m_b^2} \left( \frac{1-z}{z} \right)^2 \right]}{\int_1^{1/z} \frac{dz}{z} \mathcal{L}_{ug} \left( \frac{\tau}{z} \right) P_{qq}(z)}
\]

- \(M_W = 80\) GeV, \(\tilde{\mu}_F \simeq [0.4, 0.5] M_W\)
- \(M_W = 400\) GeV, \(\tilde{\mu}_F \simeq [0.3, 0.4] M_W\)
- \(M_W = 800\) GeV, \(\tilde{\mu}_F \simeq [0.25, 0.35] M_W\)
We have shown that theoretical results obtained in 4F and 5F schemes can be consistently understood.

We have suggested a physically-motivated scale at which predictions should be compared.

A substantial and justified agreement between 4F and 5F calculations for a given process means that both calculations can be used in different contexts.

1. The resummation effects of the initial state logs into the b-PDFs are important only at large $x$!

2. The possibly large ratios $M^2/m_b^2$ are always accompanied by universal phase space factors that lead to their suppression.

\[
Q^2(z) = (M^2 + Q^2) \frac{(1 - z)^2}{z} \frac{1}{1 - \frac{zQ^2}{M^2+Q^2}}
\]

\[
z = \frac{M^2 + Q^2}{s + Q^2}
\]

\[
Q \to \infty L_{DY} = \log \left[ \frac{M^2}{m_b^2} \frac{(1 - z)^2}{z} \right]
\]

\[
M \to 0 L_{DIS} = \log \left[ \frac{Q^2}{m_b^2} \frac{1 - z}{z} \right]
\]

Answers - summary
Charged Higgs production

- Consider Charged Higgs production in 2 Higgs Doublet Model
- Heavy Charged Higgs boson with a mass larger than a top quark would be produced in association with a top quark
Improved comparison between 4F and 5F by using motivated factorization scale in the 5F prediction instead of ad-hoc scale

Substantial agreement between predictions and reduced theoretical uncertainty

All sources of theoretical uncertainties are kept into account: PDFs, scale variation

Matched predictions with Santander matching [Harlander et al, 1112.3478]

Needed?

$$\sigma_{\text{matched}} = \frac{\sigma_{4F} + w\sigma_{5F}}{1 + w}$$

$$w = \log \frac{M_H}{M_b} - 2$$
Findings generalized to the case of two b’s in the initial state, \(bb>H\) and \(bb>Z\): a detailed comparison is interesting both for SM and BSM.

Similar logarithms and stronger suppression at the LHC.

\[ L_{\text{UNIV}}^2 = \log^2 \frac{Q_2(z)}{m_b^2} \]
Two b’s in the initial state

Findings generalized to the case of two b’s in the initial state, \(bb>H\) and \(bb>Z\): a detailed comparison is interesting both for SM and BSM.

Similar logarithms and stronger suppression at the LHC.

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L_{\text{UNIV}}^2 = \log^2 \frac{Q^2(z)}{m_b^2}
\]

plots by M. Lim
Two b’s in the initial state

- Findings generalized to the case of two b’s in the initial state, $\text{bb} > \text{H}$ and $\text{bb} > \text{Z}$: a detailed comparison is interesting both for SM and BSM
- Similar logarithms and stronger suppression at the LHC
- Top quark at 100 TeV collider is like the bottom quark at LHC

$$L_{\text{UNIV}}^2 = \log^2 \frac{Q^2(z)}{m_b^2}$$

- Relevant also for t initiated Higgs production at future high energy colliders
- Plots by J Rojo
Conclusions

- We have developed a simple method to assess the size of potentially large collinear logs in processes initiated by bottom quarks.
- Quantitative estimate of the impact of resummation for some key LHC processes: $b+V$ production, single top.
- Study applies also to DIS $b$ production.
- Do we need $b$ PDFs at the LHC? Not so desperately as one may think due to small-$x$ fraction of momentum carried by partons and to suppressed scale of collinear logs.
- Application to Charged Higgs production [M Flechl, R. Klees, M. Kraemer, MU].
- Results generalized to two $b$'s in the initial state and initial top at future colliders [with M. Lim].
- What about fragmentation functions? An analogous study for final state gluon splitting at high-$p_T$, in particular 4F versus 5F approaches is going to be delivered soon [F. Demartin, F. Maltoni, G. Ridolfi, MU].
- Next step is to look into more exclusive observables and modeling into MC event generators.
Back-up
Importance of resummation

\[ b_{\text{pdf}} \text{ has all the logs resummed} \]

\[
\int dx_1 dx_2 \, q(x_1, \mu_F^2) b(x_1, \mu_F^2) \hat{\sigma}(qb \rightarrow q't) = \int dx_1 dx_2 \, q(x_1, \mu_F^2) \tilde{b}(x_1, \mu_F^2) \hat{\sigma}(qb \rightarrow q't)
\]

\[
\tilde{b}(x, \mu_F) \sim \frac{\alpha_s}{2\pi} \log \frac{\mu^2}{m_b^2} \int_x^1 \frac{dy}{y} P_{qg} \left( \frac{x}{y} \right) g(y, \mu_f)
\]

\text{btilde is just the first log that one gets from a LO 4F calculation. The b-pd}f\text{ resums the full tower of such logs that come from higher orders in the 4F calculation.}
Importance of resummation

\[ \tilde{b}^{(1)}(x, \mu^2) = \frac{\alpha_S}{2\pi} \log \frac{\mu^2}{m_b^2} \int_x^1 \frac{dz}{z} P_{qq}(z) g \left( \frac{x}{z}, \mu^2 \right) / b^{(1)}(x, \mu^2) \]

Comparison between the first log which the one included in the LO 4F calculation of single-top, and the full resummed result given by the AP equations.

The various curves correspond to different Bjorken x's.

At small x the effect is positive, in other words b~ is a kind of bad overestimate.

At large x resummation effects are manifest.

LO approximation does not look good enough.
Importance of resummation

\[ \tilde{b}^{(2)}(x, \mu^2) = \int_x^1 \frac{dz}{z} \left[ \sum^{4F,(2)} \left( \frac{x}{z}, \mu^2 \right) \left( \frac{\alpha_S}{4\pi} \right)^2 a_{\Sigma,b}(z, \mu^2/m_b^2) \right] \]

\[ + \int_x^1 \frac{dz}{z} g^{4F,(2)} \left( \frac{x}{z}, \mu^2 \right) \left[ \left( \frac{\alpha_S}{4\pi} \right) a_{g,b}(z, \mu^2/m_b^2) + \left( \frac{\alpha_S}{4\pi} \right)^2 a_{g,b}(z, \mu^2/m_b^2) \right] \]

Comparison between the first log^2 + log which are included in the NLO 4F calculation of single-top, and the full resummed result given by the AP equations at NLO.

The various curves correspond to different Bjorken x's.

At small x also now the resummation is visible yet is very small.

At large x resummation effects are manifest.

NLO approximation does look reasonably behaved.
Importance of resummation

- Can I understand this behaviour (at least roughly)?

I write the DGLAP equation for the b-pdf:

\[
\frac{d}{d \log \mu^2} b(N, \mu^2) = \frac{\alpha_S(\mu^2)}{2\pi} \left[ \gamma_{qq}^{(0)}(N)b(N, \mu^2) + \gamma_{qg}^{(0)}(N)g(N, \mu^2) \right],
\]

\[b(N, m_b^2) = 0 \quad \text{boundary condition}\]

whose solution at LO can be easily written as:

\[b(N, \mu^2) = \gamma_{qq}^{(0)}(N)g(N, m_b^2) \left\{ \frac{\alpha_S(m_b^2)}{2\pi} \log \frac{\mu^2}{m_b^2} + \sum_{k=2}^{\infty} A_k(N) \frac{1}{k!} \left[ \frac{\alpha_S(m_b^2)}{2\pi} \log \frac{\mu^2}{m_b^2} \right]^k \right\},\]

\[A_k(N) = \left[ \gamma_{qq}^{(0)}(N) - \beta_0 \right] \left[ \gamma_{qq}^{(0)}(N) - 2\beta_0 \right] \cdots \left[ \gamma_{qq}^{(0)}(N) - (k-1)\beta_0 \right].\]

The logarithms resummed in the b-PDF are larger:

1. as \( \mu \) gets larger with respect to \( m_b \)
2. at large \( N \) \( \Leftrightarrow \) large \( x \)
Collinear log @ DIS

\[ \sigma_b(\mu^2) = \int_{y_{\text{min}}}^{y_{\text{max}}} dy \int_{Q_{\text{min}}}^{Q_{\text{max}}} dQ^2 \frac{2\pi \alpha_l \alpha_h}{y(M^2 + Q^2)^2} \left\{ [1 + (1 - y)^2] F_2^b(x, Q^2, m_b^2) - y^2 F_L^b(x, Q^2, m_b^2) + [1 - (1 - y)^2] F_3^b(x, Q^2, m_b^2) \right\} \]

\[ y = \frac{Q^2}{xS} \]
Collinear log @ DIS

Take the expression for the 4F process $\gamma^* + g \to b + \bar{b}$ at small $t$:

$$\frac{d\sigma_2}{dt} = \frac{\pi \alpha_e e_b^2 \alpha_S C_F}{16} \left[ -\frac{4z}{Q^2(t - m_b^2)} \frac{z^2 + (1 - z)^2}{2} \right] + \text{non-singular terms}$$

Integrating over $t$ gives:

$$\int_{t_-}^{t_+} dt \frac{d\sigma_2}{dt} = \frac{\pi \alpha_e e_b^2 \alpha_S C_F}{4Q^2} zP_{qg}(z) \log \frac{1 + \beta}{1 - \beta} t_\pm = m_b^2 - \frac{s + Q^2}{2}(1 \pm \beta); \quad \beta = \sqrt{1 - \frac{4m_b^2}{s}}$$

$$= \left( \frac{\pi^2 \alpha_e e_b^2 C_F}{2Q^2} \right) \frac{\alpha_S}{2\pi} zP_{qg}(z) \left[ \log \frac{m_b^2}{s} + O \left( \frac{m_b^2}{s} \right) \right],$$

i.e., doing it properly, one sees that the naively expected $\log Q^2/m_b^2$ is actually:

$$L_{\text{DIS}} \equiv \log \left[ \frac{Q^2}{m_b^2} \frac{1 - z}{z} \right] = \log \frac{M_{b\bar{b}}^2}{m_b^2}$$
Collinear log @ DIS

\[ L_{\text{DIS}} \equiv \log \left[ \frac{Q^2}{m_b^2} \frac{1 - z}{z} \right] = \log \frac{M_{b\bar{b}}^2}{m_b^2} \]

The typical values for \((1-z)/z\) lead to an enhancement of the log at HERA and \(\sim 1\) at the LHeC.