



DIS 2014 – XXII. International Workshop on
Deep-Inelastic Scattering and Related Subjects

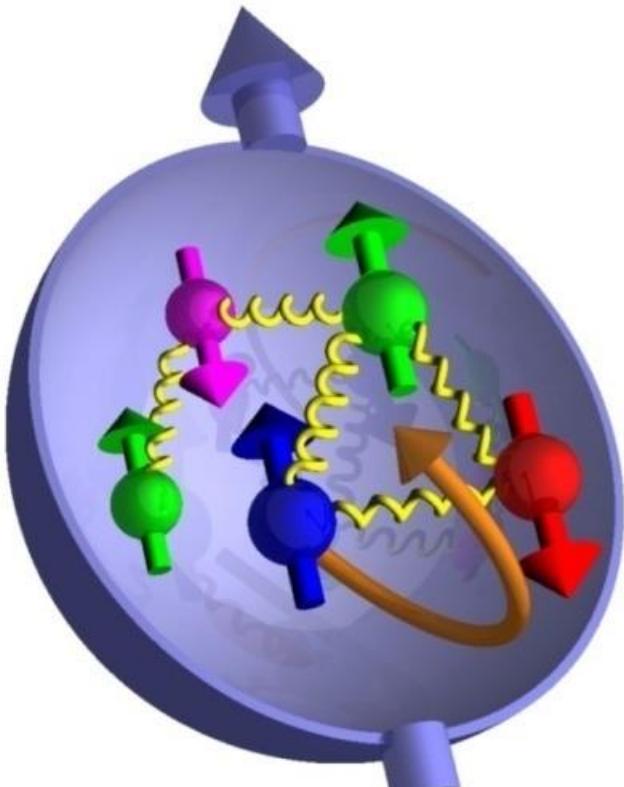
Orbital angular momentum in the nucleon

Cédric Lorcé

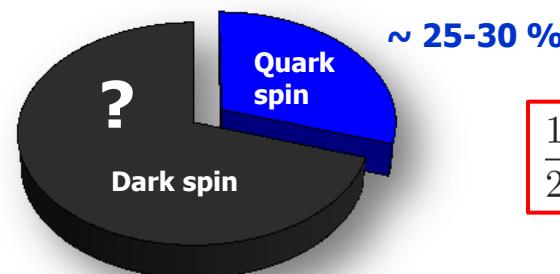
IPN Orsay - IFPA Liège



Outline



- Summary of the decompositions
- Gauge-invariant extensions
- Orbital angular momentum
- Conclusions



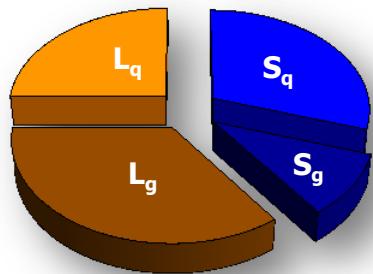
$$\frac{1}{2} = S_q + L_q + S_g + L_g$$

Angular momentum decompositions

$$\vec{p} = \frac{\partial L}{\partial \vec{v}}$$

Canonical

[Jaffe-Manohar (1990)]



$$\begin{aligned}\vec{S}_q &= \frac{1}{2} \int d^3r \psi^\dagger \vec{\Sigma} \psi \\ \vec{L}_q &= \int d^3r \psi^\dagger \vec{r} \times (-i\vec{\nabla}) \psi \\ \vec{S}_g &= \int d^3r \vec{E}^a \times \vec{A}^a \\ \vec{L}_g &= \int d^3r E^{ai} \vec{r} \times \vec{\nabla} A^{ai}\end{aligned}$$

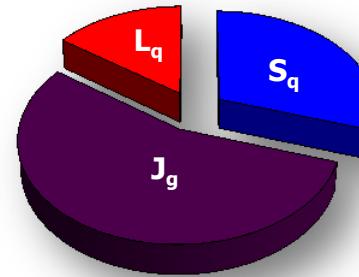
Gauge non-invariant!

$$\vec{\pi} = m\vec{v} = \vec{p} + g\vec{A}$$

Kinetic

[Ji (1997)]

$$\vec{D} = \vec{\nabla} + ig\vec{A}$$



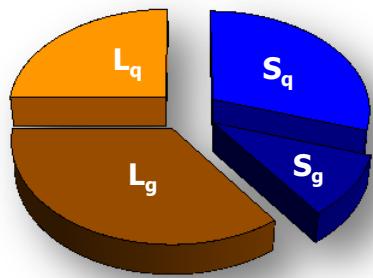
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Angular momentum decompositions

$$\vec{p} = \frac{\partial L}{\partial \vec{v}}$$

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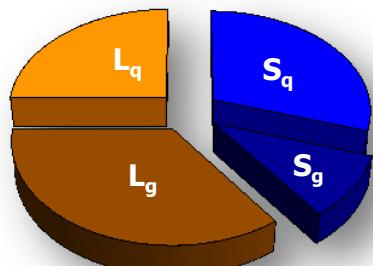


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Gauge non-invariant!

[Chen *et al.* (2008)]

$$A = A_{\text{pure}} + A_{\text{phys}}$$



$$\begin{aligned}\vec{S}_q &= \frac{1}{2} \int d^3r \psi^\dagger \vec{\Sigma} \psi \\ \vec{L}_q &= \int d^3r \psi^\dagger \vec{r} \times (-i\vec{D}_{\text{pure}}) \psi \\ \vec{S}_g &= \int d^3r \vec{E}^a \times \vec{A}_{\text{phys}}^a \\ \vec{L}_g &= \int d^3r E^{ai} \vec{r} \times \vec{D}_{\text{pure}} A_{\text{phys}}^{ai}\end{aligned}$$

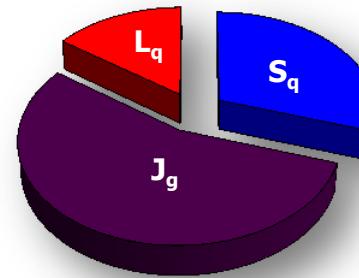
Gauge-invariant extension (GIE)

$$\vec{\pi} = m\vec{v} = \vec{p} + g\vec{A}$$

Kinetic

[Ji (1997)]

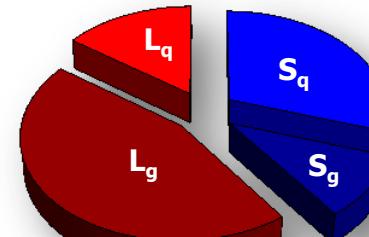
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[Wakamatsu (2010)]

$$A = A_{\text{pure}} + A_{\text{phys}}$$



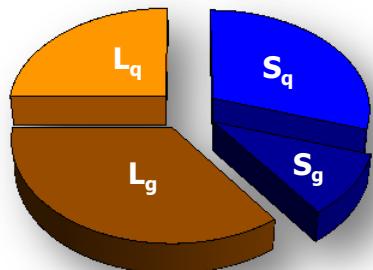
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Angular momentum decompositions

$$\vec{p} = \frac{\partial L}{\partial \vec{v}}$$

Canonical

[Jaffe-Manohar (1990)]

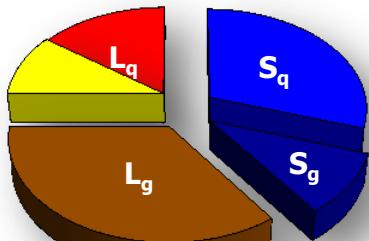


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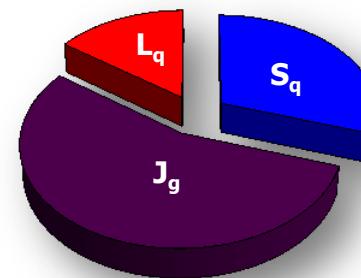
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Gauge-invariant extension (GIE)

Kinetic

[Ji (1997)]

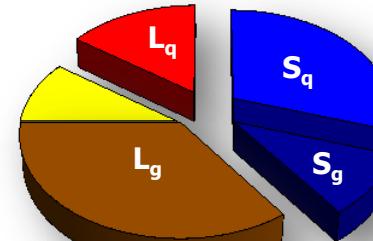
$$\vec{D} = \vec{\nabla} + ig\vec{A}$$



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[atsu (2010)]

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Stueckelberg symmetry

$$A = A_{\text{pure}} + A_{\text{phys}} = \underbrace{\bar{A}_{\text{pure}}}_{A_{\text{pure}}+C} + \underbrace{\bar{A}_{\text{phys}}}_{A_{\text{phys}}-C}$$

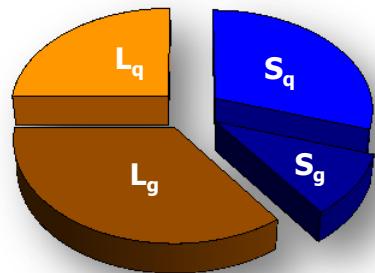
[Stoilov (2010)]
[C.L. (2013)]

Ambiguous!

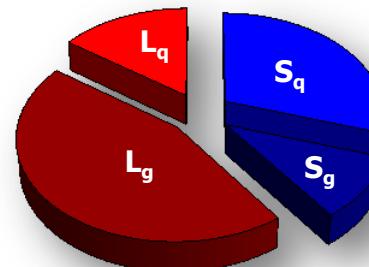
Infinitely many possibilities!

Coulomb GIE

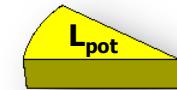
$$\vec{\mathcal{D}}_{\text{pure}} \cdot \vec{A}_{\text{phys}} = 0$$



[Chen *et al.* (2008)]

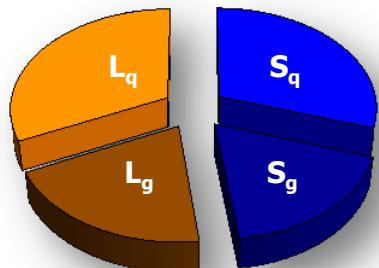


[Wakamatsu (2010)]

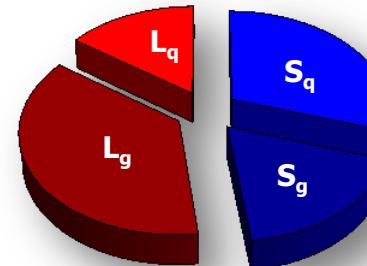


Light-front GIE

$$A_{\text{phys}}^+ = 0$$



[Hatta (2011)]
[C.L. (2013)]



Stueckelberg symmetry

$$A = A_{\text{pure}} + A_{\text{phys}} = \underbrace{\bar{A}_{\text{pure}}}_{A_{\text{pure}}+C} + \underbrace{\bar{A}_{\text{phys}}}_{A_{\text{phys}}-C}$$

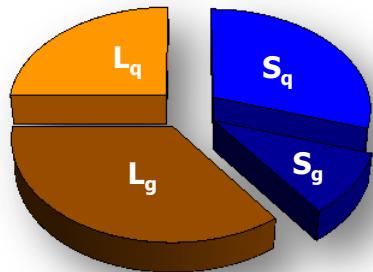
[Stoilov (2010)]
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Ambiguous!

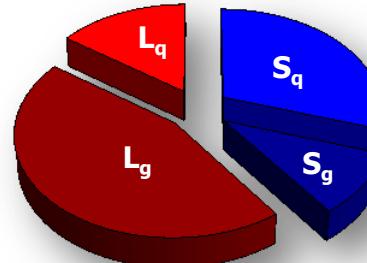
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Coulomb GIE

$$\vec{\mathcal{D}}_{\text{pure}} \cdot \vec{A}_{\text{phys}} = 0$$



[Chen *et al.* (2008)]

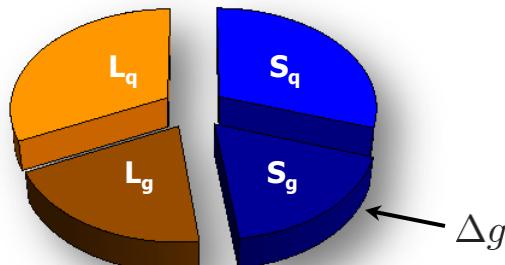


[Wakamatsu (2010)]



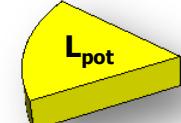
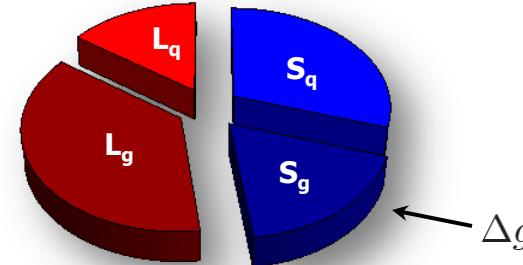
Light-front GIE

$$A_{\text{phys}}^+ = 0$$



[Hatta (2011)]
[C.L. (2013)]

Proton structure probed in IMF!



Gluon spin

$$\begin{aligned}
 \Delta g &= \int_0^1 dx \Delta g(x) \\
 &= \int_0^1 dx \frac{i}{x P^+} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle P, \Lambda | 2\text{Tr}[F^{+\alpha}(0) \mathcal{W}_{[0,z^-]} \tilde{F}^+{}_\alpha(z^-) \mathcal{W}_{[z^-,0]}] | P, \Lambda \rangle \\
 &= \frac{\epsilon^{+-}{}_{\alpha\beta}}{2P^+} \langle P, \Lambda | 2\text{Tr}[F^{+\alpha}(0) \int dz^- \frac{1}{2}\epsilon(z^-) \mathcal{W}_{[0,z^-]} F^{+\beta}(z^-) \mathcal{W}_{[z^-,0]}] | P, \Lambda \rangle
 \end{aligned}$$

Gluon helicity distribution

« Measurable », gauge invariant but **non-local**

Light-front gauge $A^+ = 0$

[Jaffe-Manohar (1990)]

$$\begin{aligned}
 &= \frac{\epsilon^{+-}{}_{\alpha\beta}}{2P^+} \langle P, \Lambda | -2\text{Tr}[F^{+\alpha}(0) A^\beta(0)] | P, \Lambda \rangle \\
 &= \frac{1}{2P^+} \langle P, \Lambda | \frac{1}{2}\epsilon^{+-}{}_{\alpha\beta} M_{g,\text{spin}}^{+\alpha\beta,\text{JM}}(0) | P, \Lambda \rangle \\
 &= \frac{\langle P, \Lambda | S_g^{z,\text{JM}} | P, \Lambda \rangle}{\langle P, \Lambda | P, \Lambda \rangle}
 \end{aligned}$$

Local fixed-gauge interpretation

Light-front GIE $A_{\text{phys}}^+ = 0$

[Hatta (2011)]

$$\begin{aligned}
 &= \frac{\epsilon^{+-}{}_{\alpha\beta}}{2P^+} \langle P, \Lambda | -2\text{Tr}[F^{+\alpha}(0) A_{\text{phys}}^{\beta,\text{Hatta}}(0)] | P, \Lambda \rangle \\
 &= \frac{1}{2P^+} \langle P, \Lambda | \frac{1}{2}\epsilon^{+-}{}_{\alpha\beta} M_{g,\text{spin}}^{+\alpha\beta,\text{Hatta}}(0) | P, \Lambda \rangle \\
 &= \frac{\langle P, \Lambda | S_g^{z,\text{Hatta}} | P, \Lambda \rangle}{\langle P, \Lambda | P, \Lambda \rangle}
 \end{aligned}$$

Non-local gauge-invariant interpretation

Kinetic and canonical OAM

Kinetic OAM (Ji)

$$L_z = \underbrace{\frac{1}{2} \int dx x [H(x, 0, 0) + E(x, 0, 0)]}_{J_z} - \underbrace{\frac{1}{2} \int dx \tilde{H}(x, 0, 0)}_{S_z}$$

= $-\int dx x G_2(x, 0, 0) = \int dx x [H(x, 0, 0) + E(x, 0, 0) + \tilde{E}_{2T}(x, 0, 0)]$

Pure twist-3

[Ji (1997)]

[Penttinen *et al.* (2000)]
 [Kiptily, Polyakov (2004)]
 [Hatta (2012)]

Quark *naive* canonical OAM (Jaffe-Manohar)

$$\mathcal{L}_z = - \int dx d^2 k_\perp \frac{\vec{k}_\perp^2}{2M^2} h_{1T}^\perp(x, \vec{k}_\perp)$$

 **Model-dependent !**

[Burkardt (2007)]
 [Efremov *et al.* (2008, 2010)]
 [She, Zhu, Ma (2009)]
 [Avakian *et al.* (2010)]
 [C.L., Pasquini (2011)]

Canonical OAM (Jaffe-Manohar)

$$\ell_z = - \int dx d^2 k_\perp \frac{\vec{k}_\perp^2}{M^2} F_{14}(x, 0, \vec{k}_\perp, \vec{0}_\perp)$$

[C.L., Pasquini (2011)]
 [C.L., Pasquini, Xiong, Yuan (2012)]
 [Hatta (2012)]
 [Kanazawa, C.L., Pasquini, Metz, Schlegel (2014)]

Model q	LCCQM			χ QSM		
	u	d	Total	u	d	Total
ℓ_z^q	0.131	-0.005	0.126	0.073	-0.004	0.069
L_z^q	0.071	0.055	0.126	-0.008	0.077	0.069
\mathcal{L}_z^q	0.169	-0.042	0.126	0.093	-0.023	0.069

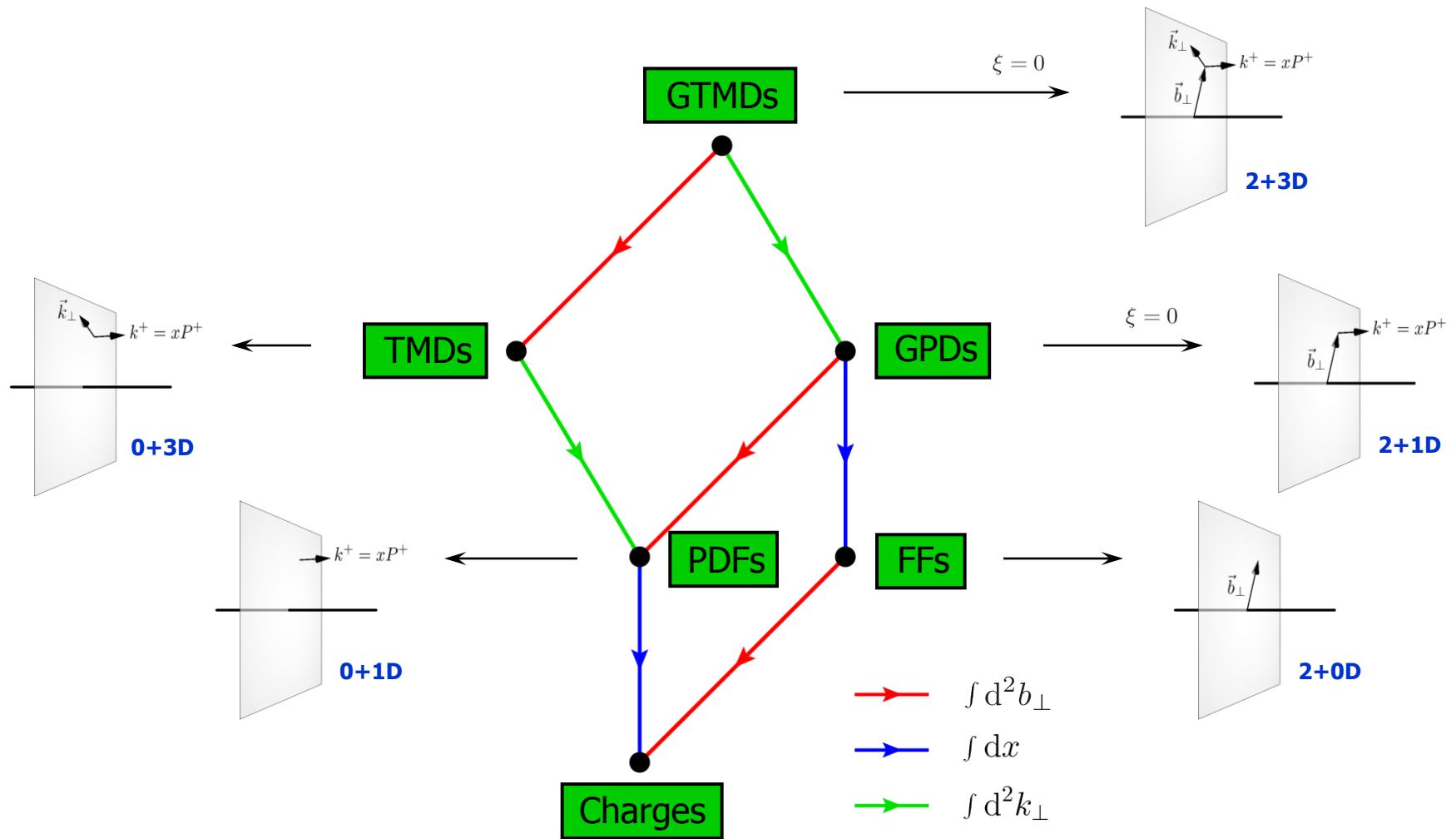
 **No gluons and not QCD EOM!**

$$\ell_z = L_z \quad \text{but} \quad \ell_z^q \neq L_z^q$$

[C.L., Pasquini (2011)]

The phase-space picture

Complete parametrizations : Quarks [Meissner, Metz, Schlegel (2009)]
Quarks & gluons [C.L., Pasquini (2013)]

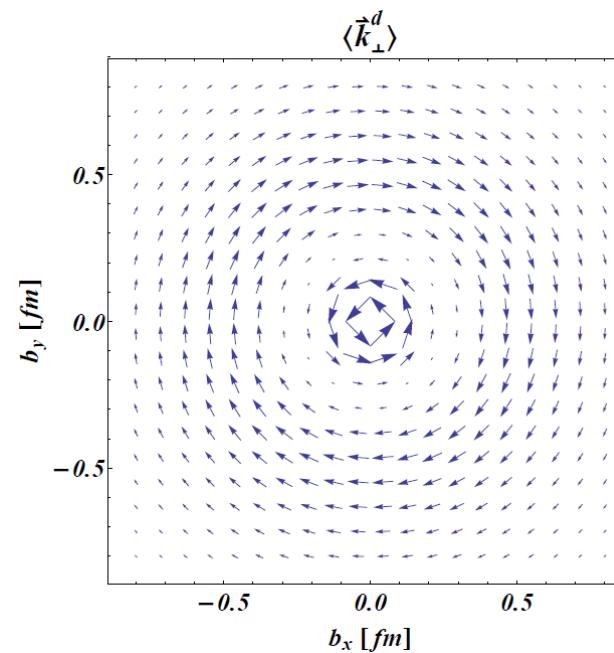
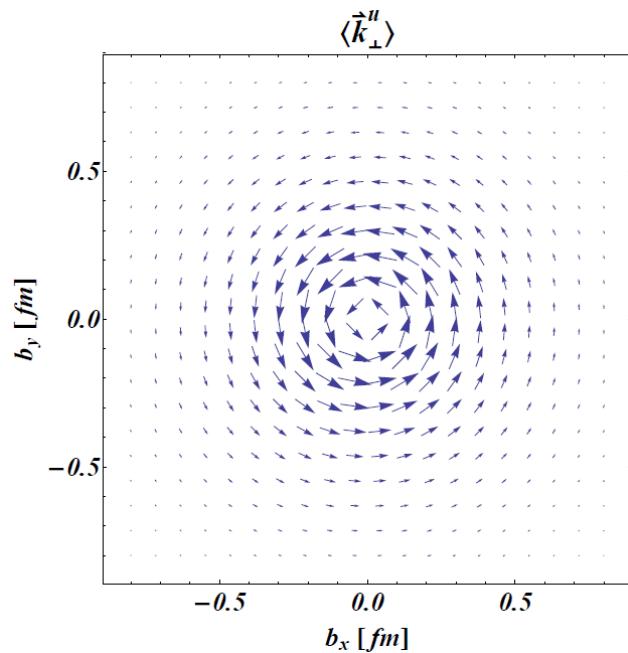


The orbital motion in a light-front quark model

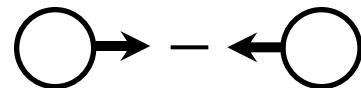
[C.L., Pasquini, Xiong, Yuan (2012)]

Average transverse quark momentum in a longitudinally polarized nucleon

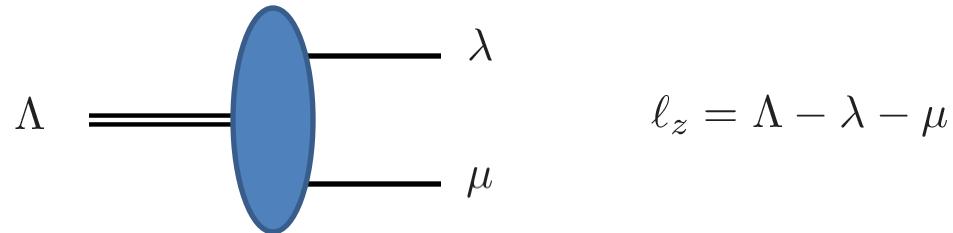
$$\langle \vec{k}_\perp \rangle(\vec{b}_\perp) = \int dx d^2 k_\perp \vec{k}_\perp \rho_{++}^{[\gamma^+]}(x, \vec{k}_\perp, \vec{b}_\perp)$$



F_{14}
« Vorticity »

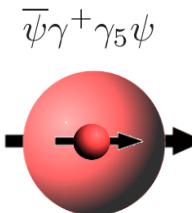


Proton spin structure



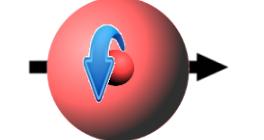
« Quark spin »

$$\langle\langle S_z^q \rangle\rangle \sim \frac{1}{2(2\pi)^3} \sum_{\Lambda, \lambda, \mu} \Lambda \lambda |\psi_{\lambda, \mu}^\Lambda|^2 \sim \langle S_z^N S_z^q \rangle$$



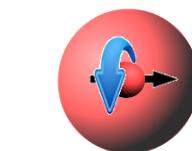
« Quark OAM »

$$\langle\langle L_z^q \rangle\rangle \sim \frac{1}{2(2\pi)^3} \sum_{\Lambda, \lambda, \mu} \Lambda l_z |\psi_{\lambda, \mu}^\Lambda|^2 \sim \langle S_z^N L_z^q \rangle$$



Quark spin-orbit correlation

$$\langle\langle C_z^q \rangle\rangle \sim \frac{1}{2(2\pi)^3} \sum_{\Lambda, \lambda, \mu} \lambda l_z |\psi_{\lambda, \mu}^\Lambda|^2 \sim \langle S_z^q L_z^q \rangle$$



OAM vs spin-orbit

OAM

$$L_z = \int d^3r [r^1 \langle\langle T^{+2} \rangle\rangle - r^2 \langle\langle T^{+1} \rangle\rangle]$$

Parametrization

$$\begin{aligned} T^{\mu\nu} &= \langle p', \Lambda' | \bar{\psi} \gamma^\mu \frac{i}{2} \overleftrightarrow{D}^\nu \psi | p, \Lambda \rangle \\ &= \bar{u}' \left[\frac{P^{\{\mu}\gamma^{\nu\}}}{2} A + \frac{P^{\{\mu}i\sigma^{\nu\}}\Delta}{4M} B \right. \\ &\quad \left. + \frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{M} C + Mg^{\mu\nu}\bar{C} + \frac{P^{[\mu}\gamma^{\nu]}}{2} D \right] u \end{aligned}$$

Relations

$$L_z = \frac{1}{2}(A + B + D) \quad [\text{Shore, White (2000)}]$$

$$= \int dx \frac{1}{2}[x(H + E) - \tilde{H}] \quad [\text{Ji (1997)}]$$

$$= - \int dx x G_2 \quad [\text{Penttinen et al. (2000)}]$$

$$= - \int dx d^2k_\perp \frac{\vec{k}_\perp^2}{M^2} F_{14} \quad [\text{C.L., Pasquini (2011)}] \\ [\text{Hatta (2012)}]$$

Spin-orbit

$$C_z = \int d^3r [r^1 \langle\langle T_5^{+2} \rangle\rangle - r^2 \langle\langle T_5^{+1} \rangle\rangle]$$

Parametrization

$$\begin{aligned} T_5^{\mu\nu} &= \langle p', \Lambda' | \bar{\psi} \gamma^\mu \gamma_5 \frac{i}{2} \overleftrightarrow{D}^\nu \psi | p, \Lambda \rangle \\ &= \bar{u}' \left[\frac{P^{\{\mu}\gamma^{\nu\}}\gamma_5}{2} \tilde{A} + \frac{P^{\{\mu}\Delta^{\nu\}}\gamma_5}{4M} \tilde{B} \right. \\ &\quad \left. + \frac{P^{[\mu}\gamma^{\nu]}\gamma_5}{2} \tilde{C} + \frac{P^{[\mu}\Delta^{\nu]}\gamma_5}{4M} \tilde{D} + Mi\sigma^{\mu\nu}\gamma_5 \tilde{F} \right] u \end{aligned}$$

Relations

$$\begin{aligned} C_z &= \frac{1}{2}(\tilde{A} + \tilde{C}) \\ &= \int dx \frac{1}{2}(x\tilde{H} - H) + \mathcal{O}(\frac{m_q}{M}) \quad [\text{C.L. (2014)}] \\ &= - \int dx x(\tilde{G}_2 + 2\tilde{G}_4) \\ &= \int dx d^2k_\perp \frac{\vec{k}_\perp^2}{M^2} G_{11} \quad [\text{C.L., Pasquini (2011)}] \\ &\quad [\text{C.L. (2014)}] \end{aligned}$$

Some figures

Valence number

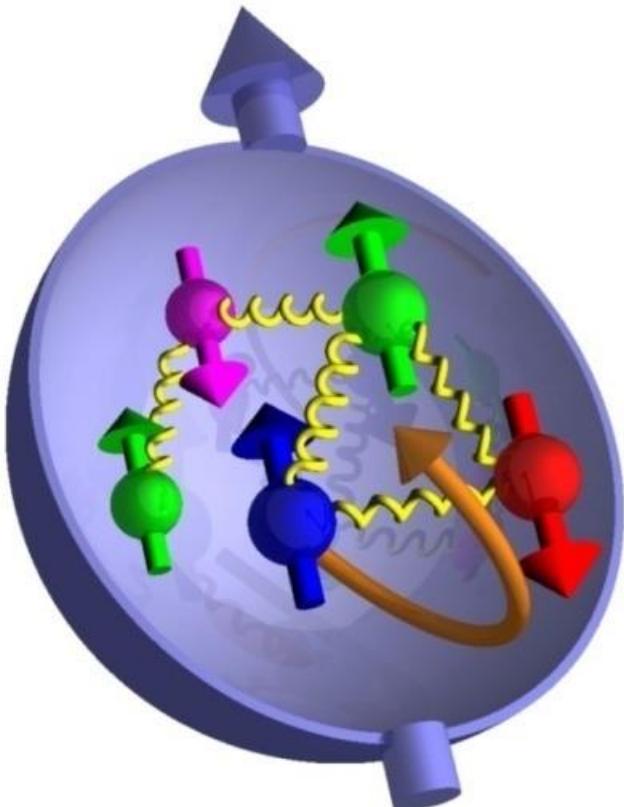
$$F_1^u(0) = 2 \quad F_1^d(0) = 1$$

	$\int dx x \tilde{H}^u$	$\int dx x \tilde{H}^d$	C_z^u	C_z^d
NQM	4/9	-1/9	-7/9	-5/9
LFCQM	0.34	-0.09	-0.83	-0.54
LF χ QSM	0.39	-0.10	-0.80	-0.55
LSS2010	0.19	-0.06	-0.90	-0.53

Conclusion :

Spin and kinetic OAM of valence quarks are anti-correlated !

The conclusions



- Kinetic and canonical decompositions are physically inequivalent and are both interesting
- Measurability requires gauge invariance but not necessarily local expressions
- All the canonical and kinetic contributions are in principle measurable (twist-3 GPDs, GTMDs?) and computable on a lattice

Backup slides

The decompositions in a nutshell

The Chen *et al.* approach

[Chen *et al.* (2008,2009)]
[Wakamatsu (2010,2011)]

$$A_\mu(x) = A_\mu^{\text{pure}}(x) + A_\mu^{\text{phys}}(x)$$

Gauge transformation (assumed)

$$A_\mu^{\text{pure}}(x) \mapsto U(x) \left[A_\mu^{\text{pure}}(x) + \frac{i}{g} \partial_\mu \right] U^{-1}(x)$$

$$A_\mu^{\text{phys}}(x) \mapsto U(x) A_\mu^{\text{phys}}(x) U^{-1}(x)$$

Pure-gauge covariant derivatives

$$D_\mu^{\text{pure}} = \partial_\mu - ig A_\mu^{\text{pure}}(x)$$

$$\mathcal{D}_\mu^{\text{pure}} = \partial_\mu - ig [A_\mu^{\text{pure}}(x),]$$

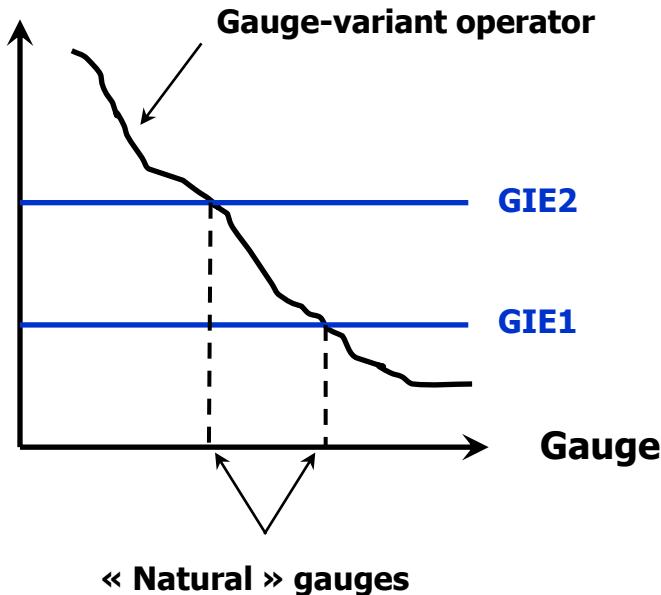
Field strength

$$F_{\mu\nu}^{\text{pure}}(x) = \frac{i}{g} [D_\mu^{\text{pure}}, D_\nu^{\text{pure}}] = 0$$

$$F_{\mu\nu}(x) = \mathcal{D}_\mu^{\text{pure}} A_\nu^{\text{phys}}(x) - \mathcal{D}_\nu^{\text{pure}} A_\mu^{\text{phys}}(x) - ig [A_\mu^{\text{phys}}(x), A_\nu^{\text{phys}}(x)]$$

The gauge-invariant extension (GIE)

[Hoodbhoy, Ji (1998)]
[Ji, Xu, Zhao (2012)]
[C.L. (2013)]



~

Lorentz-invariant
extensions

$$p^2 = m_0^2$$

$$s = E_{\text{CM}}^2$$

$$x = k_{\text{IMF}}^z / p_{\text{IMF}}^z$$

Rest

Center-of-mass

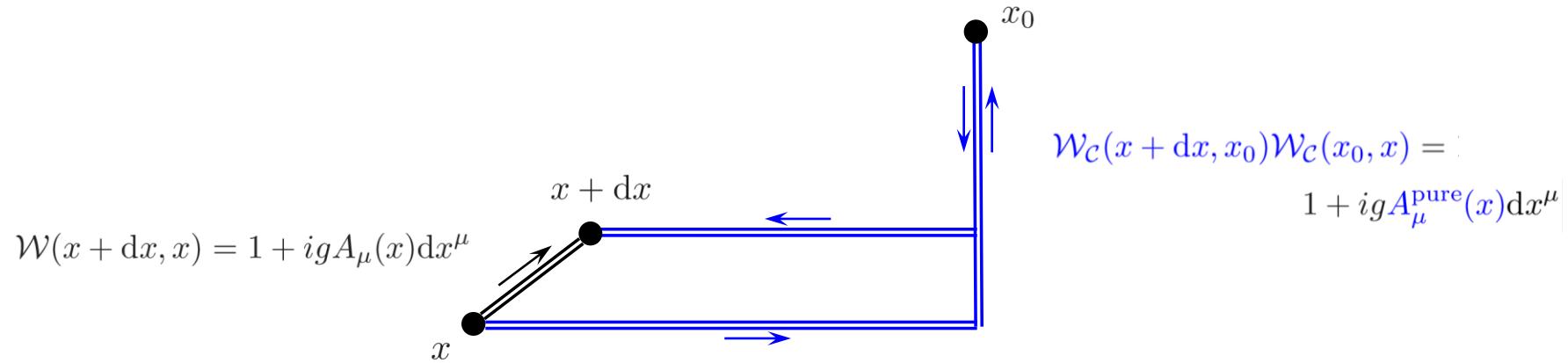
Infinite momentum

« Natural » frames

The geometrical interpretation

[C.L. (2013)]

Parallel transport



$$A_\mu^{\text{pure}}(x) = \frac{i}{g} \mathcal{W}_c(x, x_0) \frac{\partial}{\partial x^\mu} \mathcal{W}_c(x_0, x)$$

$$A_\mu^{\text{phys}}(x) = - \int_{x_0}^x \mathcal{W}_c(x, s) F_{\alpha\beta}(s) \mathcal{W}_c(s, x) \frac{\partial s^\alpha}{\partial x^\mu} ds^\beta$$

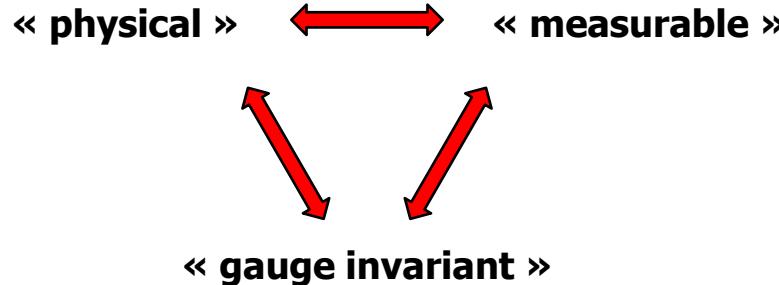


Non-local!

Path dependence → **Stueckelberg dependence**

The semantic ambiguity

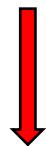
Quid ?



Observables

E.g. cross-sections

Measurable, physical, gauge invariant and **local**



Expansion scheme

E.g. collinear factorization

**Path
Stueckelberg
Background** } dependent **but fixed by the process**

Light-front
gauge links

Quasi-observables

E.g. parton distributions

« Measurable », « physical », gauge invariant but **non-local**

The observability

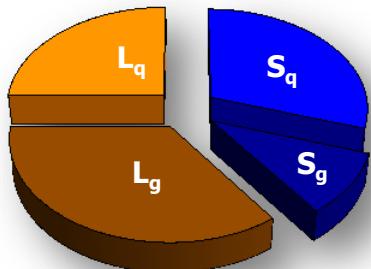
Observable

Quasi-observable

Not observable

Canonical

[Jaffe-Manohar (1990)]



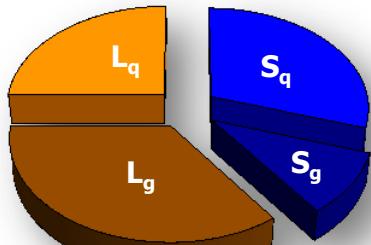
$$\vec{S}_q = \frac{1}{2} \int d^3r \psi^\dagger \vec{\Sigma} \psi$$

$$\vec{L}_q = \int d^3r \psi^\dagger \vec{r} \times (-i\vec{\nabla})\psi$$

$$\vec{S}_g = \int d^3r \vec{E}^a \times \vec{A}^a$$

$$\vec{L}_g = \int d^3r E^{ai} \vec{r} \times \vec{\nabla} A^{ai}$$

[Chen *et al.* (2008)]



$$\vec{S}_q = \frac{1}{2} \int d^3r \psi^\dagger \vec{\Sigma} \psi$$

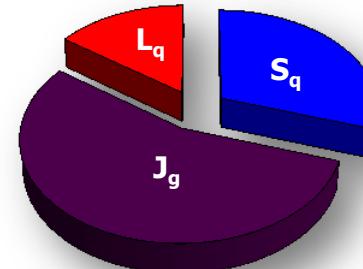
$$\vec{L}_q = \int d^3r \psi^\dagger \vec{r} \times (-i\vec{D}_{\text{pure}})\psi$$

$$\vec{S}_g = \int d^3r \vec{E}^a \times \vec{A}_{\text{phys}}^a$$

$$\vec{L}_g = \int d^3r E^{ai} \vec{r} \times \vec{\mathcal{D}}_{\text{pure}} A_{\text{phys}}^{ai}$$

Kinetic

[Ji (1997)]

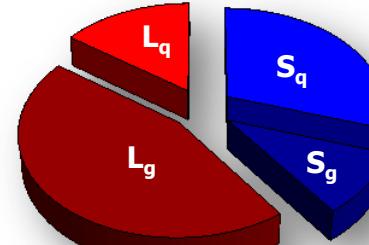


$$\vec{S}_q = \frac{1}{2} \int d^3r \psi^\dagger \vec{\Sigma} \psi$$

$$\vec{L}_q = \int d^3r \psi^\dagger \vec{r} \times (-i\vec{D})\psi$$

$$\vec{J}_g = \int d^3r \vec{r} \times (\vec{E}^a \times \vec{B}^a)$$

[Wakamatsu (2010)]



$$\vec{S}_q = \frac{1}{2} \int d^3r \psi^\dagger \vec{\Sigma} \psi$$

$$\vec{L}_q = \int d^3r \psi^\dagger \vec{r} \times (-i\vec{D})\psi$$

$$\vec{S}_g = \int d^3r \vec{E}^a \times \vec{A}_{\text{phys}}^a$$

$$\vec{L}_g = \int d^3r \vec{r} \times (\vec{E}^a \times \vec{B}^a) - \int d^3r \vec{E}^a \times \vec{A}_{\text{phys}}^a$$

The path dependence

[C.L., Pasquini, Xiong, Yuan (2012)]

[Hatta (2012)]

[Ji, Xiong, Yuan (2012)]

[C.L. (2013)]

Orbital angular momentum

$$\ell_z = \frac{\langle p, + | \hat{L}_z | p, + \rangle}{\langle p, + | p, + \rangle}$$

$$\hat{L}_z = \int d^4r \delta(r^+) \bar{\psi}(r) \gamma^+ \left(\vec{r}_\perp \times i \vec{D}_\perp^{\text{pure}} \right)_z \psi(r)$$

$$= \int d^4r \delta(r^+) \bar{\psi}_D(r) \gamma^+ \left(\vec{r}_\perp \times (-i) \vec{\nabla}_\perp \right)_z \psi_D(r)$$

$$D_\mu^{\text{pure}}(y) = \partial_\mu - ig A_\mu^{\text{pure}}(y)$$

$$= \partial_\mu - ig \left[\frac{i}{g} \mathcal{W}_{[y, y_0]} \partial_\mu \mathcal{W}_{[y_0, y]} \right]$$

$$= \mathcal{W}_{[y, y_0]} \partial_\mu \mathcal{W}_{[y_0, y]}$$

$$\psi_D(y) = \mathcal{W}_{[y_0, y]} \psi(y)$$

↳ Reference point

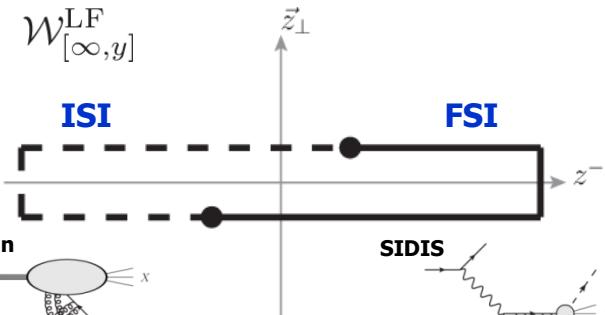
Canonical

[Jaffe, Manohar (1990)]

$$D_\mu^{\text{pure}} \stackrel{A^+=0}{=} \partial_\mu$$

$$\psi_D(r) \stackrel{A^+=0}{=} \psi(r)$$

$$\mathcal{W}_{[\infty, y]}^{\text{LF}}$$



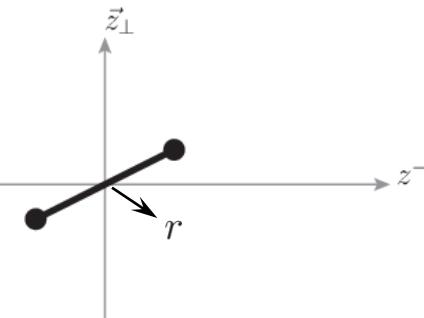
Kinetic

[Ji (1997)]

$$D_\mu^{\text{pure}}(r) = D_\mu(r)$$

$$D_\mu^{\text{pure}}(y) \neq D_\mu(y) \quad y \neq r$$

$$\mathcal{W}_{[r, y]}^{\text{straight}}$$



The quark orbital angular momentum

[C.L., Pasquini (2011)]

GTMD correlator

$$W_{\Lambda'\Lambda}^{[\Gamma]}(x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp) = \frac{1}{2} \langle p', \Lambda' | \widehat{W}^{[\Gamma]}(0, xP^+, \vec{k}_\perp) | p, \Lambda \rangle$$

Wigner distribution

$$\rho_{\Lambda'\Lambda}^{[\Gamma]}(x, \vec{k}_\perp, \vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp} W_{\Lambda'\Lambda}^{[\Gamma]}(x, 0, \vec{k}_\perp, \vec{\Delta}_\perp)$$

Orbital angular momentum

$$\begin{aligned} \ell_z &= \int dx d^2k_\perp d^2b_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z \rho_{++}^{[\gamma^+]}(x, \vec{k}_\perp, \vec{b}_\perp) \\ &= \int dx d^2k_\perp (\vec{k}_\perp \times i\vec{\nabla}_{\Delta_\perp})_z W_{++}^{[\gamma^+]}(x, 0, \vec{k}_\perp, \vec{\Delta}_\perp) \Big|_{\vec{\Delta}_\perp = \vec{0}_\perp} \\ &= \boxed{- \int dx d^2k_\perp \frac{\vec{k}_\perp^2}{M^2} F_{14}(x, 0, \vec{k}_\perp, \vec{0}_\perp)} \end{aligned}$$

↑ Unpolarized
quark density

Parametrization

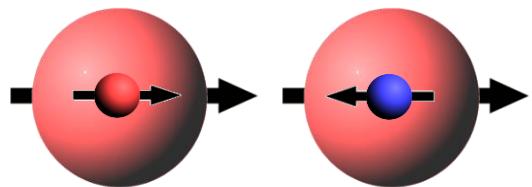
$$W_{\Lambda'\Lambda}^{[\gamma^+]} = \frac{1}{2M} \bar{u}(p', \Lambda') \left[F_{11} + \frac{i\sigma^{k_\perp +}}{P^+} F_{12} + \frac{i\sigma^{\Delta_\perp +}}{P^+} F_{13} + \frac{i\sigma^{k_\perp \Delta_\perp}}{M^2} F_{14} \right] u(p, \Lambda)$$

[Meißner, Metz, Schlegel (2009)]

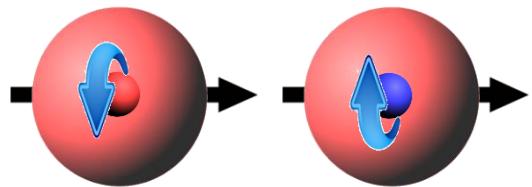
The emerging picture

Longitudinal

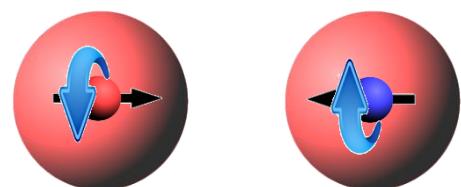
$$g_{1L} \leftrightarrow \tilde{H}$$



$$\ell_z \leftrightarrow F_{14}$$



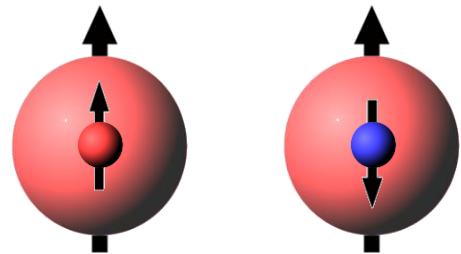
$$C_z \leftrightarrow G_{11}$$



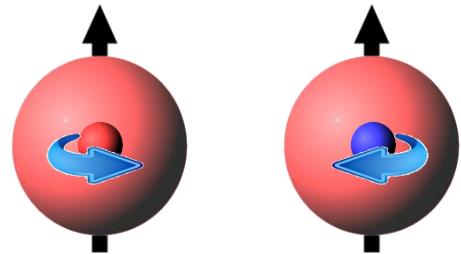
[C.L., Pasquini (2011)]

Transverse

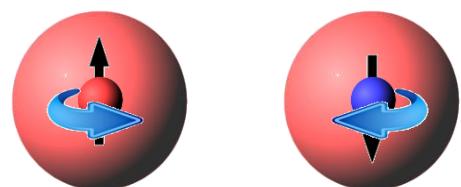
$$h_1 \leftrightarrow H_T$$



$$f_{1T}^\perp \leftrightarrow E$$



$$h_1^\perp \leftrightarrow 2\tilde{H}_T + E_T$$

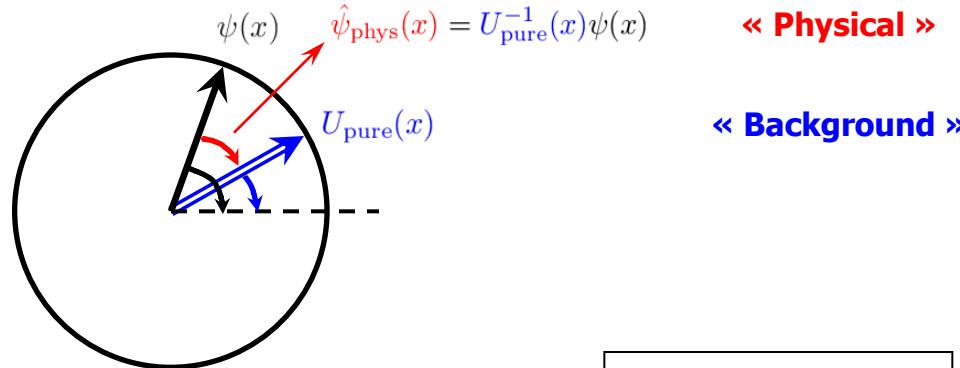


[Burkardt (2005)]
[Barone *et al.* (2008)]

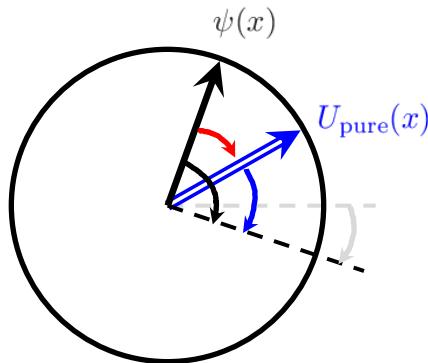
The gauge symmetry

[C.L. (2013)]

Quantum electrodynamics



Passive

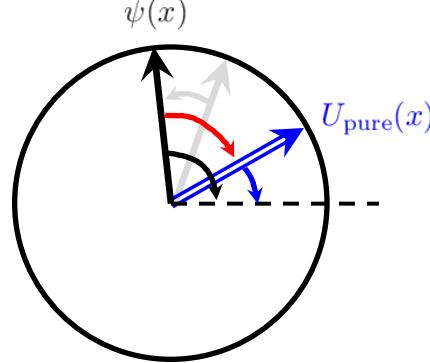


$$\psi(x) \mapsto U(x)\psi(x)$$

$$U_{\text{pure}}(x) \mapsto U(x)U_{\text{pure}}(x)$$

$$\hat{\psi}_{\text{phys}}(x) \mapsto \hat{\psi}_{\text{phys}}(x)$$

Active



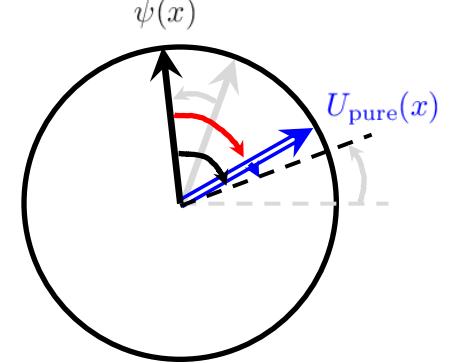
$$\psi(x) \mapsto U(x)\psi(x)$$

$$U_{\text{pure}}(x) \mapsto U_{\text{pure}}(x)$$

$$\hat{\psi}_{\text{phys}}(x) \mapsto U(x)\hat{\psi}_{\text{phys}}(x)$$

Stueckelberg

Active \times **(Passive)**⁻¹



$$\psi(x) \mapsto \psi(x)$$

$$U_{\text{pure}}(x) \mapsto U_{\text{pure}}(x)U^{-1}(x)$$

$$\hat{\psi}_{\text{phys}}(x) \mapsto U(x)\hat{\psi}_{\text{phys}}(x)$$

The phase-space distribution

[Wigner (1932)]
[Moyal (1949)]

Wigner distribution

$$\begin{aligned}\rho(r, k) &= \int \frac{dz}{2\pi} e^{-ikz} \psi^*(r - \frac{z}{2}) \psi(r + \frac{z}{2}) \\ &= \int \frac{d\Delta}{(2\pi)^2} e^{-i\Delta r} \varphi^*(k + \frac{\Delta}{2}) \varphi(k - \frac{\Delta}{2})\end{aligned}$$



Galilei covariant

- Either non-relativistic
- Or restricted to transverse position

Probabilistic interpretation

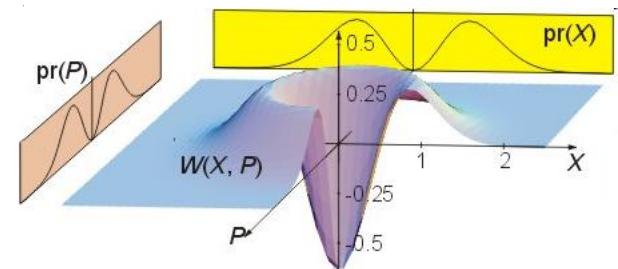
$$\int dk \rho(r, k) = |\psi(r)|^2$$



$$\rho(r, k) \geq 0$$

Heisenberg's
uncertainty relations

$$\int dr \rho(r, k) = |\varphi(k)|^2$$



Expectation value

$$\begin{aligned}\langle \hat{O} \rangle &= \int dr \psi^*(r) O(r, -i\partial_r) \psi(r) \\ &= \int \frac{dk}{2\pi} \varphi^*(k) O(i\partial_k, k) \varphi(k) \\ &= \int dr dk O(r, k) \rho(r, k)\end{aligned}$$

Position space

Momentum space

Phase space

The parametrization @ twist-2 and $\xi=0$

Parametrization : [Meißner, Metz, Schlegel (2009)]

		GTMDs			
Quark polarization		U	T_x	T_y	L
Nucleon polarization	U	F_{11}	$\frac{i}{M} (k_y H_{11} + \Delta_y H_{12})$	$-\frac{i}{M} (k_x H_{11} + \Delta_x H_{12})$	$\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} G_{11}$
	T_x	$\frac{i}{M} (k_y F_{12} + \Delta_y (F_{13} - \frac{1}{2} F_{11}))$
	T_y	$-\frac{i}{M} (k_x F_{12} + \Delta_x (F_{13} - \frac{1}{2} F_{11}))$
	L	$-\frac{i(\vec{\Delta}_\perp \times \vec{k}_\perp)_z}{M^2} F_{14}$	$\frac{1}{M} (k_x H_{17} + \Delta_x H_{18})$	$\frac{1}{M} (k_y H_{17} + \Delta_y H_{18})$	G_{14}

$$\vec{\Delta}_\perp = \vec{0}_\perp$$

$$\int d^2 k_\perp$$

TMDs

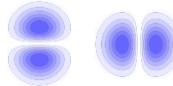
GPDs

	U	T_x	T_y	L
U	f_1	$\frac{k_y}{M} h_{1T}^\perp$	$-\frac{k_x}{M} h_{1T}^\perp$	
T_x	$\frac{k_y}{M} f_{1T}^\perp$	$h_1 + \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$\frac{k_x}{M} g_{1T}$
T_y	$-\frac{k_x}{M} f_{1T}^\perp$	$\frac{k_x k_y}{M^2} h_{1T}^\perp$	$h_1 - \frac{k_x^2 - k_y^2}{2M^2} h_{1T}^\perp$	$\frac{k_y}{M} g_{1T}$
L		$\frac{k_x}{M} h_{1L}^\perp$	$\frac{k_y}{M} h_{1L}^\perp$	g_{1L}

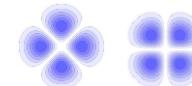
	U	T_x	T_y	L
U	H	$i \frac{\Delta_y}{2M} (2\tilde{H}_T + E_T)$	$-i \frac{\Delta_x}{2M} (2\tilde{H}_T + E_T)$	
T_x	$i \frac{\Delta_y}{2M} E$	$H_T - \frac{\Delta_x^2 - \Delta_y^2}{4M^2} \tilde{H}_T$	$-\frac{\Delta_x \Delta_y}{2M^2} \tilde{H}_T$	
T_y	$-i \frac{\Delta_x}{2M} E$	$-\frac{\Delta_x \Delta_y}{2M^2} \tilde{H}_T$	$H_T + \frac{\Delta_x^2 - \Delta_y^2}{4M^2} \tilde{H}_T$	
L				\tilde{H}



Monopole

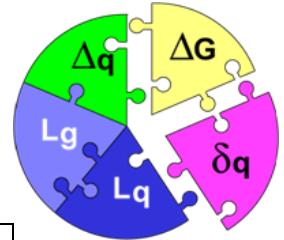


Dipole



Quadrupole

OAM and origin dependence



Naive

$$\mathcal{L}_{iz} = \vec{r}_{i\perp} \times \vec{k}_{i\perp}$$



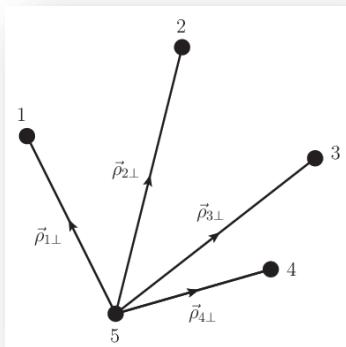
Depends on
proton position

Momentum conservation

$$\sum_{i=1}^N \vec{k}_{i\perp} = \vec{0}_\perp$$

Relative

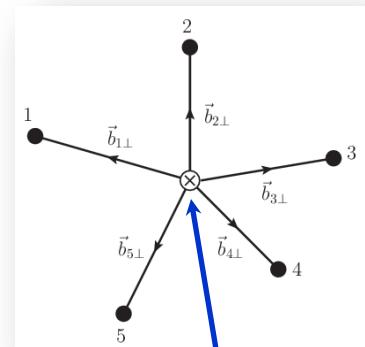
$$\ell_{iz}^{\text{rel}} = \vec{\rho}_{i\perp} \times \vec{k}_{i\perp}$$



Physical interpretation ?

Intrinsic

$$\ell_{iz}^{\text{int}} = \vec{b}_{i\perp} \times \vec{k}_{i\perp}$$



Transverse center
of momentum

$$\vec{R}_\perp = \sum_{i=1}^N x_i \vec{r}_{i\perp}$$

Equivalence $\mathcal{L}_z = \ell_z^{\text{rel}} = \ell_z^{\text{int}}$

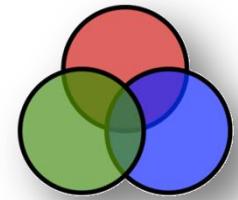
$$\sum_{i=1}^N \vec{b}_{i\perp} \times \vec{k}_{i\perp} = \sum_{i=1}^N (\vec{r}_{i\perp} - \vec{R}_\perp) \times \vec{k}_{i\perp} = \sum_{i=1}^N \vec{r}_{i\perp} \times \vec{k}_{i\perp} = \sum_{i=1}^{N-1} \vec{r}_{i\perp} \times \vec{k}_{i\perp} - \vec{r}_{N\perp} \times \sum_{i=1}^{N-1} \vec{k}_{i\perp} = \sum_{i=1}^{N-1} \vec{\rho}_{i\perp} \times \vec{k}_{i\perp}$$

Intrinsic

Naive

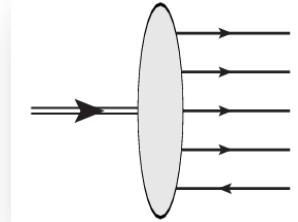
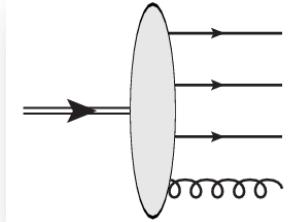
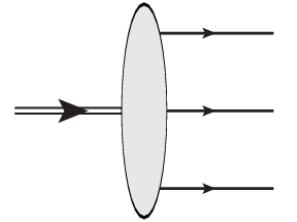
Relative

Overlap representation

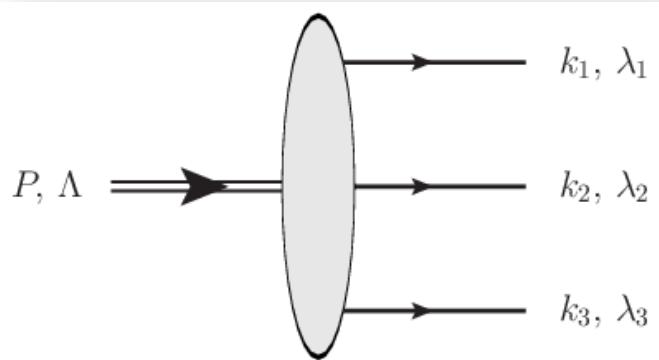


Fock expansion of the proton state

$$|p\rangle = \Psi_{qqq} |qqq\rangle + \Psi_{qqqg} |qqqg\rangle + \Psi_{qqq\bar{q}} |qqq\bar{q}\rangle + \dots$$



Fock states



Simultaneous eigenstates of

$$P^+ = \sum_{i=1}^N k_i^+$$

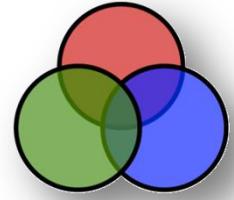
$$\vec{0}_\perp = \vec{P}_\perp = \sum_{i=1}^N \vec{k}_{i\perp}$$

λ_i

Light-front
helicity

Momentum

Overlap representation



Fock-state contributions

[C.L., Pasquini (2011)]
[C.L. *et al.* (2012)]

Kinetic OAM

GPDs

$$L_z^{N\beta,q} = \frac{1}{2} \int [dx]_N [d^2 k_\perp]_N \sum_{i=1}^N \delta_{qq_i} \left\{ (x_i - \lambda_i) |\Psi_{N\beta}^\uparrow|^2 + M x_i \sum_{n=1}^N (\delta_{ni} - x_n) \left[\Psi_{N\beta}^{*\uparrow} \frac{\overleftrightarrow{\partial}}{\partial k_n^x} \Psi_{N\beta}^\downarrow \right] \right\}$$

Naive canonical OAM

TMDs

$$\mathcal{L}_z^{N\beta,q} = -\frac{i}{2} \int [dx]_N [d^2 k_\perp]_N \sum_{i=1}^N \delta_{qq_i} \left[\Psi_{N\beta}^{*\uparrow} \left(\vec{k}_i \times \vec{\nabla}_{k_i} \right)_z \Psi_{N\beta}^\uparrow \right]$$

Canonical OAM

GTMDs

$$\ell_z^{N\beta,q} = -\frac{i}{2} \int [dx]_N [d^2 k_\perp]_N \sum_{i=1}^N \delta_{qq_i} \sum_{n=1}^N (\delta_{ni} - x_n) \left[\Psi_{N\beta}^{*\uparrow} \left(\vec{k}_i \times \vec{\nabla}_{k_n} \right)_z \Psi_{N\beta}^\uparrow \right]$$