A classification of chiral-odd pion generalized parton distributions beyond leading twist

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Extensions from DIS

**DIS**: inclusive process $\rightarrow$ forward amplitude ($t = 0$) *(optical theorem)*

*(DIS: Deep Inelastic Scattering)*

ex: $e^\pm p \rightarrow e^\pm X$ at HERA

$x \Rightarrow$ 1-dimensional structure

Structure Function

\[ = \text{Coefficient Function (hard)} \otimes \text{Parton Distribution Function (soft)} \]

**DVCS**: exclusive process $\rightarrow$ non forward amplitude ($-t \ll s = W^2$)

*(DVCS: Deep Virtual Compton Scattering)*

Fourier transf.: $t \leftrightarrow$ impact parameter

$(x, t) \Rightarrow$ 3-dimensional structure

Amplitude

\[ = \text{Coefficient Function (hard)} \otimes \text{Generalized Parton Distribution (soft)} \]

Müller et al. '91 - '94; Radyushkin '96; Ji '97
Collinear factorization
A bit more technical: DVCS and GPDs

The two steps for factorization, in a nutshell

- **momentum factorization**: light-cone vector dominance for $Q^2 \to \infty$

  \[ p_1, p_2 : \text{the two light-cone directions} \]

  \[
  \begin{align*}
  p_1 &= \frac{\sqrt{2}}{2} (1, 0, 1) \\
  p_2 &= \frac{\sqrt{2}}{2} (1, 0, -1) \\
  2 p_1 \cdot p_2 &= s \sim s_{\gamma p} \gtrsim Q^2
  \end{align*}
  \]

- **Sudakov decomposition**: $k = \alpha p_1 + \beta p_2 + k_{\perp}$

  \[
  \begin{aligned}
  \gamma^*(q) &\quad H \\
  \int d^4k &\quad k + \Delta \\
  \gamma
  \end{aligned}
  \]

  \[
  \begin{aligned}
  \int d^4k_S(k, k + \Delta) H(q, k, k + \Delta) &= \int dk^- \int dx \int dk^+ d^2k_{\perp} S(k, k + \Delta) H(q, k^-, k^- + \Delta^-)
  \end{aligned}
  \]

- **Quantum numbers factorization** (Fierz identity: spinors + color)

  \[
  \Rightarrow \quad \mathcal{M} = \text{GPD} \otimes \text{Hard part}
  \]

Müller et al. ’91 - ’94; Radyushkin ’96; Ji ’97
Collinear factorization
Twist 2 GPDs

Physical interpretation for GPDs

Emission and reabsorption of an antiquark
\( \sim \) PDFs for antiquarks
DGLAP-II region

Emission of a quark and emission of an antiquark
\( \sim \) meson exchange
ERBL region

Emission and reabsorption of a quark
\( \sim \) PDFs for quarks
DGLAP-I region
Collinear factorization
Twist 2 GPDs

Classification of twist 2 GPDs

- For quarks, one should distinguish the exchanges
  - without helicity flip (chiral-even $\Gamma'$ matrices): 4 chiral-even GPDs:
    \[ H^q \xrightarrow{\xi=0, t=0} \text{PDF } q, \ E^q, \ \tilde{H}^q \xrightarrow{\xi=0, t=0} \text{polarized PDFs } \Delta q, \ \tilde{E}^q \]
    \[ F^q = \frac{1}{2} \int \frac{dz^+}{2\pi} e^{ixP^- z^+} \langle p' | \bar{q}(-\frac{1}{2}z) \gamma^- q(\frac{1}{2}z) | p \rangle \bigg|_{z^- = 0, z_\perp = 0} \]
    \[ = \frac{1}{2P^-} \left[ H^q(x, \xi, t) \bar{u}(p') \gamma^- u(p) + E^q(x, \xi, t) \bar{u}(p') \frac{i \sigma^- \Delta^-}{2m} u(p) \right] , \]
    \[ \tilde{F}^q = \frac{1}{2} \int \frac{dz^+}{2\pi} e^{ixP^- z^+} \langle p' | \bar{q}(-\frac{1}{2}z) \gamma^- \gamma_5 q(\frac{1}{2}z) | p \rangle \bigg|_{z^- = 0, z_\perp = 0} \]
    \[ = \frac{1}{2P^-} \left[ \tilde{H}^q(x, \xi, t) \bar{u}(p') \gamma^- \gamma_5 u(p) + \tilde{E}^q(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^-}{2m} u(p) \right] . \]

- with helicity flip (chiral-odd $\Gamma'$ mat.): 4 chiral-odd GPDs:
  \[ H_T^q \xrightarrow{\xi=0, t=0} \text{quark transversity PDFs } \Delta_T q, \ E_T^q, \ \tilde{H}_T^q, \ \tilde{E}_T^q \]
  \[ \frac{1}{2} \int \frac{dz^+}{2\pi} e^{ixP^- z^+} \langle p' | \bar{q}(-\frac{1}{2}z) i \sigma^- q(\frac{1}{2}z) | p \rangle \bigg|_{z^- = 0, z_\perp = 0} \]
  \[ = \frac{1}{2P^-} \bar{u}(p') \left[ H_T^q i \sigma^- + \tilde{H}_T^q \frac{P^- \Delta^- - \Delta^- P^-}{m^2} + E_T^q \frac{\gamma^- \Delta^- - \Delta^- \gamma^-}{2m} + \tilde{E}_T^q \frac{\gamma^- P^- - P^- \gamma^-}{m} \right] . \]
Classification of twist 2 GPDs

- analogously, for gluons:

  - 4 gluonic GPDs without helicity flip:
    \[ H_g \xrightarrow{\xi=0,t=0} \text{PDF } x g \]
    \[ E_g \]
    \[ \tilde{H}_g \xrightarrow{\xi=0,t=0} \text{polarized PDF } x \Delta g \]
    \[ \tilde{E}_g \]

  - 4 gluonic GPDs with helicity flip:
    \[ H_{gT} \]
    \[ E_{gT} \]
    \[ \tilde{H}_{gT} \]
    \[ \tilde{E}_{gT} \]

(no forward limit reducing to gluons PDFs here: a change of 2 units of helicity cannot be compensated by a spin 1/2 target)
Spin transversity in the nucleon

What is transversity?

- Tranverse spin content of the proton:
  \[ |\uparrow\rangle(x) \sim |\rightarrow\rangle + |\leftarrow\rangle \]
  \[ |\downarrow\rangle(x) \sim |\rightarrow\rangle - |\leftarrow\rangle \]
  spin along \(x\) helicity state

- An observable sensitive to helicity spin flip gives thus access to the transversity \(\Delta_T q(x)\), which is very badly known (first data have recently been obtained by COMPASS)

The transversity GPDs are completely unknown

- Chirality: \(q_{\pm}(z) \equiv \frac{1}{2}(1 \pm \gamma^5)q(z)\) with \(q(z) = q_+(z) + q_-(z)\)
  - Chiral-even: chirality conserving
    \(\bar{q}_{\pm}(z)\gamma^\mu q_{\pm}(-z)\) and \(\bar{q}_{\pm}(z)\gamma^\mu\gamma^5 q_{\pm}(-z)\)
  - Chiral-odd: chirality reversing
    \(\bar{q}_{\pm}(z) \cdot 1 \cdot q_{\mp}(-z)\), \(\bar{q}_{\pm}(z) \cdot \gamma^5 \cdot q_{\mp}(-z)\) and \(\bar{q}_{\pm}(z)[\gamma^\mu, \gamma^\nu]q_{\mp}(-z)\)

- For a massless (anti)particle, chirality = (-)helicity

- Transversity is thus a chiral-odd quantity

- QCD and QED are chiral even \(\Rightarrow \mathcal{A} \sim (\text{Ch.-odd})_1 \otimes (\text{Ch.-odd})_2\)
Spin transversity in the nucleon

How to get access to transversity?

- The dominant DA for $\rho_T$ is of twist 2 and chiral-odd ($[\gamma^\mu, \gamma^\nu]$ coupling)
- Unfortunately $\gamma^* N^\uparrow \to \rho_T N' = 0$
  - this is true at any order in perturbation theory (i.e. corrections as powers of $\alpha_s$), since this would require a transfer of 2 units of helicity from the proton: impossible!
  - Diehl, Gousset, Pire '99; Collins, Diehl '00

- diagrammatic argument at Born order:

\[
\gamma^* N^\uparrow \to \rho_T N' = 0
\]

\[
\gamma^\alpha [\gamma^\mu, \gamma^\nu] \gamma_\alpha = 0
\]
Spin transversity in the nucleon

Can one circumvent this vanishing?

- This vanishing is true only at twist 2 in electroproduction: one may consider a final state with 3 particles (next slide)
- At twist 3 this process does not vanish but for consistency one needs to consider higher twist corrections both for the meson DAs and for the GPDs (next part of this talk: for simplicity we will consider the $\pi^0$ case)
Spin transversity in the nucleon

\[ \gamma N \to \pi^+ \rho_T^0 N' \] gives access to transversity

- Factorization à la Brodsky Lepage of \( \gamma + \pi \to \pi + \rho \) at large \( s \) and fixed angle (i.e. fixed ratio \( t'/s, u'/s \))
  \[ \to \] factorization of the amplitude for \( \gamma + N \to \pi + \rho + N' \) at large \( M_{\pi\rho}^2 \)

\[ \begin{array}{ccc}
\gamma & \to & \gamma' \\
T_H & \to & T_H \\
\pi & \to & \pi' \\
\rho & \to & \rho' \\
\end{array} \]

- a typical non-vanishing diagram:

- These processes with 3 body final state can give access to all GPDs:

\[ M_{\pi\rho}^2 \] plays the role of the \( \gamma^* \) virtuality of usual DVCS (here in the time-like domain) JLab, COMPASS

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see also, at large \( s \), with \( P \)omerón exchange:

R. Ivanov, B. Pire, L. Symanowski, O. Teryaev '02

R. Enberg, B. Pire, L. Symanowski '06
Beyond leading twist
Light-Cone Collinear Factorization versus Covariant Collinear Factorization

The **Light-Cone Collinear Factorization**, a self-consistent method, while non-covariant, is very efficient for practical computations

Anikon, Ivanov, Pire, Szymanowski, S.W. '09

- inspired by the inclusive case
  Ellis, Furmanski, Petronzi '83; Efremov, Teryaev '84

- axial gauge

- parametrization of matrix element along a **light-like prefered direction**
  \[ z = \lambda n \ (n = 2 p_2 / s) \].

- non-local correlators are defined along this prefered direction, with contributions arising from Taylor expansion up to needed term for a given twist order computation

- their number is then reduced to a minimal set combining equations of motion and \( n-\)indepenendency condition

- Another approach (Braun, Ball), based on non-local OPE and fully covariant but less convenient (at least at twist 3) when practically computing coefficient functions, can equivalently be used

- We have established the dictionnary between these two approaches

- **This has been explicitly checked for the** \( \gamma_T^* \rightarrow \rho_T \) **impact factor at twist 3**

Anikin, Ivanov, Pire, Szymanowski, S.W.

Beyond leading twist: $\gamma^* \rightarrow \rho$ impact factor up to twist 3 as an example

Light-Cone Collinear Factorization

- The impact factor $\Phi^{\gamma^*\rho}(\lambda_\gamma) \rightarrow \rho(\lambda_\rho)$ can be written as

$$\Phi^{\gamma^*\rho}(\lambda_\gamma) \rightarrow \rho(\lambda_\rho) = \int d^4 \ell \cdots \text{tr}[H(\lambda_\gamma)(\ell \cdots) S^{(\lambda_\rho)}(\ell \cdots)]$$

- Hard part
  - Soft part

(2-parton exchange)

(3-parton exchange)

- Soft parts:

$$S_{q\bar{q}}(\ell_q) = \int d^4 z e^{-i\ell_q \cdot z} \langle \rho(p) | \psi(0) \bar{\psi}(z) | 0 \rangle$$

$$S_{q\bar{q}q}(\ell_q, \ell_g) = \int d^4 z_1 \int d^4 z_2 e^{-i(\ell_q \cdot z_1 + \ell_g \cdot z_2)} \langle \rho(p) | \psi(0) g A_{\alpha}^{\perp}(z_2) \bar{\psi}(z_1) | 0 \rangle$$
Beyond leading twist: $\gamma^* \rightarrow \bar{\rho}$ impact factor up to twist 3 as an example

Light-Cone Collinear Factorization

- **Sudakov expansion** in the basis $p \sim p_{\rho}$, $n$ ($p^2 = n^2 = 0$ and $p \cdot n = 1$)

\[
\ell_\mu = u p_\mu + \ell_\mu^\perp + (\ell \cdot p) n_\mu, \quad u = \ell \cdot n
\]

\[
1 \quad 1/Q \quad 1/Q^2
\]

- **Taylor expansion** of the hard part $H(\ell)$ along the collinear direction $p$:

\[
H(\ell) = H(up) + \left. \frac{\partial H(\ell)}{\partial \ell_\alpha} \right|_{\ell = up} (\ell - up) \alpha + \ldots \quad \text{with} \quad (\ell - up) \alpha \approx \ell_\alpha^\perp
\]

- $l_\alpha^\perp \xrightarrow{\text{Fourier}}$ derivative of the soft term: $\int d^4 z \ e^{-i \ell \cdot z} \langle \rho(p) | \psi(0) \ i \ \partial_\alpha \bar{\psi}(z) | 0 \rangle$

- **Color + spinor factorization = Fierz transforms**: 

\[
\begin{align*}
H_{q\bar{q}} \quad S_{q\bar{q}} & \quad \xrightarrow{\ell} \quad \sum H_{q\bar{q}} \quad S_{q\bar{q}} + \sum H_{q\bar{q}} \quad S_{q\bar{q}}^\perp \\
H_{q\bar{g}} \quad S_{q\bar{g}} & \quad \xrightarrow{\ell} \quad \sum H_{q\bar{g}} \quad S_{q\bar{g}}
\end{align*}
\]
Beyond leading twist: $\gamma^* \to \beta$ impact factor up to twist 3 as an example

Light-Cone Collinear Factorization

2-body non-local correlators

- vector correlator

$$\langle \rho(p) | \bar{\psi}(z) \gamma_\mu \psi(0) | 0 \rangle \equiv m_\rho f_\rho \left[ \varphi_1(y) (e^* \cdot n)p_\mu + \varphi_3(y) e^*_T \right]$$

- axial correlator

$$\langle \rho(p) | \bar{\psi}(z) \gamma_5 \gamma_\mu \psi(0) | 0 \rangle \equiv m_\rho f_\rho i \varphi_A(y) \epsilon_{\mu \lambda \beta \delta} e^*_T p_\beta n_\delta$$

- vector correlator with transverse derivative

$$\langle \rho(p) | \bar{\psi}(z) \gamma_\mu i \partial_\alpha \psi(0) | 0 \rangle \equiv m_\rho f_\rho \varphi_1^T(y) p_\mu e^*_\alpha$$

- axial correlator with transverse derivative

$$\langle \rho(p) | \bar{\psi}(z) \gamma_5 \gamma_\mu i \partial_\alpha \psi(0) | 0 \rangle \equiv m_\rho f_\rho i \varphi_A^T(y) p_\mu \epsilon_{\alpha \lambda \beta \delta} e^*_\lambda p_\beta n_\delta,$$

where $y$ ($\bar{y} \equiv 1 - y$) = momentum fraction along $p \equiv p_1$ of the quark (antiquark) and

$$\mathcal{F} \equiv \int_0^1 dy \exp[i \, y \, p \cdot z], \text{ with } z = \lambda n$$

$\Rightarrow$ 5 2-body DAs
3-body non-local correlators

- vector correlator

\[
\langle \rho(p) | \bar{\psi}(z_1) \gamma_\mu g A^T_\alpha(z_2) \psi(0) | 0 \rangle \equiv m_\rho f^V_3 B(y_1, y_2) p_\mu e^{*T}_\alpha,
\]

- axial correlator

\[
\langle \rho(p) | \bar{\psi}(z_1) \gamma_5 \gamma_\mu g A^T_\alpha(z_2) \psi(0) | 0 \rangle \equiv m_\rho f^A_3 i D(y_1, y_2) p_\mu \varepsilon_{\alpha\lambda\beta\delta} e^{*T}_\lambda p_\beta n_\delta,
\]

where \( y_1, \bar{y}_2, y_2 - y_1 = \text{quark, antiquark, gluon momentum fraction} \)

and \( \mathcal{F}_2 \equiv \int_0^1 dy_1 \int_0^1 dy_2 \exp \left[ i y_1 p \cdot z_1 + i(y_2 - y_1) p \cdot z_2 \right] \), with \( z_{1,2} = \lambda n \)

\( \Rightarrow \) 2 3-body DAs
Beyond leading twist: $\gamma^* \rightarrow \rho$ impact factor up to twist 3 as an example

Light-Cone Collinear Factorization

Minimal set of DAs

- Number of non-perturbative quantities: a priori 7 at twist 3
  (5 2-parton DA and 2 2-parton DA)

- Non-perturbative correlators cannot be obtained perturbatively!

- One should reduce their number to a minimal set before any use of a model or any measure on the QCD lattice

- Independence w.r.t the choice of the vector $n$ defining
  - the light-cone direction $z$: $z = \lambda n$
  - the $\rho_T$ polarization vector: $e_T \cdot n = 0$
  - the axial gauge: $n \cdot A = 0$

- We have proven that 3 independent Distribution Amplitudes are necessary:
  \[
  \begin{align*}
  &\left\{ \begin{array}{ccc}
    & \text{QCD equations of motion} & 2 \text{ equations} \quad (\text{DAs from } \partial_{\perp} \text{ operators eliminated}) \\
    \text{Arbitrariness in the choice of } n & 2 \text{ equations} \\
  \end{array} \right.
  \end{align*}
  \]

- $\varphi_1(y) \leftarrow$ 2-body twist 2 correlator
- $B(y_1, y_2) \leftarrow$ 3-body genuine twist 3 vector correlator
- $D(y_1, y_2) \leftarrow$ 3-body genuine twist 3 axial correlator
Beyond leading twist: $\gamma^* \to \rho$ impact factor up to twist 3 as an example

Light-Cone Collinear Factorization: $n$–independence

$n$–independence at the amplitude level

- $\rho_T$ polarization: $e^*_{\mu} T = e^*_{\mu} - p_{\mu} e^* \cdot n$ keeping \[ \begin{cases} n \cdot p = 1 \\ n^2 = 0 \end{cases} \]

- for the full factorized amplitude:

\[ A = H \otimes S \quad \frac{dA}{dn_{\perp} \mu} = 0, \]

- rewrite hard terms in one single form, of 2-body type: use Ward identities

Example: hard 3-body $\to$ hard 2-body

\[ \text{tr} \left[ H_3 (y_1, y_2) \, p^\rho \right] \, B(y_1, y_2) = \frac{1}{y_1 - y_2} \left( \text{tr} \left[ H_2 (y_1) \right] - \text{tr} \left[ H_2 (y_2) \right] \right) B(y_1, y_2), \]

\[ (y_1 - y_2) p_\mu \quad y_1 \quad y_2 \quad 1 - y_1 \quad 1 - y_2 \]

- thus, symbolically,

\[ \frac{dS}{dn_{\perp} \mu} = 0 \]
Beyond leading twist: $\gamma^* \to \rho$ impact factor up to twist 3 as an example

Light-Cone Collinear Factorization: $n-$independence

$n-$independence from the operators

**Variation of a Wilson line**

- When implementing the two above generators, one should not forget the hidden Wilson line, entering the non-local operators!
- Wilson line $[y, x]_C$ between $x$ and $y$ along an arbitrary path $C$, defined as

$$[y, x]_C \equiv P_C \exp ig \int_x^y dx_\mu A^{\mu}(x).$$

- Variation of a Wilson line from path $C$ to path $C'$

$$\delta[y, x]_C =$$

$$-ig \int_0^1 [y, x[\sigma]]_C G_{\nu\gamma}(x[\sigma]) \delta x^{\gamma}[\sigma] \frac{dx^\nu}{d\sigma}[\sigma] [x[\sigma], x]_C d\sigma$$

$$+ ig A(y) \cdot \delta x[1] [y, x]_C - ig [y, x]_C A(x) \cdot \delta x[0],$$

$$\begin{cases} [0, 1] \to C \\ \sigma \mapsto x[\sigma] \end{cases} \text{ with } x[0] = x \text{ and } x[1] = y.$$
Beyond leading twist: $\gamma^\nu \rightarrow \beta$ impact factor up to twist 3 as an example
Light-Cone Collinear Factorization

\[ n-\text{independence from the operators} \]

\textbf{Variation of a Wilson line}

- consider now the Wilson line involved in our non-local operators, like

\[
\bar{\psi}(z) \Gamma [z, -z] \psi(-z) \quad \text{with } \Gamma \in \{ \sigma^{\alpha\beta}, 1, i\gamma^5 \}
\]

- For simplicity, take a straight line from $-z$ to $z$: $x[\tau] = \tau z$, $\tau \in [-1, 1]$.

- Consider an infinitesimal transformation $\delta z^\gamma$:

\[
\frac{\partial}{\partial z^\gamma} \left[ \bar{\psi}(z) \Gamma [z, -z] \psi(-z) \right] =
\]

\[
-\bar{\psi}(z) \Gamma [z, -z] \overrightarrow{D}_\gamma \psi(-z) + \bar{\psi}(z) \overleftarrow{D}_\gamma \Gamma [z, -z] \psi(-z)
\]

\[
-ig \int_{-1}^{1} dv \, v \bar{\psi}(z)[z, vz] z^\nu G_{\nu\gamma}(vz) \Gamma[vz, -z] \psi(-z),
\]

with $\overrightarrow{D}_\alpha = \partial_\alpha - ig A_\alpha(-z)$ and $\overleftarrow{D}_\alpha = \partial_\alpha + ig A_\alpha(z)$. 

Balitsky, Braun '89
Beyond leading twist: $\gamma^* \rightarrow \rho$ impact factor up to twist 3 as an example
Light-Cone Collinear Factorization

$n-$independence from the operators

Application to matrix elements

\[
\frac{\partial}{\partial z^\gamma} \left[ \langle \rho(p)|\bar{\psi}(z)\Gamma[z, -z]\psi(-z)|0\rangle \right] = \\
-\langle \rho(p)|\bar{\psi}(z)\Gamma[z, -z]\overleftrightarrow{D_\gamma}\psi(-z) + \bar{\psi}(z)\overleftrightarrow{D_\gamma}\Gamma[z, -z]\psi(-z)|0\rangle \\
-ig \int_{-1}^{1} dv \nu \langle \rho(p)|\bar{\psi}(z)[z, vz]z^\nu G_{\nu \gamma}(vz)\Gamma[vz, -z]\psi(-z)|0\rangle. \tag{1}
\]

- Use light-like gauge: $n \cdot A = 0$
- Thus
  \[ z^\nu G_{\nu \gamma} = z^\nu \partial_\nu A_\gamma \]
- Only the $\gamma_\perp$ index contributes non-trivially
- Thus (1) only involves matrix elements with the $\perp$ components of the field $A_\gamma$ introduced before
- One finally gets a set of two integral equations between DAs
Consider the process $A \pi^0 \rightarrow B \pi^0$
(e.g. $\gamma^* \pi^0 \rightarrow \rho \pi^0 \pi^0$, i.e. $B = \rho \pi^0$).

\[ P \equiv \frac{p_1 + p_2}{2} \quad \text{and} \quad \Delta \equiv p_2 - p_1 . \]

- **Sudakov** basis provided by $p$ and $n$ ($p^2 = n^2 = 0, p \cdot n = 1$):
  
  \[ k = (k \cdot n) p + (k \cdot p) n + k_\perp . \]

- In particular $\Delta = -2\xi p + (\Delta \cdot p) n + \Delta_\perp$.

Symmetric kinematics for $p_1$ and $p_2$:

\begin{align*}
p_1 &= (1 + \xi) p + \frac{m^2 - \frac{\Delta_\perp^2}{4}}{2(1 + \xi)} n - \frac{\Delta_\perp}{2} , \\
p_2 &= (1 - \xi) p + \frac{m^2 - \frac{\Delta_\perp^2}{4}}{2(1 - \xi)} n + \frac{\Delta_\perp}{2} ,
\end{align*}

makes $P$ longitudinal (no $\perp$ component): $P = p + (P \cdot p) n = p + \frac{m^2 - \frac{\Delta_\perp^2}{4}}{1 - \xi^2} n$. 

Kinematics and factorization

\[ A \quad \Downarrow \Delta \quad B \]

\[ \pi^0(p_1) \quad \pi^0(p_2) \]
Beyond leading twist: Chiral-odd pion GPDs beyond leading twist (up to 6)
Light-Cone Collinear Factorization

- The $p$, $\perp$, $n$ basis is natural for the twist expansion
- To implement $T$-invariance, the basis $P$, $\perp$, $n$ is more suitable
- We only consider 2- and 3-parton correlators
Beyond leading twist: Chiral-odd pion GPDs beyond leading twist (up to 6)
Light-Cone Collinear Factorization

Light-Cone Collinear Factorization

- Loop integrations:

  \[ H(\ell_i) = H(y_i p) + \frac{\partial H(\ell_i)}{\partial \ell_\alpha} \bigg|_{\ell_i = y_i p} (\ell_i - y p)_\alpha + \ldots . \]

  with \((\ell_i - y p)_\alpha = \ell_{i\alpha} + (\ell \cdot p)_n \alpha\)

- Taylor expansion of the hard part w.r.t. loop momenta \(\ell_i\)

- Using \(\int d^4 \ell_i = \int d^4 \ell_i \int dy_i \delta(y_i - \ell_i \cdot n)\) we integrate according to

  \[
  \int d^4 \ell_i = \int dy_i \times \int d(\ell_i \cdot n) \delta (y_i - \ell_i \cdot n) \times \int d^2 \ell_i \perp \times \int d(\ell_i \cdot p)
  \]

  \(\leftrightarrow\) fact. \(\leftrightarrow\) trivial \(\rightarrow\) soft-part

- Fourier transf. w.r.t. :
  - \(\ell_i \perp \) \(\Rightarrow\) non-local op. with \(\partial_\perp\) (e.g. \(\bar{\psi} \partial_\perp \psi\)) \(\Rightarrow\) correlators \(\Phi_\perp(l)\)
  - \((\ell \cdot p)n_\alpha \) \(\Rightarrow\) non-local op. with \(\partial_n^\gamma \equiv (\partial \cdot p)n^\gamma\) (e.g. \(\bar{\psi} \partial_n^\gamma \psi\)) \(\Rightarrow\) correl. \(\Phi_n(l)\)
Light-Cone Collinear Factorization

- For consistency, we stop at order 1: the $A$ field and the derivative should appear in a QCD gauge invariant way, through the covariant derivative

$$D_\mu = \partial_\mu - ig A_\mu(z).$$

- Here: number of gluons $\leq 1 \iff$ number of derivatives $\leq 1$

- Color + spinor factorization = Fierz transforms
Beyond leading twist: Chiral-odd pion GPDs beyond leading twist (up to 6)
Light-Cone Collinear Factorization

Parametrization of the non-local correlators

2-parton (with no derivative) non-local correlators

Based on $C, P, T$, this leads to the following set of 4 real GPDs:

$$\langle \pi^0(p_2)|\bar{\psi}(z) \left[ \begin{array}{c} \sigma^{\alpha\beta} \\ \frac{1}{i\gamma^5} \end{array} \right] \psi(-z)|\pi^0(p_2)\rangle = \int_{-1}^{1} dx \, e^{i(x-\xi)P.z + i(x+\xi)P.z} \times$$

$$\left[ -\frac{i}{m_{\pi}} \left( P^\alpha \Delta_\perp^\beta - P^\beta \Delta_\perp^\alpha \right) H_T + i m_{\pi} \left( P^\alpha n^\beta - P^\beta n^\alpha \right) H_{T3} - i m_{\pi} \left( \Delta_\perp^\alpha n^\beta - \Delta_\perp^\beta n^\alpha \right) H_{T4} \right]$$

\[ \begin{array}{ccc} 
\sigma^{\alpha\beta} & 1 & 0 \\
\frac{1}{i\gamma^5} & i m_{\pi} & m_{\pi} H_S \\
0 & m_{\pi} H_T & 0 \\
\end{array} \] 

twist 2 & 4 
twist 3 
twist 4
Parametrization of the non-local correlators

2-parton (with $\perp$ derivative) and 3-parton non-local correlators: $\sigma^{\alpha\beta}$ structure

Based on $C$, $P$, $T$, this leads to the following set of 12 real GPDs:

$$\langle \pi^0(p_2)|\bar{\psi}(z)\sigma^{\alpha\beta}\left\{i\frac{\partial^\gamma}{g A^\gamma(y)}\right\}\psi(-z)|\pi^0(p_1)\rangle = \int_{-1}^{1+\xi} dx e^{i(x-\xi)P\cdot z+i(x+\xi)P\cdot z}$$

$$\int d^3[x_1, 2, g] e^{iP\cdot z(x_1+\xi)-iP\cdot y x_g+iP\cdot z (x_2-\xi)}$$

$$\times \left[ i m_\pi \left( P^\alpha g^\beta g^\gamma_\perp - P^\beta g^\alpha g^\gamma_\perp \right) \left\{ \frac{T^T_1}{T_1} \right\} + \frac{i}{m_\pi} \left( P^\alpha \Delta^\beta_\perp - P^\beta \Delta^\alpha_\perp \right) \Delta^\gamma_\perp \left\{ \frac{T^T_2}{T_2} \right\} \right] \text{ (twist 3 & 5)}$$

$$+ i m_\pi \left( \Delta^\alpha_\perp g^\beta g^\gamma_\perp - \Delta^\beta_\perp g^\alpha g^\gamma_\perp \right) \left\{ \frac{T^T_3}{T_3} \right\} + i m_\pi \left( P^\alpha n^\beta - P^\beta n^\alpha \right) \Delta^\gamma_\perp \left\{ \frac{T^T_4}{T_4} \right\} \text{ (twist 4)}$$

$$+ i m_\pi^3 \left( n^\alpha g^\beta g^\gamma_\perp - n^\beta g^\alpha g^\gamma_\perp \right) \left\{ \frac{T^T_5}{T_5} \right\} + i m_\pi \left( n^\alpha \Delta^\beta_\perp - n^\beta \Delta^\alpha_\perp \right) \Delta^\gamma_\perp \left\{ \frac{T^T_6}{T_6} \right\}, \text{ (twist 5)}$$

$$\int d^3[x_1, 2, g] \equiv \int_{-1}^{1+\xi} dx_g \int_{-1}^{1} dx_1 \int_{-1}^{1} dx_2 \delta(x_g-x_2+x_1), \text{ and } \frac{\partial^\gamma_\perp}{2} \equiv \frac{1}{2} (\partial^\gamma_\perp - \partial^\gamma_\perp).$$

$$T^T_i \equiv T^T_i(x, \xi, t) \text{ and } T_i \equiv T_i(x_1, x_2, \xi, t) \ (i = 1, \cdots 6).$$
Beyond leading twist: Chiral-odd pion GPDs beyond leading twist (up to 6)
Light-Cone Collinear Factorization

Parametrization of the non-local correlators

2-parton (with \( \perp \) derivative) and 3-parton non-local correlators: \( \Box \) and \( i\gamma^5 \) structures

Based on \( C, P, T \), this leads to the following set of 4 real GPDs:

\[
\langle \pi^0(p_2) | \bar{\psi}(z) \Box \left\{ \frac{i}{g} \frac{\partial^\perp}{\partial^\perp} \right\} \psi(-z) | \pi^0(p_1) \rangle = \left\{ \begin{array}{l}
\frac{1}{-1} \int dx e^{i(x-\xi)P \cdot z + i(x+\xi)P \cdot z} \\
\int d^3[x_1, 2, g] e^{iP \cdot z(x_1+\xi) - iP \cdot y x_g + iP \cdot z(x_2-\xi)} \\
\times m_\pi \Delta^\perp \left\{ \frac{H^T_4}{T_4} \right\} .
\end{array} \right. \\
\text{(twist 4)}
\]

\[
\langle \pi^0(p_2) | \bar{\psi}(z) \Box \left\{ \frac{i}{g} \frac{\partial^\perp}{\partial^\perp} \right\} \psi(-z) | \pi^0(p_1) \rangle = \left\{ \begin{array}{l}
\frac{1}{-1} \int dx e^{i(x-\xi)P \cdot z + i(x+\xi)P \cdot z} \\
\int d^3[x_1, 2, g] e^{iP \cdot z(x_1+\xi) - iP \cdot y x_g + iP \cdot z(x_2-\xi)} \\
\times m_\pi \Delta^\perp \left\{ \frac{H^T}{T} \right\} .
\end{array} \right. \\
\text{(twist 4)}
\]
Beyond leading twist: Chiral-odd pion GPDs beyond leading twist (up to 6)
Light-Cone Collinear Factorization

Parametrization of the non-local correlators

2-parton (with long. derivative) and 3-parton non-local correlators: \( \sigma^{\alpha\beta} \) structure

Based on \( C, P, T \), this leads to the following set of 6 real GPDs:
\[
\langle \pi^0(p_2)|\bar{\psi}(z)\sigma^{\alpha\beta}\left\{ i \frac{\partial}{\partial_n^\gamma}, g A_n^\gamma(y) \right\}\psi(-z)|\pi^0(p_1) \rangle = \begin{cases} 
\int dxe^{i(x-\xi)P\cdot z + i(x+\xi)P\cdot z} \\
\int d^3[x_1, 2, g] e^{iP\cdot z(x_1+\xi)-iP\cdot y x_g+iP\cdot z (x_2-\xi)} 
\end{cases}
\]

\[ \times \left[ i m_\pi \left( P^\alpha \Delta^\beta_\perp - P^\beta \Delta^\alpha_\perp \right) n^\gamma \left\{ \frac{M^-_1}{M_1} \right\} \right. \] twist 4 & 6

\[ + i m_\pi^3 \left( P^\alpha n^\beta - P^\beta n^\alpha \right) n^\gamma \left\{ \frac{M^-_2}{M_2} \right\} \] twist 5

\[ + i m_\pi^3 \left( n^\alpha \Delta^\beta_\perp - n^\beta \Delta^\alpha_\perp \right) n^\gamma \left\{ \frac{M^-_3}{M_3} \right\} \], twist 6

\[
\int d^3[x_1, 2, g] \equiv \int_{-1+\xi}^{1+\xi} dx_g \int_{-1}^1 dx_1 \int_{-1}^1 dx_2 \, \delta(x_g-x_2+x_1), \quad \text{and} \quad \frac{\partial}{\partial_n^\gamma} \equiv \frac{1}{2}(\frac{\partial}{\partial_n^\gamma} - \frac{\partial}{\partial_n^\gamma}) \]

\[ M^-_i \equiv M^-_i(x, \xi, t) \quad \text{and} \quad M_i \equiv M_i(x_1, x_2, \xi, t) \quad (i = 1, \cdots 3). \]
Beyond leading twist: Chiral-odd pion GPDs beyond leading twist (up to 6)
Light-Cone Collinear Factorization

Parametrization of the non-local correlators

2-parton (with long. derivative) and 3-parton non-local correlators: $\mathbb{1}$ and $i\gamma^5$ structures

Based on $C, P, T$, this leads to the following set of 2 real GPDs:

\[
\langle \pi^0(p_2) | \bar{\psi}(z) \mathbb{1} \left\{ \frac{i}{g} \gamma^\mu \partial_n \right\} \psi(-z) | \pi^0(p_1) \rangle = \left\{ \begin{array}{l}
\int_{-1}^{1} dx e^{i(x-\xi)P \cdot z + i(x+\xi)P \cdot z} \\
\int d^3[x_1, 2, g] e^{iP \cdot z(x_1+\xi)-iP \cdot y x_g+iP \cdot z (x_2-\xi)}
\end{array} \right. \\
\times m_\pi^3 n^\gamma \left\{ \frac{H_S^-}{M_S} \right\}. \quad \text{(twist 5)}
\]

For the $i\gamma^5$ structure, we cannot define correlators with the needed parity:

\[
\langle \pi^0(p_2) | \bar{\psi}(z) i\gamma^5 \left\{ \frac{i}{g} \gamma^\mu \partial_n \right\} \psi(-z) | \pi^0(p_1) \rangle = 0.
\]
Beyond leading twist: Chiral-odd pion GPDs beyond leading twist (up to 6)
Light-Cone Collinear Factorization

Minimal set of GPDs

- Number of GPDs: a priori 28 up to twist 5
- Two constraints:
  - QCD equations of motion (EOM)
  - Arbitrariness of $p$ and $n$
Minimal set of GPDs: QCD equations of motion

Dirac equation in a covariant form (no inclusion of mass effects):

\[(i\not{D}\psi)_\alpha = 0 \quad \text{and} \quad (i\not{D}\bar{\psi})_\beta = 0\]

i.e. at correlator level:

\[
\langle \pi^0(p_2) | (i\not{D}\psi)\alpha(-z) \bar{\psi}_\beta(z) | \pi^0(p_1) \rangle = 0
\]

and

\[
\langle \pi^0(p_2) | \psi\alpha(-z) (i\not{D}\bar{\psi})_\beta(z) | \pi^0(p_1) \rangle = 0.
\]

\[\Rightarrow\] relations between various correlators

\[\Rightarrow\] 8 equations between GPDs.
Beyond leading twist: Chiral-odd pion GPDs beyond leading twist (up to 6)

Light-Cone Collinear Factorization

Minimal set of GPDs: $n$–independence

The implementation of $n$–independence is much more difficult here, in comparison with the case of the DAs (because of $\xi$). Under progress...
Conclusion

- **The transversity GPDs** are difficult to extract.
- In order to extract the quark transversity GPDs:
  - At twist 2, one may think of rather involved processes w.r.t. to usual DVCS or vector meson electroproduction, with 3 instead of 2 particles in the final state.
  - Another possibility is to consider vector meson electroproduction beyond leading twist.
  - This requires to classify the corresponding DAs and GPDs.
- For simplicity, we considered the $\pi^0$. 
  - In the light-cone collinear factorization framework, we introduced the relevant matrix element for:
    - 2-partons non-local correlators, with and without transverse and longitudinal derivatives.
    - 3-partons non-local correlators.
  - Their detailed parametrization is fixed by $C, P, T$.
  - This leads to the introduction of 28 real GPDs.
  - Their symmetry properties have been obtained.
  - Their reduction to a minimal set requires the use of
    - QCD equations of motions.
    - Implementation of the $n$—independence constraint.
  - The complete reduction to a minimal set is under process.
  - The next stage is to perform the same analysis for the nucleons and to use it for phenomenology.
SCHOOL: “Correlations between partons in nucleons”
ORSAY, LPT, June 30th - July 4th

https://indico.in2p3.fr/conferenceDisplay.py?ovw=True&confId=9917

- **Long lectures:**
  - Marco Stratmann, BNL (USA)
    Partons Distribution Functions and the LHC (6h)
  - Markus Diehl, DESY (Germany)
    Multi Parton Interactions (6h)
  - Cédric Lorcé, IPNO (France) and IFPA Liège (Belgium)
    Nucleon structure (4h)
  - Raju Venugopalan, BNL and Stony Brook University (USA)
    Color Glass Condensate (4h)
  - Leif Lönnblad, Lund Observatory (Sweden)
    Introduction to event generators physics (3h)
  - Abhay Deshpande, Stony Brook University (USA)
    The questions of Hadronic physics (3h)

- **Short lectures:**
  - Paolo Bartalini, CERN and Central China Normal University (China)
    CMS and ATLAS signals for MPI processes (1.5h)
  - Sarah Porteboeuf-Houssais, LPC Clermont Ferrand (France)
    ALICE signals for MPI processes (1.5h)
  - David Kosower, IPhT (France)
    Introduction to multi-gluons processes (1.5h)