Dipole amplitude with uncertainty estimate from HERA data and applications in Color Glass Condensate phenomenology

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Introduction

- CGC: QCD at high energies
- Saturation phenomena described by CGC
- Evolution in $x$ (energy): BK equation
- Saturation scale $Q_s = \text{characteristic momentum scale}$
CGC offers a consistent framework to describe small-$x$ data.

- rcBK equation gives energy (Bjorken $x$) evolution
- Non-perturbative input: dipole amplitude at $x = x_0$

**Compute**

- DIS
- Diffractive DIS
- Single inclusive hadron production in pp and pA
- Dihadron correlations, ...
Fit setup

Solve rcBK with modified MV model initial condition

\[ N_p(r, y = 0) = 1 - \exp\left[ -\frac{(r^2 Q_{sp}^2)^\gamma}{4} \ln \left( \frac{1}{r^{\Lambda_{QCD}}} + e_c \cdot e \right) \right], \]

\[ \alpha_s(r^2) = \frac{12\pi}{(33 - 2N_f) \ln \frac{4C^2}{r^2 \Lambda_{QCD}^2}}. \]

Compute \( \sigma_r \) (or \( F_2 \)), assume factorizable proton impact parameter profile

\[ \sigma_{\gamma^*p}^{T,L} = \sigma_0 \int dz |\Psi_{\gamma^*q\to q\bar{q}}^{T,L}|^2 N(r, y) \]

Fit \( Q_s^2, \sigma_0, C^2 \) and consider different parametrizations:

- MV with \( \gamma \equiv 1, e_c \equiv 1 \).
- MV\( ^\gamma \) with \( e_c \equiv 1 \), fit \( \gamma \) (AAMQS collaboration, arXiv:1012.4408).
- MV\( ^e \) with \( \gamma \equiv 1 \), fit \( e_c \) (New, T. Lappi, H.M, 1309.6963).
Fit result

Fit to HERA $\sigma_r$ data at $Q^2 < 50\text{ GeV}^2, x < 0.01$ (MV$^\gamma$ fit by AAMQS).
Only light quarks

<table>
<thead>
<tr>
<th></th>
<th>$\chi^2/N$</th>
<th>$Q_{s0}^2$ [GeV$^2$]</th>
<th>$Q_s^2$ [GeV$^2$]</th>
<th>$\gamma$</th>
<th>$C^2$</th>
<th>$e_c$</th>
<th>$\sigma_0/2$ [mb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MV</td>
<td>2.76</td>
<td>0.104</td>
<td>0.139</td>
<td>1</td>
<td>14.5</td>
<td>1</td>
<td>18.81</td>
</tr>
<tr>
<td>MV$^\gamma$</td>
<td>1.18</td>
<td>0.165</td>
<td>0.245</td>
<td>1.135</td>
<td>6.35</td>
<td>1</td>
<td>16.45</td>
</tr>
<tr>
<td>MV$^e$</td>
<td>1.15</td>
<td>0.060</td>
<td>0.238</td>
<td>1</td>
<td>7.2</td>
<td>18.9</td>
<td>16.36</td>
</tr>
</tbody>
</table>

Much better fit with modified MV models.

- Inclusion of heavy quarks: AAMQS takes separate $\sigma_{0,c}$, we do not want to introduce extra parameters $\Rightarrow \chi^2/N \sim 2$, work in progress...

- MV$^\gamma$: Fourier transform of $1 - N(r)$ ($\sim ugd$) negative at large $k$
- MV$^e$ without $\gamma$ is easier to generalize for nuclei.
Fourier transform of the ICs

- Thick lines: fundamental representation, thin lines: adjoint
- MV\textsuperscript{e}: smoother interpolation between saturation region and large-\(k\) power law behavior.
Fit result

Fitted to $Q^2 < 50 \text{ GeV}^2$, $x < 0.01$ HERA data (arXiv:0911.0884)
Fit parameters are correlated $\Rightarrow$ error sets for the dipole amplitude.
Example: $\chi^2$/d.o.f for the MV model, $Q_s^2$ and $C^2$ are correlated:
Uncertainty analysis

Hessian method: expand $\chi^2$ around the minimum $\Rightarrow$ Hessian matrix $H$

$$\chi^2 \approx \chi^2_0 + \sum_{ij} \delta a_i H_{ij} \delta a_j,$$

$H$ is computed by fitting a quadratic function

$$f(a_i, a_j) = \chi^2_0 + c_i (a_i - a^0_i)^2 + c_j (a_j - a^0_j)^2 + c_{ij} (a_i - a^0_i)(a_j - a^0_j)$$

- Diagonalize $H \Rightarrow$ obtain a basis in which parameters are uncorrelated
- In the uncorrelated basis compute error sets $S_k$ such that
  $$\chi^2(S_k) - \chi^2_0 = \text{const.}$$
- Evolve error sets using rcBK

Uncertainty of any quantity $X$ becomes

$$(\Delta X)^2 \approx \frac{1}{4} \sum_k [X(S^+_k) - X(S^-_k)]^2$$
Uncertainty analysis

Preliminary results

Dipole amplitude at initial $y = 0$ and at higher rapidities ($y = \ln \frac{1}{x}$)

$N(r, x)$

$\Delta \chi^2 \approx 35 \sim 5\sigma$

$y = 0$

$y = 2$

$y = 4$

$y = 6$

$r [\text{GeV}^{-1}]$
Single inclusive hadron production from CGC

$k_T$ factorization, gluon production:

$$\frac{d\sigma}{d^2p_Tdy} = \frac{2\alpha_s}{C_Fp_T^2} \int d^2q_T \frac{\varphi_1(q_T) \varphi_2(k_T - q_T)}{q_T^2 (k_T - q_T)^2}$$

- Convolute with PDF and FF $\Rightarrow$ hadron spectrum.
- $\varphi_i$: Unintegrated (transverse momentum dependent) gluon distribution of hadron $i$.
- $\varphi(k_T) \sim k_T^4 \int d^2re^{ik\cdot r}[1 - N(r)]$
Single inclusive hadron production from CGC

\[ \frac{d\sigma}{d^2p_T dy} = \frac{2\alpha_s}{C_F p_T^2} \int d^2q_T \frac{\varphi_{x_1}(q_T) \varphi_{x_2}(p_T - q_T)}{q_T^2 (p_T - q_T)^2} \]

Assuming that \(p_T \gg Q_s\) we get the hybrid formalism (Note: \(\varphi \sim \sigma_0/2 = \) proton DIS area).

\[ \frac{dN}{dy d^2p_T} = \frac{\sigma_0/2}{\sigma_{\text{inel}}} \frac{1}{(2\pi)^2} xg(x, Q^2) \tilde{S}(p_T) \]

\(xg\): is collinear factorization gluon distribution (computable from \(\tilde{S}\))

\(\tilde{S}\) is Fourier transform of \(1 - N(r)\) (adj. rep.).

**Normalization factor**

At RHIC (LHC) \((\sigma_0/2)/\sigma_{\text{inel}} \sim 0.4\) (0.3)
Single inclusive hadron production from CGC

MV model result is not consistent with the LHC data, modifications required.

**RHIC**

\[ p + p \rightarrow \pi^0/h^- + X, \sqrt{s} = 200 \text{ GeV}, K = 2.5 \]

**LHC**

\[ p + p \rightarrow \pi^0, h^\pm + X, \sqrt{s} = 7000 \text{ GeV}, K = 1 \]

Hybrid formalism, LO CTEQ&DSS

\[
\frac{dN}{d^2p_T dy} \text{ [1/GeV}^2]\]

\[ p_T \text{ [GeV]} \]

\[ |y| < 2.4 \]

\[ h^\pm \times 10 \]

\[ K = 1 \]

\[ y = 4 \pi^0 \times 0.01 \]

\[ y = 3.2 \pi^- \times 10 \]

\[ y = 3.3 \pi^0 \times 0.1 \]

\[ y = 3.8 \pi^0 \times 0.05 \]

\[ y = 4 \pi^0 \times 0.01 \]

\[ k_T \text{ factorization, LO DSS} \]
Initial condition for nuclei: Optical Glauber

\[ N_A(b, r) = 1 - \exp \left[ -A T_A(b) \frac{\sigma_0}{2} \left( \frac{r^2 Q_{sp}^2}{4} \right)^\gamma \ln \left( \frac{1}{\Lambda_{\text{QCD}} r + e_c \cdot e} \right) \right]. \]

No additional nuclear parameters

- \( \sigma_0 \) from DIS
- \( T_A \): standard Woods-Saxon

⇒ prediction for \( Q_s^2(b) \).

Note: average dipole cross section obtained by integrating over fluctuating nucleon positions.
From proton to nucleus $R_{pA}$

Nuclear suppression factor $R_{pA} = \frac{dN_{pA}}{N_{bind}dN_{pp}}$

Results consistent with ALICE data, get explicitly $R_{pA} \rightarrow 1$ at large $p_T$.

$p + Pb/p + p \rightarrow \pi^0 + X$, $\sqrt{s} = 5020 \text{ GeV}$, $\delta \chi^2 \approx 36$

Minimum bias, small dependence on initial condition.
Conclusions

- CGC offers a consistent framework to describe small-\(x\) observables, non-perturbative input = dipole amplitude at initial \(x_0\) required
- New initial condition MV\(^e\) fitted to HERA data
- Uncertainty estimates
- Generalization to pA collisions using only DIS data, agrees with the available \(R_{pA}\) data
BACKUPS
From proton to nucleus: $R_{pA}$, centrality dependence

Consistent with ALICE pA data ($k_T$ factorization, LO-DSS fragfun)

\[ p + Pb / p + p \rightarrow h^\pm + X, \sqrt{s} = 5020 \text{GeV} \]

MV, MV$^\gamma$ and MV$^e$ give \( \approx \) same $R_{pA}$
Energy dependence of $R_{pA}$

Midrapidity $R_{pA} \rightarrow 1$ at large $p_T$ independently of $\sqrt{s_{NN}}$.

$p + Pb / p + p \rightarrow \pi^0 + X$

$R_{pA}$ at $y = 0$ ($k_T$ factorization) and at $y = 4$ (hybrid formalism)
CGC vs pQCD (EPS09s)

CGC predicts faster centrality and especially rapidity evolution than EPS09s-NLO pQCD

$R_{pA}(p_T = 3 \text{ GeV}), \sqrt{s} = 5020 \text{ GeV}$

CGC: Hybrid formalism, LO-CTEQ PDF, LO DSS FF.
Impact parameter dependence of the saturation scale

\[ N(r^2 = 2/Q_s^2) = 1 - e^{-1/2} \]
$N(r^2 = 2/Q_s^2) = 1 - e^{-1/2}$

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure.png}
\caption{Dipole amplitude from HERA}
\end{figure}
$k_T$ factorization vs hybrid formalism

$p + p \rightarrow g + X, \sqrt{s} = 7000 \text{ GeV}$

![Graph showing the comparison between Hybrid/$k_T$-fact and CTEQ/UGD for different $p_T$ values.](image_url)
ALICE $p + Pb$ spectrum, $k_T$ factorization

$MV^\gamma$ and $MV^e$ model ICs work with ALICE data, MV does not.

$p + A \rightarrow h^\pm + X$, $\sqrt{s} = 5020$ GeV, $K = 1$

$k_T$ factorization, DATA: ALICE, 1210.4520
Rapidity dependence of $R_{pA}$: 0 – 20% vs 20 – 40%

Centrality dependence increases at forward rapidities ($y = 2, 4, 6$)

$p + Pb / p + p \rightarrow \pi^0 + X, \sqrt{s} = 5020$ GeV

Hybrid formalism, LO-CTEQ PDF, LO DSS FF.