Quark and gluon helicity flip GPDs and their cross channel analysis

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DIS 2014, 28 April -2 May 2014

Outline

1. Introduction
2. Double partial wave expansion: J^{PC} selection rules and $\mathrm{SO}(3)$ t-channel PW
3. Alternative method: effective vertices
4. $f_2(1270)$ meson exchange model for gluon helicity flip GPDs
5. Summary and Outlook

B. Pire, K. S., L. Szymanowski, S. Wallon, arXiv:1403.0803 [hep-ph] B. Pire, K. S., L. Szymanowski, S. Wallon, in preparation

Introduction

- Helicity non-flip GPDs H, E, $\tilde{H}$, $\tilde{E}$ studied in DVCS and HMP
- Much less is known on helicity flip (or transversity GPDs):

$$\hat{O}_T^{q\,+i} = \bar{\Psi}(-\lambda n/2) i\sigma^{+i} \Psi(\lambda n/2)$$
$$\hat{O}_T^{g\,+ij+} = \mathbb{S} G^{+i}(-\lambda n/2) G^{j+}(\lambda n/2)$$

- Chiral invariance: Collins and Diehl'99: quark transversity GPDs does not contribute to HMP at leading twist.
- However attempts to evade this "no-go" theorem: Enberg, Pire, Szymanowski'06: quark transversity GPDs through $\gamma^*_{\mathrm{L/T}} p \to \rho^0_{\mathrm{L}} \rho^+_{\mathrm{L/T}} n$.
- See also Goloskokov, Kroll'13.
- Case of gluon transversity GPDs: clean probe of gluon contents inside nucleon through DVCS (but not so easy to separate from observables).

Quark and gluon helicity flip GPDs

Belitsky and Mueller'00, Diehl'01

- Quark helicity flip GPDs

$$\frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ixP^+\lambda} \langle N(p')|\bar{\Psi}(-\lambda n/2) i\sigma^{+i}\Psi(\lambda n/2)|N(p)\rangle$$
$$= \frac{1}{2P^+}\bar{U}(p')\left[H_T^q i\sigma^{+i} + \tilde{H}_T^q \frac{P^+\Delta^i - \Delta^+ P^i}{m^2}\right.$$
$$\left. + E_T^q \frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2m} + \tilde{E}_T^q \frac{\gamma^+ P^i - P^+\gamma^i}{m}\right] U(p),$$

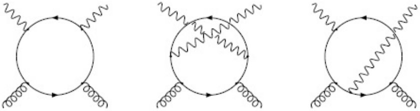
- Gluon helicity flip GPDs

$$\frac{1}{P^+}\int \frac{d\lambda}{2\pi} e^{ixP^+\lambda} \langle p'|\mathbb{S} G^{+i}(-\lambda n/2) G^{j+}(\lambda n/2)|p\rangle$$
$$= \mathbb{S}\,\frac{1}{2P^+}\frac{P^+\Delta^j - \Delta^+ P^j}{2mP^+}\bar{U}(p')\left[H_T^g i\sigma^{+i} + \tilde{H}_T^g \frac{P^+\Delta^i - \Delta^+ P^i}{m^2}\right.$$
$$\left. + E_T^g \frac{\gamma^+\Delta^i - \Delta^+\gamma^i}{2m} + \tilde{E}_T^g \frac{\gamma^+ P^i - P^+\gamma^i}{m}\right] U(p),$$

- H_T, E_T, $\tilde{H}_T$ are even functions of ξ; $\tilde{E}_T$ is odd (hermiticity + T-invariance).

More motivation: DVCS at NLO

- Belitsky, Mueller'00: gluon transversity GPDs contribution into DVCS at NLO separated by means of harmonic analysis.



One-loop diagrams for the coefficient function.

- *E.g.*: longitudinally (along and contrary to the lepton beam) polarized target cross section difference:

$$\frac{1}{2\pi}\int_0^{2\pi} d\phi \sin(3\phi)\frac{d\sigma_{\rightarrow}-d\sigma_{\leftarrow}}{d\phi} \sim \tilde{\mathcal{J}}\, d\mathcal{M}$$

$$\tilde{\mathcal{J}} = \frac{1}{2M^2}\mathrm{Im}\Big[F_2(\mathcal{H}_T^g + \frac{x}{2}(\mathcal{E}_T^g + 2\tilde{\mathcal{H}}_T^g + \tilde{\mathcal{E}}_T^g)) + F_1(x\tilde{\mathcal{H}}_T^g + \tilde{\mathcal{E}}_T^g)\Big],$$

where the corresponding CFFs are

$$\mathcal{F}_T^g = \frac{\alpha_s}{2\pi}\sum_i q_i^2 \int_{-1}^{1} dx \frac{F_T^g(x,\xi,\Delta^2)}{(x-\xi+i0)(x+\xi-i0)}.$$

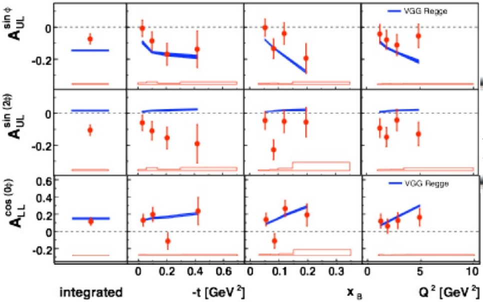
Pre-recoil data

[Nucl.Phys.B842 (2011) 265]

$$\sigma(\phi, P_z, P_l, e_l) = \sigma_{UU}(\phi, e_l)[1 + P_z \mathcal{A}_{UL}(\phi) + P_l P_z \mathcal{A}^I_{LL}(\phi)]$$

> No separate access to DVCS and Interference terms possible

$$\mathcal{A}_{UL}(\phi) \simeq \sum_{n=0}^{3} A_{UL}^{\sin(n\phi)} \sin(n\phi) \qquad \mathcal{A}_{LL}(\phi) = \sum_{n=0}^{2} A_{LL}^{\cos(n\phi)} \sin(n\phi)$$



$$A_{UL}^{\sin\phi} \propto \begin{cases} \text{DVCS : twist} - 3 \\ \text{I : twist} - 2 \end{cases}$$

$$A_{UL}^{\sin\phi} \propto F_1 \mathrm{Im}\widetilde{\mathcal{H}}$$

$$A_{UL}^{\sin 2\phi} \propto \begin{cases} \text{I : quark twist} - 3 \\ \quad \text{or gluon twist} - 2 \\ \text{DVCS : twist} - 4 \end{cases}$$

Unexpectedly large value

$$A_{LL}^{\cos 0\phi} \propto \begin{cases} \text{DVCS : twist} - 2 \\ \text{I : twist} - 2 \end{cases}$$

$$A_{LL}^{\cos 0\phi} \propto F_1 \mathrm{Re}\widetilde{\mathcal{H}}$$

Parametrizations for GPDs

- GPD modelling can be done in various representations: (DD representation, conformal PW expansions, ...)

List of non-trivial requirements:

- polynomiality
- hermiticity
- T-invariance
- positivity

Other sources of inspiration:

- evolution properties
- relation to PDFs and FFs
- analyticity
- Regge theory insight

- Should be possible to map one representation to another (as long as basic properties are satisfied).
- "Which representation is better is not a meaningful question!" (see K. Kumerički & D. Müller'09).
- The hope: get more insight from considering various GPD properties within different representations.

Conformal PW expansion for GPDs I

- Idea: expand GPDs over the conformal basis (factorization of functional dependencies)
- Main advantage: trivial solution of the LO evolution equations.

- Conformal moments of quark GPDs are defined with respect to $c_n(x,\xi) = N_n \times \xi^n C_n^{\frac{3}{2}}\left(\frac{x}{\xi}\right)$; Normalization: $\lim_{\xi\to 0} c_n(x,\xi) = x^n$.

$$m_n(\xi,t) = \int_{-1}^{1} dx\, c_n^{\frac{3}{2}}\left(\frac{x}{\xi}\right) H(x,\xi,t)\,.$$

- $c_n(x,\xi)$ form a complete basis in $[-\xi,\ \xi]$ with the weight $\left(1-\frac{x^2}{\xi^2}\right)$.
- $p_n(x,\xi)$ include the weight and θ to ensure the support:

$$p_n(x,\xi) = \xi^{-n-1}\theta\left(1-\frac{x^2}{\xi^2}\right)\left(1-\frac{x^2}{\xi^2}\right) N_n^{-1}\frac{(n+1)(n+2)}{2n+3} C_n^{\frac{3}{2}}\left(\frac{x}{\xi}\right)\,.$$

- Orthogonality of the basis: $$\int_{-1}^{1} dx\, p_n(x,\xi) c_n(x,\xi) = \delta_{mn}$$

Conformal PW expansion for GPDs II

Conformal PW expansion for GPDs:

$$H(x, \xi, t) = \sum_{n=0}^{\infty} p_n(x, \xi) m_n(\xi, t) .$$

- Allows to factorize x, ξ and t dependence of GPDs.
- Conformal moments are reproduced by this series.
- Restricted support property $\not\Rightarrow$ GPD vanishes in the outer region.
- The expansion is to be understand as an ill-defined sum of generalized functions.

Different ways to assign meaning to conformal PW expansion

1. Sommerfeld-Watson transform + Mellin-Barnes integral techniques D. Müller and A. Schäfer'05; A. Manashov, M. Kirch and A. Schafer'05;
2. - Shuvaev transform A. Shuvaev'99, J. Noritzsch'00;
   - Dual parametrization of GPDs M. Polyakov and A. Shuvaev'02;

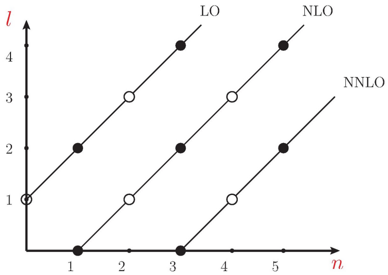
Dual Parametrization (M. Polyakov, A. Shuvaev'02):

- Mellin moments expanded in a set of suitable orthogonal polynomials. E.g. partial waves of the t-channel (t-channel refers to $\bar{h}h \to \gamma^*\gamma$):

$$N_n^{-1} \frac{(n+1)(n+2)}{2n+3} m_n(\xi, t) = \xi^{n+1} \sum_{l=0}^{n+1} B_{nl}(t) P_l\left(\frac{1}{\xi}\right)$$

Conformal PW expansion is then rewritten as:

$$H(x, \xi, t) = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \sum_{\substack{l=0 \\ \text{even}}}^{n+1} B_{nl}(t)\, \theta\left(1 - \frac{x^2}{\xi^2}\right) \left(1 - \frac{x^2}{\xi^2}\right) C_n^{\frac{3}{2}}\left(\frac{x}{\xi}\right) P_l\left(\frac{1}{\xi}\right)$$



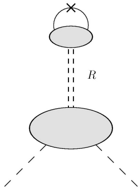
- Polynomiality implemented via Wigner-Ekkart theorem ($l \leq n+1$).
- Discrete symmetries (C, T) through the selection rules for l^{PC} (X. Ji, R. Lebed'01).
- Generalized FFs $B_{nl}(t)$ are renormalized multiplicatively.

t-channel point of view and duality

- Conformal PW expansion converges for $\xi > 1$.
- By means of the crossing relation one gets conformal PW expansion for two particle GDAs.

$$\frac{x}{\xi} \leftrightarrow 1-2z; \quad \frac{1}{\xi} \leftrightarrow 1-2\zeta; \quad t \leftrightarrow W^2$$

- Duality in the spirit of R. Dolen, D. Horn, C. Schmid'67. GPDs are presented as infinite series of t-channel Regge exchanges M. Polyakov'98:



$$\langle \pi(p')|\, \hat{O}\, |\pi(p)\rangle \sim \text{ Crossing of } \sum_{R_J} \sum_{\substack{\text{polarization}\\ \text{of } R_J}} \frac{1}{t - M_{R_J}^2}$$

$$\times \underbrace{\langle \pi(p')\pi(-p)|R_J\rangle}_{R_J\pi\pi \text{ effective vertex}} \underbrace{\langle R_J|\, \hat{O}\, |0\rangle}_{\text{F.T. of DA of } R_J} .$$

- Expansion in the t-channel PW:

$$\cos\theta_t = \frac{s-u}{\sqrt{1-\frac{4\,m^2}{t}}\,(\mathcal{Q}^2+t)} = \frac{1}{\xi\sqrt{1-\frac{4m^2}{t}}} + O(\frac{1}{\mathcal{Q}^2}),$$

Working out selection rules in J^{PC}

Ji and Lebed'01, Haegler'04, Chen and Ji'05:

- Main object: towers of twist-2 local operators

$$\hat{O}_T^{q\,\mu\nu,\,\mu_1\ldots\mu_N} = \mathbb{S}_{\{\nu\mu_1\ldots\mu_N\}}\bar{\Psi}(0)i\sigma^{\mu\nu}(i\overleftrightarrow{D}_{\mu_1})\ldots(i\overleftrightarrow{D}_{\mu_N})\Psi(0).$$
$$\hat{O}_T^{g\,\alpha\rho\,\beta\sigma\,\mu_1\ldots\mu_N} = \mathbb{S}_{\{\alpha\beta\mu_1\ldots\mu_N\}}G^{\alpha\rho}(0)(i\overleftrightarrow{D}_{\mu_1})\ldots(i\overleftrightarrow{D}_{\mu_N})G^{\beta\sigma}(0).$$

- Goal: implement P and T invariance for the parametrization of their nucleon matrix elements.
- Switch to t-channel and match J^{PC} to those of $\langle N\bar{N}|$
- Quark case: $\left(\frac{N+2}{2},\frac{N}{2}\right)\oplus\left(\frac{N}{2},\frac{N+2}{2}\right)$ representation of the Lorentz group.
- Both $P=1$ and $P=-1$ are possible.
- $\langle N\bar{N}|$: $\vec{J}=\vec{L}+\vec{S}$; $P=(-1)^{L+1}$ $C=(-1)^{L+S}$

$N\backslash J$	1	2	3	4	…	Number of FFs
0	$1^{--}_{0,2}$					2
1		$2^{++}_{1,3}$				2
2	$1^{--}_{0,2}$		$3^{--}_{2,4}$			4
3		$2^{++}_{1,3}$		$4^{++}_{3,5}$		4

$N\backslash J$	1	2	3	4	…	Number of FFs
0	1^{+-}_{1}					1
1	1^{++}_{1}	2^{-+}_{2}				2
2	1^{+-}_{1}	2^{--}_{2}	3^{+-}_{3}			3
3	1^{++}_{1}	2^{-+}_{2}	3^{++}_{3}	4^{-+}_{4}		4

Case of spin-$1/2$ hadrons

- Combinations of unpolarized GPDs suitable for $\mathrm{SO}(3)$ PW expansion:

$$H^{E\,\{q,g\}} = H^{\{q,g\}} + \frac{\Delta^2}{4m^2} E^{\{q,g\}}; \quad H^{M\,\{q,g\}} = H^{\{q,g\}} + E^{\{q,g\}}.$$

- To be expanded respectively in $P_J(\cos\theta)$ and $P'_J(\cos\theta)$.
- General method Deihl'03.

How it works for quark transversity I:

- Cross to the t-channel and consider the N-th Mellin moment of $N\bar{N}$ GDA

$$\langle N(p',\lambda')\bar{N}(\tilde{p},\lambda)|\hat{O}_T^{q\;+i,\;++\ldots+}|0\rangle$$

- Consider it in $\bar{N}N$ CMS for the helicities of nucleon and antinucleon couple to $\lambda' - \lambda = 0$ and $|\lambda' - \lambda| = 1$.
- Project out the combination with definite helicity $J_3 = \pm 1$ of the operator

$$\hat{O}_T^{q\;+1,\;++\ldots+} \pm i\hat{O}_T^{q\;+2,\;++\ldots+} \quad \Leftrightarrow \quad J_3 = \pm 1$$

- Compute explicitly to check it is to be expanded in the Wigner "small-d" rotation functions $d^J_{J_3,|\lambda'-\lambda|}$.

How it works for quark transversity II:

- Example:

$$\langle N(p',\lambda')\bar{N}(\tilde{p},\lambda)|\hat{O}_T^{q\,+(1+i2),\,++\ldots+}|0\rangle\Big|_{|\lambda'-\lambda|=1}$$

$$= \eta^{+}_{\lambda'\lambda}(\tilde{P}^{+})^{N+1}(1+\cos\theta)\Big\{\sum_{\substack{k=0\\ \text{even}}}^{N}\Big[\frac{2m}{\sqrt{\tilde{s}}}A^{q}_{T\,N+1,k}(\tilde{s})+\frac{\sqrt{\tilde{s}}}{2m}B^{q}_{T\,N+1,k}(\tilde{s})\Big]\Big(\frac{1}{2}\beta\cos\theta\Big)^{N-k}$$

$$-\sum_{\substack{k=0\\ \text{odd}}}^{N}\frac{\beta\sqrt{\tilde{s}}}{2m}\tilde{B}^{q}_{T\,N+1,k}(\tilde{s})\Big(\frac{1}{2}\beta\cos\theta\Big)^{N-k}\Big\}$$

- To be expanded in

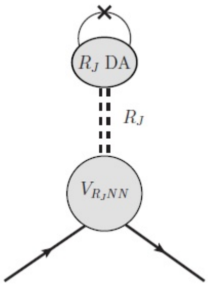
$$d^{J}_{1,1}(\theta)=\frac{1}{J(J+1)}(1+\cos\theta)\left[P'_J(\cos\theta)+\cos\theta P''_J(\cos\theta)-P''_J(\cos\theta)\right]$$

How it works for quark transversity III:

- Perform crossing back to the s-channel and conclude
  1. $$H_T^q(x,\xi,\Delta^2)+\frac{\Delta^2}{4m^2}\tilde{H}_T^q(x,\xi,\Delta^2)\pm\frac{\Delta^2}{4m^2}\tilde{E}_T^q(x,\xi,\Delta^2)$$
     is to be expanded in $P'_J(1/\xi)+\frac{1\mp\xi}{\xi}P''_J(1/\xi)$.
  2. $$\frac{\Delta^2}{4m^2}\tilde{H}_T^q(x,\xi,\Delta^2)-\frac{1}{2}E_T^q(x,\xi,\Delta^2);$$
     $$-H_T^q(x,\xi,\Delta^2)+\frac{\Delta^2}{4m^2}\tilde{H}_T^q(x,\xi,\Delta^2)-\frac{1}{2}E_T^q(x,\xi,\Delta^2).$$
     are to be expanded in $P'_J(1/\xi)$.
- J^{PC} selection rules of Ji and Lebed are respected.
- These fundamental combinations to be employed for the dual parametrization or with Mellin-Barnes approach of Mueller et al..
- More complicated results for gluon case. *C.f.* $d^J_{\pm 2,0}$, $d^J_{\pm 2,1}$, $d^J_{\pm 0,1}$.

Alternative way to proceed

- Explicit calculation of t-channel resonance exchange graphs



Ingredients

1. Minimal effective $R_J \bar{N} N$ vertices. *E.g. for* $P = (-1)^J$ *mesons*

$$V_{R_J NN} = \bar{U}(p')\left\{ \frac{g_1^{R_J}}{M_{R_J}^{J-1}} \gamma^{\mu_1} P^{\mu_2} ... P^{\mu_J} + \frac{g_2^{R_J}}{M_{R_J}^{J}} P^{\mu_1} ... P^{\mu_J} \right\} U(p)\, \mathcal{E}^*_{\mu_1 ... \mu_J}$$

2. Quark and helicity flip DAs for spin-J mesons.

- *E.g.* parametrization of gluon helicity flip DA for spin-J natural parity meson resonance:

$$
\begin{aligned}
&\langle 0|\mathbb{S} G^{+i}(-\lambda n/2) G^{j+}(\lambda n/2)|R_J(\Delta, j)\rangle \\
&= n^\alpha n^\beta \tau^{\perp}_{ij;\,\rho\sigma} \otimes\; f^{g}_{T\,R_J}\, M_R^{J-2} \left\{ \frac{1}{4}\Delta^\alpha \Delta^\beta \mathcal{E}^{\rho\sigma\nu_1\ldots\nu_{J-2}}(\Delta,j) - \frac{1}{4}\Delta^\rho \Delta^\beta \mathcal{E}^{\alpha\sigma\nu_1\ldots\nu_{J-2}}(\Delta,j) \right. \\
&\left. -\frac{1}{4}\Delta^\alpha \Delta^\sigma \mathcal{E}^{\rho\beta\nu_1\ldots\nu_{J-2}}(\Delta,j) + \frac{1}{4}\Delta^\rho \Delta^\sigma \mathcal{E}^{\alpha\beta\nu_1\ldots\nu_{J-2}}(\Delta,j) \right\} n^{\nu_1} \ldots . n^{\nu_{J-2}} \left(\frac{2}{(\Delta \cdot n)} \right)^{J-2} \\
&\times \int_{-1}^{1} dy e^{iy\lambda \frac{\Delta\cdot n}{2}}\, \Phi^{g}_{T\,R_J}(y)
\end{aligned}
$$

- DA $\Phi^{g}_{T\,R_J}(y)$ is expanded over the set of Gegenbauer polynomials:

$$
\Phi^{g}_{T\,R_J}(y) = \frac{3\; 2^J \Gamma\left(J+\frac{3}{2}\right)}{\Gamma(\frac{1}{2})\Gamma(J+3)} (1-y^2)^2 \sum_{\substack{k=J-2 \\ \text{even}}}^{\infty} (a^{g}_{T\,R_J})_k C_k^{\frac{5}{2}}(y) \;\; \text{with} \;\; (a^{g}_{T\,R_J})_{J-2} = 1
$$

- $f^{g}_{T\,R_J}$ is the normalization constant for $(J-2)$th Mellin moment.

A byproduct:

- $f_2(1270)$ exchange model for gluon helicity flip GPDs.
- Does not populate all structures (unnatural parity $\pi_2(1670)$ also needed)

$$
\begin{aligned}
H_T^g(x,\xi,\Delta^2)\Big|_{f_2(1270)} &= 0; \\
\tilde{H}_T^g(x,\xi,\Delta^2)\Big|_{f_2(1270)} &= \theta\left(1-\frac{x^2}{\xi^2}\right)\Phi_g^T\left(\frac{x}{\xi}\right)\frac{1}{|\xi|}(3\Upsilon_2(J=2,\Delta^2)); \\
E_T^g(x,\xi,\Delta^2)\Big|_{f_2(1270)} &= \theta\left(1-\frac{x^2}{\xi^2}\right)\Phi_g^T\left(\frac{x}{\xi}\right)\frac{1}{|\xi|}(12\Upsilon_1(J=2,\Delta^2)); \\
\tilde{E}_T^g(x,\xi,\Delta^2)\Big|_{f_2(1270)} &= 0.
\end{aligned}
$$

- For possible phenomenological applications we need the $N\bar{N}f_2$ couplings $g_{1,2}$ and the DA normalization constant f_g^T.

A tale of $f_2(1270)$ meson

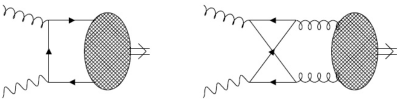
- V.M. dominance for nucleon e.m. FFs: ρ, $(\omega,\ \phi)$ isovector resp. isoscalar FF.
- Gamberg and Goldstein'01 $b_1(1235)$ exchange model for the 1st moment of quark transversity PDF. Tensor charge estimates consistent with Boffi et al.'06.
- $f_2(1270)$ contributes into $T^q_{\mu\nu}$ QCD energy-momentum tensor matrix element.

$$T^q_{\mu\nu} = \frac{1}{2}\bar{\Psi}\left(\gamma_\mu i \overleftrightarrow{D}_\nu + \gamma_\nu i \overleftrightarrow{D}_\mu\right)\Psi$$

- It describes M_2^q fraction of momentum carries by quarks
  1. pions $g_{f_2\pi\pi}$ from decay width: ok. $M_2^q|_{f_2(1270)} = 0.4$ (consistent e.g. with chiral quark model estimates)
  2. nucleons $g_{1,2}$ from NN scattering studies (see Dumbrajs'84): complete failure $M_2^q|_{f_2(1270)} = 0.02$.
- What is wrong with tensor meson dominance for $T^q_{\mu\nu}$ FFs?

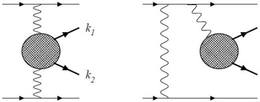
Gluon helicity flip DA normalization

- Braun and Kivel'01: hard exclusive production of tensor mesons ($q^2 = -Q^2$-large, $q'^2 = 0$)



Leading-order contributions to $\gamma^*(q) + \gamma(q') \to f_2(P)$ in the large-Q^2 limit

- 3 helicity amplitudes T_0, T_1, T_2.
- T_0 probes Φ^q and f_q ($f_q = 56$ MeV, QCD sum rules).
- T_2 receives contribution also from Φ^g_T
- Only measurements from f_2 decay into $\gamma\gamma$. ($Q^2 = 0$). $T_2(0) \gg T_0(0)$ f_g^T - big?
- Was suggested to measure $T_2(Q^2)$ from $ee \to ee\pi^+\pi^-$ for $(k_1 + k_2)^2 \sim m^2_{f_2}$



(a) (b)

Conclusions

1. Formal solution of the t-channel $\mathrm{SO}(3)$ partial wave expansion for quark and gluon helicity flip GPDs.
2. Universal result to be employed within the dual parametrization or the Mellin-Barnes approaches for GPD parametrization.
3. $f_2(1270)$ exchange model for gluon helicity flip GPDs. The best we can at the moment working out physical normalization.
4. Strongly encourage looking on higher Fourier components of the DVCS cross section.
5. More observables sensitive to gluon helicity flip GPDs with longitudinally and transversely polarized targets (under development).