ACAT-2014

A novel robust and efficient algorithm for charge particle tracking in high background flux

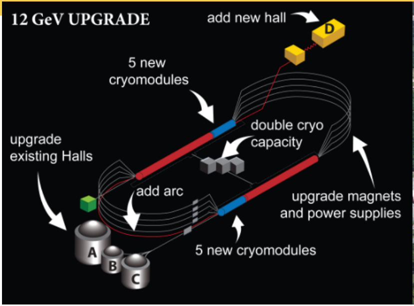
C. Fanelli[1,3], A. Del Dotto[2,3], E. Cisbani[3]

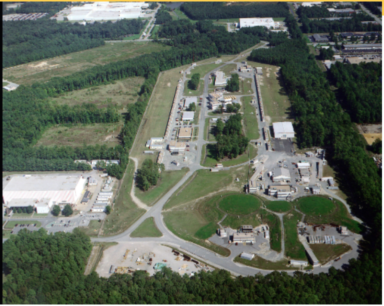
(1)Dipartimento di Fisica, Università "La Sapienza", Rome, Italy
(2)Dipartimento di Fisica, Università di Roma Tre, Rome, Italy
(3)INFN, Sezione di Roma, gruppo Sanità and Istituto Superiore di Sanità, I-00161 Rome, Italy

Outline

1. Introduction
   - JLab Facility
   - Front tracker in SBS
2. Tracking
   - Neural Network
   - Performance
3. Fitting
   - Kalman & RTS
   - Performance
4. Conclusions

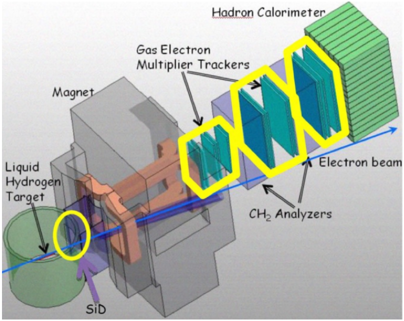
Résumé of JLab 12 GeV program





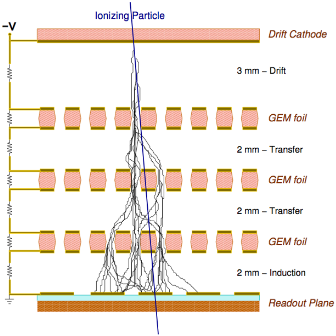
- JLab is a U.S. national laboratory (Newport News, Virginia).
- JLab is one of the world leading facility endowed with a continuous polarized electron beam delivered to the four experimental halls to investigate the nuclear and hadron physics.
- Starting 2014, doubled beam energy (from 6 to 12 GeV) with outstanding luminosity of 10^{39} evts/cm^2/s.
- Physics program: nucleon Form Factor, hadron structure, Parity measurements, Dark Matter, and many other hot physics topics.

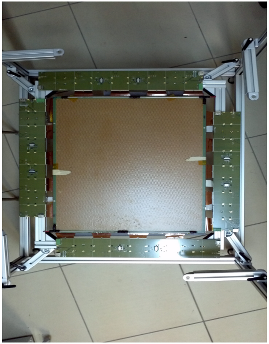
New tracking system based on GEM



- New detectors and targets have been developed to face the high luminosity and higher beam energy".
- A new hybrid tracker is under development for the new SBS spectrometer of hall A (figure) for experiments on Form Factors, Nucleon Structure· · ·
- The new tracker is made of: 6 large GEM chambers (40x150 cm^2) and 2 small (10x20 cm^2) silicon microstrip planes.

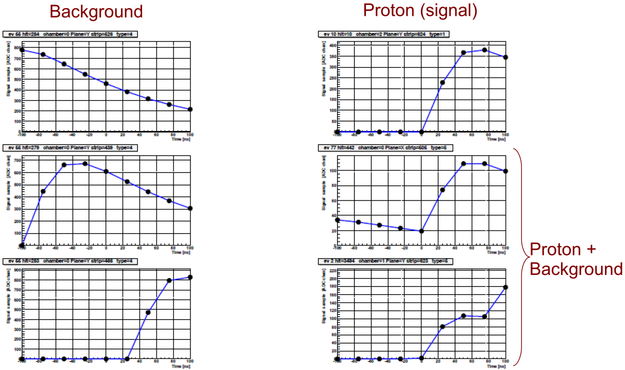
GEM (Gaseous Electron Multiplier) detectors





- Each GEM chamber is composed by 3 40 x 50 cm^2 individual modules.
- Each module has 3 GEM foils equipped with 2D strip readout.
- **Designed spatial resolution of 80 μm.**

Simulated signal



Expected 20 kHz of signal and 400 kHz/cm^2 of photon and charged particles background. Typical rate in a cone of 10 cm^2 on a single plane: 1 signal hit + 200 hits of background (mainly ghost hits arising from combinations of x and y strips).

Tracking: Description of the Approach

Methodology followed in this work

1. Take advantage of all the hardware (time and charge) information to minimize the number of hits to associate (see previous slide).
2. Employ Neural Network(NN) within a Mean Field Theory (MFT) framework to find charged particle tracks. MFT → reduce computational cost.
3. Utilize Kalman Filter for precise tracking.

Association with adaptive algorithms: Neural Network

Hopfield Network

In a NN each *neuron* has an input and gets one or more outputs.
Main ingredients:

- Neurons S_{ij}, with real activations, which can have binary 0 or 1 ($S_{ij} \rightarrow$ *connection between two points in subsequent GEM planes*);
- A matrix of synaptic weights representing their correlations T_{ijkl} ($T_{ijkl} \rightarrow$ *correlation between two subsequent connected segments*);

Idea: system of correlated neurons starting from an energy associated with each possible state. We chose an Hopfield system:

- $E(\vec{S}) = -\frac{1}{2} \sum_{ijkl} T_{ijkl} S_{ij} S_{kl}$
- updating rule: $S_{ij} = 1, \quad \text{if} \sum_{kl} T_{ijkl} S_{kl} > 0.$ (0, otherwise)

Recursive solution provides local minimum, for a global minimum necessary to add a noisy environment:
$\rightarrow$ Boltzmann machine $\rightarrow$ high computational cost.

Neural Network in Mean Field Approximation

Energy Function

$$E = -\frac{1}{2}\sum \delta_{jk}\frac{(\cos\theta_{ijl})^m}{r_{ij}+r_{jl}} S_{ij}S_{kl} + \frac{1}{2}\alpha\sum_{l\neq j} S_{ij}S_{il} + \frac{1}{2}\alpha\sum_{k\neq i} S_{ij}S_{kj} + \frac{1}{2}\beta\left(\sum S_{kl} - N\right)^2$$

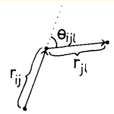
Mean Field Solution

$$S_{ij} \rightarrow V_{ij} \in [0,1]$$

$$V_{ij} = \frac{1}{2}\left[1 + \tanh\left(-\frac{\partial E(\mathbf{V})}{\partial V_{ij}}\frac{1}{T}\right)\right]$$

V are continuous value in $[0,1]$

Geometric Connections



- for(back)-ward planes
- consider hits within a reference cone (from interaction vertex (IV))

NN workflow

- Random initialization of V_{ij}
- Asynchronous updating cycle
- Stabilization of V_{ij} (Convergence and parameter settings study)
- Conversion in binary map S_{ij}
- Control with the true generated tracks

Modeling the problem with high background

1. Signal tracks come from IV in the target.
2. Background tracks (γ,e) not necessarily from IV and random (ghost) hits imply very large number of hits within a small cone.
3. Magnetic field negligible $\rightarrow$ tracks $\sim$ straight lines
4. Experimental arms endowed with 6 GEM planes at distance $\mathbf{d}_G$

Choice of Energy Function

$$E = \underbrace{-\sum_{ijl} \frac{d_G \cos\theta_{ijl}}{r_{ij} + r_{jl}} S_{ij} S_{jl}}_{cost} + \underbrace{\sum_{ijl} \frac{d_G \cos\theta_{ijl}\delta_{li} S_{ij}}{r_{ij} + r_{jl}}}_{firing\ \ threshold} + \underbrace{\alpha(\sum_{ijl \neq j} S_{ij} S_{il} + \sum_{ijk \neq j} S_{ij} S_{kj})}_{bifurcations}$$

Notes

- (1) (2) motivate the choice of the firing threshold.
- (3): Since **B** $\sim$0, we can apply an **affine scaling approach**.

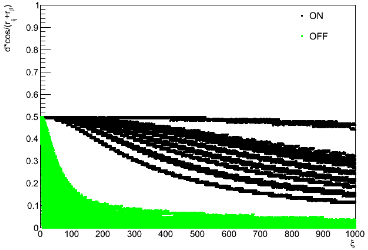
Affine scaling approach

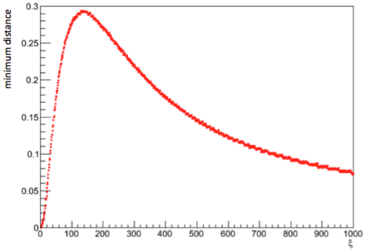
Affine ξ-scaling tansformation (2D case)

$$\hat{\mathcal{S}}_x(\xi)\frac{\cos\theta_{ijl}}{r_{ij}+r_{jl}} = \frac{\xi^2\Delta x_{ij}\Delta x_{jl}+\Delta z_{ij}\Delta z_{jl}}{\left(\sqrt{\xi^2\Delta x_{ij}^2+\Delta z_{ij}^2}+\sqrt{\xi^2\Delta x_{jl}^2+\Delta z_{jl}^2}\right)\sqrt{\xi^2\Delta x_{ij}^2+\Delta z_{ij}^2}\sqrt{\xi^2\Delta x_{jl}^2+\Delta z_{jl}^2}}$$

- Straight lines preserve linearity, only change in slope.
- Scaling scatters hits on GEM planes improving association.

$$V_{ij} \propto \tanh\left[\frac{1}{T}\left(\sum_l \hat{\mathcal{S}}_{xy}(\xi)\frac{d_G\cos^m\theta_{ijl}}{r_{ij}+r_{jl}} + \cdots - \alpha\left(\sum_{l\neq j} V_{il} + \sum_{k\neq i} V_{kj}\right)\right)\right]$$

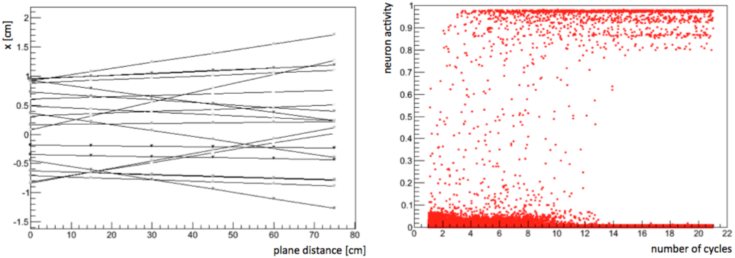




Evident region of optimal choice" around $\xi \sim$ 100-200

Testing on a toy model

- Consider 20 tracks (10 from IV) within a small cone 1x1 cm^2.



(left) Projection on the x-axis of the associated hits. (right) Neuron activity vs number of cycles, after 10 cycles neurons (connections) saturate to 1 (threshold 0.5).

Study of Network stability

NN parameter settings

- Through a digitalization procedure (considering solutions $\{S_{ij}\}$ with logic values) one can infere the following inequality:

$$\alpha|_{S_{ij}=0} \geq \frac{\sum_l \frac{d_G}{r_{ij}+r_{jl}} \cos^m \theta_{ijl} S_{jl} + \frac{d_G}{2r_{ij}} \delta_{i,n-1}\delta_{j,n}}{\sum_{l\neq j} S_{il} + \sum_{k\neq i} S_{kj}}$$

and then maximize the lower limit, $\forall\ i, j$.

- It is possible to introduce the following quantities:

$$C_{ij}^{off(on)} = \frac{\sum_l \frac{d_G}{r_{ij}+r_{jl}} \cos^m \theta_{ijl} S_{jl}}{\sum_{l\neq j} S_{il} + \sum_{k\neq i} S_{kj}}\Big|_{S_{ij}=0(1)}$$

$$F_{ij}^{off(on)} = \frac{\frac{d_G}{2r_{ij}} \delta_{i,n-1}\delta_{j,n}}{\sum_{l\neq j} S_{il} + \sum_{k\neq i} S_{kj}}\Big|_{S_{ij}=0(1)}$$

As a necessary condition for the NN convergence, the 'on' and 'off' distributions must be clearly separated and distinguishable.

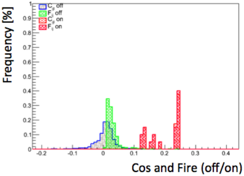
- The critical temperature T_c can be searched out a posteriori looking at a transition in energy. Then choose $T \leq T_c$ to saturate neurons.

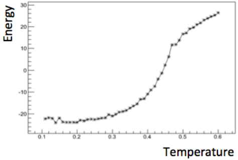
- **NN must converge to a minimum in energy E (Lyapunov).**
- **An intrinsic proportionality $\alpha \propto T_c$ is expected.**

Study of Network stability

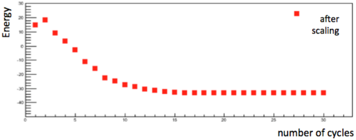
NN parameter settings

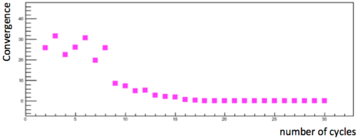
The variables defined in the previous slide are well distinguished in the two cases 'off' and 'on' (red)





Searching out a posteriori the critical temperature: plateau energy (after 20 cycles) as a function of the temperature.



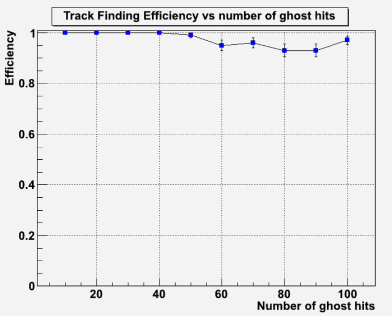


Energy (up) and Convergence (down: variation of the mean neuron activity $\frac{1}{N}\sum_{ij}|\frac{V_{ij}(n+1)-V_{ij}(n)}{V_{ij}}|$) as a function of the number of cycles.

A more realistic case

Background estimates in 5 cm^2 (1^{st} plane)

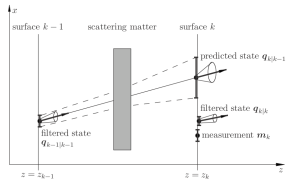
1. 1 out of vertex charged track
2. 1 photon / plane
3. (1)+(2)+charged track from IV, considering the GEM active area $\rightarrow \sim$ 100 ghost hits



As for the Efficiency, tracks with at least 5 hits correctly associated are considered.

Tracks filtering and smoothing

Kalman filter



Physical track: $\vec{x}_k = \mathbf{\Phi}_{k-1}\vec{x}_{k-1} + \vec{e}_{k-1}$
Measurement: $\vec{y}_k = \mathbf{C}_k\vec{x}_k + \vec{n}_k$

1. The covariance matrix of the posterior probability of $\mathbf{x}_k$
$$\tilde{\mathbf{P}}_k = \hat{\mathbf{P}}_k - \hat{\mathbf{P}}_k\hat{\mathbf{C}}_k^{\mathrm{T}}(\hat{\mathbf{\Gamma}}_k + \hat{\mathbf{C}}_k\hat{\mathbf{P}}_k\hat{\mathbf{C}}_k^{\mathrm{T}})^{-1}\hat{\mathbf{C}}_k\hat{\mathbf{P}}_k$$
2. The *Kalman gain matrix*
$$\hat{\mathbf{K}}_k = \tilde{\mathbf{P}}_k\hat{\mathbf{C}}_k^{\mathrm{T}}\hat{\mathbf{\Gamma}}_k^{-1}$$
3. The Kalman filtered state (the mean of the posterior probability)
$$\tilde{\mathbf{x}}_k = \bar{\mathbf{x}}_k + \hat{\mathbf{K}}_k(\mathbf{y}_k - \hat{\mathbf{C}}_k\bar{\mathbf{x}}_k)$$
4. The covariance matrix of the prior probability of $\mathbf{x}_{k+1}$
$$\hat{\mathbf{P}}_{k+1} = \hat{\mathbf{\Phi}}_k\tilde{\mathbf{P}}_k\hat{\mathbf{\Phi}}_k^{\mathrm{T}} + \hat{\mathbf{S}}_k$$
5. The mean of the prior probability for the state $k+1$
$$\bar{\mathbf{x}}_{k+1} = \hat{\mathbf{\Phi}}_k\tilde{\mathbf{x}}_k + \mathbf{b}_k$$

Definitions

$$\mathbf{x} = \begin{pmatrix} \text{x position} \\ \text{y position} \\ \text{x slope} \\ \text{y slope} \end{pmatrix} \qquad \mathbf{y} = \begin{pmatrix} \text{x strip number} \\ \text{y strip number} \end{pmatrix} \qquad \hat{\mathbf{C}} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Evolution matrix and definition of the true-value initial covariance matrix

$$\hat{\mathbf{\Phi}} = \begin{pmatrix} 1 & 0 & \Delta z & 0 \\ 0 & 1 & 0 & \Delta z \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

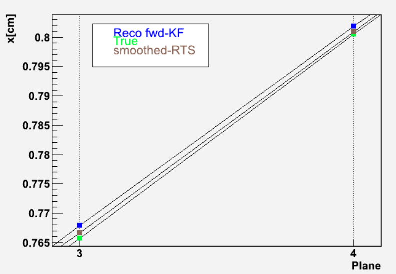
$$\hat{\mathbf{P}} = \begin{pmatrix} Var[x] & 0 & 0 & 0 \\ 0 & Var[y] & 0 & 0 \\ 0 & 0 & Var[\Delta x] & 0 \\ 0 & 0 & 0 & Var[\Delta y] \end{pmatrix}$$

RTS filter

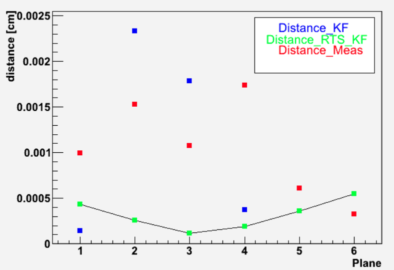
Rauh-Tung-Striebel is backward, applied after KF.

Preliminary results from KF+RTS filters

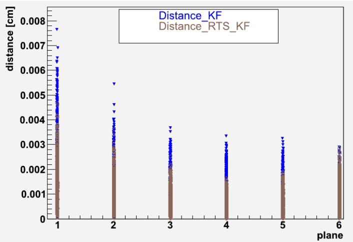
A typical associated track projected in the x-coordinate, and zoomed in the central planes 3 and 4: the true, Kalman filtered, RTS smoothed hits, the latter closer to true hits.

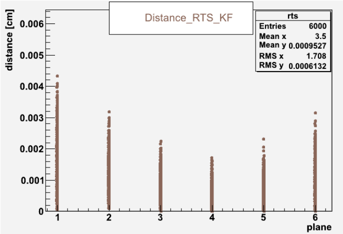


Relative to the same found track, the distance between its true hits and the GEM measured, Kalman filtered, and RTS smoothed hits.



Estimate of accuracy





- distance between KF filtered and true hits
- distance between KF+RTS filtered and true hits

The accuracy of the combined filters method is $\sim$ 10 μm

Conclusions

1. NN+Mean Field seems to be a good approach for solving the association of signal hits into tracks in a high background condition:
   → The introduced affine scaling has revealed very effective
2. Methods for accurate tuning of the NN parameters have been developed:
   →Promising results in terms of efficiency have been shown.
3. KF + RTS good candidate for precise fitting tracks:
   → High accuracy obtained in rather realistic conditions.
4. Work in progress:
   - Comparison NN chi-square algorithm.
   - Add multiple scattering in KF.
   - Study and optimization of computation time.
   - Realistic simulation (GEANT4) underway.
   - Application to real data very soon.