Six-loop calculations of the critical exponents in the ϕ^4 theory

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Outline

- R'-operation and *IRR* (Infrared Rearrangement)
- *IRR* limitations and overcoming its by R^*-operation
- definition of R^*-operation and *IR*-counterterms
- application to 6-loop ε-expansion in ϕ^4 model
- results, discussion

$$S(\varphi) = -\int d\mathbf{x} \left(\frac{1}{2}\tau\varphi(\mathbf{x})^2 + \frac{1}{2}(\vec{\nabla}\varphi(\mathbf{x}))^2 + \frac{1}{24}g(\varphi(\mathbf{x})^2)^2 \right)$$

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- This work: critical exponent η at 6-loop level

Bogoliubov–Parasyuk R' operation

Bogoliubov–Parasyuk R-operation (and incomplete R'-operation) plays significant role in multiloop calculations.

$$\delta Z = \Delta_{UV}(\gamma) = -KR'(\gamma), \quad R'(\gamma) = \gamma + \sum_{\delta \text{ is UV}} \Delta_{UV}(\delta)\gamma/\delta$$

where K is subtraction operator, for MS-scheme

$$K\left(\sum_{k=-\infty}^{\infty} a_k \varepsilon^k\right) = \sum_{k=-\infty}^{-1} a_k \varepsilon^k$$

$$d = 4 - 2\varepsilon$$

and R'-operation subtracts all *proper* UV subdiveregences.

Infrared rearrangement

counterterm $\Delta_{UV}(\Gamma) = -KR'(\Gamma)$ of the graph Γ is **polynomial on masses and external momentums**[1].

For logarithmically divergent graphs this allows one to perform infrared rearrangement:

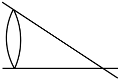
- to set some masses and external moments to zero
- or to introduce new external momenta or masses

All these operations can be done if they are does not produce unphysical **infrared divergences** and result doesn't depend on procedure details.[2]

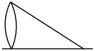
[1] **Collins J, 1975** *Nucl. Phys. B92, 477*

[2] **Vladimirov A A**, 1978 *Theor. Math. Phys.* **36**, 732

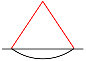
IR-divergences



Two momentum rearrangements:



$$\sim \int \frac{1}{k_1^2} \frac{1}{(q-k_1)^2} \frac{1}{k_2^2} \frac{1}{(k_1-k_2)^2} dk$$



$$\sim \int \frac{1}{k_1^2} \left(\frac{1}{k_2^2}\right)^2 \frac{1}{(q-k_1-k_2)^2} dk$$

Asymptotic of second integrand for $k_2 \sim 0$:

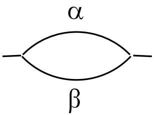
$$\frac{dk_2}{k_2^4} \sim \frac{|k_2|^{3-2\varepsilon} d|k_2|}{|k_2|^4} = |k_2|^{-1-2\varepsilon} d|k_2|$$

G-functions reduction

Limited class of massless diagrams with one external momenta can be evaluated analytically using

$$\frac{1}{(2\pi)^d}\int dk \frac{1}{k^{2\alpha}(k-p)^{2\beta}} = \frac{1}{p^{2(\alpha+\beta-\lambda-1)}} G(\alpha,\beta)$$

where $\lambda = d/2-1,\ G(\alpha,\beta) = \frac{\Gamma(\lambda+1-\alpha)\Gamma(\lambda+1-\beta)\Gamma(\alpha+\beta-\lambda-1)}{(4\pi)^{\lambda+1}\Gamma(\alpha)\Gamma(\beta)\Gamma(2\lambda+2-\alpha-\beta)}$



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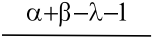

$G(\alpha,\beta) = \frac{1}{p^{2(\alpha+\beta-\lambda-1)}} G(\alpha,\beta)$

Integration by parts reduction

One of the most powerful analytical method for evaluation of diagrams depending on one external momentum (p-integral) is IBP (Integration by Parts) reduction. **IBP reduction** is the set of rules to express diagram value through the values of some predefined diagrams, calls master integrals. This procedure can be fully **automated**

We use **LiteRed** program for generating IBP and DRR[3] rules for reduction[4]. All 4-loop masters we need are available from[5] and from[6].

[3]Dimensional recurrence relations

[4]R. Lee, LiteRed, `http://www.inp.nsk.su/~lee/programs/LiteRed/`

[5]Baikov, P. A. and Chetyrkin K. G., 2010 *Nucl. Phys. B837, 186*

[6]Lee, R. N., Smirnov, A. V. and Smirnov, V. A. 2011 *Nucl. Phys. B856, 95*

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But with *IPB* reduction + *G*-function reduction and R' + *IRR* can be calculated **only 18** *UV*-counterterms of $6-$loop propagator diagrams **from 50** in ϕ^4.

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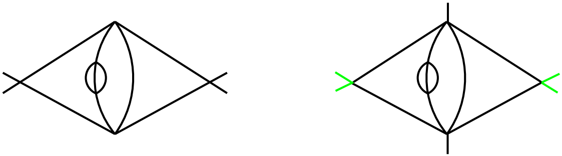
Overcoming of *IRR* restrictions using R^*



No way to calculate left diagram using R' and 4-loop *IBP* reduction.

Right diagram calculable with G-functions (no reduction required!) but with such rearrangement IR divergence arise, and to get correct result we need to use $R^{*\prime}$ operation to subtract this divergences.

R^*-operation historical overview

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  - **K.G. Chetyrkin, S.G. Gorishny, S.A. Larin and F.V. Tkachov** 1986 *Preprint* INR P-0453, Moscow.
- We employ a version of R^* operation as developed in **K.G. Chetyrkin**, *MPI-Ph/PTh*, 1991

$R^{*\prime}$ definition

$$R^{*\prime}(\Gamma) = \widetilde{R}' R'(\Gamma)$$

where R' is incomplete R-operation removing divergences in subgraphs. $\widetilde{R}'$ is IR-counterpart of R', it removes $IR-$divergences arising at choosing the wrong momentum rearrangement in IRR point of view.

$$\widetilde{R}'(\Gamma) = \Gamma + \sum_{\gamma \in W_u(\Gamma)} \Delta_{IR}(\Gamma/\gamma)\gamma, \text{ if } \Gamma \text{ is not 0-tadpole}$$

$$\widetilde{R}'(\Gamma) = \Delta_{IR}(\Gamma), \text{ if } \Gamma \text{ is 0-tadpole,}$$

Here **by 0-tadpole we denote massless tadpole**, $W_u(\Gamma)$ - set of all UV-divergent subgraphs of Γ through which external momentum of Γ can be completely passed and subgraph doesn't contain tadpoles.

$$\boxed{\Delta_{UV}(\Gamma) = -KR^{*\prime}(\Gamma)}$$

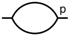
Recursive calculation of IR-counterterms

Let Γ is 0-tadpole:

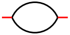
$$\Delta_{UV}(\Gamma) = -K\widetilde{R}'R'(\Gamma) = -K\Delta_{IR}R'(\Gamma)$$

$$\boxed{\Delta_{UV}(\Gamma) + \Delta_{IR}R'(\Gamma) = 0}$$

Physical sense:

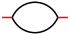
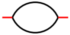


has no IR, only UV divergences

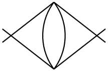
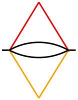
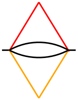


has IR and UV divergences but diagram identically equals zero

Hence

$\Delta_{UV}\Big($  $\Big) + \Delta_{IR}\Big($  $\Big) = 0$

Example 1

$\Delta_{UV}\Big($  $\Big) = -KR^{*\prime}\Big($  $\Big) = -K\widetilde{R}'R'\Big($  $\Big)$

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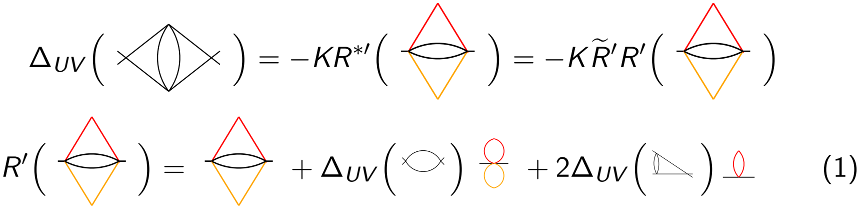

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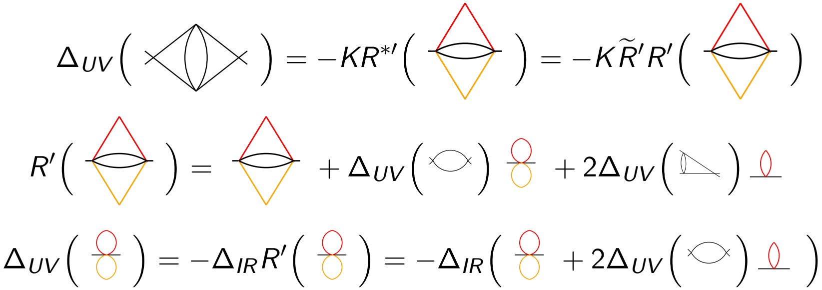
Δ_{UV} ( ) $= -KR^{*\prime}$ ( ) $= -K\widetilde{R}' R'$ ( )
R' ( ) $=$ $+\,\Delta_{UV}$ ( ) $+\,2\Delta_{UV}$ ( )
Δ_{UV} ( ) $= -\Delta_{IR} R'$ ( ) $= -\Delta_{IR}$ ($+\,2\Delta_{UV}$ ( ))

(1)

(2)

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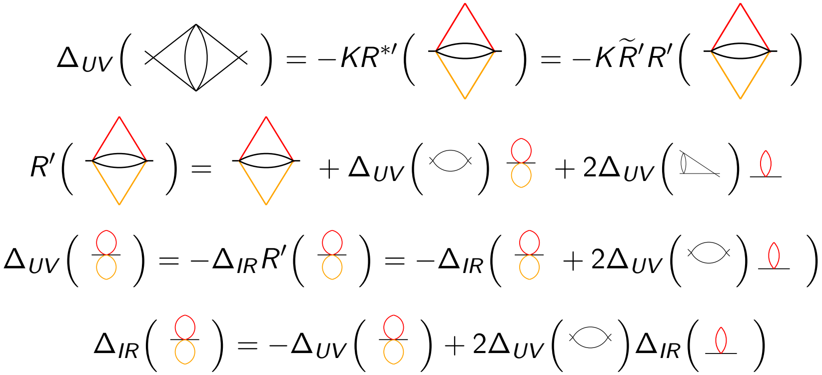

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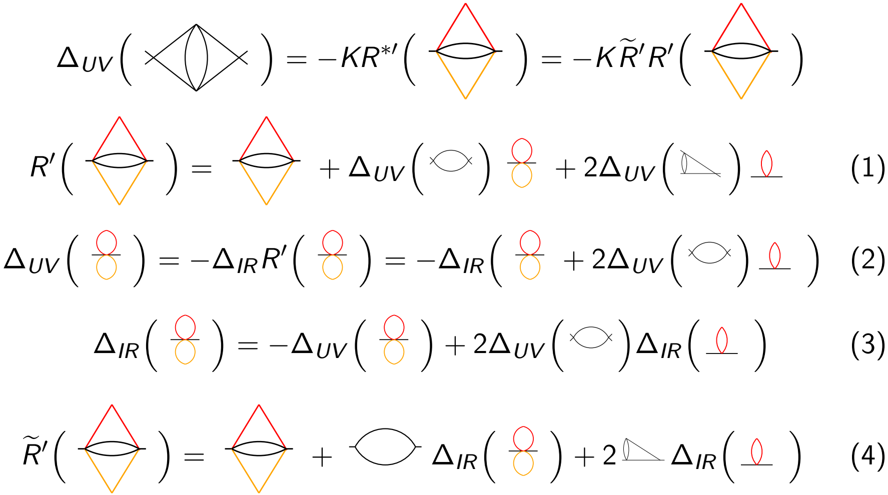


$\Delta_{UV}(\ ) = -KR^{*\prime}(\ ) = -K\widetilde{R}'R'(\ )$
$R'(\ ) = \ + \Delta_{UV}(\ )\ + 2\Delta_{UV}(\ )$ (1)
$\Delta_{UV}(\ ) = -\Delta_{IR}R'(\ ) = -\Delta_{IR}(\ + 2\Delta_{UV}(\ )\ )$ (2)
$\Delta_{IR}(\ ) = -\Delta_{UV}(\ ) + 2\Delta_{UV}(\ )\Delta_{IR}(\ )$ (3)
$\widetilde{R}'(\ ) = \ + \ \Delta_{IR}(\ ) + 2\ \Delta_{IR}(\ )$ (4)

Six-loop diagrams overview

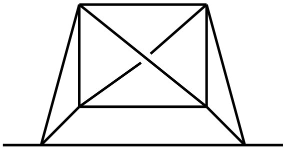
6-loop diagrams of propagator and vertex type 1-PI Green functions:

	total	factorized	primitive	4-loop reducible	4-loop irreducible
Γ_2	50	0	0	48	2
Γ_4	627	124	10	481	12

Γ_2 have been calculated. Γ_4 in progress now.

48 of 50 Γ_2-diagrams can be easily calculated using $R^{*\prime}$-operation and 4-loop IBP reduction, the last 2 requires some additional tricks

4-loop irreducible self-energy diagrams



Calculated using **reconstruction of pole part** of the 5-loop graph from known UV-counterterm.

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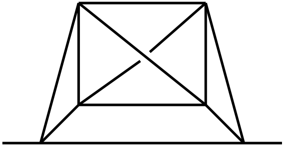

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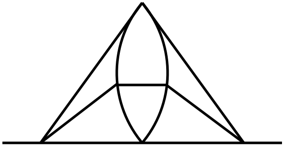


Calculated using **dual graphs**.

Calculation of graph pole part

Proposition

Consider graph Γ with $N-$loops, then its value up to $\mathcal{O}(\varepsilon^0)$ can be expressed through

- $\Delta_{UV}(\Gamma)$
- Δ_{UV} *and values of diagrams with loops count less then* N*.*

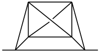
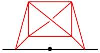
Proof.

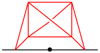
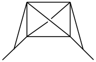
$$\Delta_{UV}(\Gamma) = -KR^{*\prime}(\Gamma) = -K(\Gamma) + (K(\Gamma) - KR^{*\prime}(\Gamma)) = -K(\Gamma) + \widehat{\Delta}_{UV}(\Gamma)$$

$$\Gamma = K(\Gamma) + \mathcal{O}(\varepsilon^0) = -\Delta_{UV}(\Gamma) + \widehat{\Delta}_{UV}(\Gamma) + \mathcal{O}(\varepsilon^0)$$

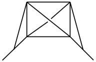
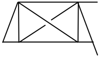
□

Calculation of graph pole part

$\Delta_{UV}\Big(\frac{1}{2}\partial^2 p$  $\Big) = \Delta_{UV}\Big(\frac{4-d}{d}$  $\Big)$

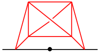
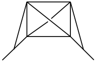
$K\frac{4-d}{d}$  $= K\frac{-\varepsilon}{2-\varepsilon}G(2, 5-5\lambda)$ 

$$\frac{-\varepsilon}{2-\varepsilon}G(2, 5-5\lambda) \sim \text{Const} + \mathcal{O}(\varepsilon),$$

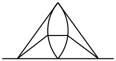
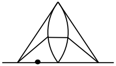
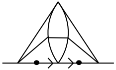
$\Delta_{UV}\Big($  $\Big) = -KR^{*\prime}\Big($  $\Big)$

where $\lambda = d/2 - 1$.

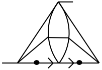
Hence

$K\frac{4-d}{d}$  $= K\frac{-\varepsilon}{2-\varepsilon}G(2, 5-5\lambda)K\Big($  $\Big)$

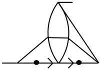
Calculation via dual diagram

$\Delta_{UV}\left(\frac{1}{2}\partial^2 p\right.$  $\left.\right) = \Delta_{UV}\left(2\frac{4-d}{d}\right.$  $\left.\right) + \Delta_{UV}\left(2\frac{4}{d}\right.$  $\left.\right)$

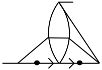
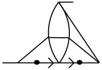
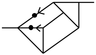
For 2nd contribution it is possible to rearrange external momenta in the following way:



Now to get the value we need to calculate this (primitive) graph:



It is possible to calculate this graph by the following well known trick
that graph in x-space is equal to dual graph in p-space

$\Big($  $\Big)_{\text{p-space}} = C \cdot \Big($  $\Big)_{\text{x-space}} = C \cdot \Big($  $\Big)_{\text{p-space}}$

Additional simplification comes from the fact that this graph is primitive
and we need only leading term from this graph so we can put all line indices equal to 1
and calculate this graph using 4-loop reduction

Results

Six-loops critical exponent η expansion, $N = 1$:

$$
\begin{aligned}
\eta &= \frac{2}{27}\varepsilon^2 + \frac{109}{729}\varepsilon^3 + \left(\frac{7217}{39366} - \frac{64}{243}\zeta_3\right)\varepsilon^4 \\
&+ \left(\frac{321511}{2125764} - \frac{16}{3645}\pi^4 - \frac{1316}{2187}\zeta_3 + \frac{1280}{729}\zeta_5\right)\varepsilon^5 \\
&+ \left(\frac{3421613}{38263752} - \frac{3136}{243}\zeta_7 + \frac{73232}{19683}\zeta_5 - \frac{181462}{177147}\zeta_3 + \frac{640}{137781}\pi^6\right. \\
&+ \left.\frac{2432}{2187}\zeta_3^2 - \frac{329}{32805}\pi^4\right)\varepsilon^6 + \mathcal{O}(\varepsilon^7)
\end{aligned}
$$

Checking the result

- each graph have calculated by all possible ways using R', $R^{*\prime}$ with all possible rearrangements and different levels of reduction

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Checking the result

- each graph have calculated by all possible ways using R', $R^{*'}$ with all possible rearrangements and different levels of reduction
- there is no $log(p)$ in Δ_{UV} and Δ_{IR}
- some of diagrams calculated numerically with Sector Decomposition
- checking through the known $1/N$-expansion of η (up to $1/N^3$ for arbitrary ε)

$1/N$-expansion critical exponent η

Vasiliev, Pis'mak and Honkonen, *Theor. Math. Phys.*, 50,N 2, p127 (1982)[7]:

$$\eta_N = \frac{\eta_1}{N} + \frac{\eta_2}{N^2} + \frac{\eta_3}{N^3} + \mathcal{O}\left(\frac{1}{N^4}\right) \quad \text{for any } \varepsilon,$$

We can expand η_N by ε. But we know η_ε up to 6-loops for any N.

$$\boxed{\eta_N \text{ expansion by } \varepsilon = \eta_\varepsilon \text{ expansion by } 1/N}$$

[7] there was a missprint in the paper, correct answer can be found in **Vasilev A. N.**, Quantum Field Renormalization Group in Critical Behavior Theory and Stochastic Dynamics, 2004

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All of 6-loops self-energy diagrams Δ_{UV} give contribution to η_3.

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We have indeed found FULL agreement to this 22 years old prediction!

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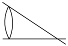
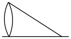
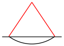
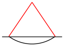
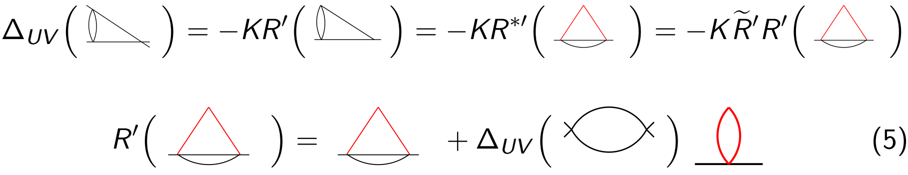
Thank you for your attention!

Backup

Example 0

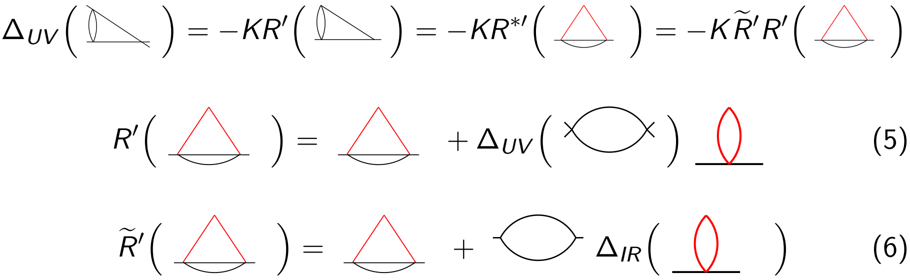
$\Delta_{UV}\Big($  $\Big) = -KR'\Big($  $\Big) = -KR^{*\prime}\Big($  $\Big) = -K\widetilde{R}'R'\Big($  $\Big)$

Example 0



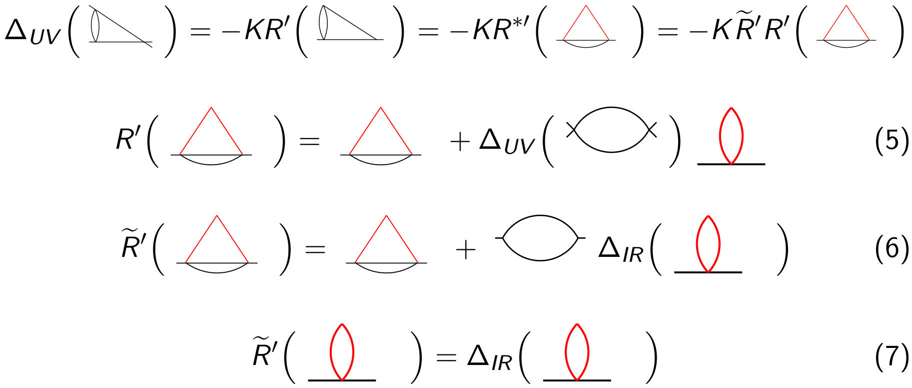
(5)

Example 0



$\Delta_{UV}(\;) = -KR'(\;) = -KR^{*\prime}(\;) = -K\widetilde{R}'R'(\;)$
$R'(\;) = \; + \Delta_{UV}(\;)$ (5)
$\widetilde{R}'(\;) = \; + \; \Delta_{IR}(\;)$ (6)

Example 0



Δ_{UV} ( ) = $-KR'$ ( ) = $-KR^{*\prime}$ ( ) = $-K\widetilde{R}'R'$ ( )
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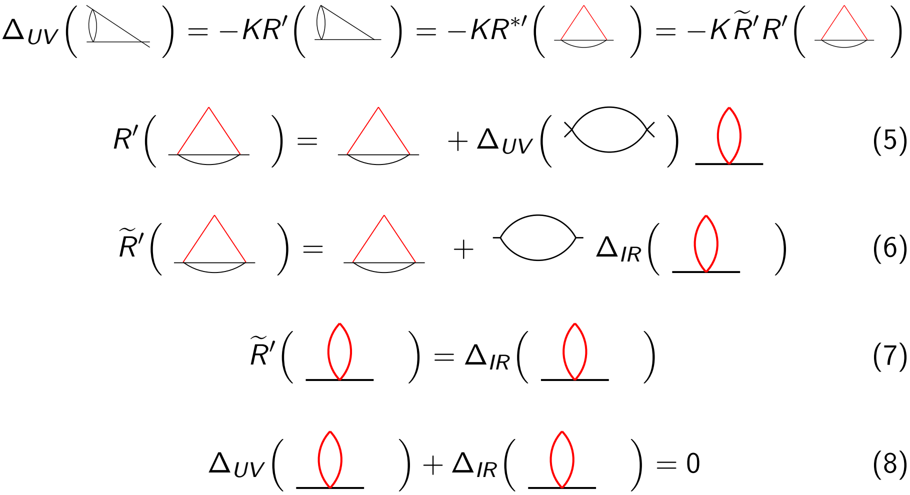
$\widetilde{R}'$ ( ) = Δ_{IR} ( ) (7)

Example 0



$\Delta_{UV}(\ ) = -KR'(\ ) = -KR^{*\prime}(\ ) = -K\widetilde{R}'R'(\ )$
$R'(\ ) = \ + \Delta_{UV}(\ )$ (5)
$\widetilde{R}'(\ ) = \ + \ \Delta_{IR}(\ )$ (6)
$\widetilde{R}'(\ ) = \Delta_{IR}(\ )$ (7)
$\Delta_{UV}(\ ) + \Delta_{IR}(\ ) = 0$ (8)

Results

Numerical six-loop critical exponent η expansion, $N = 1$:

$$\eta = 0.0740\varepsilon^2 + 0.1495\varepsilon^3 - 0.1332\varepsilon^4 + 0.8210\varepsilon^5 - 5.2014\varepsilon^6 + \mathcal{O}(\varepsilon^7)$$

Resummation using Borel transform and conformal mapping for $d = 2$, $N = 1$