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Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

Numerical multi-loop calculations with SecDec

Gudrun Heinrich

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In collaboration with Sophia Borowka, Johannes Schlenk

ACAT 2014

Czech Technical University, Prague

Particle physics after the Higgs discovery

- **the big question:** *is there something beyond the clouds (SM) ?*
- *how to find out in the absence of "smoking gun" signals ?*

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- *the key is* ***precision***
  - higher order corrections (QCD, EW)
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  - quark mass effects
  - reduction of PDF uncertainties
  - resummation
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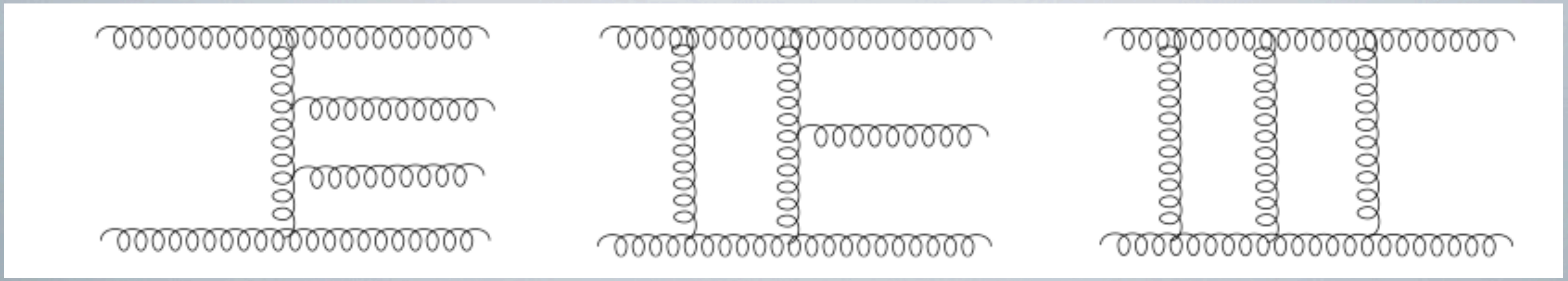
anatomy of 2 to 2 scattering at NNLO



double real | 1-loop virtual ⊗ single real | 2-loop virtual

- need efficient methods to evaluate
  2-loop amplitudes/integrals

- need various subtraction terms for singularities
  of individual contributions

Sector Decomposition

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Sector Decomposition

the **method** of sector decomposition:

- quite general algorithm [T. Binoth, GH 2000]
- factorizes poles in regulator epsilon
- applicable to loop integrals and
  IR divergent real radiation

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the program SecDec [S. Borowka, J.Carter, GH]

- extracts poles from loop integrals
  and
  more general parametric integrals
- evaluates loop integrals numerically

also in the presence of thresholds [S. Borowka, GH 2012]

public programs:

- sector_decomposition (uses Ginac) [Bogner, Weinzierl '07] (only Euclidean region)

  supplemented with CSectors [Gluza, Kajda, Riemann, Yundin '10]

  for construction of integrand in terms of Feynman parameters

- Fiesta (uses Mathematica, C) [A.Smirnov, V.Smirnov, Tentyukov, '08, '09,'13]

- **SecDec** (uses Mathematica, Fortran/C)

  [J.Carter, GH '10, S.Borowka, J.Carter, GH '12, S.Borowka, GH '13]

http://secdec/hepforge.org

SecDec is hosted by Hepforge, IPPP Durham

- Home
- Subversion
- Tracker
- Wiki

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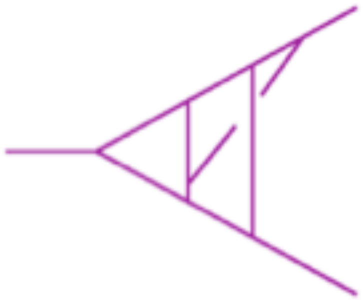
Durham University



SecDec

Sophia Borowka, Jonathon Carter, Gudrun Heinrich

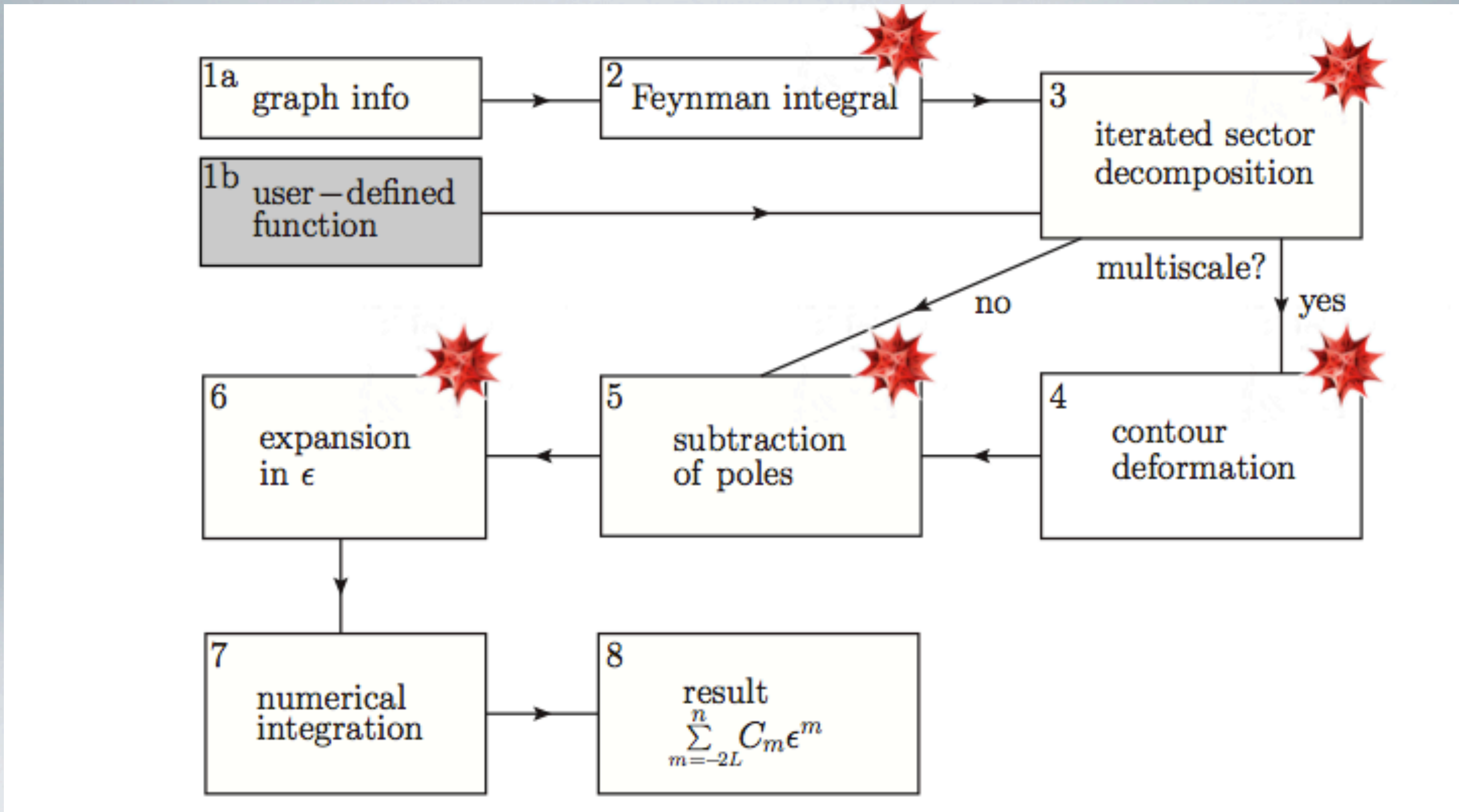
A program to evaluate dimensionally regulated parameter integrals numerically

Download Program FAQ ChangeLog

NEW: Version 2.1.6 of the program can be downloaded as SecDec-2.1.6.tar.gz (corresponds to SVN revision number 281).

Version 2.0 (of Dec 18, 2012) of the program can be downloaded as SecDec-2.0.tar.gz.

SecDec flowchart



SecDec installation and usage

- installation:

download from

http://secdec/hepforge.org

tar xzvf SecDec-2.1.tar.gz

cd SecDec-2.1

./install

installs numerical integration libraries

CUBA [T. Hahn] and BASES [S. Kawabata]

prerequisites:

Mathematica version 6 or above,
perl, Fortran/C compiler

SecDec installation and usage

- usage:

  edit two files

  - parameter.input :

    text file, contains parameters for integrand specification and numerical integration

  - graph.m :

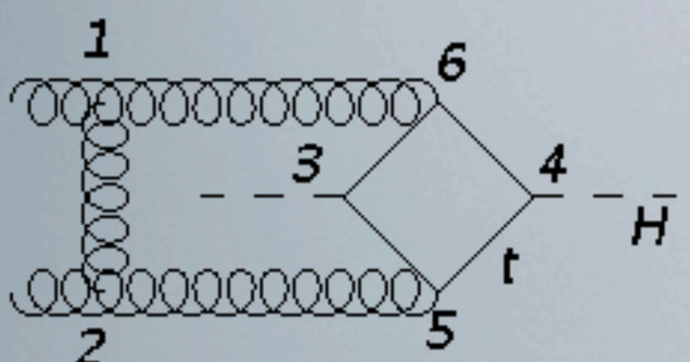
    Mathematica syntax,
    contains definition of the integrand

Example

nonplanar graph occurring in double

Higgs production at 2 loops

$(m_t \neq 0)$



$ms[1] = m_t^2$

$ms[2] = p_3^2 = p_4^2 = m_H^2$

graph definition:

HHNP1.m

```
(******* USER INPUT for construction of integrand ***************************)

(******* Use with cutconstruct=1 ********************************************)

(* If cutconstruct=1 is used give list of propagators with syntax
         {mass,vertex1,vertex2}, where masses m_i^2 must be called
         ms[i] and vertices are labeled with integer numbers*)

proplist={{0,{1,2}},{0,{1,6}},{0,{2,5}},
 {ms[1],{3,6}},{ms[1],{3,5}},{ms[1],{4,5}},{ms[1],{4,6}}};


(******* Use with cutconstruct=0 ********************************************)
(*
momlist={k1,k2};
proplist={k1^2-ms[1],(k1+p4)^2,(k1-p3)^2,((k1-k2)^2-ms[1]),
((k2+p4)^2-ms[1]),((k1-k2+p2)^2-ms[1]),((k2-p2-p3)^2-ms[1])};
numerator={1};
*)

(******* Propagator powers (optional) ***************************************)

(* give propagator powers if different from one *)

powerlist=Table[1,{i,Length[proplist]}];

(******* On-shell conditions (optional) *************************************)

(* give on-shell conditions, but PLEASE NOTE: in constructing F,
         (pi+pj)^2 will automatically be called sp[i,j]; pi^2 will be
         called ssp[i]; masses m_i^2 must be called ms[i]; *)

onshell={ssp[1]->0,ssp[2]->0,ssp[3]->ms[2],ssp[4]->ms[2]};

(******* Set Dimension ******************************************************)
(* Dimension can be changed, but symbol for epsilon must remain the same *)

Dim=4-2*eps;
```

example from gg $\to$ HH

input parameter definition:

HHNP1.input

```
################ input parameters for sector decomposition #################
#
# ############### all lines beginning with # are comments #################
#-------------------------------------------------------------------------
# insert subdirectory for the mathematica output files (will be created if
# non-existent): if not specified, a directory with the name of the graph
# given below will be created by default
subdir=2loop
#-------------------------------------------------------------------------
# if outputdir is not specified: default directory for
# the output will have integral name (given below) appended to directory
# above, otherwise specify full path for Mathematica output files here
outputdir=
#-------------------------------------------------------------------------
# graphname (can contain underscores, numbers, but should not contain commas)
graph=HHNP1
#-------------------------------------------------------------------------
# number of propagators:
propagators=7
#-------------------------------------------------------------------------
# number of external legs:
legs=4
#-------------------------------------------------------------------------
# number of loops:
loops=2
#-------------------------------------------------------------------------
# construct integrand (F and U) via topological cuts (only possible for
# scalar integrals), default is 0 (no cut construction used),
# use cutconstruct=1 for more effective treatment of hexagons
cutconstruct=1
##########################################################################
# parameters for subtractions and epsilon expansion
##########################################################################
# epsord: level up to which expansion in eps is desired
# (default is epsord=0: Laurent series is cut after finite part eps^0)
# series will be calculated from eps^(-maxpole) to eps^epsord
# note that epsord is negative if only some pole coeffs are required
epsord=0
#-------------------------------------------------------------------------
```

$$p_1^2 = p_2^2 = 0, p_3^2 = p_4^2 = m_H^2 = 125^2$$

$$m_1^2 = m_t^2 = 172^2, m_2^2 = m_H^2 = 125^2$$

```
#---------------------------------------------------------------------------
# off-shell legs p1^2,p2^2,...
pi2=0,0,15625,15625
#---------------------------------------------------------------------------
# propagator masses m1^2,m2^2,... (should be >=0)
# mi^2 = ms[i] =
ms2=29584,15625
```

Recent application of SecDec

momentum dependent $\mathcal{O}(\alpha_s \alpha_t)$ corrections to neutral Higgs boson masses in the MSSM

[S. Borowka, T. Hahn, S. Heinemeyer, GH, W. Hollik 2014]

- requires calculation of Higgs boson self-energies up to two-loop level

$$\Gamma \equiv \Delta^{-1}_{\text{Higgs}} = \begin{pmatrix} p^2 - m^2_{H,\text{tree}} + \hat{\Sigma}_H(p^2) & \hat{\Sigma}_{hH}(p^2) \\ \hat{\Sigma}_{hH}(p^2) & p^2 - m^2_{h,\text{tree}} + \hat{\Sigma}_h(p^2) \end{pmatrix}$$

- find the complex solutions of $\det \Gamma = 0$
- the masses are identified with the real parts of the solutions

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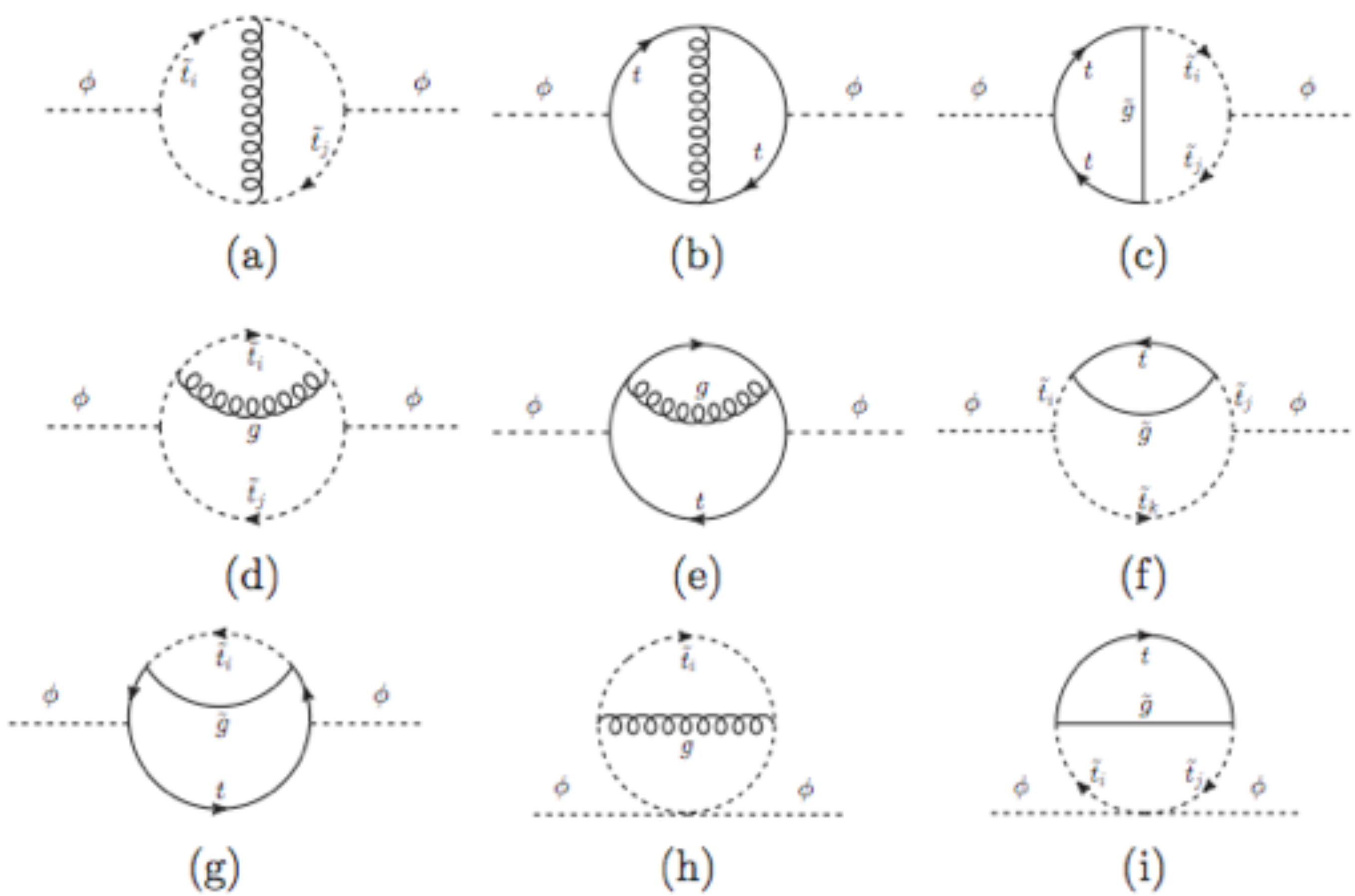
- the masses are identified with the real parts of the solutions

note: experimental precision on Higgs boson mass:

ATLAS:	$M_H^{\text{exp}} = 125.5 \pm 0.4 \pm 0.2$ GeV
CMS:	$M_H^{\text{exp}} = 125.7 \pm 0.3 \pm 0.3$ GeV

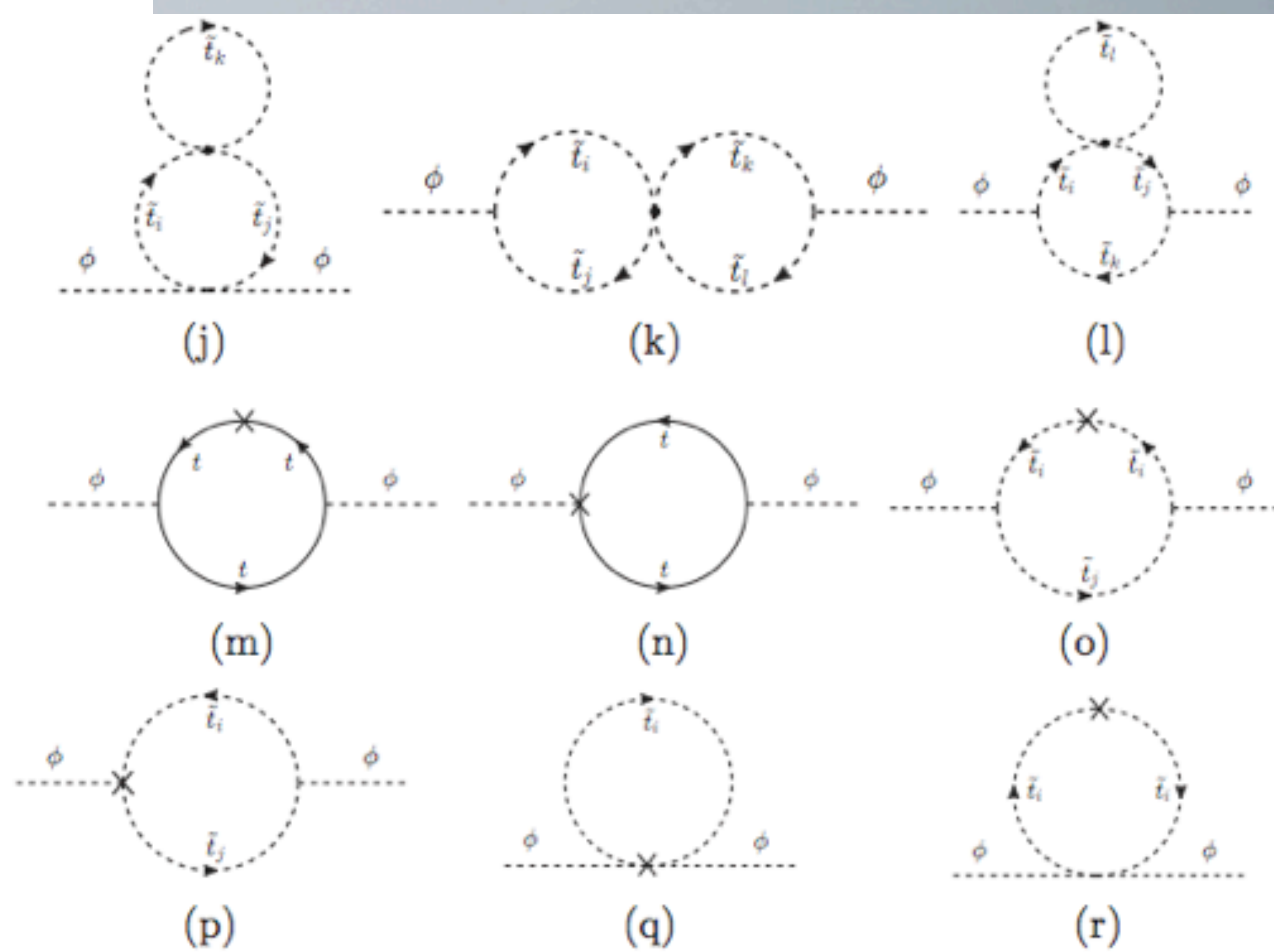
theoretical precision (MSSM): $\Delta M_h \simeq 3\,\text{GeV}$

two-loop Higgs boson selfenergy diagrams



- only few of these integrals are known analytically
- up to 4 different masses
- was technically not feasible so far:

  momentum dependence



→ numerical calculation possible with

SecDec-2.1

[S. Borowka, GH 2013]

Status of corrections in real MSSM

public programs:

FeynHiggs Frank, Hahn, Heinemeyer, Hollik, Rzehak, Weiglein '00 '03 '07 '14
SoftSusy Allanach '02 SPheno Porod '03 Staub '11
CPsuperH Carena, Choi, Drees, Ellis, Lee, Pilaftsis, Wagner '04 '09 '12
Suspect Djouadi, Kneur, Moultaka '07
H3m Kant, Harlander, Mihaila, Steinhauser '10

implemented corrections:

- 1-loop : complete
- 2-loop : $\mathcal{O}(\alpha_s\alpha_t), \mathcal{O}(\alpha_t^2), \mathcal{O}(\alpha_s\alpha_b), \mathcal{O}(\alpha_b^2), \mathcal{O}(\alpha_t\alpha_b)$ at $p^2 = 0$
- 3-loop : $\mathcal{O}(\alpha_s^2\alpha_t)$ at $p^2 = 0$

now: 2-loop $\mathcal{O}(\alpha_s\alpha_t)$ at $p^2 \neq 0$ [S. Borowka et al 2014]

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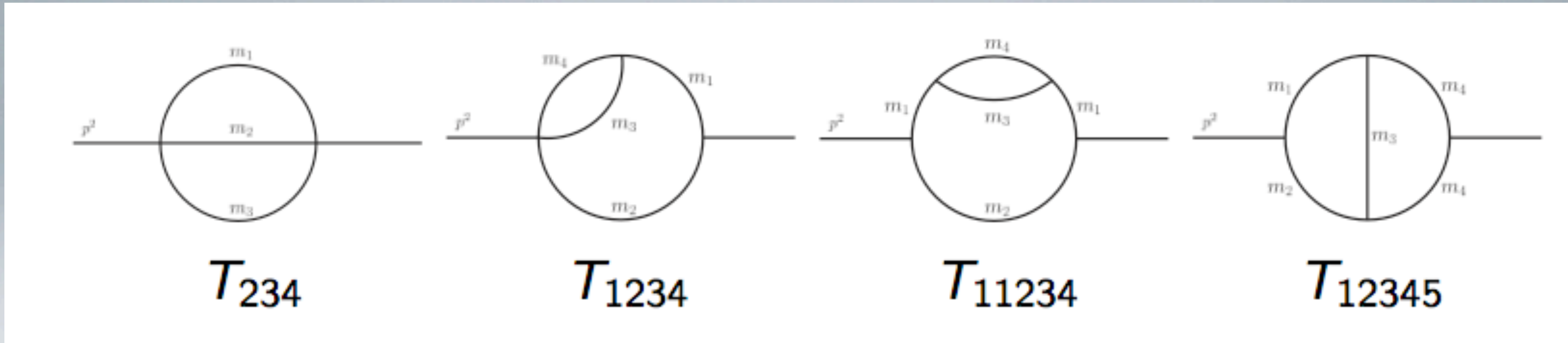
two-loop MSSM Higgs boson selfenergy diagrams

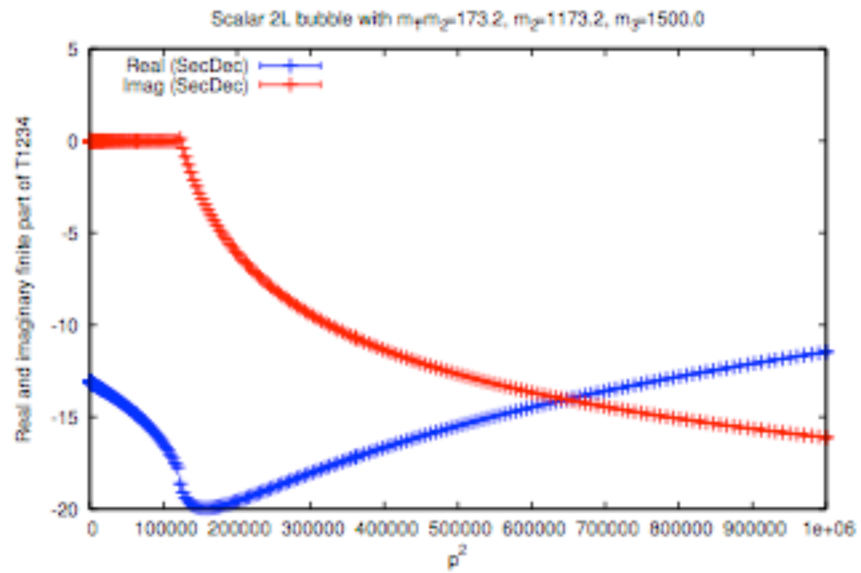
procedure:

- generate diagrams with FeynArts [T. Hahn]
- tensor reduction with TwoCalc [G.Weiglein et al]
  and FormCalc [T.Hahn et al]
- analytically unknown topologies:

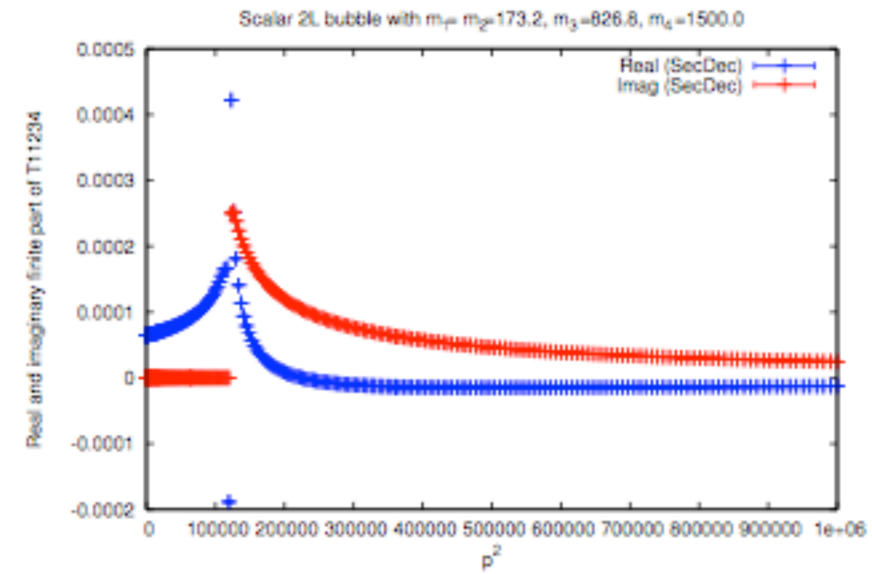


- calculated with SecDec, need 34 different mass configurations
- included in FeynHiggs

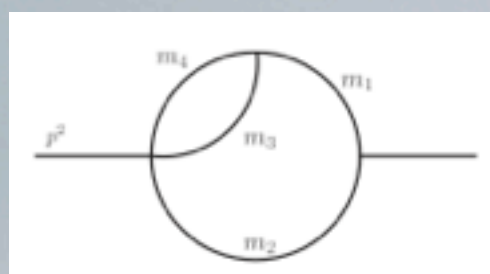
examples of results for master integrals

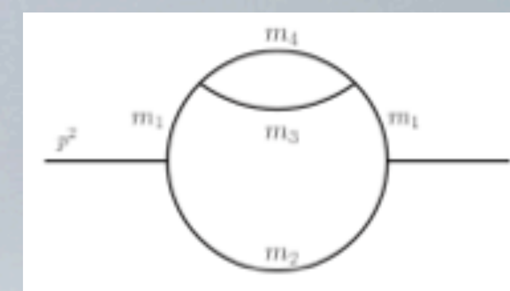


T_{1234}, finite part



T_{11234}, finite part





- values for kinematic invariants can differ by up to 14 orders of magnitude
- relative accuracy better than 10^-5, timings range between 0.01s and 100s

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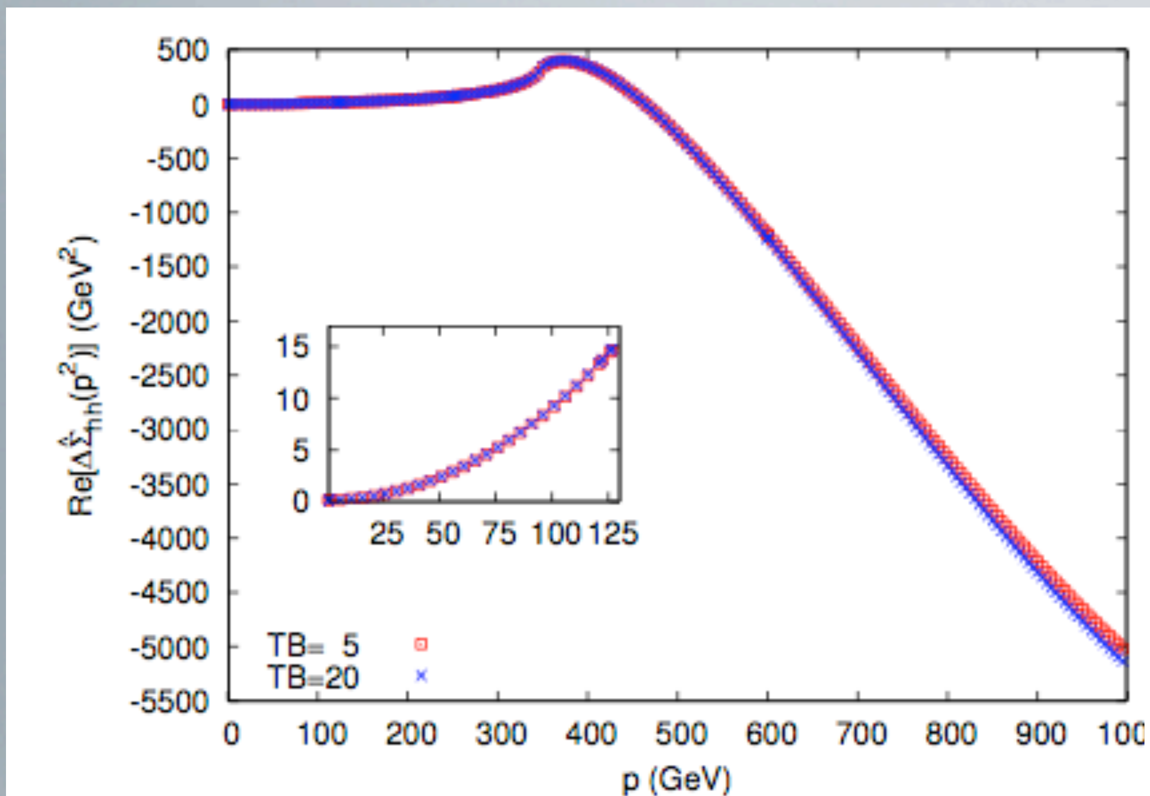
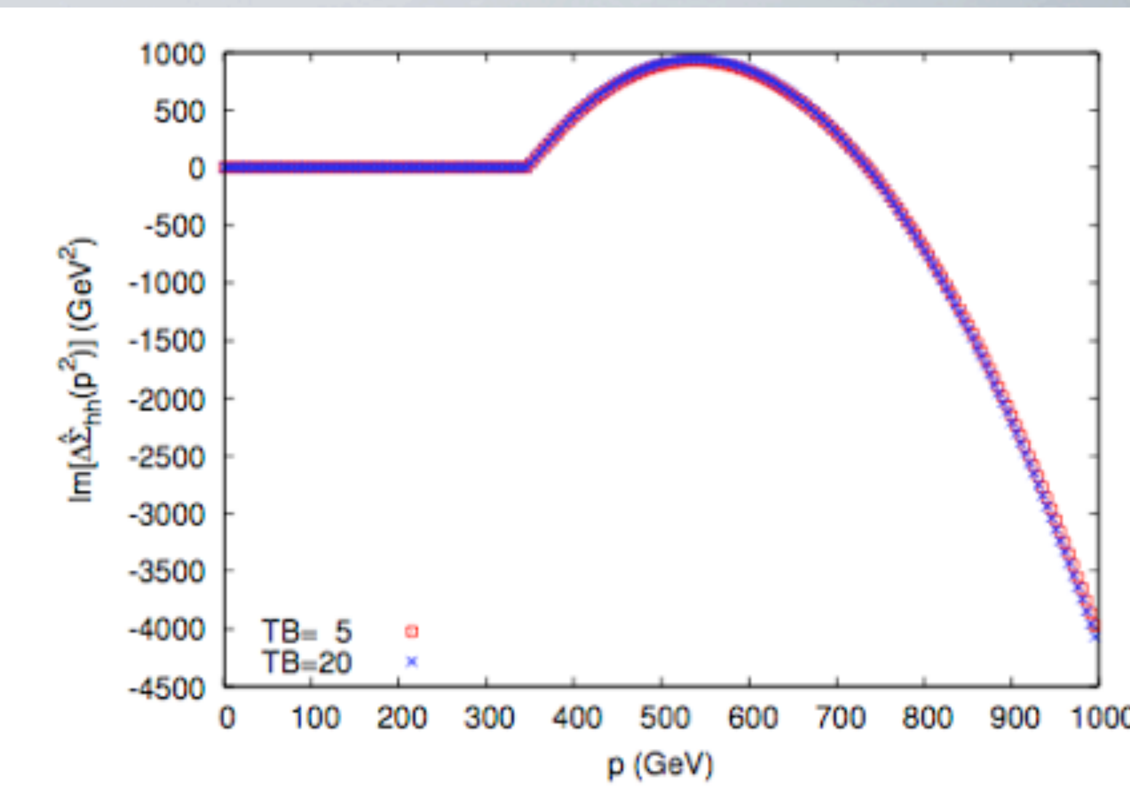
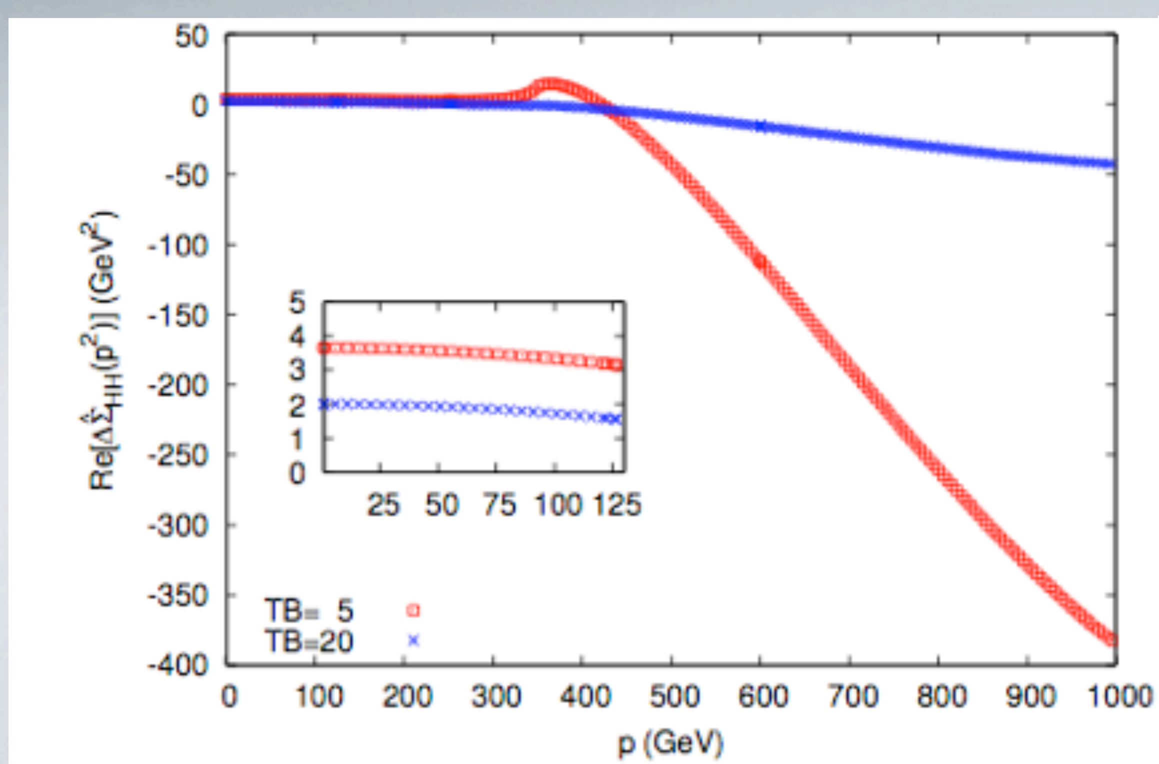
2-loop Higgs selfenergies in mh_max benchmark scenario

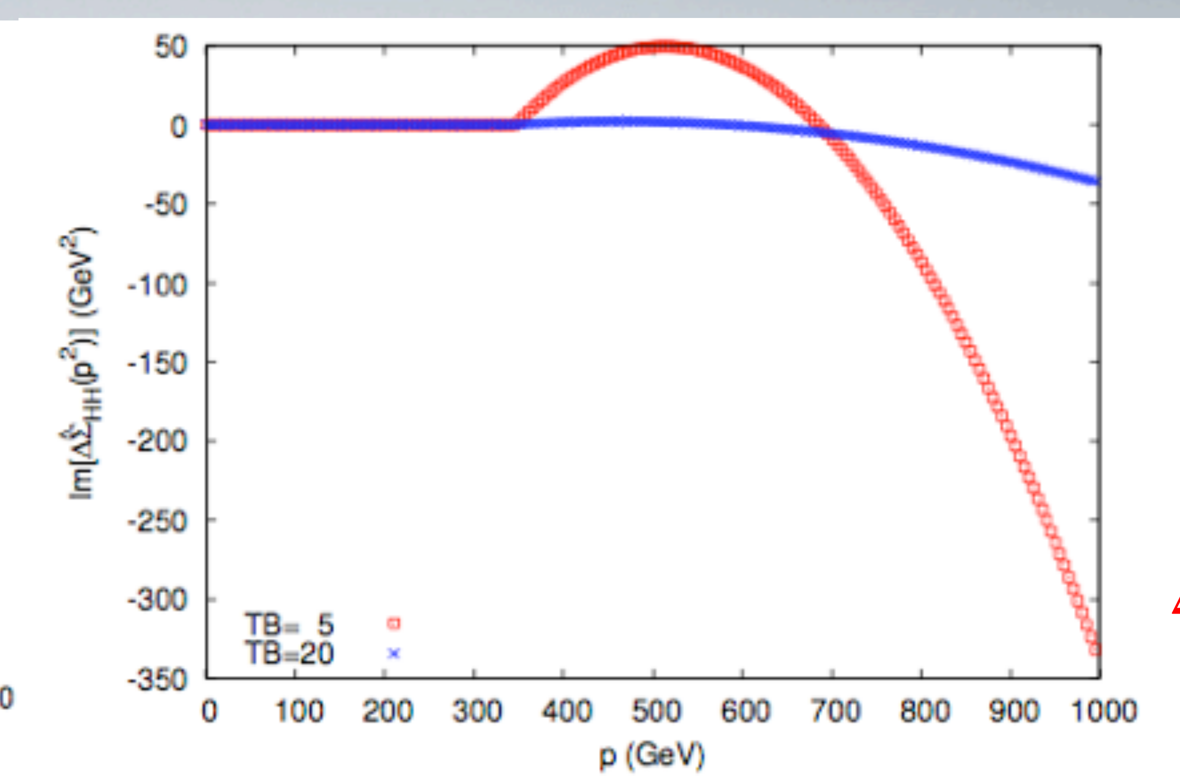
$$m_t = 173.2\,\mathrm{GeV}, \mu = 200\,\mathrm{GeV}, \mathrm{M_{Susy}} = 1\,\mathrm{TeV}, \mathrm{X_t} = 2\,\mathrm{M_{Susy}}, \mathrm{M_A} = 250\,\mathrm{GeV}, \mathrm{m_{\tilde{g}}} = 1.5\,\mathrm{TeV}$$





light Higgs h $\Delta\hat{\Sigma}_{hh}(p^2)$





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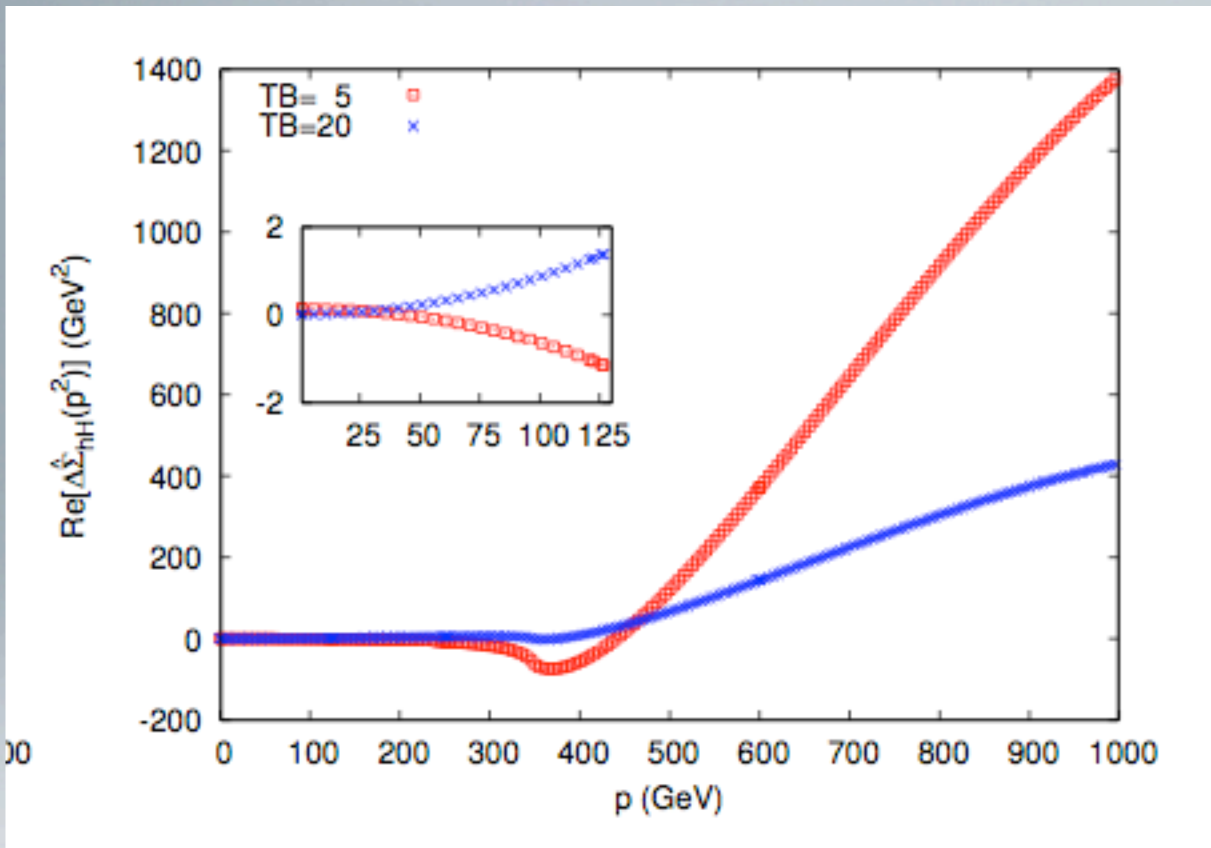
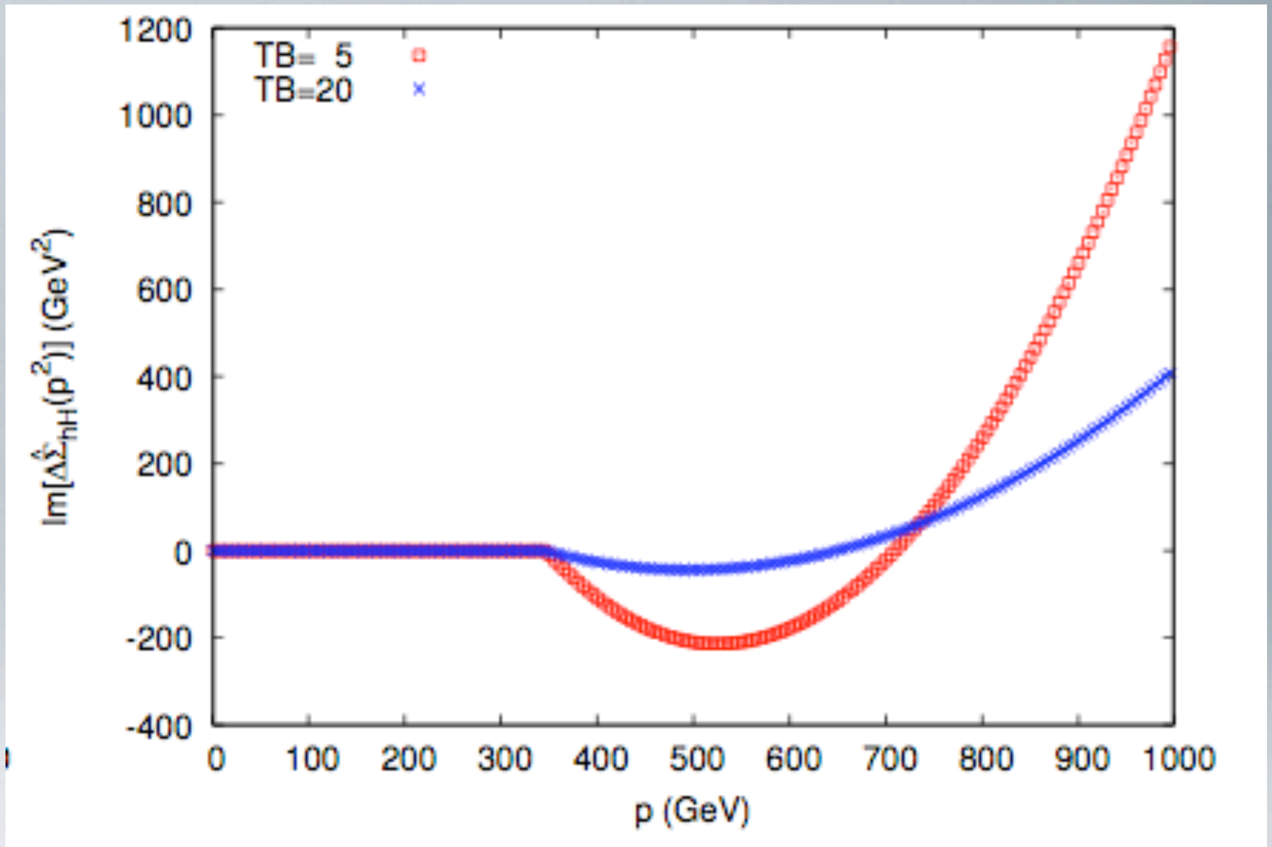
mixed term $\Delta\hat{\Sigma}_{hH}(p^2)$





calculation of mass shifts

based on solution of equation

$$\left[p^2 - m^2_{h,\text{tree}} + \hat{\Sigma}_{hh}(p^2)\right]\left[p^2 - m^2_{H,\text{tree}} + \hat{\Sigma}_{HH}(p^2)\right] - \left[\hat{\Sigma}_{hH}(p^2)\right]^2 = 0$$

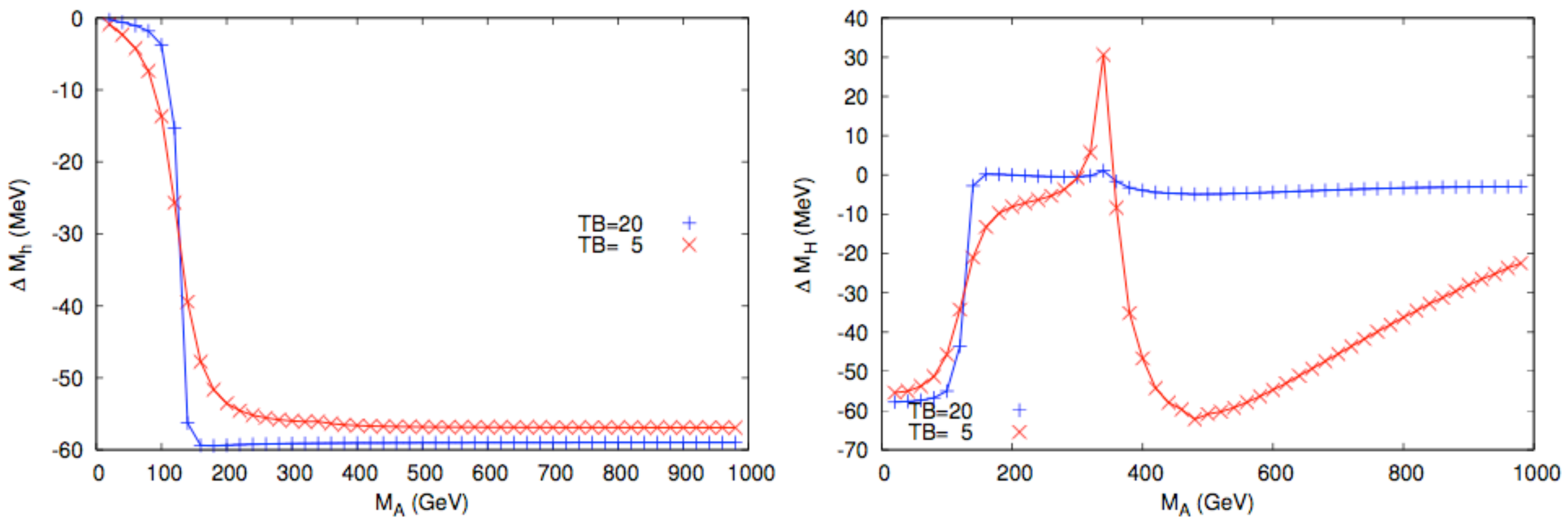
procedure:

1. compute Mh and MH without momentum dependent 2-loop corrections using FeynHiggs

2. compute momentum dependent renormalized 2-loop selfenergies at $p^2 = M_h^2$ and $p^2 = M_H^2$

3. include new selfenergy contributions as constant shifts into FeynHiggs (version 2.10.2) and find poles M_h^{new} and M_H^{new}

dependence of mass shifts on A-boson mass



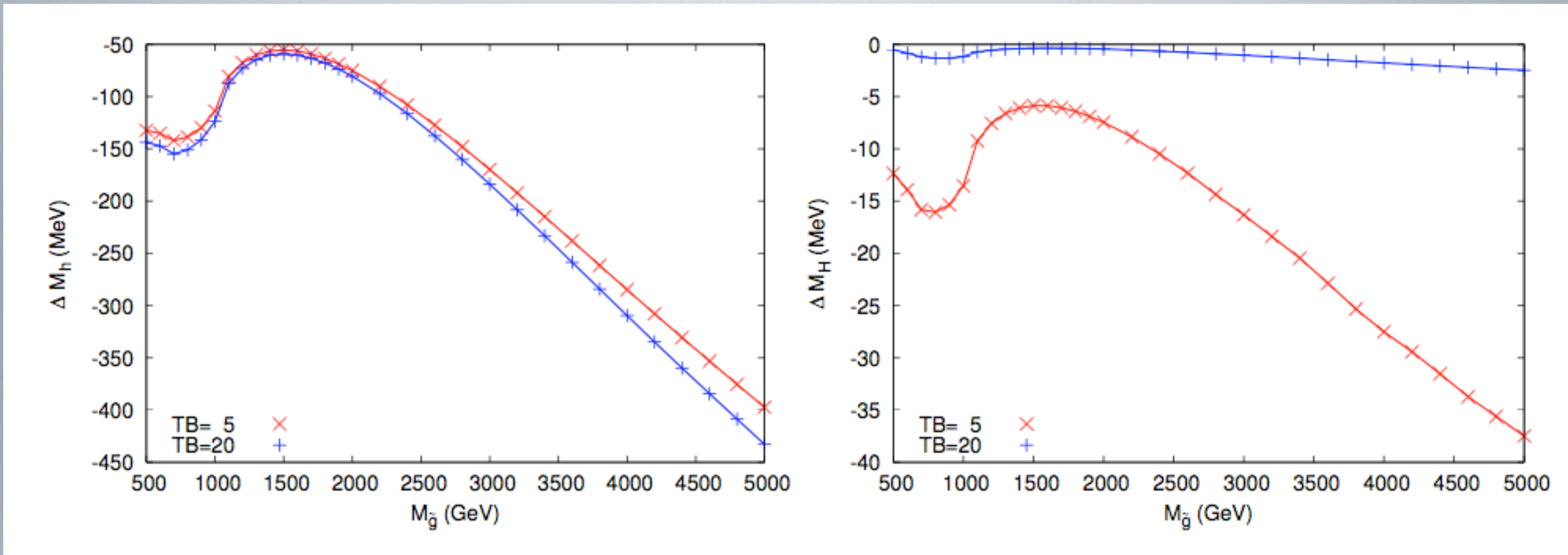
$$\Delta M_h = M_h - M_{h,0}\ ,\ \Delta M_H = M_H - M_{H,0}$$

including new momentum dependent corrections

without 2-loop momentum dependence

shift below current experimental precision, interesting for ILC precision

dependence of mass shifts on gluino mass



- (squared) logarithmic dependence of ΔM_h on gluino mass
- similar behaviour in other scenarios (e.g. light stop scenario)
- for large gluino masses, shift in Mh of the order of current experimental precision!

Summary and Outlook

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- **SecDec-2.1** has proven very useful for 2-loop problems with several mass scales
- momentum dependent corrections $\mathcal{O}(\alpha_s\alpha_t)$ to Higgs boson masses in the MSSM:

  light Higgs boson mass can get additional shift up to -600 MeV (current LHC precision) for gluino masses around 5 TeV
- corrections available in FeynHiggs version 2.10.2

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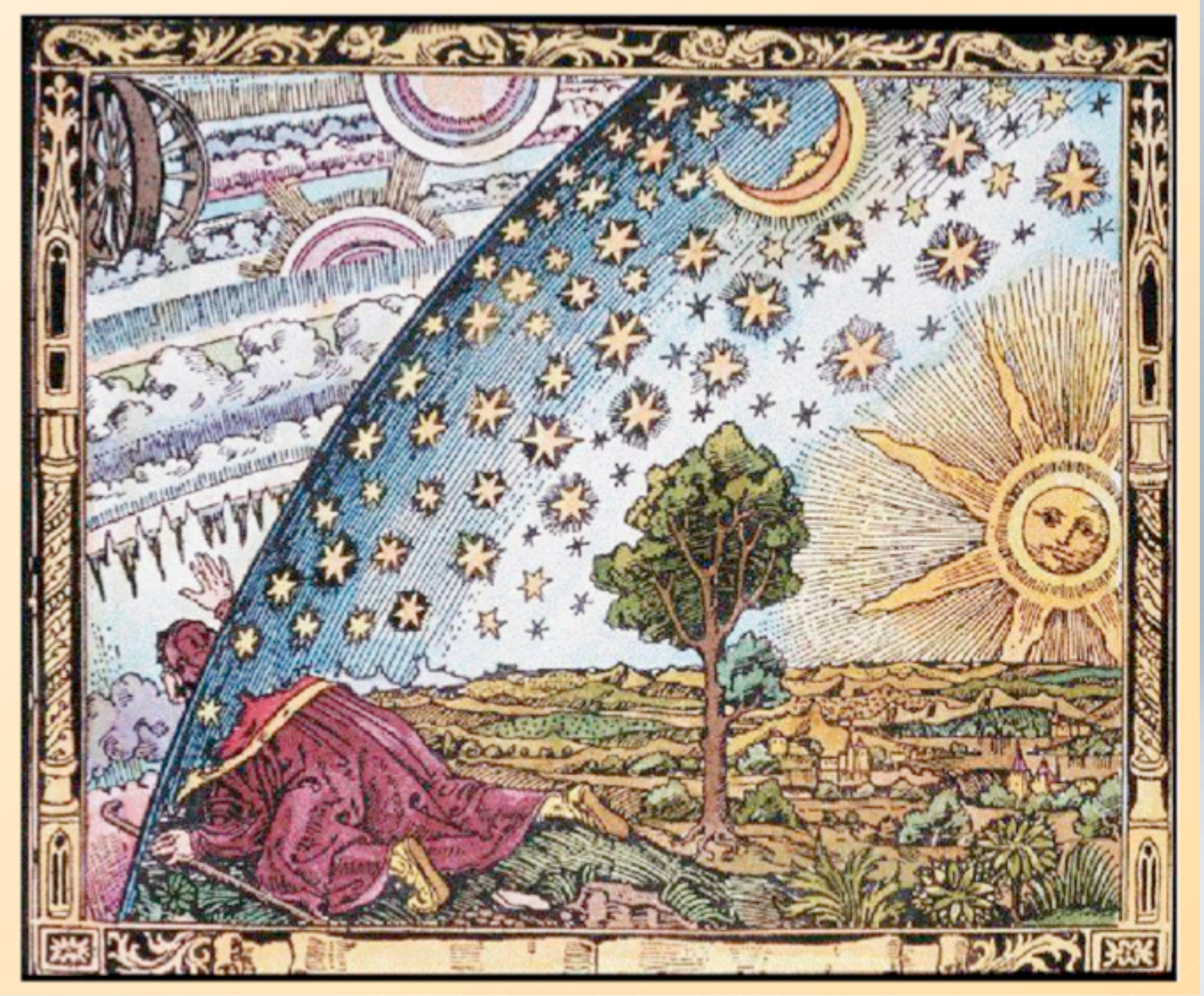
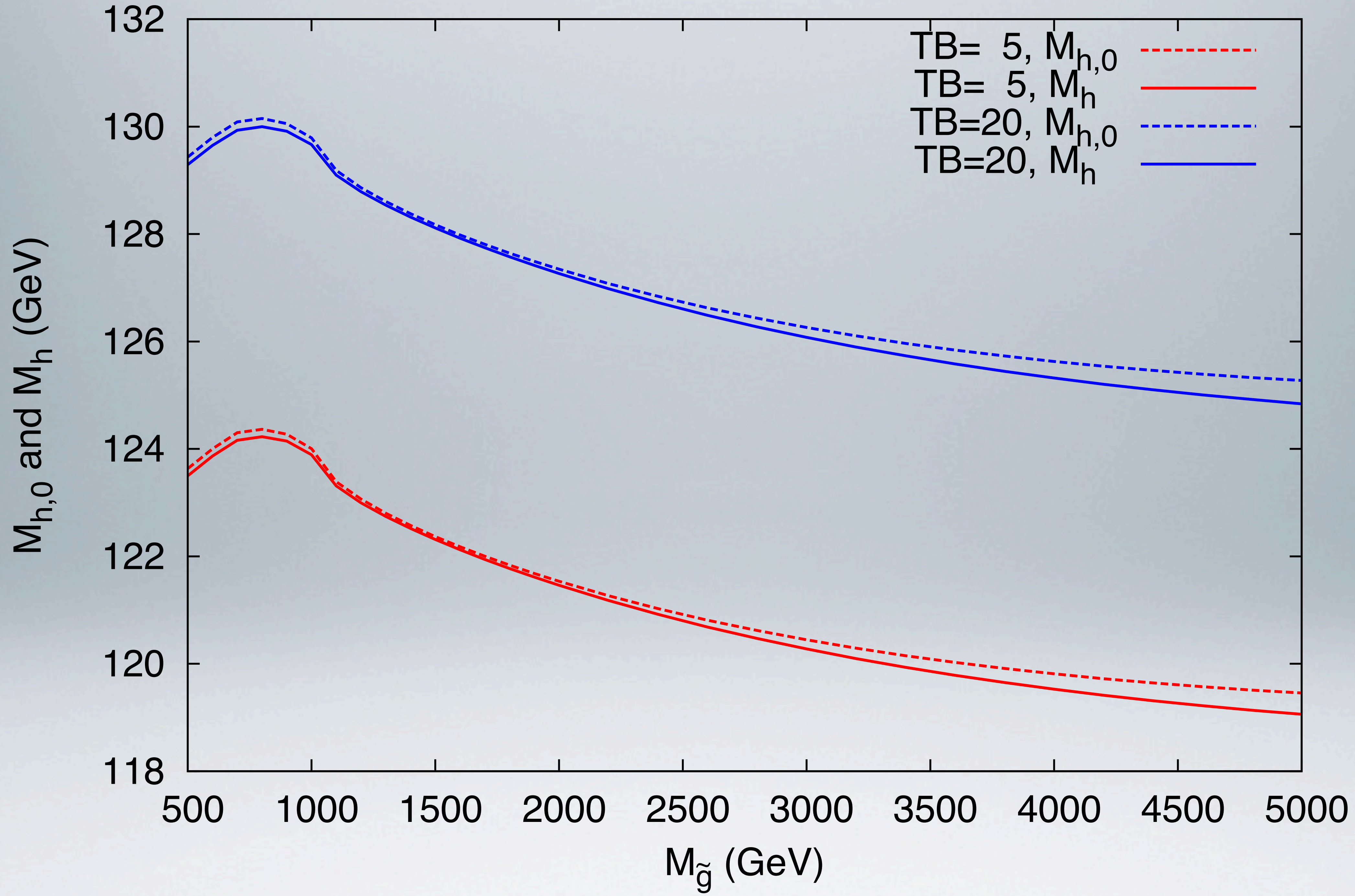
- Outlook: application of SecDec to other multi-loop problems involving several mass scales
- SecDec-3.0: faster, alternative decomposition strategy (based on algebraic geometry), axial gauge denominators, etc

http://secdec/hepforge.org



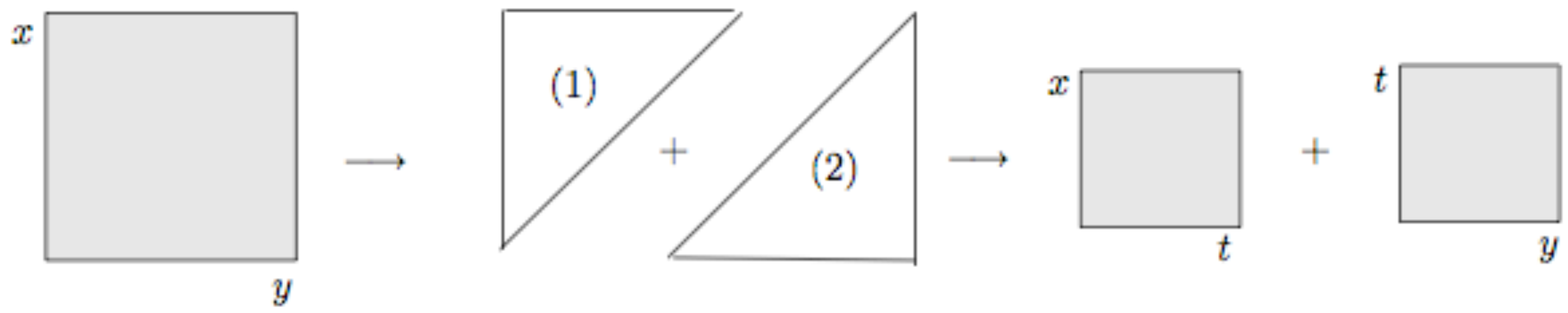
BACKUP SLIDES

impact of gluino mass



dependence on gluino mass with and without newly calculated corrections, mh_max scenario

basics of sector decomposition



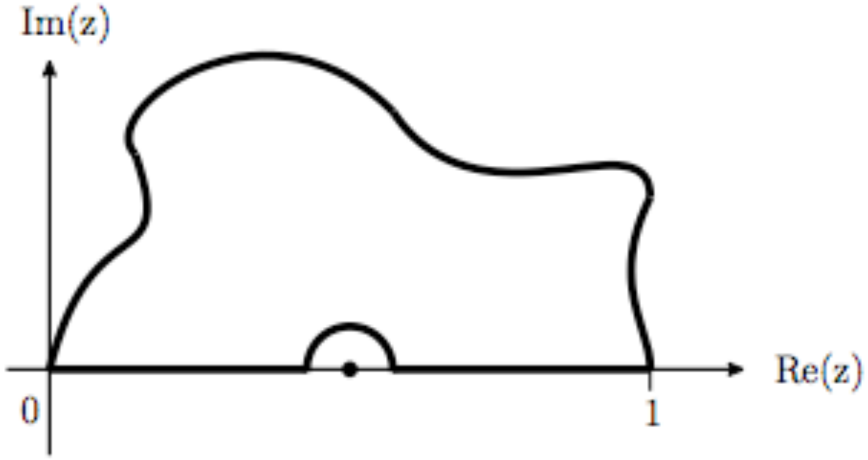
$$I = \int_0^1 dx \int_0^1 dy \, x^{-1-\epsilon} (x+y)^{-1} [\underbrace{\Theta(x-y)}_{(1)} + \underbrace{\Theta(y-x)}_{(2)}]$$

subst. (1) $y = x\,z$ (2) $x = y\,z$ to remap to unit cube

$$I = \int_0^1 dx \, x^{-1-\epsilon} \int_0^1 dz \, (1+z)^{-1} + \int_0^1 dy \, y^{-1-\epsilon} \int_0^1 dz \, z^{-1-\epsilon} (1+z)^{-1}$$

singularities are disentangled, number of integrals doubled

contour deformation



Cauchy: integral over closed contour is zero if no poles are enclosed

$$\int_0^1 \prod_{j=1}^{N} x_j \mathcal{I}(\vec{x}) = \int_0^1 \prod_{j=1}^{N} x_j \left| \frac{\partial z_k(\vec{x})}{\partial x_l} \right| \mathcal{I}(\vec{z}(\vec{x}))$$

$i\,\delta$ prescription for Feynman propagators $\Rightarrow Im(\mathcal{F})$ should be < 0

complexify:

$$\vec{z}(\vec{x}) = \vec{x} - i\,\vec{\tau}(\vec{x}) \; , \; \tau_k = \lambda\, x_k(1 - x_k)\, \frac{\partial \mathcal{F}(\vec{x})}{\partial x_k}$$

For small λ correct sign of Im part is guaranteed:

$$\mathcal{F}(\vec{z}(\vec{x})) = \mathcal{F}(\vec{x}) - i\,\lambda \sum_j x_j(1 - x_j) \left(\frac{\partial \mathcal{F}}{\partial x_j} \right)^2 + \mathcal{O}(\lambda^2)$$