Time-like Showers and Matching with Antennae

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In collaboration with R. Frederix, W. Giele, D. Kosower, T. Sjöstrand
Parton Showers

► The final answer depends on:
  • The choice of evolution variable
  • The splitting functions (finite/subleading terms not fixed)
  • The phase space map \( \frac{d\Phi_{n+1}}{d\Phi_n} \)
  • The renormalization scheme (argument of \( \alpha_s \))
  • The infrared cutoff contour (hadronization cutoff)

► Step 1, Quantify uncertainty: vary all of these (within reasonable limits)

► Step 2, Systematically improve: Understand the importance of each and how it is canceled by
  • Matching to fixed order matrix elements
  • Higher logarithms, subleading color, etc, are included

► Step 3, Write a generator: Make the above explicit (while still tractable) in a Markov Chain context \( \rightarrow \) matched parton shower MC algorithm
Based on Dipole-Antennae

- Shower off color-connected pairs of partons
- Plug-in to PYTHIA 8.1 (C++)


- Time-like QCD cascades (with massless quarks)
- 2 different shower evolution variables:
  - pT-ordering (= ARIADNE ~ PYTHIA 8)
  - Dipole-mass-ordering (~ Thrust ≠ PYTHIA 6, SHERPA)
- For each: an infinite family of antenna functions
  - Laurent series in branching invariants with arbitrary finite terms
- Shower cutoff contour: independent of evolution variable
  → IR factorization “universal”
- Several different choices for \( \alpha_s \) (evolution scale, \( p_T \), mother antenna mass, 2-loop, …)
- Phase space mappings: 2 different choices implemented
  - Antenna-like (ARIADNE angle) or Parton-shower-like: Emitter + longitudinal Recoiler
Dipole-Antenna Showers

Dipole branching and phase space

\[ s = s_{ab} = s_{arb} \]

\[ \hat{a} \rightarrow \hat{b} \]

\[ s = s_{ab} = s_{arb} \]

\[ s_{ar} \]

\[ s_{br} \]

\[ s_{ab} \]

\[ \rightarrow \]

\[ y_{ab} = S_{rb}/s \]

\[ y_{ar} = S_{ar}/s \]

\[ y_E = 0.5 \]

Type I \((p_{r2})\)

Type II \((m^2)\)

(⇒ Most of this talk, including matching by antenna subtraction, should be applicable to ARIADNE as well)

Giele, Kosower, PS: hep-ph/0707.3652
Example: Z decays

- **VINCIA and PYTHIA8** (using identical settings to the max extent possible)

\[
\alpha_s(p_T),
\]
\[
p_{\text{Thad}} = 0.5 \text{ GeV}
\]
\[
\alpha_s(m_Z) = 0.137
\]
\[
N_f = 2
\]

**Note:** the default Vincia antenna functions reproduce the $Z \rightarrow 3$ parton matrix element;

Pythia8 includes matching to $Z \rightarrow 3$

Frederix, Giele, Kosower, PS : Les Houches Proc., in preparation
Example: Z decays

► Why is the dependence on the evolution variable so small?
  • Conventional wisdom: evolution variable has huge effect
    • Cf. coherent vs non-coherent parton showers, mass-ordered vs $p_T$-ordered, etc.

► Dipole-Antenna showers resum radiation off pairs of partons
  • $\Rightarrow$ interference between 2 partons included in radiation function
  • If radiation function = dipole formula $\Rightarrow$ intrinsically coherent
    • $\Rightarrow$ angular ordering there by construction  
      Gustafson, PLB175(1986)453
  • Remaining dependence on evolution variable much milder than for conventional showers

► The main uncertainty in this case lies in the choice of radiation function away from the collinear and soft regions
  • $\Rightarrow$ dipole-antenna showers under the hood …
Starting point: “GGG” antenna functions, e.g., $gg \rightarrow ggg$:

$$f_3^0(p_a, p_r, p_b) = \frac{1}{s_i} \left[ (1 - y_{ar} - y_{rb}) \left( \frac{2}{y_{ar} y_{rb}} + \frac{y_{ar}}{y_{rb}} + \frac{y_{rb}}{y_{ar}} \right) + \frac{8}{3} \right]$$

$y_{ar} = s_{ar} / s_i$

$s_i =$ invariant mass of $i'$th dipole-antenna

Generalize to arbitrary Laurent series:

$$A(s_{ar}, s_{rb}) = \frac{4\pi\alpha_s(\mu_R)N_C}{s_{arb}} \sum_{\alpha, \beta \geq -1} C_{\alpha\beta} s_{ar}^\alpha s_{rb}^\beta$$

Singular parts fixed, finite terms arbitrary

Can make shower systematically “softer” or “harder”

- Will see later how this variation is explicitly canceled by matching
- quantification of uncertainty
- quantification of improvement by matching
The unknown finite terms are a major source of uncertainty

- DGLAP has some, GGG have others, ARIADNE has yet others, etc…
- They are arbitrary (and in general process-dependent)

\[ \alpha_s(M_Z) = 0.137, \]
\[ \mu_R = \frac{1}{4} m_{\text{dipole}}, \]
\[ p_{\text{Thad}} = 0.5 \text{ GeV} \]

Giele, Kosower, PS : hep-ph/0707.3652
Matching

**Pure Shower (all orders)**

\[
\frac{d\sigma_X}{d\mathcal{O}} \bigg|_{\text{PS}} = \int d\Phi_X \; w_X \; S(\{p\}_X, \mathcal{O})
\]

\[
S(\{p\}_X, \mathcal{O}) = \frac{\delta(\mathcal{O} - \mathcal{O}(\{p\}_X))}{X + 0 \text{ exclusive above } 1/t_{\text{had}}}
\]

\[
\int_{t_X}^{t_{\text{had}}} dt_{X+1} \sum_i \int \frac{d\Phi_{X+1}^{[i]}}{d\Phi_X} \delta(t_{X+1}) \Delta(t_X, t_{X+1}) A_i(\ldots) S(\{p\}_{X+1}, \mathcal{O})
\]

\[
X + 1 \text{ inclusive above } 1/t_{\text{had}}
\]

**A**: splitting function

**Matched shower** (including simultaneous tree- and 1-loop matching for any number of legs)

\[
\frac{d\sigma}{d\mathcal{O}} \bigg|_{\text{MS}} = \sum_{k=0}^{n} \int d\Phi_{X+k} \left( w^{(R)}_{X+k} + w^{(V)}_{X+k} \right) S(\{p\}_{X+k}, \mathcal{O})
\]

**Tree-level “real”**

**Loop-level “virtual+unresolved”**

**matching term for X+k**

**matching term for X+k**

**Giele, Kosower, PS**: hep-ph/0707.3652
Tree-level matching to X+1

1. Expand parton shower to 1st order (real radiation term)

\[ \text{PS} : \int \frac{d\Phi_X}{\int_{t_{\text{had}}}^{t_{X+0}}} \left| M_{X+0}^{(0)} \right|^2 \int_{t_{X+0}}^{t_{X+1}} \sum_i \int \frac{d\Phi_{X+1}}{d\Phi_X} \delta(t_{X+1} - t^{[i]}(\{p\}_{X+1})) A_i(...) \delta(\mathcal{O} - \mathcal{O}(\{p\}_{X+1})) \]

2. Matrix Element (Tree-level X+1 ; above \( t_{\text{had}} \))

\[ \text{ME} : \int_{t < t_{\text{had}}} \frac{d\Phi_{X+1}}{\int_{t_{X+0}}} \left| M_{X+1}^{(0)} \right|^2 \delta(\mathcal{O} - \mathcal{O}(\{p\}_{X+1})) \]

\( \Rightarrow \) Matching Term:

\[ M^T : w_{X+1}^{(R)} = \left| M_{X+1}^{(0)} \right|^2 - \sum_{i \in X \rightarrow X+1} A_i(...) \left| M_{X+0}^{(0)}(\{\hat{p}_i\}_{X+0}) \right|^2 \]

- \( \Rightarrow \) variations in finite terms (or dead regions) in \( A_i \) canceled (at this order)
- (If \( A \) too hard, correction can become negative \( \Rightarrow \) negative weights)

\[ - \frac{8\pi\alpha_s(\mu_R)N_c}{M_H^2} \left( 8 - 3C_{00} - (C_{10} + C_{01}) - C_{11}(y_{12}y_{23} + y_{23}y_{31} + y_{31}y_{12}) + \cdots \right) \left| M_{2}^{(0)} \right|^2 \]

Inverse phase space map \( \sim \) clustering
\( \{\hat{p}_i\}_X = \{\kappa_i^{-1}(\{p\}_{X+1})\}_X \)

Giele, Kosower, PS : hep-ph/0707.3652
The unknown finite terms are a major source of uncertainty
- DGLAP has some, GGG have others, ARIADNE has yet others, etc…
- They are arbitrary (and in general process-dependent)

Varying finite terms only

with

\( \alpha_s(M_Z) = 0.137 \),
\( \mu_R = 1/4m_{\text{dipole}} \),
\( p_{\text{Thad}} = 0.5 \text{ GeV} \)

Giele, Kosower, PS: hep-ph/0707.3652
1-loop matching to X

- **NLO “virtual term” from parton shower (≈ expanded Sudakov: exp=1 - ...)**

\[
\text{PS} : \quad - \int d\Phi_X |M_X^{(0)}|^2 \delta (\mathcal{O} - \mathcal{O}(\{p\}_X)) \int_{t_{X X}}^{t_{X had}} d t_{X X} \sum_i \int \frac{d \Phi_{X+1}^{[i]}}{d \Phi_X} \delta (t_{X X} - t_{[i]}(\{p\}_{X + 1})) A_i(...) 
\]

- **Matrix Elements (unresolved real plus genuine virtual)**

\[
\text{ME} : \quad \int_{t > t_{had}} d\Phi_{X+1} |M_{X+1}^{(0)}|^2 \delta (\mathcal{O} - \mathcal{O}(\{p\}_{X+1})) + \int d\Phi_X 2 \operatorname{Re}[M_X^{(0)} M_X^{(1)}] \delta (\mathcal{O} - \mathcal{O}(\{p\}_X))
\]

- **Matching condition same as before (almost):**

\[
\text{MT : } w_X^{(V)} = 2 \operatorname{Re}[M_X^{(0)} M_X^{*(1)}] + |M_X^{(0)}|^2 \sum_i \int_{t > t_{had}} \frac{d\Phi_{X+1}^{[i]}}{d\Phi_X} A_i(...) + \int d\Phi_{X+1} w_X^{(R)} 
\]

\[
= 2 \operatorname{Re}[M_X^{(0)} M_X^{*(1)}] + |M_X^{(0)}|^2 \sum_i \int_{t > t_{had}} \frac{d\Phi_{X+1}^{[i]}}{d\Phi_X} A_i(...) + \mathcal{O}(t_X / t_{had}) ,
\]

• (This time, too small A → correction negative)

- **You can choose anything for \( A_i \) (different subtraction schemes) as long as you use the same one for the shower**
Note about “NLO” matching

- Shower off virtual matching term ➔ unmatched $O(\alpha^2)$ contribution to 3-jet observables (only canceled by 1-loop 3-parton matching)

- While normalization is improved, most shapes are not

Using $\alpha_s(M_Z)=0.137$, $\mu_R=1/4m_{dipole}$, $p_{Thad} = 0.5$ GeV
What happened?

► Brand new code, so bear in mind possibility of bugs

► Naïve conclusion: tree-level matching “better” than NLO?
  • No, first remember that the shapes we look at are not “NLO”
    • E.g., 1-T appears at $O(\alpha)$ below $1-T=2/3$, and at $O(\alpha^2)$ above
    • An “NLO” thrust calculation would have to include at least 1-loop corrections to $Z \to 3$
  • So: both calculations are LO/LL from the point of view of 1-T

► What is the difference then?
  • Tree-level $Z \to 3$ is the same (LO)
  • The $O(\alpha)$ corrections to $Z \to 3$, however, are different
    • The first non-trivial corrections to the shape!
    • So there should be a large residual uncertainty $\to$ the 1-loop matching is “honest”
  • The real question: why did the tree-level matching not tell us?
    • I haven’t completely understood it yet … but speculate it’s to do with detailed balance
    • In tree-level matching, unitarity $\Rightarrow$ Virtual = - Real $\Rightarrow$ cancellations. Broken at 1 loop
    • + the tree-level curves had different normalizations (covering the shape uncertainty?)
What to do next?

► Further shower studies
  • Interplay between shower ambiguities and hadronization “tuning”
  • Shower+hadronization tuning in the presence of matching

► Go further with tree-level matching
  • Demonstrate it beyond first order (include H,Z → 4 partons)
  • Automated tree-level matching (automated subtractions)

► Go further with one-loop matching
  • Demonstrate it beyond first order (include 1-loop H,Z → 3 partons)
    • Should start to see genuine stabilization of shapes as well as normalizations

► Extend to the initial state

► Extend to massive particles
  • Massive antenna functions, phase space, and evolution

We’re only a few people working part-time, so plenty of room for collaboration
Number of partons and number of quarks

- $N_q$ shows interesting dependence on ordering variable.

Fig. 3: Number of partons (left) and number of quarks (right) at shower termination, with 2 massless quark flavors.
\[ A(s_{ar}, s_{rb}) = \frac{4\pi\alpha_s(\mu_R)N_C}{s_{arb}} \sum_{\alpha, \beta=1}^{6} C_{\alpha, \beta} s_{arb}^{\alpha} s_{arb}^{\beta} \]

<table>
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<th>GGG</th>
<th>( q\bar{q} \rightarrow qg\bar{q} )</th>
<th>( qg \rightarrow qgg )</th>
<th>( gg \rightarrow qgq )</th>
<th>( gg \rightarrow q\bar{q}'q' )</th>
<th>( gg \rightarrow ggq )</th>
<th>( C_{-1} )</th>
<th>( C_{-10} )</th>
<th>( C_{0-1} )</th>
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<th>ARIADNE</th>
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<tr>
<th>ARIADNE2 (reparametrization of ARIADNE functions à la GGG, for comparison)</th>
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Table 1: Laurent coefficients for massless LL QCD antennae (\( \hat{a} \hat{b} \rightarrow \omega \omega \)). The coefficients with at least one negative index are universal (apart from a reparametrization ambiguity for gluons). For “GGG” (the defaults in VINCIA), the finite terms correspond to the specific matrix elements considered in [4]. In particular, the \( qg\bar{q} \) antenna absorbs the tree-level \( Z \rightarrow qg\bar{q} \) matrix element [5] and the \( qa \) antennae absorb the tree-level \( h^0 \rightarrow qa \rightarrow qaa \) and \( h^0 \rightarrow qa \rightarrow qaa \) matrix elements [6]. The
Monte Carlo Tools for Beyond the Standard Model Physics

3rd workshop: MARCH 10-11, 2008 (CERN)

Note: both days will be full days. Participants who don’t want to miss anything should plan on arriving Mar 9 and leaving Mar 12.

Organizing committee: Georges Azuelos, Christophe Grojean, Jay Hubisz, Borut Kersevan, Joe Lykken, Fabio Maltoni, Konstantin Matchev, Filip Moortgat, Steven Mrenna, Maxim Perelstein, Peter Skands:
mc4bsm_AT_phys_ufl.edu

Main Focus: Alternative (non-MSSM) TeV scale physics models

http://theory.fnal.gov/mc4bsm/