Colour-dipole cascades involving initial-state partons

[MCnet meeting in Lund]

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Extension of the Colour Dipole Model (CDM):
final–initial and initial–initial dipole radiation pattern.

Survey of first dipole-shower results.


http://www.sherpa-mc.de/
Parton showers ... recent developments.

- New physics challenges (LHC), rewrites of PYTHIA/HERWIG codes plus enormous progress in the techniques of combining (N)LO calculations with parton showers led to an intensive overhaul of existing formulations.

Efforts aim at ...

- achieving better analytic control.
- gaining better understanding of systematic uncertainties.
- providing (easier/more consistent) merging/matching with LO/NLO calculations.
- going beyond common approximations (LL, large $N_C$, include small-$x$)?

New $1 \rightarrow 2$ splittings showers, for PYTHIA and HERWIG, and new shower formulation based on Catani–Seymour dipole factorization.

Still other ways to identify/pick leading logs of multiple QCD emissions? Yes. $2 \rightarrow 3$ splittings. ➡️ VINCIA. And, of course, the successful Lund CDM as implemented in ARIADNE.
The Colour Dipole Model (CDM)

Alternative to conventional Altarelli–Parisi parton showers // same principles.

- soft gluons intrinsically correct
- colour coherent emission

QCD antenna pattern

\[ d\sigma = \frac{8}{3\pi} \frac{d\omega_g}{\omega_g} dy_g = \frac{4}{3\pi} \frac{dx_1 dx_3}{(1 - x_1)(1 - x_3)} \]

- corrigible for hard bremsstrahlung (spin!)
- recall PS: ME corr. + conar conditions

- semiclassical probabilistic picture for dipole cascade

  \( q\bar{q} \) emits \( g \), in turn \( q'g\bar{q}' \) may emit softer \( g' \)

  \( q'\bar{q}' \) contribution neglected \((1/N_C^2 \text{ supp.})\)

- onshell kinematics; no momentum reshuffling
- emitters almost independent of rest of tree

- \( 2 \rightarrow 3 \) splittings somewhat more complicated (more cases)
- gluon splitting not naturally, easily included
CDM for timelike evolution

Pioneering works: Azimov, Dokshitzer, Khoze, Troyan / Gustafson, Pettersson, Andersson, Lönnblad

- Splitting functions ($g$ emit):
  \[ \frac{d\sigma_{2\to3}}{dx_1 dx_3} = \frac{2\alpha_s}{3\pi} \left[ 1 + \frac{1}{8}\Theta(g_{1,3}) \right] \frac{x_1^2 + \Theta(g_1) + x_3^2 + \Theta(g_3)}{(1 - x_1)(1 - x_3)} \]

- Evolution variables
  \[ p_t^2 = M^2(1 - x_1)(1 - x_3), \quad y = \frac{1}{2} \ln \frac{1 - x_1}{1 - x_3} \]

- Sudakov exponentiation
  \[ \frac{dP}{dp_t^2 dy} = \frac{d\sigma_{2\to3}}{dp_t^2 dy} \exp \left( -\int_{p_t^2}^{p_{t,\text{max}}^2} dk_t^2 \int_{y_{\text{min}}(k_t^2)}^{y_{\text{max}}(k_t^2)} dy' \frac{d\sigma_{2\to3}}{dk_t^2 dy'} \right) \]

- Ordering
  \[ p_{t,i}^2 > p_{t,j}^2 \]

- Recoils
  \[ \frac{d\sigma_{qq\to q\bar{q}Q\bar{Q}}}{dx_1 dx_3} = \frac{N_F\alpha_s}{8\pi} \frac{(1 - x_2)^2 + (1 - x_3)^2}{1 - x_1} \]

- Gluon splitting (competition)
  \[ q\bar{q} \text{ Kleiss} \]

\[ qg, gg \text{ minimize } p_\perp \text{'s} \]
Lund CDM for DIS & hadron–hadron collisions


QCD cascade is not divided in IS and FS.

- Struck quark pointlike. Hadron remnant extended.
- Suppression of the emission of small wavelengths from extended antennae.
- Only a fraction of remnant’s light-cone momentum participates in emission of $p_\perp$ (extra tunable parameter).

Leif Lönnblad: Small-x effects in $W +$ jets at the Tevatron (CERN-TH/95-212)

- Boson not intrinsically coupled.
- Boson’s transverse momentum through recoil transfer according to a phase-space measure.
- 1st emission also ME corrected.
Gluon emission from colour dipoles

Differential cross section for a $\tilde{k}\tilde{\ell} \rightarrow k\ell$ dipole splitting:

$$d\mathcal{P}_{\tilde{k}\tilde{\ell} \rightarrow k\ell} = \frac{d\sigma_{0 \rightarrow k\ell}}{d\sigma_{0 \rightarrow \tilde{k}\tilde{\ell}}} = \frac{\alpha_s(\mu_R)}{2\pi} D_{\tilde{k}\tilde{\ell} \rightarrow k\ell}(p_\perp, y) \frac{dp_\perp^2}{p_\perp^2} dy$$

Studying the pure final-state case: $\tilde{k} = k = f$, $\tilde{\ell} = \ell = \bar{f}'$, $f = q, g$

$$d\Gamma_{0 \rightarrow f g \bar{f}'} \cong d\Gamma_{0 \rightarrow f \bar{f}'} \frac{C\alpha_s}{2\pi} \hat{D}_{f \bar{f}' \rightarrow f g \bar{f}'}(p_\perp, y, \varphi) \frac{dp_\perp^2}{2\pi} dy \frac{d\varphi}{y^2}$$

using dipole phase-space and matrix-element factorization:

$$d\Phi_{0 \rightarrow f g \bar{f}'}(p_0; p_f, p_g, p_{\bar{f}'}) = d\Phi_{0 \rightarrow f \bar{f}'}(\tilde{p}_0 = p_0; \tilde{p}_f, \tilde{p}_{\bar{f}'}) \frac{ds_{f g} ds_{g f'}}{16\pi^2 M^2} \frac{d\varphi}{2\pi}$$

$$|\mathcal{M}_{0 \rightarrow f g \bar{f}'}|^2 \cong 8\pi\alpha_s C \hat{D}_{f \bar{f}' \rightarrow f g \bar{f}'} |\mathcal{M}_{0 \rightarrow f \bar{f}'}|^2$$

Instead of redefining ISR in terms of FSR,

$\tilde{k} \tilde{\ell} \rightarrow \begin{cases} \tilde{k} g_f \tilde{\ell} & : \text{gluon emission,} \\ q g_i \tilde{\ell} & : \text{quark emission, provided that } \tilde{k} = \tilde{q}_i , \\ \tilde{k} g_i \tilde{q} & : \text{antiquark emission, provided that } \tilde{\ell} = q_i . \end{cases}$
Gluon emission from colour dipoles

Generalizing the kinematic framework:

- momentum balances, \( \varsigma_0 = \pm 1 \) (+ for II dipoles), \( \tilde{\varsigma}_m, \varsigma_m = \pm 1 \) (+ outgoing, − incoming),

\[
-\varsigma_0 \tilde{p}_0 = \tilde{\varsigma}_k \tilde{p}_k + \tilde{\varsigma}_\ell \tilde{p}_\ell, \quad -\varsigma_0 p_0 = s_k p_k + s_g p_g + s_\ell p_\ell
\]

- dipole invariant masses,

\[
\tilde{p}_0^2 = p_0^2 = M^2 = s_{kg} + s_{g\ell} + s_{k\ell} = s_{kg\ell} = -Q^2
\]

- invariant energy fractions and parton-system invariant masses,

\[
x_m = \frac{2 p_m p_0}{p_0^2}, \quad s_{mn} = (s_m p_m + s_n p_n)^2, \quad s_{mnr} = (s_m p_m + s_n p_n + s_r p_r)^2
\]

Lorentz invariant evolution variables:

\[
p_\perp^2 = \left| \frac{s_{kg} s_{g\ell}}{s_{kg\ell}} \right|, \quad y = \frac{1}{2} \ln \left| \frac{s_{g\ell}}{s_{kg}} \right|; \quad |s_{kg}| = |M| p_\perp e^{-y}, \quad |s_{g\ell}| = |M| p_\perp e^{+y}
\]
Gluon emission from colour dipoles

Similarly apply to II dipoles:

\[
\begin{align*}
    d\sigma_{\bar{\nu}'(g)\to 0g(0\bar{q})} & \simeq d\sigma_{\bar{\nu}'(\bar{q})\to 0}(dy_{cm}) \frac{f_{\bar{\nu}'(g)}(x_{\pm}, \mu_F)}{f_{\bar{\nu}'(\bar{q})}(\bar{x}_{\pm}, \bar{\mu}_F)} \frac{f_i(x_{\mp}, \mu_F)}{f_i(\bar{x}_{\mp}, \bar{\mu}_F)} \frac{M^4}{s_{\bar{\nu}'i(\bar{q}i)}(p_|, y)} \\
    & \times \frac{\xi C\alpha_s}{2\pi} \hat{D}_{\bar{\nu}'i(\bar{q}i)\to \bar{\nu}'g(\bar{q}g,i)}(p_|, y) \, dp_\perp^2 \, dy
\end{align*}
\]

\[y_{cm} = \ln(x_+/x_-)/2, \quad \bar{y}_{cm} = \ln(\bar{x}_+/?/\bar{x}_-)/2\]

And to FI dipoles as well:

\[
\begin{align*}
    d\sigma_{0i(0g)\to fg(f\bar{q})} & \simeq d\sigma_{0i(0\bar{q})\to f}(f_i(g)) \frac{f_i(g)(x_{\pm}, \mu_F)}{f_i(q)(\bar{x}_{\pm}, \bar{\mu}_F)} \frac{Q^4}{[s_{fg(f\bar{q})}(p_|, y) + Q^2]^2} \\
    & \times \frac{\xi C\alpha_s}{2\pi} \hat{D}_{f_i(f\bar{q})\to fg(\bar{q}g,i)}(p_|, y) \, dp_\perp^2 \, dy
\end{align*}
\]
Colour dipole shower for hadronic collisions

- Formulate IS emission completely perturbatively through colour dipoles (partly) spanned by incoming parton lines.
- Radiation that is associated to initial, initial-final and final colour lines.
- Keep beam remnants outside evolution as long as hadronization has not set in.

Construction principles of perturbative CDM:
- new dipole types: $\bar{q}iq_i$, $giq_i$, $giGi$ and $qfq_i$, $qfGi$, $gfGi$.
- radiation pattern in terms of $2 \rightarrow 3$ splittings.
- generalization of the kinematics setup to the new cases $\rightarrow$ dipole phase-space factorization and invariant evolution variables.
- dipole ME factorization $\rightarrow$ re-calculate or use crossing symmetry of FF dipole MEs or use antenna functions.
- probabilistic interpretation of Sudakov form factor based on dipole splitting cross sections.
- large $N_C$ limit, onshell kinematics, for all ISR apply backward evolution.
Initial-state dipole evolution at a glance: $\bar{v}'i \rightarrow \bar{v}'g\bar{i}$

- Invariant transverse momentum and rapidity

$$p_{\perp}^2 = \left| \frac{s_{\bar{v}'g} s_{gi}}{s_{\bar{v}'gi}} \right| = \frac{\hat{t}}{M^2}, \quad y = \frac{1}{2} \ln \left| \frac{s_{gi}}{s_{\bar{v}'g}} \right| = \frac{1}{2} \ln \frac{\hat{u}}{\hat{t}}$$

- Phase space, $a = \hat{s}_{\text{max}}/M^2 \leq S/M^2$

$$|y| \leq \text{arcosh} \left( \frac{a - 1)M}{2p_{\perp}} \right) \leq \ln \left( \frac{(a - 1)M}{p_{\perp}} \right) = \ln \frac{1}{z}$$

- Dipole splitting function for gluon emission off $\pi$ dipoles,

$$D_{\bar{v}'i \rightarrow \bar{v}'g\bar{i}}(p_{\perp}, y) = \frac{f_{\bar{v}'}(x_{\pm}, \mu_F) f_i(x_{\mp}, \mu_F)}{f_{\bar{v}'}(\tilde{x}_{\pm}, \tilde{\mu}_F) f_i(\tilde{x}_{\mp}, \tilde{\mu}_F)} \xi_{\{F\}}^{\{A\}} C_{\{F\}}^{\{A\}} \frac{x_{\bar{v}'}^{n_{\bar{v}'}}(p_{\perp}, y) + x_i^{n_i}(p_{\perp}, y)}{[x_{\bar{v}'}(p_{\perp}, y) + x_i(p_{\perp}, y) - 1]^2}$$

$$\leq N_{\text{PDF}} \xi_{\{F\}}^{\{A\}} C_{\{F\}}^{\{A\}} \left\{ \begin{array}{c} 2 \\ a + 1 \end{array} \right\} \equiv D_{\bar{v}'i \rightarrow \bar{v}'g\bar{i}}^{\text{approx}}(p_{\perp}, y)$$

$$n_{q, g} = 2, 3; \quad \left\{ \begin{array}{l} \ldots \text{for quark dipoles} \\ \ldots \text{else} \end{array} \right\}$$
Initial-state dipole evolution at a glance:

**Invariant transverse momentum and rapidity**

\[ p_\perp^2 = \left| \frac{s_{qg_i} s_{g_i}}{s_{qg_i}} \right| = -\frac{\hat{t}\hat{s}}{M^2}, \quad y = \frac{1}{2} \ln \left| \frac{s_{g_i}}{s_{qg_i}} \right| = \frac{1}{2} \ln \frac{\hat{s}}{-\hat{t}} \]

**Phase space,**

\[ a = \frac{s_{\text{max}}}{M^2} \leq S/M^2 \]
\[ \text{arsinh} \left( \frac{M}{2p_\perp} \right) \leq y \leq \ln \frac{aM}{p_\perp} = \ln \frac{1}{z} \]

**Dipole splitting function for quark emission off II dipoles,**

\[ D_{\bar{q}_i \to qg_i}(p_\perp, y) = \frac{f_g(x_\pm, \mu_F) f_i(x_\mp, \mu_F)}{f_q(\bar{x}_\pm, \bar{\mu}_F) f_i(\bar{x}_\mp, \bar{\mu}_F)} T_R \frac{x_q^2(p_\perp, y) + x_i^{n_i}(p_\perp, y)}{[1 + x_q(p_\perp, y)]^2} \]

\[ \leq N_{\text{PDF}} T_R \left\{ \begin{array}{c} 2 \\ a + 1 \end{array} \right\} \equiv D_{\bar{q}_i \to qg_i}^{\text{approx}}(p_\perp, y) \quad n_{q,g} = 2, 3; \]

\[ \{ \ldots \text{for quark dipoles} \}
\[ \{ \ldots \text{else} \} \]
Final–initial dipole evolution at a glance: $f_i \rightarrow fg_i$

- Invariant transverse momentum and rapidity

$$p_{\perp} = \left| \frac{s_{fg} s_{gi}}{s_{fgi}} \right| = \frac{\hat{s} \hat{t}}{-Q^2}, \quad y = \frac{1}{2} \ln \left| \frac{s_{gi}}{s_{fg}} \right| = \frac{1}{2} \ln \left( -\frac{\hat{t}}{\hat{s}} \right)$$

- Phase space,

$$a = \frac{s_{\text{max}}}{Q^2} \leq x_B^{-1} - 1$$

$$\ln z = -\ln \frac{aQ}{p_{\perp}} \leq y \leq \text{arsinh} \left( \frac{Q}{2p_{\perp}} \right)$$

- Dipole splitting function for gluon emission off FI dipoles,

$$D_{fi \rightarrow fg i}(p_{\perp}, y) = \frac{f_i(-x_i x_B, \mu_F)}{f_i(x_B, \tilde{\mu}_F)} \xi \{ F \} C \{ F \} \left[ x_f(p_{\perp}, y) |^{n_f} + x_i(p_{\perp}, y) |^{n_i} \right] x_i^2(p_{\perp}, y)$$

$$\leq N_{PDF} \xi \{ F \} C \{ F \} \left\{ \frac{2}{2(a + 1)} \right\} \equiv D_{fi \rightarrow fg i}^{\text{approx}}(p_{\perp}, y)$$

$$n_{q,g} = 2, 3; \quad \left\{ \begin{array}{l} \ldots \text{for quark dipoles} \\ \ldots \text{else} \end{array} \right\}$$
Final–initial dipole evolution at a glance:  \( f q_i \rightarrow f g_i \bar{q} \)

- **Invariant transverse momentum and rapidity**
  \[
P_{\perp}^2 = \left| \frac{s_f g_i s_{g_i \bar{q}}}{s_f g_i \bar{q}} \right| = \frac{\hat{u} \hat{t}}{Q^2}, \quad y = \frac{1}{2} \ln \left| \frac{s_{g_i \bar{q}}}{s_{f g_i}} \right| = \frac{1}{2} \ln \frac{\hat{t}}{\hat{u}}
\]

- **Phase space,** \( a = \hat{s}_{\text{max}}/Q^2 \leq x_B^{-1} - 1 \)
  \[
  \text{arcosh} \left( \frac{Q}{2 P_{\perp}} \right) \leq |y| \leq \text{arcosh} \left( \frac{(a+1)Q}{2 P_{\perp}} \right) \leq \ln \left( \frac{(a+1)Q}{P_{\perp}} \right) = \ln \frac{1}{z}
\]

- **Dipole splitting function for antiquark emission** off FI dipoles,
  \[
  D_{f q_i \rightarrow f g_i \bar{q}}(p_{\perp}, y) = \frac{f_{g_i}(-x_{g_i} x_B, \mu_F)}{f_{q_i}(x_B, \tilde{\mu}_F)} T_R \left[ x_f(p_{\perp}, y) |n_f + x_q^2(p_{\perp}, y) \right]^{n_{g_i}(p_{\perp}, y)}
\]

\[
\leq \mathcal{N}_{PDF} T_R \left\{ \max\{2, a + 1\} \right\} \equiv D_{f q_i \rightarrow f g_i \bar{q}}^{\text{approx}}(p_{\perp}, y)
\]

\( n_{q,g} = 2, 3; \quad \{\ldots \text{for quark dipoles}\} \quad \{\ldots \text{else}\} \)
Sudakov exponentiation, kinematics and showering

Rule of thumb: no-branching probability exponentiates

\[ \Delta(p_{\perp, \text{stt}}, p_{\perp}^2) = \exp \left\{ - \int_{p_{\perp}^2}^{p_{\perp, \text{stt}}} \frac{dk_{\perp}^2}{k_{\perp}^2} \mathcal{I}(k_{\perp}^2) \right\}, \]

\[ \mathcal{I}(k_{\perp}^2) = \frac{\alpha_s[\mu_R(k_{\perp})]}{2\pi} \sum_{\{k' \to k, g, \ell\}} \int_{y-(k_{\perp}, a)}^{y+(k_{\perp}, a)} dy \, D_{k' \to k, g, \ell}(k_{\perp}, y) \]

Differential probability that branching occurs at \( p_{\perp}^2 \)

\[ \frac{dP}{dp_{\perp}^2} = \frac{d\Delta(p_{\perp, \text{stt}}, p_{\perp}^2)}{dp_{\perp}^2} = \frac{\mathcal{I}(p_{\perp}^2)}{p_{\perp}^2} \Delta(p_{\perp, \text{stt}}, p_{\perp}^2) \]

Monte Carlo method (Veto Algorithm) yields values for evolution variables.

Set up kinematics: use light-cone variables and recoil strategies.

Shower algorithm fully defined by fixing:

- renormalization and factorization scale choices.
- the initializing scale in dependence on the hard process.
- the maximal phase space of a single emission globally.
- the iteration procedure to generate the cascade and the cut-off(s).
Results for pure final-state cascading

Testbed: hadron production in electron–positron annihilations @ LEP1

- Durham differential jet rates as a function of the jet-resolution parameter $y_{cut}$.
Examples of event-shape observables: 1-thrust (left panel) and sphericity.

Results for hadronic collisions

Testbed: inclusive production of Drell–Yan lepton pairs @ Tevatron Run I

- Boson transverse-momentum distribution and its peak region in $e^+e^- + X$.
Results for hadronic collisions

Testbed: Drell–Yan pair production @ LHC, compared to SHERPA predictions

- Pseudo-rapidity (left) and transverse-momentum (right) distribution of the first jet.
- The 1st emission in the dipole shower is ME corrected per construction.
Correlations ... inclusive jet production @ Tevatron


- Dijet azimuthal decorrelation measured by DØ at Run II.
- Idea: test QCD radiation pattern.

Jet production: Dijet azimuthal decorrelation

SHERPA

Tevatron Run 2

75 < p_{T,max} < 100 GeV

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Correlations ... inclusive jet production @ Tevatron


- Dijet azimuthal decorrelation measured by DØ at Run II.
- Idea: test QCD radiation pattern.

Jet production : Dijet azimuthal decorrelation

Jet production : Dijet azimuthal decorrelation

Jet production : Dijet azimuthal decorrelation

Jet production : Dijet azimuthal decorrelation

Tevatron Run 2

SHERPA

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Correlations ... inclusive jet production @ Tevatron


- Dijet azimuthal decorrelation measured by DØ at Run II.
- Idea: test QCD radiation pattern.

Jet production : Dijet azimuthal decorrelation

1/(σ_dijet) dσ/dΔφ_dijet

- DØ Run 2 data (2005)
- Dipole shower, II sc., SL
- Dipole shower, QCD sc., SL
- Dipole shower, low default sc., SL
- Dipole shower, default sc., SL

130 < p_T,max < 180 GeV

Tevatron Run 2
Correlations ... inclusive jet production @ Tevatron


- Dijet azimuthal decorrelation measured by DØ at Run II.
- Idea: test QCD radiation pattern.

Jet production : Dijet azimuthal decorrelation

Dipole shower, II sc., SL
Dipole shower, QCD sc., SL
Dipole shower, low default sc., SL
Dipole shower, default sc., SL

\[ \Delta \phi_{\text{dijet}} \]

\[ \frac{1}{\sigma_{\text{dijet}}} \frac{d\sigma}{d\Delta \phi_{\text{dijet}}} \]

Run 2 data (2005)
DØ Run 2 data (2005)

Tevatron Run 2

180 < \( p_{T,\text{max}} \)

\( \Delta \phi_{\text{dijet}} \) [rad]

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Results for hadronic collisions

Testbed: inclusive jet production @ Tevatron Run I

Jet production: Dijet mass

\[ \text{mod}(\eta) < 1 \quad R_\parallel > 0.7 \]

DØ Run 1 data (1999)
Dipole shower, default sc., full PSP, SL

Colour coherence test: \( \eta \) 3rd jet

\( \text{mod}(\eta_{1,2}) < 0.7 \quad R_\parallel > 0.7 \quad \text{mod}(\eta_{1,2}) > 2.79 \text{rad} \quad E_{T,1(2,3)} > 110(10) \text{GeV} \)

CDF Run 1 detector-level data (1994)
Dipole shower, default sc., restr. PSP, SL
Dipole shower, default sc., full PSP, SL

\[ \frac{\text{(MC-data)}}{\text{data}} \]

-0.2 0 0.2
-4 -3 -2 -1 0 1 2 3 4
\( \eta_3 \)

-0.2 0 0.2
200 400 600 800 1000 1200 1400
M [GeV]

-0.2 0 0.2
-0.2 0 0.2

Results for hadronic collisions

Testbed: inclusive jet production @ Tevatron Run II

- Discrepancy between MC and CDF data: arXiv:0710.2372.
- Was interested what the new dipole shower would predict.