Beam divergence near IP and beam-beam effect

B. Jeanneret

ABP BB-meeting

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Goal of the study

- Evaluate the impact of the betatronic divergence of the 'strong' beam in the presence of a crossing angle and with considering the longitudinal distribution of the bunches (question raised by Stephane in view of HL-LHC)

- The existing 6D lens with crossing-angle disregards the divergence

- An estimator of the importance of the effect is presented

Parameters

	Nominal	High Lum
Bunch population	2×10^{11}	3×10^{11}
β_X*	0.5 m	0.2 m
β_y*	0.5 m	0.05 m
σ_z	0.075 m	0.075 m
$\Phi_{crossing}$	160 μrad	720 μrad
ε_n	3.75 μm	2.5 μm
σ*	16 μm	32 μrad
σ'_x*	8.2 μm	41 μm

2D - differential beam-beam kick, gaussian beams

$\sigma_x \neq \sigma_y$ Basseti-Erskine $z1 = x + iy \;,\; z2 = x\frac{\sigma_y}{\sigma_x} + iy\frac{\sigma_x}{\sigma_y}$

$$dk = \frac{\sqrt{2\pi}Nr_0}{A\gamma\sigma_s}\left[w(\frac{z_1}{B}) - e^{-C(x,y)}w(\frac{z_2}{B})\right]e^{-s^2/2\sigma_s^2}ds$$

$A = \sqrt{2\pi(\sigma_x^2 - \sigma_y^2)}$ $B = \sqrt{2(\sigma_x^2 - \sigma_y^2)}$ $C(x,y) = \frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}$

w(z) is the modified complex error function

Formula valid y>0, $\sigma_x > \sigma_y$, and stable for $(\sigma_x - \sigma_y)/\sqrt{\sigma_x\sigma_y} > 0.01$

$\sigma_x = \sigma_y$

$$dk = \sqrt{\frac{\pi}{2}}\frac{Nr_0}{\gamma\sigma_s r}\left[1 - e^{-r^2/2\sigma^2}\right]e^{-s^2/2\sigma_s^2}ds$$

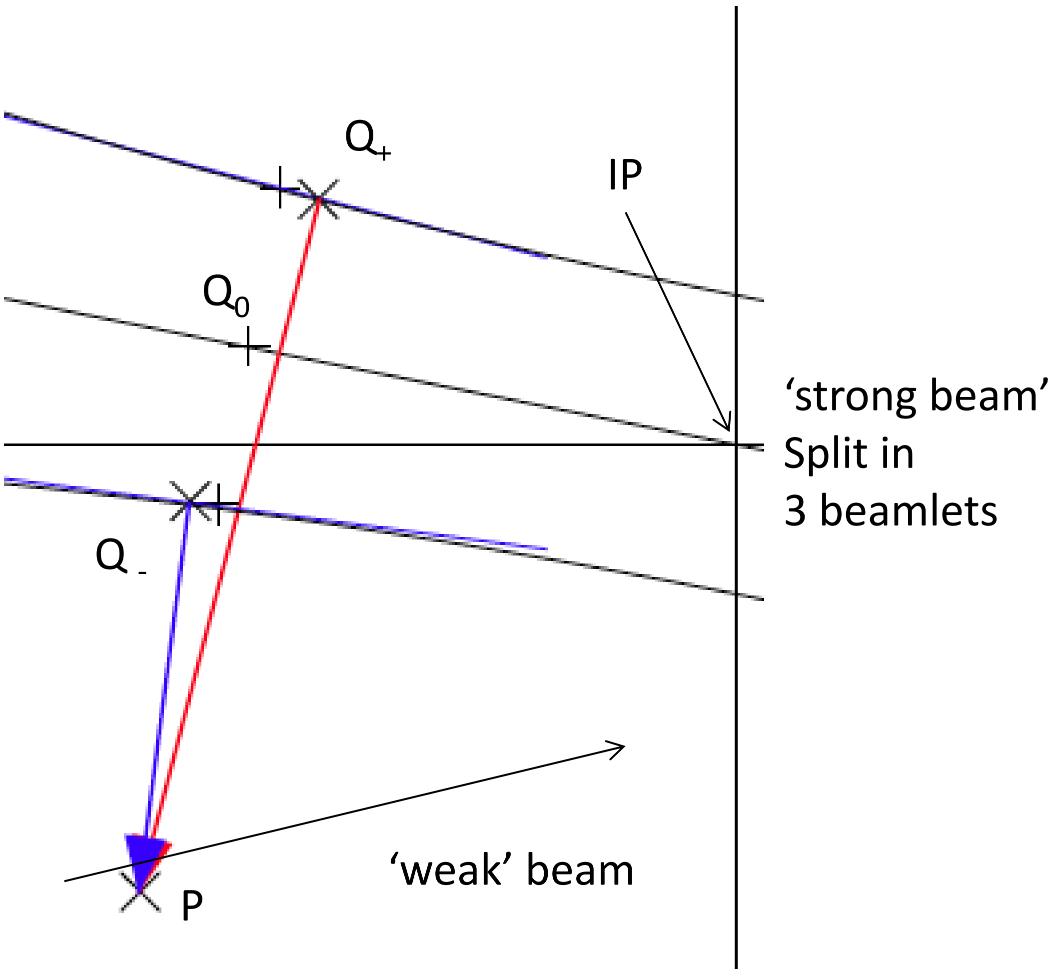
Small r :

$$dk = \frac{Nr_0 r}{\gamma\sigma^2}\frac{1}{\sqrt{2\pi}\sigma_s}e^{-s^2/2\sigma_s^2}ds$$

Method used

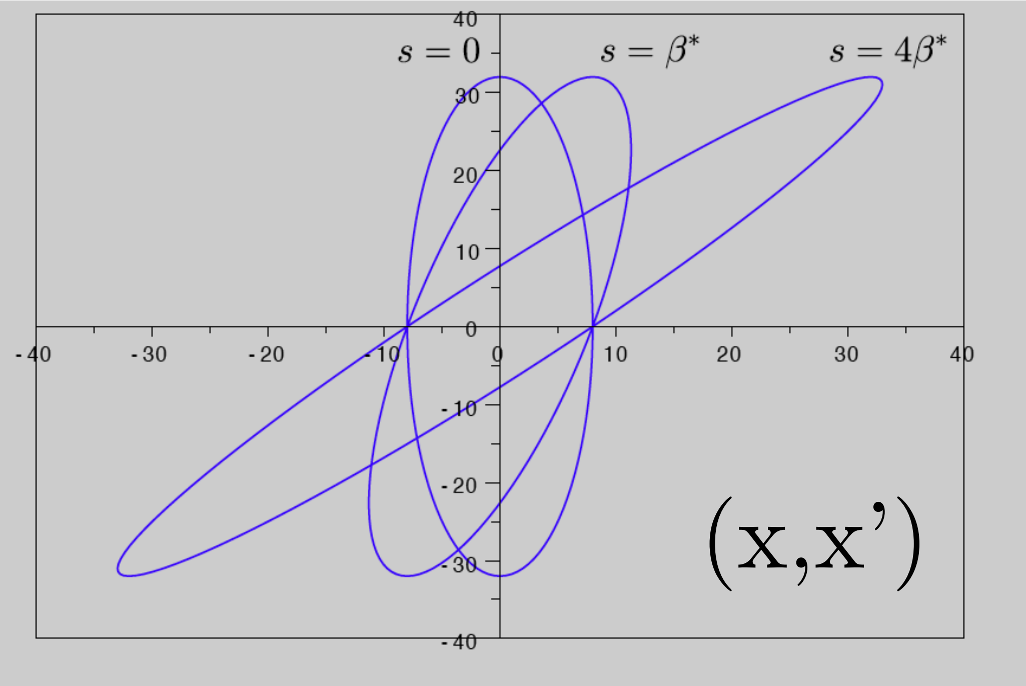
- In the crossing plane, the ‘Strong’ beam is split in 3 beamlets of
  - Null average divergence
  - Negative average divergence
  - Positive average divergence
- Each beamlet is parametrised by its average and r.m.s. value (we can compute a kick properly only for 2D-gaussian beams (Bassetti-Erskine)
- The three corresponding bb-kicks are added
- A tracking is made along the longitudinal coordinate

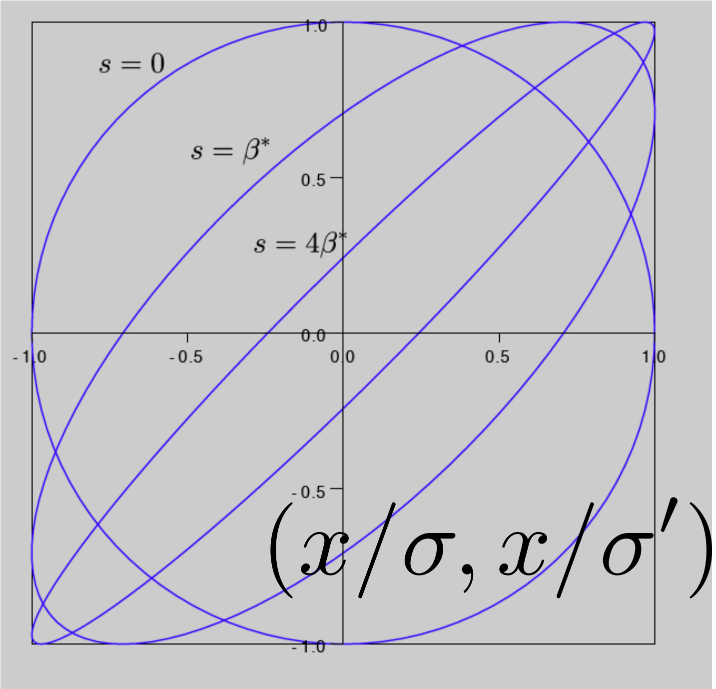
Beam divergence considered or not



- The kick is ⊥to the strong beamlet axis
- W/o div, $Q_{0,\pm}$ (+) are aligned with P
- With div (×) they are not :
  - The distances to P are changed
  - $Q_{\pm}$ move longitudinally, kick intensity is different

Phase space near the IP, High Lum

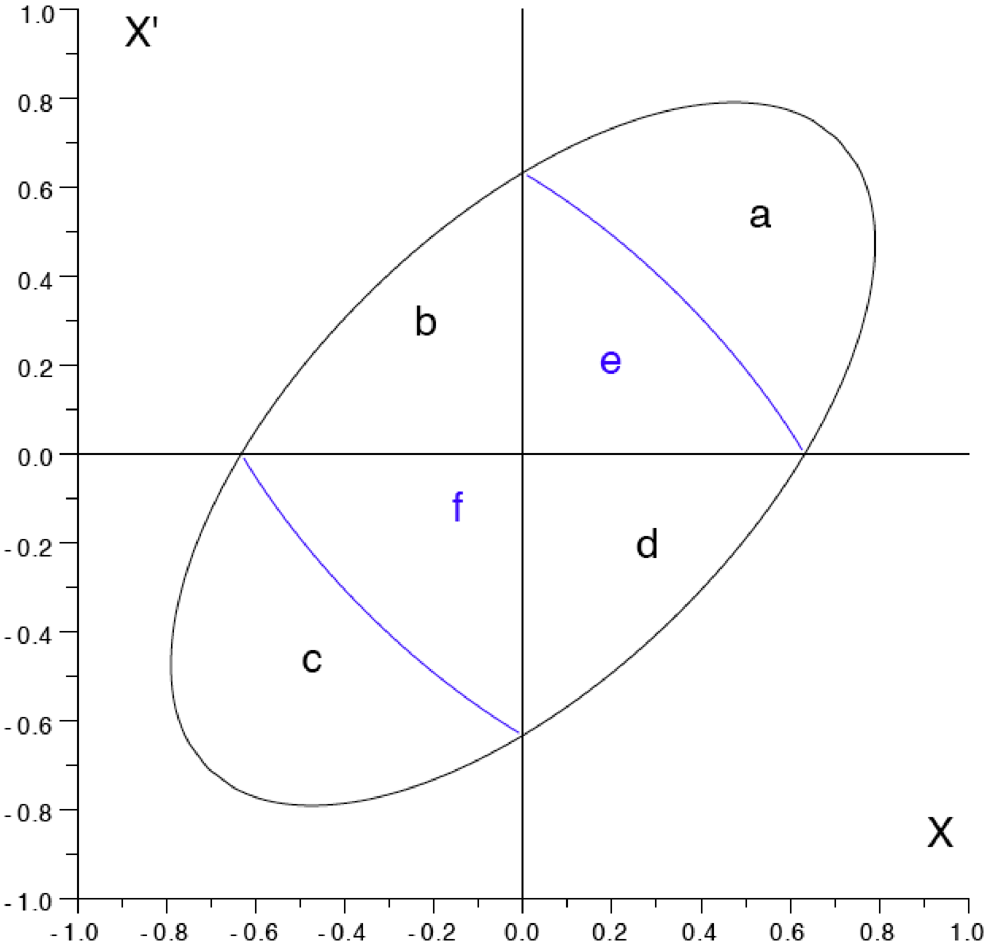




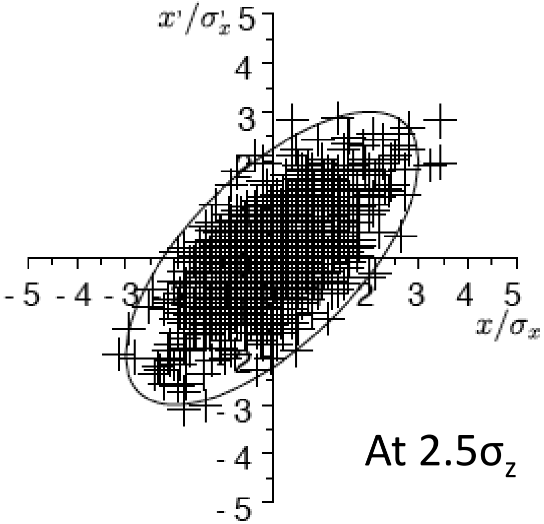
$$\begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix} = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \beta^* & \alpha^* \\ -\alpha^* & \gamma^* \end{pmatrix} \begin{pmatrix} 1 & 0 \\ s & 1 \end{pmatrix}$$

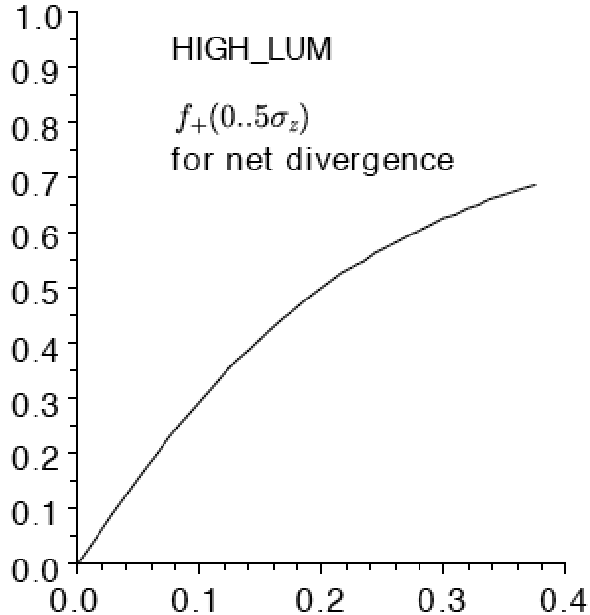
$$\sigma(s) = \sqrt{\epsilon_n \beta / \gamma_{\text{rel}}} \quad , \quad \sigma'(s) = \sqrt{\epsilon_n \gamma / \gamma_{\text{rel}}}$$

Building distributions with divergence



- Build a 2D-gauss distribution with x/x' correlation
- With the 4 sub-samples : a,b,c,d
  - Build e : with d and x'→-x'
  - Build f : with b and x→-x'
  - Sub-sample 1 with <x>=<x'>=0 : central area : b+e+f+d
  - Sub-sample 2 with <x> > 0 & <x'> > 0 : a-e
  - Sub-sample 3 with <x> < 0 & <x'> < 0 : c-f
- Get <x>,<x'>,σ(x), σ(x') for 1,2,3 as a function of z

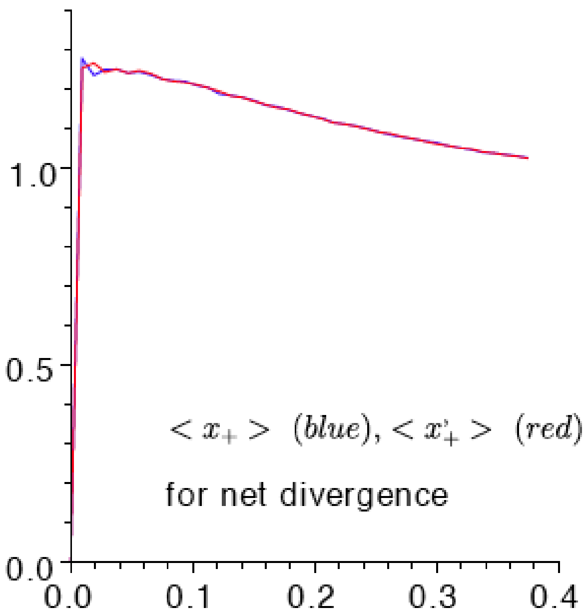


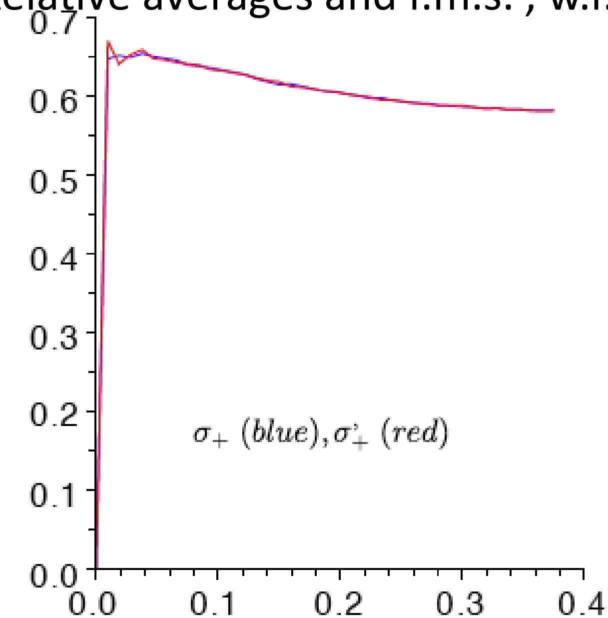


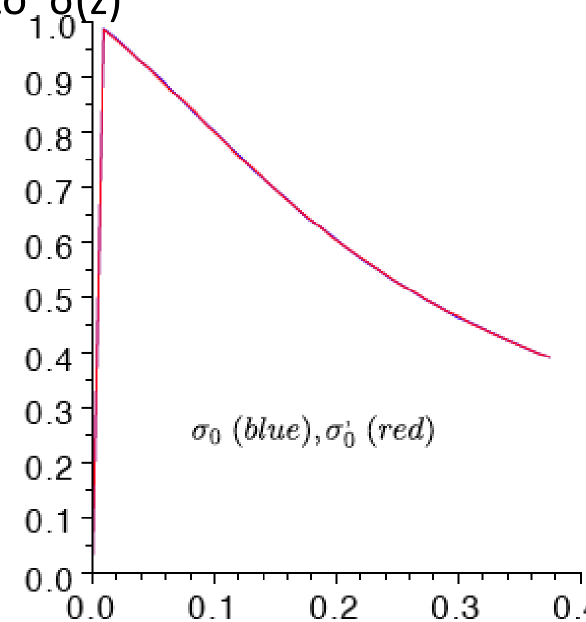
Monte-Carlo filling
of x-x' 'normalized'
phase-space
As a function of z.

Fraction for
sub-sample 1 : 1-f_+
sub-sample 2 and 3 :f_+ /2

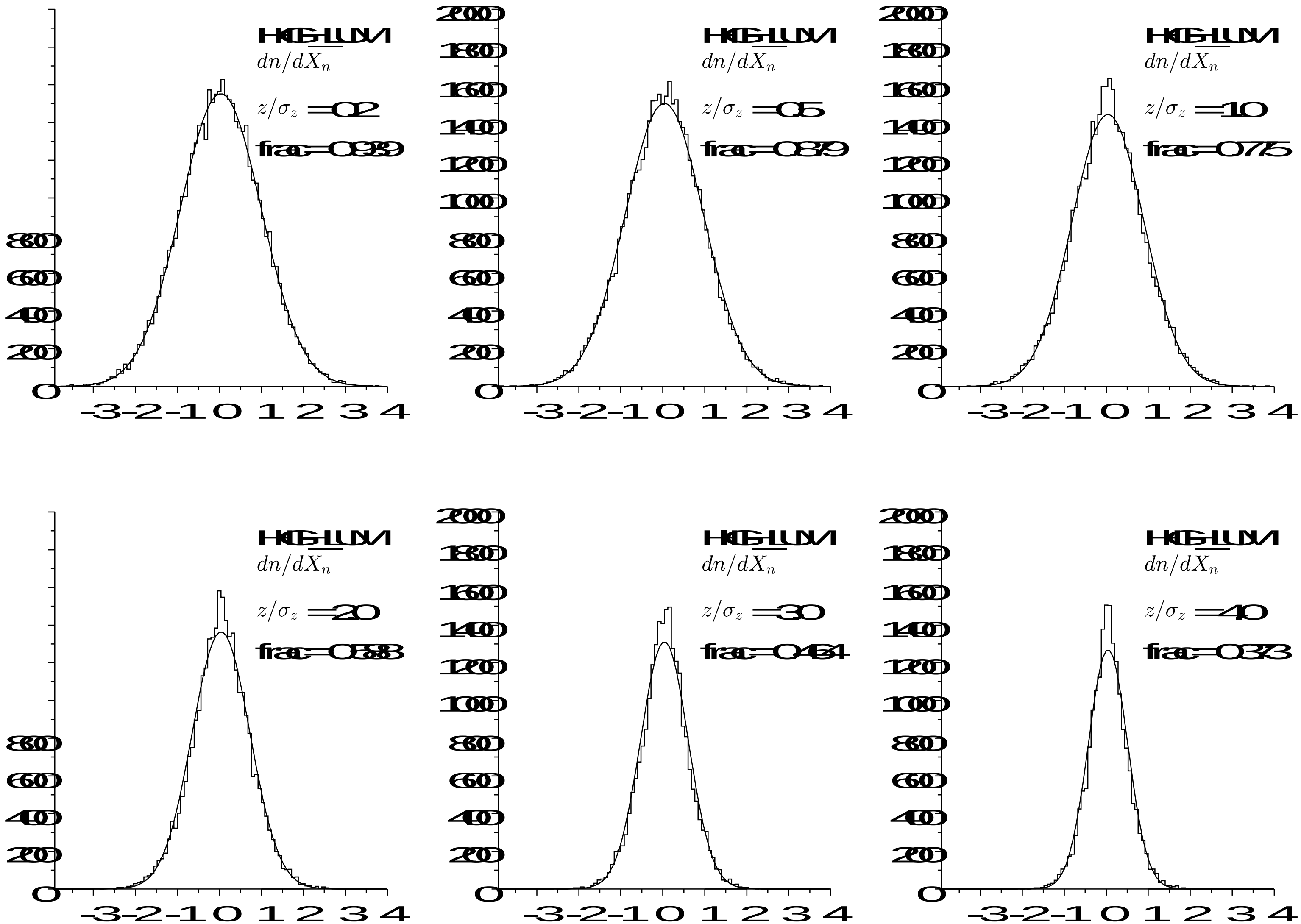
Relative averages and r.m.s. , w.r.t to $\sigma(z)$





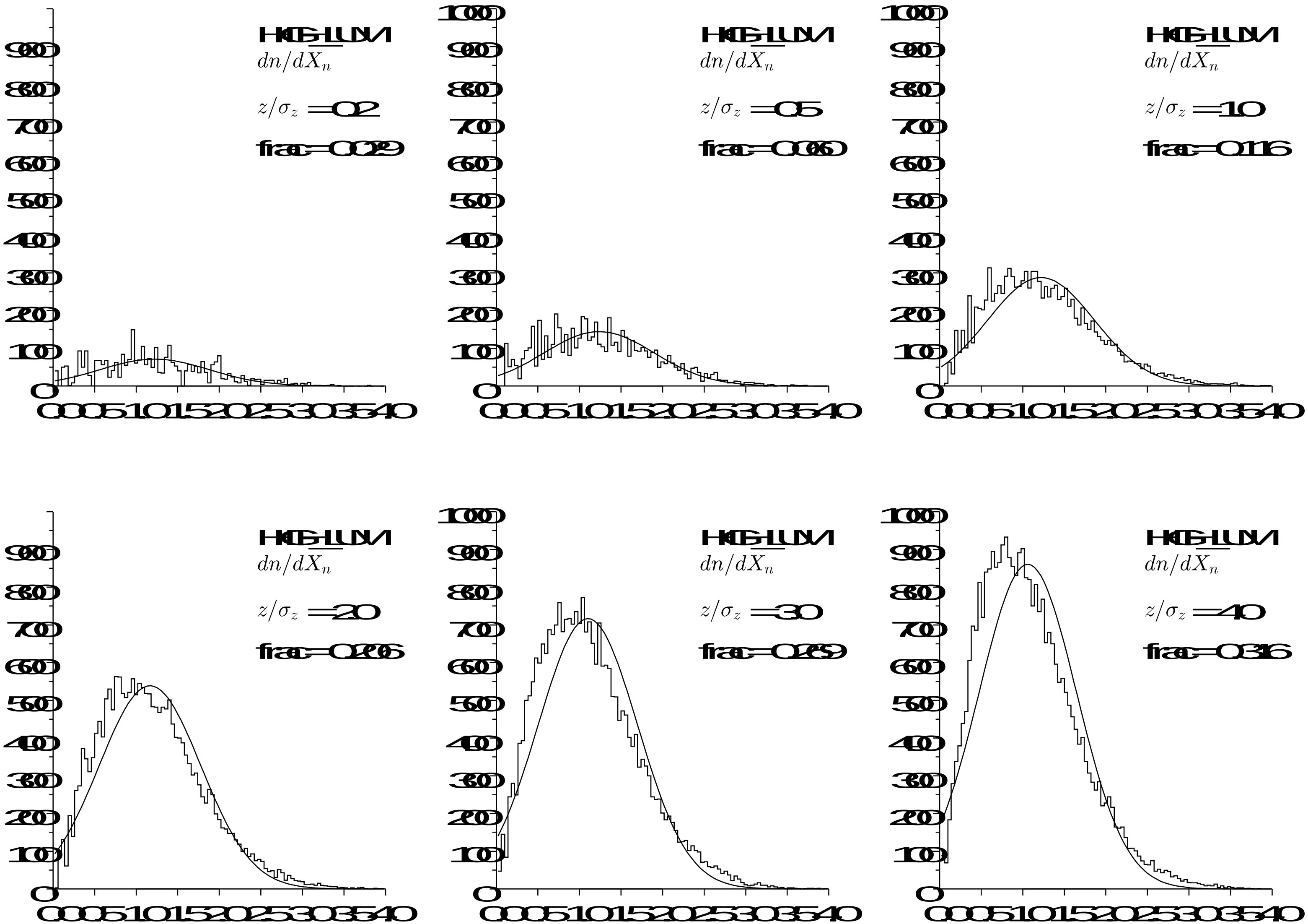


Central beamlet



Abscissa : x/σ_x

Beamlet of positive divergence

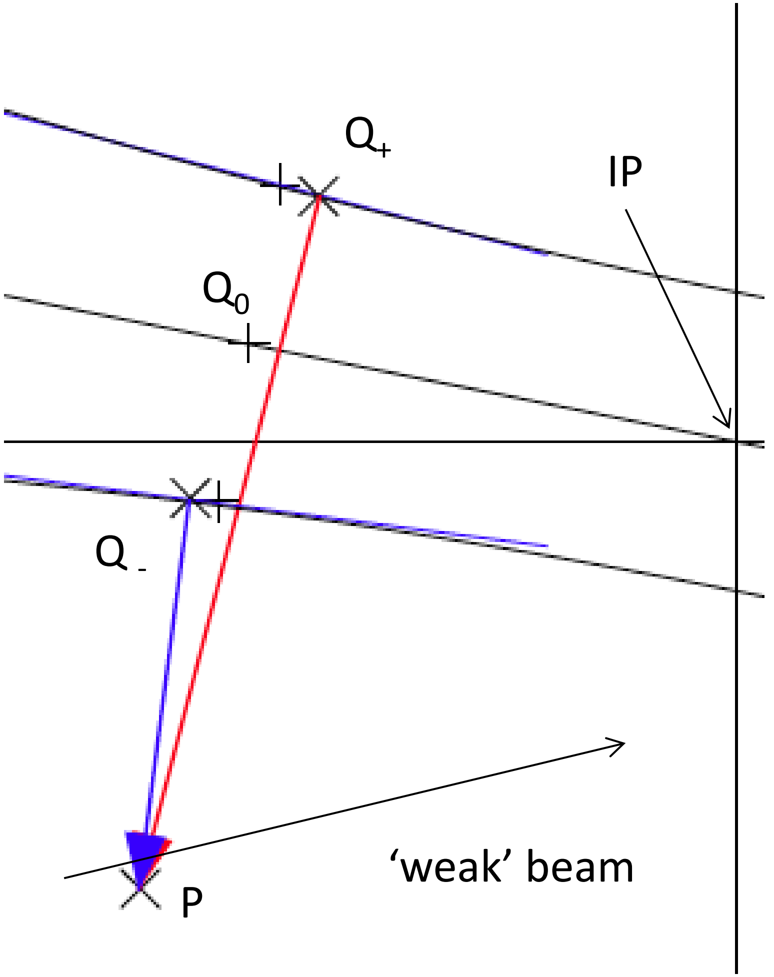


Abscissa : x/σ_x

Method used - II

- The decomposition in 3 gaussian beamlets is not perfect
- To compare adequately the two cases (div / no div)
  - The same beamlet decomposition is used for both cases
  - For the no-div case, $\langle x_{\pm}' \rangle = \langle x_0' \rangle = 0$

Tracking



- At P, with $Q_{0,\pm}P$, get

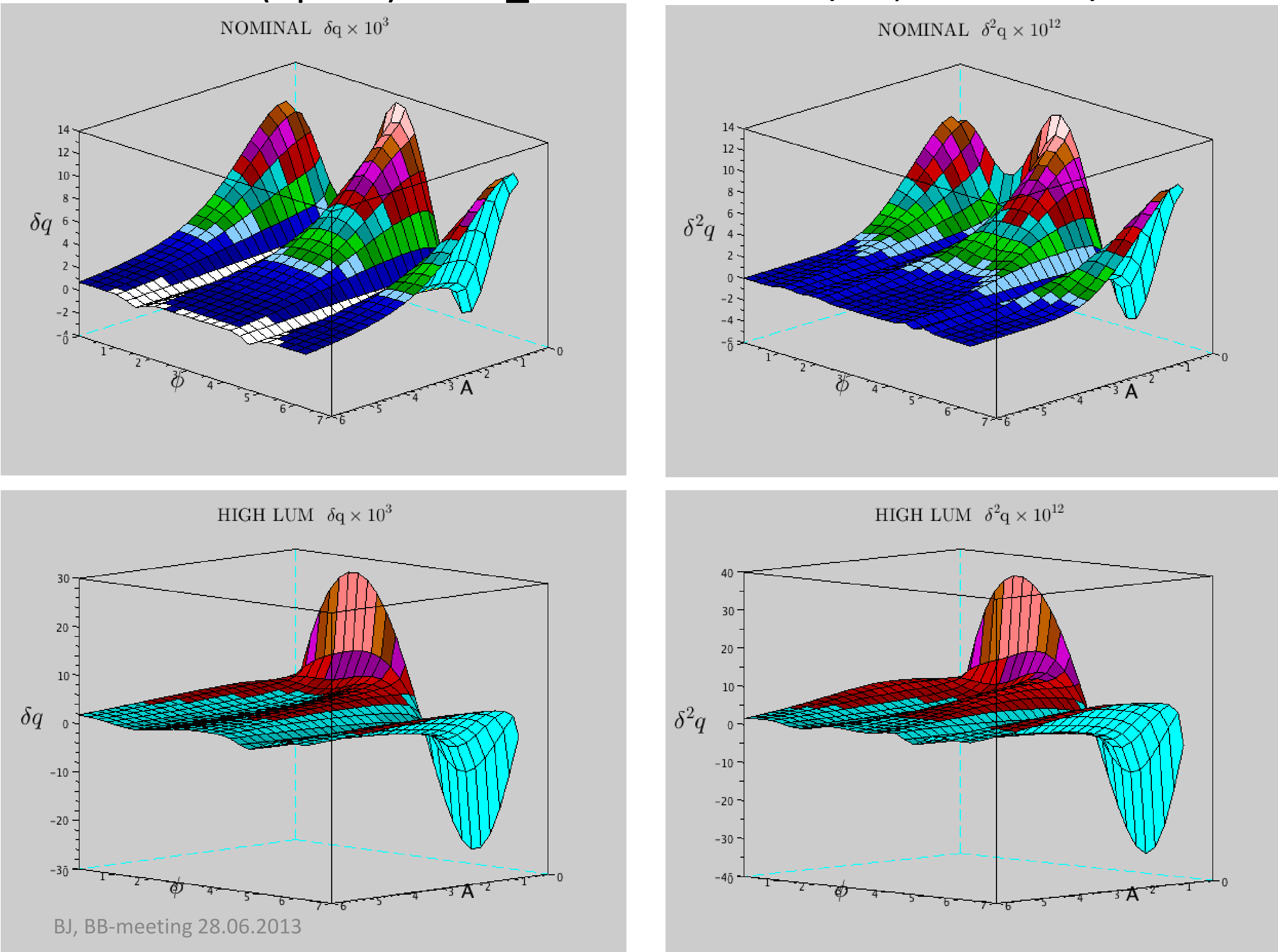
$$d\vec{k} = d\vec{k}_- + d\vec{k}_0 + d\vec{k}_+$$

- Update x', then x for step ds
- Iterate …
- This over $-4\sigma_s \rightarrow 4\sigma_s$
- And for A= [0 .. 6]×σ_x and ϕ = [0 .. 2π]
- Do everything twice
  - With divergence
  - Without divergence

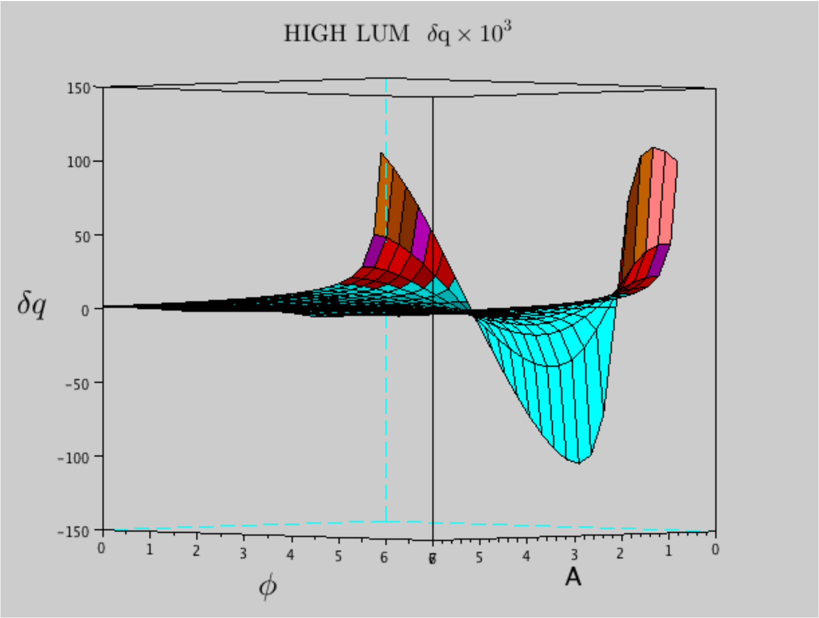
Results

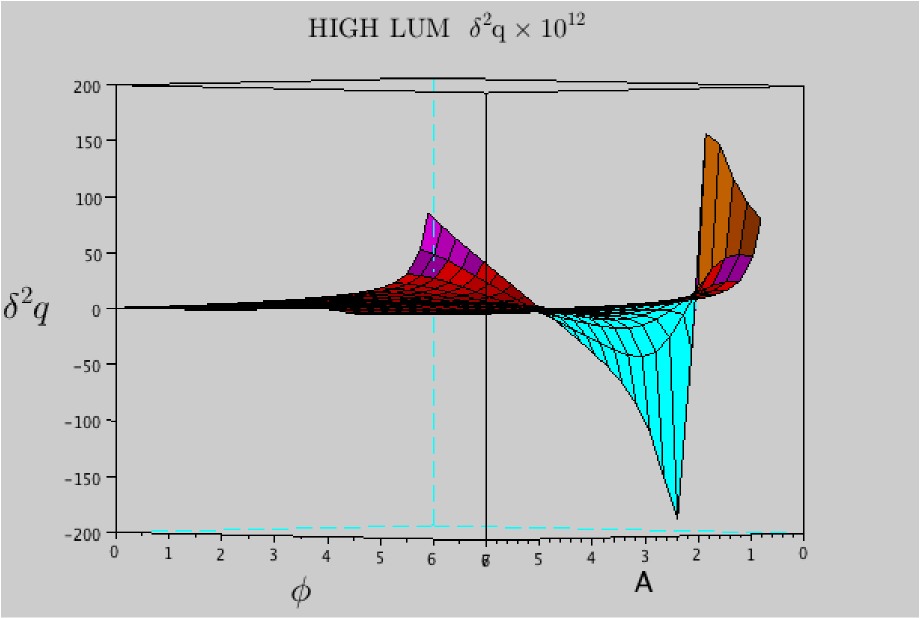
- Start with (x_0, x_0') at IP
- Drift back to $-4\sigma_z$
- Track to $+4\sigma_z$
- Drift back to IP : (x_1, x_1')
- Compute raw δQ as angle between (x_0, x_0') and (x_1, x_1')
- Get $\delta^2 Q = \delta Q_{div} - \delta Q_{no\text{-}div}$

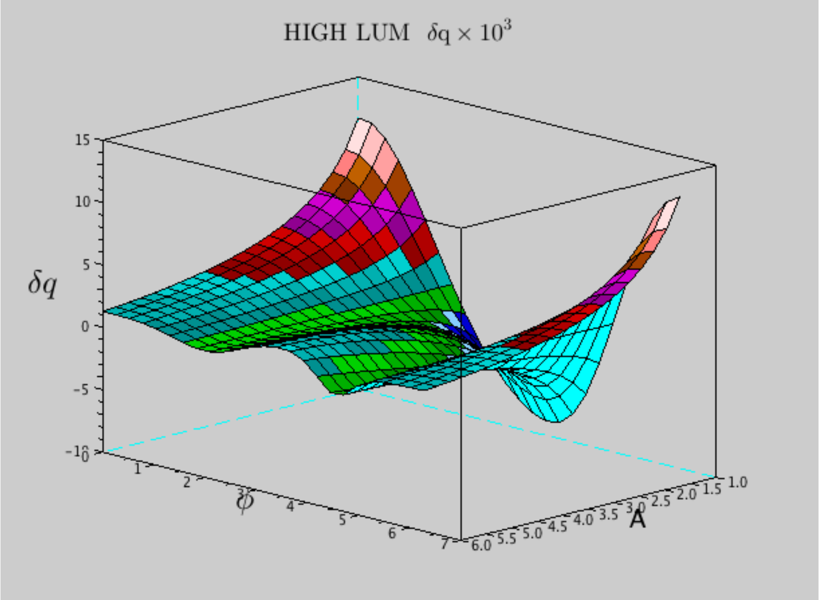
s = 0 (δp = 0) dQ_Thin : nom 6.5 o/oo, HL 19.5 o/oo

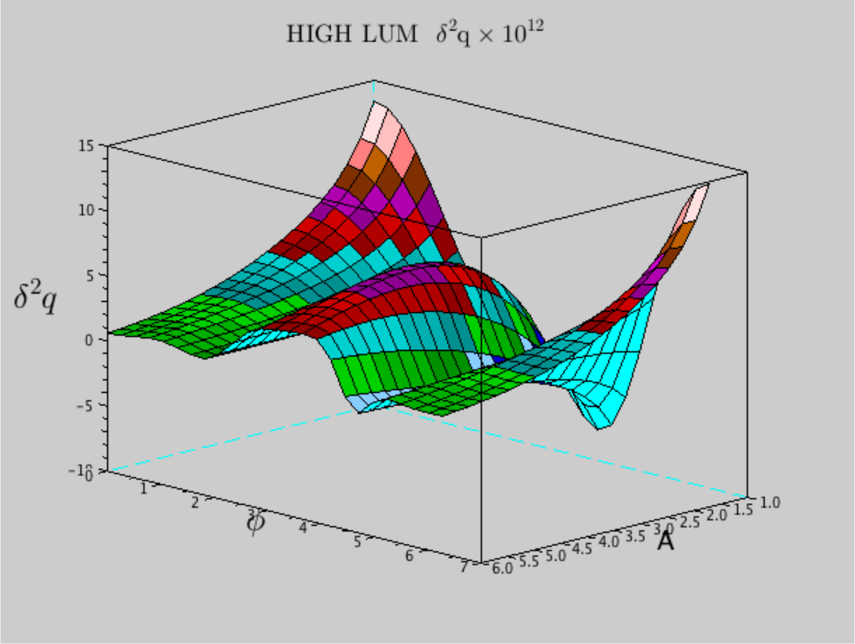


s = 1σ_s dQ_Thin : HL 19.5 o/oo



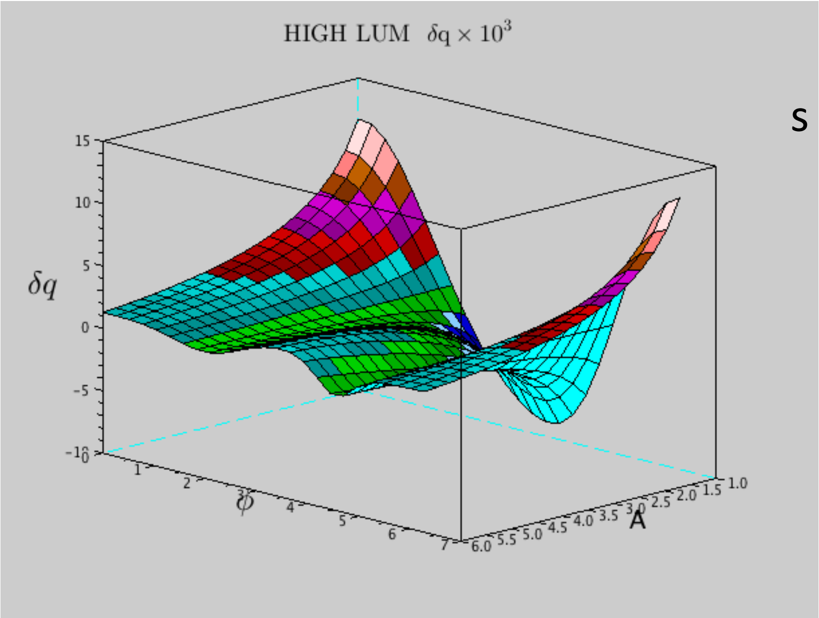


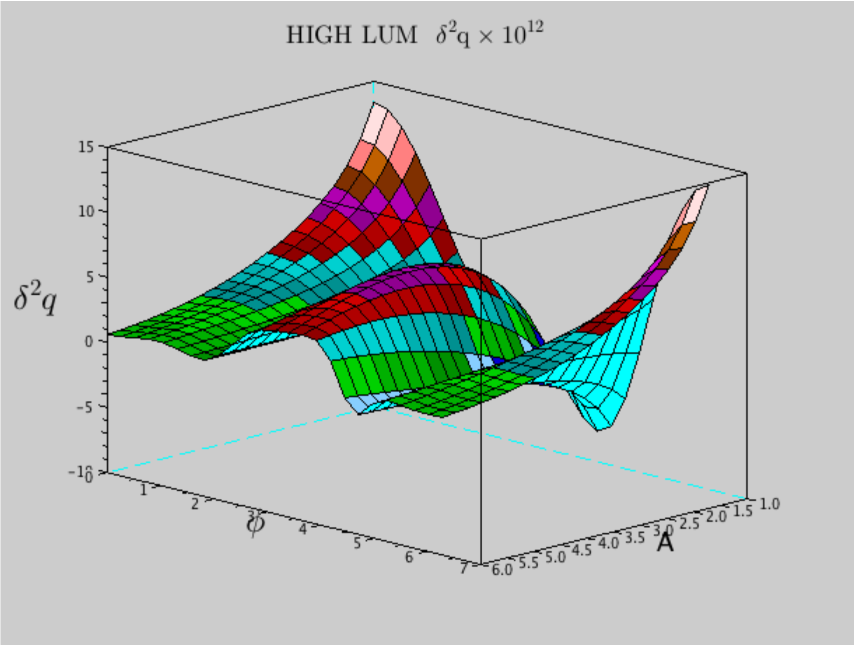




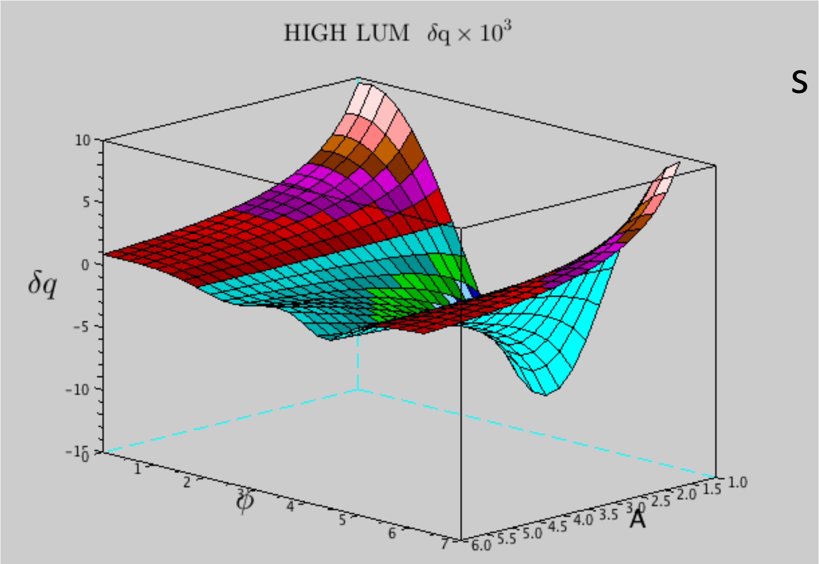
s = 1,2 σ_s dQ_Thin : HL 19.5 o/oo

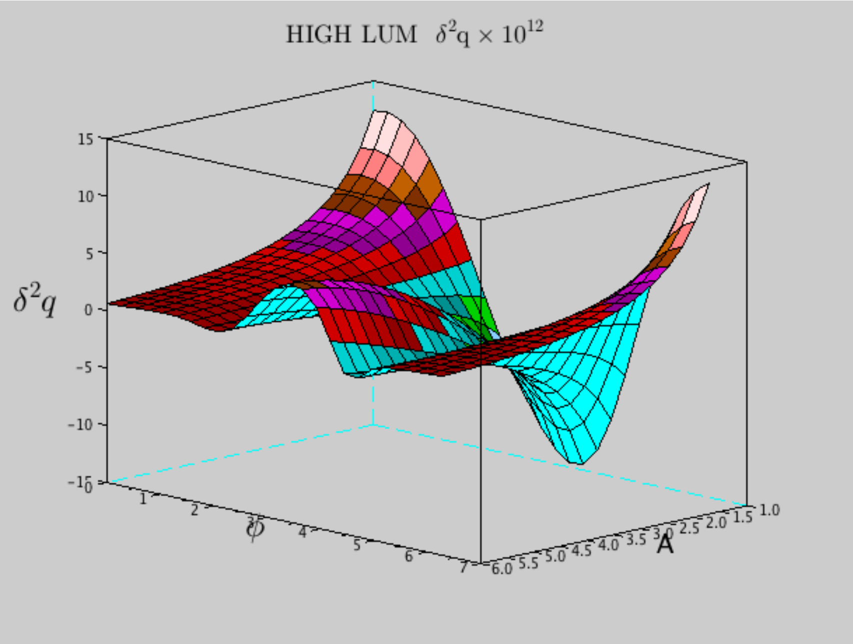
s = $1\sigma_s$



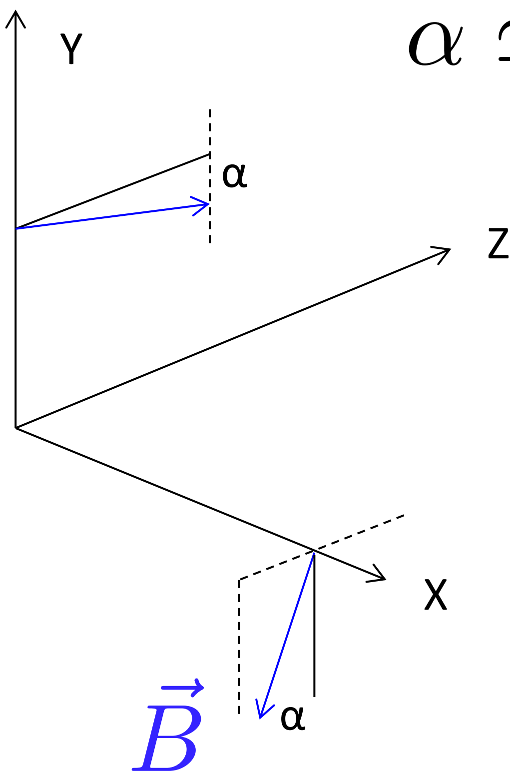


s = $2\sigma_s$





Divergence and vertical plane



$$\alpha \simeq \sigma{*}'_y$$

Diverging fraction, averaged over z : $f_{div} \cong 0.5$

$$\delta^2 k = \delta k - \delta k_0 = \frac{f_{\rm div}}{1+2f_{\rm div}} \frac{\alpha^2}{2} \delta k_0 \simeq \frac{\alpha^2}{8} \delta k_0$$

$$\vec{B} = B_0 \times (0, \cos\alpha, \sin\alpha)$$

$\vec{E}$ Independent of α

$$\Rightarrow \quad \delta k \sim (1 - \alpha^2/4)$$

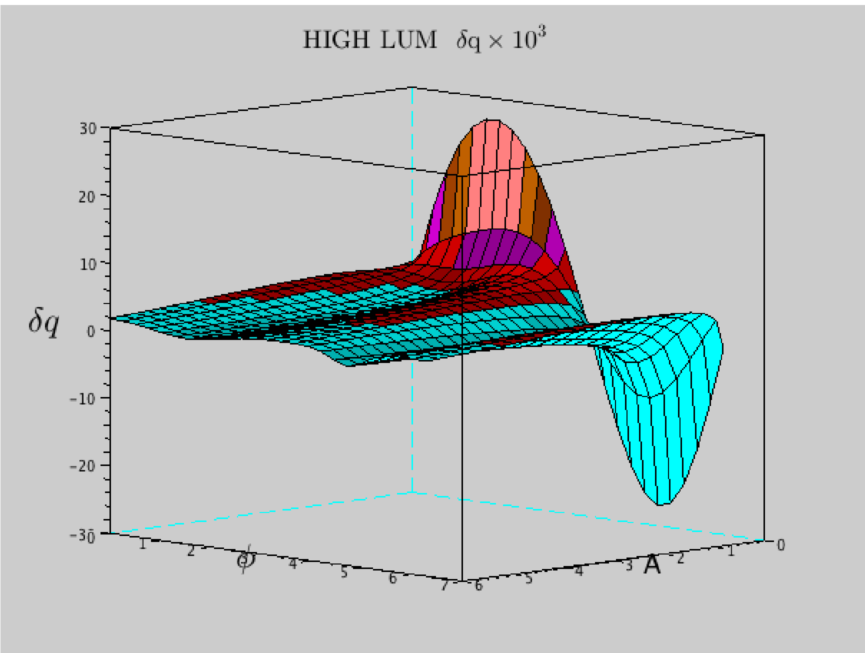
- Nominal : $\alpha = 3\times10^{-5} \rightarrow \alpha^2/8 = 0.13\times10^{-9}$
- High-Lum : $\alpha = 8\times10^{-5} \rightarrow \alpha^2/8 = 0.80\times10^{-9}$

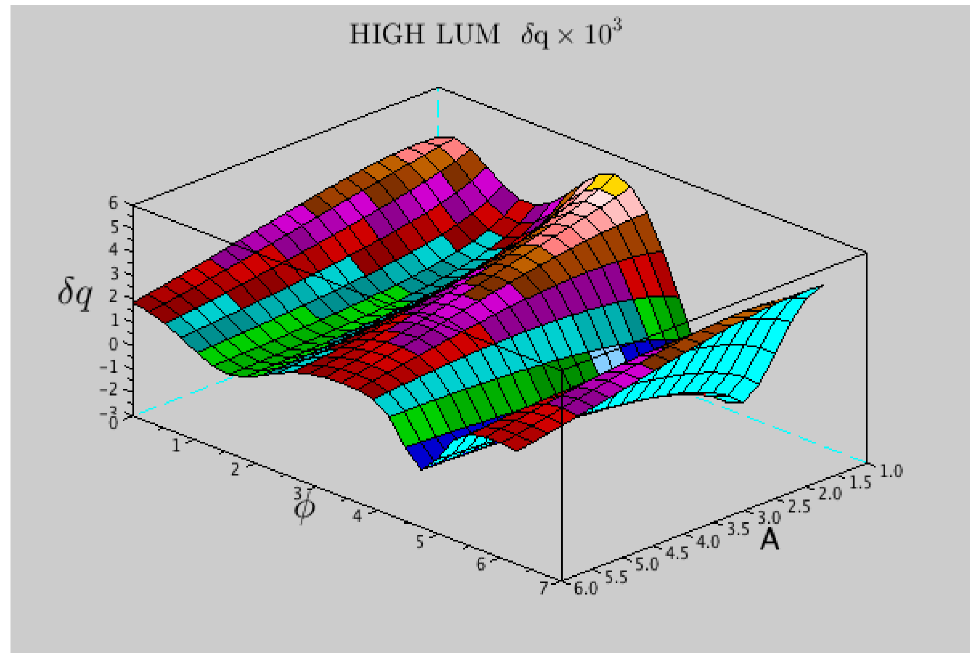
Results about divergence

- Considering the tune variations
  - In the crossing plane :
    - The difference between the two cases, divergence considered or not considered is $\delta^2 q_{rel} < 2\times10^{-9}$, both with NOMINAL and HL.
    - This difference similar when the average longitudinal position of the test particle w.r.t. to the strong bunch is changed ($0 \rightarrow 2\sigma_s$).
  - In the other plane :
    - The effect is 10× smaller with NOMINAL ,i.e. $\delta^2 q_{rel} \cong 0.13 \times10^{-9}$
    - The effect is 2× smaller with HL ,i.e. $\delta^2 q_{rel} \cong 0.8 \times10^{-9}$

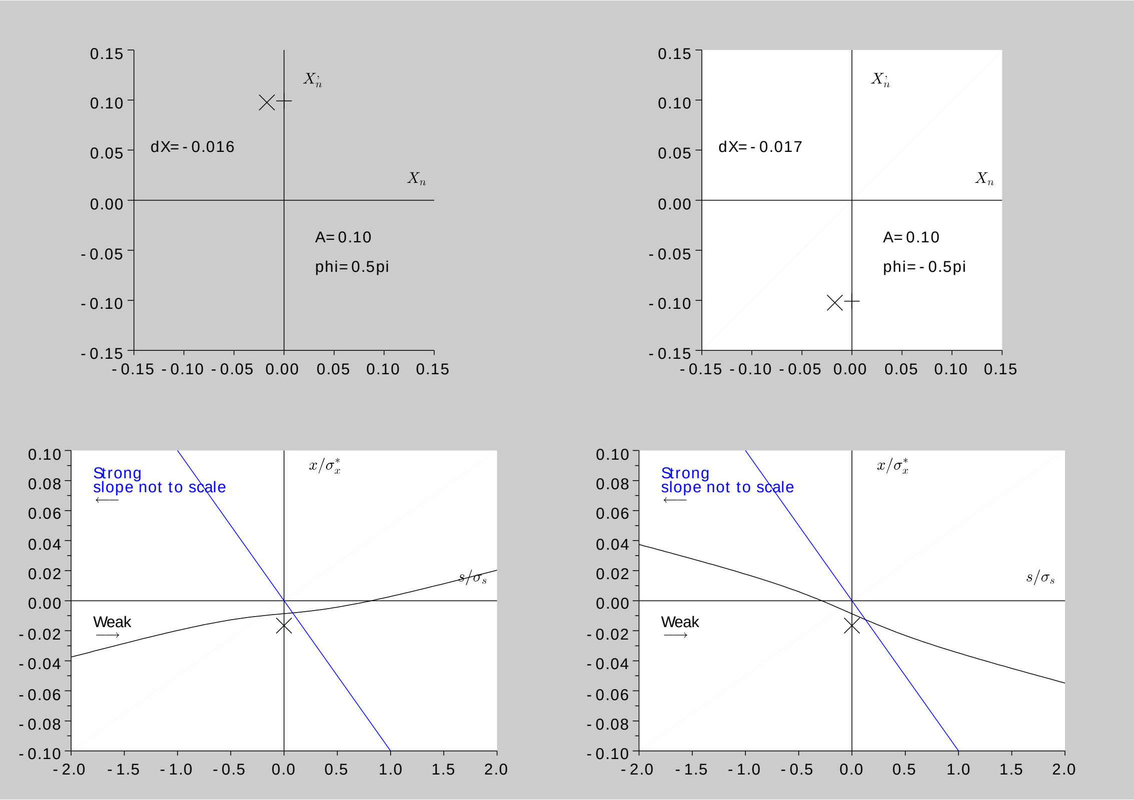
Small amplitude distortions, (independent of divergence effect)

The apparent dQ excursion at small amplitude and phase space angle ±π/2





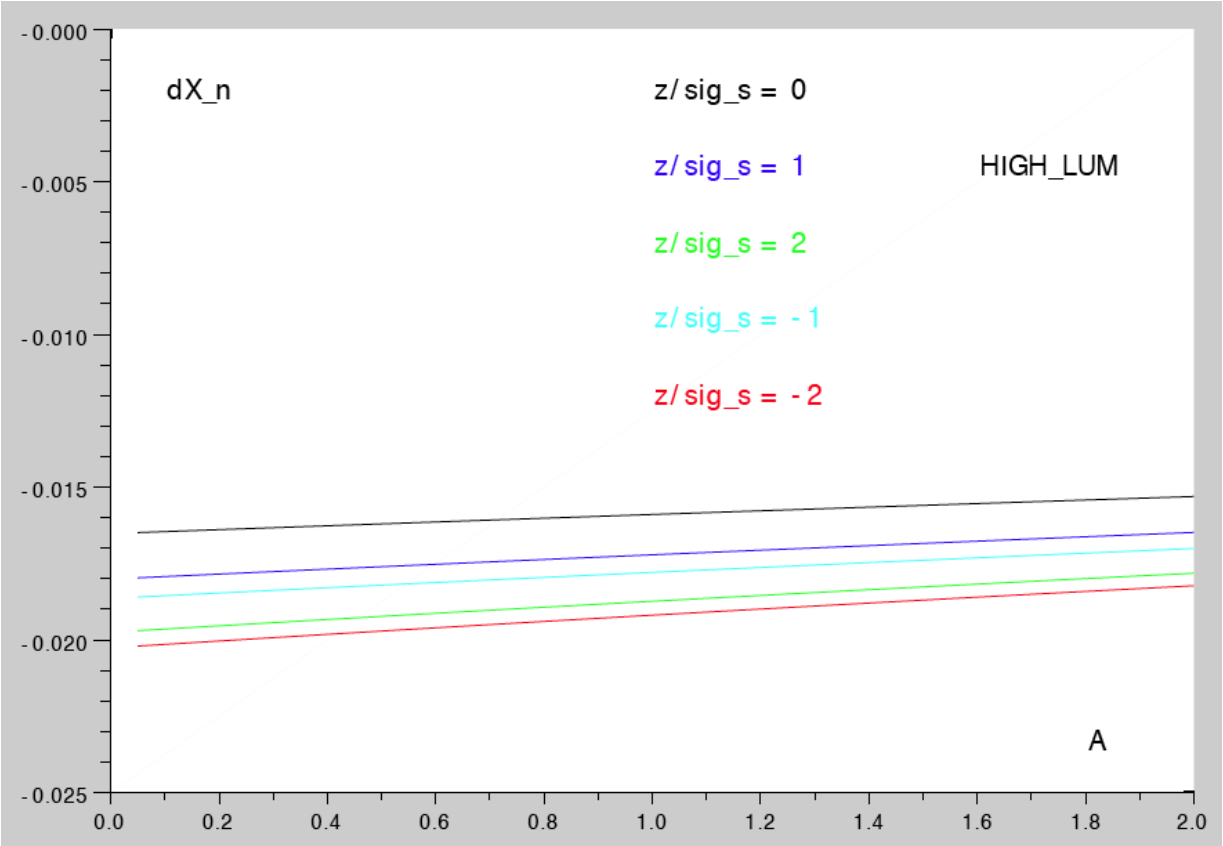
- The raw dQ grows without limit at towards small amplitude (x=0, x' ≠ 0), particularly marked with HL
- What happens ?



High_LUM

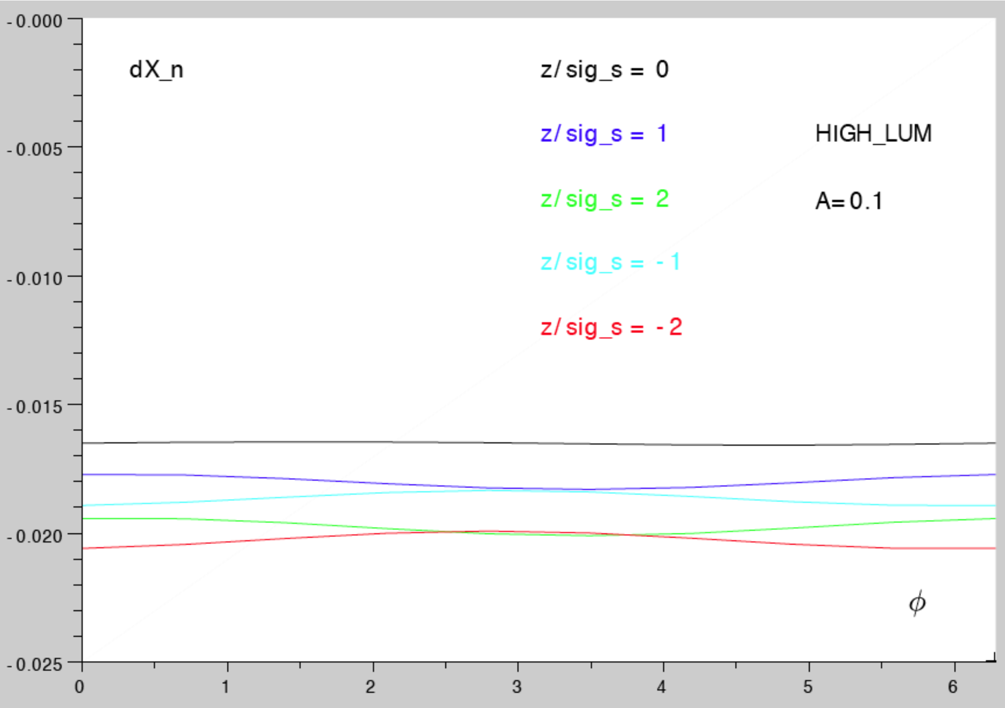
- With bunch length considered and crossing angle
  - A δX appear with tacking, with the same sign whatever the phase angle
  - So, this is an orbit effect
  - At High-LUM : δX= -0.016 σ*
  - The same applies to the other beam → collision mismatch of 3%σ*
  - Problematic ?

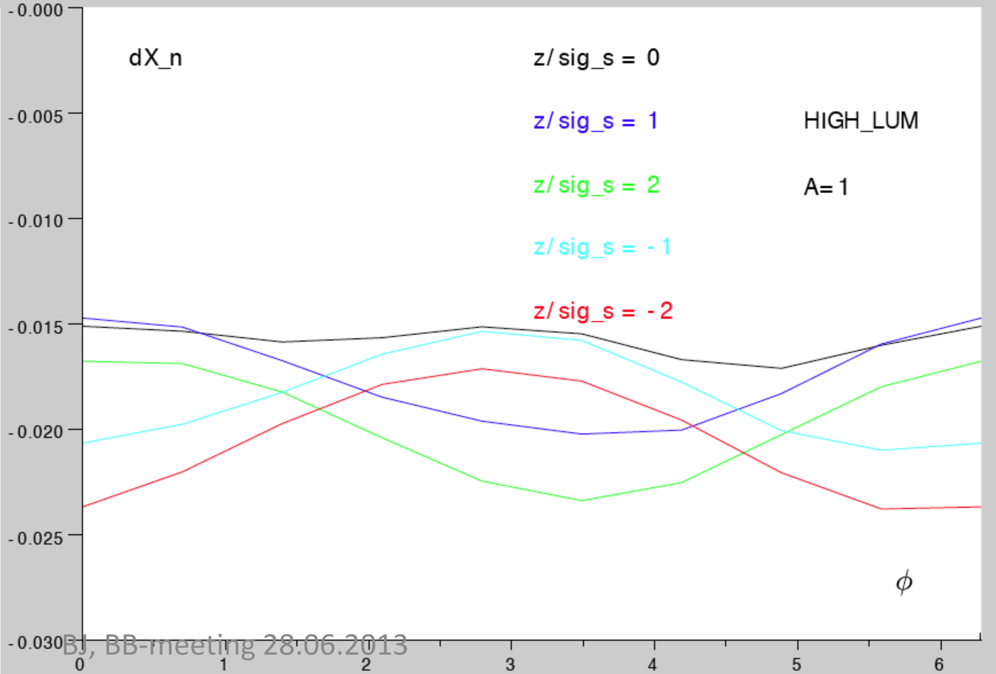
Beam displacement High-LUM



- Vary A, Φ=π/2
- Slight variations
  - with amplitude
  - and z-displacement (δp)

Beam displacement High-LUM - II

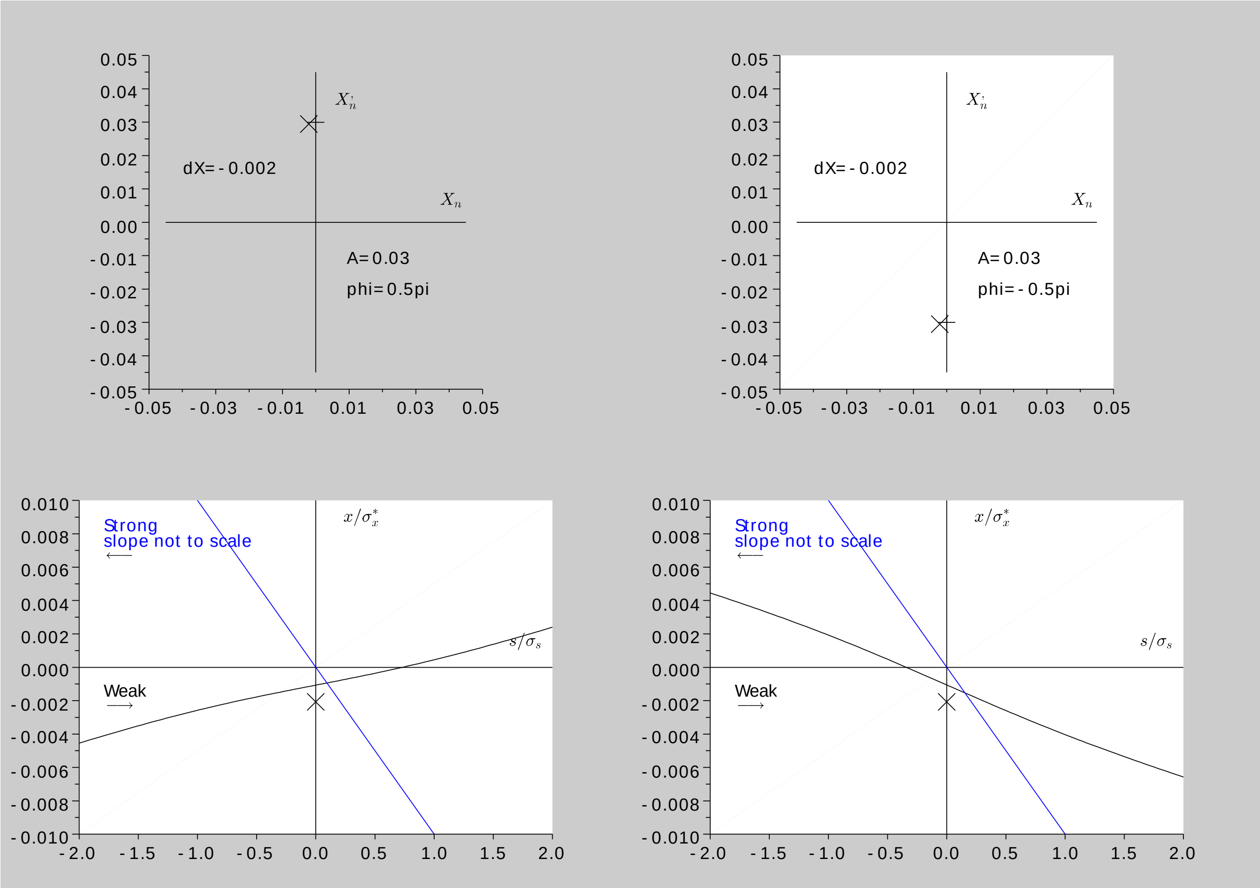




- Vary Φ
- Slight variations
  - with Φ (1% of σ)
  - with z-displacement (δp)

(2 ± 0.5) % σ* at A=1

Nominal



- Much smaller effect, of 2 o/oo σ*

Summary

- The effect of the betatronic divergence can be safely neglected ($\delta^2Q/\delta Q < 2 \times 10^{-9}$) with both nominal and high luminosity collision parameters
- Small orbit effect (partly amplitude & phase dependent) visible with ‘thick lens’ beam-beam tracking