(Higgs) Effective Field Theories

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The Nobel Prize in Physics 2013
François Englert, Peter W. Higgs

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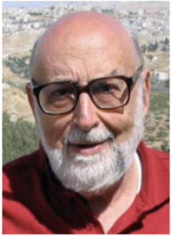
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Peter W. Higgs

The Nobel Prize in Physics 2013 was awarded jointly to François Englert and Peter W. Higgs *"for the theoretical discovery of a mechanism that contributes to our understanding of the origin of mass of subatomic particles, and which recently was confirmed through the discovery of the predicted fundamental particle, by the ATLAS and CMS experiments at CERN's Large Hadron Collider"*

Pre-LHC Experimental Summary

- Precision electroweak measurements and flavor physics consistent with the standard model and the GIM mechanism to high accuracy. [BaBar, BELLE, Fermilab, LHCb]
- Rare decays such as $B \to X_s \gamma$ which occur at loop level in the standard model agree with theory to within the errors (sub 10%).
- These were supposed to occur at rates at factors of 100 times the standard model rate (until they did not) in models such as SUSY.
- Flavor physics sector and CKM unitarity works well, including second order weak processes such as $K^0 - \overline{K}^0$, $B^0 - \overline{B}^0$ and $B_s^0 - \overline{B}_s^0$ mixing.
- Tevatron results on jets, Drell-Yan, etc. consistent with SM. [t asymmetry?]
- No direct tests on mechanism of EW symmetry breaking, but many indirect checks from precision EW, etc.

LHC Summary

- Standard Model provides a good description of all observations so far at the LHC at 7 and 8 TeV.
- A particle has been seen with a mass $M_h \sim 126$ GeV consistent with the Higgs boson of the standard model
- 0^+ quantum numbers favored
- Production rate times branching ratios consistent with the standard model, but with large error bars.
- No evidence for any new particles, mini black-holes, dimensions, etc. up to energies of ~ 1 TeV

HEFT

Assume that at the scale $M_h \sim 125$ GeV, the standard model including the scalar doublet H is a good description.

SM fields can be used to describe the physics. All new physics is then parameterized by higher dimensional gauge invariant operators made of standard model fields.

Can include additional particles as heavy fields, but the key point is that the EW symmetry breaking is the Higgs mechanism.

Can use non-linear realization of $SU(2) \times U(1)$ plus a light scalar. Discussed in other talks. Important to test this. Moves away from the Higgs mechanism assumption, but needs a light scalar.

At dimension 5, we have $\Delta L = 2$ operators which give neutrino masses. The scale Λ_5 is very high and does not affect Higgs physics.

For Higgs physics, the dominant effect is from dimension six operators due to new physics at some scale Λ, which is taken to be ~ 1 TeV.

If Λ is much higher than this, then the effects of NP become too small to see. [B violation.]

EFT

Fields are three generations of fermions Q, U, D, L, E, the scalar doublet H, and $SU(3) \times SU(2) \times U(1)$ gauge fields.

$$L = L_{SM} + \sum \frac{1}{\Lambda^n} L^{(4+n)} = L_{SM} + \frac{1}{\Lambda^2} L^{(6)} + \dots$$

L_{SM} contains dimension 4 interactions $\overline{\psi}\psi H$, $(H^\dagger H)^2$, $\overline{\psi} \not{D} \psi$, etc. and the dimension two mass term

$$L = -\lambda \left(H^\dagger H - \frac{v^2}{2} \right)^2 = -\lambda (H^\dagger H)^2 + \lambda v^2 H^\dagger H - \frac{1}{4} \lambda v^4$$

v is the only dimensionful parameter in L_{SM} (classically), and

$$m_f \propto v, \qquad M_{W,Z} \propto v$$

$m_p \propto \Lambda_{\mathrm{QCD}}$ (origin of mass)

Power Counting

$$L^{(6)} \sim \frac{m_H^2}{\Lambda^2} \qquad L^{(8)} \sim \frac{m_H^4}{\Lambda^4} \qquad L^{(6)} \times L^{(6)} \sim \frac{m_H^4}{\Lambda^4}$$

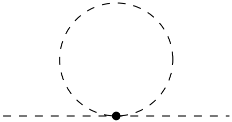
- Essential for using an EFT — it gives you an expansion parameter.
- The ... in L are there, otherwise there would be infinities.

$$\frac{m_H^4}{\Lambda^4} \times \infty \text{ is not small}$$

- Need to use a mass independent subtraction scheme such as $\overline{\text{MS}}$.

Otherwise

$$L^{(6)} \sim \frac{\Lambda^2_{\text{cutoff}}}{\Lambda^2} \qquad L^{(8)} \sim \frac{\Lambda^4_{\text{cutoff}}}{\Lambda^4} \qquad L^{(6)} \times L^{(6)} \sim \frac{\Lambda^4_{\text{cutoff}}}{\Lambda^4}$$



Compute the Higgs mas correction which has a quadratic divergence:

$$\propto \lambda\, m_H^2 \left[\frac{1}{\epsilon} + \log \frac{\mu^2}{m_H^2}\right] \propto m_H^2$$

No $\Lambda^2_{\text{cutoff}}$ term in dim reg.

Power Counting

Amplitudes and anomalous dimensions obey power counting:

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C^{(6)} \propto C^{(6)}$$
$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C^{(8)} \propto C^{(8)} + \left[C^{(6)}\right]^2$$

In the SM, because of the dimension two operator $H^\dagger H$, have

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C^{(4)} \propto C^{(4)} + m_H^2 C^{(6)} + \ldots$$
$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C^{(6)} \propto C^{(6)} + m_H^2 C_8 + \ldots$$
$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C^{(8)} \propto C^{(8)} + \left[C^{(6)}\right]^2 + m_H^2 C_{10} + \ldots$$

SM parameter RG evolution affected.

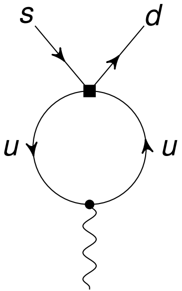
Equations of Motion

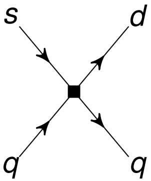
Used to eliminate operators with derivatives:

$$D^\mu F_{\mu\nu} = g\, j_\nu$$

$$g\, \overline{d}\gamma^\mu T^A P_L s\, D^\mu F^A_{\mu\nu} \to g^2\, \overline{d}\gamma^\mu T^A P_L s\, \overline{q}\gamma^\mu T^A q$$

Penguin operators:





Eliminate $v \cdot D$ terms in HQET.

Equations of Motion (contd)

H.D. Politzer: NPB172 (1980) 349

Operator conversions done by making field redefinitions, since

$$L(\phi + \epsilon\, f(*)) = L(\phi) + \epsilon\, \frac{\delta L}{\delta \phi}\, f(*) + \dots$$

- Change of variables in a (functional) integral
- *S*-matrix unchanged
- Green's functions can change
- Have to be a bit careful, since usually one computes 1PI graphs, and the *S*-matrix includes non-1PI graphs.
- Can induce operators for which there is no direct 1PI graph such as the *LR* four-quark operators.

EOM:

$$E_i = 0$$

RG equations:

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} O_i = -\gamma_{ji}\, O_j + \zeta_{ri}\, E_r \qquad \mu \frac{\mathrm{d}}{\mathrm{d}\mu} E_i = -\Gamma_{ji}\, E_j$$

ζ_r can be gauge and scheme dependent.

RG evolution consistent with equations of motion — can evolve and use EOM or use EOM and evolve.
Prefer

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C_i = \gamma_{ij}\, C_j$$

because there can be non-linear terms in the RGE.

Effective Lagrangian

$$L = L_{SM} + L^{(6)} + \dots \qquad\qquad L^{(6)} = \sum_i C_i \frac{Q_i}{\Lambda^2}$$

- Assume C_i are arbitrary because we do not know the underlying theory
- A given theory has specific value for each C_i — they are correlated, including the C_i in $L^{(8)}$, etc.
- If we know something, e..g that the theory preserves baryon number, then we can constrain some C_i.
- One cannot assume that the underlying theory gives certain specific values for C_i — equivalent to making assumptions about the dynamics over which one has no control.

Chiral Perturbation Theory

A. Pich, Chiral perturbation theory, Rept. Prog. Phys. 58 (1995) 563-610

χPT: a theory with $SU(3) \times SU(3)$ spontaneously broken to $SU(3)$.

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4 + \ldots$$

$$\mathcal{L}_2 = \frac{f^2}{4} \langle D_\mu U^\dagger D^\mu U + U^\dagger \chi + \chi^\dagger U \rangle$$

$$\begin{aligned}
\mathcal{L}_4 = & \; L_1 \langle D_\mu U^\dagger D^\mu U \rangle^2 + L_2 \langle D_\mu U^\dagger D_\nu U \rangle \langle D^\mu U^\dagger D^\nu U \rangle \\
& + L_3 \langle D_\mu U^\dagger D^\mu U D_\nu U^\dagger D^\nu U \rangle + L_4 \langle D_\mu U^\dagger D^\mu U \rangle \langle U^\dagger \chi + \chi^\dagger U \rangle \\
& + L_5 \langle D_\mu U^\dagger D^\mu U \left(U^\dagger \chi + \chi^\dagger U \right) \rangle + L_6 \langle U^\dagger \chi + \chi^\dagger U \rangle^2 \\
& + L_7 \langle U^\dagger \chi - \chi^\dagger U \rangle^2 + L_8 \langle \chi^\dagger U \chi^\dagger U + U^\dagger \chi U^\dagger \chi \rangle \\
& - i L_9 \langle F_R^{\mu\nu} D_\mu U D_\nu U^\dagger + F_L^{\mu\nu} D_\mu U^\dagger D_\nu U \rangle + L_{10} \langle U^\dagger F_R^{\mu\nu} U F_{L\mu\nu} \rangle \\
& + H_1 \langle F_{R\mu\nu} F_R^{\mu\nu} + F_{L\mu\nu} F_L^{\mu\nu} \rangle + H_2 \langle \chi^\dagger \chi \rangle \, .
\end{aligned}$$

χPT

χPT: the Lagrangian has f, M, and low-energy constants L_i.

The parameters are fixed at certain values in QCD.

The parameters are fixed at certain (other values) in a $SU(5)_{\text{color}}$ gauge theory

The parameters are fixed at certain (yet other values) in a linear σ-model (the Higgs sector)

The χPT expressions for $\pi - \pi$ scattering, etc. hold in all cases. The power of the method is that

$$A = a_2 p^2 + b_4 p^4 \ln p^2/\mu^2 + a_4 p^4 + \ldots$$

There are more amplitudes than parameters at a fixed order in p^n, and the non-analytic terms are calculable.

Reasons for using EFT

The constraints of gauge invariance, analyticity, unitarity, and crossing are automatically included.

To a given order in $1/\Lambda$ (or p), the theory is determined by a finite number of constants, rather than general functions — i.e. the scattering amplitudes.

Turned out to be much more efficient — compare the results of χPT with those using current algebra.

Dimension Six Operators

Grzadkowski et al. JHEP 1010 (2010) 085

Buchmuller and Wyler, Nucl.Phys. B268 (1986) 621

X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I\varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p\gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G_{\mu\nu}^A G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar l_p\tau^I\gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p\gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W_{\mu\nu}^I W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r)\tau^I\widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p\gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar q_p\tau^I\gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p\gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W_{\mu\nu}^I B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I\varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p\gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W_{\mu\nu}^I B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p\gamma^\mu d_r)$

$(\bar{L}L)(\bar{L}L)$		$(\bar{R}R)(\bar{R}R)$		$(\bar{L}L)(\bar{R}R)$	
Q_{ll}	$(\bar{l}_p\gamma_\mu l_r)(\bar{l}_s\gamma^\mu l_t)$	Q_{ee}	$(\bar{e}_p\gamma_\mu e_r)(\bar{e}_s\gamma^\mu e_t)$	Q_{le}	$(\bar{l}_p\gamma_\mu l_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{uu}	$(\bar{u}_p\gamma_\mu u_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{lu}	$(\bar{l}_p\gamma_\mu l_r)(\bar{u}_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar{q}_p\gamma_\mu\tau^I q_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{dd}	$(\bar{d}_p\gamma_\mu d_r)(\bar{d}_s\gamma^\mu d_t)$	Q_{ld}	$(\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$	Q_{eu}	$(\bar{e}_p\gamma_\mu e_r)(\bar{u}_s\gamma^\mu u_t)$	Q_{qe}	$(\bar{q}_p\gamma_\mu q_r)(\bar{e}_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t)$	Q_{ed}	$(\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{u}_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar{u}_p\gamma_\mu u_r)(\bar{d}_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{u}_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar{u}_p\gamma_\mu T^A u_r)(\bar{d}_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar{q}_p\gamma_\mu q_r)(\bar{d}_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar{q}_p\gamma_\mu T^A q_r)(\bar{d}_s\gamma^\mu T^A d_t)$

$(\bar{L}R)(\bar{R}L)$ and $(\bar{L}R)(\bar{L}R)$		B-violating	
Q_{ledq}	$(\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar{q}_p^j u_r)\varepsilon_{jk}(\bar{q}_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar{q}_p^j T^A u_r)\varepsilon_{jk}(\bar{q}_s^k T^A d_t)$	$Q_{qqq}^{(1)}$	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\varepsilon_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar{l}_p^j e_r)\varepsilon_{jk}(\bar{q}_s^k u_t)$	$Q_{qqq}^{(3)}$	$\varepsilon^{\alpha\beta\gamma}(\tau^I\varepsilon)_{jk}(\tau^I\varepsilon)_{mn}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(3)}$	$(\bar{l}_p^j\sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar{q}_s^k\sigma^{\mu\nu} u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$

59 baryon number conserving operators, not including flavor indices. Field redefinitions (equations of motion) used to eliminate operators.

Eight classes: X^3, ϕ^6, $\phi^4 D^2$, $X^2\phi^2$, $\psi^2\phi^3$, $\psi^2 X\phi$, $\psi^2\phi^2 D$, ψ^4.

Naive Dimensional Analysis

Georgi, AM: NPB234 (1984) 189

$$f^2\Lambda^2 \left(\frac{\psi}{f\sqrt{\Lambda}}\right)^a \left(\frac{H}{f}\right)^b \left(\frac{yH}{\Lambda}\right)^c \left(\frac{D}{\Lambda}\right)^d \left(\frac{gX}{\Lambda^2}\right)^e$$

with $\Lambda \sim 4\pi f$.

$$\frac{f^2}{\Lambda^4} g^3 X^3, \quad \frac{\Lambda^2}{f^4} H^6, \quad \frac{1}{f^2} H^4 D^2, \quad \frac{1}{\Lambda^2} g^2 X^2 H^2,$$
$$\frac{1}{f^2} y\psi^2 H^3, \quad \frac{1}{\Lambda^2} y\psi^2 gXH, \quad \frac{1}{f^2} \psi^2 H^2 D, \quad \frac{1}{f^2} \psi^4$$

Differs from just using $1/\Lambda^2$ for all operators by factors of 4π.

$$L^{(6)} = \sum_i C_i \, Q_i = \sum_i \widehat{C}_i \, \widehat{Q}_i \qquad L = L_{SM} + L^{(6)} + L^{(8)} + \ldots$$

Just a trivial rescaling. Two versions equivalent.

- NDA estimate is that $\widehat{C}_i$ order unity. Works in a strongly interacting theory such as QCD. In weakly interacting theories, there are small dimensionless parameters that you have to account for (such as e).
- C_i, $\widehat{C}_i$ are arbitrary because we do not know the underlying theory

Anomalous Dimension Matrix

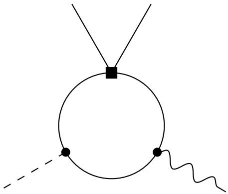
To determine non-analytic terms in the EFT

$$\gamma \propto \left(\frac{g^2}{16\pi^2}\right)^{n_1} \left(\frac{\lambda}{16\pi^2}\right)^{n_2} \left(\frac{y^2}{16\pi^2}\right)^{n_3}, \qquad N = n_1 + n_2 + n_3$$

Even for a one-loop anomalous dimension, $N = 0, 1, 2, 3, 4$.

N — perturbative order.

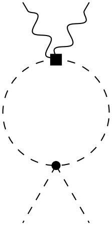
γ ranges from tree to 4-loop order, even though it comes from a one-loop graph.



$gy\psi^2 XH$ – ψ^4 mixing.

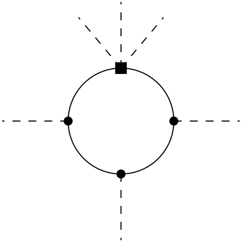
This is an $N = 0$ term.

Anomalous Dimension Matrix



$X^2H^2 - X^2H^2$ mixing.

This is an $N = 1$ term with $\gamma \propto \lambda$



$H^6 - y\psi^2H^3$ mixing.

This is an $N = 2$ term with $\gamma \propto y^4$

γ_{68}

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C_{\substack{eB\\pr}} = \frac{1}{16\pi^2}\left[4g_1 N_c\,(y_u + y_q) C^{(3)}_{\substack{lequ\\prst}}\,[Y_u]_{ts}\right] + \ldots$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C_{\substack{eW\\pr}} = \frac{1}{16\pi^2}\left[-2g_2 N_c\, C^{(3)}_{\substack{lequ\\prst}}\,[Y_u]_{ts}\right] + \ldots$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C_{\substack{uB\\pr}} = \frac{1}{16\pi^2}\left[4g_1 (y_e + y_l)\, C^{(3)}_{\substack{lequ\\stpr}}\,[Y_e]_{ts}\right] + \ldots$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} C_{\substack{uW\\pr}} = \frac{1}{16\pi^2}\left[-2g_2 C^{(3)}_{\substack{lequ\\stpr}}\,[Y_e]_{ts}\right] + \ldots,$$

$\ldots$ is non-γ_{68} terms.
Class 6 are the dipole operators:

$$Q_{\substack{eB\\pr}} = (\bar{l}_{p,a}\sigma^{\mu\nu} e_r) H^a\, B_{\mu\nu}$$

$$\begin{aligned} Q^{(3)}_{lequ} &= (\bar{l}^j_p \sigma_{\mu\nu} e_r)\epsilon_{jk}(\bar{q}^k_s \sigma^{\mu\nu} u_t) = -4(\bar{l}^j_p e_r)\epsilon_{jk}(\bar{q}^{k\alpha}_s u_{\alpha t}) - 8(\bar{l}^j_p u_{\alpha t})\epsilon_{jk}(\bar{q}^{k\alpha}_s e_r) \\ &= -4 Q^{(1)}_{lequ} - 8(\bar{l}^j_p u_{\alpha t})\epsilon_{jk}(\bar{q}^{k\alpha}_s e_r) \end{aligned}$$

Anomalous Dimension Matrix

		g^3X^3	H^6	H^4D^2	$g^2X^2H^2$	$y\psi^2H^3$	$gy\psi^2XH$	ψ^2H^2D	ψ^4
		1	2	3	4	5	6	7	8
g^3X^3	1	g^2	0	0	1	0	0	0	0
H^6	2	$g^6\lambda$	λ, g^2, y^2	$g^4, g^2\lambda, \lambda^2$	$g^6, g^4\lambda$	$\lambda y^2, y^4$	0	$\lambda y^2, y^4$	0
H^4D^2	3	g^6	0	g^2, λ, y^2	g^4	y^2	g^2y^2	g^2, y^2	0
$g^2X^2H^2$	4	g^4	0	1	g^2, λ, y^2	0	y^2	1	0
$y\psi^2H^3$	5	g^6	0	g^2, λ, y^2	g^4	g^2, λ, y^2	$g^2\lambda, g^4, g^2y^2$	g^2, λ, y^2	λ, y^2
$gy\psi^2XH$	6	g^4	0	0	g^2	1	g^2, y^2	1	1
ψ^2H^2D	7	g^6	0	g^2, y^2	g^4	y^2	g^2y^2	g^2, λ, y^2	y^2
ψ^4	8	g^6	0	0	0	0	g^2y^2	g^2, y^2	g^2, y^2

Structure of anomalous dimension matrix. Jenkins, Trott, AM: 1309.0819

$$N = L + w - \sum_k w_k \equiv L + \Delta. \qquad w \text{ is the NDA weight of operator}$$

True in any field theory.

$X^2 H^2$ Operators

Grojean, Jenkins, Trott, AM: JHEP 1304 (2013) 016

59 operators. Look at

$$\mathcal{O}_G = \frac{g_3^2}{2\,\Lambda^2} H^\dagger\, H\, G^A_{\mu\nu} G^{A\,\mu\nu}, \qquad \widetilde{\mathcal{O}}_G = \frac{g_3^2}{2\,\Lambda^2} H^\dagger\, H\, G^A_{\mu\nu} \widetilde{G}^{A\,\mu\nu},$$

$$\mathcal{O}_B = \frac{g_1^2}{2\,\Lambda^2} H^\dagger\, H\, B_{\mu\nu} B^{\mu\nu}, \qquad \widetilde{\mathcal{O}}_B = \frac{g_1^2}{2\,\Lambda^2} H^\dagger\, H\, B_{\mu\nu} \widetilde{B}^{\mu\nu},$$

$$\mathcal{O}_W = \frac{g_2^2}{2\,\Lambda^2} H^\dagger\, H\, W^a_{\mu\nu} W^{a\,\mu\nu}, \qquad \widetilde{\mathcal{O}}_W = \frac{g_2^2}{2\,\Lambda^2} H^\dagger\, H\, W^a_{\mu\nu} \widetilde{W}^{a\,\mu\nu},$$

$$\mathcal{O}_{WB} = \frac{g_1\, g_2}{2\,\Lambda^2} H^\dagger\, \tau^a\, H\, W^a_{\mu\nu} B^{\mu\nu}, \qquad \widetilde{\mathcal{O}}_{WB} = \frac{g_1\, g_2}{2\,\Lambda^2} H^\dagger\, \tau^a\, H\, W^a_{\mu\nu} \widetilde{B}^{\mu\nu}.$$

When you expand them out in the broken phase, $H \to h + v$, get $h \to \gamma\gamma$, $h \to \gamma Z$ and $gg \to h$.

The $\widetilde{O}$ operators are CP and P odd.

Constraint on c_{WB} from the S parameter.

$$c_{WB} = -\frac{1}{8\pi}\frac{\Lambda^2}{v^2}S.$$

The hgg amplitudes get contributions from c_G and $\widetilde{c}_G$.

For $h\gamma\gamma$,

$$c_{\gamma\gamma} = c_W + c_B - c_{WB}, \qquad \widetilde{c}_{\gamma\gamma} = \widetilde{c}_W + \widetilde{c}_B - \widetilde{c}_{WB}$$

For $h\gamma Z$,

$$\begin{aligned} c_{\gamma Z} &= c_W \cot\theta_W - c_B \tan\theta_W - c_{WB} \cot 2\theta_W, \\ \widetilde{c}_{\gamma\gamma} &= \widetilde{c}_W \cot\theta_W - \widetilde{c}_B \tan\theta_W - \widetilde{c}_{WB} \cot 2\theta_W \end{aligned}$$

$$\frac{\Gamma(h \to \gamma\gamma)}{\Gamma^{\rm SM}(h \to \gamma\gamma)} \simeq \left|1 - \frac{4\pi^2 v^2 c_{\gamma\gamma}}{\Lambda^2 I^\gamma}\right|^2 + \left|\frac{4\pi^2 v^2 \tilde{c}_{\gamma\gamma}}{\Lambda^2 I^\gamma}\right|^2,$$

and $I^\gamma \approx -1.64$.

The $\tilde{c}$ terms do not interfere with the standard model amplitude.

Similar expressions for $gg \to h$ and $h \to \gamma Z$.

Did a complete calculation of the anomalous dimension matrix of the eight X^2H^2 operators. This is the γ_{44} submatrix of the full 59×59 matrix.

The values of $c_i(M_h)$ (i.e. the low-scale values) determine the Higgs production and decay rates

Considered these operators in an earlier work, and an explicit model that produced these operators.
AM, M.B. Wise, PLB636 (206) 107, PRD74 (2006) 035009

An exactly solvable model that produces the O_W, O_{WB} and O_B operators. AM: PLB 726 (2013) 347

$$c_W = \frac{(\lambda_1/\lambda_3)}{48 \log \frac{\Lambda_3^2}{\langle \Phi \rangle}}, \qquad c_B = \frac{(\lambda_1/\lambda_3) Y_S^2}{12 \log \frac{\Lambda_3^2}{\langle \Phi \rangle}}, \qquad c_{WB} = \frac{(\lambda_2/\lambda_4) Y_S}{24 \log \frac{\Lambda_4^2}{\langle \Phi \rangle}}$$

$X^2 H^2$ Anomalous Dimension

λ coupling and y_t enter.

In a gauge theory, the operators

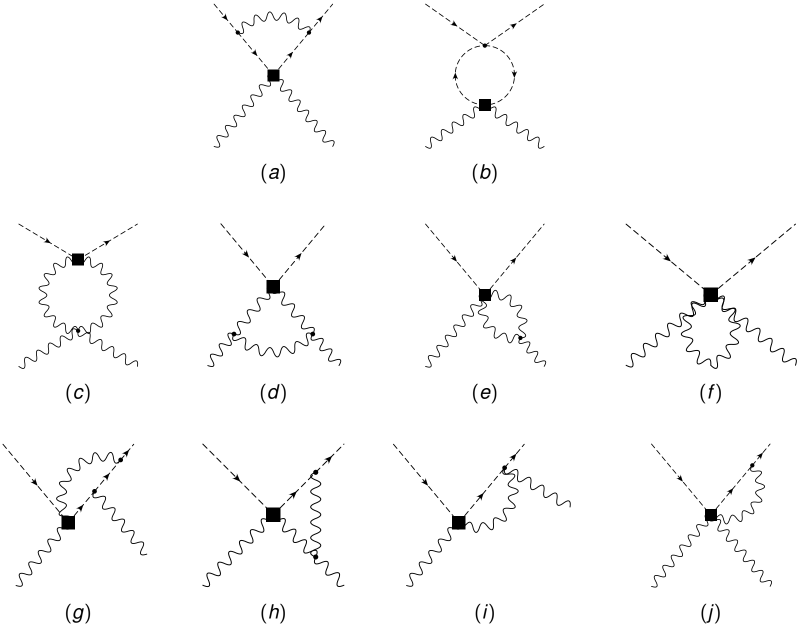
$$\mathcal{O}_+ = \sum \frac{\beta(g)}{2g} F^A_{\mu\nu} F^{A\,\mu\nu}, \qquad \mathcal{O}_- = g^2 F^A_{\mu\nu} \widetilde{F}^{A\,\mu\nu},$$

are not multiplicatively renormalized to all orders in perturbation theory.

At one-loop,

$$g^2 F^A_{\mu\nu} F^{A\,\mu\nu}, \qquad g^2 F^A_{\mu\nu} \widetilde{F}^{A\,\mu\nu},$$

are not renormalized.


(a)
(b)
(c)
(d)
(e)
(f)
(g)
(h)
(i)
(j)

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} c_G = \gamma_G \, c_G,$$

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \begin{bmatrix} c_B \\ c_W \\ c_{WB} \end{bmatrix} = \gamma_{WB} \begin{bmatrix} c_B \\ c_W \\ c_{WB} \end{bmatrix},$$

where the anomalous dimensions are

$$\gamma_G = \frac{1}{16\pi^2} \left[-\frac{3}{2} g_1^2 - \frac{9}{2} g_2^2 + 12\lambda + 2Y \right],$$

$$\gamma_{WB} = \frac{1}{16\pi^2} \begin{bmatrix} \frac{1}{2} g_1^2 - \frac{9}{2} g_2^2 + 12\lambda + 2Y & 0 & 3g_2^2 \\ 0 & -\frac{3}{2} g_1^2 - \frac{5}{2} g_2^2 + 12\lambda + 2Y & g_1^2 \\ 2g_1^2 & 2g_2^2 & -\frac{1}{2} g_1^2 + \frac{9}{2} g_2^2 + 4\lambda + 2Y \end{bmatrix},$$

and

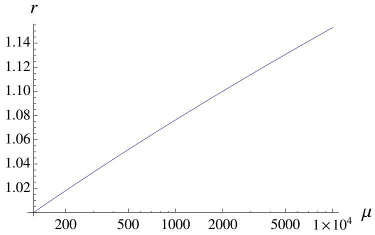
$$Y = \mathrm{Tr} \left[3 Y_u^\dagger Y_u + 3 Y_d^\dagger Y_d + Y_e^\dagger Y_e \right] \approx 3 y_t^2.$$

and similarly for the *CP*-odd operators.

Largest contribution is the Yukawa coupling, and can be integrated exactly.

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} r(\mu) = \frac{3 y_t^2(\mu)}{8\pi^2} r(\mu) \,.$$

Only ratios of $r(\mu)$ enter, so the overall scale of r is irrelevant.
A plot of $r(\mu)$ normalized so that $r(\mu = 125\,\mathrm{GeV}) = 1$



The correction is about 8% to the amplitude for $\mu = 1$ TeV.

Can use an approximate numerical integration:

$$\mathbf{c}(M_h) = \frac{r(M_h)}{r(\Lambda)} \left[1 - \gamma_{WB}(Y \to 0) \; \log \frac{\Lambda}{M_h} \right] \mathbf{c}(\Lambda).$$

This equation is accurate to about 3% for Λ less than 10 TeV.

Remember that there is an overall $1/\Lambda^2$, so if the new physics scale is much larger than a few TeV, the corrections are not observable.

$$\frac{r(\Lambda)c_{\gamma\gamma}(M_h)}{r(M_h)} = \left[1 + \frac{3}{32\pi^2}\left(g_1^2 + 3g_2^2 - 8\lambda\right)\log\frac{\Lambda}{M_h}\right]c_{\gamma\gamma}(\Lambda)$$
$$+ \frac{1}{8\pi^2}\left(3g_2^2 - 4\lambda\right)\log\frac{\Lambda}{M_h}c_{WB}(\Lambda),$$
$$\frac{r(\Lambda)c_{\gamma Z}(M_h)}{r(M_h)} = \left[1 + \frac{1}{32\pi^2}\left(g_1^2 + 7g_2^2 - 24\lambda\right)\log\frac{\Lambda}{M_h}\right]c_{\gamma Z}(\Lambda)$$
$$+ \frac{1}{8\pi^2}\left(g_1 g_2 + 4g_2^2\cot 2\theta_W - 4\lambda\cot 2\theta_W\right)\log\frac{\Lambda}{M_h}c_{WB}(\Lambda)$$
$$- \frac{b_{0,1}g_1^2 - b_{0,2}g_2^2}{16\pi^2}\left(c_{\gamma\gamma}(\Lambda)\sin 2\theta_W + c_{\gamma Z}(\Lambda)\cos 2\theta_W\right)\log\frac{\Lambda}{M_h}.$$

S-Parameter

$$S = -\frac{8\,\pi\,v^2}{\Lambda^2} c_{WB}(M_h)\,.$$

$$c_{WB}(M_h) = \frac{r(M_h)}{r(\Lambda)}\,c_{WB}(\Lambda)\,\left[1 + \frac{g_1^2 - 9\,g_2^2 - 8\lambda}{32\,\pi^2}\,\log\frac{\Lambda}{M_h}\right] - \frac{r(M_h)}{r(\Lambda)}\,\frac{1}{8\,\pi^2}\,\left[g_2^2\,c_W(\Lambda) + g_1^2\,c_B(\Lambda)\right]\,\log\frac{\Lambda}{M_h},$$

K. Hagiwara, S. Ishihara, R. Szalapski, and D. Zeppenfeld, PRD48 (1993) 2182

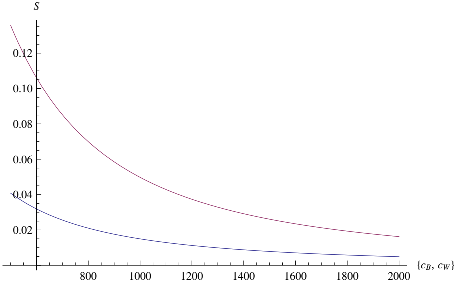
S. Alam, S. Dawson, and R. Szalapski, PRD57 (1998) 1577

$$S = -\frac{8\,\pi\,v^2}{\Lambda^2}\left(c_{WB}(\Lambda) - \frac{1}{8\,\pi^2}\,\left[g_2^2\,c_W(\Lambda) + g_1^2\,c_B(\Lambda)\right]\,\log\frac{\Lambda}{M_h}\right),$$

From finite parts of graphs in broken theory.

$$\mu_{\gamma\gamma} \simeq 1 - 0.02\, S \log \frac{\Lambda}{M_h} + 2.7 \left(\frac{1\ \text{TeV}}{\Lambda} \right)^2 c_{\gamma\gamma}(\Lambda)$$

Experimental limit $|S| \lesssim 0.1$.



Plot of S at $\mu = m_H$ assuming $S = 0$ at Λ, from c_B and c_W, i.e. coefficients of c_B and c_W.

Brief Experimental Summary

ATLAS: $\mu_{\gamma\gamma} = 1.6 \pm 0.3$ CMS: $\mu_{\gamma\gamma} = 0.77 \pm 0.27$

Naive combination of these results (not recommended) gives

$$\mu_{\gamma\gamma} \simeq 1.14 \pm 0.2$$

If due to $c_{\gamma\gamma}$:

$$\frac{v^2}{\Lambda^2} c_{\gamma\gamma}(M_h) \simeq -0.08,\ 0.003 \pm 0.003$$

The second solution is preferred. The first solution is when $c_{\gamma\gamma}$ switches the sign of the standard model $h \to \gamma\gamma$ amplitude.

The experiments are sensitive to these effects if the new physics scale Λ is near a few TeV.

Work in Progress

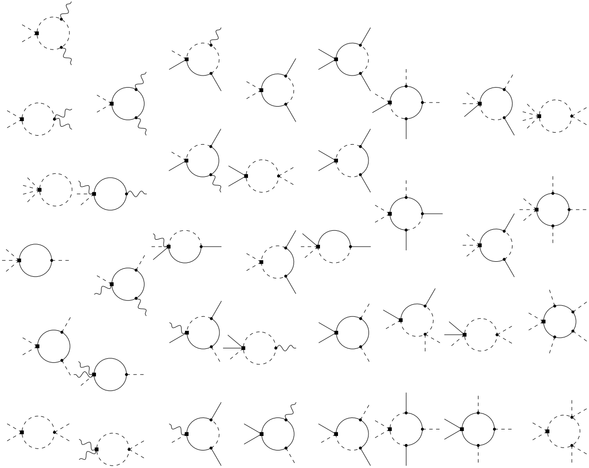
Jenkins, Trott, AM: 1308.2627

Computed the complete contribution to the RGE of SM parameters from the 59 dimension six operators. These terms contribute at order m_H^2/Λ^2, just like the direct $L^{(6)}$ contributions.

$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}\lambda = \frac{m_H^2}{16\pi^2}\left[12C_H + \left(-32\lambda + \frac{10}{3}g_2^2\right)C_{H\Box} + \left(12\lambda - \frac{3}{2}g_2^2 + 6g_1^2 y_H^2\right)C_{HD} + 2\eta_1 + 2\eta_2 \right.$$
$$\left. + 12g_2^2 c_{F,2}C_{HW} + 12g_1^2 y_H^2 C_{HB} + 6g_1 g_2 y_H C_{HWB} + \frac{4}{3}g_2^2 C^{(3)}_{\substack{Hl\\tt}} + \frac{4}{3}g_2^2 N_c C^{(3)}_{\substack{Hq\\tt}}\right],$$
$$\mu\frac{\mathrm{d}}{\mathrm{d}\mu}m_H^2 = \frac{m_H^4}{16\pi^2}\left[-4C_{H\Box} + 2C_{HD}\right],$$

where $\eta_{1,2}$ are Y terms. Similarly corrections to $Y_{e,u,d}$ running.

Work in Progress



approx 1/4 the graphs needed.

Work in Progress

- Computed the complete contribution to the RGE of SM parameters from the 59 dimension six operators. These terms contribute at order m_H^2/Λ^2, just like the direct $L^{(6)}$ contributions. Jenkins, Trott, AM: 1308.2627
- Computed the terms in the 59×59 anomalous dimension matrix with $n_g = 0$ and $n_\lambda \geq 1$ Jenkins, Trott, AM: 1308.2627
- Computed the terms in the 59×59 anomalous dimension matrix with $n_g = 0$ keeping the full Yukawa dependence. Jenkins, Trott, AM
- Results valid for any number of fermion generations.
- Computing the gauge dependent terms, which will give the full one-loop 59×59 anomalous dimension matrix.

Elias-Miró, Espinosa, Masso, Pomarol, JHEP1308 (2013) 033, 1308.1879

Conclusions

- Higgs EFT are a good way to analyze the data (not the only way).
- Lots of work being done, as you will hear over the next 3 days.
- Experimental data can constrain new physics at a TeV
- This is complementary to direct searches
- If any deviations seen, then have to be careful about interpretation, because of mixing. What you see at the low scale is (in general) not the same as what is there at the high scale.