HIGGS CROSS-SECTION
WHAT IS NEXT?

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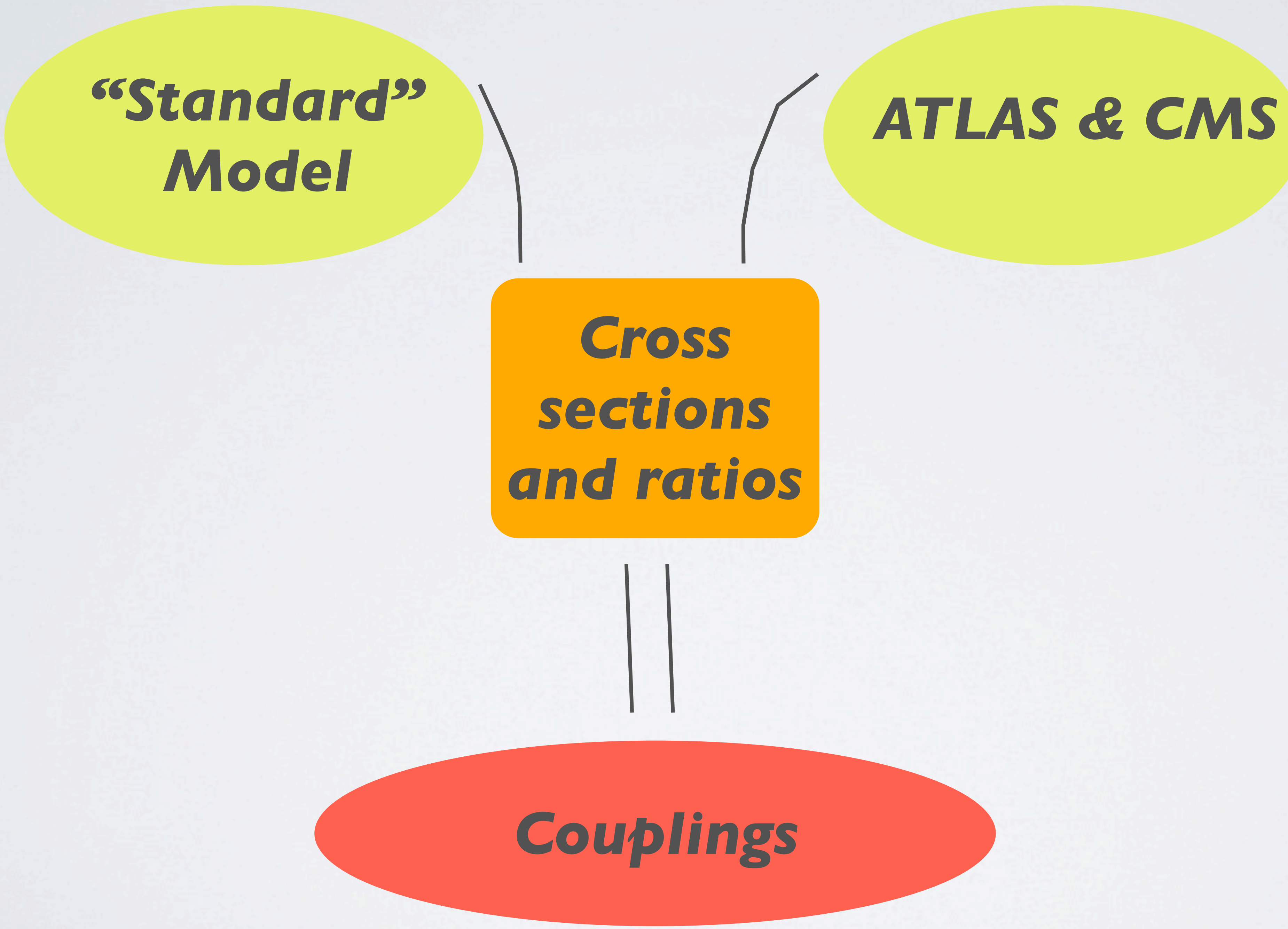
INTRODUCTION

- Discovery of Higgs boson initiated a new era of precision physics
- Current measurements of Higgs couplings are limited by statistical uncertainties.
- In two decades of LHC operations they will be limited by experimental systematics and theory uncertainties.
- Can we do something about it as theorists?

HIGGS COUPLINGS

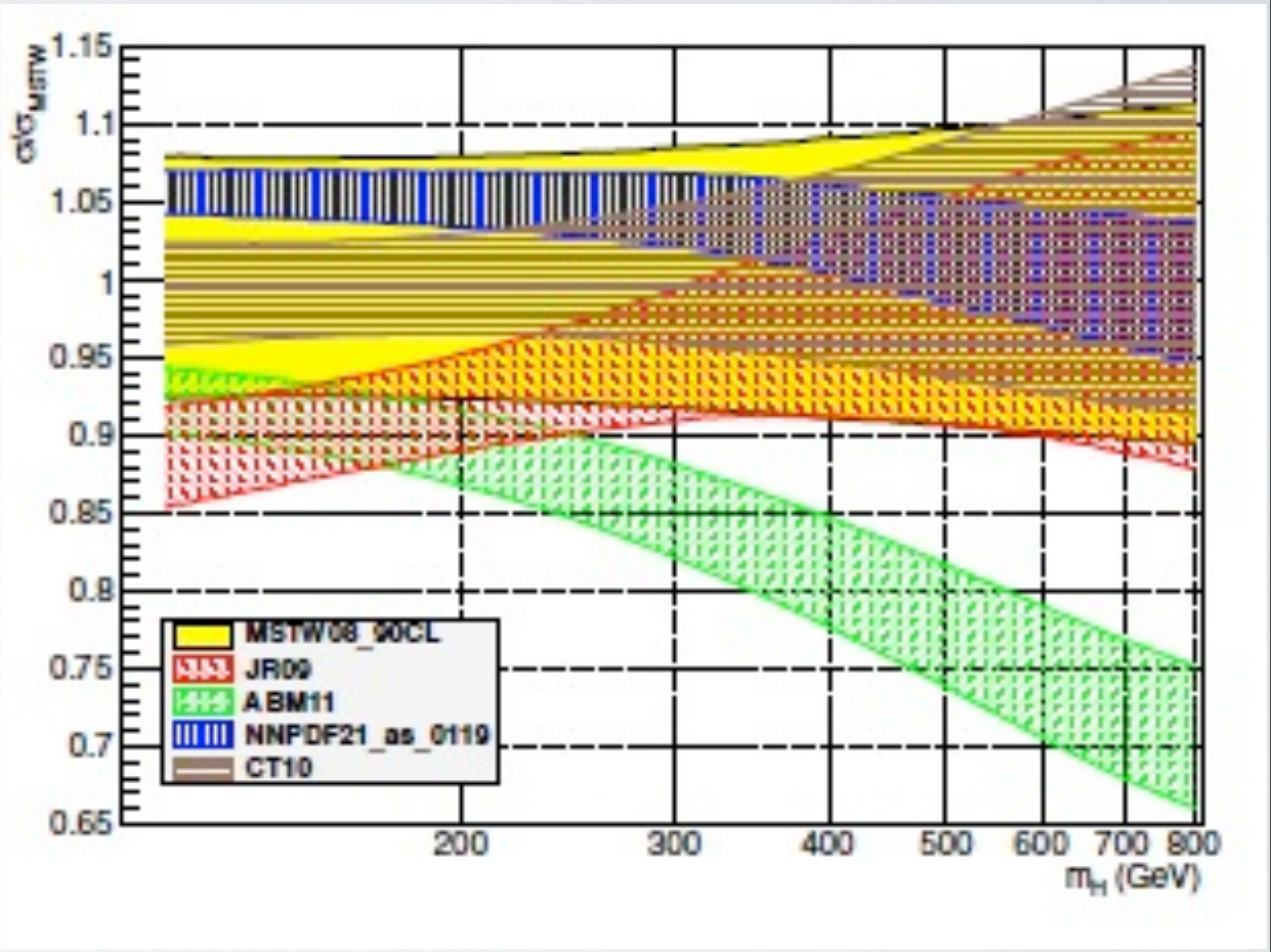


HOW PRECISE ARE THEORY CROSS-SECTIONS?

- An active field of research for almost 25 years now.
- Both continuous and discrete progress.
- Higgs cross-sections are a theory laboratory for perturbative methods.
- Excellent status given the complexity of the task (theory uncertainties below 20%).
- Room for improvement.

PDF UNCERTAINTIES

- Five NNLO pdf sets
- 68% confidence level uncertainties show discrepancies
- Enlarge uncertainties, e.g. adopting a 90%CL
- Important: high precision measurements of top and other SM cross-sections at the LHC.



CONVERGENCE

known to NNLO

$$\sigma^{SM}_{gg\to H} = C^2\alpha_s(\mu_r)^2\left[X_0\left(\mu_r,\mu_f\right) + \alpha_s(\mu_r)X_1\left(\mu_r,\mu_f\right) + \alpha_s(\mu_r)^2X_2\left(\mu_r,\mu_f\right) + \ldots\right]$$

$$\Gamma^{SM}_{H\to gg} = C^2\alpha_s(\mu_r)^2\left[Y_0\left(\mu_r\right) + \alpha_s(\mu_r)Y_1\left(\mu_r\right) + \alpha_s(\mu_r)^2Y_2\left(\mu_r\right) + \alpha_s(\mu_r)^3Y_3\left(\mu_r\right) + \ldots\right]$$

known to NNNLO

Harlander,Kilgore; CA, Melnikov; Neerven,Ravindran,Smith

$$\frac{\sigma^{SM}_{gg\to H}}{\Gamma^{SM}_{H\to gg}} = \frac{1+0.72+0.28+\ldots}{1+0.65+0.20+0.002+\ldots}$$

Baikov, Chetyrkin

- Slow perturbative convergence
- but many orders in perturbation theory are known

IN THIS TALK

- Dissecting the NLO gluon fusion cross-section
- NNLO, resummations, precision at NNNLO
- NNNLO? Methods and some results.

NLO QCD CORRECTIONS

cross-section for gluon fusion via a heavy (top) quark:

$$\sigma \sim \mathcal{L}_{gg}(\mu) \times \left(\frac{\alpha_s(\mu)}{\pi}\right)^2$$

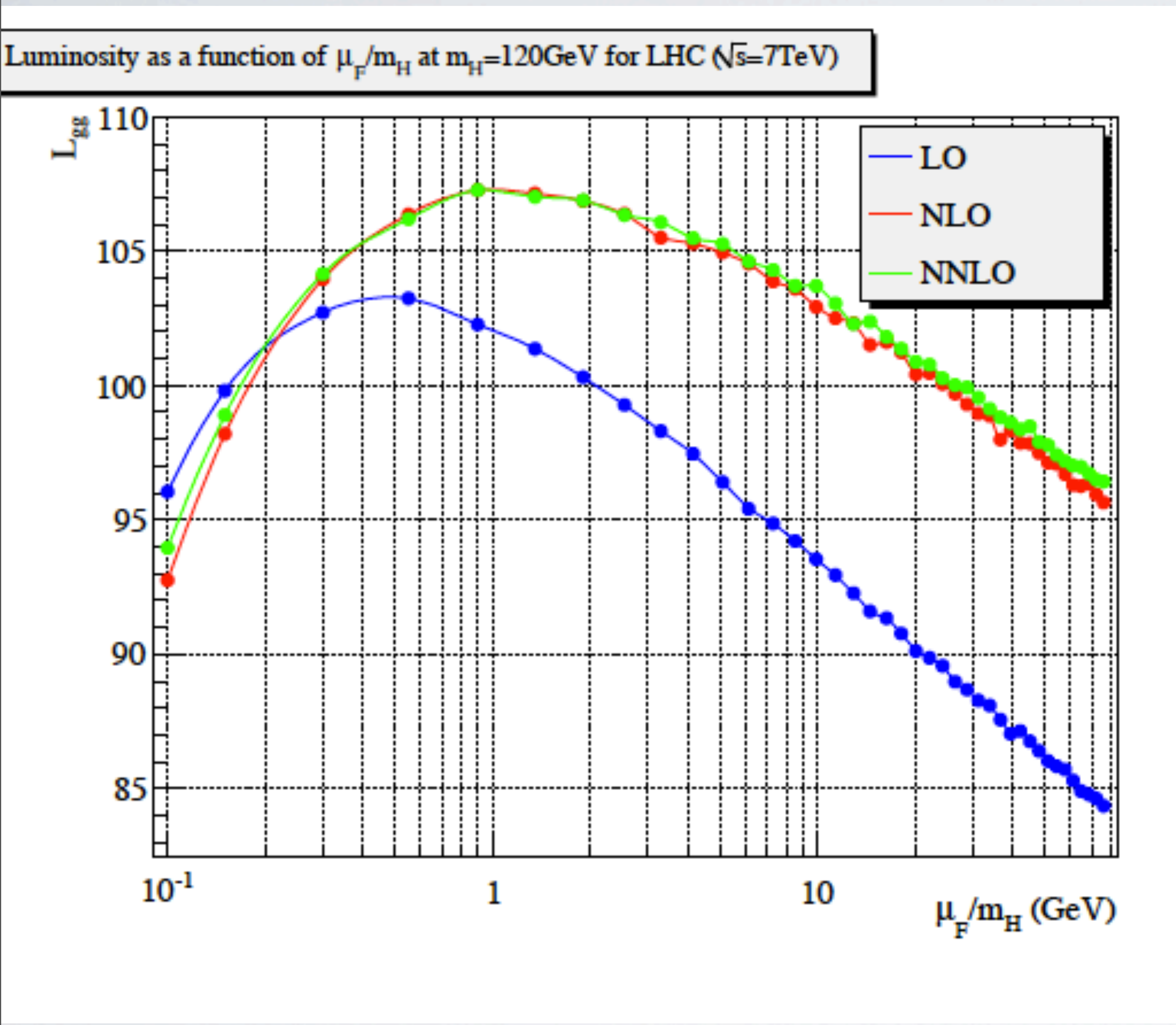
$$\left\{1 + \frac{\alpha_s(\mu)}{\pi}\left[\left(N_c\frac{\pi^2}{3} + \frac{11}{2}\right) + 2\log\left(\frac{\mu^2}{p^2_{T_{max}}}\right) N_c \mathrm{Coll}\left(\frac{p^2_{T_{max}}}{M_h^2}\right) + \int d\theta \mathrm{Reg}\left(\frac{p^2_{T_{max}}}{M_h^2}, \theta\right)\right]\right\}$$

Soft real and virtual corrections $\pi^2, \log\left(\frac{\mu^2}{p^2_{T_{max}}}\right)$

$\frac{11}{2} = 2\,C_1$ Wilson coefficient of Heavy Quark Effective Theory (~ UV nature)

$\mathrm{Reg}\left(\frac{p^2_{T_{max}}}{M_h^2}, \theta\right) \to 0,$ hard, vanishes in $p_t, \theta, \pi - \theta \to 0$

GLUON-GLUON LUMINOSITY



Lgg(Mh=120GeV, LHC7, MSTW08)

- Very stable from NLO to NNLO
- Within 5% from LO for a light Higgs boson at the LHC for reasonable factorization scales.
- ~ 20% higher than LO for very very large factorization scales

LARGE K-FACTORS

$$\frac{\text{NLO}}{\text{LO}} \sim (80\% - 105\%) \left\{ 1 + 4\% \left[9.876 \; + \; 5.5 \right] + \ldots \right\}$$

NLO/LO gluons and alpha_s — π^2 — Wilson coefficient

Bound to have a large K-factor of at least 1.5 due to pi's and the Wilson coefficient

Milder K-factor if gluon fusion is mediated through a light quark (bottom) as, for example, in large tan(beta) MSSM.

$$\frac{\text{NLO}}{\text{LO}} \sim (80\% - 105\%) \left\{ 1 + 4\% \left[9.876 \; + \; 0.9053 \right] + \ldots \right\}$$

π^2 — Two-loop bottom amplitude.

LARGE K-FACTORS (II)

$$\frac{\text{NLO}}{\text{LO}} \sim (80\% - 105\%) \left\{ 1 + \frac{\alpha_s(\mu)}{\pi} \left[\ldots + 6 \log \left(\frac{\mu^2}{p^2_{T_{max}}} \right) \text{Coll} \left(\frac{p^2_{T_{max}}}{M_H^2} \right) + \ldots \right] \right\}$$

NLO/LO gluons and alpha_s

- Logarithmic enhancement at threshold (small pt,max)
- Integrable: reliable perturbative expansion for inclusive cross-sections.
- The mu scale is arbitrary, but no need to be senseless.

$$M_H = 120\,\text{GeV}\ @\text{LHC7} \rightsquigarrow < p_t > \sim 35\,\text{GeV}$$

$$\frac{\text{NLO}}{\text{LO}} \sim (80\% - 105\%) \begin{cases} \left\{ 1 + 4\% \left[9.876 + 5.5 + \mathcal{O}(15.) \right] + \ldots \right\} & \mu = M_h \\ \left\{ 1 + 4\% \left[9.876 + 5.5 + \mathcal{O}(1.) \right] + \ldots \right\} & \mu = \frac{M_h}{4} \end{cases}$$

π^2 — *Wilson coefficient* — *Pt-Log*

NLO/LO gluons and alpha_s

PERTURBATIVE CONVERGENCE?

- Three main worries from the NLO calculation:
  - Large NLO Wilson coefficient ~15-20%
  - Pi^2 = 2 x Nc x (Pi^2/6) term ~ 30-40%
  - Large logs { 2 x Nc x Log(pt^2/mu^2)} of transverse momentum (sensitive to mu) ~1% - 80%

- Comforting that the NNLO corrections are mild.
  The Wilson coefficient has a regular perturbative expansion.

At NNLO:

Wilson coefficient

$$C \sim 1 + (4\%) \cdot 5.5 + (4\%)^2 \cdot 10.$$

Chetyrkin, Kniehl, Steinhauser

PERTURBATIVE CONVERGENCE?

- Half of Pi^2 belongs to a different Wilson coefficient when matching to SCET. It ``exponentiates''. We are left to explain the other half, which is a smaller (half) concern.

At NNLO and beyond:

Ahrens, Becher, Neubert

$$1 + \frac{\alpha_s}{\pi} \cdot (\pi^2) + \ldots \sim e^{\frac{\alpha_s}{\pi} \cdot \left(\frac{\pi^2}{2}\right)} \left(1 + \frac{\alpha_s}{\pi} \left[\frac{\pi^2}{2}\right] \ldots\right)$$

- Logs due to soft radiation exponentiate and can be resummed with NNLL accuracy at all orders.

Can also calculate their contribution at N3LO.

Catani, de Florian, Grazzini

Moch, Vogt

Ahrens, Becher, Neubert, Yang

Ball, Bonvini, Forte, Marzani, Ridolfi

CONTROVERSIES

- What are the “right” parton densities?
- What are the “right” renormalization and factorization scales?
- What are the “dominant” logarithmic contributions?

WHICH LOGS ARE IMPORTANT?

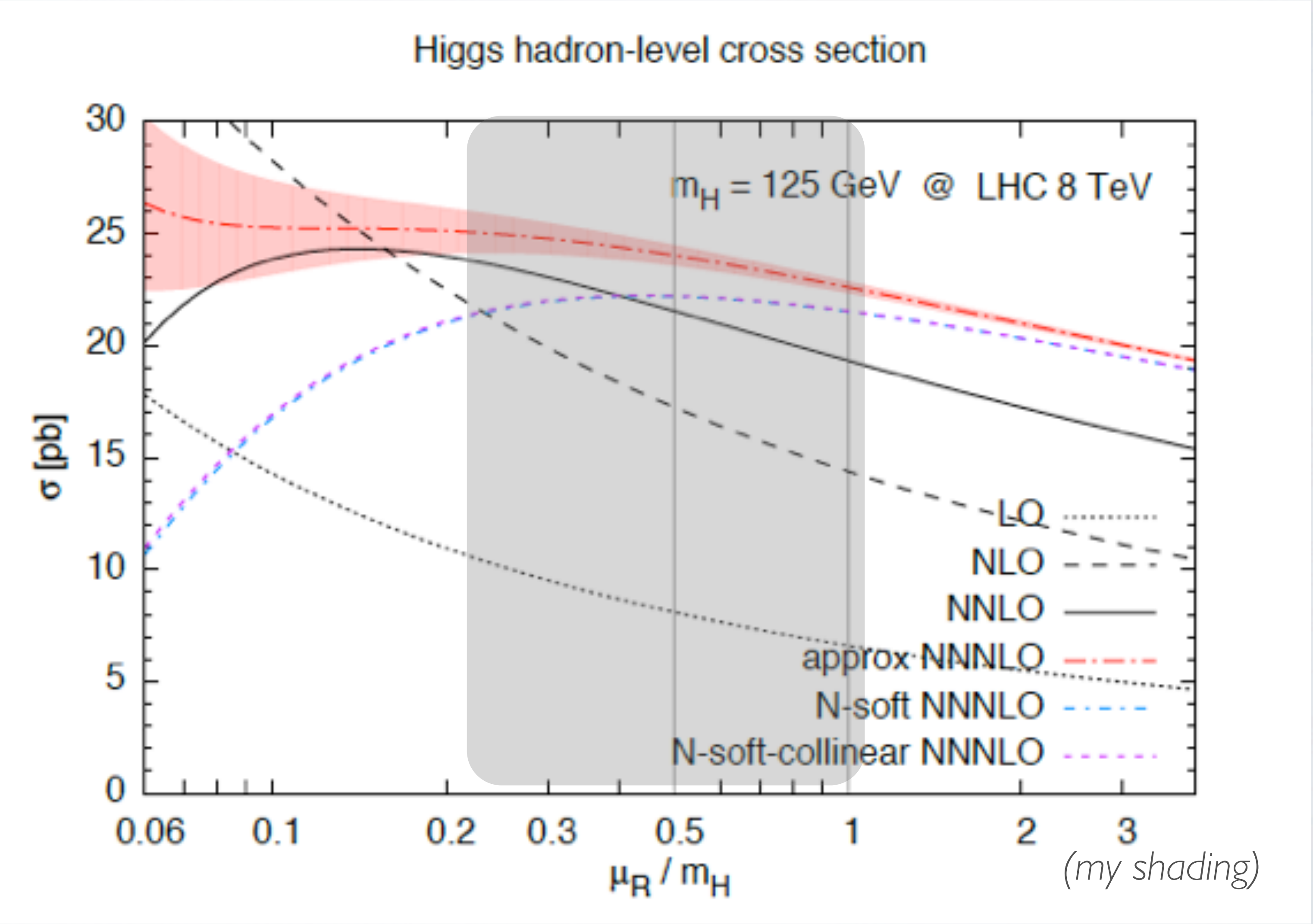
$$\log\left(1-\frac{M_H^2}{s}\right) \quad \log\left(\frac{p_{T_{max}}^2}{M_H^2}\right) \quad \log\left(\log\left(\frac{s}{M_H^2}\right)\right) \ldots$$

Many options. All equivalent in the strict soft limit. More than one, all formally correct, resummations/log expansions by various authors. Differ by subleading terms which turn out to be important…

THRESHOLD LOGS BEYOND NNLO

- Keeping different forms of threshold logs at N3LO and their associated subleading terms leads to different magnitudes of corrections.

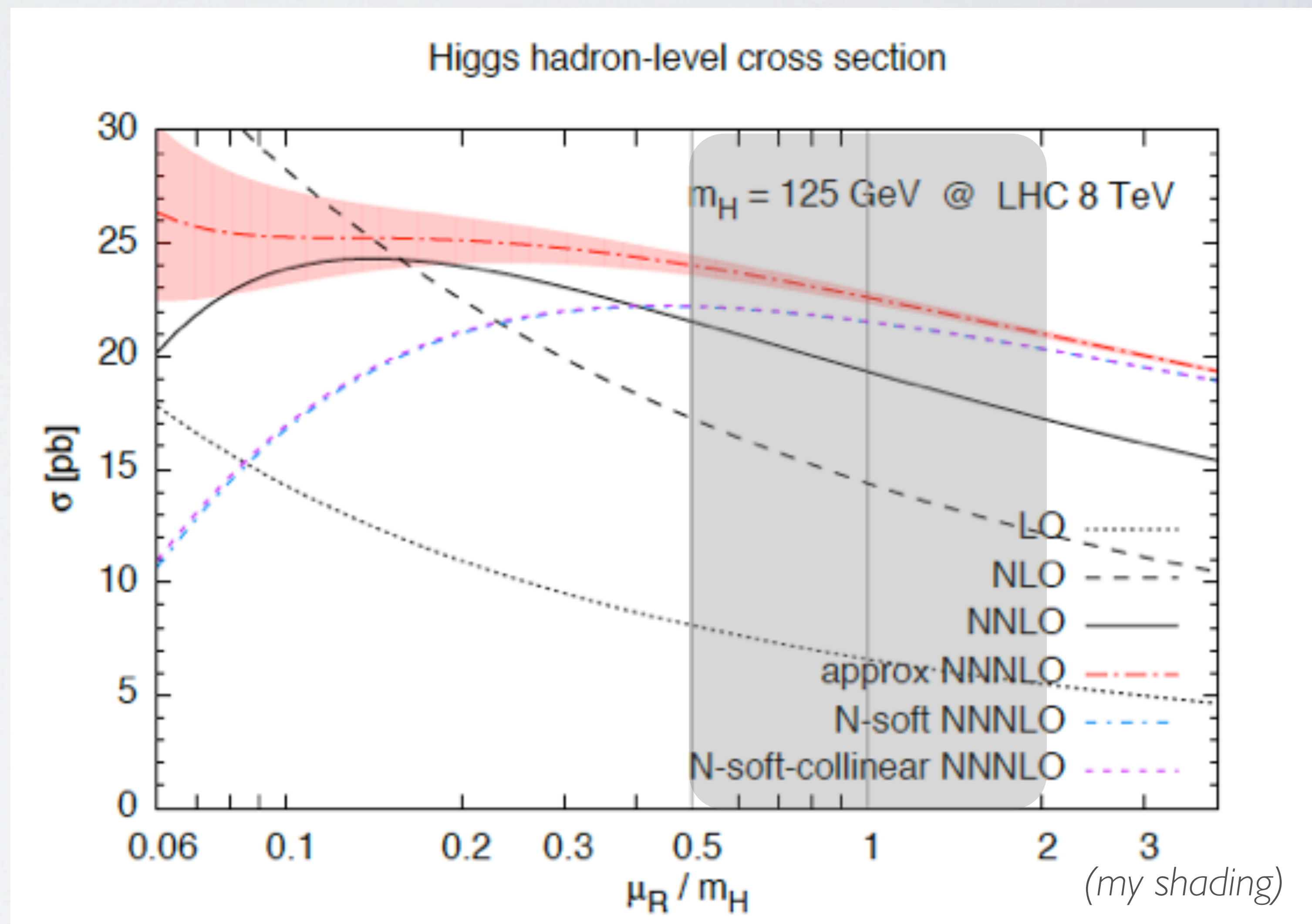
- The N3LO log terms of *Ball et al* lie above the NNLO uncertainty band and the N3LO log terms of *Moch and Vogt.*



Ball, Bonvini, Forte, Marzani, Ridolfi

THRESHOLD LOGS BEYOND NNLO

- These differences can be exacerbated with the choice of renormalization and factorization scale



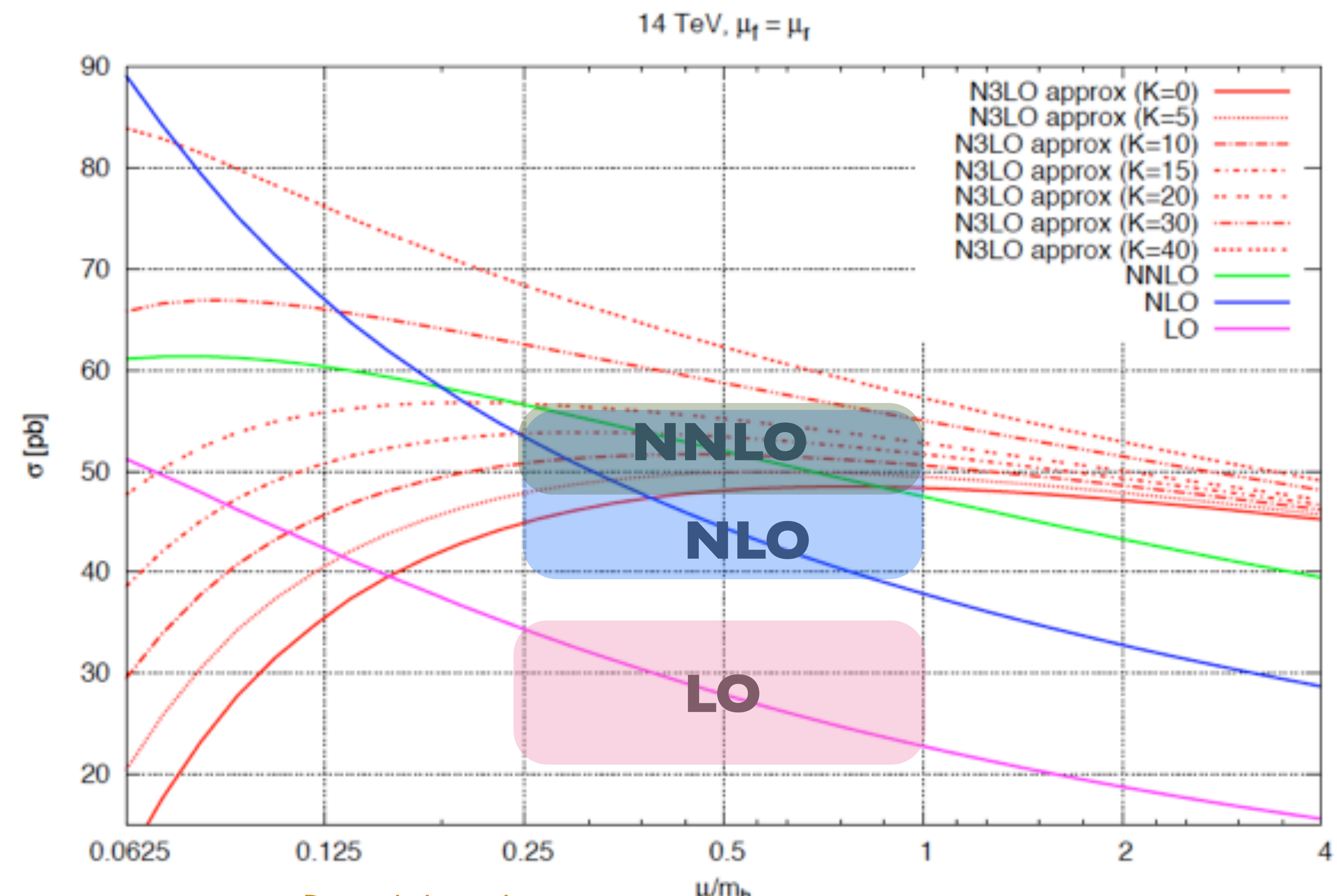
Ball, Bonvini, Forte, Marzani, Ridolfi

BEYOND NNLO

- I am skeptical of the benefit of including threshold logs and the improvement offered by threshold resummation
- ...if we do not have control over the size of "subleading" terms.
- Is "subleading terms" a misnomer in practice?
- We need to calculate the FULL N3LO coefficient or to devise a method to compute many of the terms of the threshold expansion at N3LO.

ULTIMATE N3LO PRECISION

- Collinear and UV counterterms at N3LO by Buehler and Lazopoulos
- Byproduct: Renormalization and factorization scale variation at N3LO.
- Also included threshold logs by Moch and Vogt.
- and scaled by a factor K the NNLO threshold subleading contributions as an estimate of the N3LO equivalent terms.



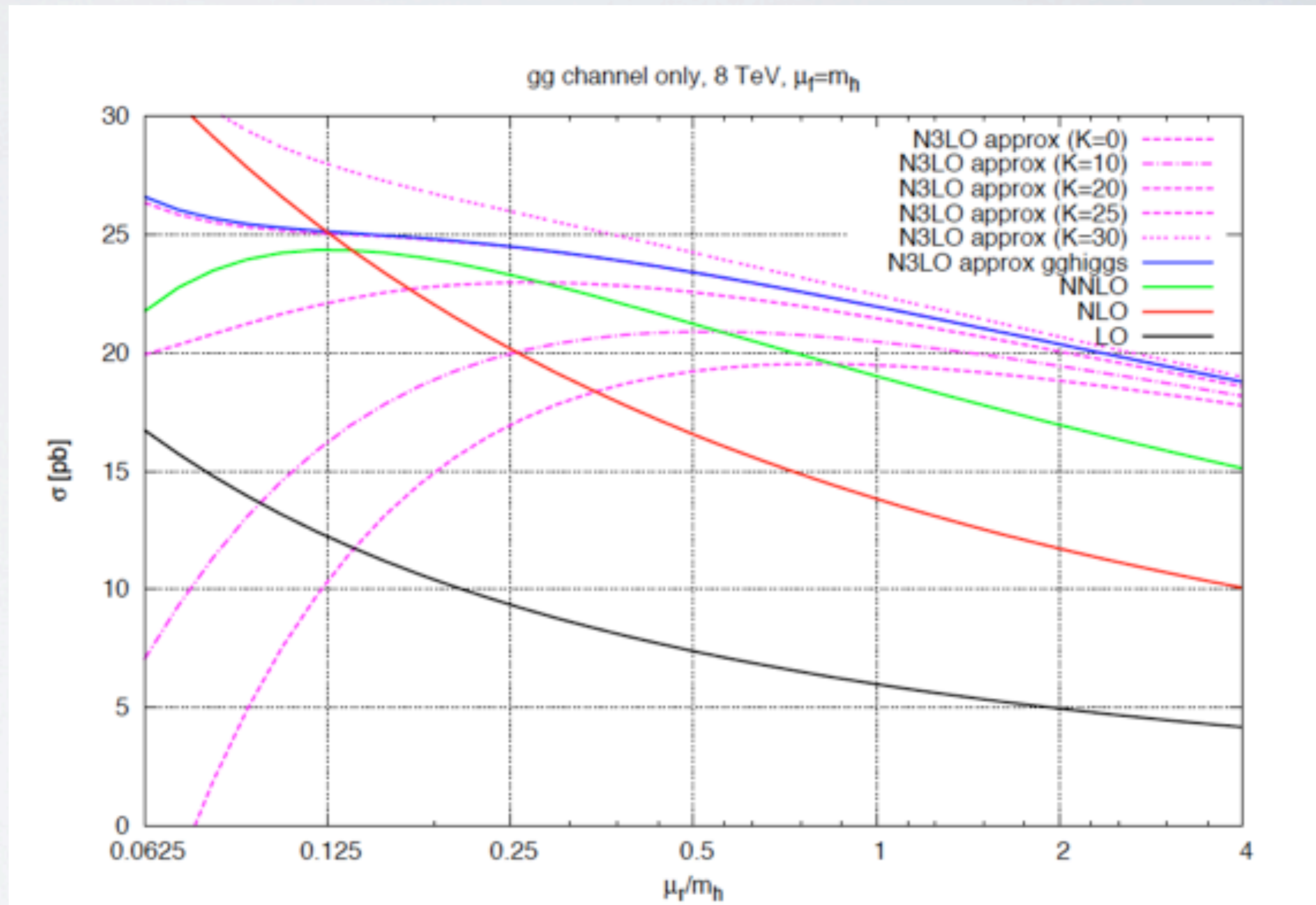
Buehler, Lazopoulos

$$\tilde{\sigma}_{ij}^{(n,m)}(x) = a_{ij}^{(n,m)}\,\delta(1-x) + \sum_k b_{ij}^{(n,m),k}\,\mathcal{D}_k(1-x) + c_{ij}^{(n,m)}(x)\,,$$

$$a_{ij}^{(3,0)} = K\,a_{ij}^{(2,0)}\,, \qquad f_i \otimes f_j \otimes c_{ij}^{(3,0)}(z) = K\left(f_i \otimes f_j \otimes c_{ij}^{(2,0)}(z)\right)$$

ULTIMATE N3LO PRECISION

- Buehler and Lazopoulos find a tiny dependence at N3LO on the factorization scale (per mile)
- Renormalization scale can vary (still) significantly, from 2% to 8% depending on the size of "subleading" threshold N3LO terms.
- Subleading terms of Ball et al at N3LO are as big as their counterparts at NNLO.



Buehler, Lazopoulos

A PATH TO NNNLO

REVERSE UNITARITY,
STRATEGY OF REGIONS,
THRESHOLD EXPANSIONS,
ORDERING MULTIPLE INTEGRATIONS,
DIFFERENTIAL EQUATIONS,
MELLIN-BARNES,
DIMENSIONAL SHIFTS,...

EXISTING NNLO METHODS

- Pioneering work by van Neerven et al in Drell-Yan (~1990).

  - computing the inclusive cross-section in the soft limit $z \equiv \frac{M_V^2}{\hat{s}} \to 1.$
  - followed by complete calculation for arbitrary partonic energy.

- Additional techniques for Higgs production

  - Soft limit (*Catani, de Florian, Grazzini; Harlander,Kilgore*)
  - Systematic method for threshold expansion and resuming of the series (*Harlander,Kilgore*)

$$\hat{\sigma}_{RR} = (1-z)^{-1-4\epsilon}\left[a_1 + a_2(1-z) + a_3(1-z)^2 + \ldots\right]$$

$$\hat{\sigma}_{RV} = (1-z)^{-1-4\epsilon}\left[b_1 + b_2(1-z) + b_3(1-z)^2 + \ldots\right] + (1-z)^{-1-2\epsilon}\left[c_1 + c_2(1-z) + c_3(1-z)^2 + \ldots\right]$$

REVERSE UNITARITY

Melnikov, CA

- Convert phase-space integrals into loop integrals.

$$\delta\left(p^2 - M^2\right) \to \frac{i}{p^2 - M^2} - c.c.$$

can almost forget about it

- Use IBP identities and the Laporta algorithm to reduce phase-space integrals into master integrals

$$\int d^d k \frac{\partial}{\partial k_\mu} \frac{q^\mu}{k^2 \ldots} = 0 \rightsquigarrow I_1 + I_2 + \ldots = 0$$

- Simplification for cut propagators.

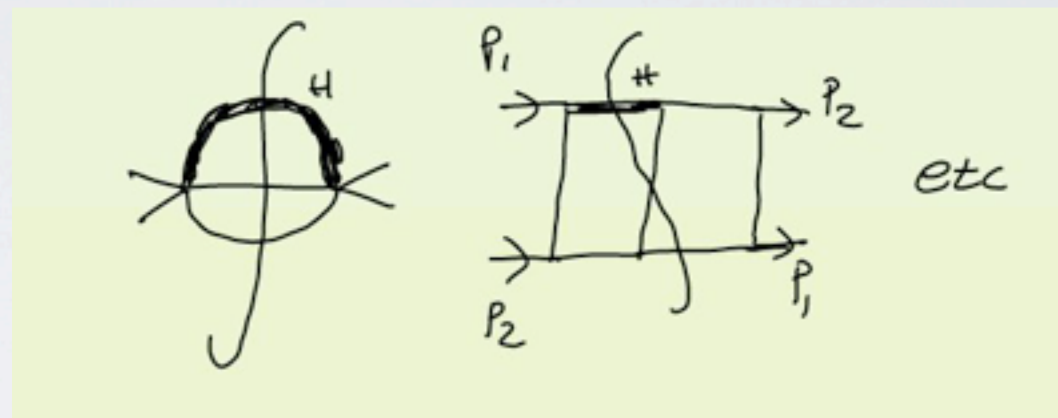
$$\left(\frac{i}{\not{k}^2}\right)^n \to 0, \quad n = 0, -1, 2, \ldots$$

- Few remaining master integrals. Solved using differential equations, derived and solved in the same way as for loop master integrals (*Kotikov; Gehrmann, Remiddi, Smirnov, Veretin, ...*)

REVERSE UNITARITY

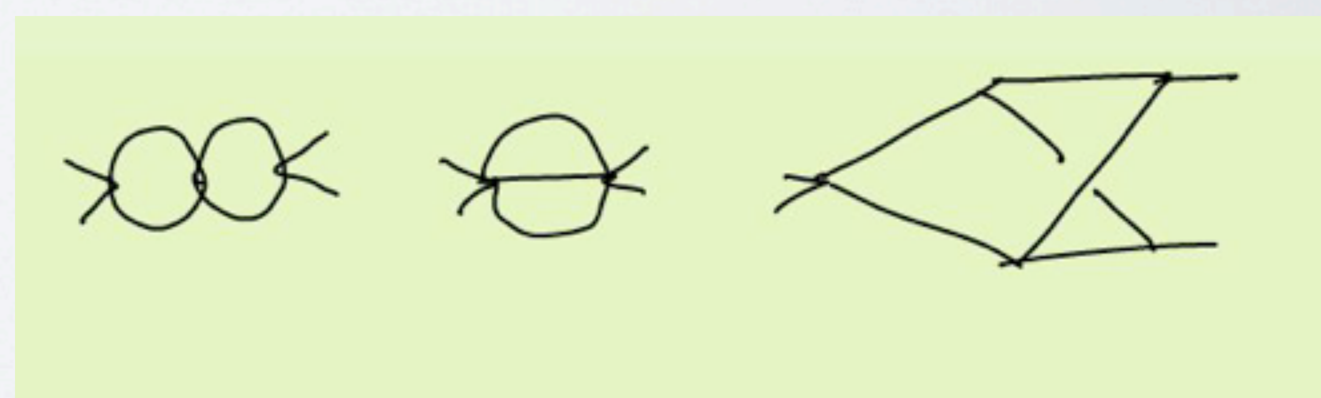
Melnikov, CA

- 18 double real-radiation master integrals



- 7 real-virtual master integrals

- 3 double-virtual master integrals for the two-loop form factor



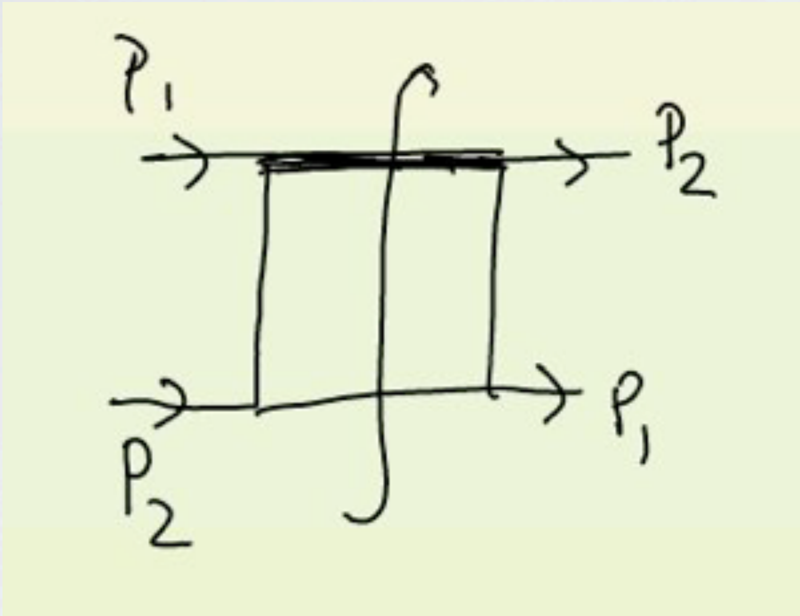
FROM NNLO TO NNNLO

	NLO	NNLO	NNNLO
topologies	1	11	217
master integrals per topology	1	~5	~25
total number of master integrals	1	18	~1000
	integrations over real radiation tree-level graphs		

- *Sheer magnitude of such a calculation is frightening*
- *But, we can hope in sharpening our methods*

THE NLO REAL RADIATION EXAMPLE

Consider the NLO real radiation topology:



$$I\left[\nu_1,\nu_2\right] = \int d^d k \frac{\delta\left((p_{12}-k)^2 - M_V^2\right)\delta\left(k^2\right)}{\left[(k-p_1)^2\right]^{\nu_1}\left[(k-p_2)^2\right]^{\nu_2}}$$

two-scale integral

$$\nu_1,\nu_2 = \ldots,-2,-1,0,1,2,\ldots$$

Scaling of the gluon momentum: $k = \bar{z}\quad l, \quad \bar{z} \equiv 1 - z = 1 - \frac{M_V^2}{\hat{s}}$

(no approximation made)

$$I\left[\nu_1,\nu_2\right] = \bar{z}^{1-\nu_{12}-2\epsilon} I_{reg}\left[\nu_1,\nu_2\right]$$

exponent X divergent

= LOGS

$$I_{reg}\left[\nu_1,\nu_2\right] = \int d^d l \frac{\delta\left((l-p_{12})^2\right)\delta\left(l^2\right)}{\left[(l-p_1)^2\right]^{\nu_1}\left[(l-p_2)^2\right]^{\nu_2}}$$

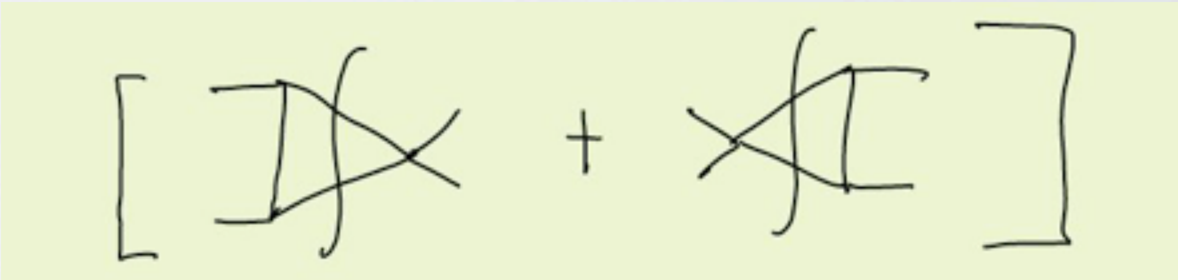
one-scale integral

THE NLO REAL RADIATION EXAMPLE

$$I_{reg}\left[\nu_1,\nu_2\right] = \int d^d l \frac{\delta\left((l-p_{12})^2\right)\delta\left(l^2\right)}{\left[(l-p_1)^2\right]^{\nu_1}\left[(l-p_2)^2\right]^{\nu_2}}$$

Trivial to perform the integration over the rescaled momentum.
But, let's resist the temptation.

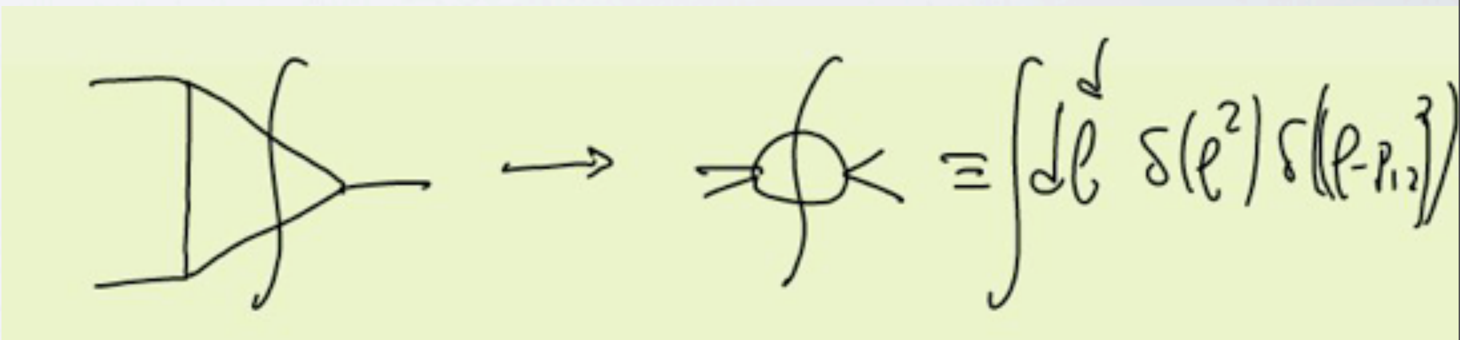
$$(l-p_1)^2 + (l-p_2)^2 = (l-p_{12})^2 + l^2 - p_{12}^2 \rightsquigarrow$$



Double cut of one-loop form factor integrals

REVERSE UNITARITY:

$$\delta\left(l^2\right), \delta\left((l-p_{12})^2\right) \to \frac{i}{l^2}, \frac{i}{(l-p_{12})^2} \quad \rightsquigarrow$$



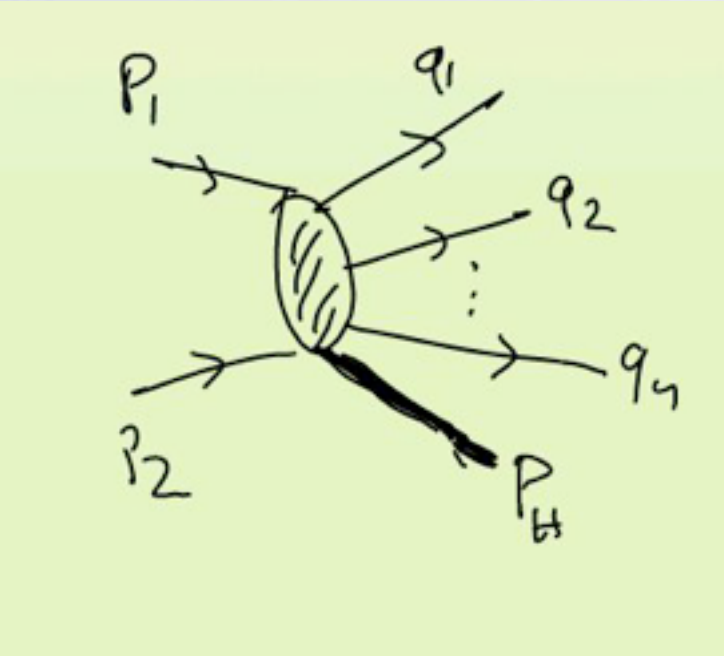
One master integral:
two massless particle phase-space measure

FIRST LESSONS

- A rescaling of gluon momenta which captures their behavior in the soft limit leads to phase-space integrals which depend only on a single kinematic scale (at NLO).
- Reverse unitarity and integration by parts minimize the amount of integrations (down to one integral).
- Calculation is almost entirely algebraic (=algorithmic).

MULTIPLE REAL EMISSION

$$I = \int d^d q_1 \ldots d^d q_N \delta(q_1^2) \ldots \delta(q_N^2) \delta\left((p_{12} - q_{12\ldots N})^2 - M_V^2\right) \left|\mathcal{M}^2\right|^2$$



reverse unitarity

$$I = \int \frac{d^d q_1 \ldots d^d q_N}{q_1^2 \ldots q_N^2 \left((p_{12} - q_{12\ldots N})^2 - M_V^2\right)} \left|\mathcal{M}^2\right|^2$$

SCALING: $q_i \to \bar{z} q_i$ ***(no approximation made yet)***

$$I = \bar{z}^{N(d-2)-1} \int \frac{d^d q_1 \ldots d^d q_N}{q_1^2 \ldots q_N^2 \left((p_{12} - q_{12\ldots N})^2 - z q_{12\ldots N}^2\right)} \left|\mathcal{M}\right|^2 (\bar{z} q_i, p_1, p_2)$$

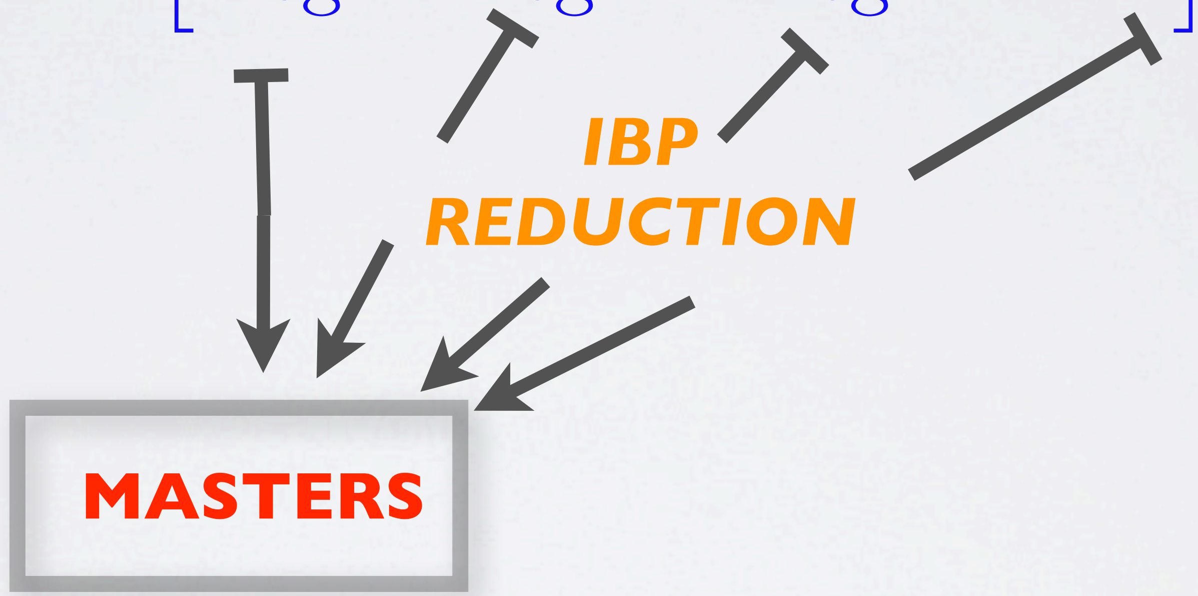
Correct asymptotic behavior

New integral depends on z. But it is regular at z=1.
Can be expanded INSIDE the integration sign.

MULTIPLE REAL RADIATION

Taylor expanding the integrand:

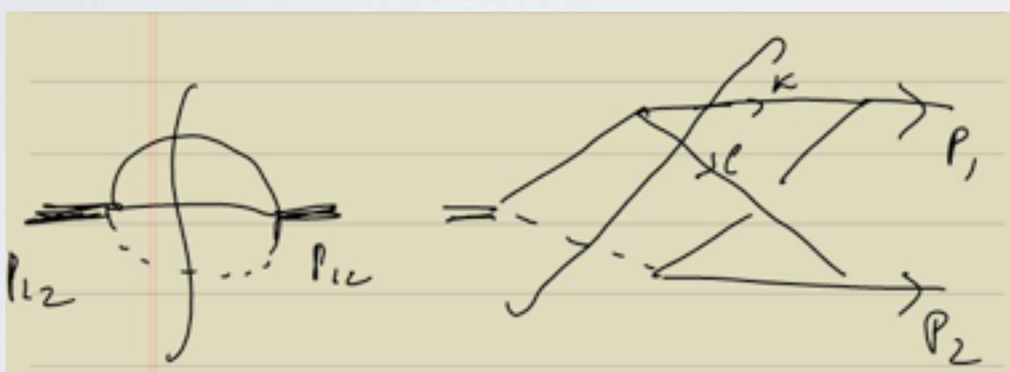
$$I = \bar{z}^{2N\epsilon-1}\left[I^{(0)}_{\text{reg}} + I^{(1)}_{\text{reg}}\bar{z} + I^{(2)}_{\text{reg}}\bar{z}^2 + \ldots\right]$$



- Integrals of sub-leading terms within a topology reduce to the *same master integrals* as the ones making up the strict soft limit!
- Computing more terms in the series expansion is an algebraic problem
- no new master integrals emerge.

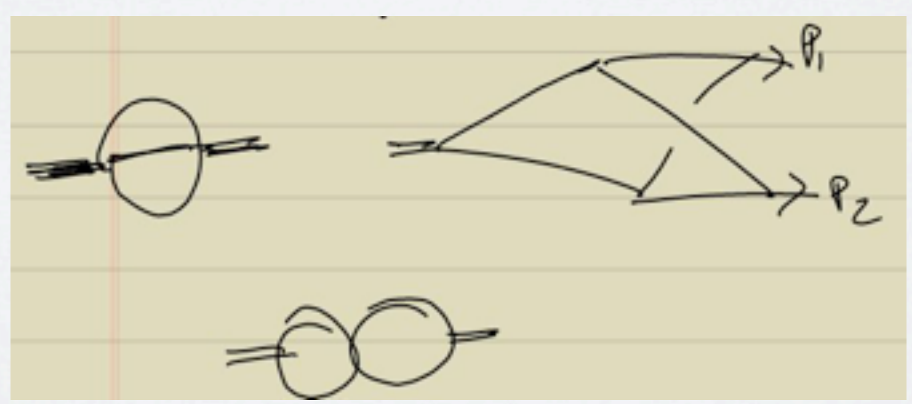
DOUBLE REAL RADIATION AT NNLO

- 18 master integrals for a generic value of z.
- Two master integrals for the expansion around the soft limit:



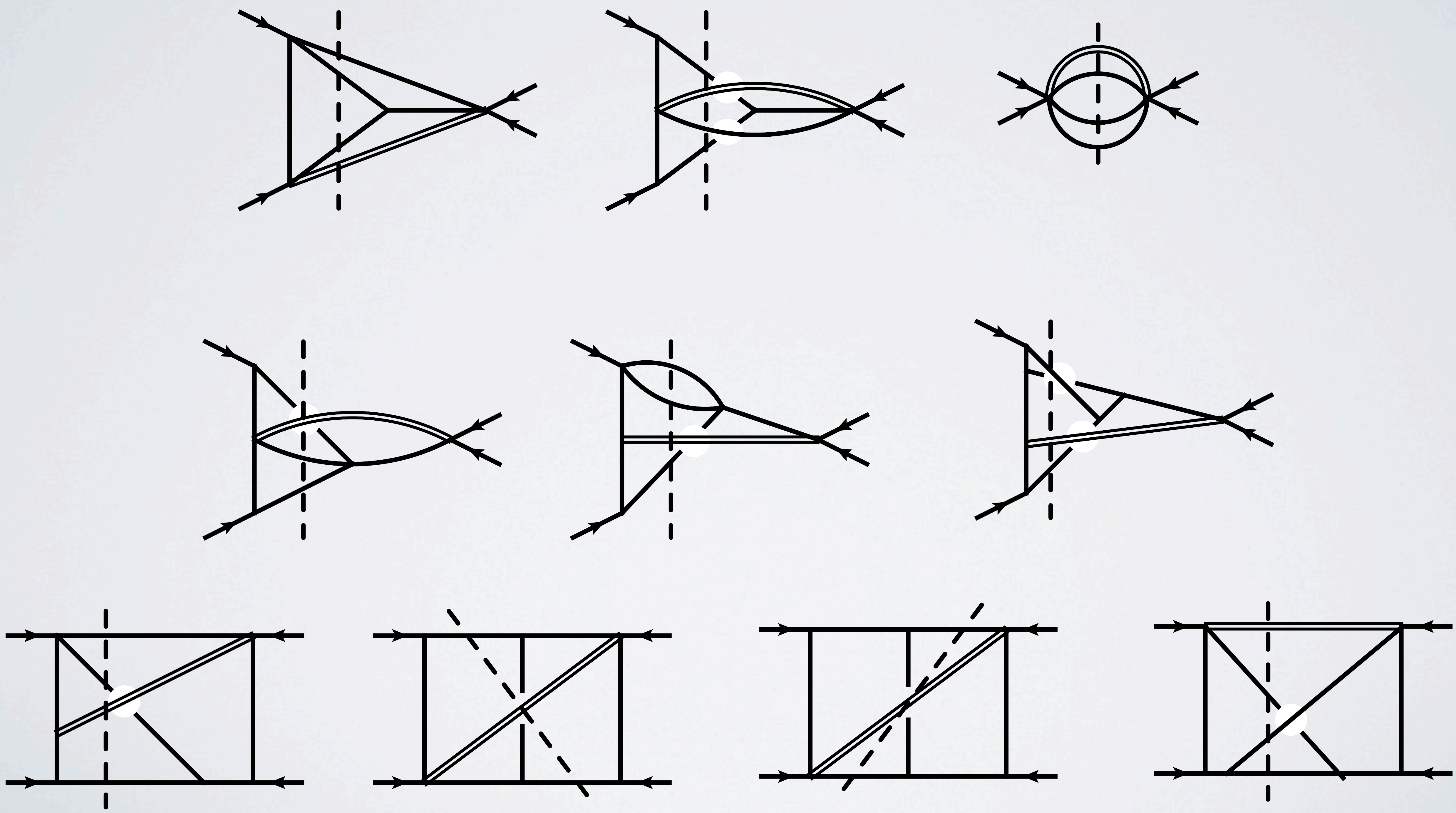
$$- - - - \equiv \left. \frac{1}{(p_{12}-k+l)^2 - z(k+l)^2} \right|_{z=1}$$

- Recall the master integrals for the two-loop form factor:



- They are of similar nature (coincide in the "wrong" limit $\mathbf{z=0}$).

10 Triple Real Soft Masters @ N3LO



$$\int \text{Calculation of Master Integrals}\; d\Phi_4$$

- Almost at our wit's end... used every trick we knew for multidimensional integrals and developed some new tricks and techniques.

- Especially useful: mathematical methods brought to phenomenology by Claude Duhr
  - an algorithm to order multiple integrations
  - algebra of differentials of multiple polylogarithms (symbol and coproduct)

- transformations from Euler-type integrals to Mellin-Barnes integrals and back to Euler-type integrals.

- Shifting the number of space-time dimensions, thus changing the infrared structure of the integrals.

THE SOFT TRIPLE REAL CROSS-SECTION

$$
\begin{aligned}
\sigma^{S(0)}_{g\,g\to H+g\,q\,\bar{q}} = \frac{2^5}{3^7}\frac{1}{8(N_c^2-1)^2}(4\pi\alpha_S)^3\Phi_4^S(\epsilon)C_AC_Fc_H^2N_f \\
\times\Bigg\{\frac{153090}{\epsilon^4}-\frac{1604043}{\epsilon^3}+\frac{1}{\epsilon^2}\Big(-29160\zeta_2+4903902\Big) \\
+\frac{1}{\epsilon}\Big(-204120\zeta_3+321732\zeta_2-4833675\Big)-874800\zeta_4+2252124\zeta_3-911088\zeta_2 \\
+203535+\epsilon\Big(-2711880\zeta_5-233280\zeta_2\zeta_3+9651960\zeta_4-6290136\zeta_3-492210\zeta_2 \\
+1667109\Big)+\epsilon^2\Big(-9360360\zeta_6-816480\zeta_3^2+29921076\zeta_5+2573856\zeta_2\zeta_3 \\
-26589060\zeta_4-4323186\zeta_3+4693212\zeta_2+1294731\Big) \\
+2C_AC_F\Bigg[\frac{167670}{\epsilon^4}-\frac{1743039}{\epsilon^3}+\frac{1}{\epsilon^2}\Big(-29160\zeta_2+5267592\Big)+\frac{1}{\epsilon}\Big(-204120\zeta_3 \\
+321732\zeta_2-5183163\Big)-874800\zeta_4+2252124\zeta_3-911088\zeta_2+337959 \\
+\epsilon\Big(-2711880\zeta_5-233280\zeta_2\zeta_3+9651960\zeta_4-6290136\zeta_3-492210\zeta_2+1651749\Big) \\
+\epsilon^2\Big(-9360360\zeta_6-816480\zeta_3^2+29921076\zeta_5+2573856\zeta_2\zeta_3-26589060\zeta_4 \\
-4323186\zeta_3+4693212\zeta_2+1284491\Big)\Bigg]+\mathcal{O}(\epsilon^3)\Bigg\}
\end{aligned}
$$

[Duhr,Dulat,Mistlberger, CA]

EXPANDING REAL-VIRTUAL CONTRIBUTIONS

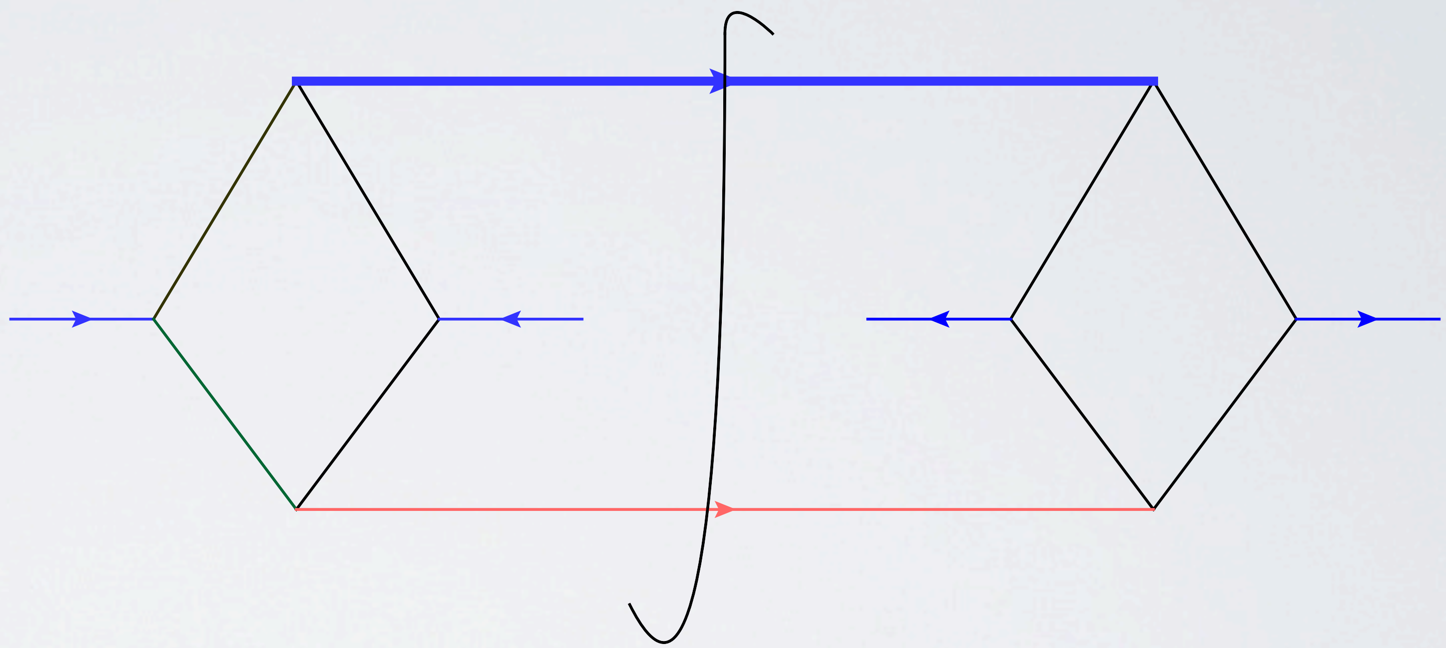
[Duhr,Dulat,Herzog,Mistlberger,CA]

$$\int d^d k \sum \stackrel{?}{=} \sum \int d^d k$$

- Extending this method for the threshold expansion of integrals with both loop and phase-space variables is not straightforward.
- Loop momenta have an unbounded range; all hierarchies for the magnitude of the components of the loop momenta and the external momenta are possible.
- We need expansions of loop integrals which are convergent in the entire phase-space.
- And we would like these representations to be in momentum space so that we can perform a threshold expansion at the integrand before we perform the integration.

AN EXAMPLE

- Let's try to integrate the one-loop squared over phase-space.
- We can find an expression for the box function, through analytic continuations etc, which can be expanded around the limit of soft gluon emissions.
- But such convergent series representations are not guaranteed to be discovered for more complicated one-loop (pentagons) or two-loop subgraphs.



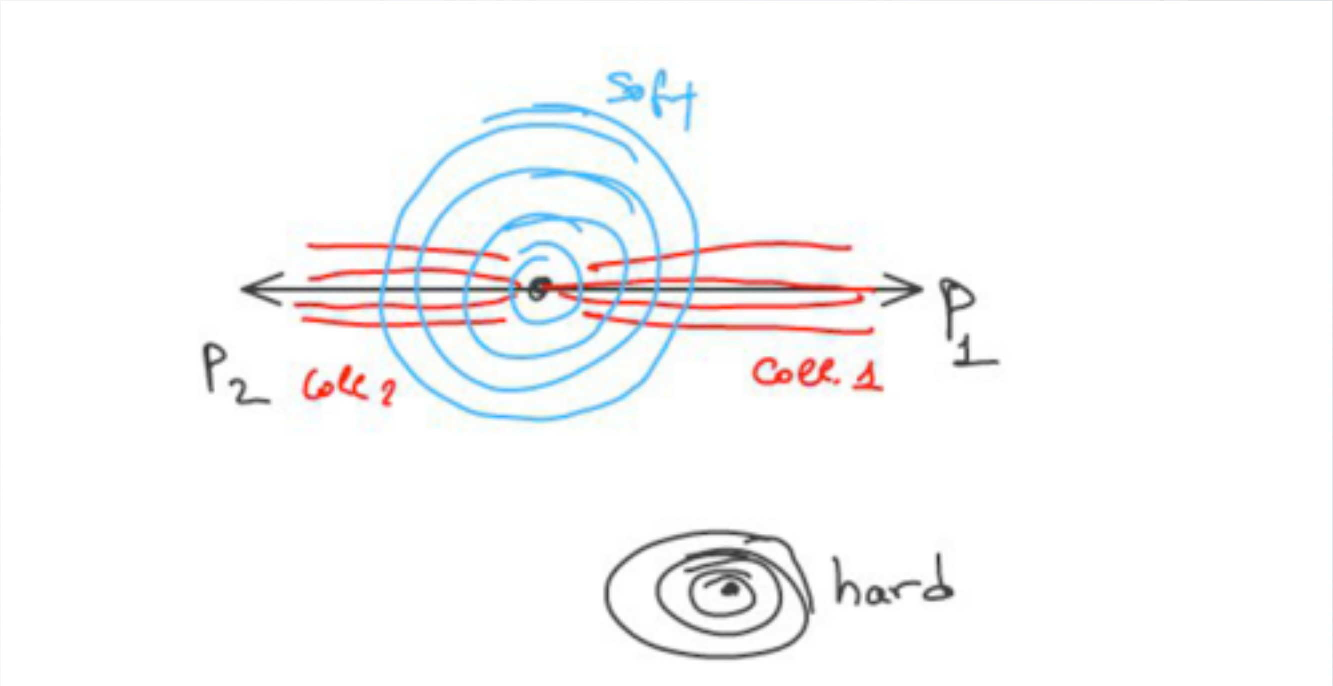
$$\mathrm{Box}(-t,-u,s) = \frac{2c_\Gamma}{\epsilon^2}\Gamma(1+\epsilon)\Gamma(1-\epsilon)\frac{\left(-\frac{tu}{s}\right)^{-\epsilon}}{tu}$$
$$-\frac{2c_\Gamma}{\epsilon(1+\epsilon)}\frac{t^{-\epsilon-1}}{s}\,{}_2F_1\left(1,1+\epsilon;2+\epsilon;\frac{u}{s}\right)$$
$$-\frac{2c_\Gamma}{\epsilon(1+\epsilon)}\frac{u^{-\epsilon-1}}{s}\,{}_2F_1\left(1,1+\epsilon;2+\epsilon;\frac{t}{s}\right)$$
$$-\frac{2c_\Gamma}{\epsilon(1+\epsilon)}\frac{(-s)^{-\epsilon}}{s^2}\,F_2\left(2+\epsilon;1+\epsilon,1+\epsilon;2+\epsilon,2+\epsilon;\frac{u}{s},\frac{t}{s}\right)$$

EXPANSION BY REGIONS

- An examination of Landau equations and their Norton-Coleman picture reveals three singular surfaces/point: collinear 1, collinear 2 and soft.
- We find internal and normal coordinates to the singular surfaces.
- And expand the integrand moving further and further away from the three singular surfaces.
- We also expand **the integrand** around a point which is far away from any singularity (hard region)
- This procedure yields the four terms that we have found before.

Singular surfaces

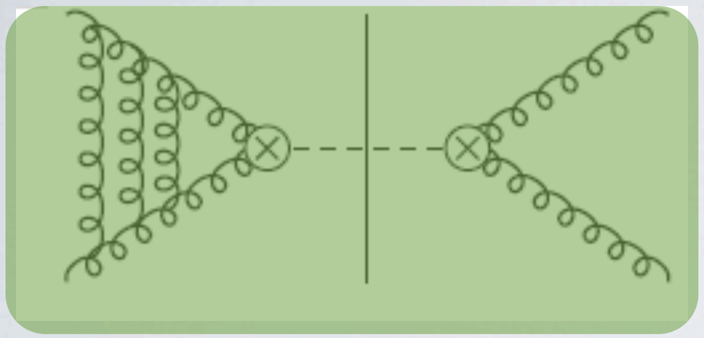
$$k^\mu = 0 \quad k^\mu = a p_1^\mu \quad k^\mu = b p_2^\mu$$



EXPANSION BY REGIONS AND REVERSE UNITARITY

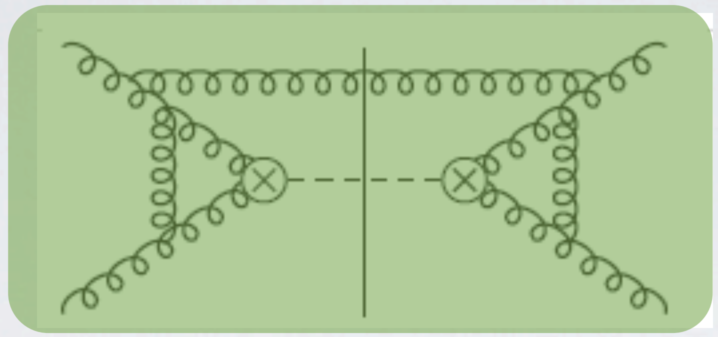
- We do not have yet a mathematical proof that an expansion by regions as defined by the infrared singular surfaces of loop integrals should yield the complete answer for the integral.
- But it seems to be the case in all examples that we have seen so far.
- With this conjecture, we have a method to perform a threshold expansion at the integrand level of loop integrals.
- We can readily combine it with reverse unitarity to obtain the result of mixed phase-space and loop integrations.
- Have applied it to the computation of real-virtual squared contributions.

PROGRESS AT N3LO



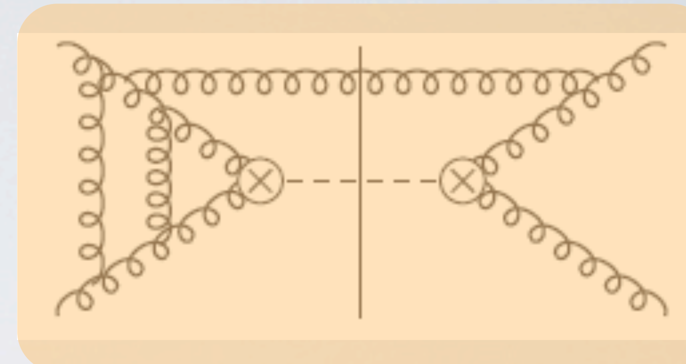
Triple-Virtual

Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser; Gehrmann, Glover, Huber, Ikilerli, Studerus



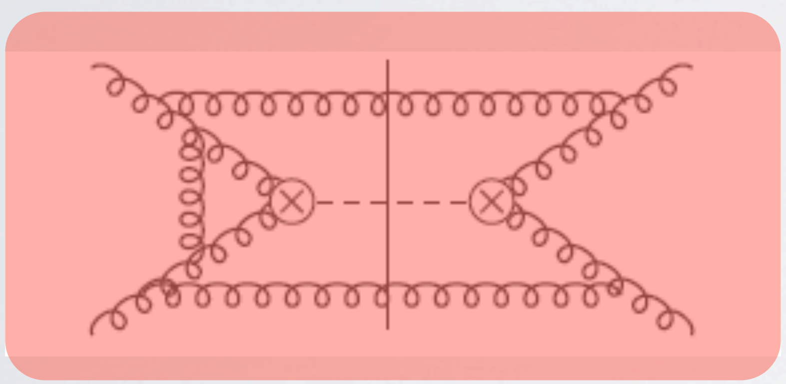
Squared Real-Virtual

Duhr, Dulat, Herzog, Mistlberger, CA

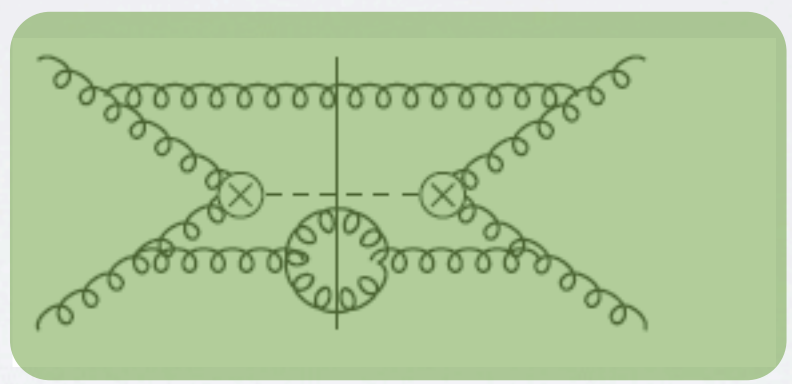


Real-Double Virtual

Duhr, Gehrmann;
Li, Xing Zhu



Double Real-Virtual



Triple Real

Duhr, Dulat, Mistlberger, CA

Convolution of splitting functions and NNLO

Buehler, Lazopoulos;
Buehler, Duhr, Herzog, CA;
Pak, Rogal, Steinhauser;
Hoeschele, Hoff, Pak, Steinhauser, Ueda

to be continued...