

# Small- $x$ gluon from exclusive $J/\psi$ production

$$\sigma(\gamma p \rightarrow p + J/\psi) \text{ \& } d\sigma/dy(pp \rightarrow p + J/\psi + p)$$

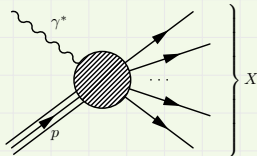
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LIVERPOOL

Stephen Jones

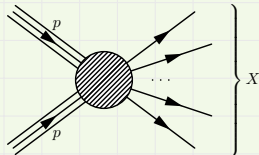
H1 Collaboration Meeting  
12th September

# Forward physics: Exclusive & Diffractive Processes

Inclusive Processes - Included in global analyses

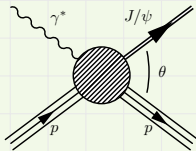


DIS

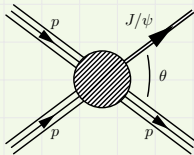


Inclusive  $pp$  Scattering

Exclusive & Diffractive Processes



Exclusive HVM Electroproduction



Ultrapерipheral HVM production

# Outline

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## Motivation

- Inclusive DIS poorly constrains  $xg(x, \mu^2)$  at low scales ( $\mu^2 < 6 \text{ GeV}^2$ ) and small- $x$  ( $\sim 10^{-4}$ )
- Exclusive processes such as exclusive HVM production are a sensitive probe of the gluon in this domain

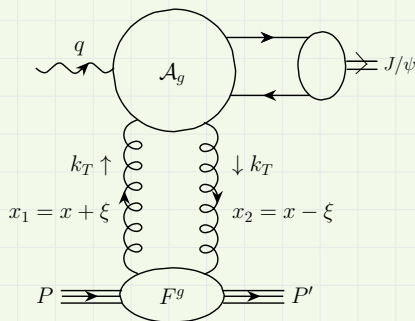
## Contents

- Review of elastic  $J/\psi$  photo(electro)-production
- Extracting the gluon density at low  $\mu^2$ , small- $x$
- Photoproduction data from LHCb events ( $pp \rightarrow p + J/\psi + p$ )
- Results & predictions
- Future theory improvements

Based on work with:

Alan Martin, Misha Ryskin and Thomas Teubner [ [arXiv:1307.7099](https://arxiv.org/abs/1307.7099)[hep-ph] ]

# $J/\psi$ Photo(electro)-production



## General Setup & Assumptions

- Factorises:  $\phi_{c\bar{c}}^\gamma \otimes T_{c\bar{c}+p} \otimes \phi_{c\bar{c}}^{J/\psi}$
- Non-relativistic  $J/\psi$  wave function  $\propto \Gamma_{ee}$
- $\Gamma_{ee}$ ,  $M_{J/\psi}$  from experiment
- Relativistic corrections  $\mathcal{O}(4\%)$
- At LO:  $k_T^2 \ll \bar{Q}^2$ , leading log approximation (LLA)
- Scale:  $\bar{Q}^2 = (Q^2 + M_{J/\psi}^2)/4$
- $Q^2 = -q^2$  photon virtuality
- $W$  CM energy of  $\gamma^* p$

## $J/\psi$ Photo(electro)-production at LO

### Amplitude ( $k_T$ Factorisation)

$$A \propto \int \frac{dk_T^2}{2k_T^4} \left( \frac{1}{\bar{Q}^2} - \frac{1}{\bar{Q}^2 + k_T^2} \right) \alpha_s(k_T^2) f(x, k_T^2) \xrightarrow{\text{LLA}} \frac{\alpha_s(\bar{Q}^2) x g(x, \bar{Q}^2)}{2\bar{Q}^4}$$

### Result (LLA, Im Part)

$$\frac{d\sigma}{dt}(\gamma^* p \rightarrow J/\psi p) \Big|_{t=0} = \frac{\Gamma_{ee} M_{J/\psi}^3 \pi^3}{48\alpha} \frac{\alpha_S(\bar{Q}^2)^2}{\bar{Q}^8} [xg(x, \bar{Q}^2)]^2 \left( 1 + \frac{Q^2}{M_{J/\psi}^2} \right)$$

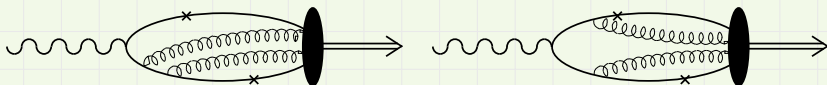
[ Ryskin 1993 ]

- $x = (Q^2 + M_{J/\psi}^2)/(W^2 + Q^2)$
- Can restore  $t$  dependence assuming form  $\exp(-bt)$
- $b = b_0 + 4\alpha' \ln(W/W_0)$  is the *slope* parameter
- Proton-pomeron intercept ( $b_0$ ) and pomeron slope ( $\alpha'$ ) fitted from experiment

# Non-relativistic $J/\psi$ Formation (Theory Uncertainty)

## Hoodbhoy Study [ Hoodbhoy 1997 ]

- Accounts for Fermi motion of the  $c\bar{c}$  pair
  - Work in  $J/\psi$  rest frame using Coulomb gauge
  - Expand in powers of heavy quark relative velocity upto  $\mathcal{O}(v^2)$
  - Necessary to include extra gluon fields to maintain gauge invariance
  - Procedure accounts for largest contribution from Fermi motion



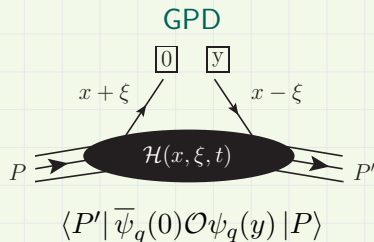
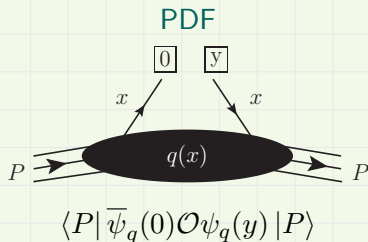
## Result

- Correction factor due to Fermi motion  $\approx 0.96$  (for cross-section)
- Our study neglects these corrections

# Skewing (Theory Uncertainty)

## GPDs

- Strictly, process does not couple to diagonal gluon but to non-diagonal Generalised PDF (GPDs)



## Shuvaev Transform

- In small- $x$  and  $\xi$  limit GPDs are related to PDFs via Shuvaev transform [ Shuvaev et. al 1999 ]

# Shuvaev Transform

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## Outline

- Anomalous dimensions of Gegenbauer moments,  $G_N$  of  $H(x, \xi)$ , are equal to anomalous dimensions of conventional Mellin moments,  $M_N = \int_0^1 x^N H(x, 0) dx$
- Polynomiality:  $G_N = \sum_{n=0}^N c_n^N \xi^{2n}$ , allows all Gegenbauer moments to be determined  $\mathcal{O}(\xi^2)$  from conventional PDFs ( $c_0^N = M_N$ )<sup>1</sup>

## Regge-based assumption

- No singularities in the right half plane ( $j > 1$ ) in the space-like ( $\xi < |x|$ ) domain of diagonal small- $x$  input distributions

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<sup>1</sup>reduces to  $\mathcal{O}(\xi)$  at NLO



## Shuvaev Transform (II)

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- Shuvaev transform is an inverse (integral) transformation which determines  $x$  dependence of  $H(x, \xi)$  for a given small  $\xi$  from diagonal  $H(x, 0)$

$$H_g(x, \xi) = \int_{-1}^1 dx' \left[ \frac{2}{\pi} \text{Im} \int_0^1 \frac{ds(x + \xi(1 - 2s))}{y(s)\sqrt{1 - y(s)x'}} \right] \frac{d}{dx'} \left( \frac{g(x')}{|x'|} \right)$$

With  $y(s) = \frac{4s(1-s)}{x + \xi(1-2s)}$

- Computationally expensive to compute for realistic input distributions
- GPD grids obtained from PDFs via Shuvaev transform available  
[ [Martin et al. 2009](#) ]

## Skewing & Real Part

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- We use 'maximal skewing' limit ( $\xi = x$ ), pure power PDF ( $xg \sim x^{-\lambda}$ ) in transform which gives skewing factor

$$R_g = \frac{2^{2\lambda+3} \Gamma(\lambda + \frac{5}{2})}{\sqrt{\pi} \Gamma(\lambda + 4)}$$

- $\mathcal{O}(20 - 30\%)$  error on cross section compared to full transform  
[ Harland-Lang 2013 ]
- $\mathcal{O}(10 - 15\%)$  on gluon parameters

### Real Part

- Real contribution included via dispersion relation assuming in the low  $x$  region  $A \propto x^{-\lambda} + (-x)^{-\lambda}$

$$\frac{\text{Re}A}{\text{Im}A} \simeq \frac{\pi}{2} \lambda$$

## Beyond LO Approach

- 'Unintegrated' PDF 
$$f(x, k_T^2, \mu^2) = \frac{\partial [xg(x, k_T^2) \sqrt{T(k_T^2, \mu^2)}]}{\partial \ln k_T^2}$$
- $T$  is a Sudakov Factor representing the probability that no additional gluons emitted in DGLAP evolution from  $k_T^2$  to  $\mu^2$  destroy the rapidity gap

$$T(k_T^2, \mu^2) = \exp \left[ \frac{-C_A \alpha_s(\mu^2)}{4\pi} \ln^2 \left( \frac{\mu^2}{k_T^2} \right) \right]$$

- Above some IR scale  $Q_0^2$  up to kinematic upper bound perform explicit  $k_T^2$  integration in the last step of the evolution

$$\text{Im}A \sim \text{IR Part} + \int_{Q_0^2}^{(W^2 - M_{J/\psi}^2)/4} \frac{dk_T^2}{k_T^2} \frac{\alpha_s(\mu^2)}{\bar{Q}^2(\bar{Q}^2 + k_T^2)} f(x, k_T^2, \mu^2)$$

## IR Part and Scale Choices

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### IR Part

- For  $k_T < Q_0$  assume linear behaviour of gluon at small  $k_T^2$

$$xg(x, k_T^2) \sqrt{T(k_T^2, \mu^2)} = xg(x, Q_0^2) \sqrt{T(Q_0^2, \mu_{\text{IR}}^2)} k_T^2 / Q_0^2$$

- Gives IR Part:

$$\ln \left( \frac{\bar{Q}^2 + Q_0^2}{\bar{Q}^2} \right) \frac{\alpha_s(\mu_{\text{IR}}^2)}{\bar{Q}^2 Q_0^2} xg(x, Q_0^2) \sqrt{T(Q_0^2, \mu_{\text{IR}}^2)}$$

### Scale Choice

- Scale choice ambiguity remains (is extracted gluon e.g.  $\overline{\text{MS}}$ ?)
- Choose  $\mu^2 = \max(k_T^2, \bar{Q}^2)$  and  $\mu_{\text{IR}}^2 = \max(Q_0^2, \bar{Q}^2)$
- Scale in IR Part matches lowest scale in integral
- $Q_0^2 = 1 \text{ GeV}^2$  (fit relatively insensitive to this)
- Electroproduction typically contributes at higher scale

## Fitting Procedure

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- Try two different ansätze for small- $x$  gluon

### LO Approach

- Power law:  $xg(x, \mu^2) = Nx^{-\lambda}$  with  $\lambda = a + b \ln(\mu^2/0.45\text{GeV}^2)$

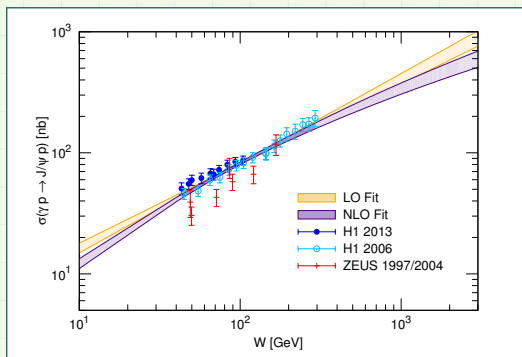
### Beyond 'LO' Approach

- Resum leading  $(\alpha_s \ln(1/x) \ln \mu^2)^n$  contributions
- $xg(x, \mu^2) = Nx^{-a}(\mu^2)^b \exp \left[ \sqrt{16N_c/\beta_0 \ln(1/x) \ln(G)} \right]$
- $G = \ln(\mu^2/\Lambda_{\text{QCD}}^2) / \ln(Q_0^2/\Lambda_{\text{QCD}}^2)$  with  $\Lambda_{\text{QCD}} = 200 \text{ MeV}$

### Fitting Procedure

- Non-linear  $\chi^2$  fit to H1 and ZEUS exclusive data
- Compute full  $\sigma(\gamma^* p \rightarrow J/\psi + p)$  vs  $N, a, b$
- Minimise  $\chi^2$  iteratively
- Obtain best fit  $N, a, b$  and full covariance matrix for error estimate

# HERA ( $\gamma^* p \rightarrow J/\psi + p$ )



## Parameters

	LO	'NLO'
$N$	1.24	0.26
$a$	0.05	-0.10
$b$	0.08	-0.15
$\chi_{d.o.f}^2$	1.2	1.3

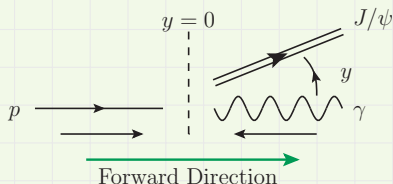
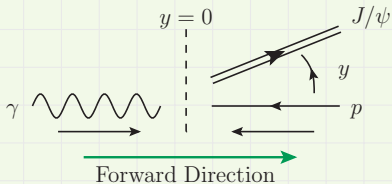
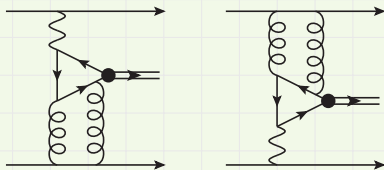
- Update to MNRT fit  
[ Martin et al. 2008 ]

- Probes gluon for  $10^{-4} \lesssim x \lesssim 10^{-2}$
- **Note:** Electroproduction data included in fit (not shown here)

# LHCb Data

- Measured  $d\sigma(pp \rightarrow p + J/\psi + p)/dy$  vs  $y$

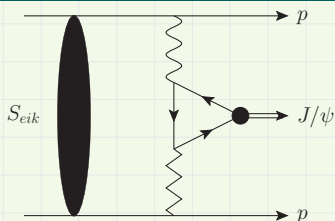
- Aliasing problem extracting  $\sigma(\gamma p \rightarrow J/\psi + p)$

 $W_+$  $W_-$ 

- $(W_{\pm})^2 = M_{J/\psi} \sqrt{s} \exp(\pm|y|)$

## Survival factors

- For  $pp \rightarrow p + J/\psi + p$  non-negligible interactions between spectator quarks
- Can populate rapidity gap
- Event not selected



## KMR Model

$$S^2 = \langle S^2(b_t) \rangle = \frac{\int \sum_i |\mathcal{M}_i(s, b_t^2)|^2 \exp[-\Omega_i(s, b_t^2)] d^2 b_t}{\int \sum_i |\mathcal{M}_i(s, b_t^2)|^2 d^2 b_t}$$

- $\mathcal{M}_i$  - process dependent matrix elements
- $b_t$  - impact parameter,  $\Omega_i$  - 'universal' proton opacities

[ Khoze et al. 2002 ] [ Khoze et al. 2013 ]



## KMR Model

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- Fitted to diffractive  $pp$  and  $p\bar{p}$  data:
  - $\sigma_{\text{tot}}$  - Total cross section ( $\sigma_{\text{el}} + \sigma_{\text{inel}}$ )
  - $d\sigma/dt$  - Elastic cross section
  - $\sigma_{\text{lowM}}^{\text{D}}$  - Low mass dissociation ( $pp \rightarrow N^* + p$ )
  - $d\sigma/d(\Delta\eta)$  - High mass dissociation
- Data from:
  - CERN ISR 1975–1980
  - CERN SPS 1982–1993
  - TEVATRON (CDF, DØ) 1990–2012
  - TOTEM 2011–2013
  - ATLAS 2012
- Two-channel eikonal model with one ‘effective pomeron’
- Proton wave function written as superposition of two diffractive Good-Walker eigenstates  $|p\rangle = \sum_i a_i |\phi_i\rangle$  with  $i = 1, 2$

## KMR Model (II)

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- Strictly, use an opacity matrix  $\Omega_{ik}$  corresponding to one-pomeron-exchange between states  $\phi_i$  and  $\phi_k$
- Observables in terms of GW eigenstates depend on this opacity e.g.

$$\sigma_{\text{inel}} = \int d^2 b_t \sum_{i,k} |a_i|^2 |a_k|^2 (1 - \exp[-\Omega_{ik}(b_t)])$$

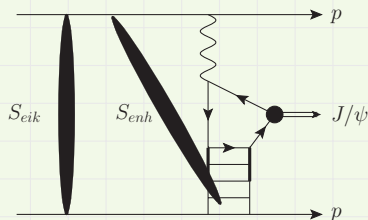
- Each GW eigenstate  $|\phi_i\rangle$  independently parametrised by a form factor

$$F_i(t) = \exp \left[ -(b_i(c_i - t))^{d_i} + (b_i c_i)^{d_i} \right]$$

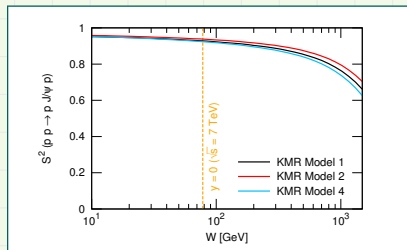
- 3 parameters per eigenstate + 1 relative weighting
- 'Effective' pomeron has energy dependent coupling to eigenstates
- 6 pomeron trajectory parameters: intercept ( $\Delta$ ), slope ( $\alpha'$ ) and couplings (gives  $b_0 = 4.9$ ,  $\alpha' = 0.06$  for  $b$  slope)

## KMR Model (III)

- Survival factors reasonably certain ( $\mathcal{O}(5\%)$  difference between KMR models)
- Less certain for high rapidity



- Include this possibility using method of KMR [Ryskin et al. 2009]
- Find small effect from including  $S_{enh}$  (see later)



- Possibility of 'enhanced rescattering'
- Interaction between spectator quarks and parton in ladder

## Photon Flux

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$$\frac{dn}{dk} = \frac{\alpha}{\pi k} \int_0^\infty dq_T^2 \frac{q_T^2 F_p^2(q_T^2)}{(t_{\min} + q_T^2)^2}$$

- $k$  - photon energy
- $q_T$  - photon trans. momentum
- $t_{\min}$  - kinematic  $q^2$  cut-off

- Proton form factor:

$$F_p(q_T^2) = \left(1 + \frac{t_{\min} + q_T^2}{0.71 \text{ GeV}^2}\right)^{-2}, \quad t_{\min} \approx \frac{(x_\gamma m_p)^2}{1 - x_\gamma}$$

- Photon flux consistent with KMR model
  - Similar to equivalent photon approximation (EPA)
  - But: neglect terms  $\propto$  anomalous magnetic moment of the proton

### Accuracy

- Neglected terms  $\propto q_T^2$  have no singularity at  $q_T^2 \rightarrow 0$
- Contributions from  $q_T \sim 1/R_p$  are concentrated at small  $b_t$ , suppressed by large opacities

$(\gamma p \rightarrow J/\psi + p)$  from  $(pp \rightarrow p + J/\psi + p)$

$$\frac{d\sigma(pp)}{dy} = S^2(W_+) \left( k_+ \frac{dn}{dk_+} \right) \sigma_+(\gamma p) + S^2(W_-) \left( k_- \frac{dn}{dk_-} \right) \sigma_-(\gamma p)$$

- Contribution from  $W_+$  and  $W_-$  due to  $\gamma p/p\gamma$  ambiguity
- Absorptive corrections  $S^2(W)$ , depends on  $W$
- Cancellation between photon flux in cross section and denominator of  $S^2$

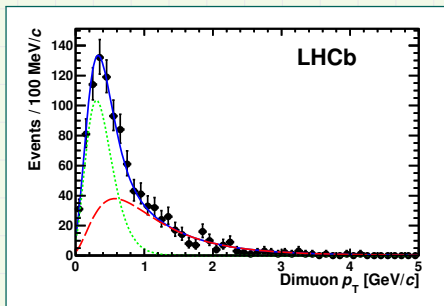
### Fit Accuracy

- Directly fit to  $d\sigma(pp)/dy$  data
- Undesirable dependence on  $\sigma_-(\gamma p)$  unavoidable (Pb-p may improve this)

# LHCb & CDF

## LHCb

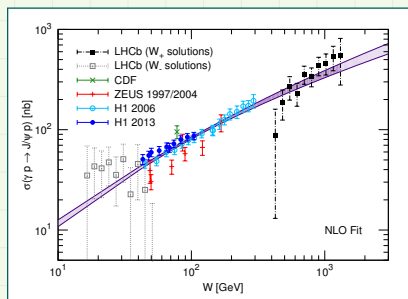
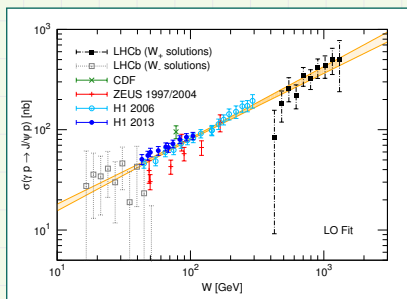
- Non-exclusive background from fitting  $p_T$  of events with 3–8 forward tracks and linearly extrapolating to 2
- Background higher  $p_T$  component may contain significant part of odderon contribution
- LHCb provide 10 points for rapidities  $2 < y < 4.5$  [LHCb 2013]



## CDF

- CDF provide 1 point for  $y = 0$  (no  $W_{\pm}$  ambiguity) [CDF 2009]
- May include odderon contribution (point not included in our fit)

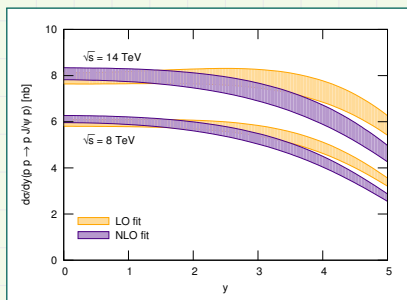
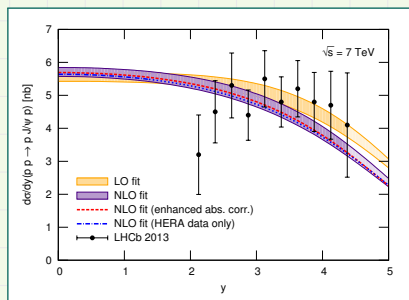
# HERA & LHCb ( $\gamma^* p \rightarrow J/\psi + p$ )



- Extends to  $x \sim 10^{-6}$
- LHCb  $W_{\pm}$  points calculated with our  $S^2$ ,  $dn/dk$ ,  $\sigma_{-}(\gamma p)$
- **Recall:** LO and 'NLO' parameters have different meaning

	LO	'NLO'
$N$	1.27	0.25
$a$	0.05	-0.10
$b$	0.08	-0.15
$\chi_{d.o.f}^2$	1.1	1.2

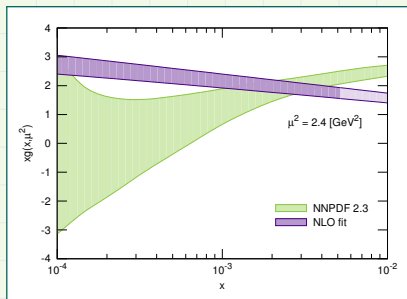
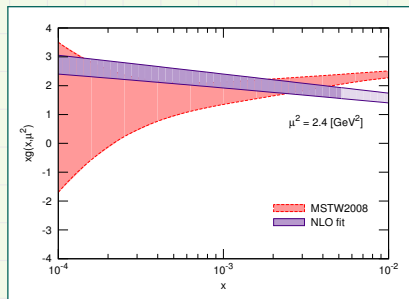
# HERA & LHCb ( $pp \rightarrow p + J/\psi + p$ )



- Can use fitted gluon for  $d\sigma(pp)/dy$  prediction (stable at 14 TeV)
- $W_-$  component accounts for  $\mathcal{O}(30 - 40\%)$  of  $d\sigma(pp)/dy$
- Including 'enhanced rescattering' has small effect on prediction
- HERA fit is consistent with LHCb data (LHCb not constraining)
- New LHCb data will soon reduce errors and provide stronger constraints

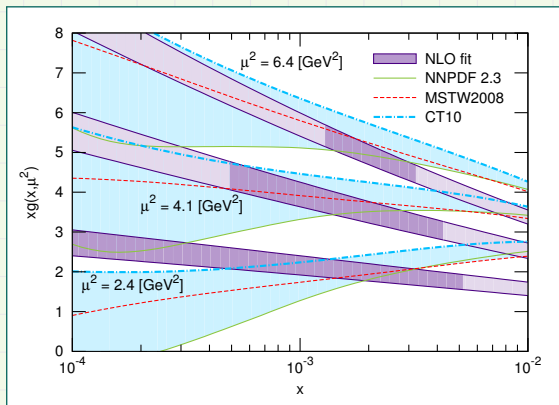


## Gluon Fit



- Fitted gluons below global partons for higher  $x \sim 10^{-2}$
- LHCb data provides support for fit down to  $x \sim 10^{-6}$
- 14 TeV data will probe even lower  $x$

## Gluon Fit (II)

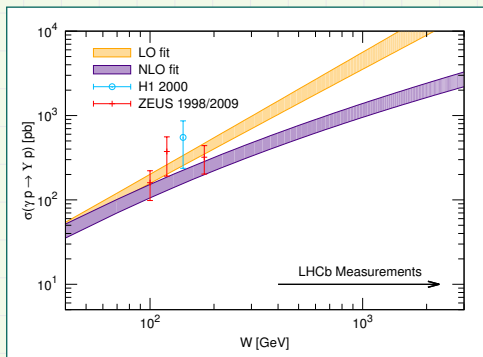


- Scale dependence from  $k_T^2$  integral and HERA electroproduction data

- $J/\psi$  data diminish the huge uncertainty on global gluons at low scale & small- $x$

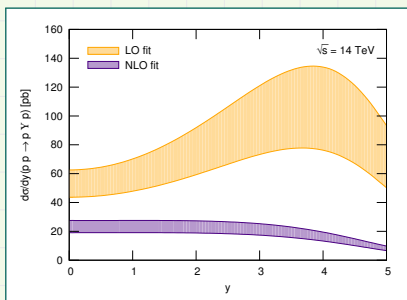
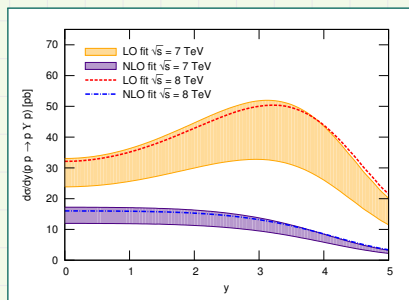
## $\Upsilon$ Postdiction ( $\gamma p \rightarrow \Upsilon + p$ )

- Can use gluon fit extracted from  $J/\psi$  data to make  $\Upsilon$  prediction
- $\sigma(\gamma p)$  and  $d\sigma(pp)/dy$  as for  $J/\psi$
- $S^2$  for  $\Upsilon$  used



- Very little data available for comparison
- Need high energy  $\Upsilon$  to determine scale dependence
- Huge discrepancy between extrapolated LO and 'NLO' fits
- LHCb data to come...

## $\Upsilon$ Prediction ( $pp \rightarrow p + \Upsilon + p$ )



- 'NLO' gluon parametrisation grows  $1/x$  and  $\ln(\mu^2)$  less steep than  $xg \propto x^{-\lambda}$
- Large discrepancy between LO and 'NLO' due to poor high energy constraints on scale behaviour
- Large real and skewing corrections when  $W_-$  is small
- But:  $W_-$  component only  $\mathcal{O}(15 - 20\%)$  of  $d\sigma(pp)/dy$

# Scale Uncertainty

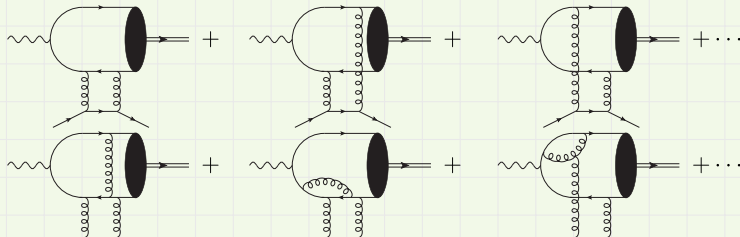
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## Observations from MNRT Study

- Varying  $\mu$  from  $\mu/2$  to  $2\mu$  in Sudakov factor ( $T$ ) gives small effect
  - Inclusion of  $T$  at all only slightly alters the behaviour of the gluon
  - Reason:  $T$  mostly contributes in  $k_T^2$  integral where derivative is taken
- Varying  $\mu$  in  $\alpha_s$  causes strong scale dependence
- Varying scale in both  $\alpha_s$  and  $T$  simultaneously from  $\mu/2$  to  $2\mu$  gives large effect  $\pm 20\%$ 
  - Even including scale variation still obtain better gluon certainty than global partons!
  - Most scale variation is absorbed into the normalisation of the gluon
  - $x$ -behaviour reasonably stable

## Full NLO Contribution

- Currently working on full NLO matrix element in collinear factorisation  
[ Ivanov et al. 2004 ]
- Quark contribution starts & many more gluon diagrams



- Better estimation of scale & check of scale variation
- Clear scheme choice ( $\overline{\text{MS}}$ )
- Possibility to extend NRQCD treatment (e.g... 1-loop QCD)

# Conclusion

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## Summary

- Updated MNRT gluon fit to include recent LHCb data [ [Martin et al. 2008](#) ]
- Provided predictions for  $J/\psi$  photoproduction @ 8 & 14 TeV
- Provided predictions for  $\Upsilon$  photoproduction @ 7, 8 & 14 TeV

## Issues

- Considerable scale uncertainty remains
- Not a complete NLO analysis (but main kinematic effects included)
- Can not directly identify extracted gluon with e.g.  $\overline{\text{MS}}$  partons

## Future

- Work under way to include NLO gluon diagrams and quark coupling [ [Ivanov et al. 2004](#) ]
- More  $pp \rightarrow p + J/\psi + p$  data on the way from LHCb (and others?)
- Pb-p data, reduced  $W_-$  component
- $pp \rightarrow p + \Upsilon + p$  data will have strong ability to constrain scale dependence
- Excellent opportunity to utilise new exclusive data to constrain small- $x$  PDFs

## References

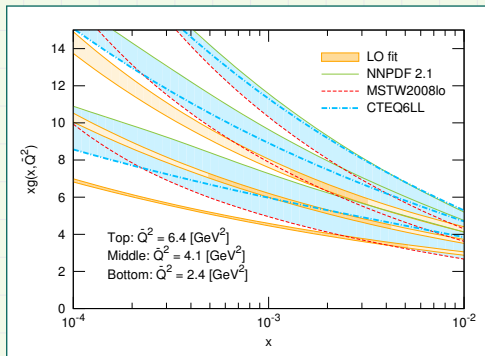
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Extra

# LO Gluon Fit



- Decreased uncertainty
- Steeply rising behaviour as for global partons
- Receive sizeable correction when including NLO effects (Like global partons)