Quark and Lepton Yukawa couplings from Minimum Principle

Gino Isidori
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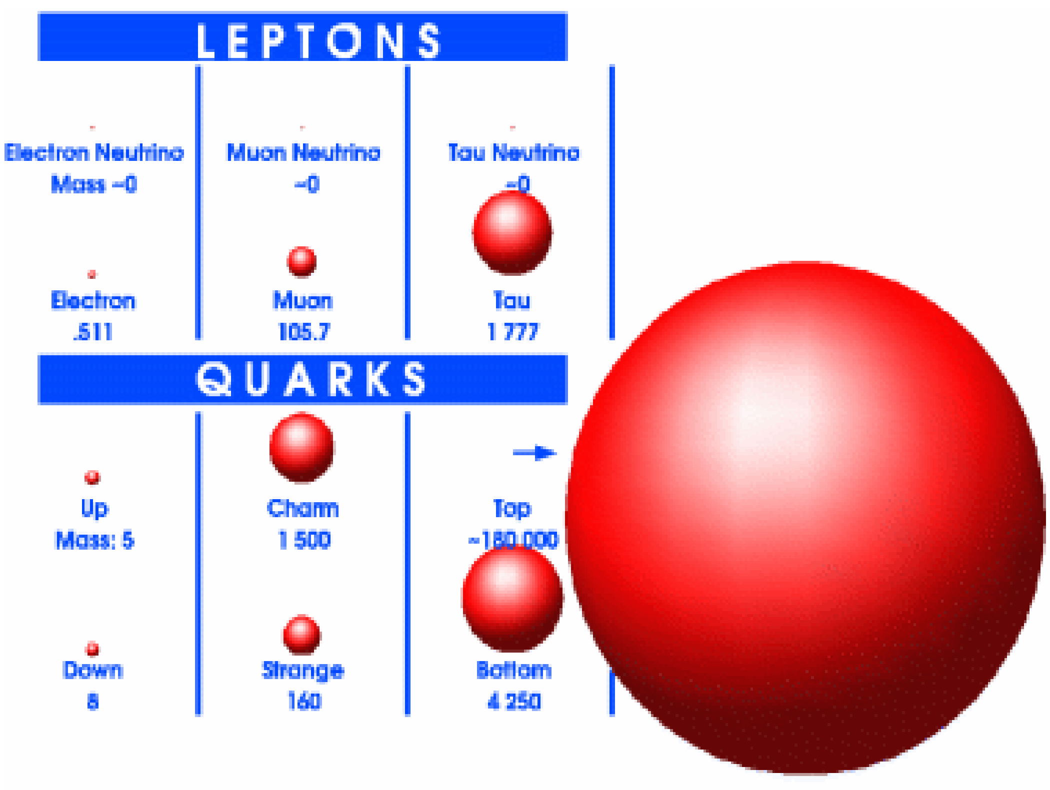
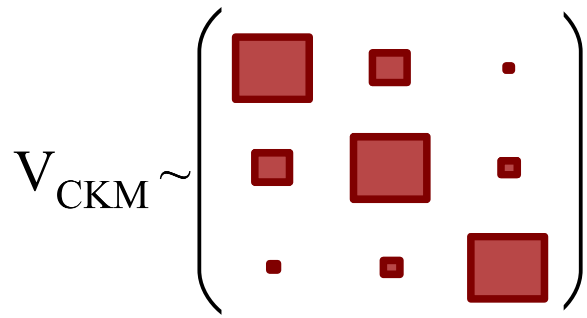
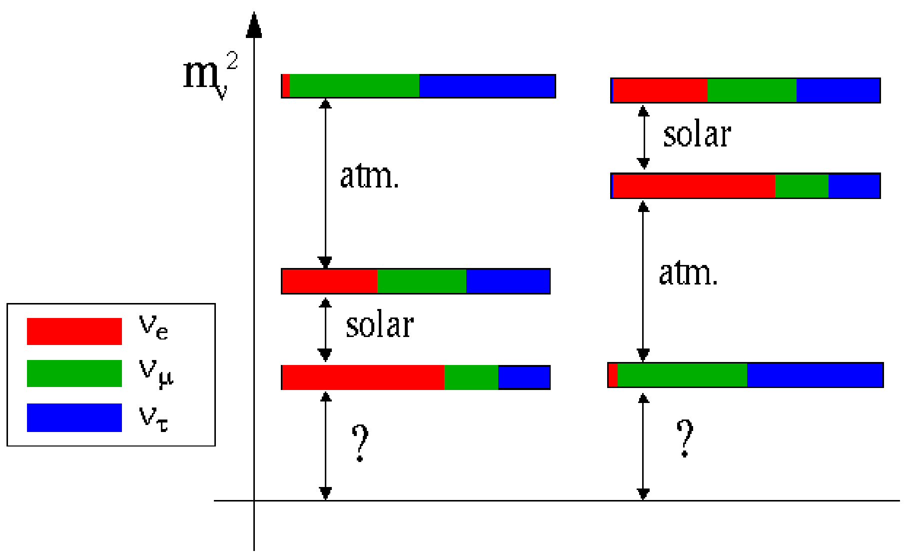
- Introduction [*anarchy vs. symmetry*]
- A short digression: $U(3)^3$ and $U(2)^3$ symmetries in the quark sector
- Some open problems
- Dynamical Yukawa's from a Minimum Principle
- Conclusions

Introduction [*anarchy vs. symmetry*]







Finding a rational explanation for the observed pattern of quark and lepton mass matrices (eigenvalues & mixing) is one of the key open problems in particle physics

Introduction [*anarchy vs. symmetry*]

Anarchy
+
Anthropic selection

("*Chance & Necessity*" [J. Monod])

- A new way of thinking in particle physics, motivated by the hierarchy problems in Λ_{cosmo} and m_h

The symmetric way

("*The book of nature is written in terms of circles, triangles and other geometrical figures...*" [G. Galilei])

- Main road of particle physics so far

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- A new way of thinking in particle physics, motivated by the hierarchy problems in Λ_{cosmo} and m_h
- Many unanswered questions:

  It works well for $m_{u,d}$
  maybe also for m_t & ν mixing,
  but what about CKM and the other masses? Why 3 generations?....

- No clear direction for future searches

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The symmetric way

("*The book of nature is written in terms of circles, triangles and other geometrical figures…*" [G. Galilei])

- Main road of particle physics so far
- It works well in the Yukawa sector (*several possible options*), less evident, but not excluded, in the neutrino case
- "large" flavor symmetry + "small" breaking is an interesting hypothesis that fits well with all available data [*including the lack of deviations from SM*] and could possibly tested in the near future.

A short digression:

$U(3)^3$ & $U(2)^3$ symmetries in the quark sector

▸ $U(3)^3$ & $U(2)^3$ *symmetries in the quark sector*

$$\boxed{U(3)^3 = U(3)_Q \times U(3)_U \times U(3)_D}$$

- Largest flavor symmetry group compatible with the SM gauge symmetry
- MFV = minimal breaking of $U(3)^3$ by $(3,\underline{3})$ terms [*SM Yukawa couplings*]

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virtue

- Naturally small effects in FCNC observables (assuming TeV-scale NP)

problems

- No explanation for *Y* hierarchies (masses and mixing angles)

In the explicit framework of low-energy SUSY

- No explanation for small CPV flavor-conserving observables (edms)
- Enhanced hierarchy problem due to the strong LHC bounds on "1st & 2nd gen. partners"

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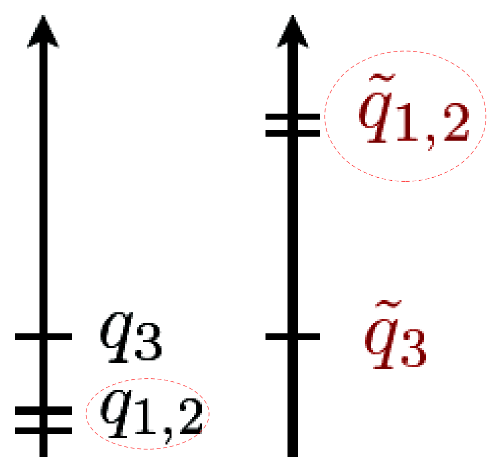
$$\boxed{U(2)^3 = U(2)_Q \times U(2)_U \times U(2)_D \text{ flavor symmetry}}$$

acting on 1st & 2nd generations

- The exact symmetry limit is good starting point for the SM quark spectrum ($m_u=m_d=m_s=m_c=0$, $V_{CKM}=1$) → we only need small breakings terms
- The small breaking ensures small effects in rare processes
- In the SUSY context, this symmetry allows a large mass gap among light and 3rd generations squarks (*natural SUSY*), and corresponding small edms (for heavy 1st & 2nd gen. squarks).



A closer look to $U(2)^3$ *& its (minimal) breaking pattern:*

The symmetry is a good approximation to the SM quark spectrum (exact symmetry for $m_u=m_d=m_s=m_c=0$, $V_{CKM}=1$), hence we only need to introduce small breaking terms

Unbroken

$$U(2)^3 = U(2)_Q \times U(2)_U \times U(2)_D$$

$$\downarrow$$

$$Y_u = y_t \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & 1 \end{array}\right] \qquad Y_d = y_b \left[\begin{array}{c|c} 0 & 0 \\ \hline 0 & 1 \end{array}\right] \qquad M_{\mathrm{squarks}} = \left[\begin{array}{c|c} m_{\mathrm{heavy}} & 0 \\ \hline 0 & m_3 \end{array}\right]$$

Barbieri, G.I., Jones-Perez, Lodone, Straub, '11

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The symmetry is a good approximation to the SM quark spectrum (exact symmetry for $m_u=m_d=m_s=m_c=0$, $V_{CKM}=1$), hence we only need to introduce small breaking terms

Minimal set of breaking terms necessary to reproduce the quark spectrum, while keeping small FCNCs beyond SM:

$$\boxed{V \sim (2,1,1) \qquad O(\lambda^2 \sim 0.04)}$$

Leading breaking term:
connection 3rd gen. $\rightarrow$ light gen.

$$U(2)^3 = U(2)_Q \times U(2)_U \times U(2)_D$$

$$Y_u = y_t \left[\begin{array}{c|c} 0 & c_u V \\ \hline 0 & 1 \end{array}\right] \qquad Y_d = y_b \left[\begin{array}{c|c} 0 & c_d V \\ \hline 0 & 1 \end{array}\right]$$

$\rightarrow$

$$(V_{ts}{}^2 + V_{td}{}^2)^{1/2} = (V_{cb}{}^2 + V_{ub}{}^2)^{1/2} = = O(\lambda^2)$$

Barbieri, G.I., Jones-Perez, Lodone, Straub, '11

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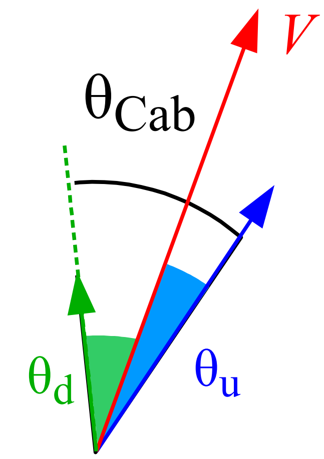

Minimal set of breaking terms necessary to reproduce the quark spectrum, while keeping small FCNCs beyond SM:

$$\boxed{V \sim (2,1,1) \qquad O(\lambda^2 \sim 0.04)}$$

$$\Delta Y_u \sim (2,2,1) \qquad m_c,\ m_u,\ \theta_u \quad O(y_c \sim 0.006)$$

$$\Delta Y_d \sim (2,1,2) \qquad m_s,\ m_d,\ \theta_d \quad O(y_s < 0.001)$$

$$U(2)^3 = U(2)_Q \times U(2)_U \times U(2)_D$$

$$Y_u = y_t \begin{bmatrix} \Delta Y_u & c_u V \\ 0 & 1 \end{bmatrix} \qquad Y_d = y_b \begin{bmatrix} \Delta Y_d & c_d V \\ 0 & 1 \end{bmatrix} \quad \rightarrow \quad \begin{aligned} |V_{us}| &\approx |\theta_u - \theta_d| \\ |V_{td}/V_{ts}| &= \theta_d \\ |V_{ub}/V_{cb}| &= \theta_u \end{aligned}$$

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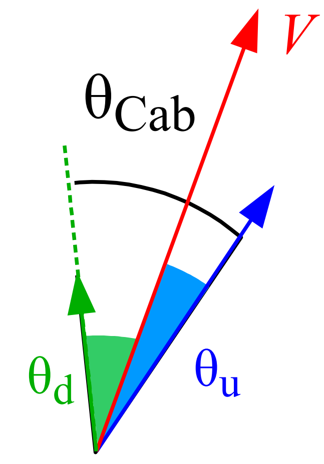

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$\Delta Y_d \sim (2,1,2)$ $\quad m_s, m_d, \theta_d \quad O(y_s < 0.001)$

$$U(2)^3 = U(2)_Q \times U(2)_U \times U(2)_D$$

The assumption of a single (2,1,1) breaking term [*= a single spurion connecting the light generations to the third one*] ensures a MFV-like protection of FCNCs

The protection is as effective as MFV at large tanβ or general (non-linear) MFV, where $U(3)^3 \to U(2)^3 x U(1)$

Feldmann, Mannel, '08
Kagan *et al*. '09

Some open problems

Open problems

I. A potential problem of the $U(2)^3$ approach and, more generally, of any approach attributing a special role to the hierarchies in the Yukawa sector, is the problem of neutrino masses (*under the hypothesis we are interested to describe in a common/unified way quark and lepton sectors*):

- Why neutrino mixing angles are not as small as in the quark sector? Why the mass hierarchies in the neutrino sector are not as large?

II. A problem common to both $U(3)^3$ and $U(2)^3$ is their non-compatibility with (standard) GUT groups (*if we believe GUTs play some role at high energies*)

III. Most important, both in $U(3)^3$ and in $U(2)^3$ the breaking terms are put in "by hands" (*non-dynamical spurion analysis*)

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To extend the idea of large flavor symmetry group with small breaking to the neutrino sector we need to assume a different initial symmetry for Dirac and Majorna sectors (*or a different initial breaking of some larger flavor symmetry*)

Blankenburg, G.I., Jones-Perez, '12

Small parameters in the Neutrino (Majorana) mass matrix:

$$\zeta = \left| \frac{\Delta m^2_{\rm sol}}{\Delta m^2_{\rm atm}} \right|^{1/2} = 0.174 \pm 0.007 \,,$$

$$s_{13} = |(U_{\rm PMNS})_{13}| = 0.15 \pm 0.02 \,,$$

$$M_\nu^+ M_\nu \xrightarrow[\zeta,\, s_{13} \to 0]{} m_\nu^2\, I + \Delta m_{\rm atm}^2\, \Sigma \xrightarrow[\Delta m_{\rm atm}^2 << m_\nu^2]{} m_\nu^2\, I$$

$$\Sigma \approx \frac{1}{2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$\boxed{\text{O(3) symmetry}}$

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Explicit potentials

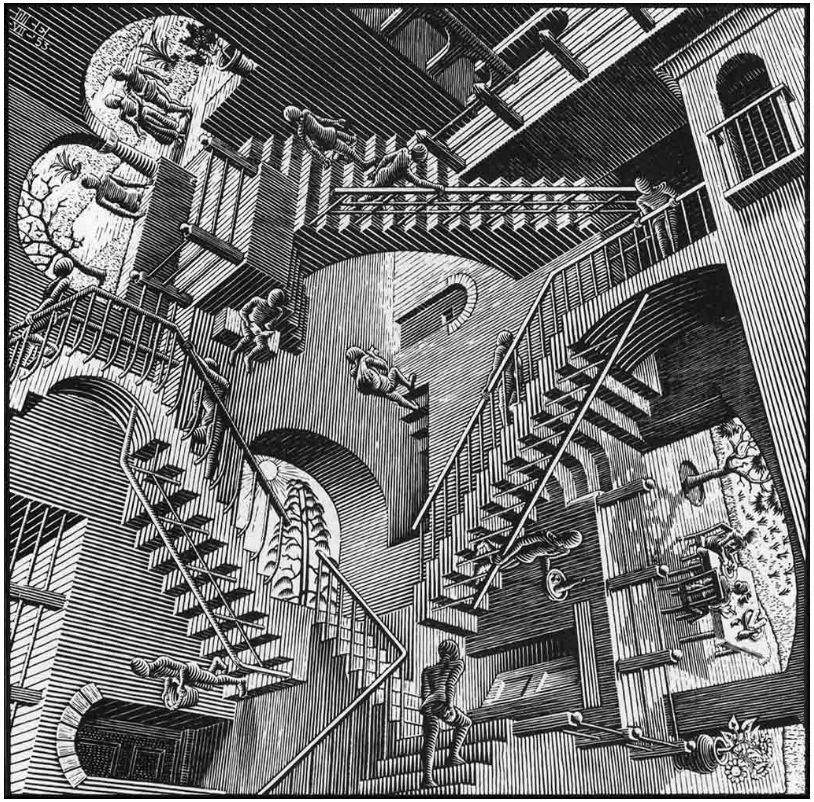
Feldmann *et al.* '09
Alonso, Gavela, *et al.* '11-'13
Nardi '11; Espinosa, Fong, Nardi '12

Gauging of $U(3)^3$ & $U(2)^3$

Albrecht, Feldmann, Mannel, '09
Grinstein, Redi, Villadoro, '09
D'Agnolo & Straub, '11

- $Y \sim \mathrm{diag}(0,0,1) + V_{CKM} = I$, stable solution of renormalizable potentials
- Maximal ν mixing possible with 2 heavy RH neutrinos [*with renorm. potential*]

Dynamical Yukawa's from a Minimum Principle



Dynamical Yukawa's from a Minimum Principle

Let's consider first a type-I model:

- SM field content enlarged by 3 heavy right-handed neutrinos (N)
- Largest flavor symmetry compatible with SM gauge group + non-vanishing N masses [*ignoring flavor-conserving* U(1) *phases*]: $SU(3)^5 \times O(3)_N$

$$-\mathcal{L}_Y = \bar{q}_L \underline{Y_D} H D_R + \bar{q}_L \underline{Y_U} \tilde{H} U_R + \bar{\ell}_L \underline{Y_E} H E_R + \bar{\ell}_L \underline{Y_\nu} \tilde{H} N + \text{h.c.} + \frac{M}{2} N^T N$$

Let's then assume that both quark and lepton Yukawa couplings are *dynamical fields* of $SU(3)^5 \times O(3)_R$ and that their values are determined by a *minimization principle* (e.g. the potential minimum)

↓

The "natural solutions" [*i.e. solution requiring no tuning in the parameters of the potential*] are the configurations preserving maximally unbroken subgroups.

Michel & Radicati, '69
Cabibbo & Maiani, '69

Dynamical Yukawa's from a Minimum Principle

The Michel-Radicati theorem (a sketch):

- $V = f[\, I_i(Y)\,]$ $I_i(Y)$=invariants of the group G built out of the Y's
- The space spanned by Y is infinite, but the manifold spanned by the I_i has boundaries, corresponding to the subgroups of G

E.g.: G=SU(3), I_1=Det(Y), I_2=Tr(Y^2) $\rightarrow$ $I_2 \geq (54\, I_1^{\,2})^{1/3}$

$I_2 = (54\, I_1^{\,2})^{1/3}$
only if Y invariant under SU(2)xU(1)

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- Extrema of V characterized by $\partial V/\partial Y_j = \partial V/\partial I_i \times J_{ij} = 0$ where $J_{ij} = \partial I_i/\partial Y_j$

- Extrema of V (partially) independent from its structure if J has low rank $\rightarrow$ "natural extrema" corresponding to <u>maximally unbroken subgroups</u>.

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"natural solutions" associated to maximally unbroken subgroups:

I) $SU(3)_L \times SU(3)_R \xrightarrow{Y \sim 3\times\underline{3}} SU(3)_{L+R}$ or $SU(2)_L \times SU(2)_R \times U(1)$

II) $SU(3)_L \times O(3)_N \xrightarrow{Y \sim 3\times 3} O(3)_{L+N}$

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degenerate light ν's

"chiral" solution:
Y ~ diag(0,0,1)

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Quarks: $SU(3)_Q \times SU(3)_U \times SU(3)_D \quad \rightarrow \quad SU(2)_Q \times SU(2)_U \times SU(2)_D \times U(1)_3$

"chiral" solution + $V_{CKM} = I$

Leptons: $SU(3)_E \times SU(3)_L \times O(3)_N \quad \rightarrow \quad SU(2)_E \times U(1)_{L+N}$

"chiral" charged leptons + degenerate light neutrinos
+ non-trivial PMNS [related to the orientation of O(3) in SU(3)]

Dynamical Yukawa's from a Minimum Principle

Alonso, Gavela, G.I., Maiani, '13

$SU(3)^5$

LR LL

$Y_{U,D,E} \propto \mathrm{diag}(0,0,1)$ $\left(\begin{array}{c}\text{unbroken} \\ SU(2)_L \times SU(2)_R\end{array}\right)$

$|M_\nu \propto Y_\nu Y_\nu^T| \propto \mathrm{diag}(1,1,1)$ $\left(\begin{array}{c}\text{unbroken} \\ O(3)_L\end{array}\right)$

Two important comments:

- The assumption of seesaw of type-I can be relaxed
  [$O(3)_L$ "natural solution" also if $SU(3)_L$ is broken by $M_\nu \sim 6$ of $SU(3)_L$]

- The structure of the "initial" group can be made compatible with GUTs
  [e.g.: $SU(3)_{10} \times SU(3)_5 \times SU(3)_1$ in $SU(5)_{gauge}$]

Dynamical Yukawa's from a Minimum Principle

Alonso, Gavela, G.I., Maiani, '13

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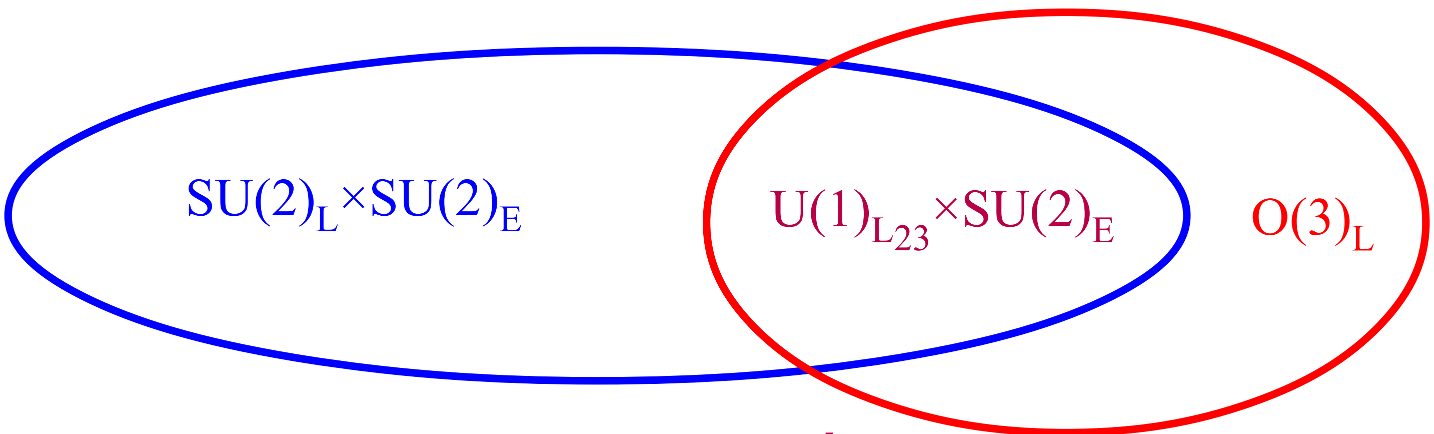
LR LL

$Y_E \propto \mathrm{diag}(0,0,1)$ $\left(\begin{array}{c}\text{unbroken}\\ SU(2)_L \times SU(2)_E\end{array}\right)$

$|M_\nu \propto Y_\nu Y_\nu^T| \propto \mathrm{diag}(1,1,1)$ $\left(\begin{array}{c}\text{unbroken}\\ O(3)_L\end{array}\right)$



A "natural orientation" of $O(3)_L$ vs. $U(2)_L$ preserving an unbroken U(1) symmetry implies a <u>π/4 mixing angle</u> in the <u>PMNS matrix.</u>

Dynamical Yukawa's from a Minimum Principle

Alonso, Gavela, G.I., Maiani, '13

$SU(3)^5$

LR LL

$$Y_E \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \left(\begin{array}{c} \text{unbroken} \\ SU(2)_L \times SU(2)_E \end{array} \right)$$

$$M_\nu \propto Y_\nu Y_\nu^T \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \left(\begin{array}{c} \text{unbroken} \\ O(3)_L \end{array} \right)$$

same basis

$$Y_\nu \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & -i\frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & i\frac{1}{\sqrt{2}} \end{pmatrix}$$

Residual $U(1)_{L_{23}}$ symmetry:

$$Y_\nu \rightarrow \exp(i\alpha\lambda'_3)\, Y_\nu\, \exp(-i\alpha\lambda_7)$$

$$\lambda'_3 = \mathrm{diag}(0,1,-1)$$

Dynamical Yukawa's from a Minimum Principle

$SU(3)^5$

LR

LL

$$Y_E \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} \text{unbroken} \\ SU(2)_L \times SU(2)_E \end{pmatrix}$$

$$M_\nu \propto Y_\nu Y_\nu^T \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \begin{pmatrix} \text{unbroken} \\ O(3)_L \end{pmatrix}$$

same basis

Sub-leading $U(2)_L$ breaking
resolving 1-2 degeneracy

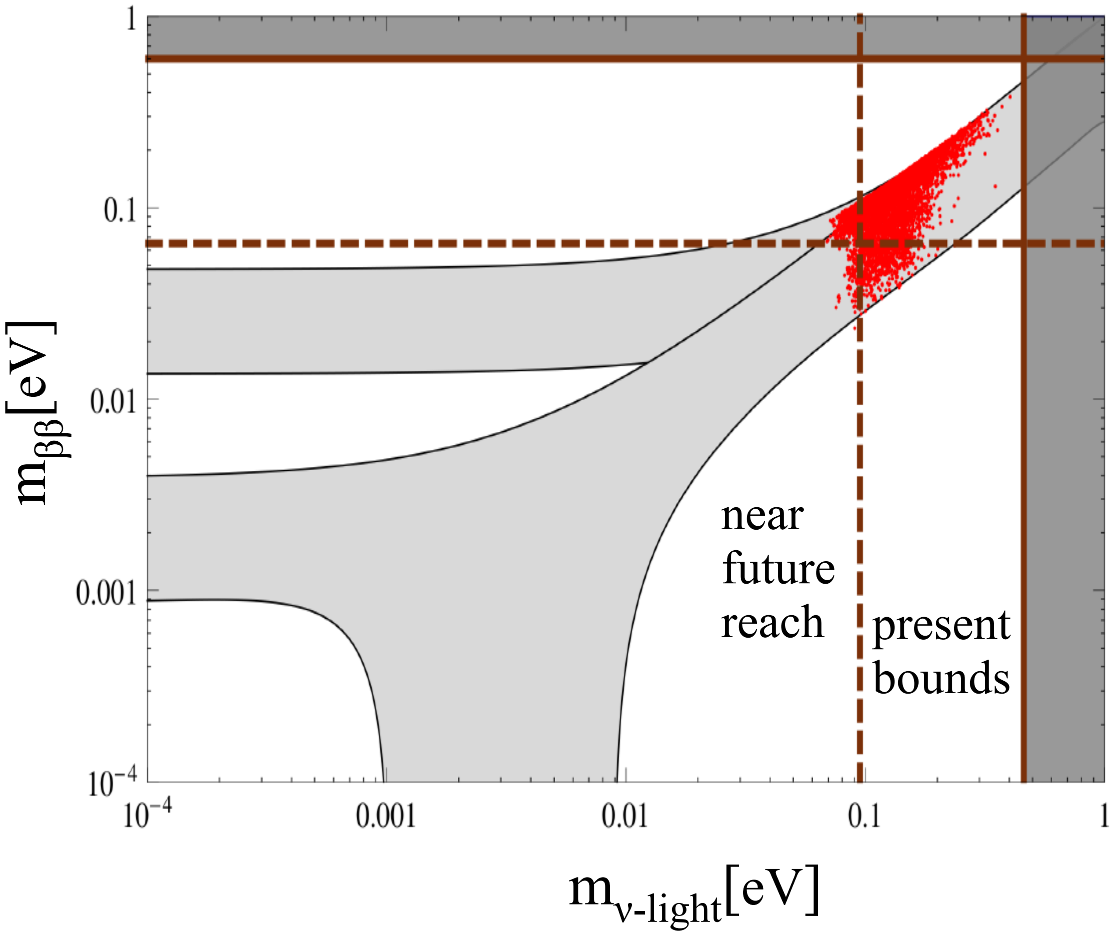
$$\frac{m_\mu}{m_\tau} = O(\varepsilon)$$

$$|s_{13}| \sim O(\varepsilon), \quad |s_{12}| \sim O(1)$$

$$\frac{m_\mu}{m_\tau} \sim 0.06 \ < \ \frac{\Delta m_{atm}{}^2}{m_\nu{}^2} = O(\varepsilon) \ < \ |s_{13}| \sim 0.2$$

If all this is correct...

$SU(3)^5$ $\xrightarrow{LL}$



$$M_\nu \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad \left(\begin{array}{c}\text{unbroken}\\ O(3)_L\end{array}\right)$$

$+$

$$\frac{\Delta m_{atm}^{\ 2}}{m_\nu^{\ 2}} = O(\varepsilon)$$

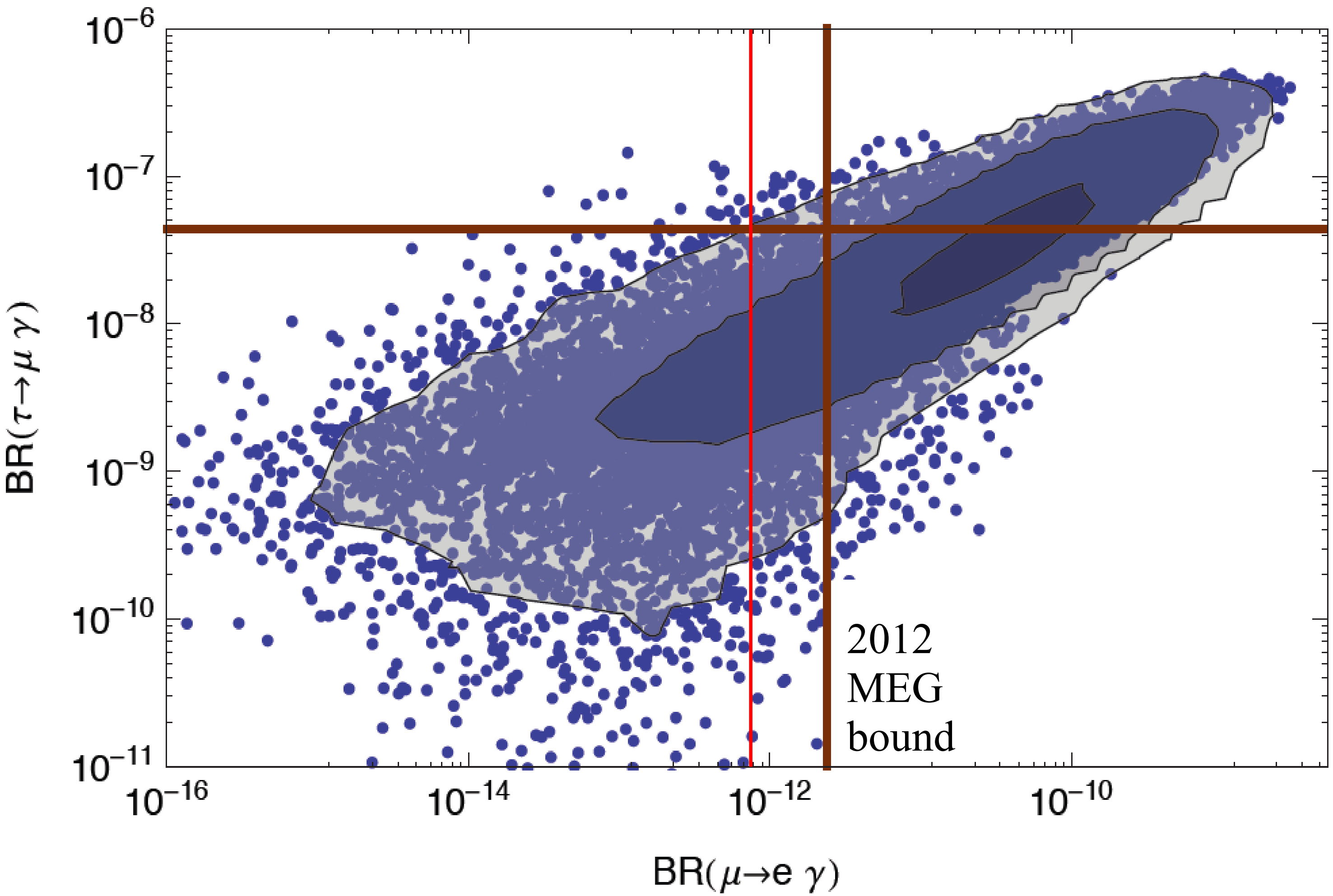
→ 0ν2β decay experiments (and maybe KATRIN) should be very close to observe a positive signal

If all this is correct...

→ $0\nu 2\beta$ decay experiments should be very close to observe a positive signal

… and if we add (low-energy) SUSY

→ LFV in charged leptons ($\mu \to e\gamma$) may also be close to present exp. bounds:



N.B.: LFV rates affected by a larger uncertainty

[BR ~ $1/\widetilde{m}^4$]

Conclusions

- The apparently different structure of quark and lepton mixing matrices could be well understood in terms of "natural solutions" of a large non-Abelian flavor symmetry broken by dynamical Yukawa fields → residual $SU(2)_L \times SU(2)_R$ chiral symmetry for Dirac (Yukawa) mass terms + O(3) symmetry in the neutrino sector.

- Predictions of the un-perturbed solution:
  → Vanishing masses for first two generations of quarks & leptons + trivial CKM
  → Degenerate neutrinos + $\theta_{23}=\pi/4$, $\theta_{12}=O(1)$, $\theta_{13}=0$.

- This is an excellent first-order approximation to the observed patter of quark and lepton mass matrices
  → this hypothesis can soon be tested by $0\nu2\beta$ decay experiments
  → worth to investigate a dynamical theory for the "perturbations" (that so far is still missing...)