Flavour Symmetries: a status report

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Electroweak symmetry breaking
flavour and dark matter after the Higgs discovery

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Flavour problem

flavour sector of the SM
[minimally extended to include
massive Majorana neutrinos]

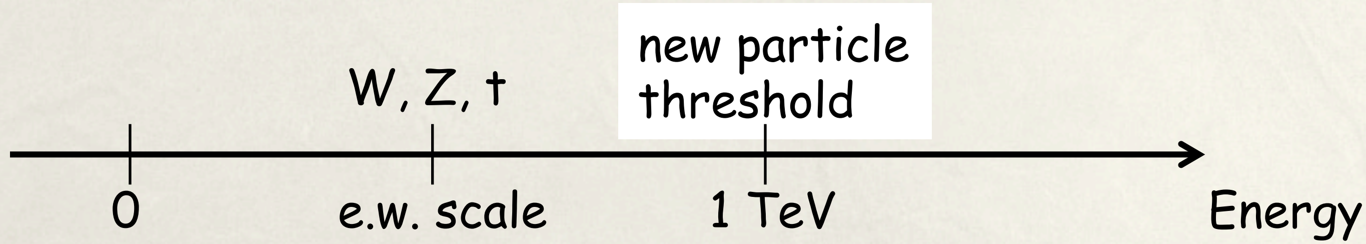
⟷

22 parameters: $\mathcal{Y}$
18 measured
(+ 1 mass + 3 phases)

two aspects

1 origin of $\mathcal{Y}$

2



how to avoid large FCNC and CPV induced by the new particles?

less urgent? no evidence for a new threshold so far

this talk: main focus on 1, but also comments on 2

not a systematic review of models, rather a reappraisal of few well-known ideas
as an introduction to more specialized presentations
[-> Hagedorn, Isidori, Paradisi...]

approaches

fundamental theory

↓

[symmetry and/or dynamical principle]

↓

$\mathcal{Y}$

1 $\mathcal{Y}$ should be deduced from first principles

most striking fact: nothing approaching a standard theory of $\mathcal{Y}$, despite decades of experimental progress and theoretical efforts

2 $\mathcal{Y}$ are due to chance

many variants

bottom-up: anarchy, FN models, fermions in extra dimensions

top-down: fundamental theory with a landscape of ground states

observed $\mathcal{Y}$ are environmental and cannot be fully predicted ⟷ relative sizes of solar planetary orbits

assumptions

- knowledge of statistical distribution of $\mathcal{Y}$ in the fundamental theory
- the observed $\mathcal{Y}$ are typical

[any anthropic selection?]

relevant questions

- how typical are the $\mathcal{Y}$ we observe?
- which is the statistical distribution of $\mathcal{Y}$ in the fundamental theory?

Flavor symmetries

largest possible flavour symmetry is obtained in the limit $\mathcal{Y} = 0$

$$G_{MFV} = U(3)^5$$

[SM particle content]

observed fermion masses and mixing angles break G_{MFV} completely (up to the hypercharge and, possibly, B-L)

$$G_{MFV} \supseteq G_f \rightarrow H_f$$

for any realistic flavour symmetry

in most predictive models the breaking is spontaneous, by a set of ‹scalar fields›

$$\varphi \rightarrow \varphi_g$$

under G_f

‹φ› determined by minimizing an energy functional V(φ) invariant under G_f

$$V(\varphi_g) = V(\varphi)$$

‹φ›, absolute minimum of V(φ), breaks G_f down to H_f

Yukawas promoted to dynamical variables

$$y(\varphi/\Lambda_f)$$

observed Yukawa couplings

$$y(<\varphi>/\Lambda_f)$$

huge number of possibilities: choice of G_f (global, local, continuous, discrete,….)
choice of representations for scalars φ and fermions

any empirical evidence for G_f from the quark sector?

perhaps $G_f = G_{MFV}$ or some variant of it [see Isidori talk]

$G_f = U(1)_{FN}$

[Froggatt, Nielsen 1979]

mass ratios and mixing angles are small, hierarchical parameters

$$\frac{m_u}{m_t} << \frac{m_c}{m_t} << 1 \qquad \frac{m_d}{m_b} << \frac{m_s}{m_b} << 1 \qquad |V_{ub}| << |V_{cb}| << |V_{us}| \equiv \lambda < 1$$

easily reproduced by $G_f = U(1)_{FN}$

mass ratios and mixing angles are powers of a small SB parameter λ

$U(1)_{FN}$ broken by

$$\lambda = \frac{<\varphi>}{\Lambda_f} \approx 0.2$$

flavon	Q_{FN}
φ	-1

$$y_u = F_{U^c} Y_u F_Q$$
$$y_d = F_{D^c} Y_d F_Q$$

call this map "hierarchy"

$$F_X = \begin{pmatrix} \lambda^{FN(X_1)} & 0 & 0 \\ 0 & \lambda^{FN(X_2)} & 0 \\ 0 & 0 & \lambda^{FN(X_3)} \end{pmatrix}$$

$Y_{u,d} \approx O(1)$

undetermined by $U(1)_{FN}$

$FN(X_i)$ are $U(1)_{FN}$ charges

$(X = Q, U^c, D^c)$

not a mere book-keeping

take $FN(Q_1) > FN(Q_2) > FN(Q_3) \geq 0$

$$\left(V_{u,d}\right)_{ij} \approx \frac{F_{Q_i}}{F_{Q_j}} < 1 \quad (i < j)$$

$$V_{CKM} = V_u^+ V_d$$

$$V_{ud} \approx V_{cs} \approx V_{tb} \approx O(1)$$

$$V_{ub} \approx V_{td} \approx V_{us} \times V_{cb}$$

[O.K. within a factor of 2]

independently from the specific charge choice

correct orders of magnitude of V_{ij} reproduced by e.g.

$FN(Q) = (3,2,0)$

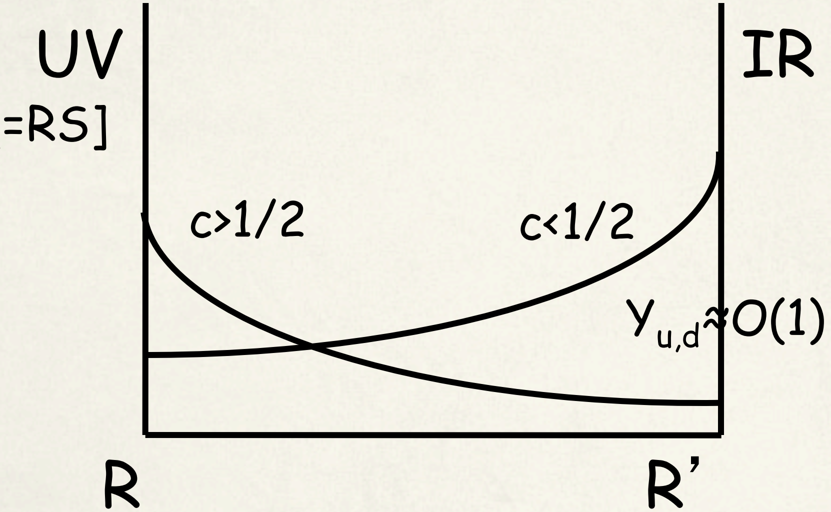
correct orders of magnitude of quark/charged lepton mass ratios [up to a couple of moderate tunings] reproduced by e.g.

$FN(U^c) = FN(E^c) = FN(Q) = (3,2,0)$
$FN(D^c) = FN(L) = (2,0,0)$

charge assignment compatible with SU(5) gauge unification

is a symmetry really needed ?

split fermions in a warped Extra Dimension [=RS]
[a flat dimension works equally well]



$$ds^2 = \left(\frac{R}{z}\right)^2 \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

$$M_{Pl}^{-1} \approx R \le z \le R' \approx (TeV)^{-1}$$

assign a bulk mass $c_i = M_i R$ to each fermion X_i and introduce random, O(1) Yukawa couplings $Y_{u,d}$ between bulk fermions and a Higgs localized at the IR brane

Yukawa couplings $y_{u,d}$ of zero-mode fermions as in FN [1 flavon and FN(X_i) ≥ 0]

$$y_u = F_{U^c} Y_u F_Q \qquad y_d = F_{D^c} Y_d F_Q$$

$$F_{X_i} = \sqrt{\frac{1-2c_i}{1-(R/R')^{1-2c_i}}} \quad \begin{cases} \approx O(1) & c_i < 1/2 \\ \approx (R/R')^{c_i - 1/2} << 1 & c_i > 1/2 \end{cases}$$

	Q	U^c	D^c
1	0.643	0.671	0.643
2	0.583	0.528	0.601
3	0.317	-0.460	0.601

[fit to c_i - Huber 0303183]

no symmetry:
hierarchy produced by geometry

Same pattern arises when matter chiral multiplets X_i of the MSSM are coupled to a superconformal sector in some finite energy range [Nelson-Strassler 0006251]

dangerous FCNC

the "hierarchy" map can support a Maximal Flavour Symmetry similar to G_{MFV}

flavour group felt by quarks can be as large as G_{MFV}, but there are more spurions
[Davidson, Isidori, Uhlig 0711.3376]

$$F_Q, F_U{}^c, F_D{}^c, Y_u, Y_d$$

true flavour symmetry can be weaker, dep. on the way "hierarchy" is realized,
as e.g. in FN models [Dudas, von Gersdorff, Parmentier, Pokorski 1007.5208]
maximal symmetry applies to RS models [RS-GIM Agashe, Perez, Soni 0408134]

one concrete example

$$O_K^4 = (\bar{s}_L d_R)(\bar{s}_R d_L)$$

contributions to ε_K are both chiral and RG enhanced

arises from $\frac{1}{\Lambda_{NP}^2}(\bar{Q}F_Q^+\gamma_\mu F_Q Q)\,(\bar{D}^c F_{D^c}^+\gamma^\mu F_{D^c} D^c)$ ⇒ $C_K^4 \approx \frac{1}{\Lambda_{NP}^2}\frac{1}{\langle Y_d\rangle^2}\frac{2m_d m_s}{v^2}$

$$\mathrm{Im}(C_K^4) < (160\times 10^3\ TeV)^{-2}$$
$$\mathrm{Im}(C_K^4) \approx \mathrm{Re}(C_K^4)$$

⇒ $\langle Y_d\rangle \Lambda_{NP} > 20\ TeV$

confirmed by explicit computation in RS
$O_K{}^4$ from tree-level KK gluon exchange
[also neutron EDM -> M_{KK}>O(10) TeV]

$$M_{KK} > (22\pm 6)\ TeV$$

[Csaki, Falkowski, Weiler 0804.1954
Von Gersdorff 1311.2078]

FCNC and/or CPV not sufficiently suppressed
if there is New Physics at the TeV scale

some lessons from the quark sector

- Pattern of quark masses and mixing angles well-explained by a hierarchy map: underlying $Y_{u,d}$ are O(1)
  hierarchy realized in several different frameworks: FN, RS, NS,....
  symmetry is not a necessary ingredient

- correct order-of-magnitude predictions

- compatible with SU(5) GUTs

- compatible with/incorporated in known solutions to the hierarchy problem

- additional ingredients needed to control the new sources of FC/CPV arising from New Physics at the TeV scale

alignment
universality
...

$$F_Q \quad F_{D^c} \quad F_{U^c}$$

$$Y_d \rightarrow Y_u$$

some symmetry ?

- large number of independent O(1) parameters:
  test of statistical distributions

additional constraints?

- present precision in quark mass/mixing parameters

testable predictions beyond order-of-magnitude accuracy ?

the lepton sector

anything special, requiring a symmetry?

3 examples from a longer list…

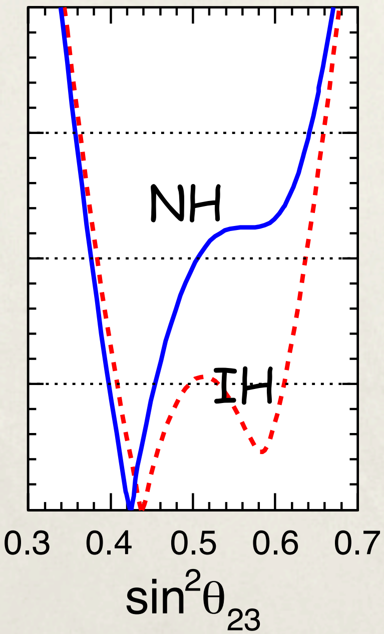
$\vartheta_{13} \approx 0.15$ rad $\approx 9^0$

$\vartheta_{13} = 0$?

$$\sin^2\vartheta_{13} = \begin{cases} 0.0234^{+0.0022}_{-0.0018} \; [NH] \\ 0.0239^{+0.0021}_{-0.0021} \; [IH] \end{cases}$$

ruled out 10σ away from 0

$\sin^2\vartheta_{23} = 1/2$?



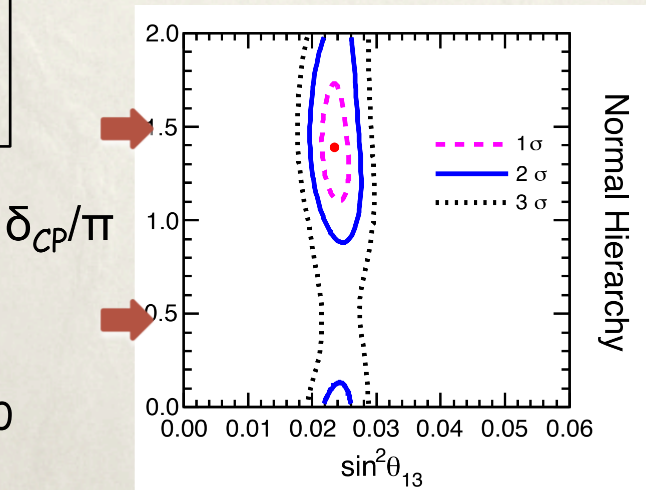
hint for non maximal ϑ_{23}

[global fit: Capozzi, Fogli, Lisi, Marrone, Montanino, Palazzo 1312.2878 see also: G.Garcia, Maltoni, Salvado, Schwetz 1209.3023]

δ_{CP} = ?

first hint for non-trivial δ_{CP} maximal?

[see alsoT2K: 1311.4750 and 1311.4114]



- no evidence for big hierarchies in neutrino mixing angles
  clear hierarchy only in the charged lepton masses

⇒

$$F_{E_1^c} << F_{E_2^c} << F_{E_3^c}$$

$$F_{L_1} \approx F_{L_2} \approx F_{L_3}$$

[independently on whether neutrinos are Majorana or Dirac]

several possibilities [here focus on Majorana neutrinos]:

- **Anarchy** [Hall, Murayama, Weiner 1999 De Gouvea, Murayama 1204.1249]

$$\vartheta_{23} \approx \vartheta_{12} \approx O(1) \quad \text{O.K.} \qquad \vartheta_{13}\ ?$$

$$\Delta m_{12}^2 \approx \Delta m_{13}^2 \quad ?$$

$$F_{L_1} = F_{L_2} = F_{L_3}$$

$$m_\nu \propto \begin{pmatrix} O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \\ O(1) & O(1) & O(1) \end{pmatrix}$$

mixing angles and mass ratios from random O(1) quantities

$$|U_{PMNS}| \approx \begin{pmatrix} 0.8 & 0.5 & 0.2 \\ 0.4 & 0.6 & 0.6 \\ 0.4 & 0.6 & 0.8 \end{pmatrix}$$

consistent with data

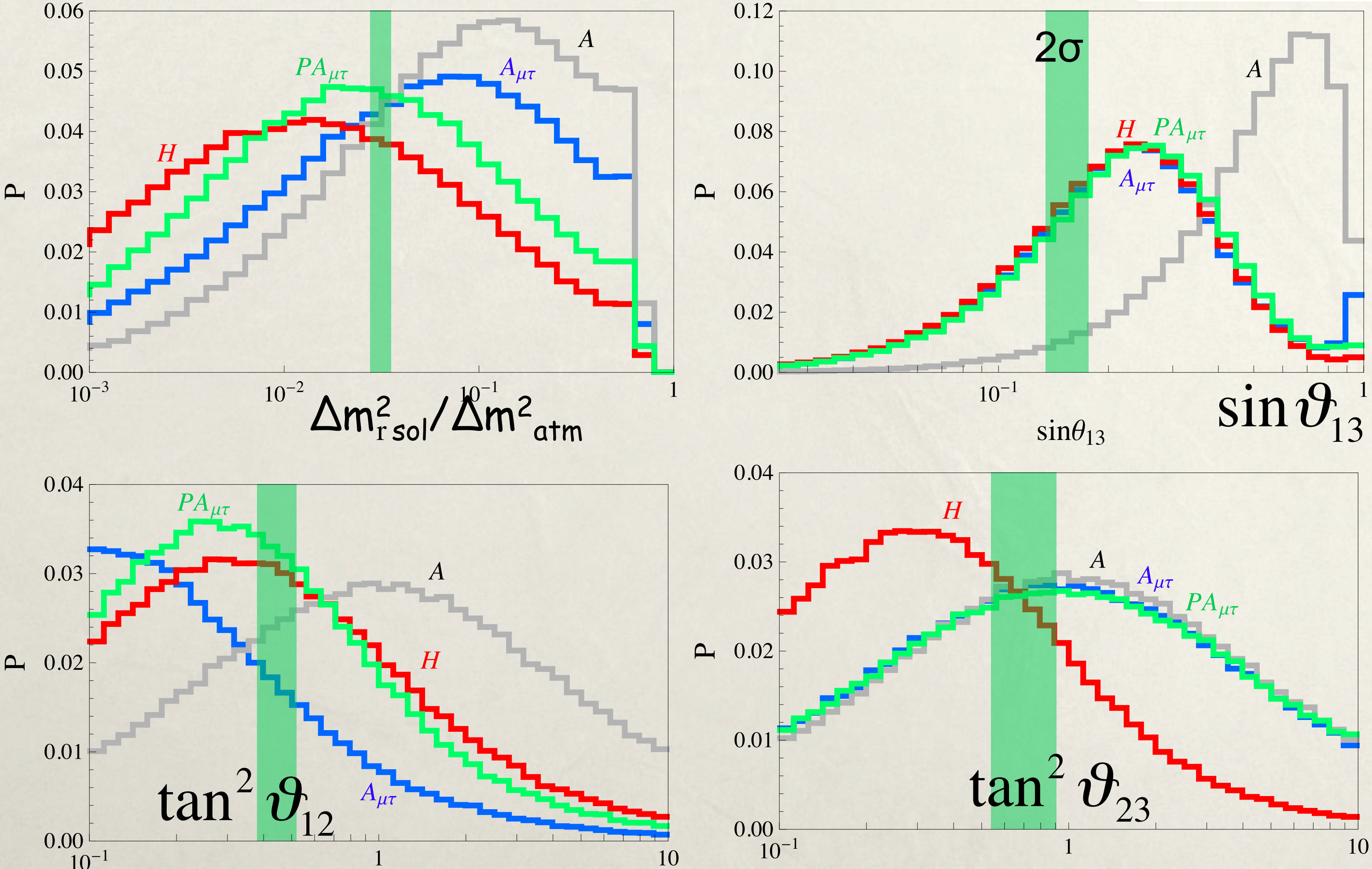
$\vartheta_{13} \approx 0.15$ rad and the hint for non maximal ϑ_{23} have strengthened the case for anarchy

- viable $U(1)_{FN}$ models, quarks and leptons treated on equal foot
- compatible with SU(5) unification
- difficult to go beyond order-of-magnitude predictions

	$FN(L)$	λ
A	(0,0,0)	
$A_{\mu\tau}$	(1,0,0)	0.25
$PA_{\mu\tau}$	(2,0,0)	0.35
H	(2,1,0)	0.45

$$F(L_i) = \lambda^{FN(L_i)}$$

[Altarelli,F,Masina, Merlo 1207.0587]



constraints from lepton flavour violation

take the limit $m_\nu = 0$

if MFV applied, we would expect no LFV [y_e diagonal]

⟷

in our setup, in general F_{E^c}, F_L, Y_e do not commute [not even when F_L is universal] LFV expected at some level

dominant LFV dipole operator

$$L_{dip} = \frac{e}{\Lambda_{NP}^2} E^c(\sigma_{\mu\nu}F^{\mu\nu})\underbrace{(F_{E^c}Y_eY_e^+Y_eF_L)}_{\text{not diagonal}}(H^+L)$$

when $y_e = F_{E^c}Y_eF_L$ diagonal

Explicit computation in RS

[Agashe, Blechman, Petriello 0606021
Csaki, Grossman, Tanedo, Tsai 1004.2037]

$$BR(\mu \to e\gamma) < 5.7\times 10^{-13}$$

$$M_{KK} > O(10)\ TeV$$

➡ F_L universality is not enough

a sufficient condition for the absence of LFV: F_{E^c}, Y_e, F_L diagonal in the same basis

for instance:

$$F_L \propto 1 \qquad F_{E^c} \propto Y_eY_e^+$$

[M.C. Chen and Yu, 08042503
Perez, Randall 0805.4652]

are there models of lepton masses that already include such conditions ?

G_f = discrete flavor symmetry

$$U_{PMNS} = U^0_{PMNS} + \text{corrections}$$

some simple pattern, exactly reproduced by a flavor symmetry

- well motivated before 2012

$$U^0_{PMNS} = U_{TB} \equiv \begin{pmatrix} 2/\sqrt{6} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

T r i b i m a x i m a l
M i x i n g

discrete flavor symmetries showed very efficient to reproduce U^0_{PMNS}

- still justified today?

$$U_{TB} \approx \begin{pmatrix} 0.82 & 0.58 & 0 \\ -0.41 & 0.58 & -0.71 \\ -0.41 & 0.58 & 0.71 \end{pmatrix}$$

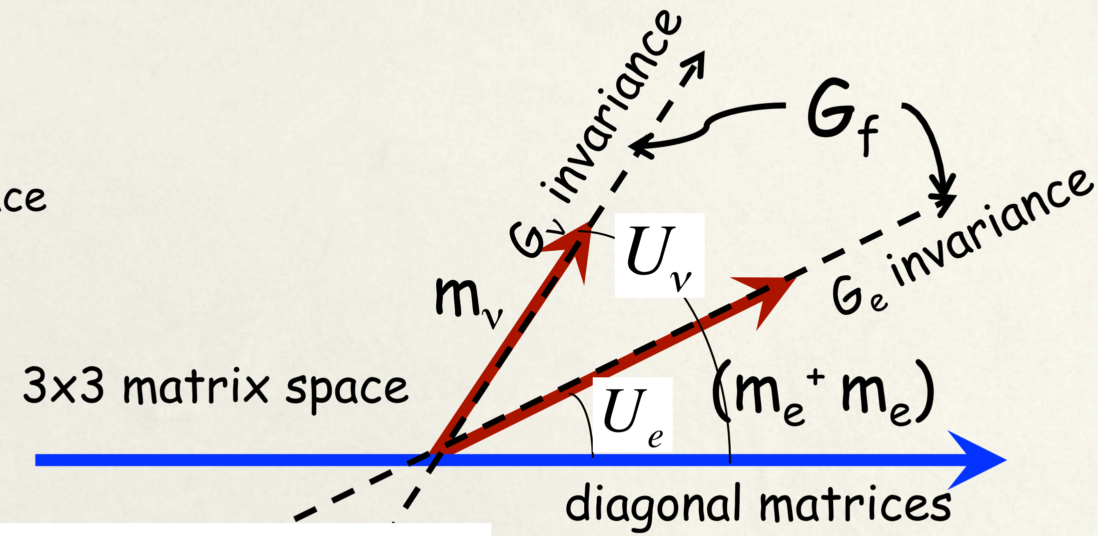
$$|U_{PMNS}| = \begin{pmatrix} 0.80 \div 0.85 & 0.51 \div 0.59 & 0.13 \div 0.18 \\ 0.21 \div 0.54 & 0.42 \div 0.73 & 0.58 \div 0.81 \\ 0.22 \div 0.55 & 0.41 \div 0.73 & 0.57 \div 0.80 \end{pmatrix}$$

[3σ ranges from Gonzalez-Garcia, Maltoni, Salvado, Schwetz 1209.3023]

Mixing patterns U^0_{PMNS} from discrete symmetries

$$U_{PMNS} = U_e^+ U_\nu$$

misalignment in flavour space
from symmetry breaking



➡ 4 predictions $\vartheta^0_{12} \quad \vartheta^0_{23} \quad \vartheta^0_{13} \quad \delta^0 (\mathrm{mod}\ \pi)$

-- Majorana neutrinos imply $G_\nu \leq Z_2 \times Z_2$ discrete

-- smallest group leading to TBM: $S_4 \approx$ (A_4+accidental symmetry) in model building

-- general feature $U_{PMNS} = U^0_{PMNS} + O(u) \quad u \equiv \langle \varphi \rangle / \Lambda < 1$

-- neutrino masses fitted, not predicted.

expectation for $U^0_{PMNS}=U_{TB}$

$$\begin{cases} \vartheta^0_{13} = 0 \\ \vartheta^0_{23} = \dfrac{\pi}{4} \end{cases} \Rightarrow \begin{cases} \vartheta_{13} = \mathrm{O(few\ degrees)} \\ \vartheta_{23} = \mathrm{close\ to}\ \dfrac{\pi}{4} \end{cases}$$

not to spoil the agreement with ϑ_{12}

wrong!

1 add large corrections $u \approx O(\vartheta_{13}) \approx 0.2$

- predictability is lost since in general correction terms are many
- new dangerous sources of LFV if NP is at the TeV scale

in a class of SUSY realizations [F, Hagedorn, Lin, Merlo, 2008-2009] [See talk by Paradisi]

$$\frac{BR(l_i \to l_j \gamma)}{BR(l_i \to l_j \nu_i \bar{\nu}_j)} = \frac{6 m_W^4 \alpha_{em}}{\pi m_{SUSY}^4} \left[\left| w_{ij}^{(1)} u^2 \right|^2 + \frac{m_j^2}{m_i^2} \left| w_{ij}^{(2)} u \right|^2 \right]$$

$w^{(1,2)}_{ij}$ are known O(1) functions of SUSY parameters

$BR(\mu \to e\gamma) < 5.7 \times 10^{-13}$	
$m_{SUSY} > 1.6$ TeV	u=0.05
$m_{SUSY} > 3.2$ TeV	u=0.10
$m_{SUSY} > 4.7$ TeV	u=0.15

2 change discrete group G_f

- solutions exist
  special forms of Trimaximal mixing

G_f	$\Delta(96)$	$\Delta(384)$	$\Delta(600)$
α	$\pm\pi/12$	$\pm\pi/24$	$\pm\pi/15$
$\sin^2\vartheta_{13}^0$	0.045	0.011	0.029

α "quantized" by group theory

$\delta^0 = 0, \pi$ (no CP violation)

$$U^0 = U_{TB} \times \begin{pmatrix} \cos\alpha & 0 & e^{i\delta}\sin\alpha \\ 0 & 1 & 0 \\ -e^{-i\delta}\sin\alpha & 0 & \cos\alpha \end{pmatrix}$$

F.F., C. Hagedorn, R. de A.Toroop
hep-ph/1107.3486 and hep-ph/1112.1340
Lam 1208.5527 and 1301.1736
Holthausen, Lim and Lindner 1212.2411
Neder, King, Stuart 1305.3200
Holthausen, Lim 1306.4356
Hagedorn, Meroni, Vitale 1307.5308]

[See talk by Hagedorn]

3 relax symmetry requirements

[Hernandez,Smirnov 1204.0445]

G_e as before

$G_\nu = Z_2$

2 predictions:
2 combinations of $\vartheta_{12}^0 \quad \vartheta_{23}^0 \quad \vartheta_{13}^0$

leads to testable sum rules

$$\sin^2\vartheta_{23} = \frac{1}{2} + \frac{1}{\sqrt{2}}\sin\vartheta_{13}\cos\delta_{CP} + O(\sin^2\vartheta_{13})$$

[He, Zee 2007 and 2011, Grimus, Lavoura 2008, Grimus, Lavoura, Singraber 2009, Albright, Rodejohann 2009, Antusch, King, Luhn, Spinrath 2011, King, Luhn 2011, G. Altarelli, F.F., L. Merlo and E. Stamou hep-ph/1205.4670]

4 include CP in the SB pattern

$$G_{CP} = G_f \rtimes CP$$

[F. F, C. Hagedorn and R. Ziegler 1211.5560, 1303.7178 Ding,King,Luhn,Stuart 1303.6180]

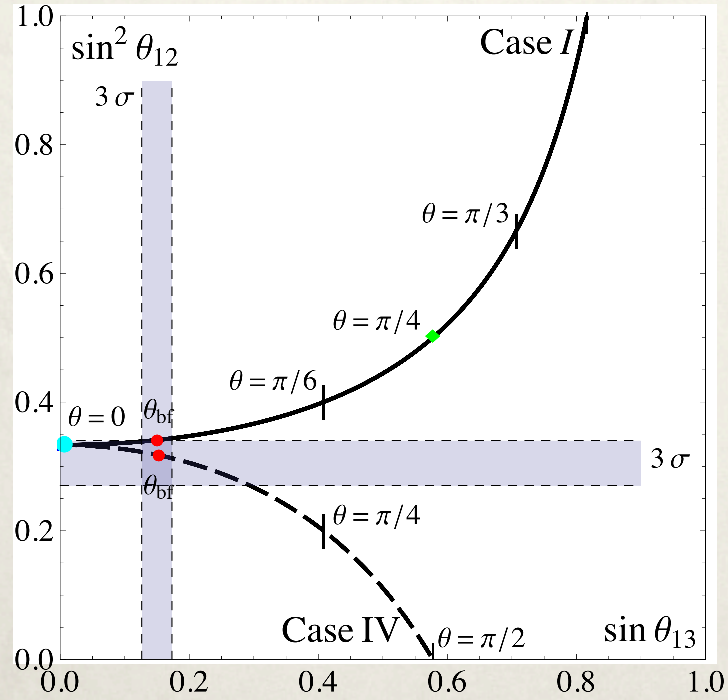
$$G_e \qquad G_\nu = Z_2 \times CP$$

$$\sin^2\vartheta_{23}^0 = \frac{1}{2} \qquad |\sin\delta^0| = 1 \qquad \sin\alpha^0 = 0 \qquad \sin\beta^0 = 0$$

mixing angles and CP violating phases

$$(\vartheta_{12}^0, \vartheta_{23}^0, \vartheta_{13}^0, \delta^0, \alpha^0, \beta^0)$$

predicted in terms of a single real parameter $0 \le \vartheta \le 2\pi$



2 examples with $G_f = S_4$ $G_e = Z_3$

Conclusions

- flavour symmetries are a useful tool in our quest of the origin of $\mathcal{Y}$
  but no compelling and unique picture have emerged so far.
  Present data can be described within widely different frameworks
  [despite the constant, impressive progress on the experimental side]

- simple schemes (e.g. “hierarchy”) with a minimal amount of structure can
  well reproduce the main features of $\mathcal{Y}$ in both quark and lepton sectors
  main drawback:
  we typically learn that $\mathcal{Y}$ are consistent with some statistical distribution
  [no precise questions/no precision tests allowed]

- if there is new physics at the TeV scale
  chance is no more enough to explain the highly suppressed FCNC and CPV
  and more structure is needed in both quark and lepton sectors.

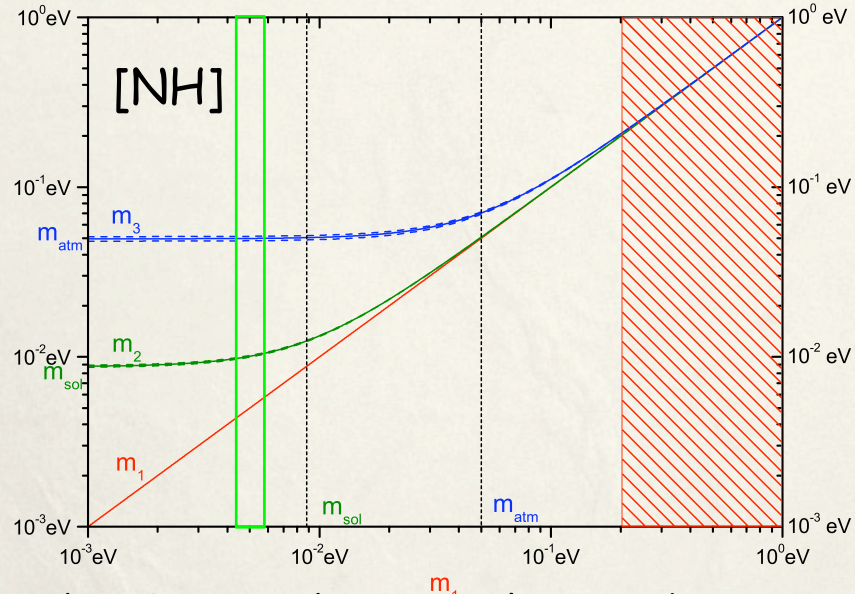
back up slides

predictions based on $G_f = A_4 \times Z_3 \times U(1)_{FN}$ [+ SEE-SAW] [Altarelli, F 2005]

lepton mixing is TB, by construction, plus NLO corrections of order $0.005 < u < 0.05$
at the LO neutrino mass spectrum depends on two complex parameters
there is a sum rule among (complex) mass eigenvalues $m_{1,2,3}$

$$\frac{1}{m_3} = \frac{1}{m_1} - \frac{2}{m_2}$$

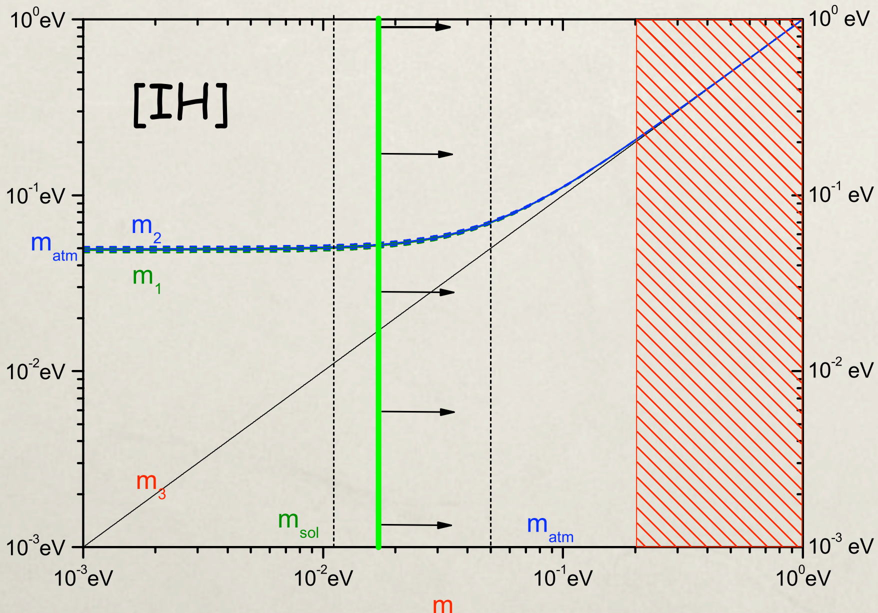
both normal [NH] and inverted [IH] hierarchy are allowed



in the NH case the sum rule
completely determines the spectrum

$$m_1 \approx 0.005\ eV \quad m_2 \approx 0.01\ eV \quad m_3 \approx 0.05\ eV$$

$$|m_{ee}| \approx 0.007\ eV$$

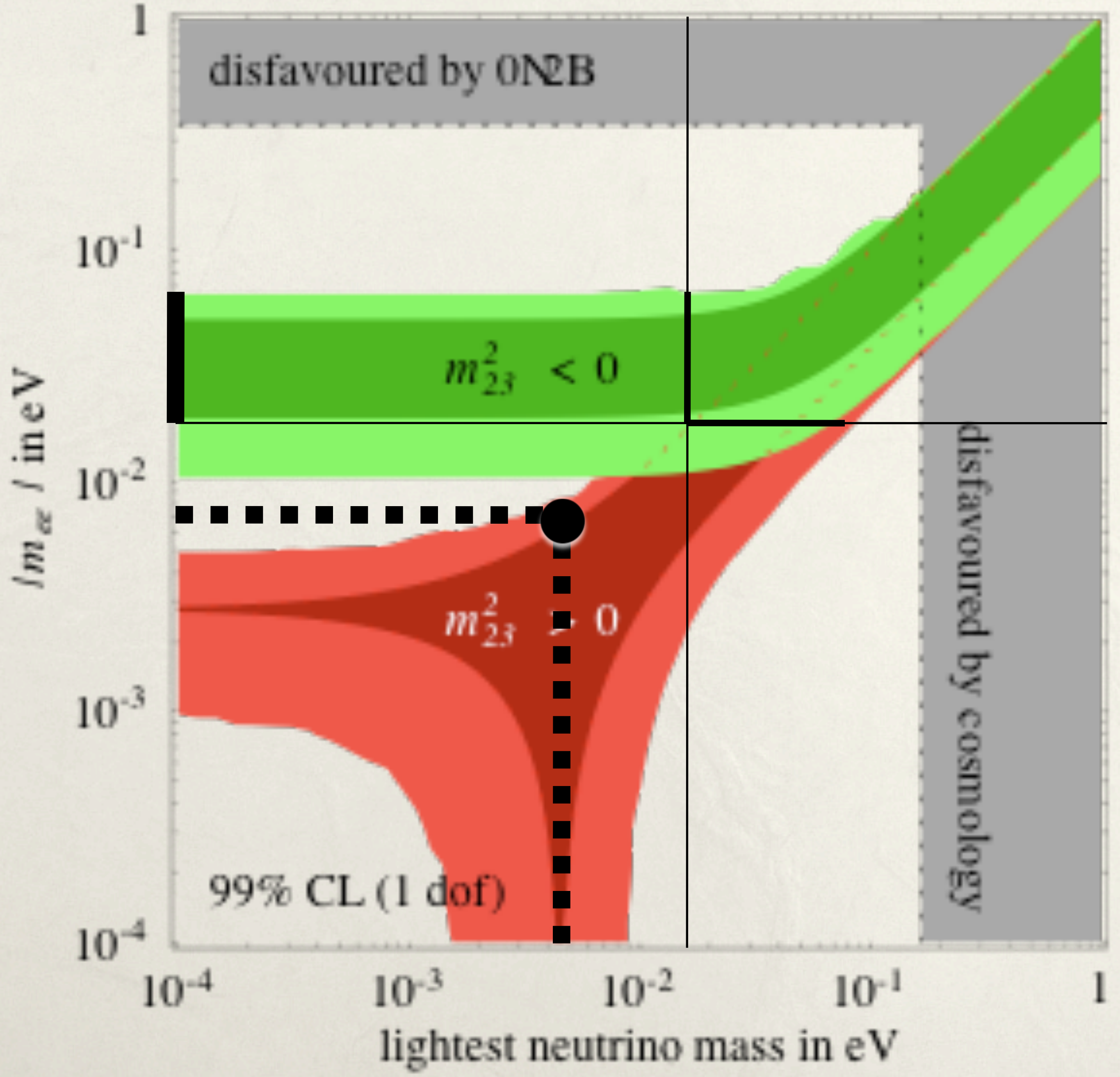


in the IH case the sum rule provides
a lower bound on m_3

$$m_3 \geq 0.017\ eV$$

$$|m_{ee}| \geq 0.017\ eV$$

NLO corrections are negligible for NH and for IH close to the lower bound



disfavoured by 0N2B
disfavoured by cosmology
$m_{23}^2 < 0$
$m_{23}^2 > 0$
99% CL (1 dof)
$|m_{ee}|$ in eV
lightest neutrino mass in eV
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}

Additional tests: LFV from 1-loop SUSY particle exchange

in a class of SUSY realizations [F, Hagedorn, Lin, Merlo, 2008-2009]

$$\frac{BR(l_i \to l_j \gamma)}{BR(l_i \to l_j \nu_i \bar{\nu}_j)} = \frac{6 m_W^4 \alpha_{em}}{\pi m_{SUSY}^4} \left[\left| w_{ij}^{(1)} u^2 \right|^2 + \frac{m_j^2}{m_i^2} \left| w_{ij}^{(2)} u \right|^2 \right]$$

$w^{(1,2)}_{ij}$ are known O(1) functions of SUSY parameters

$$BR(\mu \to e\gamma) \approx BR(\tau \to \mu\gamma) \approx BR(\tau \to e\gamma)$$

independently from $u \approx \vartheta_{13}$

cfr. MFV [Cirigliano, Grinstein, Isidori, Wise 2005]

$$\left(\frac{R_{\mu e}}{R_{\tau\mu}}\right) \approx \left| \frac{2}{3} r \pm \sqrt{2} \sin \vartheta_{13} e^{i\delta} \right|^2 < 1$$

$$r \equiv \frac{\Delta m_{sol}^2}{\Delta m_{atm}^2}$$

present (expected) sensitivity to m_{SUSY}

Assuming $w^{(1,2)}_{ij} = 1$

BR(μ->eγ) < 1.2×10^{-11} (10^{-13})	
m_{SUSY} > 255 (820) GeV	u=0.005
m_{SUSY} > 0.7 (2.5) TeV	u=0.05

BR(μ->eee) < 10^{-12} (10^{-13})	
m_{SUSY} > 140 (225) GeV	u=0.005
m_{SUSY} > 400 (700) GeV	u=0.05

[F.F. and A. Paris 1005.5526]

CR^{Ti}(μ->e) < (10^{-18})	
m_{SUSY} > (2.3) TeV	u=0.005
m_{SUSY} > (6.6) TeV	u=0.05

m_{susy} in the region of interest for LHC

[also Hagedorn, Molinaro, Petcov 0911.3605]

Leptogenesis

if ν^c_i transform in a 3-dim irreducible representation of G_f then ε_i=0 in the exact symmetry limit u=0.

→ ε_i = 0 at the LO

$\varepsilon_i \neq 0$ from the NLO corrections

$$\varepsilon_i \approx \frac{u^2}{16\pi} \qquad \text{[NH]}$$

$$\varepsilon_i \approx \frac{u^2}{16\pi r} \qquad \text{[IH]} \qquad r \equiv \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}} \approx \frac{1}{30}$$

$\varepsilon_i \geq 10^{-6}$ to produce an acceptable baryon asymmetry

$$u \geq \begin{cases} 0.01 & \text{[NH]} \\ 0.002 & \text{[IH]} \end{cases}$$

in agreement with expected range of u

[Jenkins, Manohar 0807.4176
Bertuzzo, Di Bari, FF, Nardi 0908.0161
Hagedorn, Molinari, Petcov 0908.0240]

Quark masses – grand unification

quarks assigned to the same A_4 representations used for leptons?

	q	u^c	c^c	t^c	d^c	s^c	b^c
A_4	3	1	1''	1'	1	1''	1'

- fermion masses from dim ≥ 5 operators, e.g. $\dfrac{\tau^c \varphi_T l H_d}{\Lambda}$ good for leptons, but not for the top quark
- naïve extension to quarks leads diagonal quark mass matrices and to $V_{CKM}=1$ departure from this approximation is problematic [expansion parameter (VEV/Λ) too small]

possible solution within T', the double covering of A_4

[FHLM1]

$$S^2 = R \quad R^2 = 1 \quad (ST)^3 = T^3 = 1$$

24 elements

representations: 1 1' 1'' 3 2 2' 2''

	$\begin{pmatrix} u & d \\ c & s \end{pmatrix}$	$\begin{pmatrix} u^c \\ c^c \end{pmatrix}$	$\begin{pmatrix} d^c \\ s^c \end{pmatrix}$	$(t \quad b)$	t^c	b^c	η	ξ''
T'	2''	2''	2''	1	1	1	2'	1''

[older T' models by
Frampton, Kephard 1994
Aranda, Carone, Lebed 1999, 2000
Carr, Frampton 2007
similar U(2) constructions by
Barbieri, Dvali, Hall 1996
Barbieri, Hall, Raby, Romanino 1997
Barbieri, Hall, Romanino 1997]

- lepton sector as in the A_4 model
- t and b masses at the renormalizable level (τ mass from higher dim operators)

at the leading order

$$m_{u,d} \propto \begin{pmatrix} 0 & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}$$

33>>22,23,32

$$\langle \eta \rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}$$

$$m_t, m_b > m_c, m_s \neq 0$$

$$V_{cb}$$

- masses and mixing angles of 1st generation from higher-order effects
- despite the large number of parameters two relations are predicted

$$\sqrt{\frac{m_d}{m_s}} = |V_{us}| + O(\lambda^2)$$

$0.213 \div 0.243 \qquad 0.2257 \pm 0.0021$

$$\sqrt{\frac{m_d}{m_s}} = \left|\frac{V_{td}}{V_{ts}}\right| + O(\lambda^2)$$

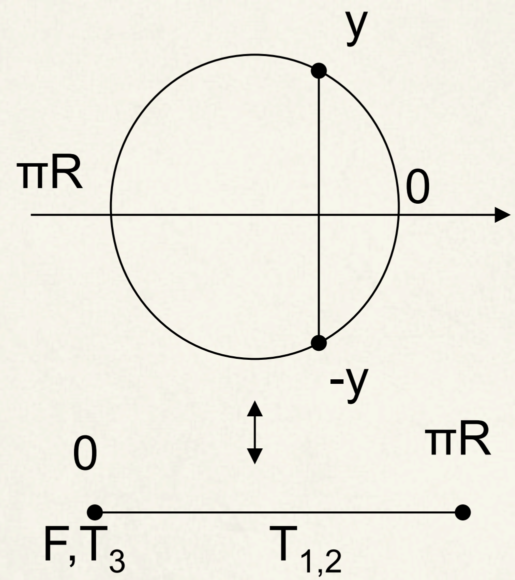
$0.208^{+0.008}_{-0.006}$

- vacuum alignment explicitly solved
- lepton sector not spoiled by the corrections coming from the quark sector

other option:

[AFH]

SUSY SU(5) in $5D=M_4 x(S^1 x Z_2)$
+
flavour symmetry $A_4 x U(1)$



DT splitting problem solved
via SU(5) breaking induced by compactification

dim 5 B-violating operators forbidden!
p-decay dominated by gauge boson exchange (dim 6)

unwanted minimal SU(5) mass relation $m_e=m_d^T$ avoided by assigning $T_{1,2}$ to the bulk

the construction is compatible with A_4!

	N	F	T_1	T_2	T_3	H_5	$H_{\bar{5}}$
$SU(5)$	1	$\bar{5}$	10	10	10	5	$\bar{5}$
A_4	3	3	1''	1'	1	1	1'

reshuffling of singlet reps.

unsuppressed top Yukawa coupling $T_3 T_3$

realistic quark mass matrices
by an additional U(1) acting on $T_{1,2}$

neutrino masses from see-saw
compatible with both normal and
inverted hierarchy

TB mixing + small corrections

A_4 as a leftover of Poincare symmetry in D>4

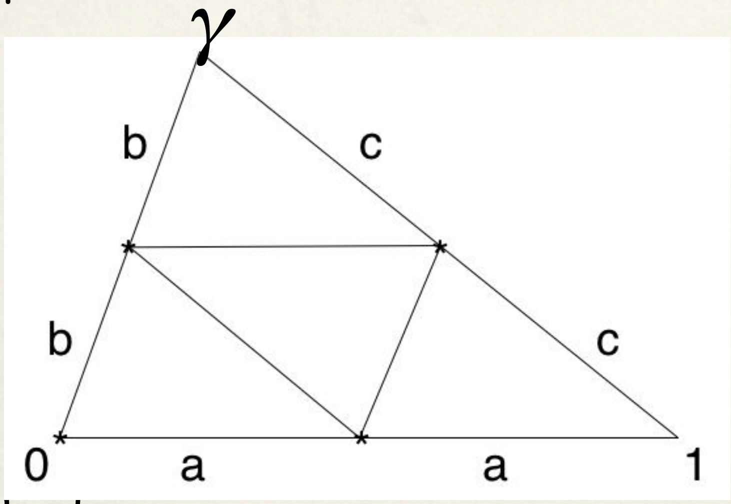
[AFL]

D dimensional
Poincare symmetry:
D-translations x SO(1,D-1)

⟺

usually broken by
compactification down to 4 dimensions:
4-translations x SO(1,3) x …

a discrete subgroup of the (D-4) euclidean group = translations x rotations
can survive in specific geometries



Example: D=6

2 dimensions
compactified on T^2/Z_2

$$z \to z+1$$
$$z \to z+\gamma$$
$$z \to -z$$

four fixed points

$$\text{if} \quad \gamma = e^{i\frac{\pi}{3}}$$

compact space is a regular tetrahedron
invariant under

$$S: \quad z \to z+\frac{1}{2}$$ [translation]

$$T: \quad z \to \gamma^2 z$$ [rotation by 120^0]

[subgroup of 2 dim Euclidean group = 2-translations x SO(2)] the four fixed points (z_1,z_2,z_3,z_4) are permuted under the action of S and T

$$S: \qquad (z_1,z_2,z_3,z_4) \rightarrow (z_4,z_3,z_2,z_1)$$
$$T: \qquad (z_1,z_2,z_3,z_4) \rightarrow (z_2,z_3,z_1,z_4)$$

S and T satisfy

$$S^2 = T^3 = (ST)^3 = 1$$

the compact space is invariant under a remnant of 2-translations x SO(2) isomorphic to the A_4 group

Field Theory

brane fields $\varphi_1(x)$, $\varphi_2(x)$, $\varphi_3(x)$, $\varphi_4(x)$ transform as 3 + (a singlet) under A_4

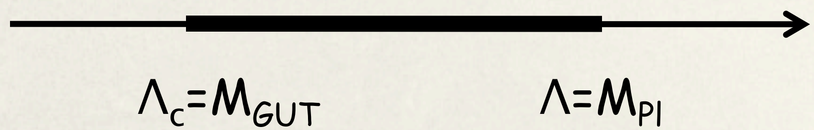
The previous model can be reproduced by choosing l, e^c, μ^c, τ^c, $H_{u,d}$ as brane fields and φ_T, φ_S and ξ as bulk fields.

other realizations of Anarchy (II)

Nelson-Strassler [0006251 "Suppressing Flavor Anarchy"]

Anarchy can arise when matter chiral supermultiplets X_i of the MSSM are coupled to a superconformal sector in some finite energy range

e.g.



large positive anomalous dimensions for X_i:

$$\frac{\gamma_i}{2} \equiv d(X_i) - 1 > 0$$

$$K = \sum_i Z_i X_i^+ X_i + \ldots \qquad Z_i(\Lambda_c) = \underbrace{Z_i(\Lambda)}_{1} \left(\frac{\Lambda_c}{\Lambda}\right)^{-\gamma_i}$$

Anarchy through wave function renormalization: $X_i \to F_{X_i} X_i$

$$w = Y_{ij} X_i X_j H + \ldots \to (F_{X_i} Y_{ij} F_{X_j}) X_i X_j H + \ldots$$

$$F_{X_i} = \left(\frac{\Lambda_c}{\Lambda}\right)^{\frac{\gamma_i}{2}} < 1$$

[as in FN with a single flavon and positive FN charges]

no underlying flavour symmetry

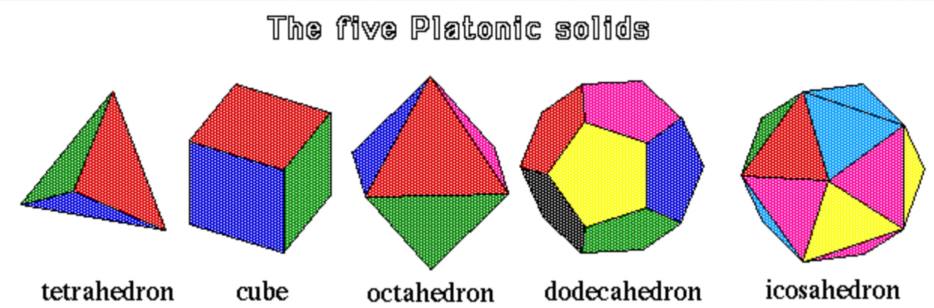
[an anomaly free R symmetry is generated dynamically at the IR stable fixed point: $\dim(X_i)=2/3\, R(X_i)$]

anomalous dimensions γ_i calculable when gauge group and field content are known

[Polland, Simmons-Duffin 0910.4585]

discrete flavour symmetries leading to previous LO mixing

the (proper) symmetry groups of the Platonic solids



duality		group	order	n
tetrahedron	tetrahedron	A_4	12	3
cube	octahedron	S_4	24	4
dodecahedron	icosahedron	A_5	60	5

they are all generated by two elements: S and T

$$S^2 = (ST)^3 = 1 \qquad T^n = 1$$

[a longer sequence? The (infinite, discrete) modular group Γ is also generated by S and T satisfying $S^2=(ST)^3=1$ and possesses an infinite serie of finite subgroups Γ/Γ_n (Γ_n being the principal congruence subgroup of level n). For n=3,4,5 we recover the symmetry groups of the Platonic solids]

What is the best 1st order approximation to lepton mixing?

in the quark sector

$$V_{CKM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + O(\vartheta_C)$$

[Wolfenstein 1983]

in the lepton sector

$$U_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + \ldots$$

agreement of ϑ_{12} suggests that only tiny corrections [$O(\vartheta_C^2)$] are tolerated. If all corrections are of the same order, then

$\vartheta_{13} \approx O(\vartheta_C^2)$ expected

$$U_{PMNS} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} + \ldots$$

can be reconciled with the data through a correction of $O(\vartheta_C)$, for instance a rotation in the 12 sector [from the left side]

$\vartheta_{13} \approx O(\vartheta_C)$ expected

[quark-lepton complementarity ?] [Smirnov; Raidal; Minakata and Smirnov 2004]

$\vartheta_{23} - \pi/4 \approx O(\vartheta_C^2)$

common feature: $\vartheta_{23} \approx \pi/4$ [maximal atm mixing]

… or anarchical U_{PMNS} ? [Hall, Murayama, Weiner 1999]

Minimal Flavor Violation [MFV]

$$G_f = SU(3)_l \times SU(3)_{e^c} \times \ldots$$

[D'Ambrosio, Giudice, Isidori, Strumia 2002
Cirigliano, Grinstein, Isidori, Wise 2005]

the largest G_f

$$l = (\bar{3},1) \qquad e^c = (1,3)$$

$$\varphi \equiv \begin{cases} y_e = (3,\bar{3}) \\ Y = (6,1) \end{cases}$$

G_f broken only by the Yukawa coupling of L_{SM} and L_5

y_e and Y can be expressed in terms of lepton masses and mixing angles

$$y_e = \sqrt{2}\frac{m_e^{diag}}{v} \qquad Y = \frac{\Lambda_L}{v^2} U^* m_\nu^{diag} U^+$$

diagonal elements $\left[\mathcal{M}(\langle\varphi\rangle)\right]_{ii}$ are of the same size as in A_4x...
similar lower bounds on the scale M

$$\left[\mathcal{M}(\langle\varphi\rangle)\right]_{ij} = \beta\,(y_e Y^+ Y)_{ij} + \ldots$$

$$= \sqrt{2}\beta \frac{(m_l)_{ii}}{v} \frac{\Lambda_L^2}{v^4}\left[\Delta m^2_{sol} U_{i2} U^*_{j2} \pm \Delta m^2_{atm} U_{i3} U^*_{j3}\right] + \ldots$$

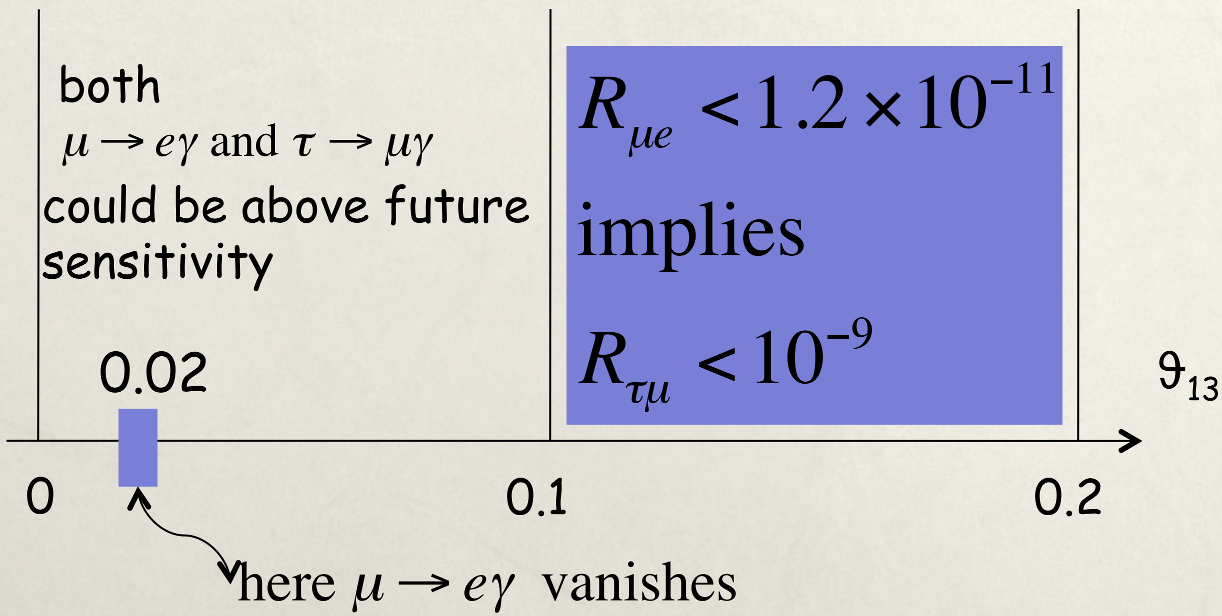
+ for normal hierarchy
- for inverted hierarchy

a positive signal at MEG $10^{-11} < R_{\mu e} < 10^{-13} \div 10^{-14}$ always accommodated [but for a small interval around $\vartheta_{13} \approx 0.02$ where $R_{\mu e}=0$]

non-observation of R_{ij} can be accommodated by lowering Λ_L

$$\left(\frac{R_{\mu e}}{R_{\tau\mu}}\right) \approx \left|\frac{2}{3} r \pm \sqrt{2}\sin\vartheta_{13} e^{i\delta}\right|^2 < 1 \qquad r \equiv \frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}$$

[Cirigliano, Grinstein, Isidori, Wise 2005]



MFV [scale M can be of order 1 TeV]

both $\mu \to e\gamma$ and $\tau \to \mu\gamma$ could be above future sensitivity

0.02 here $\mu \to e\gamma$ vanishes

$R_{\mu e} < 1.2 \times 10^{-11}$

implies

$R_{\tau\mu} < 10^{-9}$

ϑ_{13}

0 0.05 0.1 0.2

only $\mu \to e\gamma$ can be above experimental sensitivity

disfavoured by A_4

SUSYxA_4 [scale M can be of order 1 TeV]