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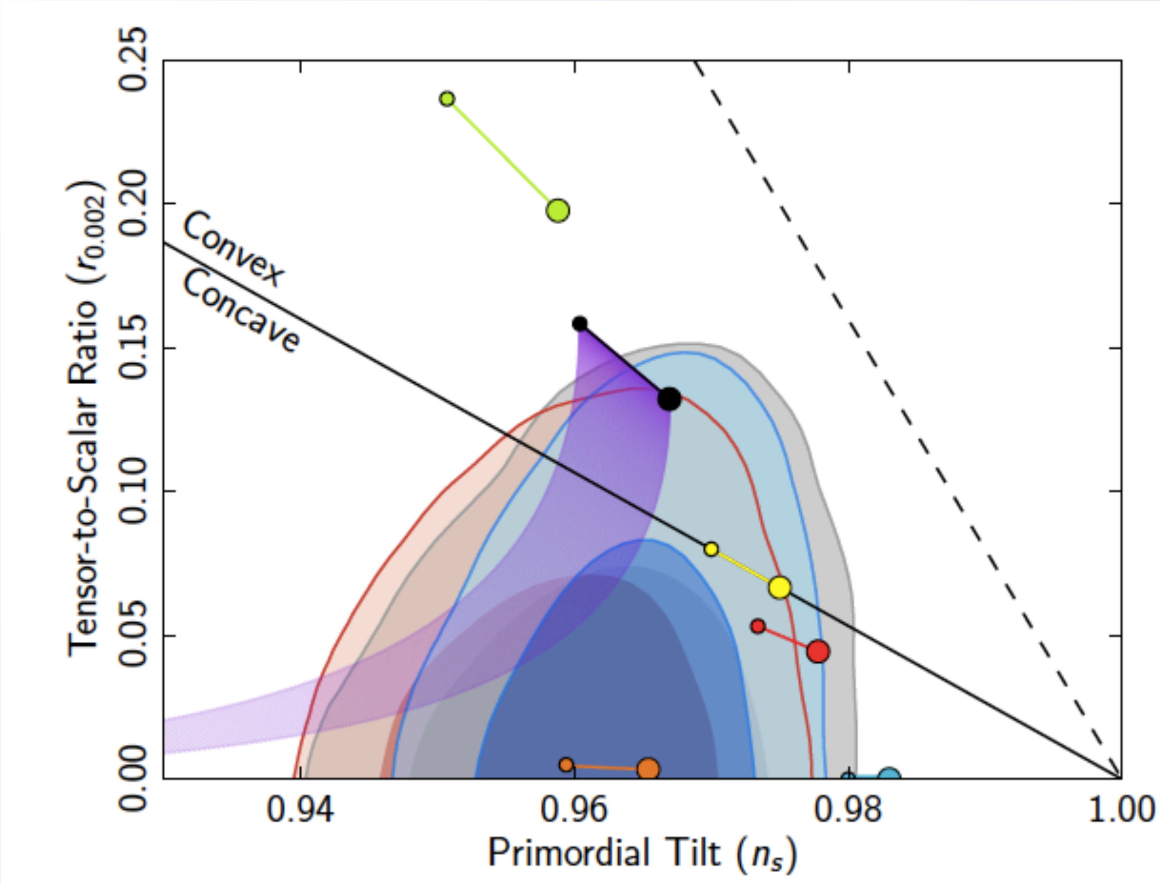
Galileons and inequivalent representations of the conformal group

with M. Serone and E. Trincherini: 1306.2946 (JHEP)
with S. Dubovsky, A. Nicolis, M. Serone, G. Trevisan and E.Trincherini,
to appear

Planck: deviation from scale-invariance at 5σ !

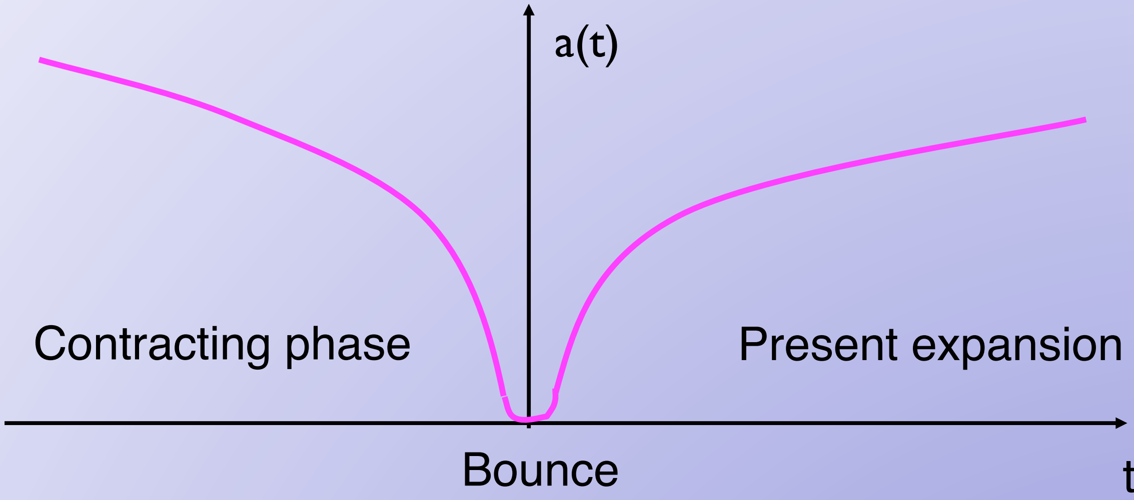


Harrison
Zeldovich

$$\langle \zeta_{\vec{k}} \zeta_{\vec{k}'} \rangle = (2\pi)^3 \delta(\vec{k} + \vec{k}') \frac{A}{k^3} \cdot k^{n_s - 1} \longrightarrow$$ Something dynamical

Higgs in 2012, inflaton in 2013?

Well…
Reasonable alternatives are on the market
Tilt does not discriminate



They (all?) require a violation of
Null Energy Condition:

$$T_{\mu\nu} k^{\mu} k^{\nu} \geq 0 \quad \longrightarrow \quad \dot{H} \leq 0$$

Usually associated with instabilities, but Galileons are an exception

Outline

- Galileons and NEC violation for non-standard cosmologies
- Galileons as **dilatons**. Non-linear representations of the conformal group
- 2 standard representations: mapping between them
- **Superluminality** constraints are different!
- Inequivalence: **different representations give different physics**

Galileon

Nicolis, Rattazzi, Trincherini, 08

Galilean symmetry: $\pi(x) \to \pi(x) + c + b_\mu x^\mu$

The **lowest dim** operators have EOM with 2 derivatives per field: $\frac{\delta \mathcal{L}_\pi}{\delta \pi} = F(\partial_\mu \partial_\nu \pi)$

$$\mathcal{L}^{(n)} \sim \partial^{2n-2} \pi^n$$

In analogy with: $x(t) \to x(t) + x_0 + v_0 t$ $\qquad \mathcal{L}^{(2)} = -\frac{1}{2}(\partial \pi)^2$

There are only 5 in total (in 4d)!

$$\mathcal{L}^{(1)} = \pi \qquad \mathcal{L}^{(2)} = (\partial \pi)^2$$

$$\mathcal{L}^{(3)} = (\partial \pi)^2 \Box \pi \qquad \mathcal{L}^{(4)} = (\partial \pi)^2 [(\Box \pi)^2 - \partial_\mu \partial_\nu \pi \partial^\mu \partial^\nu \pi]$$

$$\mathcal{L}^{(5)} = (\partial \pi)^2 [(\Box \pi)^3 - 3 \Box \pi \partial_\mu \partial_\nu \pi \partial^\mu \partial^\nu \pi + 2 \partial_\mu \partial_\nu \pi \partial^\nu \partial^\alpha \pi \partial_\alpha \partial^\mu \pi]$$

Galileon → Dilaton

Extend Galilean symmetry + Poincare' to **conformal group SO(4,2)**

$$\delta\pi_D = (1 - x^\mu \partial_\mu \pi) c\,,$$
$$\delta\pi_{K_\mu} = (-2x_\mu - x^2 \partial_\mu \pi + 2x_\mu x^\nu \partial_\nu \pi) b^\mu$$

Galileon is Wigner contraction of SO(4,2), π -> 0

$$g^{(\pi)}_{\mu\nu} \equiv e^{2\pi(x)} \eta_{\mu\nu}$$

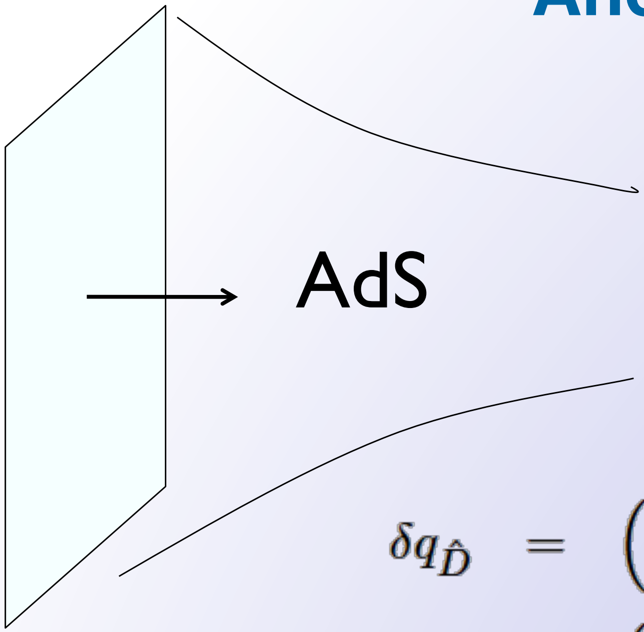
Conformal Galileons

$$\mathcal{L}_{\pi 1} = -e^{-4\pi}\,, \qquad = -\sqrt{-g}\,,$$
$$\mathcal{L}_{\pi 2} = -L^2 e^{-2\pi} (\partial\pi)^2\,, \qquad = -L^2 \frac{\sqrt{-g}}{6} R$$
$$\mathcal{L}_{\pi 3} = L^4 (\partial\pi)^2 \left(-[\Pi] + \frac{1}{2}(\partial\pi)^2\right)\,,$$

(Komargodski – Schwimmer 11)

$$\mathcal{L}_{\pi 4} = L^6 e^{2\pi} (\partial\pi)^2 \left(-[\Pi]^2 + [\Pi^2] - \frac{1}{2}(\partial\pi)^2 [\Pi] - \frac{1}{2}(\partial\pi)^4\right)$$
$$= -L^6 \frac{\sqrt{-g}}{4} \left(-\frac{7}{36} R^3 + R (R_{\mu\nu})^2 - (R_{\mu\nu})^3\right)$$

Another representation



In the AdS/CFT context we know that a brane will non-linearly realize SO(4,2).

But in a **different** way:

$$\delta q_{\hat{D}} = \Big(1 - \frac{1}{L} x^\mu \partial_\mu q\Big) c\,,$$

$$\delta q_{\hat{K}_\mu} = \Big(-2x_\mu - L\,\partial_\mu q (e^{2q/L} - 1) - \frac{1}{L} x^2 \partial_\mu q + \frac{2}{L} x_\mu x^\nu \partial_\nu q \Big) b^\mu$$

$$\mathcal{L}_{q1} = -\,e^{-4q/L}\,, \qquad = -\,e^{-4q/L}\,,$$

$$\mathcal{L}_{q2} = -\,e^{-4q/L}\sqrt{1 + e^{2q/L}(\partial q)^2}\,, \qquad = -\sqrt{-G}\,,$$

$$\mathcal{L}_{q3} = L\,\gamma^2[q^3] - L\,e^{-2q/L}[Q] + e^{-4q/L}(\gamma^2 - 5) \qquad = L\sqrt{-G}\,K\,,$$

De Rham, Tolley 10

…….

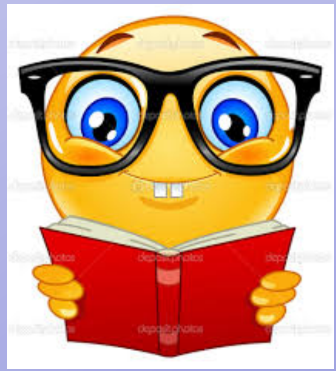
Weyl and DBI representation

Different representations

1. What is the relation between the two representations?

2. What happens to the Galileons?

3. Do they give the same physics?



Coset construction

Bellucci, Ivanov, Krivonos, 02

Weyl

$$g = e^{y^\mu P_\mu} e^{\pi D} e^{\Omega^\mu K_\mu}$$

~ CCWZ

DBI

$$g = e^{x^\mu P_\mu} e^{q\hat{D}} e^{\Lambda^\mu \hat{K}_\mu}$$

$$\hat{K}_\mu \equiv \frac{1}{\sqrt{2}L} K_\mu + \frac{L}{\sqrt{2}} P_\mu \,, \quad \hat{D} \equiv \frac{1}{\sqrt{2}L} D$$

"Straightforward". Inverse Higgs: a **single Goldstone**

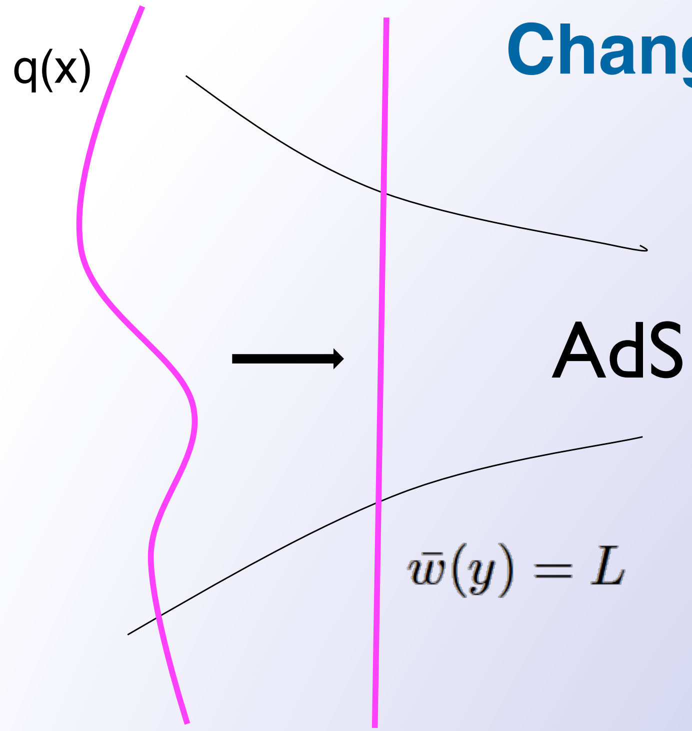
$$\lambda_\mu = \Lambda_\mu \frac{\tan(\Lambda/\sqrt{2})}{\Lambda/\sqrt{2}} \,, \quad \Lambda = \sqrt{\Lambda^\mu \Lambda_\mu}$$

$$\Omega_\mu(y) = \frac{1}{2} e^{\pi} \partial_\mu \pi(y)$$

$$\lambda_\mu(x) = \frac{\partial_\mu q(x)\, e^{q(x)/L}}{1 + \sqrt{1 + e^{2q(x)/L} (\partial q(x))^2}}$$

$$y^\mu = x^\mu + L e^{q(x)/L} \lambda^\mu(x) \,, \quad \pi(y) = \frac{q(x)}{L} + \log(1 + \lambda^2(x)) \,, \quad \Omega_\mu(y) = \frac{1}{L} \lambda_\mu(x)$$

Change of coordinates



$$(x^\mu, z) \to (y^\mu, w)$$

AdS -> Weyl representation

$$g_{\mu 5} = 0\,, \qquad g_{55} = L^2/w^2$$

$$x^\mu = y^\mu + F^\mu(y,w)\,, \qquad z = w\, e^{G(y,w)}$$

$$F^\mu = -\frac{w^2}{2} e^{G(y,w)+\pi(y)} \eta^{\mu\nu} \partial_\nu \pi(y)\,, \qquad G = \pi(y) - \log\left(1 + w^2 \frac{e^{2\pi(y)}}{4} (\partial\pi(y))^2\right)$$

$$L\, e^{q(x)/L} = L\, e^{\pi(y)} \left(1 + \frac{L^2}{4} e^{2\pi(y)} (\partial\pi)^2\right)^{-1}$$

$$ds^2 = \frac{L^2}{w^2}\left(g_{\mu\nu}(y,w)\, dy^\mu dy^\nu + dw^2\right) \qquad g_{\mu\nu} = \eta_{\mu\nu} e^{-2\pi(y)} + \mathcal{O}(w^2)$$

Rattazzi and Zaffaroni 00

Galileon map

What happens to Galileons? They are mapped into each other since they give 2nd order EOM and this does not depend on the **gauge**

$$\begin{pmatrix} \mathcal{L}_{\pi 1} \\ \mathcal{L}_{\pi 2} \\ \mathcal{L}_{\pi 3} \\ \mathcal{L}_{\pi 4} \\ \mathcal{L}_{\pi 5} \end{pmatrix} = \begin{pmatrix} 0 & \frac{1}{2} & \frac{7}{64} & -\frac{1}{24} & -\frac{1}{192} \\ 0 & 0 & -\frac{1}{16} & -\frac{1}{12} & -\frac{1}{48} \\ 4 & 0 & -\frac{11}{8} & 0 & -\frac{1}{8} \\ 0 & 0 & -\frac{3}{2} & 2 & -\frac{1}{2} \\ 0 & -96 & 21 & 8 & -1 \end{pmatrix} \begin{pmatrix} \mathcal{L}_{q1} \\ \mathcal{L}_{q2} \\ \mathcal{L}_{q3} \\ \mathcal{L}_{q4} \\ \mathcal{L}_{q5} \end{pmatrix}$$

Invertible

- Weyl kinetic term into a q3, q4 and q5
  - Nambu-Goto into all πi

Galileons less "exotic"?

S-matrix

$$y^\mu = x^\mu + L e^{q(x)/L} \lambda^\mu(x)\,, \qquad \pi(y) = \frac{q(x)}{L} + \log(1+\lambda^2(x))$$

$$\lambda_\mu(x) = \frac{\partial_\mu q(x)\, e^{q(x)/L}}{1+\sqrt{1+e^{2q(x)/L}(\partial q(x))^2}}$$

Complicated field redefinition but "standard" **once expanded in series**

For example

$$\mathcal{L}_{NG} = \frac{1}{L^4}(-\mathcal{L}_{q1} + \mathcal{L}_{q2}) = \frac{1}{L^4}\left(\frac{1}{2}\mathcal{L}_{\pi 2} - \frac{1}{4}\mathcal{L}_{\pi 3} + \frac{1}{16}\mathcal{L}_{\pi 4} - \frac{1}{96}\mathcal{L}_{\pi 5}\right)$$

$$\pi \to \pi + \frac{1}{2}\pi^2 - \frac{1}{4}L^2(\partial\pi)^2 \quad \ldots$$

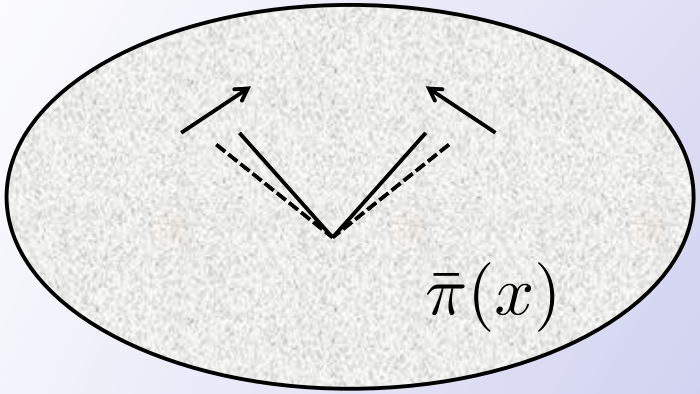
$$\mathcal{A}_{DBI}(2\to 2) = \mathcal{A}_{Weyl}(2\to 2)$$

But…

Superluminality

Absence of superluminality around background to constrain "healthy" EFTs

Arkani-Hamed etal 04



The correction to the lightcone must always go in the **subluminal** direction

$$\mathcal{L} = \partial^\mu \pi \partial_\mu \pi + \frac{c_3}{\Lambda^4}(\partial_\mu \pi \partial^\mu \pi)^2 + \dots$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{c_1}{\Lambda^4}(F_{\mu\nu} F^{\mu\nu})^2 + \frac{c_2}{\Lambda^4}(F_{\mu\nu} \tilde{F}^{\mu\nu})^2 + \dots$$

$c_1 \;\; c_2 \;\; c_3 > 0$

Galileon Superluminality

$$\mathcal{S}_\pi = \int d^4x\,\sqrt{-g}\big[-f^2 e^{2\pi}(\partial\pi)^2 + \frac{f^3}{\Lambda^3}(\partial\pi)^2\Box\pi + \frac{f^3}{2\Lambda^3}(\partial\pi)^4\big]$$

Around π = 0 consider a weak stationary $\nabla^2\pi_0 \simeq 0$

$$\delta_2\mathcal{L} = f^2\, G^{\mu\nu}\,\partial_\mu\delta\pi\,\partial_\nu\delta\pi + \dots \qquad G_{\mu\nu} \simeq \eta_{\mu\nu} + \frac{4}{3}\frac{1}{H_0^2}\partial_\mu\partial_\nu\pi_0 \qquad H_0^2 = \frac{2\Lambda^3}{3f}$$

Perturbations around a generic solution will be superluminal

Measurable effects unless new physics at H_0

No CTC. No standard Lorentz invariant UV completion

But we get it even starting from Nambu-Goto which has no superluminality

Back to Galileons: free theory paradox

In the limit π -> 0 or q -> 0 both representations go back to the Galileons (group contraction)

Our map induces a map of **Galileons into themselves** with field redefinition

$$y^\mu = x^\mu - L\partial^\mu q \ ,$$

$$\frac{\partial \pi}{\partial y^\mu} = \frac{1}{L}\frac{\partial q}{\partial x^\mu} \ ,$$

$$\pi(y) = \frac{q(x)}{L} - \frac{1}{2}(\partial q)^2$$

De Rham, Fasiello, Tolley 13

$$\mathcal{L}_{G\pi 2} = -\frac{1}{2}(\partial\pi)^2 \to \mathcal{L}_{Gq2} - \mathcal{L}_{Gq3} + \frac{1}{2}\mathcal{L}_{Gq4} - \frac{1}{6}\mathcal{L}_{Gq5}$$

$$\partial^2\bar{q}/\Lambda^3 \ll 1$$

$$\Box q - \frac{2}{\Lambda^3}\Box\bar{q}\,\Box q + \frac{2}{\Lambda^3}\partial^\mu\partial^\nu\bar{q}\,\partial_\mu\partial_\nu q = 0$$

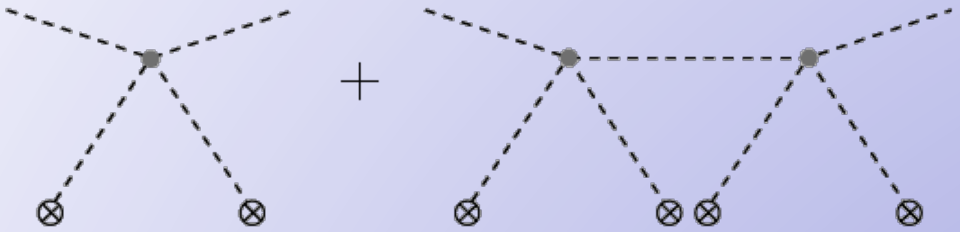
$$G_{\mu\nu} = \left(1 - \frac{2}{\Lambda^3}\Box\bar{q}\right)\eta_{\mu\nu} + \frac{2}{\Lambda^3}\partial_\mu\partial_\nu\bar{q}$$

$$\delta c_s \sim \frac{\partial^2\bar{q}}{\Lambda^3}$$

Superluminality knows more

Theories have the same S-matrix, how can they have different propagation around a background?

The propagation resums an infinite number of diagrams



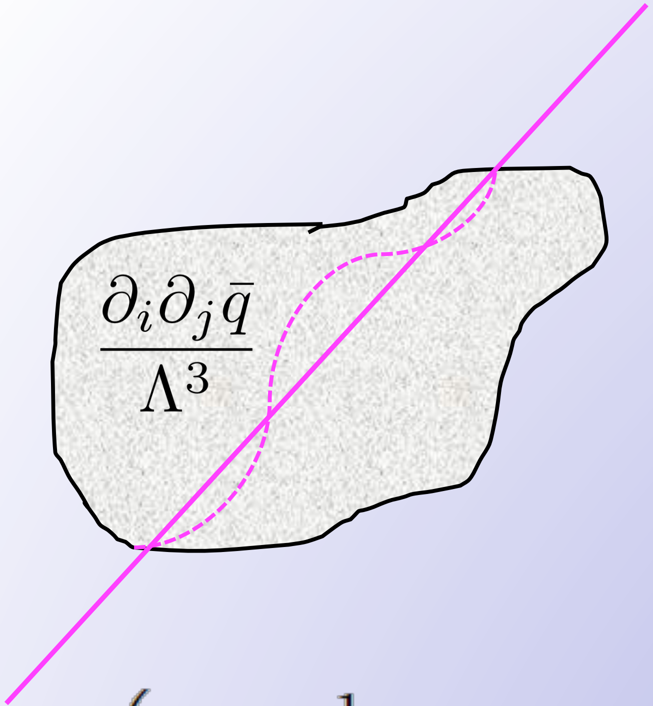
Non – perturbative statement $q(x) \propto e^{i\vec{k}\cdot\vec{x} - i(1+\delta c_s)kt}$ …and not its expansion

Superluminality measurable only when $\frac{\partial^2 \bar{q}}{\Lambda^3} L \gtrsim \frac{1}{\omega} \quad \Rightarrow \quad \frac{\omega \partial \bar{q}}{\Lambda^3} \gtrsim 1$

Expansion parameter in field redefiniton is: $\frac{\partial^2 q}{\Lambda^3}$

The field redefinition cannot be expanded

Non-local field redefinition



$$\delta y = \frac{1}{\Lambda^3} \int d\lambda \, \partial^2 \bar{q} = \frac{1}{\Lambda^3} \partial \bar{q} \Big|_{\lambda_i}^{\lambda_f}$$

No asymptotic effect

$$\pi(q(y)) J(y) = \left(q(y) + \frac{1}{2\Lambda^3} (\partial q(y))^2 + \frac{1}{2\Lambda^6} \partial^\mu \partial^\nu q(y) \partial_\mu q(y) \partial_\nu q(y) + \ldots \right) J(y)$$

$$\frac{1}{\Lambda^3} \omega \partial \bar{q} \geq 1$$

$$\mathcal{L}_{source} = q \left(y^\mu + \frac{1}{\Lambda^3} \partial^\mu \bar{q}(y) \right) J(y)$$

The two theories are not equivalent. Different local operators. Different UV completion

Conclusions

1. Different non-linear representations of the conformal group
   Weyl and **DBI**

2. Complicated field redefinition maps Weyl Galileons into DBI Galileons

3. S-matrix equivalent theories, but superluminality…

4. **Inequivalent** when the whole series of operators matters

5. The symmetry breaking pattern is not enough. How general?

6. **NEC-violating scenarios** with no pathologies around that solution

Stable NEC violation

Nicolis, Rattazzi, Trincherini 09
PC, Nicolis, Trincherini 10

Let us study solutions: **SO(4,2) → SO(4,1)**

$$\mathcal{S}_\pi = \int d^4x \sqrt{-g}\left[f^2 e^{2\pi}(\partial\pi)^2 + \frac{f^3}{\Lambda^3}(\partial\pi)^2\Box\pi + \frac{f^3}{2\Lambda^3}(\partial\pi)^4\right]$$

$$e^{\pi_{\rm dS}} = -\frac{1}{H_0 t} \qquad -\infty < t < 0 \qquad H_0^2 = \frac{2\Lambda^3}{3f}$$

$$\begin{aligned} T_{\mu\nu} &= -f^2 e^{2\pi}\left[2\partial_\mu\pi\partial_\nu\pi - g_{\mu\nu}(\partial\pi)^2\right] \\ &- \frac{f^3}{\Lambda^3}\left[2\,\partial_\mu\pi\partial_\nu\pi\Box\pi - \left(\partial_\mu\pi\,\partial_\nu(\partial\pi)^2 + \partial_\nu\pi\,\partial_\mu(\partial\pi)^2\right) + g_{\mu\nu}\,\partial_\alpha\pi\,\partial^\alpha(\partial\pi)^2\right] \\ &- \frac{f^3}{2\Lambda^3}\left[4(\partial\pi)^2\partial_\mu\pi\partial_\nu\pi - g_{\mu\nu}(\partial\pi)^4\right]. \end{aligned}$$

Scale invariance: $T^{\mu\nu} = \tau^{\mu\nu}\dfrac{1}{t^4} \longrightarrow \rho = 0 \qquad p \propto -\dfrac{1}{t^4}$

Stable perturbations! $\dfrac{f^2}{H_0^2 t^2}\left[-(\partial\delta\pi)^2 + \dfrac{4}{t^2}\delta\pi^2\right]$

$$\delta\pi \sim 1/t$$

$$\delta\pi \sim t^4$$

Genesis

Negligible gravity…

Brutal violation of NEC: $\dot{H} = -\frac{1}{2M_P^2}(\rho + p) \propto \frac{1}{t^4}$ $\quad H = -\frac{1}{3}\frac{f^2}{M_P^2}\frac{1}{H_0^2 t^3} + c$

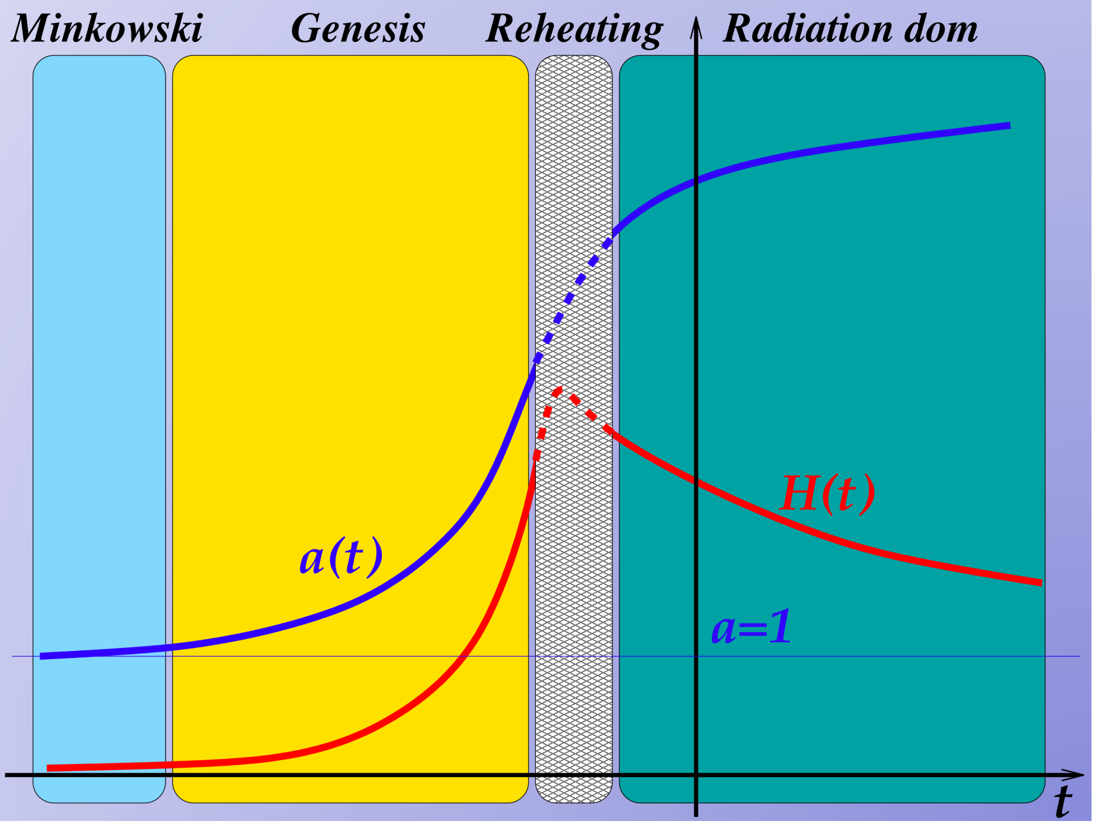
Can start in contraction or expansion

The solution is an attractor

…gravity comes back

$$a(t) \sim \exp\left[\frac{8f^2}{3H_0^2 M_{\rm Pl}^2}\frac{1}{(t_0 - t)^2}\right]$$

Theory reaches its cut-off: reheating?



Scale invariance from fake de Sitter

$$g_{\mu\nu}^{(\pi)} \equiv e^{2\pi(x)} \eta_{\mu\nu}$$

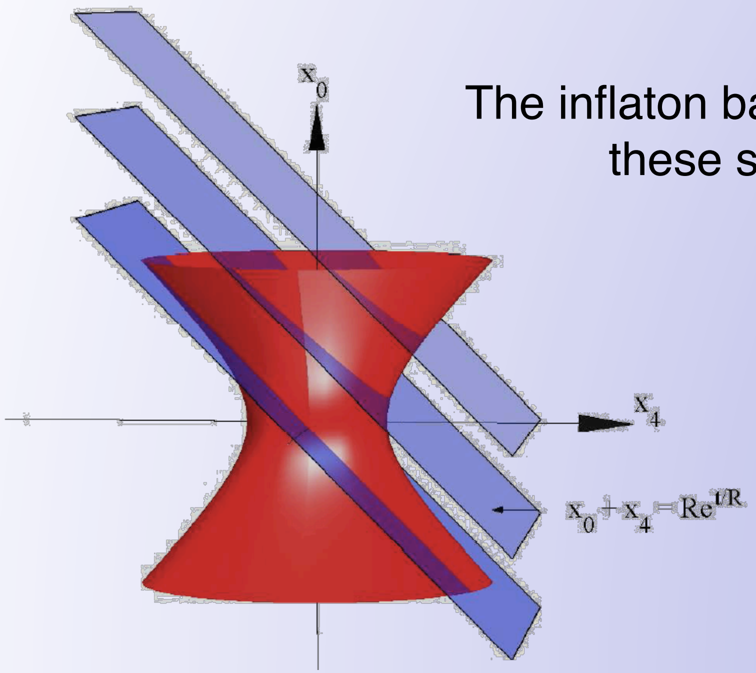
A spectator field will behave as in dS !

$$ds^2 = \frac{1}{H^2\eta^2}(-d\eta^2 + d\vec{x}^2)$$

All the isometries of dS: 3 rotations + 3 translations + dilation

+ 3 special conformal

$$\eta \to \eta - 2\eta(\vec{b}\cdot\vec{x})\ , \quad x^i \to x^i + b^i(-\eta^2 + \vec{x}^2) - 2x^i(\vec{b}\cdot\vec{x})$$



The inflaton background breaks these symmetries

But a test scalar during inflation will not: second field mechanisms

BACKUP slides

Subluminal Genesis

with Hinterbichler, Khoury, Nicolis, Trincherini 12

Soft breaking of SO(4,2) to dilation

$$\mathcal{S}_\pi = \int d^4x\,\sqrt{-g}\left[f^2 e^{2\pi}(\partial\pi)^2 + \frac{f^3}{\Lambda^3}(\partial\pi)^2\Box\pi + \frac{f^3}{2\Lambda^3}(1+\alpha)(\partial\pi)^4\right]$$

$$\frac{f^2}{H_0^2 t^2}\left[\dot{\tilde{\pi}}^2 - \frac{3-\alpha}{3(1+\alpha)}(\nabla\tilde{\pi})^2\right] \qquad \rho + p = -\frac{2f^2}{H_0^2 t^4}\frac{3+\alpha}{3(1+\alpha)}$$

~~NEC~~ and stability and $c_S < 1$

Dilation is enough to preserve scale invariance of χ

Higher order correlation functions are different

Genesis: SO(4,2) → SO(4,1)
Subluminal: ISO(3,1) x dilations → ISO(3) x dilations

Higher Galileons and coupling with gravity

• **Coupling with gravity is not unique** for Galileons > 3

• In general EOM are not second order. There are (non-unique) choices of coupling to avoid it.

Deffayet, Esposito-Farese, Vikman 09

E.g. $$\mathcal{L}_4 = \frac{1}{2}\sqrt{-g}e^{-2\pi}(\partial\pi)^2\left(-[\Pi]^2+[\Pi^2]+\frac{1}{2}(\partial\pi)^2[\Pi]-\frac{1}{2}(\partial\pi)^4+\frac{1}{4}(\partial\pi)^2R\right)$$

This is not required for our cosmological solution (E_{ghost} >> 1/t)

• The stress-energy tensor is not uniquely defined

• For the couplings we tried, no way to avoid DGP term around Minkowski. Always on verge of superluminality.

Non-renormalization property: loops of Galileons only induce operators with 2 or more derivatives on each leg. Also the copuling with gravity is radiatively stable

Luty, Porrati and Rattazzi 03

Non-linearly realized symmetries

The inflaton background breaks the symmetry. **Spontaneously**.

We expect the symmetry to be still there to regulate soft limit (q → 0) of correlation functions (Ward identities)

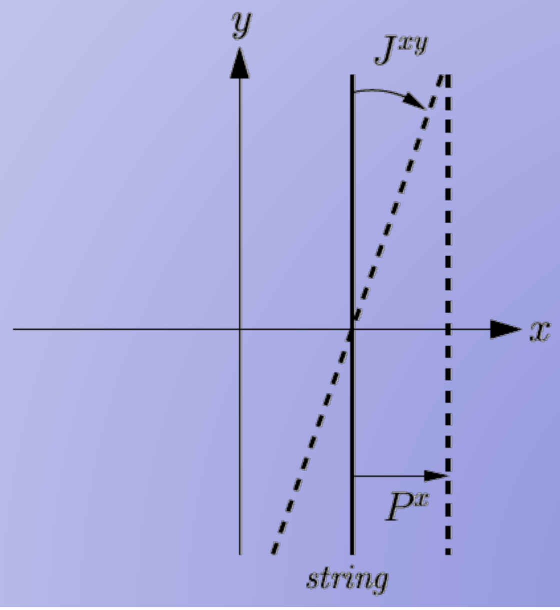
$$\lim_{q \to 0} q^\mu \Gamma^{(n)}_{J_\mu}(q; p_1, \ldots, p_n) = \delta\Gamma^{(n)}(p_1, \ldots, p_n)$$

For example. Soft emission of π's

For space time symmetries:
number of Goldstones ≠ broken generators

Manohar Low 01

We expect Ward identities to say something about higher powers of q



Grazie Riccardo!

DEEP influence!