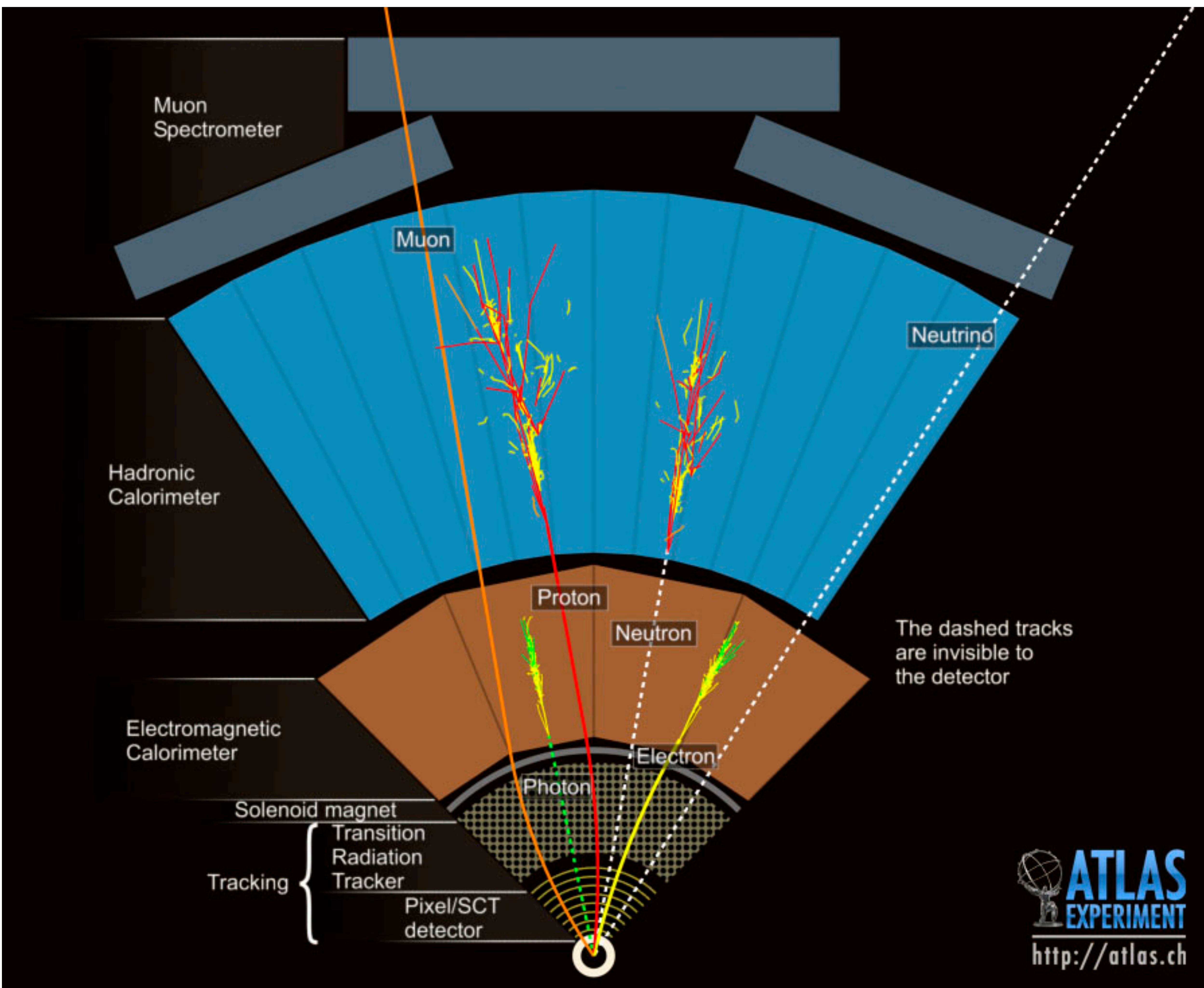
Interactions of Radiation with Matter

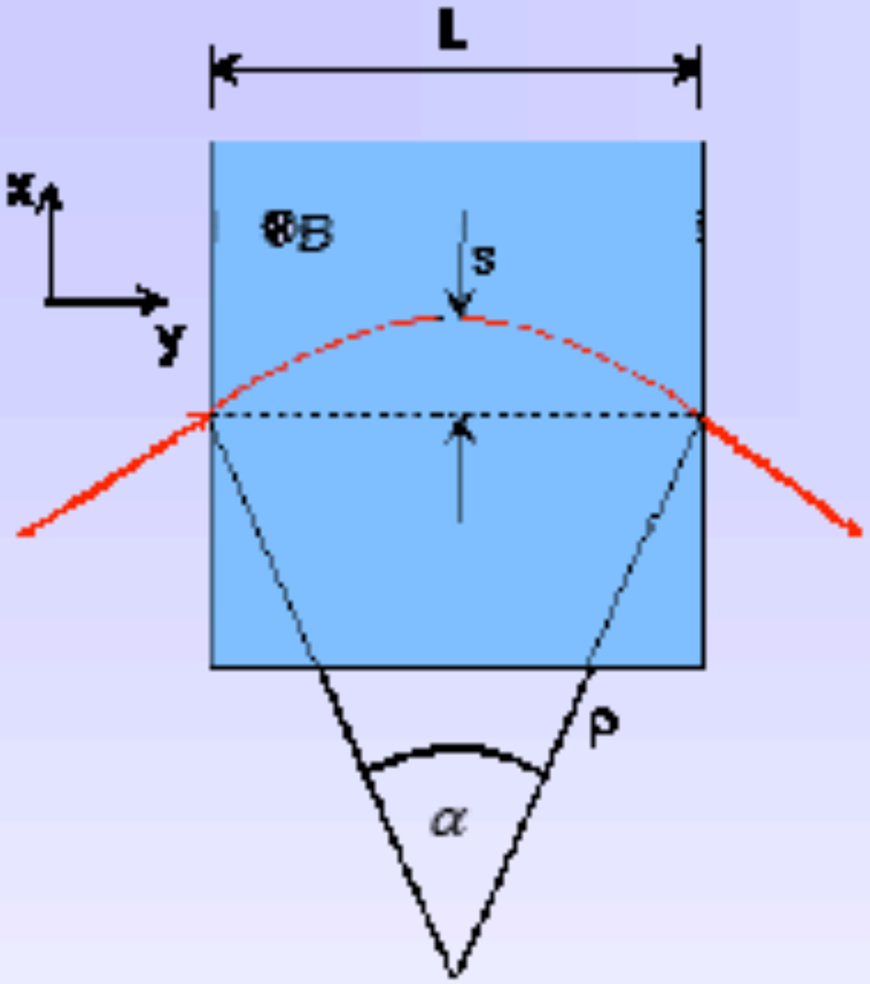
- Need to measure:
  - *Particle momenta: bending in B-Field,*
  - *or Particle energy: absorb and measure response to estimate energy.*
- We measure the charged particles via EM interaction.
- We measure "stable" particles – travel through the detector. $\gamma c\tau > metres \quad e, \mu, \pi, K, p$

  - *Some short-lived particles also seen.*

$$\tau, B, D \text{ with } \gamma c\tau > \; from(\sim 0.1 - few)\, mm$$


Muon
Spectrometer
Muon
Neutrino
Hadronic
Calorimeter
Proton
Neutron
The dashed tracks
are invisible to
the detector
Electromagnetic
Calorimeter
Electron
Photon
Solenoid magnet
Transition
Radiation
Tracker
Tracking
Pixel/SCT
detector
http://atlas.ch

Momentum measurement



We measure only p-component transverse to B field !

$$p_T = qB\rho \quad \rightarrow \quad p_T\ (\mathrm{GeV/c}) = 0.3B\rho \quad (\mathrm{T \cdot m})$$

$$\frac{L}{2\rho} = \sin\alpha/2 \approx \alpha/2 \quad \rightarrow \quad \alpha \approx \frac{0.3L \cdot B}{p_T}$$

$$s = \rho(1-\cos\alpha/2) \approx \rho\frac{\alpha^2}{8} \approx \frac{0.3}{8}\frac{L^2 B}{p_T}$$

the sagitta s is determined by 3 measurements with error *s(x)*:

$$s = x_2 - \frac{x_1 + x_3}{2} \qquad \left.\frac{\sigma(p_T)}{p_T}\right|^{meas.} = \frac{\sigma(s)}{s} = \frac{\sqrt{\frac{3}{2}}\sigma(x)}{s} = \frac{\sqrt{\frac{3}{2}}\sigma(x) \cdot 8p_T}{0.3 \cdot BL^2} \qquad \boxed{\left.\frac{\sigma(p_T)}{p_T}\right|^{meas.} \propto \frac{\sigma(x) \cdot p_T}{BL^2}}$$

for N equidistant measurements, one obtains (R.L. Gluckstern, NIM 24 (1963) 381)

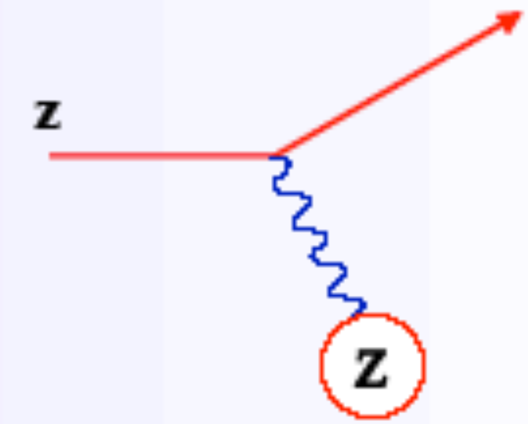
$$\left.\frac{\sigma(p_T)}{p_T}\right|^{meas.} = \frac{\sigma(x) \cdot p_T}{0.3 \cdot BL^2}\sqrt{720/(N+4)} \qquad (\text{for } N \geq \sim 10)$$

- Charged particles can:
  - *Ionize Atoms*
  - *Excite Atoms (and de-excite via fluorescence, scintillation)*
  - *Generate Cerenkov radiation (and transition radiation)*
  - *Bremsstrahlung gamma rays*
- Gamma Rays can:
  - *Generate photo-electrons (X-rays)*
  - *Compton Scatter*
  - *Pair Produce*
- Hadrons can:
  - *Interact with nucleus generating hadrons (charged and neutral)*

Interaction of charged particles

Scattering

An incoming particle with charge z interacts elastically with a target of nuclear charge Z.
The cross-section for this e.m. process is



$$\frac{d\sigma}{d\Omega} = 4zZr_e^2\left(\frac{m_e c}{\beta p}\right)^2 \frac{1}{\sin^4 \theta/2}$$ Rutherford formula

- Approximation
  - Non-relativistic
  - No spins

- Average scattering angle $\langle\theta\rangle = 0$
- Cross-section for $\theta \to 0$ infinite !

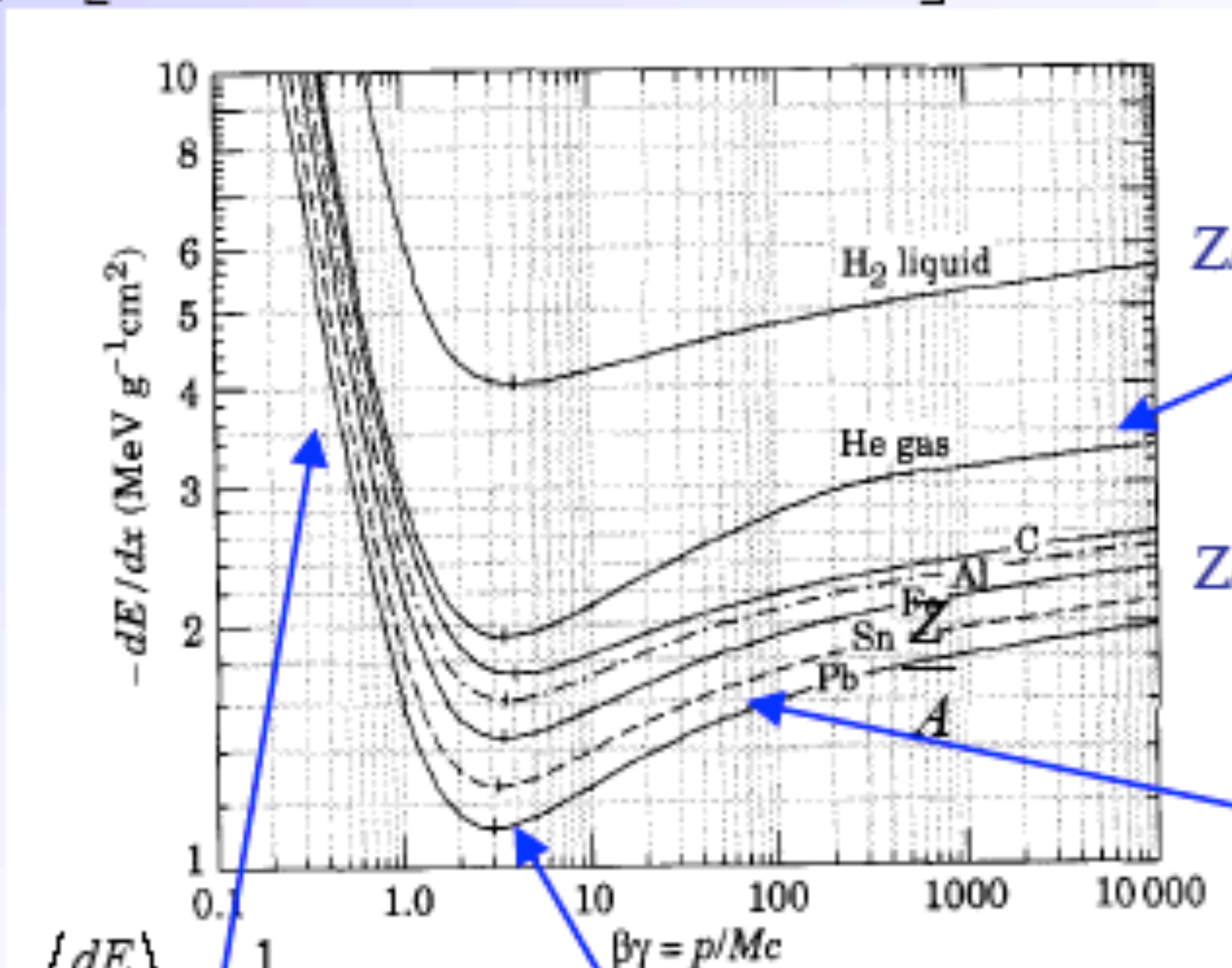
- Scattering does not lead to significant energy loss

Interaction of charged particles

Energy loss by Ionisation only → Bethe - Bloch formula

$$\left\langle \frac{dE}{dx} \right\rangle = -4\pi N_A r_e^2 m_e c^2 z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \ln \frac{2m_e c^2 \gamma^2 \beta^2}{I^2} T^{\max} - \beta^2 - \frac{\delta}{2} \right]$$

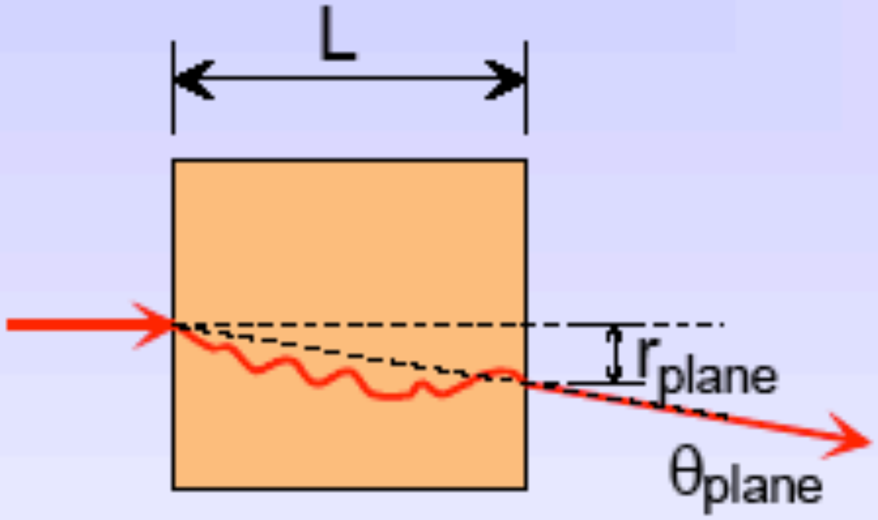
- dE/dx in [MeV g^{-1} cm^2]
- valid for "heavy" particles ($m \geq m_\mu$).
- dE/dx depends only on β, independent of m !
- First approximation: medium simply characterized by Z/A ~ electron density

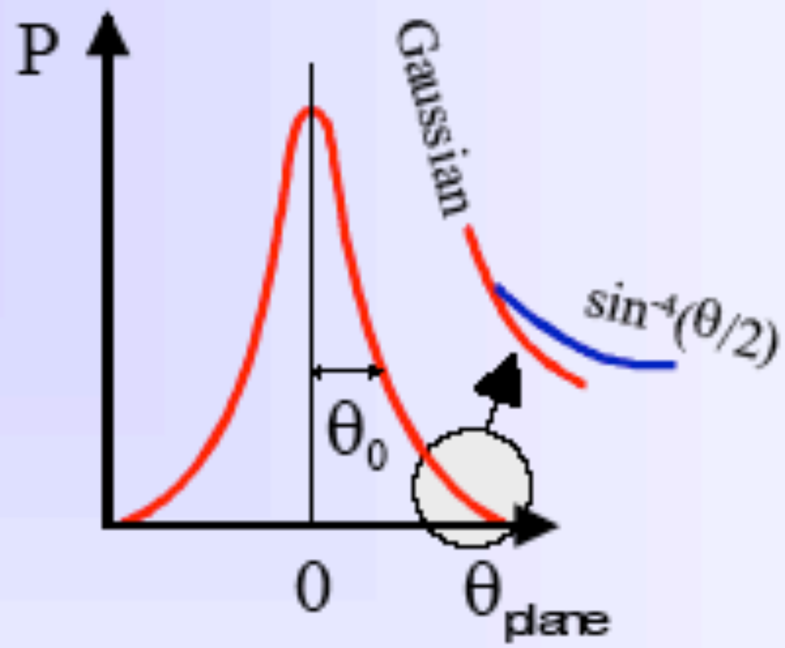


Interaction of charged particles

In a sufficiently thick material layer a particle will undergo …

- **Multiple Scattering**

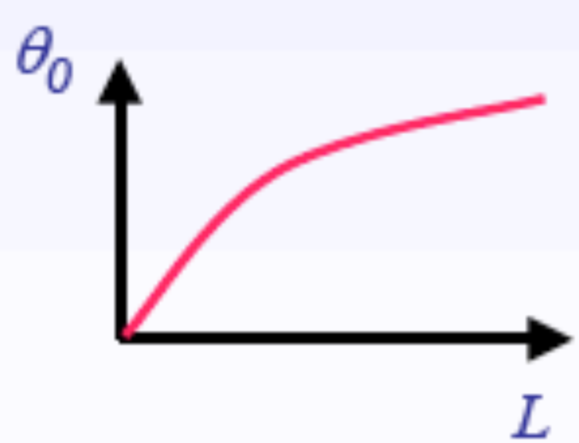




$$\theta_0 = \theta_{plane}^{RMS} = \sqrt{\langle \theta_{plane}{}^2 \rangle}$$

$$= \frac{1}{\sqrt{2}} \theta_{space}^{RMS}$$

Approximation $\theta_0 \propto \frac{1}{p}\sqrt{\frac{L}{X_0}}$ X_0 is radiation length of the medium (discuss later)



Interaction of charged particles

Back to momentum measurements:

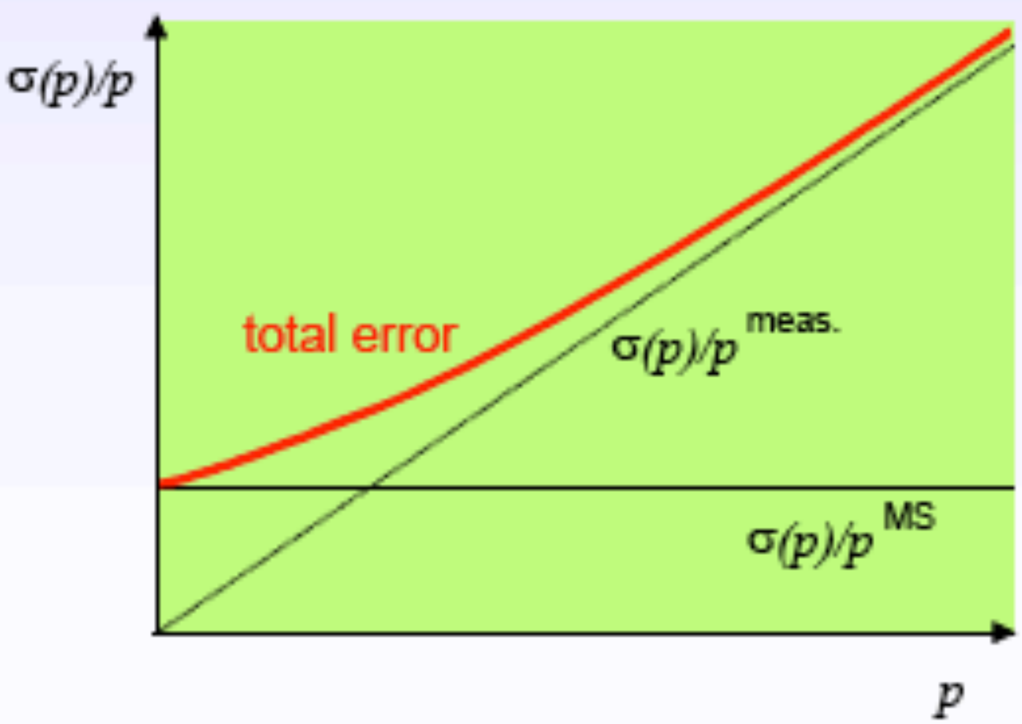
What is the contribution of multiple scattering to $\dfrac{\sigma(p)}{p_T}$?

remember $\dfrac{\sigma(p)}{p_T} \propto \sigma(x) \cdot p_T$

$\sigma(x)\Big|^{MS} \propto \theta_0 \propto \dfrac{1}{p}$

$$\left.\frac{\sigma(p)}{p_T}\right|^{MS} = \text{constant}$$, i.e. independent of p !

More precisely: $$\left.\frac{\sigma(p)}{p_T}\right|^{MS} = 0.045 \frac{1}{B\sqrt{LX_0}}$$



Example:

$p_t = 1$ GeV/c, $L = 1$m, $B = 1$ T, $N = 10$

$\sigma(x) = 200$ µm: $$\left.\frac{\sigma(p_T)}{p_T}\right|^{meas.} \approx 0.5\%$$

Assume detector (L = 1m) to be filled with 1 atm. Argon gas (X_0 = 110m),

$$\left.\frac{\sigma(p)}{p_T}\right|^{MS} \approx 0.5\%$$

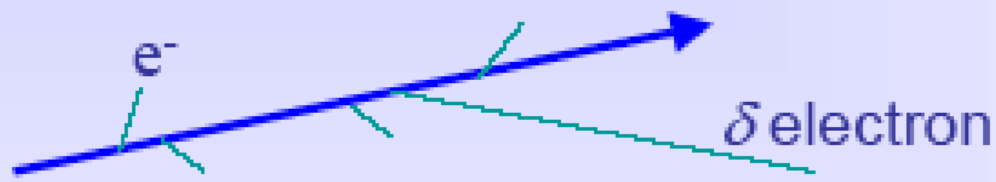
Interaction of charged particles

Real detector (limited granularity) can not measure <*dE/dx*> !

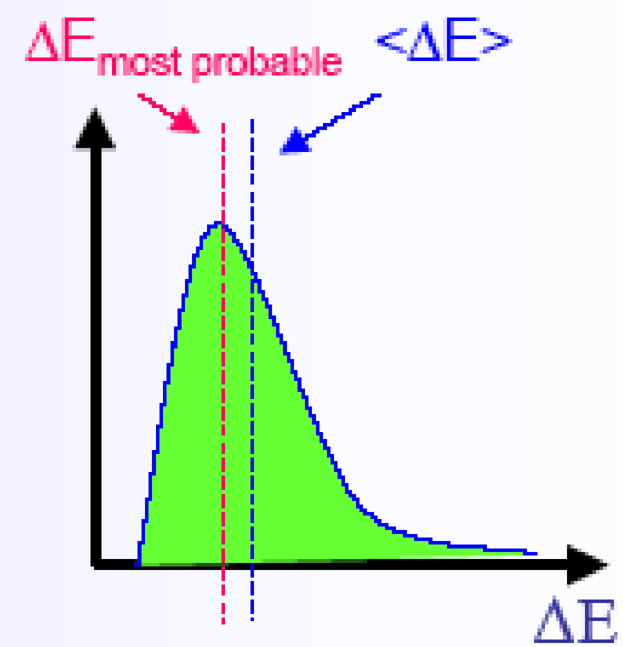
It measures the energy ΔE deposited in a layer of finite thickness δx.

For thin layers or low density materials:

→ Few collisions, some with high energy transfer.



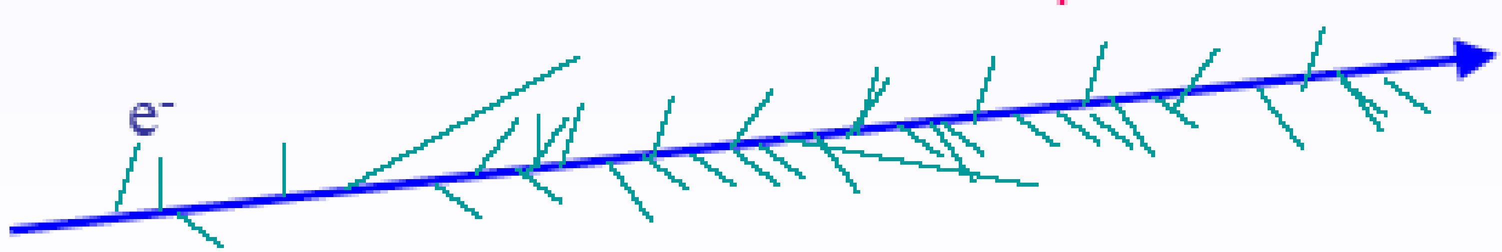
→ Energy loss distributions show large fluctuations towards high losses: "Landau tails"

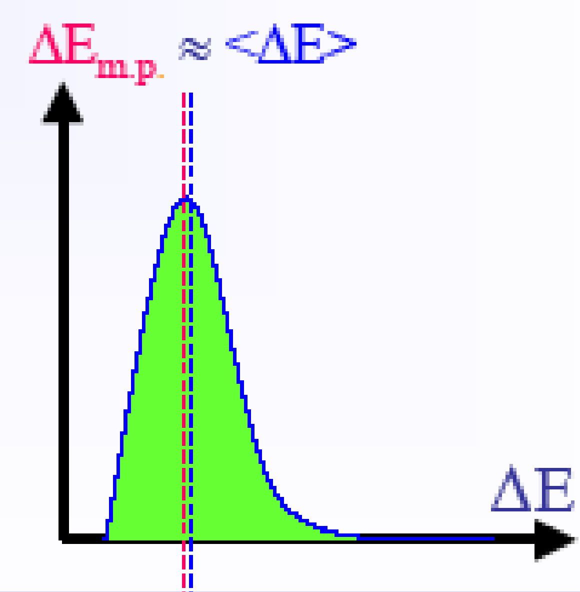


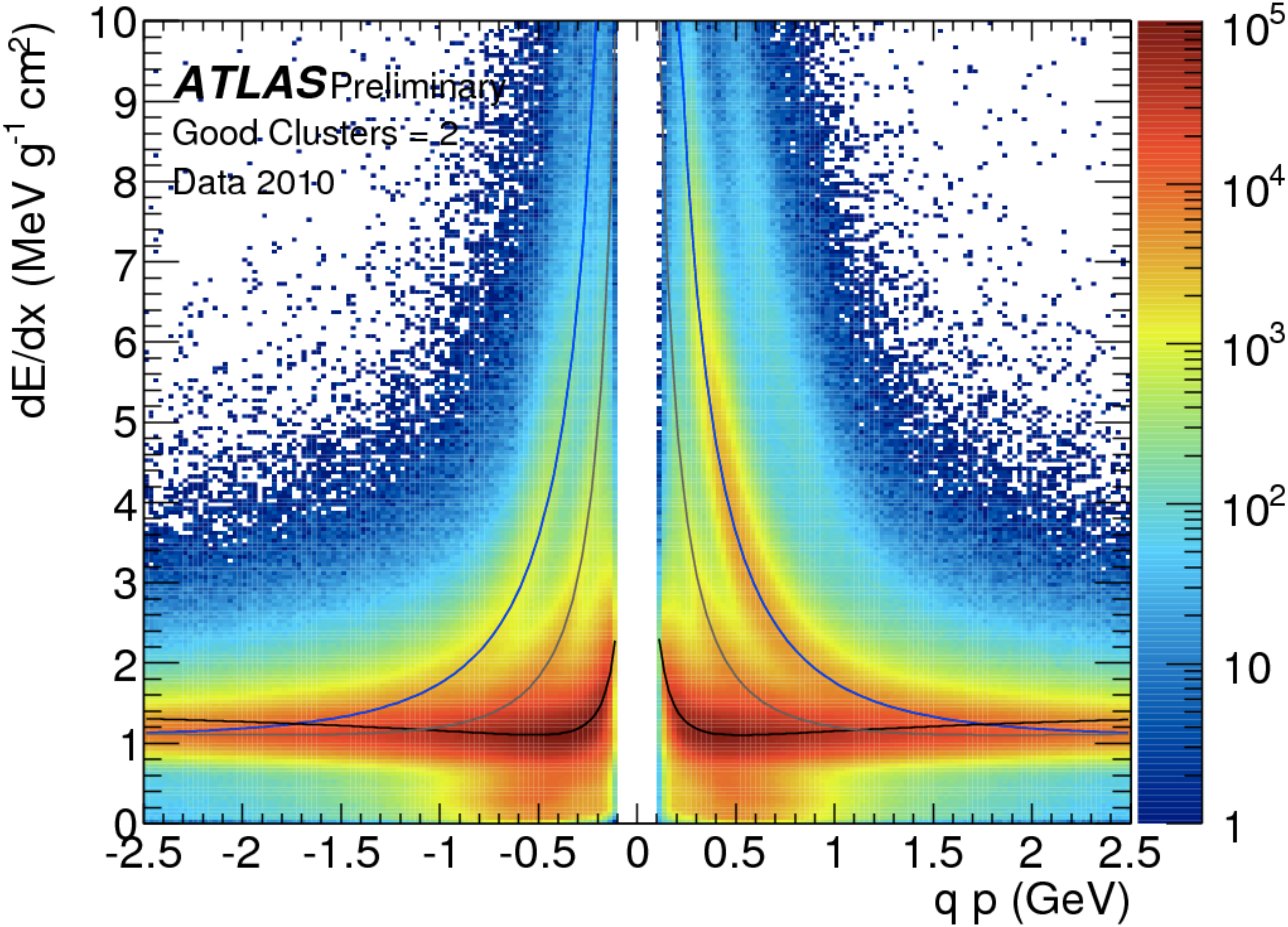
Example: Si sensor: 300 µm thick. $\Delta E_{m.p} \sim 82$ keV $\langle \Delta E \rangle \sim 115$ keV

For thick layers and high density materials:

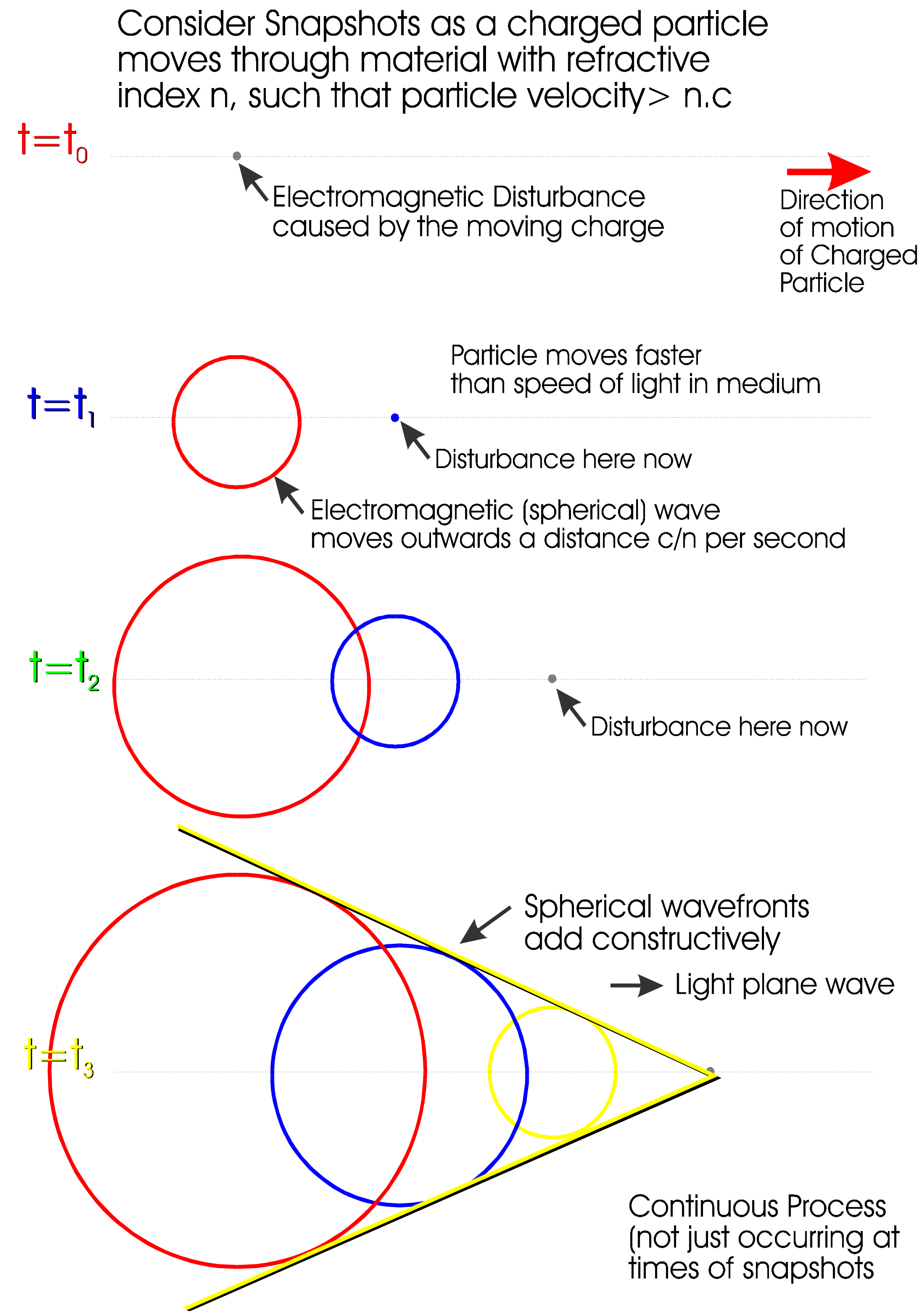
→ Many collisions.
→ Central Limit Theorem → Gaussian shaped distributions.






ATLAS Preliminary
Good Clusters = 2
Data 2010
dE/dx (MeV g^{-1} cm^2)
q p (GeV)
0
1
2
3
4
5
6
7
8
9
10
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
2.5
1
10
10^2
10^3
10^4
10^5

Cerenkov Light



Transition Radiation: Similar physics but at boundary of materials

Photo-electric Effect

In order to be detected, a photon has to create charged particles and / or transfer energy to charged particles

- **Photo-electric effect:** (already met in photocathodes of photodetectors)

$$\gamma + \text{atom} \rightarrow \text{atom}^{+} + e^{-}$$

Only possible in the close neighborhood of a third collision partner → photo effect releases mainly electrons from the K-shell.

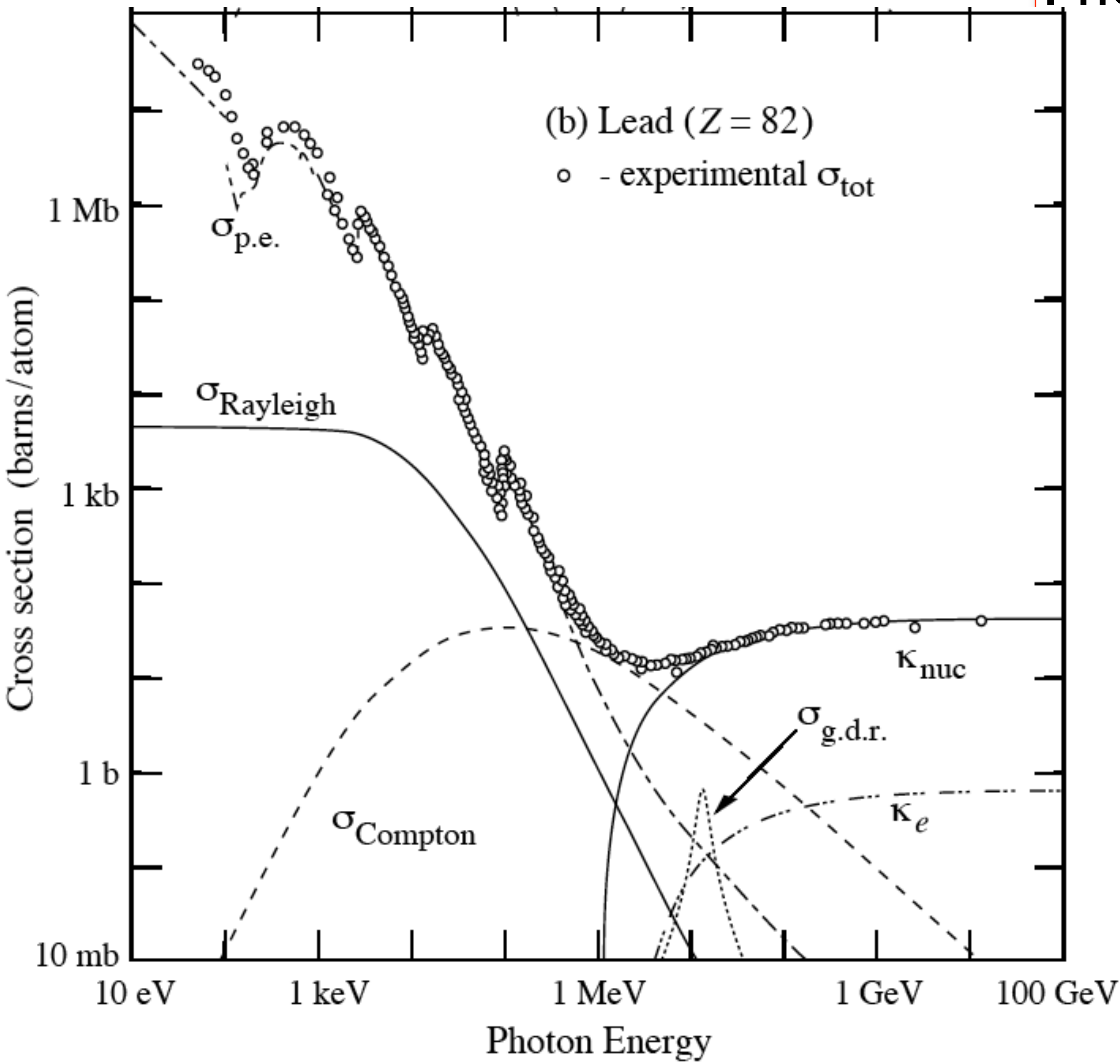
Cross section shows strong modulation if $E_\gamma \approx E_{shell}$

$$\sigma^{K}_{photo} = \left(\frac{32}{\varepsilon^7}\right)^{\frac{1}{2}} \alpha^4 Z^5 \sigma^{e}_{Th} \qquad \varepsilon = \frac{E_\gamma}{m_e c^2} \qquad \sigma^{e}_{Th} = \tfrac{8}{3}\pi r_e^2 \quad \text{(Thomson)}$$

At high energies (ε>>1)

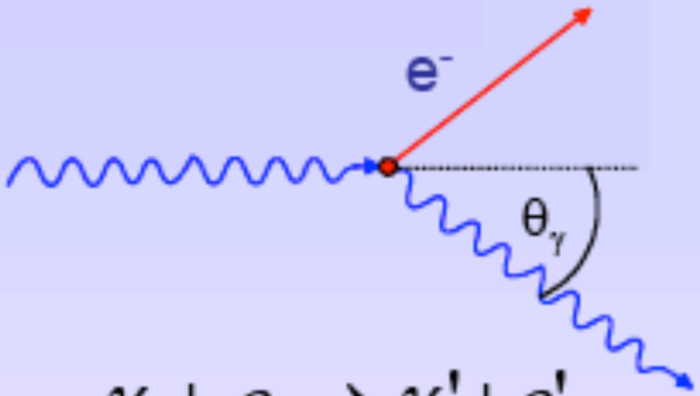
$$\sigma^{K}_{photo} = 4\pi r_e^2 \alpha^4 Z^5 \frac{1}{\varepsilon} \qquad \boxed{\sigma_{photo} \propto Z^5}$$

Photo-Electric Effect



Compton Scattering

Compton scattering:

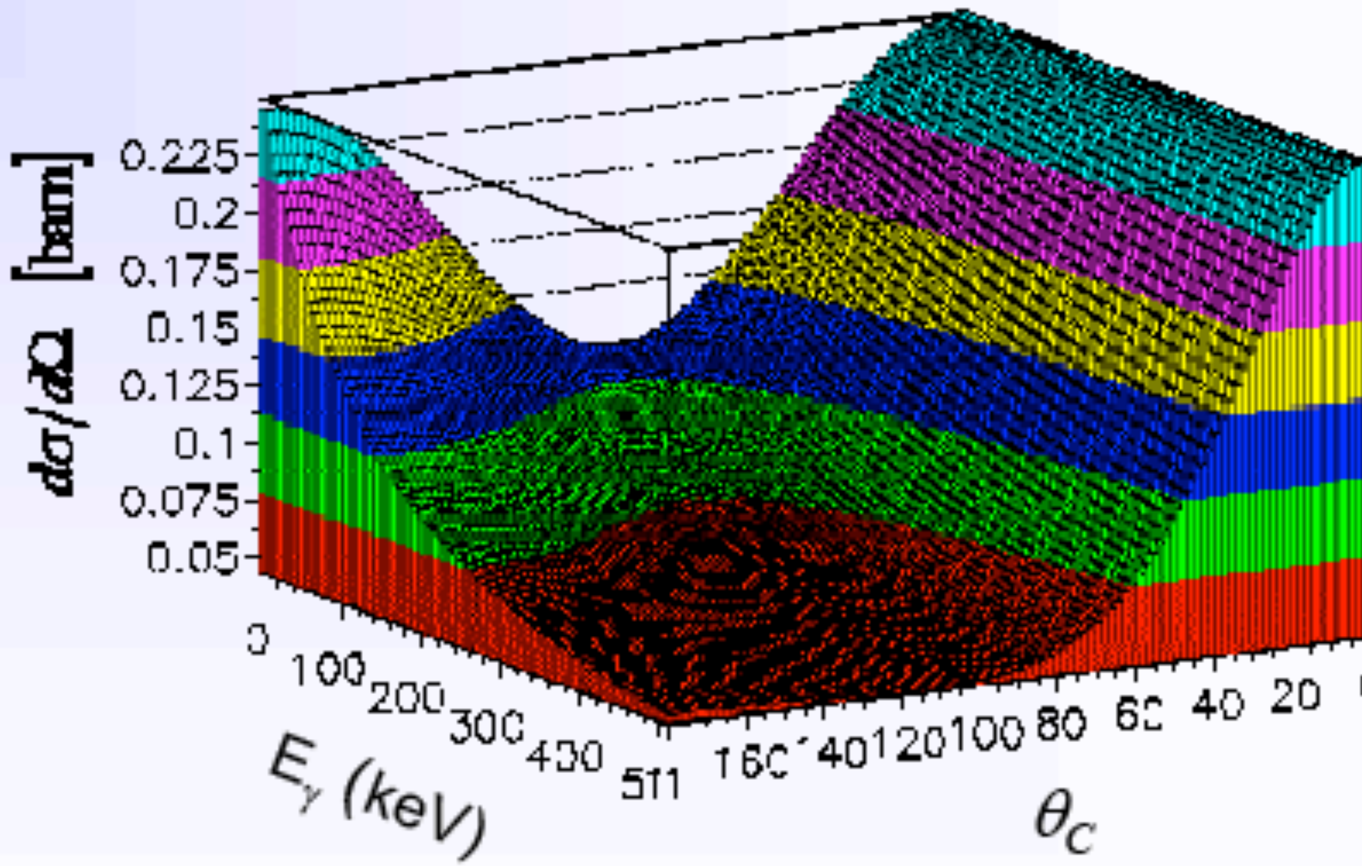


$$\gamma + e \rightarrow \gamma' + e'$$

$$E'_\gamma = E_\gamma \frac{1}{1+\varepsilon(1-\cos\theta_\gamma)}$$

$$E_e = E_\gamma - E'_\gamma$$

Compton cross-section (Klein-Nishina)



Assume electron as quasi-free.

Klein-Nishina $\frac{d\sigma}{d\Omega}(\theta,\varepsilon)$ →

At high energies approximately

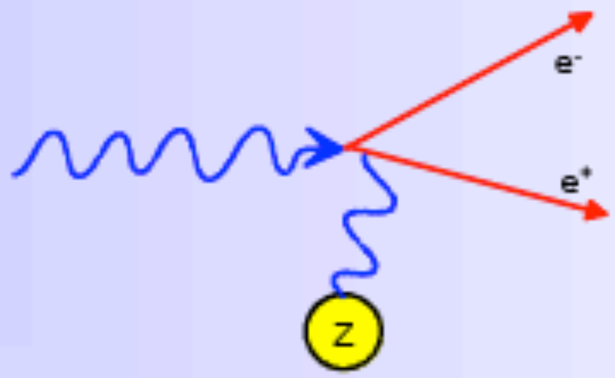
$$\sigma_c^e \propto \frac{\ln\varepsilon}{\varepsilon}$$

Atomic Compton cross-section:

$$\sigma_c^{atomic} = Z \cdot \sigma_c^e$$

Pair Production

Pair production



$$\gamma + nucleus \rightarrow e^+e^- + nucleus$$

Only possible in the Coulomb field of a nucleus (or an electron) if $E_\gamma \geq 2m_e c^2$

Cross-section (high energy approximation)

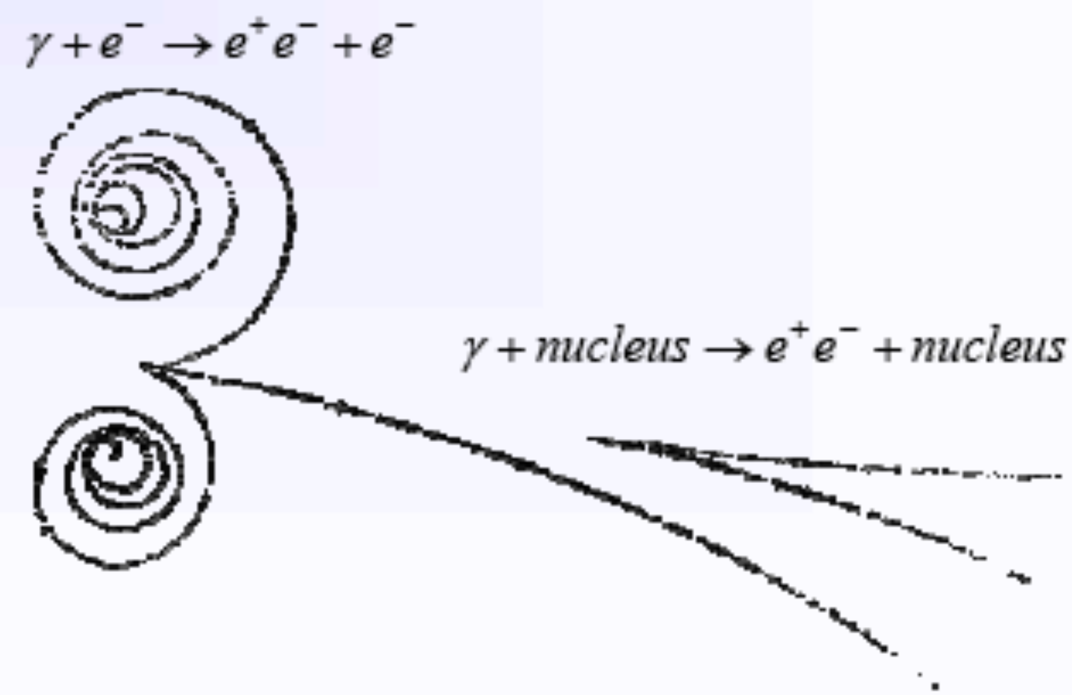
$$\sigma_{pair} \approx 4\alpha r_e^2 Z^2 \left(\frac{7}{9} \ln \frac{183}{Z^{\frac{1}{3}}} \right)$$ independent of energy !

$$\approx \frac{7}{9} \frac{A}{N_A} \frac{1}{X_0}$$

$$\approx \frac{A}{N_A} \frac{1}{\lambda_{pair}}$$

$$\lambda_{pair} = \frac{9}{7} X_0$$

Energy sharing between e⁺ and e⁻ becomes asymmetric at high energies.

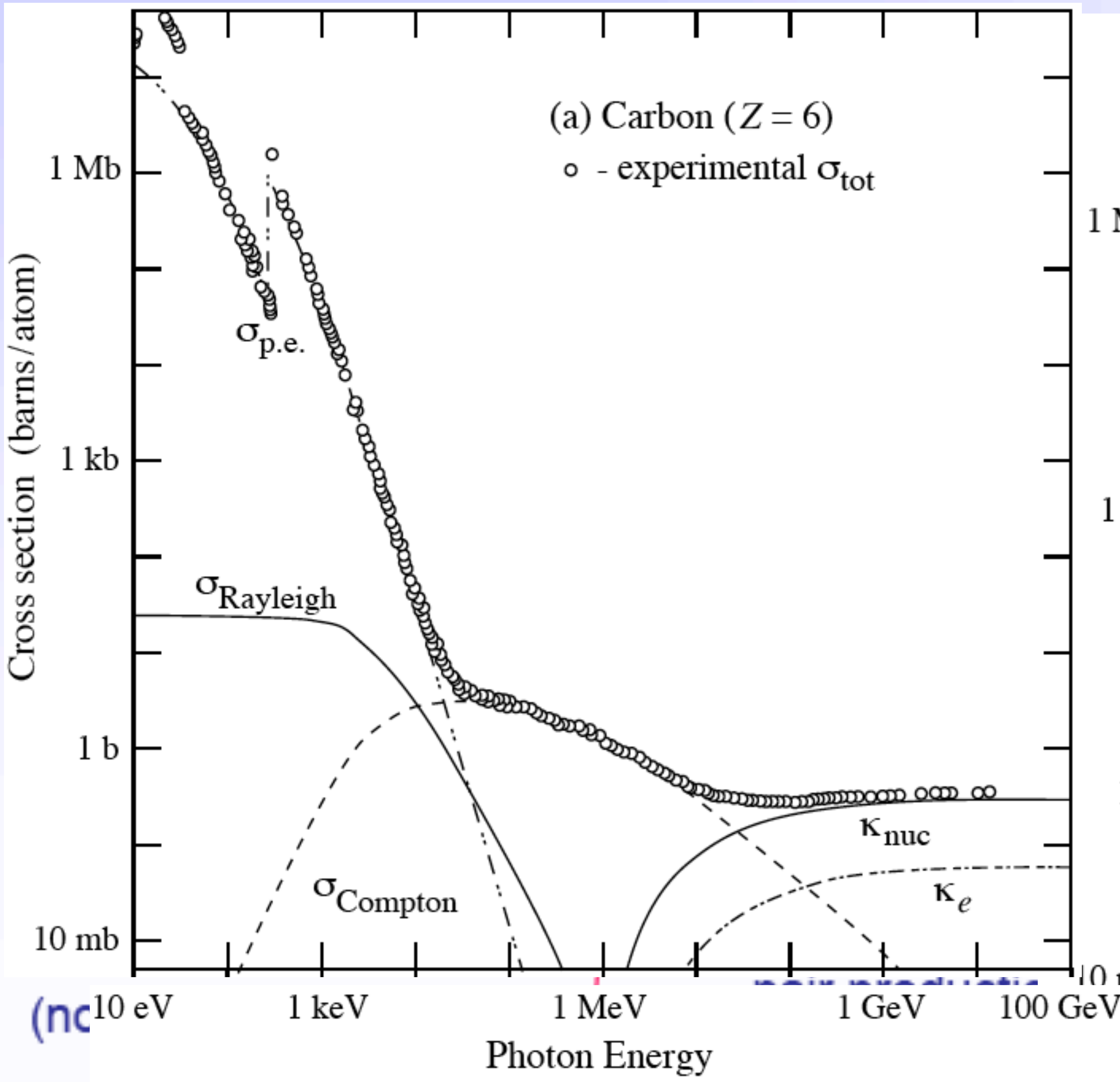


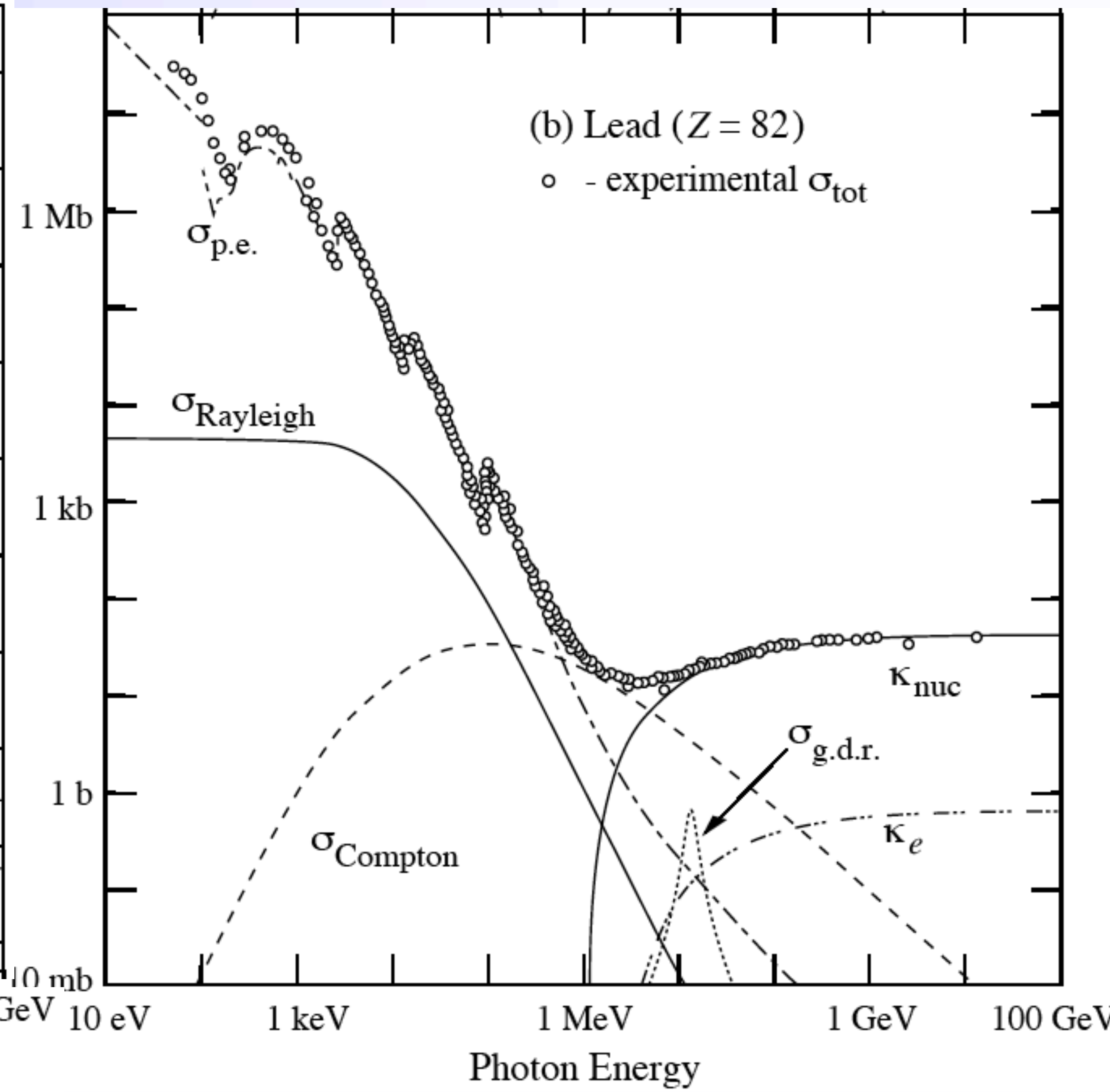
Interaction of photons

Photon Attenuation

In summary: $I_\gamma = I_0 e^{-\mu x}$

μ: mass attenuation coefficient $\mu_i = \frac{N_A}{A}\sigma_i \quad \left[cm^2/g\right]$ $\quad \mu = \mu_{photo} + \mu_{Compton} + \mu_{pair} + \ldots$

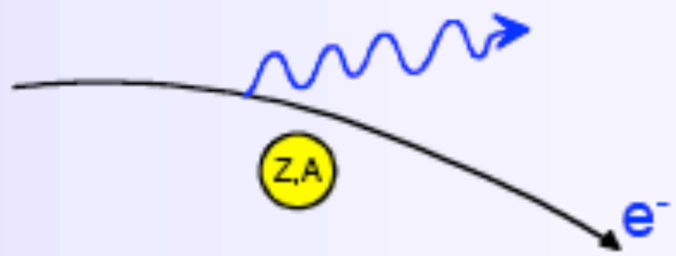




EM Charged Particles

Energy loss by Bremsstrahlung

Radiation of real photons in the Coulomb field of the nuclei of the absorber medium



$$-\frac{dE}{dx} = 4\alpha N_A \frac{Z^2}{A} z^2 \left(\frac{1}{4\pi\varepsilon_0}\frac{e^2}{mc^2}\right)^2 E \ln\frac{183}{Z^{1/3}} \propto \frac{E}{m^2}$$

Effect plays a role only for $e^{\pm}$ and ultra-relativistic μ (>1000 GeV)

For electrons:
$$-\frac{dE}{dx} = 4\alpha N_A \frac{Z^2}{A} r_e^2 E \ln\frac{183}{Z^{1/3}}$$

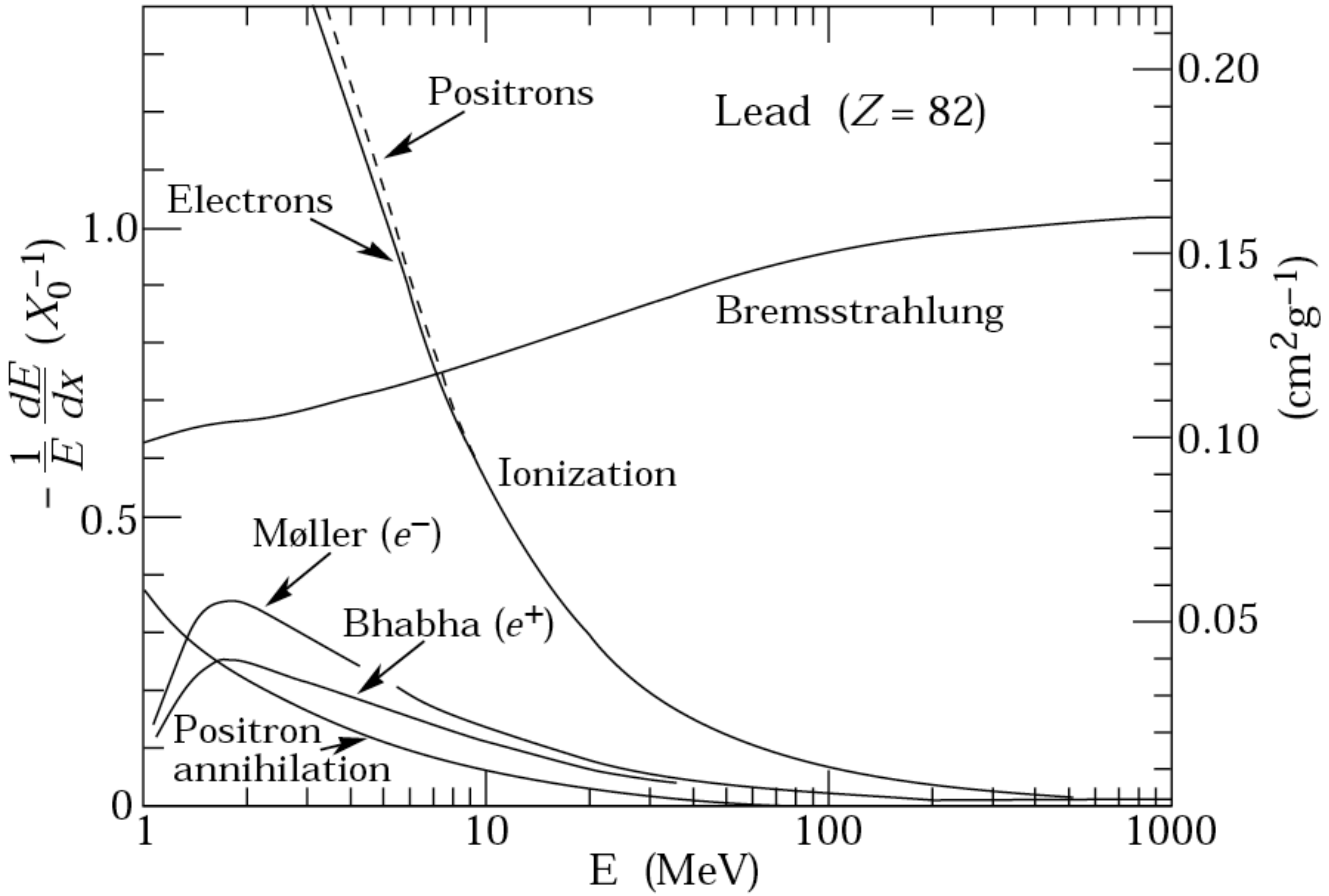
$$\boxed{-\frac{dE}{dx} = \frac{E}{X_0}} \longrightarrow E = E_0 e^{-x/X_0}$$

$$X_0 = \frac{A}{4\alpha N_A Z^2 r_e^2 \ln\frac{183}{Z^{1/3}}}$$

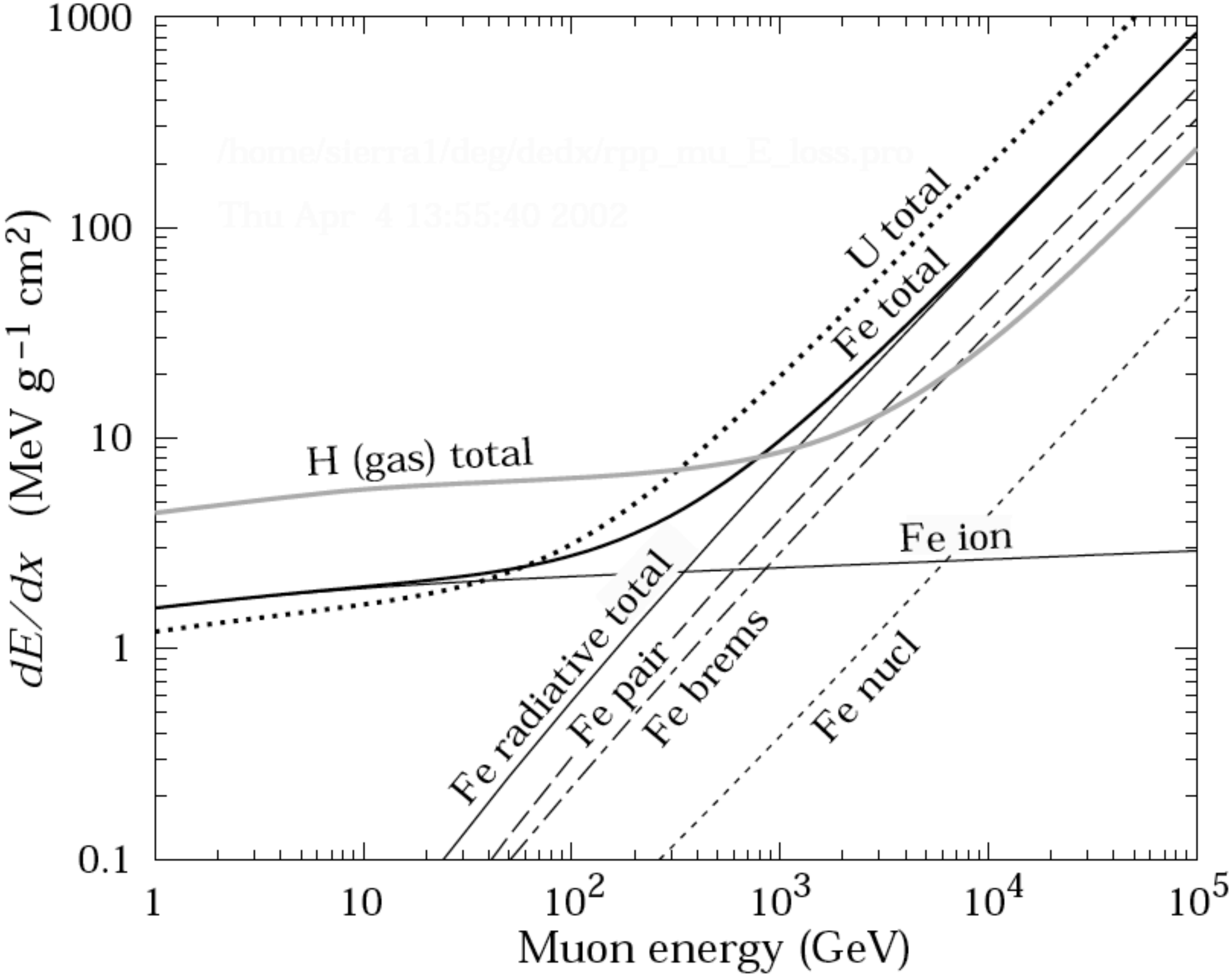
radiation length [g/cm²]

(divide by specific density to get X_0 in cm)

Electron Energy Loss



Compared with muons

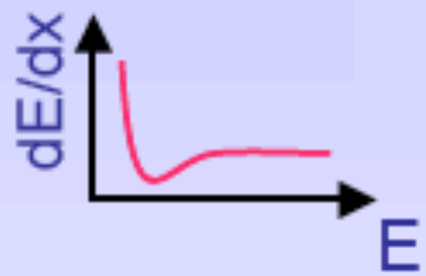


Photon Interactions

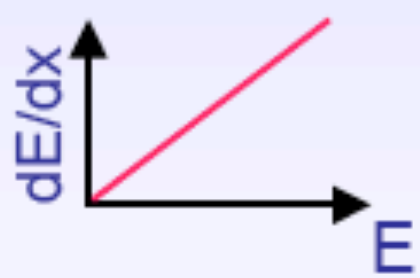
Reminder: basic electromagnetic int

e+ / e-

- Ionisation



- Bremsstrahlung



γ

- Photoelectric effect

- Compton effect

- Pair production

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Calorimetry

Calorimtery

- Basic mechanism for calorimetry in paricle physics: formation of
  - ⇨ electromagnetic
  - ⇨ or hadronic showers.

- Finally, the energy is converted into ionization or excitation of the matter.

Charge

Scintillation light

Cerenkov light

- Calorimetry is a "destructive" method. The energy and the particle get absorbed!

- Detector response $\propto$ E

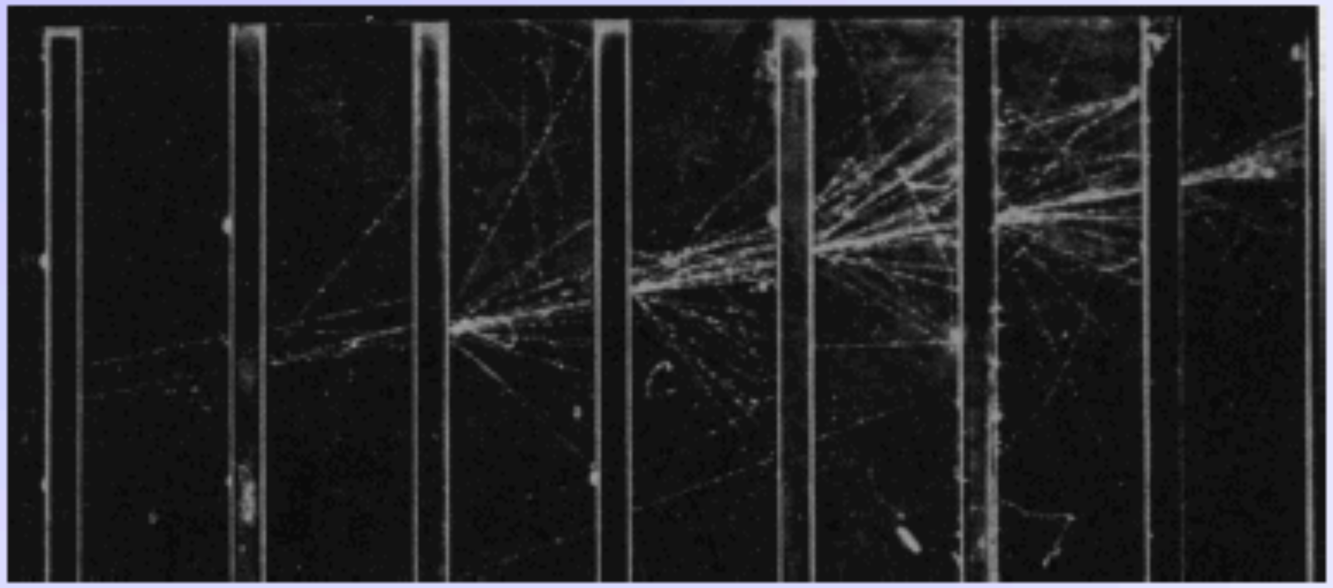
- Calorimetry works both for
  - ⇨ charged ($e^{\pm}$ and hadrons)
  - ⇨ and neutral particles (n,γ)

Complementary information to p-measurement

Only way to get direct kinematical information for neutral particles

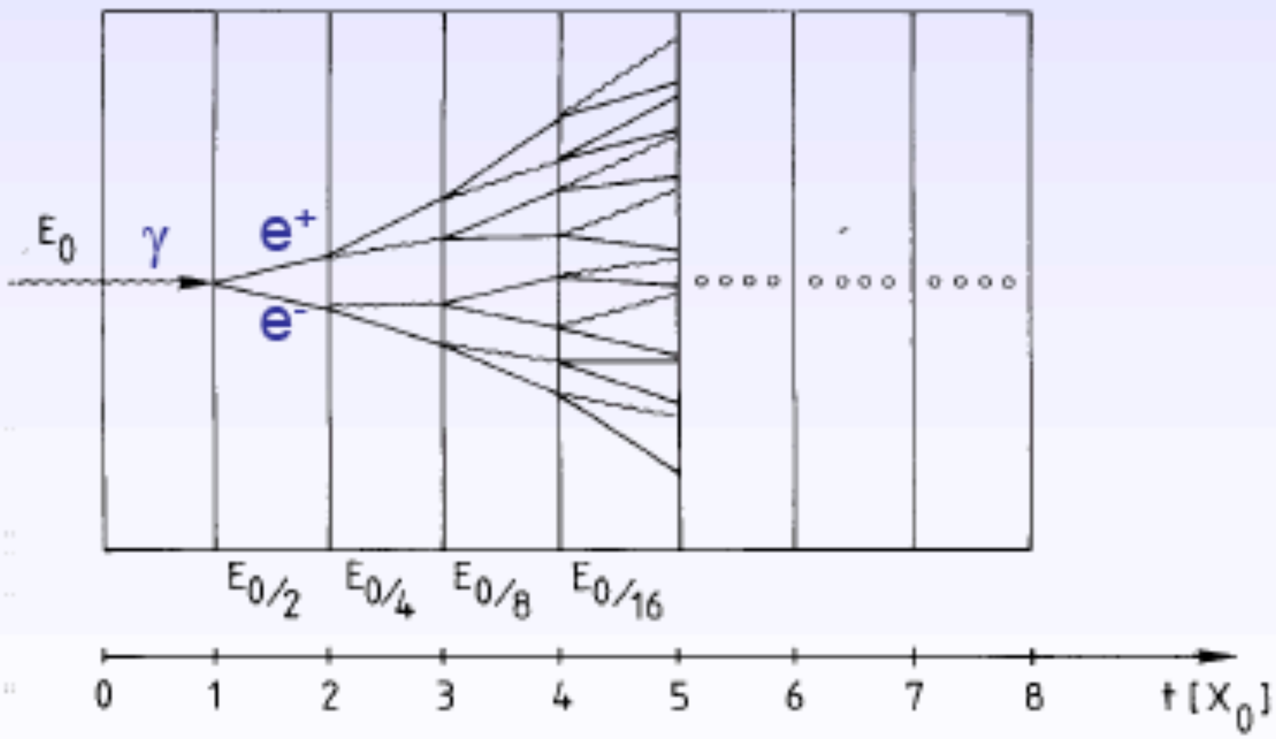
Electromagnetic cascades (showers

Electromagnetic Showers



Electron shower in a cloud chamber with lead absorbers

Simple qualitative model



- Consider only Bremsstrahlung and (symmetric) pair production.
- Assume: $X_0 \sim \lambda_{pair}$

$$N(t) = 2^t \qquad E(t)/\,particle = E_0 \cdot 2^{-t}$$

Process continues until $E(t)<E_c$

$$N^{total} = \sum_{t=0}^{t_{max}} 2^t = 2^{(t_{max}+1)} - 1 \approx 2 \cdot 2^{t_{max}} = 2\frac{E_0}{E_c}$$

$$t_{max} = \frac{\ln E_0/E_c}{\ln 2}$$

After $t = t_{max}$ the dominating processes are ionization, Compton effect and photo effect → absorption of energy.

Critical Energy

Critical energy E_c

$$\frac{dE}{dx}(E_c)\bigg|_{Brems} = \frac{dE}{dx}(E_c)\bigg|_{ion}$$

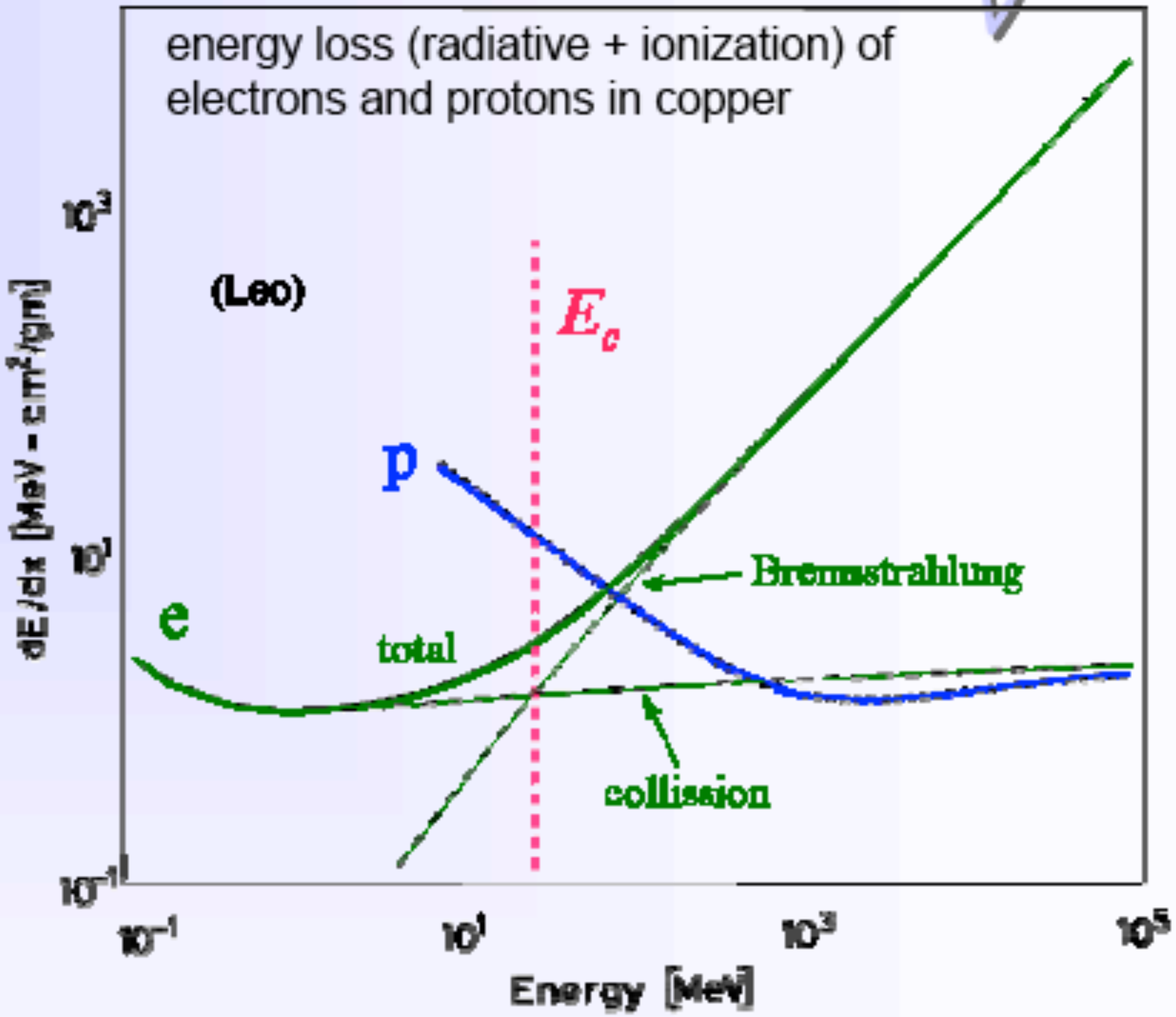
For electrons one finds approximately:

$$E_c^{solid+liq} = \frac{610 MeV}{Z+1.24} \qquad E_c^{gas} = \frac{710 MeV}{Z+1.24}$$

$E_c(e^-)$ in Cu(Z=29) = 20 MeV

For muons $E_c \approx E_c^{elec}\left(\frac{m_\mu}{m_e}\right)^2$

$E_c(\mu)$ in Cu $\approx$ 1 TeV

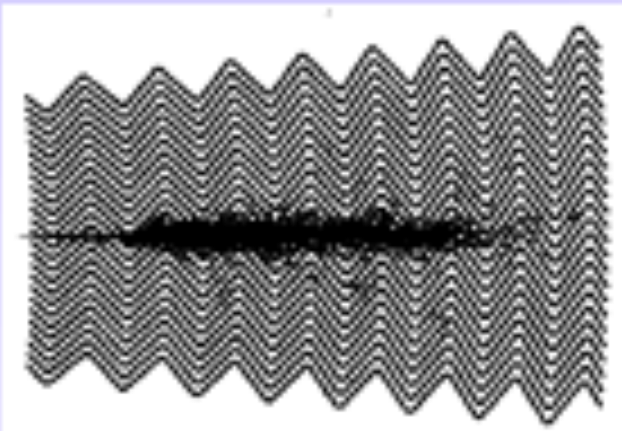


Unlike electrons, muons in multi-GeV range can traverse thick layers of dense matter.
Find charged particles traversing the calorimeter ? → most likely a muon → Particle ID

Example ECAL - sampling

ATLAS electromagnetic Calorimeter

Accordion geometry absorbers immersed in Liquid Argon



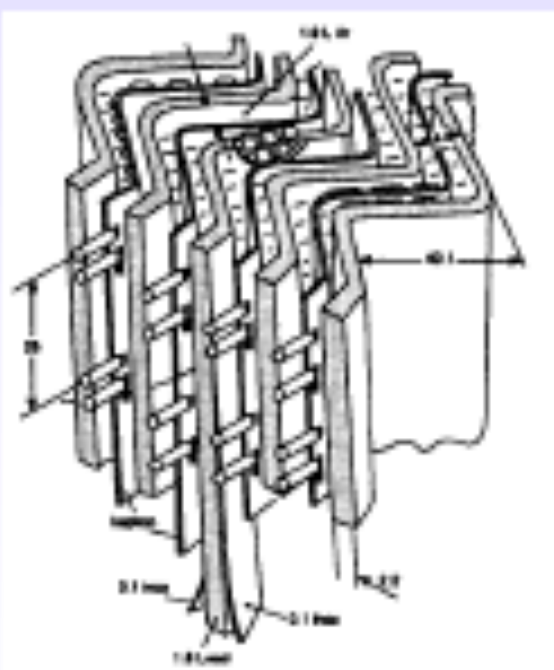
Liquid Argon (90K)

+ lead-steal absorbers (1-2 mm)

+ multilayer copper-polyimide readout boards

→ Ionization chamber.

1 GeV E-deposit → 5 x10^6 e^-



- Accordion geometry minimizes dead zones.
- Liquid Ar is intrinsically radiation hard.
- Readout board allows fine segmentation (azimuth, pseudo-rapidity and longitudinal) acc. to physics needs

Test beam results $\sigma(E)/E = 9.24\%/\sqrt{E} \oplus 0.23\%$

Spatial resolution ≈ 5 mm / $\sqrt{E}$

Electromagnetic Showers (II)

Electromagnetic cascades

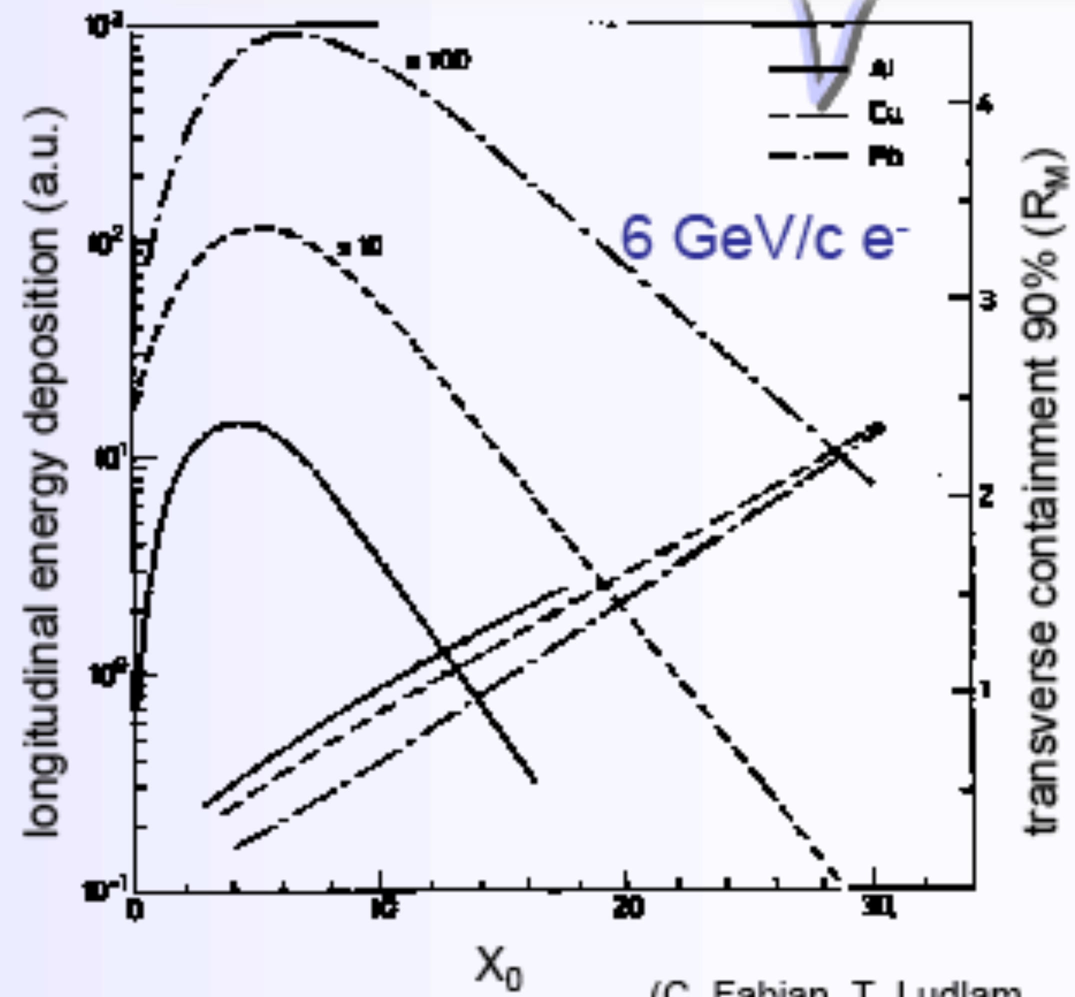
- **Longitudinal shower development**

$$\frac{dE}{dt} \propto t^{\alpha} e^{-t}$$

Shower maximum at $t_{\max} = \ln \frac{E_0}{E_c} \frac{1}{\ln 2}$

95% containment $t_{95\%} \approx t_{\max} + 0.08Z + 9.6$

Size of a calorimeter grows only logarithmically with E_0



(C. Fabjan, T. Ludlam, CERN-EP/82-37)

- Transverse shower development

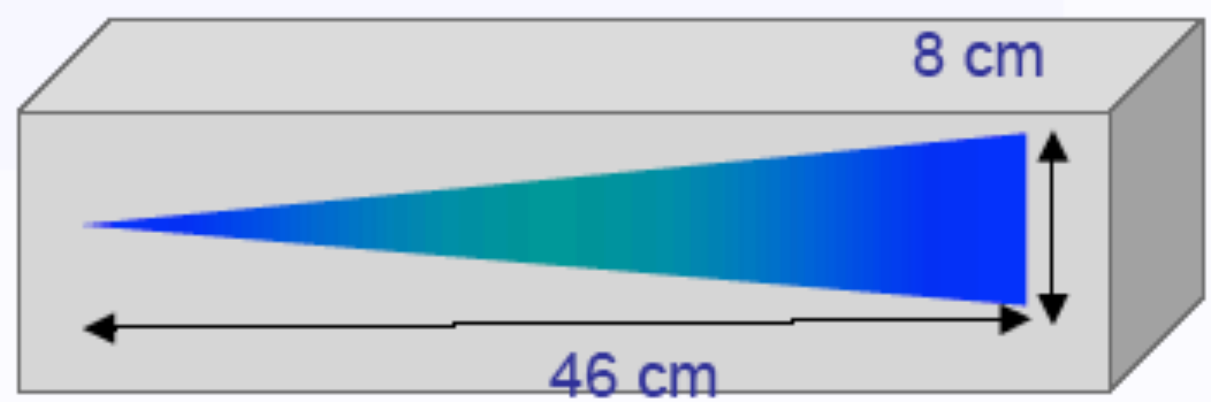
95% of the shower cone is located in a cylinder with radius 2 R_M

Molière radius $R_M = \frac{21\,\text{MeV}}{E_c} X_0 \quad [g/cm^2]$

Example: E_0 = 100 GeV in lead glass

E_c=11.8 MeV $\rightarrow t_{max} \approx 13$, $t_{95\%} \approx 23$

$X_0 \approx 2$ cm, $R_M = 1.8 \cdot X_0 \approx 3.6$ cm



Energy resolution of a calorimeter

EM Calorimeter Energy Resolution

$N^{total} \propto \frac{E_0}{E_c}$ total number of track segments

$T \propto \frac{E_0}{E_c} X_0$ total track length

$T_{det} = F(\xi) T \qquad \xi \propto \frac{E_{cut}}{E_c}$ detectable track length (above energy E_{cut})

$\frac{\sigma(E)}{E} \propto \frac{\sigma(T_{det})}{T_{det}} \propto \frac{1}{\sqrt{T_{det}}} \propto \frac{1}{\sqrt{E_0}}$ holds also for hadron calorimeters

More general: $$\frac{\sigma(E)}{E} = \frac{a}{\sqrt{E}} \oplus b \oplus \frac{c}{E}$$

Also spatial and angular resolution scale like $1/\sqrt{E}$

stochastic term (see above)

'constant term'

- inhomogenities
- bad cell inter-calibration
- non-linearities

Quality factor !

'noise term'

- Electronic noise
- radioactivity
- pile up

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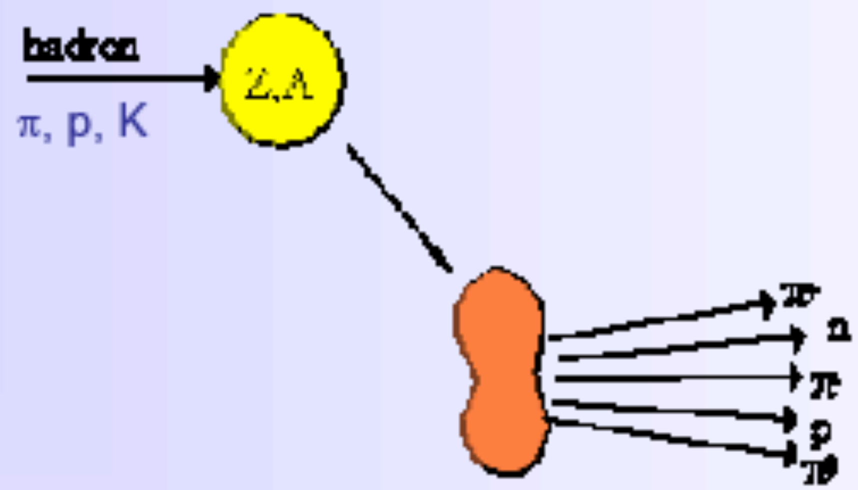
Nuclear Interactions

The interaction of energetic hadrons (charged or neutral) with matter is determined by inelastic nuclear processes.



Excitation and finally break-up of nucleus → nucleus fragments + production of secondary particles.

multiplicity $\propto \ln(E)$

$p_t \approx 0.35$ GeV/c

For high energies (>1 GeV) the cross-sections depend only little on the energy and on the type of the incident particle (π, p, K…).

$$\sigma_{inel} \approx \sigma_0 A^{0.7} \quad \sigma_0 \approx 35\, mb$$

In analogy to X_0 a hadronic absorption length can be defined

$$\lambda_a = \frac{A}{N_A \sigma_{inel}} \propto A^{\frac{1}{4}} \quad \text{because} \quad \sigma_{inel} \approx \sigma_0 A^{0.7}$$

similarly a hadronic interaction length

$$\lambda_I = \frac{A}{N_A \sigma_{total}} \propto A^{\frac{1}{3}} \qquad \lambda_I < \lambda_a$$

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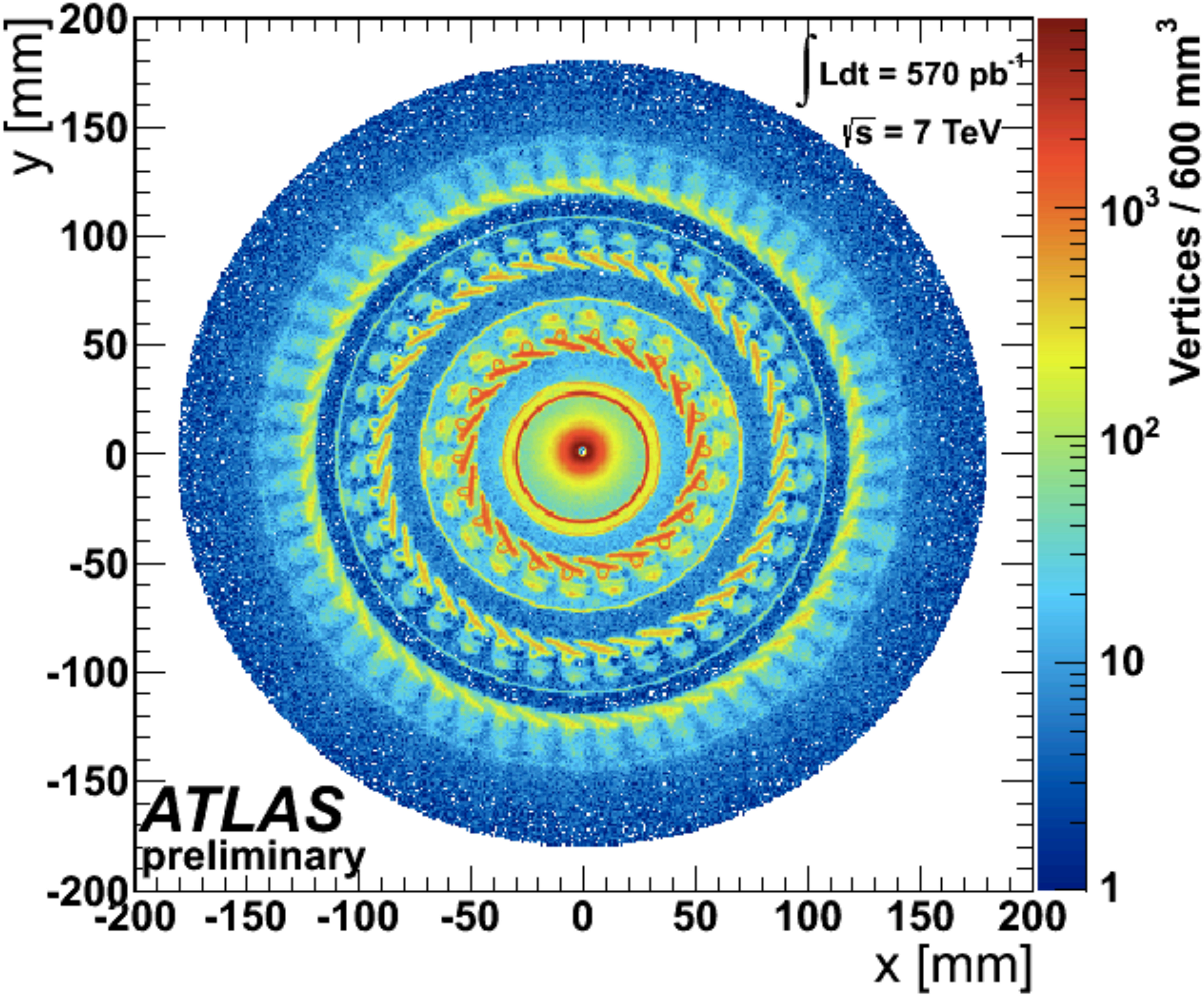


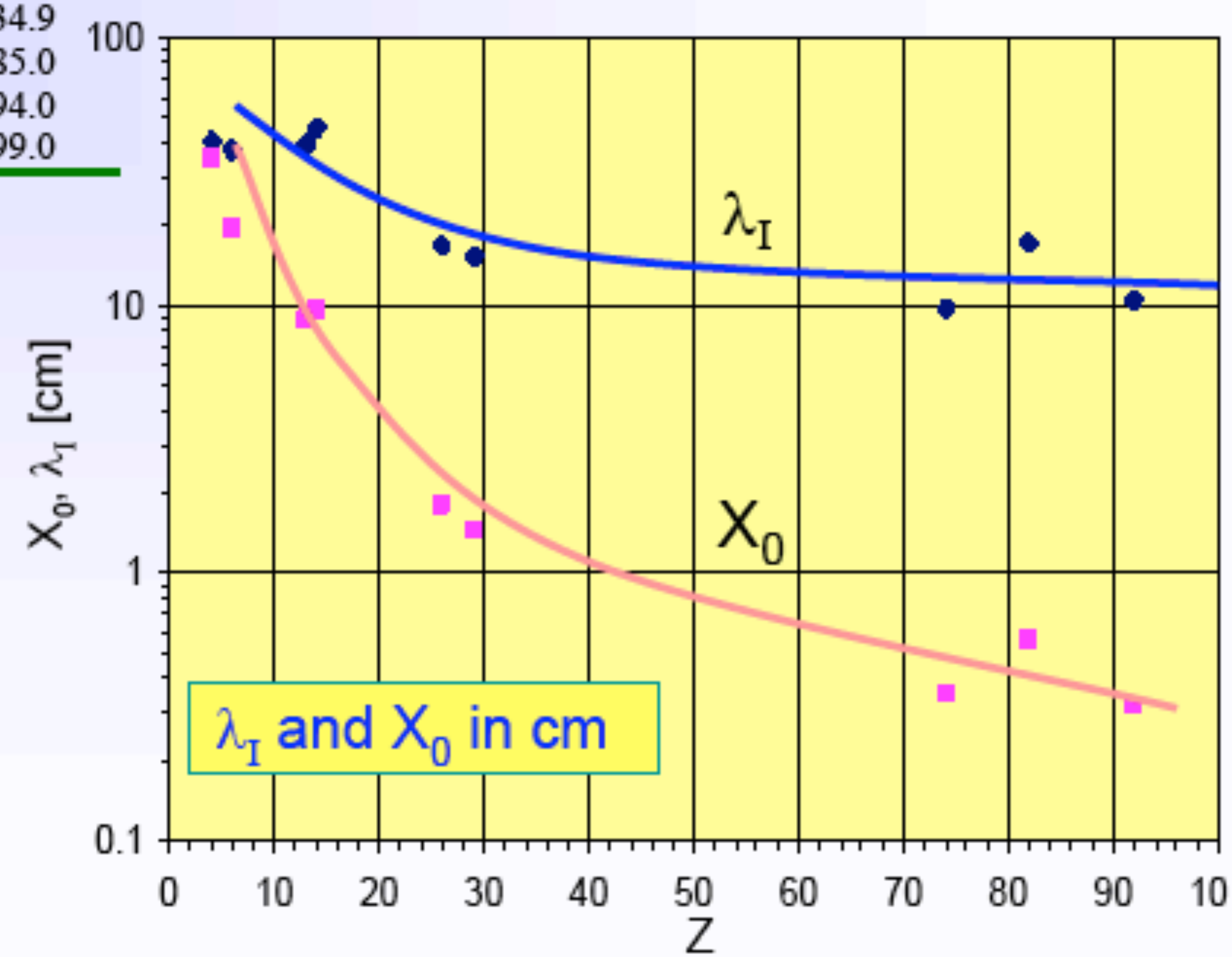
$\int Ldt = 570\ pb^{-1}$
$\sqrt{s} = 7$ TeV
ATLAS
preliminary
y [mm]
x [mm]
Vertices / 600 mm^3
200
150
100
50
0
-50
-100
-150
-200
1
10
10^2
10^3

Interaction of charged particles

EM vs Hadronic Showers

Material	Z	A	ρ [g/cm³]	X_0 [g/cm²]	λ_I [g/cm²]
Hydrogen (gas)	1	1.01	0.0899 (g/l)	63	50.8
Helium (gas)	2	4.00	0.1786 (g/l)	94	65.1
Beryllium	4	9.01	1.848	65.19	75.2
Carbon	6	12.01	2.265	43	86.3
Nitrogen (gas)	7	14.01	1.25 (g/l)	38	87.8
Oxygen (gas)	8	16.00	1.428 (g/l)	34	91.0
Aluminium	13	26.98	2.7	24	106.4
Silicon	14	28.09	2.33	22	106.0
Iron	26	55.85	7.87	13.9	131.9
Copper	29	63.55	8.96	12.9	134.9
Tungsten	74	183.85	19.3	6.8	185.0
Lead	82	207.19	11.35	6.4	194.0
Uranium	92	238.03	18.95	6.0	199.0

For Z > 6: $\lambda_I > X_0$



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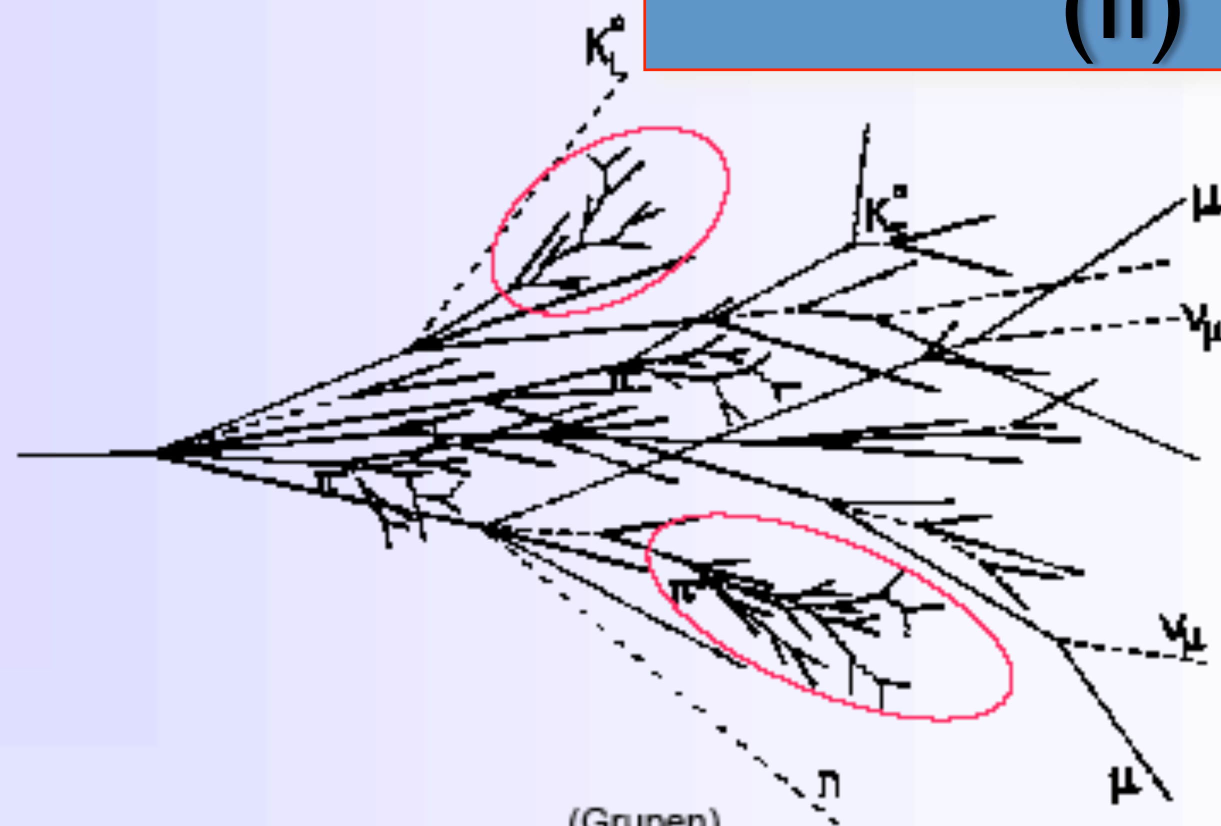
Hadronic Showers (II)

Hadronic cascades

Various processes involved.
Much more complex than
electromagnetic cascades.



(Grupen)

A hadronic shower contains two components:

hadronic + **electromagnetic**

Hadronic component:

- charged hadrons p, $\pi^{\pm}$, $K^{\pm}$,
- nuclear fragmets ….
- breaking up of nuclei (binding energy)
- neutrons, neutrinos, soft γ's, muons

Electromagnetic component:

neutral pions $\rightarrow 2\gamma$
$\rightarrow$ electromagnetic cascades

$$n(\pi^0) \approx \ln E(GeV) - 4.6$$

example $E = 100$ GeV: $n(\pi^0) \approx 18$

invisible energy $\rightarrow$ large energy fluctuations $\rightarrow$ limited energy resolution

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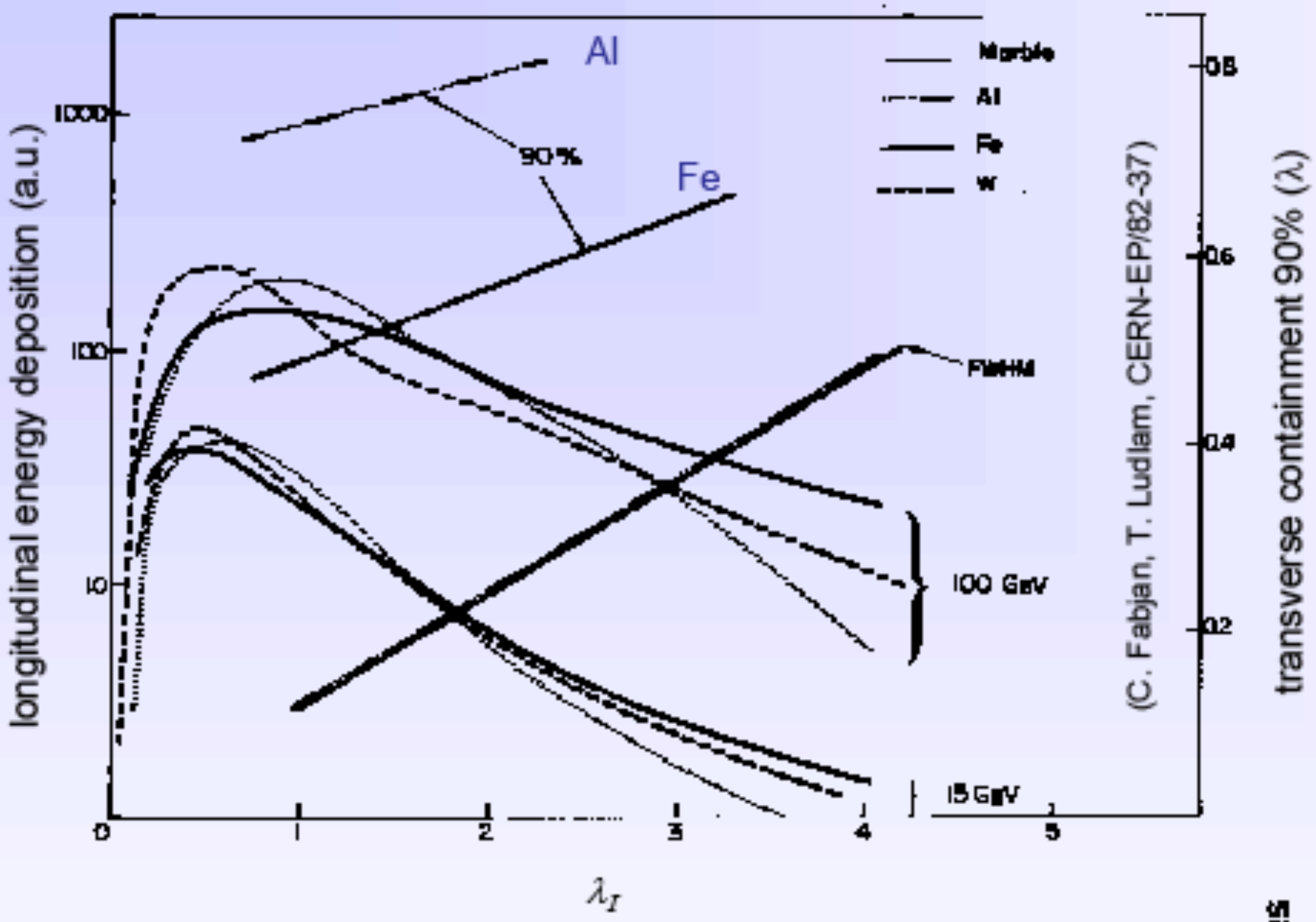
Hadronic Showers (III)

Hadronic cascades

- Longitudinal shower development



$$t_{\max}[\lambda_I] \approx 0.2 \ln E[GeV] + 0.7$$

$$t_{95\%}[cm] \approx a \ln E + b$$

Ex.: 100 Gev in iron (λ_i =16.7 cm)

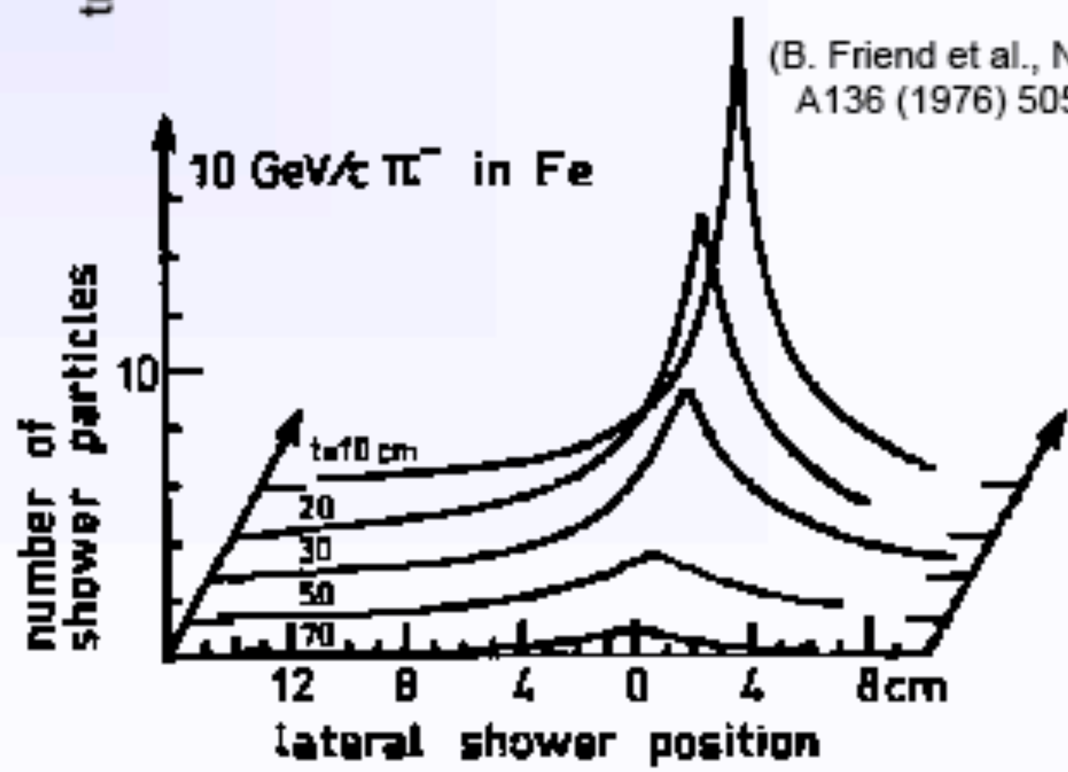
a = 9.4, b = 39

$\rightarrow t_{max}$ = 1.6 λ_i = 27 cm

$\rightarrow t_{95\%}$ = 4.9 λ_i = 80 cm

- Laterally shower consists of core + halo.
  95% containment in a cylinder of radius λ_I.

(B. Friend et al., NIM A136 (1976) 505)



Hadronic showers are much longer and broader than electromagnetic ones !